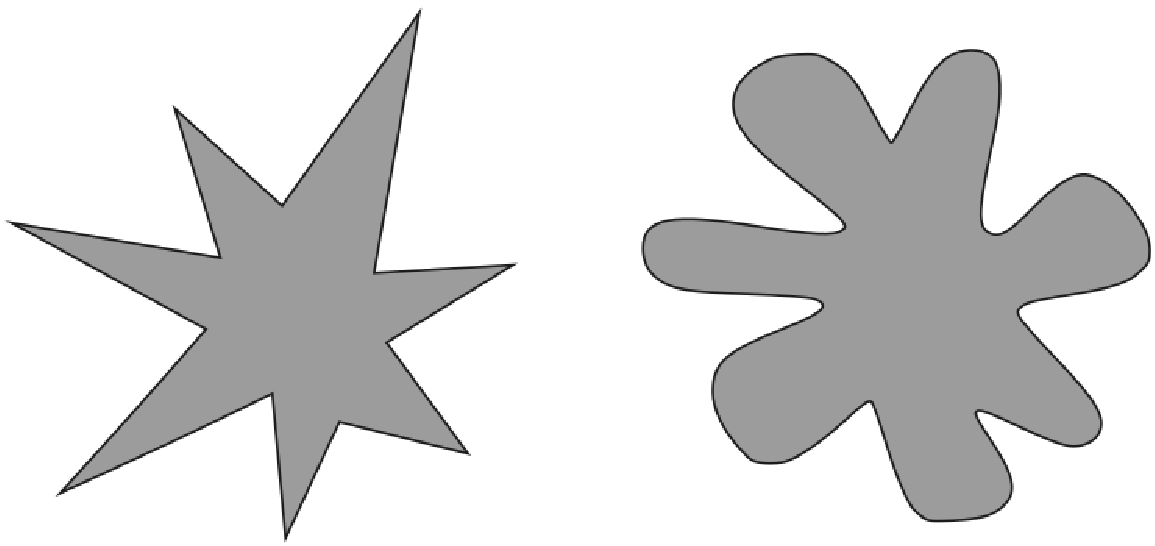
Week 8 Day 1: Revisiting the Martian Alphabet

Your Name: ____

May 16, 2022

Remembering back...

In week one of this course we did an activity exploring if you could correctly identify the letters of the Martian alphabet. You were presented with the two figures below and asked to decide which was "Bouba" and which was "Kiki" (Ramachandran, 2007).



We tabulated how many students correctly guessed that "Kiki" was the figure on the left. Next, we simulated what we would have expected to see if everyone was randomly guessing which figure went with which word.

1. If everyone was randomly guessing, what would be the probability of correctly identifying "Kiki"?

Today, we are going to investigate how we could use statistical inference to determine how often we would expect to get a statistic like the one we got on Day 1 of the class, if everyone was just randomly guessing.

Chi-Squared Goodness-of-fit Test

Specifically, we will be carrying out what is called a "Chi-Squared goodness-of-fit test." The big idea behind this test is to compare what proportions we would have expected to get under the null hypothesis to what we actually got.

In our case, we are assuming that there is an equal chance at guessing correctly or incorrectly.

2. So, under H_0, what would the values of p be?

Correct Guess	Incorrect Guess
$p =$	$p =$

3. If the table above represents what is assumed to be true under H_0, what do you believe the alternative hypothesis would be?

Chi-Squared Statistic

The idea behind the Chi-Squared statistic is to compare the frequencies (of correct and incorrect guesses) we observed to what we would have expected to see if H_0 was true.

This calculation requires two pieces of information:

- p
- sample size

Once we have each of these, we can create a table of what frequencies we would have expected to see if H_0 was true.

4. Fill out the expected table below!

Expected # of Correct Guesses	Expected # of Incorrect Guesses

Next, we compare each of our observed frequencies to what we would have expected if H_0 was true. We compare how far "off" our observed frequencies are from what was expected in a very specific way, using the following calculation:

$$\frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

This formula should look similar to calculating the percent change, except that we are squaring the numerator.

5. Fill out the table for our class' observed frequencies.

Observed # of Correct Guesses	Observed # of Incorrect Guesses

6. Using the formula above, calculate how far "off" the observed frequencies for our class' guesses are from what we expected under H_0.

By adding the two values from #5 together, you have the Chi-Squared (or X^2) statistic!

7. Calculate the value of the X^2 statistic for our classes guesses.

$X^2 =$

8. Take another look at the X^2 formula. What type of statistics will we **always** obtain? *Hint:* These statistics are similar to the ANOVA F-statistic!

Sampling Distribution of X^2 Statistics

In order for us to calculate our p-value—the probability of observing an X^2 statistic as or more extreme than we got, if the null was true—we need a distribution of X^2 statistics that could have happened if H_0 was true.

Like all of our previous topics, there are two ways we can obtain this **sampling distribution**:

- using computer simulation
- using mathematical theory

Let's see how each of these works!

Simulated / Permuted Null Distribution

On Day 2 of class we simulated a distribution of **proportions** that could have happened under the null hypothesis. Today, we are turning our attention to how simulating what X^2 statistics we could have gotten if the null hypothesis was true.

To carry out **one** simulation we need to do the following steps:

Step 1: Specify what value of p is assumed under H_0:

Step 2: Specify how many **trials** should be used when flipping a coin:

Step 3: Specify which response goes with each side of a coin:

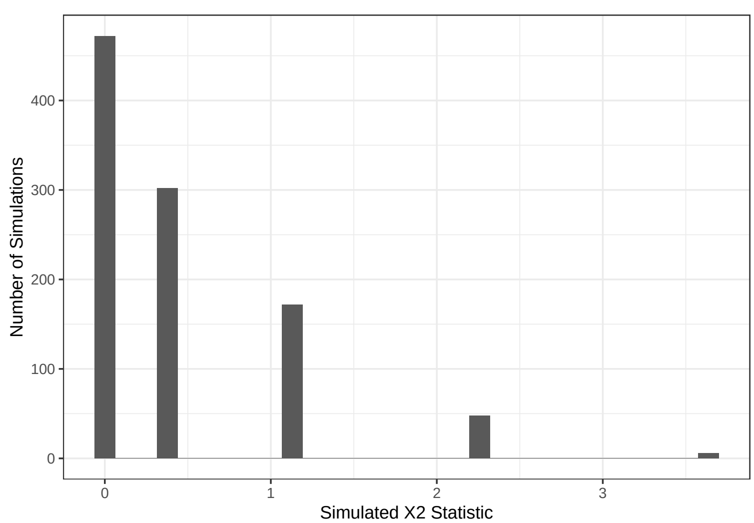
Step 4: Flip a coin (or simulate flipping a coin) the number of times specified in Step 2. Tabulate how many responses went in each category:

Simulated # of Correct Guesses	Simulated # of Incorrect Guesses

Step 5: Calculate the X^2 statistic for the simulation

Step 6: Plot the simulated X^2 statistic on the distribution

Alright, after carrying out this process, I obtained the following distribution.



9. Draw a line where the observed X^2 statistic falls on this distribution.

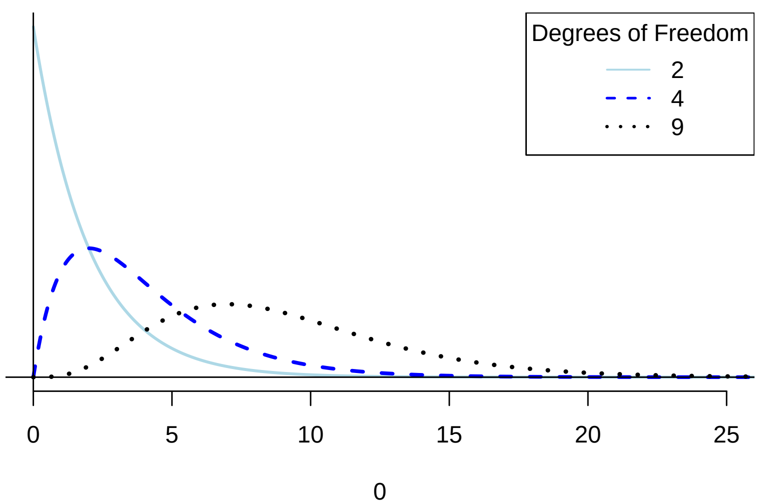
10. Estimate the p-value for testing if we were all randomly guessing which figure was "Kiki."

11. Based on your p-value, what would you conclude for your null and alternative hypotheses from #2 and #3?

Theory-based Null Distribution

For the Chi-Squared goodness-of-fit test we've been discussing, it can be mathematically shown that the distribution of X^2 statistics is approximately a χ^2 distribution with $k-1$ degrees of freedom. Similar to an ANOVA, k represents the number of groups in our response.

Degrees of freedom represent the amount of "free" information in a given situation. In our situation, we have two groups, so $k-1$ would be 1. This should make sense since we know the probabilities must sum to 1 or 100%. If I know what the probability of a correct guess is, I also know the probability of a correct guess. So, only one probability is "free" or unconstrained.

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pchisq(_____, df = 1, lower.tail = FALSE)
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Deer Habitat & Fire