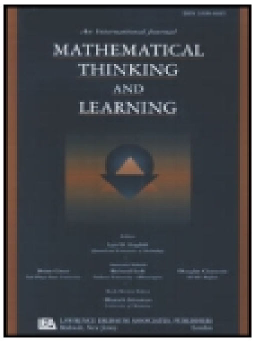

Mathematical Thinking and Learning

ISSN: 1098-6065 (Print) 1532-7833 (Online) Journal homepage: https://www.tandfonline.com/loi/hmtl20

The Social Construction of Authority Among Peers and Its Implications for Collaborative Mathematics Problem Solving

Jennifer M. Langer-Osuna

To cite this article: Jennifer M. Langer-Osuna (2016) The Social Construction of Authority Among Peers and Its Implications for Collaborative Mathematics Problem Solving, Mathematical Thinking and Learning, 18:2, 107-124, DOI: 10.1080/10986065.2016.1148529

To link to this article: https://doi.org/10.1080/10986065.2016.1148529

Published online: 12 Apr 2016.

MATHEMATICAL THINKING AND LEARNING
2016, VOL. 18, NO. 2, 107–124
http://dx.doi.org/10.1080/10986065.2016.1148529

Routledge
Taylor & Francis Group

The Social Construction of Authority Among Peers and Its Implications for Collaborative Mathematics Problem Solving

Jennifer M. Langer-Osuna

Stanford University

ABSTRACT

This article describes a study of how students construct relations of authority during dyadic mathematical work and how teachers' interactions with students during small group conferences affect subsequent student dynamics. Drawing on the influence framework (Engle, Langer-Osuna, & McKinney de Royston, 2014), I examined interactions when students appropriated their peers' ideas during collaborative mathematical problem solving and noted that each moment tended to follow particular interactions around authority. Notably, social and intellectual forms of authority became linked in ways that were directly related to how students' ideas and behaviors were evaluated by the teacher. I close by discussing how the study of authority and influence offers fertile analytic ground to generate new understandings about collaborative student work in mathematics classrooms.

The purpose of this article is to explore relationships of authority that children develop when they engage in small group collaborative mathematics problem solving in elementary classrooms and to study the catalytic effect that teacher interactions with students can have on such relationships. This article takes as its central argument that the study of children's positions of authority relative to one another is central to understanding how students author, share, debate, and, ultimately, collaboratively construct mathematical solutions to problems.

With the US Common Core Standards movement shifting expectations of what it means to do school mathematics, placing greater emphasis on particular collective practices, such as argumentation and sharing problem-solving strategies (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010; cf., Sengupta-Irving, 2014), there is an urgent need to further our understanding of how students convince one another of their ideas and how teachers might affect that process. There is much we already know about student interactions during mathematical activity. Some interactions facilitate the development of children's mathematical thinking (e.g., Chapin, O'Connor, & Anderson, 2009; Hogan, Nastasi, & Pressley, 2000; Stein, Engle, Smith, & Hughes, 2008; Webb & Mastergeorge, 2003). These interactions include explaining mathematical ideas; sharing these ideas with one another; attending to, making sense of, critiquing, and building on peers' ideas; revising one's own thinking; and seeking and offering help.

For decades, group work has been championed as a particularly fruitful activity structure to support such interactions among peers (e.g., Davidson & Kroll, 1991; Johnson & Johnson, 1989; Sharan, 1990; Slavin, 1995; Webb, 1989, 1991, 2009; Yackel, Cobb, & Wood, 1991). Yet, group work is enormously challenging for both students and teachers. While decades of research has shown that group work is a promising activity structure (Davidson & Kroll, 1991; Johnson & Johnson, 1989;

CONTACT Jennifer M. Langer-Osuna jmlo@stanford.eduProvide 236 CERAS, Stanford University, 520 Galvez Mall, Stanford, CA 94305, USA.

Color versions of one or more of the figures in the article can be found online at www.tandfonline.com/hmtl

Sharan, 1990; Slavin, 1995) and that particular interactions during group work support (Webb, 1989, 1991; Yackel et al., 1991) or hinder (Barron, 2003; Good, McCaslin, & Reys, 1992) learning among peers, the facilitation of successful group work in mathematics classrooms remains a struggle. A principle obstacle arises from the ease with which dynamics between young learners can fall prey to issues of status. Elizabeth Cohen defined status as "an agreed-on rank order where it is generally felt to be better to be high than low rank" (Cohen, 1994, p. 23). Status can be thought of as a relationship of power among peers. That power can be academic, as in status, derived from perceived smartness, or social, as in status derived from popularity.

Status relations among students can derail collaborative work. For instance, participation in group work is often unequal in fairly predictable ways. Students with the highest levels of academic and social status have the highest levels of engagement in collaborative work and those who are least engaged tend to have lower academic and social status (Chiu & Khoo, 2003; Cohen & Lotan, 1997; Mulryan, 1994). Mulryan (1992, 1995) found that high-achieving students controlled group sessions whereas low-achieving students behaved passively. In addition, the robust work on complex instruction has highlighted the role of status in organizing both participation levels and opportunities to learn and achieve (e.g., Boaler, 2008; Boaler & Staples, 2008; Cohen, 1994; Featherstone, Crespo, Jilk, Oslund, & Parks, 2011). Even among those who participate in group work, whose ideas are attended to or ignored, are linked to relationships of power (Engle, Langer-Osuna, & McKinney de Royston, 2014; Sengupta-Irving, 2014). These linkages are observable through the spatial orientation of bodies within groups that locate who spatially blocked from the core collective work (Dookie & Esmonde, 2012) and which speakers have captured the audience's gaze (Engle et al., 2014; Leander, 2002).

Although group work occurs largely in the absence of the teacher, the teacher can intervene in ways that affect subsequent group dynamics. Teachers can set particular norms for group work that foster shared inquiry and offer tasks that afford collaborative effort. Further, when mathematics teachers directly attend to issues of status, engagement becomes more equitable. For example, in complex instruction, teachers explicitly seek authentic, public, or semi-public moments to elevate the mathematical contributions of low status students, raising their relative status levels. This can balance participation levels as well as shift peers' perceptions as to who can do mathematics (Boaler, 2008; Boaler & Staples, 2008; Cohen, 1994; Featherstone et al., 2011).

In this article, I frame the interactional processes related to status from the perspective of positioning theory. That is, students interactionally position themselves and one another with academic and social power that can affect collaborative mathematical work. Davies and Harré (1990) defined the term positioning as "the discursive process whereby selves are located in conversations as observably and subjectively coherent participants in jointly produced story-lines" (p. 48). Harre (1991) later stated:

> A person is positioned in this or that social location as a speaker in a discourse when the story-line which is unfolding makes available to them only a certain limited repertoire of possible contributions to that conversation. For example, when A says to B "I'm sorry you're not feeling too well. Can I get you anything?" A positions him or herself as the active and helpful member of the duo and positions B as passive and helpless. As A and B are now discursively constituted each has an appropriate repertoire of further sayings and doings available proper to their several positions. (p. 58)

In mathematics classrooms, the jointly produced story-lines are largely organized around mathematical competence (e.g., Cobb, Gresalfi, & Hodge, 2009; Enyedy et al., 2008; Gresalfi, Martin, Hand, & Greeno, 2009; Hand, 2010; Horn, 2008; Turner, Dominguez, Maldonado, & Empson, 2013). Classrooms that develop local storylines that broaden possibilities to enact competence support engagement by a greater range of students. For example, framing mistakes as opportunities for reflection and revision position students willing to share and examine their ideas as enacting a form of competence in doing mathematics (Gresalfi et al., 2009). Yet, even in classroom explicitly designed to broaden conceptions of mathematical competence, individual students nevertheless develop very different positions of authority relative to one another (Esmonde & Langer-Osuna, 2013; Langer-Osuna, 2011; Wood, 2013).

This article builds on efforts to understand relationships of power among students by examining a case of collaborative problem solving from the perspective of how students position themselves and one another with different forms of authority. This article contributes toward our understanding of the mechanisms by which relationships of power are assumed during mathematical activity, shaping student engagement and the construction of solutions to mathematics problems. In particular, this article explores, through a single revelatory case, the construction of and relations between social and intellectual forms of authority as drivers of the collaborative problem-solving process, and the role of the teacher in affecting this process.

The organization of this investigation is as follows: I first describe the conceptual framework utilized to make visible the discursive negotiation of mathematical ideas and positions of authority among peers. I then describe the context of the focal case and the nature of the data sources. Third, I present the findings of an analysis focused on the construction of a mathematical solution path through the lens of authority and the role of the teacher in that process. Finally, I relate the findings back to the research on student-centered and discussion-oriented mathematics classrooms that ground this article and I discuss implications for future research.

Conceptual framework

I draw on the influence framework (Engle et al., 2014) to understand the relationships of authority among peers during collaborative mathematics work. The influence framework emerged from a case of a heated, student-led debate in an elementary science class wherein one student garnered far more influence in the debate than would be merited given the relative strength of his arguments. Drawing on a synthesis of related literature and a moment-to-moment interactional analysis of the case, the influence framework explains how particular interactions lead to influence; that is, the uptake and travel of particular ideas. The framework posits that influence derives out of the social negotiation of key interaction types, which include interactions around (a) the perceived merit of each participant's contributions, (b) each participant's degree of perceived authority, (c) each participant's access to the conversational floor, and (d) each participant's degree of spatial privilege.

Although the original influence framework focused only on intellectual authority, I focus here on two forms of authority. First, I understand intellectual authority in terms of positioning students as credible sources of information pertinent to the particular task at hand (Engle & Conant, 2002). Recent work has shown that interactions around intellectual authority can affect which mathematical ideas become influential during group work (Amit & Fried, 2005). For instance, students positioned as credible sources of information are likely to get their ideas attended to, positively evaluated, and accepted (Amit & Fried, 2005; Cohen & Lotan, 1997; Inglis & Mejia-Ramos, 2009; Kurth, Anderson, & Palinscar, 2002; Sengupta-Irving, Redman, & Enyedy, 2013). Further, students positioned with intellectual authority are also more likely to be spatially attended to, gain and maintain the conversational floor without interruption, and have their ideas evaluated as high quality (Erickson, 2005; Kurth et al., 2002; Sengupta-Irving, 2014). This occurs regardless of whether the ideas are productive; that is, ideas that might be incorrect or potentially misleading might be positively evaluated and ultimately accepted (Inglis & Mejia-Ramos, 2009). Conversely, students with potentially good ideas who are positioned as lacking intellectual authority are often unable to gain the conversational floor or have their ideas attended to (Kurth et al., 2002).

Intellectual authority is but one form of authority constructed among students. A second form fundamental to student-led collaborative work is a social form of authority that I term directive authority, the authority to issue directives to peers in the management of group dynamics (Wood, 2013; Wood & Kalinec, 2012). Wood and Kalinec (2012) argued that because mathematics learning occurs through engagement in social practices, it inherently contains both social and academic aspects of talk. They use the term "action-oriented subjectifying talk" to define talk that tells others to do something, similar to the idea of issuing directives. They found that such talk could affect opportunities for students to engage in productive mathematical talk. In their analysis of students collaborating on a mathematics problem, one student's frequent directives toward another student strongly limited the "managed" student's opportunities to engage in mathematical talk. Wood (2013) argued that these sorts of interactions position the managed student with the micro-identity of the "menial worker," which closed down the managed student's opportunities for learning. Wood and colleagues noted that this link between peer directives and learning opportunities is particularly problematic because most educators would characterize directives as part of being on task, as they relate to managing the work being accomplished. Langer-Osuna (2011) found that how group members perceive the validity of a peer's directive authority is fundamental to the group's dynamics. In particular, Langer-Osuna found that when a female group leader issued directives, male peers took the use of the directives as inappropriate, rejecting the group leader's legitimacy and further marginalizing her from the group project. The same peers took up the male group leader's similar directives very differently, positioning him as smart and helpful, bringing him to a more central position in the group's work. While empowering students with a more active role in the learning process is a worthwhile goal, too little is known about authority relationships among students, and how to mitigate problems that arise when students are asked to take on positions of power, typically embodied only in the teacher, in relation to their peers.

This article builds on earlier work on the influence framework in key respects. First, I apply the framework, developed in an elementary science classroom, to mathematics collaborative work for the purpose of refining the framework for its utility in analyzing mathematics classrooms. Second, the original case was almost totally led by students. That is, the teacher had almost no role in organizing the debate among students. The present case was more typical of collaborative work among peers, wherein the teacher moved from group to group throughout the lesson allowing large amounts of time during which students worked in the absence of the teacher in addition to several moments during which the teacher came by to confer with the group. The relative presence of the teacher allowed me to refine the framework to consider how interactions between students and teacher affect influence. Third, as in many mathematics classrooms, students in the focal case utilized a shared poster to record their collaborative work. In the original case, the argument was fully oral. Thus, influence was marked by verbal uptake of peers' ideas, and peers had to convince one another to adopt such ideas. Here, the shared poster becomes an influential artifact because one or the other student can potentially control the poster and what is written down as part of the solution path without necessarily convincing their peers. In this sense, more social forms of relational power come to the fore.

Introducing the case

This article focuses on a single revelatory case of student authority relations during small group work. I chose to focus on this case first because the negotiation of both social and intellectual forms of authority were particularly clear within this dyad and made visible what may be occurring among students more generally, albeit in subtler ways. That is, there were few attempts to make sense of one another's ideas, but many attempts to control the collective work socially and intellectually. Second, because the ideas accepted from the partner's work were marked by confusion rather than clarity, this case allowed for an analysis of what processes additional to mathematical sense-making explained how students constructed a final solution path. Last, the teacher conferred with this particular dyad often, allowing for several analytic events of student dynamics before and after the teacher met with them.

The focal dyad included Ana,[1] a bilingual Latina and designated English Language Learner and Jerome, an African-American boy. Students were paired and asked to solve the following problem collaboratively:

> Avenue Crest Elementary is planning to design a fruit or vegetable garden. We have decided that it is best to divide our garden site into square sections that are one meter on each side. We will use four meters of rope to rope off the first section, and we'll only need 3 meters of rope for each additional section. How many meters of rope do we need if we plan on creating a garden with ten sections if the sections are in a single row?

A worksheet with the problem, a poster board, scissors, and glue were distributed to each dyad. Each pair of students was asked to cut the problem from the worksheet, glue it onto the poster, and then collaboratively solve the problem using the shared poster to record their solution.

In order to solve this linguistically complex word problem, students must first decipher the text and then reason about the perimeter of squares with shared edges. Here, 10 squares that share at least one edge make up a rectangular garden site. Students must figure out how much total rope is needed to partition the square sections. Students were expected to enter the problem by drawing a representation of the garden that met the problem constraints and, from there, calculate the total amount of rope needed. A subsequent question asked students to generalize a pattern and compute the total amount of rope needed for a garden with 20 or 100 sections. Here I focused on the first part of the task—drawing the representation and calculating the total amount of rope needed for the garden with 10 sections in a single row.

The classroom teacher, Ms. Grand, was participating in a professional development (PD) study, *Language in Mathematics*,[2] which focused on classroom practices meant to support English learners' development of mathematics academic language[3] during student-led discussions. In the early parts of the professional development, efforts focused on the design of open-ended mathematics tasks and on encouraging a classroom context with shared authority such that students had the opportunity to engage meaningfully with one another's mathematical ideas. In later parts of the professional development, efforts focused on the development of math academic language and ways of explicitly addressing the linguistic complexity of word problems. On this focal day, Ms. Grand attempted to address both aspects of the PD for the first time. Ms. Grand chose an open-ended task and addressed issues of language while moving from group to group and, at times, addressing the entire class, by asking questions that directed students to re-read and explain particular sentences or phrases within the problem.

Data sources and analytic methods

Data analyzed in this study were extracted from a video record of one 90-minute mathematics lesson in a fifth grade classroom in the Southeastern United States. A single camera was mounted on a tripod that pointed downward onto the focal dyad, capturing their interactions, as well as shared materials, facial gestures, and body movement. Students had become accustomed to the presence of video cameras in their classroom since the start of the school year. A table mic was taped in between the students to capture their talk. Student work products were also collected. For this focal day, the work product was the shared poster with the students' problem solution.

The video was uploaded to Studiocode video analysis software and chunked into two main analytic events, namely, (a) interactions between teacher and students when the teacher approached the group and (b) partner work in the absence of the teacher. As shown in Table 1, speech and actions relevant to the following interactions were coded within each analytic event: (a) signifying potential changes in influence, (b) evaluating the merit of a conjecture, (c) evaluating the merit of a student's behavior, (d) the intellectual authority of the speaker, and (e) the directive authority of the speaker.

[1]Pseudonyms.

[2]Language in Mathematics was a five-year research study funded by the Institute for Educational Sciences.

[3]Math academic language refers to discipline-specific ways of using language effectively to participate in school mathematics. The linguistic structures used in mathematics classrooms—such as its technical vocabulary and the use of multiple semiotic systems including oral and written language, symbols and visual representations—differ from the everyday language of social interactions outside of school (Schleppegrell, 2007).

Table 1. Codes for interactions.

Framework Component	Definition	Example
Influence	The student's idea is socially positioned as having become part of (or rejected from) the solution path	A particular students' contribution becomes written onto the shared poster as part of the final answer. "Yeah, that's right! Let's write that down." "No, erase that."
Intellectual merit of a student's conjectures	The student's argument is socially positioned as being of (high or low) quality	"Good idea." "That makes sense to me." "That's just dumb."
Social merit of a student's behavior	The student's words and actions are socially positioned as being (on or off)-task	"I like how Tracy is ready to learn." "Pay attention." "Get back to work, Drew."
Intellectual authority	The student is evaluated, acts, or is treated as a credible source of information (or as lacking such credibility)	"Will you explain it to me?" "She knows what she's talking about." "You don't know." "Anybody could have done that."
Directive authority	The student is evaluated, acts, or is treated as having (or not) the right to issue directives to group members	In response to a directive: "Ok, I'll do that." "You can't tell me what to do!"

Data were analyzed at the level of proposal negotiation units (Engle et al., 2014). A proposal negotiation unit (PNU) is the set of interactions that begins with an utterance putting forth a particular proposal around one of the coded components (e.g., offering an idea to be evaluated or issuing a directive) and ends with an utterance that marks the ultimate uptake, altering, or rejection of that proposal (e.g., idea or directive) that settles into a new socially constructed reality for the dyad. For example, a proposal might include, "I know what to do. Let's add the numbers." This would be coded as a bid for intellectual authority. A response such as, "Yeah" proceeded by the partner adding the numbers would be coded as a successful uptake of that bid, establishing a social reality wherein the first speaker is positioned with intellectual authority. Conversely, a response such as, "No way!" would be coded as a rejection of that bid, establishing a social reality wherein the first speaker's intellectual authority is demoted.

Proposal negotiation units were coded in three parts: at the proposal bid, during the intervening negotiations that may have altered the original proposal, and at the final uptake, coded as promoting or demoting any of the components. Frequencies of promotions or demotions were tabulated per PNU and per focal student, with net strength (promotions minus demotions) as a measure of the thickening of particular social positions (Holland & Lave, 2001; Wortham, 2004). Holland and Lave (2001) used the term "thickening" to refer to the process of being positioned in a particular way, across many events, such that the particular subject position stabilizes as part of the person's social identity. Here, the net strength represents how stable, or thickened, a particular position has become for a particular student.

Qualitative analysis of student interactions was used to illustrate particular patterns in greater depth, with a total of 89 PNUs identified for this analysis. Each PNU was coded in relation to one of the interactional types: influence, intellectual authority, directive authority, intellectual merit, and social merit.

Results

I first offer an overview of Ana and Jerome's interactions as related to the influence framework to give a sense of how mathematical ideas and positions of authority were constructed and ultimately thickened over the course of the lesson between the two focal students. I then zoom into moments of influence by detailing the construction of Ana and Jerome's solution path and the particular interactional moments that led to each influential idea—to each moment where the next step was recorded onto the shared poster thus becoming part of their shared solution. Finally, I describe how relationships of authority that led to these moments of influence were interactionally constructed by Ana and Jerome in relation to teacher–student interactions in the evaluation of Ana's or Jerome's behavior or ideas.

An overview of Ana and Jerome's interactions in relation to the influence framework

Ana garnered far more power than Jerome did across all types of interactions, including being positioned with directive and intellectual authority, having her ideas and behaviors positively evaluated, and ultimately being more influential. Table 2 shows the frequency of kinds of interactions (as PNUs) between Ana and Jerome across the entirety of their group work.

Ana's ideas were evaluated positively 13 times and never demoted. Jerome's ideas, on the other hand, were evaluated positively and demoted an equal number of times. If one were to take the net value of positive and negative positionings as a representation of the perceived strength of their ideas overall, Ana's ideas were strongly positive with a net strength of 13 promotions, while Jerome essentially netted no intellectual merit to his ideas. One simple possibility is that Ana had better or more productive ideas than Jerome. The analysis that follows shows that this was not the case.

Ana was positioned with intellectual authority 15 times, and had her intellectual authority demoted only once. Jerome, in contrast, was positioned with intellectual authority twice and likewise had his position of intellectual authority demoted twice. In this sense, Ana's positioning with intellectual authority in relation to Jerome thickened over time with a net strength of 14 promotions. Jerome, on the other hand, essentially had no perceived intellectual authority when considering the net of both positive and negative positionings. Thus, Jerome had no intellectual power relative to Ana in terms of the strength of his ideas or his position as someone with something credible to say about the mathematics problem.

I observed an even more dramatic trend in terms of the relative social power between Ana and Jerome. Ana was positioned as behaving like a "good student"—an on-task student—twice and positioned as off-task once. While not particularly strong, Ana's position relative to social merit thickened positively, with a net strength of one promotion. Jerome was positioned as on-task once and off-task 10 times. Jerome's net strength relative to the evaluation of his behaviors thickened in a negative state. That is, with a net strength of −9, Jerome was positioned as behaving problematically. Again one possibility is that Ana's behaviors relative to Jerome's were simply far more normative. Again, the following analysis shows this not to be the case.

Ana was positioned with directive authority 20 times and demoted (that is, an attempt to issue a directive to Jerome was rejected) once. In contrast, Jerome's directive authority was promoted three times and demoted four times. Ana issued three times as many directives toward Jerome than Jerome issued to Ana, and was almost always successful. With respect to both social merit and directive (social) authority, Jerome's net positionings thickened in a negative manner. With respect to the right to issue directives, Ana was positioned particularly powerfully, with a net strength of 19 promotions to Jerome's −1.

Finally, Ana was able to garner considerable influence relative to Jerome. In particular, Ana's ideas were accepted as part of the solution path (that is, as influential) six times, while Jerome's ideas were accepted as part of the solution path twice—once for the original representation and once as a correction of Ana's computation. While Jerome's initial contribution should be seen as particularly

Table 2. Frequencies of uptake (represented as positive number) and rejections (represented as negatives in parentheses) across interactions about Ana or Jerome.

	Intellectual Merit	Intellectual Authority	Social Merit	Directive Authority	Influence
Ana	13 (0)	15 (−1)	2 (−1)	20 (−1)	6
Jerome	3 (−3)	2 (−2)	1 (−10)	3 (−4)	2

substantial, as it served as the representational basis of the rest of their work, interactionally, his contribution was ridiculed by Ana and, at one point, dismissed by the teacher, in ways that proved important to their ongoing work throughout the lesson.

In order to understand how these relationships were constructed between these two students, I turn the analytic lens to moments of influence—to interactional moments when a particular idea became part of the shared solution path—and I trace these moments to particular kinds of interactions among Ana and Jerome, as well as between the students and Ms. Grand.

Teacher evaluations in subsequent interactions round authority

Teacher evaluations of on- and off-task behaviors (social merit) affected subsequent peer interactions around directive authority. In particular, the teacher's evaluation of Jerome's behaviors as off-task afforded Ana the authority to issue directives to Jerome. Over time, Ana's directive authority and intellectual authority became linked through the increasingly mathematical nature of her directives. Finally, the teacher's attempts to share authority with Ana and Jerome, in particular the authority to evaluate the intellectual merit of their own ideas, has unintended consequences. That is, Ana and Jerome interpreted the teacher's reluctance to critique Ana's ideas as a positive evaluation of the merit of Ana's ideas, further strengthening her intellectual authority. Moments of influence (the uptake of particular ideas as part of the solution path) occurred subsequent to interactions around authority and explain how particular ideas, while riddled with confusion, were accepted as correct by Ana and Jerome.

Relations between perceived behavior and authority

At the start of the group activity, a student handed out the needed materials to one of the two partners in each dyad. The student handed to Ana the materials and Ana began to cut and glue the word problem onto the shared poster. Her partner, Jerome, had no available way of being on-task during this time. Jerome divided his attention between watching Ana work and engaging in an off-topic discussion with another classmate. Within the first few minutes of the group activity, the teacher publicly positioned Jerome as off-task three times, and as needing to focus on Ana. Students in most dyads were similarly waiting for their partner to ready the poster, but the teacher only publically noted Jerome.

The following excerpt illustrates one of the moments when the teacher positioned Jerome as off-task and, in the subsequent interaction, Ana began to issue directives to Jerome. Initially, Jerome resisted Ana's telling him to read the problem as the next step of the activity. However, being positioned as off-task, relative to Ana, for the third time, Ana was able to leverage the teacher's evaluation to direct Jerome essentially to do the next step. This was the first successful attempt to issue Jerome a directive and it drew directly on the early demotions of Jerome's social merit. At the start of Excerpt 1, Ana was busy gluing the problem onto the shared poster and Jerome waited for her while sitting at his desk. At three points, the teacher positioned Jerome as off-task and needing to "focus on Ana."

Ana positioned herself with directive authority by successfully enacting discursive moves that served to tell Jerome what to do. Ana supported her right to do so by drawing on what she had already contributed to the joint work, "I already glued. Glued and cut it. Read now." While this might be seen as an invitation for collaboration by offering Jerome the opportunity to participate, Jerome's resistance, "Why I have to read?" and his audible sighing against Ana's insistence highlight the directive nature of Ana's talk. Her utterance, "Rea:::d" was declarative rather than an invitation, followed by a sharper directive "Read now," which she uttered directly after making claims about her having cut and glued the problem.

After Jerome read the problem to himself, Ana directed him to draw a representation of the problem, which marked the first contribution to the shared solution as seen in Excerpt 2.

At Ana's directive, Jerome read the problem and drew a representation of what the garden "looks like." He began by drawing a rectangle and partitioning it, but after drawing four vertical partitions

Excerpt 1

0:03:10	Teacher:	[Walks by group, taps Jerome on back.] Get your pencil.
	Ana: [simultaneous]	[Glues problem to group poster.]
0:03:12	Jerome:	[Scoots chair closer to Ana, watches Ana glue problem then turns away to off-camera group.] Can I work with them?
[. . ..]		
0:04:14	Teacher:	[Approaches dyad.]
	Ana: [simultaneous]	[Oriented away from Jerome, toward another classmate.] And obviously—
0:04:17	Teacher:	[To Jerome] And Jerome, you're with Ana. I need you focusing on her, okay?
	Jerome: [simultaneous]	[Lightly bangs desk and grimaces, rolls pencil up and down desk.]
0:04:30	Ana:	[Pushes poster toward Jerome.]
0:04:41	Ana:	Rea::::d.
0:04:42	Jerome:	[Places hand on poster and pulls it closer.] Why I have to read? [Glances at poster then turns to off-camera group.]
0:05:28	Teacher:	Jerome, please focus on Ana.
0:05:30	Ana:	[Looks at Jerome and moves poster toward the middle of them.]
	Jerome: [simultaneous]	[Turns back to Ana, sighs.]
0:05:35	Ana:	Read. I already glued. . . . Glued and cut it. Read now.
0:05:37	Jerome:	[Pulls poster closer to him and reads to himself.]

Excerpt 2

0:09:22	Ana:	[Points to poster indicating 'go.'] Read.
0:09:24	Jerome:	[Reads the problem to himself again, lifts head.]
0:09:29	Ana:	Draw what it looks like.
0:09:32	Jerome:	[Draws rectangle and adds partitions.] 1, 2, 3, 4, 5. [Adds horizontal line bisecting rectangle.] 2, 4, 6, 8, 10.
0:10:10	Ana:	[Looks down to poster.] What are you drawing?
0:10:14	Jerome:	[Continues drawing, recounting the sections quietly.] It looks like a ladder. [Smiles, looks up.]
0:10:27	Ana:	Mm Hmm. [Turns poster around toward her; looks away disinterested.]

that created 1 X 5 array, he ran out of space. He then drew a horizontal line bisecting the drawing, creating a 2 X 5 array with a total of 10 sections (see Figure 1). Neither student took notice of the constraint in the problem that the garden's 10 sections had to be laid out "in a single row." This was Jerome's one of only two influential moments, although it is a substantial contribution as the remainder of the pair's answer built on this representation.

Excerpt 3 demonstrates Ana's emerging directive authority more dramatically. At this point in the process, Jerome had drawn a representation of the garden as a 2 X 5 array. No further progress had yet been made, and Ana's contribution remained limited to cutting and pasting the problem onto the shared poster. At this point, each had done something to contribute to the completion of the assigned task but only Jerome had contributed to the problem solution. However, Ana continued to draw on earlier evaluations of Jerome's off-task behavior to position herself with directive authority.

Ana continued to insist that Jerome had not contributed to the shared task, drawing repeatedly on Jerome's earlier positioning by the teacher as someone who was off-task and needed to focus on Ana. Although Jerome displayed surprise, frustration, and resistance to Ana, he nevertheless accepted her downgrading of his contributions and accepted her directives. Jerome's doing so further strengthened Ana's directive authority, in a sense solidifying a social position of power over Jerome during this task.

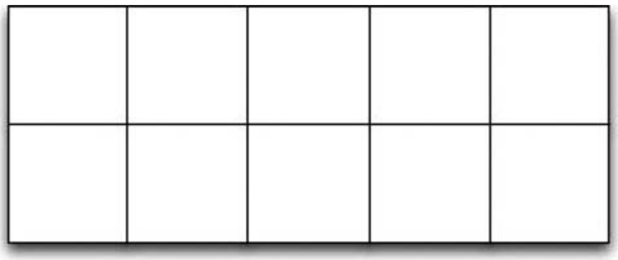

Figure 1. Jerome draws first representation of the garden.

Excerpt 3

0:16:39	Ana:	[Pushes poster to Jerome.] You're doing it!
0:16:41	Jerome:	[Looks at Ana.] Okay, I'm writing, so… [Pulls out neckband and starts playing with it.]
0:16:43	Ana:	You're doing it! You haven't done nothing yet!
0:16:45	Jerome:	What?
0:16:46	Ana:	All you've done is this. [Mimics the original drawing while grimacing.] Which is like… a kindergartner could do that!
0:14:51	Jerome:	Oka::y.
0:14:53	Ana:	I'm not helping you. You do it. [Taps on poster.] You think. Cuz I've been doing—
0:14:56	Jerome:	[Smacks his pencil down.] That's what. … Okay! That's what I'm doing. [Reads problem silently to himself.]

Excerpt 4

0:14:05	Teacher:	So, so if I asked you to draw a picture of this [points to word problem and reads], "We have decided that it is best to divide our garden site into square sections that are 1 meter on each side," What is a square section with one meter on each side look like? [Gaze shifts to Jerome.]
0:14:12	Ana: [simultaneously]	A square.
	Jerome: [simultaneously]	[Quietly.] Like this. [Points to one square of the partitioned 2X5 drawing.]
0:14:14	Ana:	[Draws a separate, individual square on the side of poster. Then draws tic marks on each side of square, as teacher watches.] One, one, one, one.
0:14:16	Jerome:	[Keeps finger pointed on square of partitioned 2X5 drawing; shifts gaze to teacher.] Like a gate.
0:14:17	Teacher:	[To Jerome, pointing at Ana's drawing.] Does that look … does that look right? Is that a square with one meter on each side?
0:14:22	Teacher:	[Shifts gaze to Ana.]
	Ana:	Mm-Hmmm.
0:14:25	Teacher: [simultaneously]	Okay… okay.
	Ana: [simultaneously]	There's one on each side.
0:14:27	Teacher: [simultaneously]	So that…
	Jerome: [simultaneously]	[To himself.] In a single row.
0:14:29	Teacher:	That [pointing to Ana's drawing] looks good to me.

Relations between perceived merit and authority

Excerpt 4 illustrates the next series of interactions, during which the students struggled to represent square sections with one meter on each side. The teacher joined the group at various moments to confer with them on this work. In the following excerpt, Ms. Grand attempted to support student thinking without directly answering their questions or instructing them on how to represent the roped sections of the garden.

When the teacher approached the group, she asked the students to describe a square section with one meter on each side. Both Ana and Jerome bid for the floor simultaneously by offering their respective drawings (see Figure 2). Jerome pointed to one of the square sections of his 2 X 5 array,

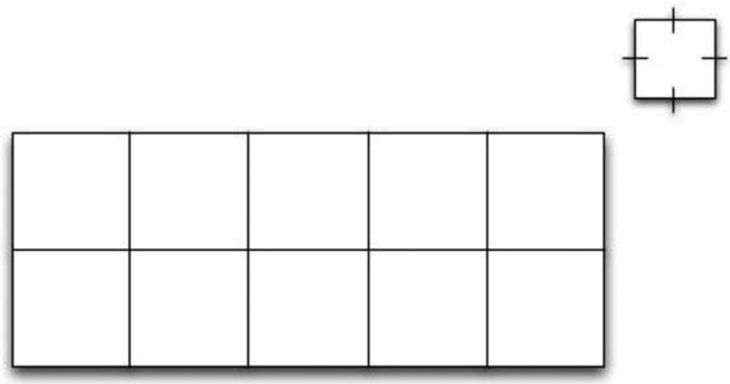

Figure 2. Ms. Grand privileges Ana's top right drawing over Jerome's gesture toward one of his drawing's squares.

while Ana drew a new square, outside of the array, with tic marks representing one meter on each side. The teacher oriented toward Ana's drawing, not Jerome's, and positively evaluated its intellectual merit by saying, "That looks good to me." Ms. Grand directed attention toward Ana's drawing, and invited Jerome to evaluate Ana's drawing positively, asking Jerome, "… does that look right? Is that a square with one meter on each side?" This occurred despite the fact that Jerome's drawing, which he referred to in response to Ms. Grand's question, was, at that point, the dyad's principle representation of the garden.

Again, we see the teacher's evaluation (this time of ideas rather than of behaviors) become integrative for the students regarding how they positioned themselves and each other with forms of authority. In the next excerpt, Ana's already strong directive authority becomes linked with increased intellectual authority, and we see a shift in the nature of Ana's directives to Jerome. Ana's directives originally functioned to regulate Jerome's on-task behavior. As the session progressed, Ana began to regulate not only whether and in what ways Jerome was on-task but also what problem-related ideas he was to implement. In essence, her directives began to include mathematical ideas; Jerome's uptake of these directives served to position Ana further as a credible source of information.

In the next two excerpts, Ana directs Jerome's behaviors related to the task. Jerome voices his opposition to her choices at 19:32 (Excerpt 5) and indicates confusion at 20:23 (Excerpt 6). However, he ultimately takes up her directives, positioning her ideas as influential, as the two accepted them as part of the solution on the shared poster. As Ana's intellectual authority thickened, Jerome positioned her as a credible source of information in other ways, such as increasingly seeking her help and asking her to clarify and evaluate his implementation of her directives.

In Excerpt 5, Ana incorporated her "good" idea by drawing an exact replica of her tic-marked square inside one of the partitions that made up the 2 X 5 rectangular representation of the garden (see Figure 3). Jerome took up the idea without contestation and started to replicate this move across other partitions. While the teacher may have intended to direct the student's focus to more conceptual aspects of Ana's representation, such as attention to the tic marks as "one meter on each side," the students took a more concrete, literal interpretation of the teacher's suggestion to utilize Ana's drawing by replicating squares with four marked sides inside each of the sections of the partitioned 2 X 5 rectangle.

Excerpt 5

0:19:25	Ana:	[Bangs on rectangle.] Erase this.
0:19:32	Jerome:	So you just going to make a big mess of this drawing. [Starts erasing his rectangle.]
0:19:56	Ana:	Wait! These are sections, ok? We have 4 meters in each one. [Draws individual square within rectangle with tick mark on each side.] One, one, one, one.
0:20:16	Jerome:	[Stands and starts drawing on second row of rectangle.] Yeah I can do that.

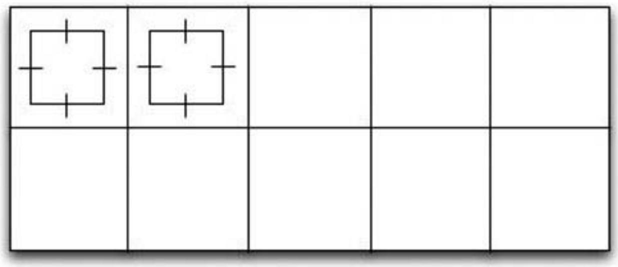

Figure 3. Jerome and Ana implement Ana's square into the main representation.

In the next major interactional moment, while Jerome continues Ana's move of drawing internal squares with four marked sides, Ana recalls from the problem that "additional sections" have three meters on each side, rather than four, and offers a revision to her idea. In Excerpt 6, Ana stops Jerome and redirects his efforts to draw tic marks on only three of the four sides of the internal squares in the bottom row.

Ana recalled that not all sides of all square sections required four meters of rope. Her insight led her to stop Jerome from fully implementing her previous directive and instead to draw tic marks on only three sides for the internal squares of the second row of the 2 X 5 array. As becomes apparent in Excerpt 7, Ana treated the first row of the dyad's drawing as if it were the first section of the garden as worded in the problem, and the second row as if it were the "additional section." Because the problem stated that only the first section required four meters of rope (the entire first row in their depiction), Ana found a way to represent only three meters of rope for the next "section" (that is, the entire second row). Although Jerome appeared to be confused by Ana's idea, as illustrated by his reply (an emphatic "huh?!"), he did not challenge her, but rather accepted Ana's directive to incorporate this revised idea as part of the solution path. (See Figure 4.)

Excerpt 6

0:20:21	Ana:	There were some. … Do three.
0:20:23	Jerome:	Huh?
0:20:24	Ana:	Three only, okay? Only four on that one. [Points to squares on top row.] It says. Cuz… [looking back at the problem] and then the rest of them. [Returns to drawing.]
0:20:25	Jerome:	[Draws three tick marks on the five internal squares on the second row.]

Excerpt 7

0:37:56	Teacher:	And then what does this say down here? [Points to latter part of problem on poster.]
0:37:59	Ana:	[Reads off the poster]: "How many meters of rope will you need if you plan on creating a garden with ten sections. …" [Looks up to teacher.]
0:38:05	Teacher:	Keeping reading.
0:38:07	Ana:	"…if the sections are in a single row?"
0:38:09	Teacher:	In a single row. So where's your single row? What does that mean? A single row?
0:38:14	Ana:	[Puts hands up parallel to each other.] Like one… one row.
0:38:17	Teacher:	One row. Okay. So where's that [in their drawing]?
0:38:21	Ana:	[Swipes finger downward over first column of drawing. Looks back at teacher and smiles unsure.]
0:38:25	Teacher:	[Looks at Jerome.] All right, are you good? You think?
0:38:27	Jerome:	Yeah.
	Ana: [simultaneously]	[Looks at Jerome, then at teacher.] Yes.
0:38:28	Teacher:	Yeah? Okay. If you think that's good then we'll go ahead to the next one. [Taps worksheet then leaves group.]

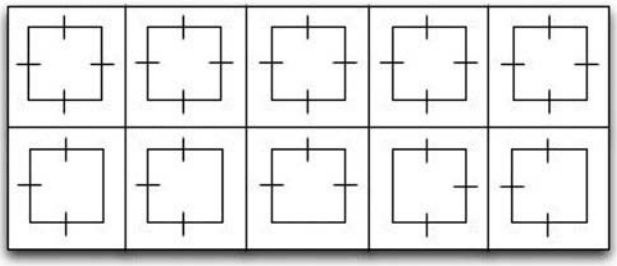

Figure 4. "Yeah, I can do that!"

Shared authority and linguistic struggle

The teacher next joined the group and pushed for an explanation, wherein several areas of confusion, especially uncertainty about the phrase "rope to rope," the mixing up of the terms "sections" and "single row," and which sections have four or three meters of rope, came to the fore. Excerpts 7 and 8 are representative of several turns with the teacher and Ana wherein Ana's linguistic struggles become increasingly apparent. Yet, even though these struggles become apparent, the teacher does not explicitly address them. Instead, the teacher attempts to share the authority to evaluate what makes sense with the students, which they mistakenly interpret as a positive evaluation.

Ana struggled with key terms in the problem, including, as seen in Excerpt 7, the idea that each column in their 2 X 5 array represented the "single row" configuration of the garden site (see Figure 5). The teacher did not clarify the term "a single row," but instead redirected the group to carefully re-read the instructions, define a single row, and point to its location in their representation. Ultimately, Ms. Grand's response, "Okay. If you think that's good then we'll go ahead to the next one," left the evaluation of the correctness of this choice up to Ana and Jerome, although Ana's facial expression (a slight, unsure smile) suggested she was unsure of the term "row." In both prior and subsequent interactions with the teacher, Ana also indicated confusion about the term "section" and the phrase "of rope to rope off." Ms. Grand never directly explained the meaning of the terms to Ana or Jerome, but rather implied that it was up to the students to decide whether their explanations for these terms made sense in the context of the problem. While this was part of her effort to create a classroom context with shared authority, it was perceived by Ana and Jerome as a positive evaluation of their explanations. That is, in subsequent interactions, they accepted Ana's explanation of rows as if it were correct.

Continuing with the logic that one column in the partitioned 2 X 5 rectangle represented "a single row," Ana then realized they had drawn five "single rows" and, confusing "rows" and "sections" once again,

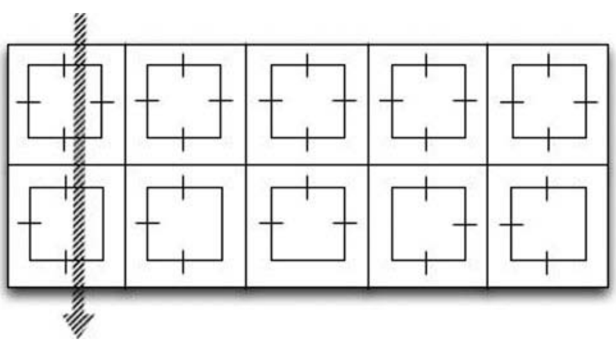

Figure 5. Ana swipes with finger down, "This is one row."

Excerpt 8

0:38:52	Ana:	Ms. Grand? I think this is one row [moves finger up and down first column]. This is one row. [Keeps moving finger over columns.] Two row. Three row.
0:38:59	Teacher:	That's one row. Tell me?
0:39:01	Ana:	And then we keep going on until we get the10 rows.
0:39:03	Teacher:	Okay. [Walks away.]
0:39:06	Ana:	[Draws long rectangle, then nine vertical lines.]
0:39:26	Jerome:	Which one is that? Why are you doing that?
0:39:38	Ana:	It's 10 sections. We only have five sections. Look one row, two row, three row, four row, five row. We have five rows there.

Excerpt 9

0:40:22	Ana:	Ok, that's five rows. And then we need another five. Ok? So, 35 plus 35. [Writes on poster.]
0:40:30	Jerome:	It's 70. I did that from my homework last night. …
0:40:39	Ana:	Okay now that's the complete answer [makes sharp horizontal hand gesture, as if "that's it"] 70.
0:40:42	Jerome:	We got it!

decided that they needed five more rows in order to represent the 10 "sections" of the garden. In Excerpt 8, Ana calls Ms. Grand once again to join their group and offers a tentative explanation of her reasoning.

Although Ms. Grand continued to press for explanation, she left the evaluation of the correctness of the assertion to Ana and Jerome. Once the teacher left the group, Ana drew a new rectangle partitioned into 10 sections. Jerome asked Ana to explain this choice, and she responded by explaining that the original partitioned rectangle had only five sections, fluidly moving between the terms "row" and "section" as if they were interchangeable and explaining that her new representation now contained 10 sections. Adding the 35 tick marks,[4] and then doubling this amount, Ana reached her final solution of 70 meters of rope. Excerpt 9 illustrates this process and the final solution.

Jerome expressed confusion regarding Ana's contributions to the problem solution and, at times, requested explanations. Yet, Jerome did not push Ana's thinking or challenge her explanations, but rather repeatedly took up her ideas whether or not he was ultimately convinced. Indeed, he joined Ana in celebration at arriving at the final answer, taking ownership of the process by phrasing it collectively, "We got it!"

Discussion

This article examines how students constructed relationships of authority during partner work and its effect on the collaborative problem-solving process. Moment-to-moment analysis of a focal dyad's interactions with and in the absence of the teacher highlighted three main points. (1) Interactions that position a student with the right to issue directives to peers (directive authority) can become linked to interactions that position the same student as a credible source of mathematical knowledge (intellectual authority). (2) Both directive and intellectual forms of authority affect which ideas become influential during collaborative mathematics problem solving. (3) Teacher evaluations of student ideas and behaviors, even perceived evaluations, shape students' subsequent interactions around authority. (See Figure 6.)

The finding that directive authority can become linked to both intellectual authority and influence connects with Wood's (2008) work, which showed how directive authority enabled or constrained opportunities to engage in mathematical sense making. Wood examined micro-moments of positioning and their effect on students' math talk. Some positions, such as "the explainer," shaped better and more sustained discursive opportunities for math talk than other positions, such as "the menial worker," which shut down discursive possibilities for students' math talk. As in Wood's findings, Jerome's

[4]Five squares with four tic-marked sides represent 20 meters of rope, plus five squares with three tic-marked sides represent 15 meters of rope, totaling 35 meters of rope.

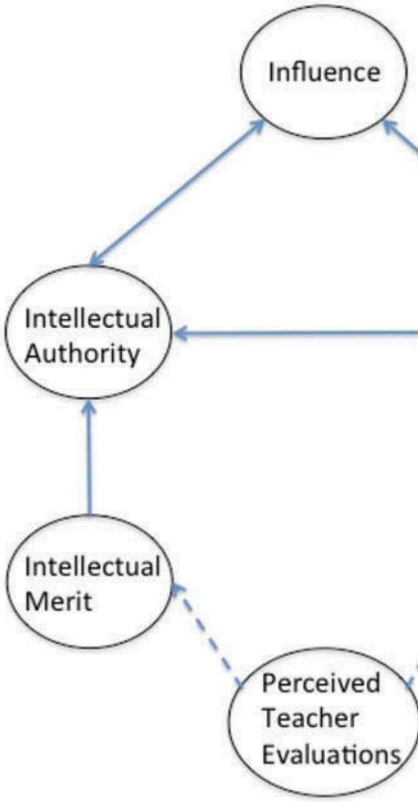

Figure 6. Interactional components affecting influence.

opportunities to engage in math sense making were constrained by Ana's directive authorities. That is, Jerome's contributions to the problem solution were mainly in the form of following Ana's directives that increasingly included implementing her mathematical ideas. In addition, her position of authority made it increasingly less likely that Jerome would push back on her ideas and thus closed down opportunities for sense-making for the dyad as a whole.

The results here go further than the dyad's opportunity for sense-making in general and Jerome's opportunities in particular. Ana's increased directive authority ultimately positioned her with increased intellectual authority and increased influence. Beyond the concern that both students had opportunities to engage in sense-making and thus opportunities to learn mathematics, the construction of the mathematics between them was compromised by Ana's undue authority over the management of the dyad. Too much social power (in the form of directive authority) compromised not only fair opportunities for engagement, but also the problem-solving process itself and the mathematics constructed between the two students.

This article also highlights the complexity of the teacher's role during small group interactions, especially the discursive consequences of even perceived (if not actual) teacher evaluations of student ideas and behaviors. Early evaluations of Jerome's behavior, what I refer to as interactions around his social merit, were discursively taken up by Ana and Jerome to position Ana with the authority to issue directives to Jerome. This finding resonates with Battey's (2013) study that showed that teachers' focus on student behavior can constrain opportunities for engagement in mathematical work. Here, evaluations of student behavior were not the only consequential form of teacher-student interactions. Teacher–student interactions that served (or were perceived to serve) to evaluate student ideas also affected subsequent interactions around authority and influence. Specifically, when Ms. Grand made particular discursive moves meant to hand intellectual authority to students such as, “Well, if you think that’s okay then go ahead,” students subsequently interacted as though Ms. Grand had positively evaluated the merit of those ideas. An implication here is that though relationships of authority, both directive and intellectual are largely constructed among students within their small group dynamics, teacher evaluations, even perceived evaluations, are the discursive building blocks.

This last finding brings up new questions about how teachers intervene in student group work. Much of the research in this area suggest that teachers largely step back and allow students to assume responsibility for learning (Ding, Piccolo, Kulm, & Xiaobao, 2007). In this focal case, Ms. Grand attempted many of the moves suggested by the literature in asking questions to focus students’ attention, eliciting their thinking, and inviting Ana and Jerome to continue to think through the areas of confusion together. While these moves are critical to supporting productive collaborative work in mathematics, results here illuminate the need for more research on how these teacher moves are discursively appropriated by students in subsequent work in both productive and unproductive ways.

Theoretically, these findings build on the influence framework and contribute to our understanding of interactions that lead to moments of influence during collaborative student-led mathematical work. The influence framework (Engle et al., 2014) posits that interactions related to influence include access to and maintenance of the conversational floor, being spatially privileged and attended to, having ideas evaluated as meritorious, and being positioned with intellectual authority. This article attempts to zoom in and unpack interactions around authority, showing that students develop multiple kinds of relationships of authority that impact the problem solving process. Like the original work that developed the influence framework, the analysis reported here affirms the link between interactions around intellectual merit (the evaluation of ideas) and being positioned with intellectual authority, as well as the link between intellectual authority and influence. The results reported here also serve to refine the influence framework to suggest that directive authority can lead to influence and can become linked to and support the development of intellectual authority in ways that bypass the need for mathematical sense-making. The analysis here further suggests that teacher interventions that serve to evaluate students’ ideas and behavior affect subsequent student interactions around authority. In particular, students draw on those perceived evaluations to justify or reject their own and one another’s bids for either directive or intellectual authority. This paper serves as a springboard for further theorizing about the nature of collaborative work and the interactional forces that shape the mathematics that students construct through solving problems jointly.

Acknowledgments

The author would like to acknowledge the collaborative effort and invaluable contributions from Walter Secada, Mary Avalos, Naomi Iuhasz-Velez, and Edwing Medina. A version of this paper was presented with Naomi Iuhasz-Velez at the 2013 annual meeting of the American Education Research Association.

Funding

This work was supported by a grant from The Institute of Educational Sciences (IES), Award Number R305A100862, and a postdoctoral fellowship from the National Academy of Education/Spencer. The content of this article does not necessarily reflect the views or policies of the National Academy of Education/Spencer, IES or the U.S. Department of Education.

References

Amit, M., & Fried, M. N. (2005). Authority and authority relations in mathematics education: A view from an eighth grade classroom. *Educational Studies in Mathematics*, *58*, 145–168. doi:10.1007/s10649-005-3618-2

Barron, B. (2003). When smart groups fail. *The Journal of the Learning Sciences*, *12*(3), 307–359. doi:10.1207/S15327809JLS1203_1

Battey, D. (2013). “Good” mathematics teaching for students of color and those in poverty: The importance of relational interactions within instruction. *Educational Studies in Mathematics*, *82*(1), 125–144. doi:10.1007/s10649-012-9412-z

Boaler, J. (2008). Promoting “relational equity” and high mathematics achievement through an innovative mixed-ability approach. *British Educational Research Journal*, *34*(2), 167–194. doi:10.1080/01411920701532145

Boaler, J., & Staples, M. (2008). Creating mathematical futures through an equitable teaching approach: The case of Railside School. *The Teachers College Record*, *110*(3), 608–645.

Chapin, S. H., O’Connor, C., & Anderson, N. (2009). *Classroom discussions: Using math talk to help students learn, grades K-6*. Sausalito, CA: Math Solutions.

Chiu, M. M., & Khoo, L. (2003). Rudeness and status effects during group problem solving: Do they bias evaluations and reduce the likelihood of correct solutions? *Journal of Educational Psychology*, *95*(3), 506–523. doi:10.1037/0022-0663.95.3.506

Cobb, P., Gresalfi, M., & Hodge, L. L. (2009). An interpretive scheme for analyzing the identities that students develop in mathematics classrooms. *Journal for Research in Mathematics Education*, *40*(1), 40–68.

Cohen, E. G. (1994). Restructuring the classroom: Conditions for productive small groups. *Review of Educational Research*, *64*(1), 1–35. doi:10.3102/00346543064001001

Cohen, E. G., & Lotan, R. A. (1997). Producing equal-status interaction in the heterogeneous classroom. *American Educational Research Journal*, *32*(1), 99–120. doi:10.3102/00028312032001099

Davidson, N., & Kroll, D. L. (1991). An overview of research on cooperative learning related to mathematics. *Journal for Research in Mathematics Education*, *22*(5), 362–365. doi:10.2307/749185

Davies, B., & Harré, R. (1990). Positioning: The discursive production of selves. *Journal for the Theory of Social Behaviour*, *20*(1), 43–63. doi:10.1111/jtsb.1990.20.issue-1

Ding, M., Piccolo, D., Kulm, G., & Xiaobao, L. (2007). Teacher interventions in cooperative-learning mathematics classes. *Journal of Educational Research*, *100*(3), 162–175. doi:10.3200/JOER.100.3.162-175

Dookie, L., & Esmonde, I. (2012). Understanding influence in collaborative group work: The importance of artifacts. In *Proceedings of the 10th International Conference of the Learning Sciences (ICLS 2012)* (Vol. 2, pp. 286–290). Sydney, Australia: International Society of the Learning Sciences.

Engle, R. A., & Conant, F. R (2002). Guiding principles for fostering productive disciplinary engagement: Explaining an emergent argument in a community of learners classroom. *Cognition and Instruction*, *20*(4), 399–483. doi:10.1207/S1532690XCI2004_1

Engle, R. A., Langer-Osuna, J., & McKinney de Royston, M. (2014). Toward a model of influence in persuasive discussions: Negotiating quality, authority, privilege, and access within a student-led argument. *Journal of the Learning Sciences*, *23*(2), 245–268. doi:10.1080/10508406.2014.883979

Enyedy, N., Rubel, L., Castellón, V., Mukhopadhyay, S., Esmonde, I., & Secada, W. (2008). Revoicing in a multilingual classroom. *Mathematical Thinking and Learning*, *10*(2), 134–162. doi:10.1080/10986060701854458

Erickson, F. (2005). *Talk and social theory: Ecologies of speaking and listening in everyday life*. Cambridge, UK: Polity.

Esmonde, I., & Langer-Osuna, J. (2013). Power in numbers: Student participation in mathematical discussions in heterogeneous spaces. *Journal for Research in Mathematics Education*, *44*(1), 288–315. doi:10.5951/jresematheduc.44.1.0288

Featherstone, H., Crespo, S., Jilk, L., Oslund, J., & Parks, A. (2011). *Smarter together! Collaboration and equity in elementary math classroom*. Reston, VA: National Council of Teachers of Mathematics.

Good, T. L., McCaslin, M., & Reys, B. J. (1992). Investigating work groups to promote problem solving in mathematics. *Advances in Research on Teaching*, *3*, 115–160.

Gresalfi, M., Martin, T., Hand, V., & Greeno, J. (2009). Constructing competence: An analysis of student participation in the activity systems of mathematics classrooms. *Educational Studies in Mathematics*, *70*(1), 49–70. doi:10.1007/s10649-008-9141-5

Hand, V. M. (2010). The co-construction of opposition in a low-track mathematics classroom. *American Educational Research Journal*, *47*(1), 97–132. doi:10.3102/0002831209344216

Harre, R. (1991). The discursive production of selves. *Theory & Psychology*, *1*(1), 51–63. doi:10.1177/0959354391011004

Hogan, K., Nastasi, B. K., & Pressley, M. (2000). Discourse patterns and collaborative scientific reasoning in peer and teacher-guided discussions. *Cognition and Instruction*, *17*(4), 379–432. doi:10.1207/S1532690XCI1704_2

Holland, D., & Lave, J. (2001). *History in person. Enduring Struggles: Contentious practice, intimate identities*. Santa Fe, NM: School for Advanced Research Press.

Horn, I. S. (2008). Turnaround students in high school mathematics: Constructing identities of competence through mathematical worlds. *Mathematical Thinking and Learning*, *10*(3), 201–239. doi:10.1080/10986060802216177

Inglis, M., & Mejia-Ramos, J. P. (2009). The effect of authority on the persuasiveness of mathematical arguments. *Cognition and Instruction*, *27*(1), 25–50. doi:10.1080/07370000802584513

Johnson, D. W., & Johnson, R. T. (1989). *Cooperation and competition: Theory and research*. Edina, MN: Interaction Book Company.

Kurth, L. A., Anderson, C. W., & Palinscar, A. S. (2002). The case of Carla: Dilemmas of helping all students to understand science. *Science Education*, *86*(3), 287–313. doi:10.1002/sce.10009

Langer-Osuna, J. (2011). How Brianna became bossy and Kofi came out smart: Understanding the trajectories of identity and engagement for two group leaders in a project-based mathematics classroom. *Canadian Journal of Science, Mathematics and Technology Education*, *11*(3), 207–225. doi:10.1080/14926156.2011.595881

Leander, K. M. (2002). Silencing in classroom interaction: Producing and relating social spaces. *Discourse Processes*, *34*(2), 193–235. doi:10.1207/S15326950DP3402_4

Mulryan, C. M. (1992). Student passivity during cooperative small groups in mathematics. *The Journal of Educational Research*, *85*(5), 261–273. doi:10.1080/00220671.1992.9941126

Mulryan, C. M. (1994). Perceptions of intermediate students' cooperative small-group work in mathematics. *The Journal of Educational Research*, *87*(5), 280–291. doi:10.1080/00220671.1994.9941255

Mulryan, C. M. (1995). Fifth and sixth graders' involvement and participation in cooperative small groups in mathematics. *The Elementary School Journal*, *95*, 297–310. doi:10.1086/461804

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics*. Washington, DC: Authors.

Schleppegrell, M. J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly*, *23*(2), 139–159. doi:10.1080/10573560601158461

Sengupta-Irving, T. (2014). Affinity through mathematical activity: Cultivating democratic learning communities. *Journal of Urban Mathematics Education*, *7*(2), 31–54.

Sengupta-Irving, T., Redman, E., & Enyedy, N. (2013). Re-storying practice: Using stories about students to advance mathematics education reform. *Teaching and Teacher Education*, *31*, 1–12. doi:10.1016/j.tate.2012.10.007

Sharan, S. (1990). *Cooperative learning: Theory and research*. Westport, CT: Praeger Publishers.

Slavin, R. E. (1995). *Cooperative learning: Theory, research, and practice* (2nd ed.). Boston, MA: Allyn & Bacon.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning*, *10*(4), 313–340. doi:10.1080/10986060802229675

Turner, E., Dominguez, H., Maldonado, L., & Empson, S. (2013). English learners' participation in mathematical discussion: Shifting positionings and dynamic identities. *Journal for Research in Mathematics Education*, *44*(1), 199–234. doi:10.5951/jresematheduc.44.1.0199

Webb, N. M. (1989). Peer interaction and learning in small groups. *International Journal of Educational Research*, *13*(1), 21–39. doi:10.1016/0883-0355(89)90014-1

Webb, N. M. (1991). Task-related verbal interaction and mathematics learning in small groups. *Journal for Research in Mathematics Education*, *22*(5), 366–389. doi:10.2307/749186

Webb, N. M. (2009). The teacher's role in promoting collaborative dialogue in the classroom. *British Journal of Educational Psychology*, *79*(1), 1–28. doi:10.1348/000709908X380772

Webb, N. M., & Mastergeorge, A. M. (2003). The development of students' helping behavior and learning in peer-directed small groups. *Cognition and Instruction*, *21*(4), 361–428. doi:10.1207/s1532690xci2104_2

Wood, M. B. (2013). Mathematical micro-identities: Moment-to-moment positioning and learning in a fourth-grade classroom. *Journal for Research in Mathematics Education*, *44*(5), 775–808. doi:10.5951/jresematheduc.44.5.0775

Wood, M. B., & Kalinec, C. A. (2012). Student talk and opportunities for mathematical learning in small group interactions. *International Journal of Educational Research*, *51–52*, 109–127. doi:10.1016/j.ijer.2011.12.008

Wortham, S. (2004). From good student to outcast: The emergence of a classroom identity. *Ethos*, *32*(2), 164–187.

Yackel, E., Cobb, P., & Wood, T. (1991). Small-group interactions as a source of learning opportunities in second-grade mathematics. *Journal for Research in Mathematics Education*, *22*(5), 390–408. doi:10.2307/749187