Educational Studies in Mathematics (2020) 104:333–349
https://doi.org/10.1007/s10649-020-09962-3

"So what are we working on?": how student authority relations shift during collaborative mathematics activity

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Published online: 29 July 2020

Abstract
This paper explores peer interactions in an elementary mathematics classroom (ages 9–10) where the teacher intentionally shared authority with her students and supported them in learning to share authority with one another. Authors examine how students shifted between shared, concentrated, and contested social and intellectual authority relations in partner and small group work during a three-week unit on place value. Findings show that (a) students were able to share both social and intellectual authority, and did so often; (b) the distribution of social authority was more dynamic than that of intellectual authority; and (c) when groups shifted into shared intellectual authority, shifts were usually preceded by a student making some aspect of the collaborative task public. We connect these findings to research on authority in mathematics classrooms that serve racially and linguistically minoritized students and offer directions for future work.

Keywords Authority · Collaboration · Group work · Elementary

1 Introduction

The purpose of this paper is to explore peer authority relations in a mathematics classroom designed to let students make sense of mathematics together. In classrooms that afford students the agency to author mathematical ideas and manage student-led, collaborative tasks, the distribution of authority shifts from unilaterally held by the teacher to shared among teacher and students. While mathematics classroom research has begun to examine how authority

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becomes shared between teacher and students (e.g., Hamm & Perry, 2002; Gerson & Bateman, 2010), little research has examined authority relations among students themselves.

Agency and authority are closely related ideas. Weber (1947) defines authority as:

"the probability that certain specific commands (or all commands) from a given source will be obeyed by a given group of persons." (p. 139).

The sources of commands are agentic acts; that is, directives arise from a person in authority exercising their agency in ways that others take up and obey. Agency is also antithetical to authority. A person with agency has a freedom to act (Cobb, 1995) that can serve to resist the commands or expectations of others.

Mathematics teachers tend to hold a great deal of authority in their classrooms (Amit & Fried, 2005; Herbel-Eisenmann, Wagner, & Cortes, 2010). Herbel-Eisenmann et al. (2010) found that mathematics classroom discourse was significantly organized around stance bundles, language related to interpersonal relations, which functioned to position the teacher with several forms of authority. Indeed, they found that mathematics classroom discourse was particularly organized around such stance bundles, more so than even other kinds of intellectual activity. That is, mathematics classrooms are unique in terms of how centrally authority relations play out and, traditionally, this authority is held unilaterally by the teacher. Yet, as central as authority is to the teaching and learning of mathematics, it remains little understood.

Mathematics teachers' authority includes directing students without necessarily offering reasons, asserting what must be done, deciding the content to be discussed, and evaluating what ideas are right or wrong (Alrø & Skovsmose, 2006; Wagner & Herbel-Eisenmann, 2014). Teachers' authority takes on both social and intellectual dimensions:

"...The teacher has two overlapping roles - namely as director of the social organisation and interactions in the classroom (i.e., social controller) and as director of the mathematical tasks and work activity of the classroom (i.e., task controller). This distinction corresponds to the traditional separation between being 'in authority' (social regulator) and being 'an authority' (knowledge expert)." (Ernest, 2008, p. 42).

In this sense, social authority is always operating in classrooms, occurring whenever humans are interacting. Intellectual authority relations are at play when individuals are engaged in intellectual work, defined in schools as engaging in academic tasks.

Traditionally, a student's role in mathematics classrooms has largely been to obey the teacher or textbook; but the role of student in mathematics classrooms has shifted as students' own mathematical thinking has taken on greater importance in instructional activities, such as classroom discussions and collaborative small group problem-solving (Boaler & Greeno, 2000). Across these activities, students are invited to invent strategies that help them solve problems, to communicate and justify their ideas, and to make sense of and evaluate the ideas of their peers. These invitations afford students the agency to make sense of, and make decisions about, mathematics problems. In doing so, they take on positions of intellectual authority: authors, evaluators, and co-directors of mathematical activities (Yackel & Cobb, 1996). These roles, centered on student agency and intellectual autonomy, are not possible in the authoritarian mathematics classroom. In this sense, the movement toward student-centered mathematics classrooms is anti-authoritarian. Yet, peer authority relations also position students with the power to marginalize one another's participation, especially in group work. Given the central role of authority in mathematics classroom discourse, a deeper exploration of how authority becomes distributed in student-centered classrooms is needed.

Teachers share authority with students through invitations to make sense of mathematics as intellectually autonomous agents and to direct mathematical activity. Hamm and Perry (2002) found that a teacher shared authority with students by asking questions that requested students' ideas and explanations, rather than questions with known answers. Teachers share authority with students through the use of particular kinds of discursive moves, such as eliciting and probing student thinking or asking students to express agreement or disagreement with the ideas of their peers (Stein, Engle, Smith, & Hughes, 2008). Gerson and Bateman (2010) found that teachers share authority with students when they attend to students' ideas and invite students to author, evaluate, and connect ideas. In addition to particular kinds of talk moves, studies have found that norms that include mutual respect, responsibility, trust, and teamwork, as well as the use of collaborative participation structures such as group roles, also support shared intellectual authority between teacher and students (Langer-Osuna, 2015). Teachers bid to share authority with students by inviting them to author and evaluate mathematical ideas, and generally make agentic choices that, in part, direct the intellectual work-at-hand. Students can and often do take up these bids to share authority, resulting in the co-construction of a student-centered, sense-making mathematics classroom community.

When teachers share authority with students, they expect students to, in turn, share authority with one another. This is particularly true during student-led partner and small group work, where students are expected to drive not only the mathematical work but also the collaborative dynamics—turn-taking, attention, consideration of ideas, and influence—in shared, inclusive ways. That is, students are expected to take on and share both intellectual (mathematical) and social authority with one another (Cobb, 1995; Langer-Osuna, 2016). How students negotiate relations of authority affects not only participation patterns and individual learning opportunities but also the nature of the mathematical discussion itself (Esmonde & Langer-Osuna, 2013) and the construction of the problem's solution path (Langer-Osuna, 2016). For example, Langer-Osuna (2016) traced that a student was able to garner undue influence while solving an open-ended problem with a partner through social dominance. As the student's directives became increasingly mathematical in nature, social authority became intertwined with intellectual authority in ways that directed an erroneous problem solution.

Cobb (1995) refers to interactions that mark concentrated or shared authority as univocal and multivocal, respectively. In univocal interactions, "the perspective of one child dominates" (Cobb, 1995, p. 42). The student roles available include the partner who explains their solution and the partner who accepts it. Cobb (1995) found that univocal interactions rarely led to learning opportunities for either student. In multivocal interactions, "both children insist that their own reasoning is valid" (Cobb, 1995, p. 42). Such interactions might include a series of challenges where authority is actively contested among peers, attempts are made to make sense of and incorporate one another's thinking, or both. These interactions can become complicated by how students interpret their peers' mathematical challenges or expressions of disagreement. For example, in Langer-Osuna and Avalos (2015), whereas some students focused on making sense of ideas during peer discussion, other students responded to challenges as if they were personal slights, shutting down opportunities for mathematical inquiry as they attempted to "save face." Cobb (1995) found that, when students were able to effectively communicate, multivocal interactions were usually productive for learning. How students distribute authority in small groups—whether univocal or multivocal—is related to patterns of participation and relationships of power.

Strategies to remediate the effects of differential power relations have focused on the role of the teacher to equalize status among peers in ways that enable shared participation (Cohen & Lotan, 1997). Less is known about how students attempt to manage authority relations on their own. While collaborative dynamics often can and do fall apart (Barron, 2003; Langer-Osuna, 2016), they also can and do become genuinely and productively shared. Barron (2003) found that groups that were productive for learning took into consideration one another's proposals, rather than ignoring or rejecting their peers' proposals off-hand. These kinds of interactions are related to the establishment of shared authority, because multiple ideas are able to make it onto the conversational floor as worthy of consideration. How do students come to establish and maintain shared authority? This question is central to our paper.

A greater understanding of how students negotiate authority relations through peer interactions during collaborative mathematical activity would benefit the field. For one, the establishment of shared authority is fundamentally related to the co-construction of learning opportunities. Authority is a gatekeeper to conceptual learning; learning opportunities in mathematics are those that support students' authorship of ideas in order to develop or build on understanding (Walkowiak, Pinter, & Berry, 2017). This paper focuses on the establishment of shared authority as the social regulation of opportunity to learn. Second, a better understanding of what students do to manage peer work enables the noticing and naming of the assets that students bring to collaboration. Understanding these assets can serve as a foundation for strategies to support, deepen, and build on students' existing strengths. Third, a better understanding of the mechanics of collaboration allows these mechanics to become teachable. Much like teachers must learn how to share authority with their students, students must also learn how to share authority with one another. Social interactions that foster shared social and intellectual activity are complex; unpacking the complexity can make collaboration more manageable for teachers and students alike. Fourth, there is a theoretical vacuum in studies of mathematics classroom authority, typically focused on the establishment, maintenance, and consequences of teacher authority. Research by Cobb and colleagues, as well as more recent research on whole-class mathematical discussions, has focused on how teachers share authority with students by inviting them to co-direct the classroom's mathematical work (Amit & Fried, 2005; Cobb, Gresalfi, & Hodge, 2009; Gerson & Bateman, 2010; Gresalfi & Cobb, 2006; Wagner & Herbel-Eisenmann, 2014; Hamm & Perry, 2002). Relatively little work, however, has theorized how students negotiate the social and intellectual forms of authority afforded to them in collaborative activity. In this paper, we examine how students distributed both social and intellectual authority and how these distributions, which we treated as interactional states, shifted over time across small group activity.

2 Methods

2.1 Study context

This study is situated within a broader research-practice partnership between a university research team and an instructional team of five teachers at an elementary school in Northern California that served predominantly bilingual Latinx and Pacific Islander students from a low-income neighborhood. The goal of the broader partnership was to support teachers in implementing collaborative mathematical activity, using the Contexts for Learning Mathematics (CFL) instructional units (Fosnot, 2007) as a curricular resource to support this work. These units draw on relevant story-based contexts to engage students in small group and whole class inquiry-based activity. The teachers involved in the study worked to create classroom contexts in which students were expected to author and evaluate mathematical ideas and to share this authority with one another productively.

For this study, we focus on the fourth grade classroom (typically ages 9 and 10 in the USA) where the teacher taught the unit, "The T-Shirt Factory: Place Value, Addition and Subtraction" (Fosnot, 2007). To support both collaborative skills and student agency, the teacher framed collaboration as "productive partnerships" wherein students were expected to reflect on and make choices about with whom to partner, where to work, and which manipulatives or strategies to use to make progress on mathematical tasks. In this sense, students had agency in managing their collaborative dynamics, offering the researchers the opportunity to capture and examine how students elected to do so. During the three-week period of data collection, the class reflected on the productivity of their partnerships regularly, identified useful language for collaborating (e.g., responding to one another's ideas, revoicing), and discussed the physical postures that created shared space for working together (e.g., eye contact, sitting facing, or side-by-side). Thus, students developed repertoires for managing collaborative dynamics and were largely able to decide when and how to deploy them.

2.2 Data sources

We focus on 13 videos of student-led group work collected during the focal unit in the fourth-grade classroom. The unit lasted 3 weeks, of which the research team videotaped 11 days. While the videos contained many contextual differences, such as different tasks and small groups made up of two, three, or four students, we see these variations as natural to classroom life and thus informative as data. We selected this particular classroom for analysis because authority had been purposefully and explicitly delegated to students, and it afforded insight into how students chose to manage, distribute, and negotiate that authority among themselves during collaborative work. While all the teachers participating in the broader study sought similar aims in the research-practice partnership, this particular teacher reported the most experience in facilitating collaborative mathematical work, and our own observations in her classroom over the course of the partnership corroborated this claim.

2.3 Analytic approaches

The purpose of our analyses was to examine shifts in the distribution of social and intellectual authority among students engaged in collaborative mathematical activity and to examine patterns in how students shifted into shared intellectual authority in particular. Below we detail our analytic approach to the data.

2.3.1 Operationalizing social and intellectual authority among students and their distribution

We draw on extant work to define ways in which intellectual and social authority relations among individuals are made evident in interaction. *Intellectual authority* relations are enacted through interactions that position students as credible sources of mathematical information (Engle & Conant, 2002; Engle, Langer-Osuna, & McKinney de Royston, 2014). This includes moments where a particular student's help is sought, or moments where a particular student is driving the intellectual work of the group. S*ocial authority* relations are enacted through interactions that position students as having the right to issue directives to their peers (Langer-Osuna, 2016). This includes moments where a particular student tells a peer(s) what to do and they obey.

These forms of authority were then operationalized in relation to their distribution among students in a small group (see Table 1). Extant literature on small group work in mathematics tends to focus on either the goal of shared (joint or dialogic) dynamics or the problems of concentrated (domination/marginalization) dynamics (e.g., Yackel & Cobb, 1996; Kotsopoulos, 2014). With these ideas in mind, we began the coding process with *shared* and *concentrated* distributions of authority and found that not all interactions in the corpus fit these established definitions. Through constant comparison (Glaser, 1965), we developed two additional ways of characterizing the distribution of authority, which we refer to as *contested* and *disbanded*.

Interactions representing shared intellectual authority include events where more than one student was treated as credible sources of mathematical ideas. An example of shared intellectual authority would be two students contributing to the mathematical work, such as making sense of a problem together by offering their own and responding to one another's mathematical thinking. Similarly, interactions representing shared social authority include events where more than one student directed the behavior of others in ways that were successfully taken up by peers. Examples of shared social authority include events wherein multiple students take

Table 1 Social and Intellectual Authority Code Definitions

Distribution	Social	Intellectual
Shared	Multiple students' bids to manage their own and others' participation are taken up. This includes voiced negotiation of roles, and distribution or management of tasks. *Example: One student suggests that the group decide roles. Another student agrees and suggests they all write their names on the worksheet. Students begin writing names and discussing roles.*	Multiple students' bids to contribute to the intellectual work are taken up. Disagreement about a mathematical idea or solution is in the service of reaching consensus. *Example: One student suggests that the group use blocks to solve the problem, and others agree. Another student suggests that they then count by tens, and the group begins to do so.*
Concentrated	Bids to manage participation are taken up only in relation to one student. This includes instances where only one student successfully issues directives in the group. *Example: One student assigns roles to others in the group. The same student directs another student to get folders for everyone and that student leaves the table to retrieve folders.*	Bids to lead the intellectual work are only taken up in relation to one student. This includes instances where only one student's mathematical contributions are considered in the group. *Example: One student declares that the group will use blocks to solve. Other suggestions are rejected, and the group proceeds with blocks.*
Contested	Multiple bids to manage participation are rejected such that there is no settled authority. *Example: One student tells another to go retrieve cubes. The partner refuses and tells the first student to write her name on the shared paper. The first student replies, "You do it."*	Multiple bids to author ideas, offer help or lead the work are rejected such that there is no settled authority. *Example: One student suggests using blocks to solve the problem, which others reject. Another student declares the group should start with her number, which the group ignores, and others make competing suggestions.*
Disbanded	n/a[1]	When the collaboration disbands into independent or off-task activity. *Example: All three group members chat about music.*

[1] We theorize social authority relations as operating at all times in the classroom.

part in the management of the work, making joint decisions about the distribution of tasks or co-managing off- and on-task behavior.

Interactions representing concentrated intellectual authority include events where exactly one student offers intellectual contributions that are considered by their peers. For example, one student contributes their thinking, which is written down by a peer or otherwise taken up by the group repeatedly. Similarly, interactions that mark concentrated social authority include events where one student controls the management of the group. Examples of concentrated social authority include dynamics where one student unilaterally assigns tasks to peers or otherwise directs the behavior of the other small group peers.

Contested intellectual and social authority events include dynamics where the right to manage behavior or contribute ideas is explicitly disputed across a series of interactions. For example, a student might issue a directive to a peer; that peer could reject the directive and even explicitly admonish the student or issue a new directive in return. Such dynamics could continue through several turns. When it did so, we coded that event as contested social authority. If contested bids focused on the intellectual nature of the work, such as contested bids to offer an answer or explain an idea, we coded such events as contested intellectual authority. Finally, we also included disbanded intellectual authority as a coded interactional state to capture moments when students were simply not engaged in intellectual work.

2.3.2 Defining the unit of analysis and Coding for distribution of authority

In line with interaction analytic methods (Jordan & Henderson, 1995), we coded video data collectively through iterative viewing and discussion sessions, which we describe below. Three of the authors coded through multiple sessions of collective viewing until all coders reached consensus, both in terms of how the instances were bounded (start and endpoint) and the code applied.

We began by defining the unit of analysis as instances bounded by shifts in the distribution of authority within the small group; that is, a shift in authority distribution indicated the end of one analytic unit and beginning of another. Social and intellectual authority distributions continued temporally as established until a shift was identified. Because we were interested in understanding the distribution of authority relations among students in the absence of the teacher, any instances of teacher interactions with the group were coded as such and removed from the data set. The data set thus included coded temporal events, which were mostly contiguous within sessions and which represented interactions outside of teacher supervision.

In this sense, each unit is a coded temporal event. For social authority, we coded (a) shared authority, (b) concentrated authority, and (c) contested authority. For intellectual authority, we coded (a) shared authority, (b) concentrated authority, (c) contested authority, and (d) disbanded.

We coded the data set in two rounds. The first round focused on intellectual authority relations. The second round focused on social authority relations. We made this choice because the complexity of the data required that we attend to only one kind of interactional event at a time. For intellectual authority, we focused on students’ talk and action related to the mathematical nature of the task. For social authority, we watched the same videos and focused on students’ talk and action related to the management of peer behavior. This resulted in two rows of data on our video analysis software (see Fig. 1a). Once

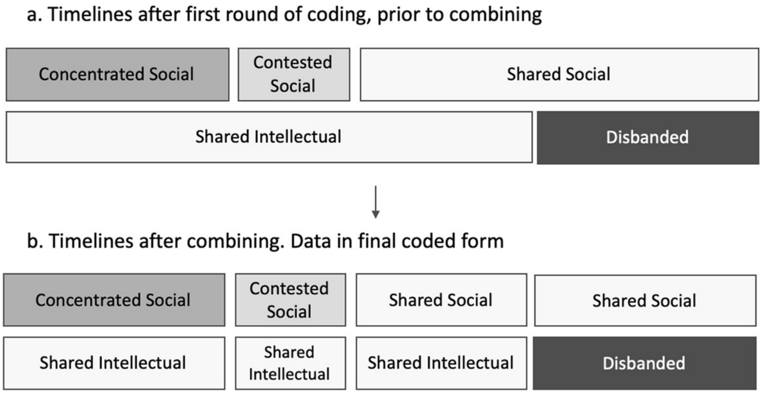


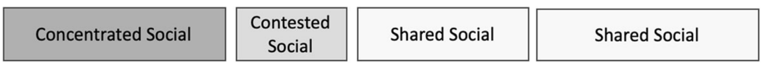
Fig. 1 Example of stacked coded social and intellectual authority units prior to (**a**) and then after (**b**) combining social and intellectual authority distributions into the final coded units

all events were defined, we coded each unit in relation to the distribution of authority (i.e., shared, concentrated, contested, disbanded).

We then combined both social and intellectual coded events according to their temporal overlap in order to capture the two authority relations in the same unit (see Fig. 1b). Because shifts could occur in relation to one kind of authority, but not the other, these new combined units were bounded by any shift in *either* intellectual or social authority. For example, the coded instance of shared intellectual authority (white) in Fig. 1a is transformed into three consecutive instances of shared intellectual authority because of the concurrent shifts in social authority in Fig. 1b. This means that the distribution of social authority shifted three times among peers, while the distribution of intellectual authority remained stable. Organizing the data this way allowed us to investigate shifts in both social and intellectual authority distributions in relation to one another.

2.3.3 Quantifying how students distributed authority in collaborative activity and exploring the likelihood of shifts

Once all qualitative coding of the video was complete, time-stamped instances were compiled and exported to R (R Core Team, 2018) to investigate patterns that arose in how groups shifted from one distribution state to another. We were interested in understanding what kinds of shifts were likely, with particular interest in understanding shifts into shared intellectual authority. We chose to represent the data as Markov chains, a probabilistic model where the likelihood of a future state is calculated as depending on the immediately previous state (Gagniuc, 2017). In our analysis, the states represent coded distributions of authority among students at the moment-to-moment level. In Fig. 2, for example, the Markov chain illustrates that shared social



Fi . 2 Sample timeline clip used to develop a Markov chain

Table 2 Contingency table of initial to subsequent state of intellectual authority

		Subsequent state				
		Concentrated	Contested	Disbanded	Shared	Total
Initial state	Concentrated	15 50.0%	6 20.0%	4 13%	5 17%	30 100%
	Contested	7 18%	18 47%	6 16%	7 18%	38 100%
	Disbanded	1 6%	4 22%	11 61%	2 11%	18 100%
	Shared	2 5%	5 12%	5 12%	29 71%	41 100%
	Total	25 20%	33 26%	26 21%	43 34%	127 100%

authority was preceded once by contested social authority and once by shared social authority. That is, shared social authority was preceded 50% of the time by shared social authority and 50% of the time by contested social authority. We examined these likelihoods across the entire data set to understand patterns in how students shifted from each state to another, and the relative stability or malleability of each kind of authority distribution.

We represented the coded data of each of the 13 videos as a Markov chain, which were then combined in order to create a transition matrix—a type of contingency table—to represent the transition of states within each construct. The transition matrix (see Tables 2 and 3 in the Findings section) allowed us to see the likelihoods of students shifting into and out of particular authority distributions overall. Each row of this matrix represents an initial state of the construct and each column represents a subsequent state. The intersections in the transition matrix represent the probability of moving from one state (represented in the rows) to another (represented in the columns). From this, we extracted simple descriptive statistics.

Table 3 Contingency table of initial to subsequent state of social authority

		Subsequent state			
		Concentrated	Contested	Shared	Total
Initial state	Concentrated	10 32.3%	15 48.4%	6 19.4%	31 100%
	Contested	11 21.2%	20 38.5%	21 40.4%	52 100%
	Shared	8 18.2%	17 38.6%	19 43.2%	44 100%
	Total	29 22.9%	52 40.9%	46 36.2%	127 100%

2.3.4 Examining student interactions as they shifted into shared intellectual authority

Ultimately, student-centered, collaborative mathematics classrooms aim to foster productive intellectual activity among learners. Given this goal, we were particularly interested in understanding how students shifted into shared intellectual authority during collaborative work. To explore these moments in our dataset, we identified the 14 instances wherein students shifted into shared intellectual authority. Four of the authors independently wrote analytic memos on each of these instances, focusing on the nature of students' interactions, including their talk and nonverbal communications such as gaze, body positions, and use of resources, immediately preceding the shift into shared intellectual authority. We then discussed our memos as a research team, where we hypothesized particular patterns. We then returned to the videos explicitly looking for both confirming and disconfirming evidence about each of the hypothesized patterns. One hypothesized pattern was consistently held across the instances, which we detail in the Findings section.

3 Findings

Overall, the study found that (a) both intellectual and social forms of authority were most often shared or contested among students; (b) distributions of social authority shifted more often than intellectual authority; and (c) students publicly named some aspect of the collaborative task before groups shifted into shared intellectual authority.

3.1 Intellectual and social authority were most often shared or contested

Overall, social and intellectual authority relations shifted across shared, concentrated, and contested distributions. Both intellectual and social authority were shared or contested by students more frequently than that they became concentrated in one student during group work. Intellectual authority was most frequently shared among students (32.3%) and often contested (29.9%). Social authority was most frequently contested (40.9%) and often shared (34.6%). Less often, intellectual or social authority (23.6% and 24.4%, respectively) was concentrated in one student. Students were disbanded in 14.2% of instances.

3.2 Distributions of authority shifted often within groups

The distribution of authority was not static within groups. The distribution of authority shifted multiple times within each video in the data sample. Instances of authority distribution ranged greatly in duration from fleeting to lasting several minutes (1–367 s; median = 33 s) and collaborative sessions ranged from seven to 33 (median = 9) shifts in authority distribution. To illustrate the shifts in social and intellectual authority we found in the data, consider the chain of coded events shown in Fig. 3. During six minutes of collaboration, this group of students distributed social and intellectual authority each as shared, contested, and concentrated, and also experienced episodes of disbanded work. Episodes varied in length from fleeting (7–10 s) to more sustained (56–71 s). During this brief window of collaboration, the group shifted their distribution of social and intellectual authority ten times.

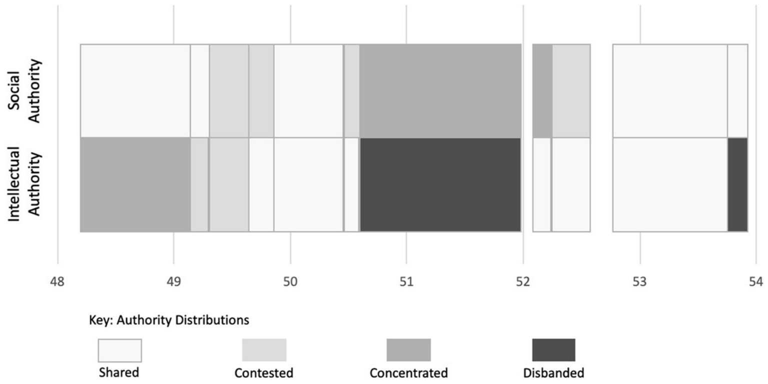


Fig. 3 Example of Markov chain of social and intellectual authority episodes from a collaborative work session video. Minutes of the video are denoted on the x-axis. Distributions of authority are indicated with color: concentrated (dark gray), contested (light gray), shared (white), and disbanded (black). Gaps in the timeline (e.g., around minute 52) indicate places where the teacher was present

3.3 Patterns of stability within intellectual authority

We found that the distribution of intellectual authority within collaborative groups was stable from instance to instance compared with the distribution of social authority, which was more likely to change. This suggests that social and intellectual authorities are exercised by students differently during collaborative work. Here we focus on the relative stability of intellectual authority distributions. In the section that follows, we shift our attention to social authority distributions.

For each instance of intellectual authority, the subsequent state was most likely to be maintaining the initial state (see Table 3). That is, when intellectual authority was concentrated, it was most likely (50.0%) to continue to be concentrated, contested was mostly to stay contested (47.4%), disbanded was most likely to stay disbanded (61.1%), and shared was most likely to remain shared (70.7%). The distribution of intellectual authority was stable, though not unchangeable, with maintenance consistently the greatest probability. Of these, shared intellectual authority, the goal of collaborative problem-solving classroom environments, was the most stable state of all.

These patterns suggest that the distribution of intellectual authority was challenging to disrupt. Even when intellectual authority was contested and students actively sought to redistribute authority, continued contestation was nearly as likely as all other outcomes combined.

3.4 Patterns of malleability within social authority and the role of contestation

The patterns of change for social authority relations were more complex than those for intellectual authority, with several probable pathways emerging (see Table 3). As with intellectual authority, there was a strong likelihood of maintaining the distribution of social authority from instance to instance. For concentrated and contested social authority distributions, stability was the second most likely outcome, at 32.3% and 38.5%, respectively. While for shared social authority, arguably the most desirable state, stability was most likely (43.2%).

Social authority shifted more often than intellectual authority, and three probable pathways for shifting the distribution of social authority were present in the data. First, concentrated social authority, in which only one member of the group held social authority, was most likely to lead to contested social authority (48.4%), in which students actively sought to redistribute authority. Students were more than twice as likely to contest concentrated social authority as concentrated intellectual authority (48.4% versus 20%).

Second, when social authority was being contested, the most likely subsequent state was shared social authority (40.4%). This suggests that contestation is likely to be a productive pathway for redistributing social authority across more group members; contestation led to concentrated social authority nearly half as often (21.2%). By contrast, when students contested intellectual authority, the most likely subsequent state was continued contestation and all other states were approximately equally likely.

Third, while shared social authority was most likely to remain stable, it often led to contested social authority (38.6%). Together with the previous pattern, this suggests a reflexive relationship between contested and shared social authority, with groups moving fluidly between these states. The prevalence in the data as a whole of contested authority raises questions about its function in redistributing authority within collaborative groups. For all initial states of social authority (concentrated, contested, and shared), there was a strong probability (40.9% overall) of the next state being contested. Contestation was the most frequent state for social authority, and even when social authority was equitably distributed, groups frequently slipped into states of contestation in which one or more students attempted to assert authority and were rejected by others. Contesting authority seemed to be an important driver of shifting authority relations, particularly shifting to shared social authority.

3.5 Getting to shared intellectual authority

Our findings indicate that intellectual authority, even under conditions that foster sharing it, can frequently be disputed or concentrated, or even dissolved. Given the prevalence of these distributions, how did groups shift into sharing intellectual authority? Despite the relative stability of intellectual authority distributions, there were 14 instances where students shifted into shared intellectual authority from another state. While this represents a small sample, these instances are worth examining for what they might reveal about the mechanisms that enable students to shift into shared authority, which was then likely to be maintained.

The strongest pattern in the data, occurring in 12 out of 14 cases where groups shifted into shared intellectual authority, showed students publicly naming the work at hand. For example, students clarified the nature of the task out loud or asked for clarification through a question, such as, "So what are we working on?" In other cases, students named the number that the group was to represent or made a public bid to get a new number to work on. In another example, a student bid to publicly share her work, telling her group mates, "These are my totals." What these cases had in common was that the task itself, or aspects of the work, became public to the group. Subsequently, students shifted into shared authority; that is, there was evidence that more than one student led the mathematical work. Just prior to these public bids, groups were either contesting authority (3 of 12 cases) or were engaged in either independent or disbanded work (9 of 12 cases).

Consider the following excerpt, which occurred at the start of small group activity (Table 4). Before the excerpt below, students were individually making sticks of 10 with unifix cubes.

Table 4 Excerpt of small group peer interactions

Turn	Speaker	Words/actions	Analytic note	Code
1	Student A	(Looks up toward peers) So we are trying to, like, make 80? Like 10, 20, 30… We are trying to copy Nathan?	Request for clarification to group makes task public *Coded as disbanded, but marks start of shift*	disbanded intellectual authority
2	Student B	(continues building 10 sticks) I have no idea.		
3	Student A	Trying to copy Nathan?		
4	Student C	(continues building 10 sticks) No		
5	Student B	(counting cubes on 10-sticks) 4, 5		
6	Student C	(looks over to and leans in toward Student B) I am doing the black	Public statements about participation in task	
7	Student A	I call red		
8	Student B	1, 2 (takes blocks handed to him by Student C and adds them to his 10-stick)	Resources explicitly shared	
9	Student C	(returns to his 10-sticks and begins counting cubes) 5, 6, 7		
10	Student A	(looks up at group mates) So I got…	Shared intellectual authority established	Shared intellectual authority
11	Student C	(continues counting) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10	*Coded as shared intellectual authority*	
12	Student A	(looks across all 10-sticks) We got 40? 40?		
13	Student B	(reaches over to Student B and begins counting his 10-sticks) 10, 20, 30, 40, 50		
14	Student C	(begins to verify the counts on each stick) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11. (turns toward Student B) OK, I got to get rid of this one because I got 11 (hands over extra cube to Student B, who adds it to the 10-stick he is building) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. (continues to count each of the 5 10-sticks)		
15	Student A	(hands over a 10-stick she built to Student C, who takes it and adds to his set) Look, I made that. I call red again (begins new 10-stick with red cubes) 2, 3…		
16	Student C	(picks up a red 10-stick from his set, leans in toward Student B, showing him his red 10-stick) The color red. It is my favorite color		
17	Student B	(displays the red and green 10-stick he built) Look it, I like red and green. (adds 10-stick to Student C’s set)		

This activity was coded as disbanded intellectual authority because students were not yet in a collective orientation, as it was the start of group work and, in this session, students began by working individually. A student then asked questions meant to clarify what they were supposed to be working on (Turn 1), which began the interactional shift (Turns 2–9) from disbanded into shared intellectual authority (Turns 10–17). Shown in the excerpt below (organized by talk turns), this public naming brings them into shared work (beginning Turn 10).

The excerpt above begins with Student A asking for clarification for what the group was meant to be working on (turns 1, 3). While her peers did not directly answer and indeed pushed back on the question (turns 2, 4), it nevertheless served to make the work public to the group. In turns 6–8, students began to shift toward shared work, making their participation in the shared task public, as well. Specifically, in turn 6, Student C stated that he would be building 10-sticks using the black cubes, a signal that he was claiming a particular role in creating a shared set of sticks. This is followed by Student A who then claimed the red cubes (turn 7). The joint effort continued as Student C handed over cubes to Student B, who used them for his own stick building (turn 8). By turns 10 and 12, Student A began to more explicitly frame the activity as shared, requesting a sense of where they are in representing number ("We got 40? 40?). By turns 10–12, shared intellectual authority had been established. Student B then turned toward Student A and began to verify his peer's representation, counting the cubes on his 10-stick, then turned to verify his own 10-sticks. When he realized one of his 10-sticks had an extra cube, he handed the cube over to his peer, who took it up seamlessly and added it to the 10-stick he was constructing. Spatially, all students were oriented toward each other as a triad and the work was jointly constructed. After some conversation about preferred colors, Student B also added a 10-stick to Student C's set, as Student A built one more 10-stick. Together, they constructed a representation for the number 80.

As in this excerpt, nearly all of the instances where students shifted from disbanded or contested intellectual authority into shared intellectual authority began with a public naming—through requests or statements—of the task they were to work on. This public naming seemed to serve the function of orienting group members toward the collective task.

3.6 Limitations in the study

The data included students working together in groups of two, three, or four. Each problem-solving session lasted between 16 and 38 min. The variations in the kinds and lengths of the sessions created limitations in the data. The number of students in a session affects the complexity of authority negotiations and could, in part, determine which distributions are more likely. This may also affect the number of transitions per session, which affects claims about the dynamic nature of these negotiations. Furthermore, we removed instances in which adults were present in collaborative interactions because we were interested in unmediated peer interactions. However, we acknowledge that the presence of adults could have affected subsequent peer interactions, and future research could examine the influence of the teacher on the distributions of authority in collaborative group work. Finally, the original qualitative coding was highly interpretive in nature. Biases were reduced by triangulating coding across multiple coders and through multiple iterations of discussion and coding.

4 Discussion

In collaborative classrooms, students are afforded the agency to make sense of, and make decisions about, mathematics problems. In doing so, they take on positions of authority and are expected to share both intellectual (mathematical) and social authority with one another (Cobb, 1995; Langer-Osuna, 2016). Prior work has focused on how teachers intervene on student power relations (e.g., Cohen & Lotan, 1997) and how teachers hold authority (Herbel-Eisenmann et al., 2010), but here we look at student authority relations and how students manage such relations (cf., Barron, 2003). How do students come to establish and maintain shared authority? This paper explored how students distributed both social and intellectual authority and how these distributions shifted over time across small group activity. We focused in particular on how students discursively shifted into shared intellectual authority. We discuss our findings below and connect these findings to extant work.

4.1 The malleability of social authority versus stability of intellectual authority

The distribution of social authority was found to be more dynamic than intellectual authority, which was relatively stable in its distribution. These contrasting patterns raise questions about their potential roots and about the value of malleability versus stability. Stability has value in a collaborative setting when the distribution of authority is shared, but malleability is a greater asset when authority is concentrated or contested, opening doors for moving into shared authority. In this sense, neither stability nor malleability is inherently more advantageous for supporting collaborative problem-solving.

One possible explanation is that patterns of malleability within social authority might reflect children's social reality in which they are expected to regularly negotiate complex dynamics of power among peers both inside and outside of schooling contexts (Davies, 1982). Conflict is common, and children develop ways to interrupt a friend or sibling's dominance, gain entrance into a game where they were initially left out, or invite others to participate in play (Davies, 1982). These skills are learned both with and without direct support from adults. This is in contrast to societal- and school-based assumptions about intelligence, and mathematical competence in particular, as hierarchical and fixed (Boaler, 2002) and thus perhaps less likely to be negotiated. Nevertheless, in the focal classroom as in other mathematical learning contexts, the explicit norms of collaboration focused more on sharing the intellectual aspects of the work rather than on negotiating the management of participation or resources. In this sense, students had fewer mathematics classroom-based interactional resources for negotiating social authority than intellectual authority. Teaching students how to interact in ways that make collaborative problem solving a truly joint enterprise, while emphasizing multiple competencies, can support students in sharing authority. Our data provide insight into two potential avenues for how students might move into shared authority worthy of additional exploration.

4.2 Contestation and getting to shared authority

Students frequently contested authority, the most common distribution of social authority and the second most common for intellectual authority. Prior work suggests that the contestation of social authority could serve to disrupt positionalities in ways that reshape collaborative dynamics (Esmonde & Langer-Osuna, 2013; Langer-Osuna, 2011; Langer-Osuna, Gargroetzi, Munson, & Chavez 2020). Our findings indicate a similar pattern, with contested social authority being most likely followed by shared social authority. Contestation, particularly of social authority, can be seen as a potentially productive state, serving as a mechanism for redistributing authority in more equitable ways. In this way, contestation can serve to support opportunities to learn by redistributing toward multivocality (Cobb, 1995). Opportunities to reason about and contribute to a shared task support conceptual understanding (Walkowiak et al., 2017).

The potential productivity of contestation presents some implications for practice. In other work, we have found that off-task interaction during collaborative problem solving can serve productive functions for the group, recruiting new members into the collaboration, resisting domination from one member, and gaining access to an existing collaboration for a student previously not included, among other outcomes (Langer-Osuna et al., 2020). Just as off-task behavior can be interpreted by teachers as goofing off, contestation can be viewed as bickering, and both could be understood as counterproductive to the mathematical goals of the lesson. However, given that these states can, counterintuitively, support productive collaboration, one implication for teachers is to allow students opportunities to engage in productive forms of contestation, or social struggle. Further research is needed to understand when and how teachers might intervene to support the disruption of inequitable distributions of authority and, alternatively, what signals may exist for productive contestation.

For intellectual authority, students were able to shift into shared distributions on a small number of occasions, and our findings indicated one pattern worthy of further exploration. In the vast majority of these instances, in the moments preceding the shift into shared authority, a student publicly named the shared work by asking for clarifications (e.g., "So what are we working on?"), stating the work to be done (e.g., "We are supposed to get a new card"), or making their own process the object of consideration (e.g., by counting manipulatives aloud). It is worth noting that these instances did not include moments, as were found in the rest of the data set, in which students exhorted each other to collaborate (e.g., "She [teacher] said we were supposed to work together"), which in practice functioned more as bids to concentrate authority than to share it. Indeed, a degree of hesitancy often characterized these bids, which opened the door for others to confirm, revise, or build upon the bid. The utility of public invitations to shared work raises the question of whether and how such moves could be taught to students to make them part of a repertoire of discursive tools for getting to shared work.

Funding information This study was funded by Private Donor Grant to the Center to Support Excellence in Teaching (CSET) at Stanford's Graduate School of Education.

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