Activity 7A: Introduction to ANalysis Of VAriance

Introduction to ANalysis Of VAriance

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Learning Outcomes

- Summarize and visualize quantitative data for three or more groups.
- Compare the centers and spreads between three or more groups.
- Understand the components of an F-statistic, both visually and computationally.

Terminology review

Thinking back to last week, we covered how we could do comparisons for

(1) a difference in two means

- Analyzing a difference in two means requires the observations in each group are independent

(2) the mean of the differences.

- Analyzing the mean difference requires there are **paired** (two) observations for each observational unit

Movies Released in 2020

Today we're going to use a data set we explored in Week 2, to visualize the distribution of IMDB movie ratings. The dataset is comprised of the following variables collected on each movie:

Variable	Description
`Movie`	Title of the movie
`averageRating`	Average IMDb user rating score from 1 to 10
`numVotes`	Number of votes from IMDb users
`Genre`	Categories the movie falls into (e.g., Action, Drama, etc.)
`2020 Gross`	Gross profit from movie viewing
`runtimeMinutes`	Length of movie (in minutes)

Comparing Many Groups

Last week, we could have used these data to investigate if there were differences in IMDb scores between **two** genres (e.g., Action and Drama). This week, however, we are going to expand our analysis to more than two groups!

Below is a table summarizing the number of observations (movies) in the data set for each genre. We can see that most of the movies fall in the Action, Adventure, Comedy, Documentary, Drama, Horror, and Thriller/Suspense categories. So, let's focus our analysis with these genres (removing the others).

Genre	n
Action	14
Adventure	16
Black Comedy	4
Comedy	23
Documentary	26
Drama	75
Horror	19
Multiple Genres	1
Musical	5
Romantic Comedy	3
Thriller/Suspense	29

Visualizing a Single Categorical and a Single Quantitative Variable

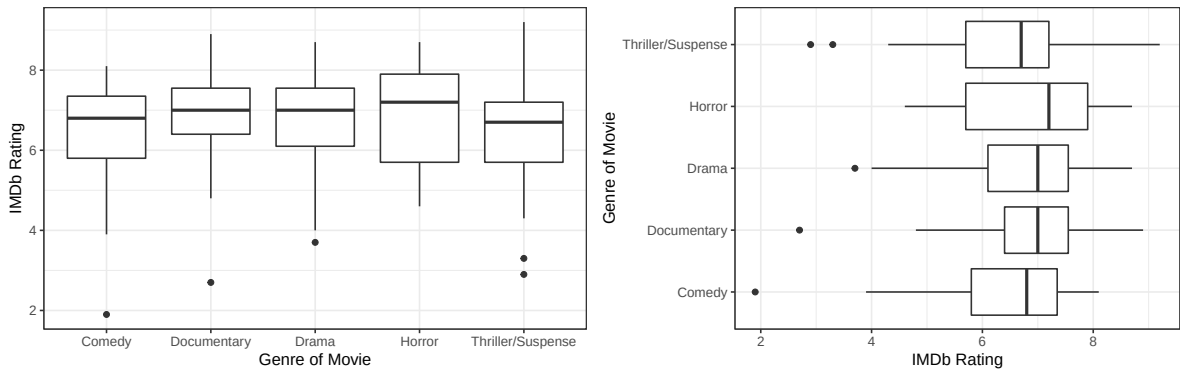
For a categorical variable that has more than two groups, we can use the **same** visualization techniques as we did for a categorical variable with two groups.

1. Think back to last week, what were two ways we visualized one numerical variable and one categorical variable?

Previously, we used side-by-side boxplots and faceted histograms.

Side-by-Side Boxplots

The boxplot of movie budgets (in millions) by content rating is plotted using the code below. The boxplots are presented in both orientations, horizontal stacking and vertical stacking, so you can pick whichever orientation you prefer. :)



Answer the following questions about the box-plots above.

2. Which genre has the highest center?

Horror genre movies have the highest median IMDb rating.

3. Which genre has the largest spread?

The spread appears similar across all movie genres, but Thriller/Suspense and Horror appear to have the largerst spread.

4. Which genre has the most skewed distribution?

Drama and Comedy movies appear to both be skewed left (longer whisker to the left).

Summary Statistics

Let's obtain a more complete picture of how different these groups are with summary statistics. Our familiar friend `favstats()` can help us compare summary statistics across different groups.

Like before, the rating of the film is the response and the genre is the explanatory variable. So, our code looks like:

```
favstats(averageRating ~ Genre,
         data = movie_ratings)
```

Genre	min	Q1	median	Q3	max	mean	sd	n	missing
Comedy	1.9	5.8	6.8	7.35	8.1	6.413	1.413	23	0
Documentary	2.7	6.4	7.0	7.55	8.9	6.835	1.204	26	0
Drama	3.7	6.1	7.0	7.55	8.7	6.729	1.149	75	0
Horror	4.6	5.7	7.2	7.90	8.7	6.826	1.370	19	0
Thriller/Suspense	2.9	5.7	6.7	7.20	9.2	6.317	1.536	29	0

Use the output from the `favstats()` function to answer the following questions:

4. Report the mean rating for each genre. Use appropriate notation.

$\bar{x}_{\text{Comedy}} = 6.413$, $\bar{x}_{\text{Documentary}} = 6.834$, $\bar{x}_{\text{Drama}} = 6.729$

$\bar{x}_{\text{Horror}} = 6.826$, $\bar{x}_{\text{Thriller/Suspense}} = 6.317$

5. Which genres have the largest difference in their mean rating?

Documentaries have the highest mean rating at 6.835 while Thriller/Suspense movies have the lowest mean ratings at 6.317 for a difference in mean ratings of 0.518.

6. Which genre has the largest standard deviation in ratings?

Thriller/Suspense movies have the largest standard deviation in ratings ($s_{\text{Thriller/Suspense}} = 1.536$).

7. Which genre has the smallest standard deviation in ratings?

Drama movies have the smallest standard deviation in ratings ($s_{\text{Drama}} = 1.149$).

8. How many times larger is your answer in #6 than your answer in #7?

$\frac{s_{\text{Thriller/Suspense}}}{s_{\text{Drama}}} = \frac{1.536}{1.149} = 1.337$ times larger.

9. Which genre has the largest sample size? What is the formula for the standard deviation of a mean (aka standard error)? What effect does sample size have on the standard error?

Drama movies has the largest sample size ($n_{\text{Drama}} = 75$).

The standard deviation of the mean is our standard error calculated by $SE = \frac{s}{\sqrt{n}}$. Therefore, with a larger sample size, you are dividing by a larger number and your standard error is decreasing / smaller.

Introducing a New Statistic

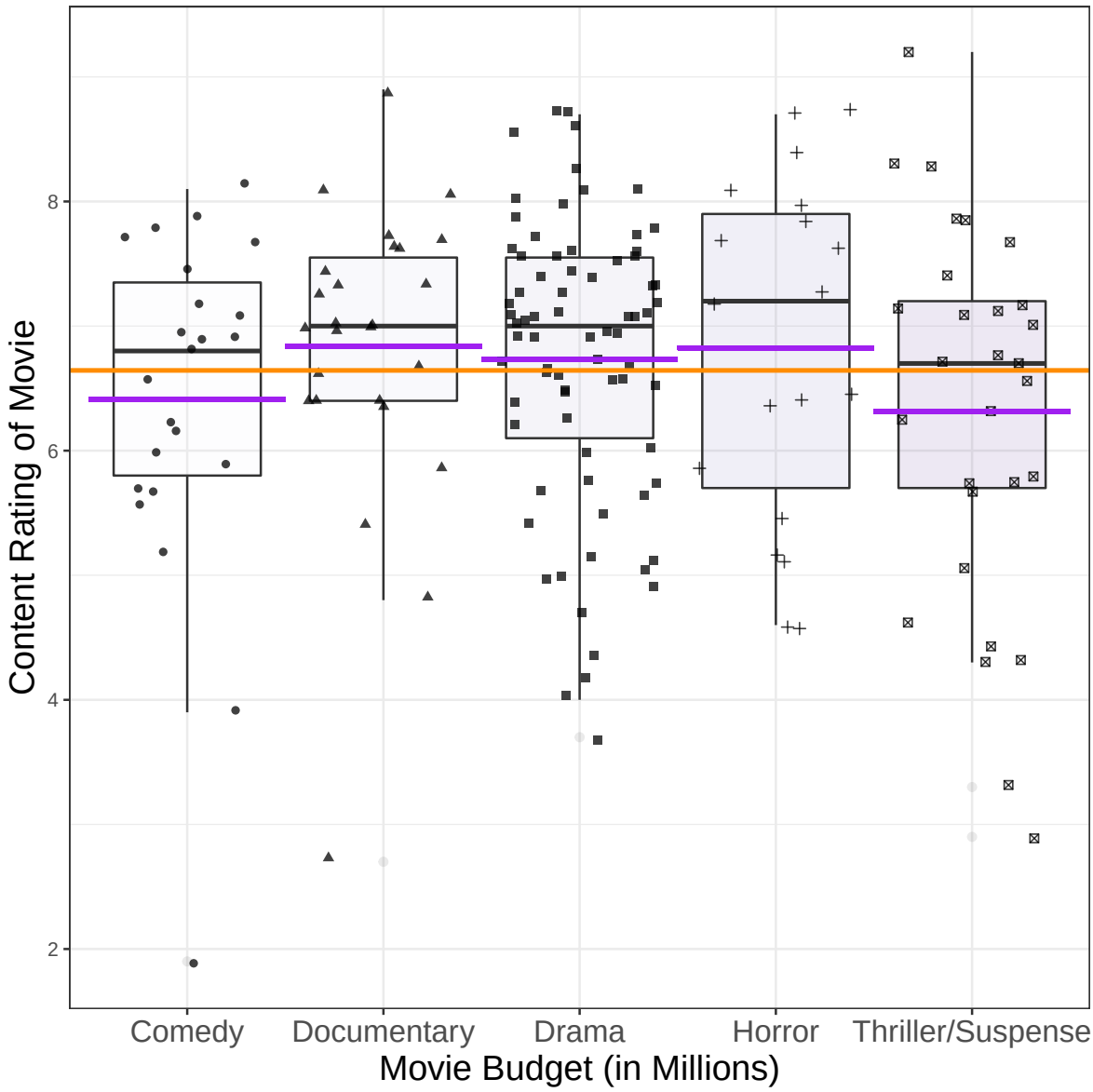
In an ANOVA, there are more than two groups that we wish to compare how different the means are from each other. We could make every comparison of two means (Drama - Action, Horror - Documentary, Comedy - Adventure, etc.), but how would we use these numbers to summarize how different **all** of the groups are from each other?

Enter the F-statistic! An F-statistic summarizes two quantities:

- How different the means of the groups are from each other
- How different the observations in each group are from the mean of their group

To me, an F-statistic makes more sense if I visualize what these pieces mean. In the plot below, I've added three pieces,

- Individual points within each group (these are the movies)
- An orange line across the entire plot
- A purple line across each group



10. What does the orange line across the entire plot represent?

The orange line is the overall mean IMDB rating across all genres.

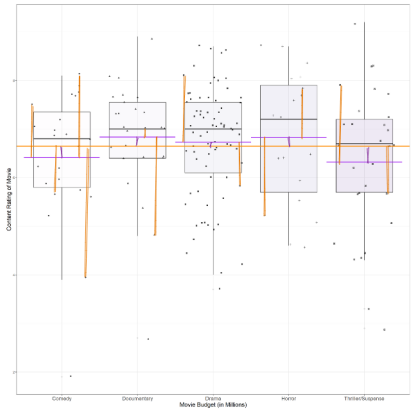
11. What do the purple lines across each group's boxplot represent? *Hint:* The purple dashed line is different from the black solid line!

The purple line is the mean IMDB rating for each group individually (note: the black solid line is the median IMDB rating).

Components of an F-statistic

The two components of an F-statistic are called the *sum of squares between groups* (SSG) and the *sum of squares of the errors* (SSE). Let's break down what each of these mean.

The **SSG** compares each group's mean to the overall mean. As its name indicates, these differences are then **squared** and added together.



12. Draw vertical lines on the plot above, indicating which values are being compared when calculating the SSG.

See vertical purple lines on sketch above from the mean of each group to the overall mean.

The **SSE** is similar to a "residual," it measures how far an observation is from the mean of that group. As its name indicates, these differences are **squared** and then added together.

13. Draw vertical lines on the plot above, indicating which values are being compared with calculating the SSE.

See vertical orange lines on sketch above from each individual point/observation to the group mean for that observation's group.

There is one final part to an F-statistic. We take each of these quantities (SSG, SSE) and divide them by their respective degrees of freedom. The degrees of freedom are calculated based on (1) the number of items available and (2) the number of statistics that need to be calculated.

For the SSG, we have k groups and we need to calculate the overall mean. So, our resulting degrees of freedom are $k - 1$.

14. How many degrees of freedom does the `Genre` variable have?

$df_{Genre} = k - 1$ $df_{Genres} = 5 - 1 = 4$

For the SSE, we have n observations and we need to calculate k group means. So, our resulting degrees of freedom are $n - k$.

15. How many degrees of freedom does the SSE for our content rating analysis have?

$df_{Error} = n - k$ $df_{Error} = 172 - 5 = 167$

Now, putting all of these pieces together, we can obtain the magical F-statistic using the following formula:

$$\frac{\frac{SSG}{k-1}}{\frac{SSE}{n-k}} = \frac{MSG}{MSE}$$

16. Can an F-statistic be negative?

No, an F-statistic cannot be negative because we square the distances between the group means and overall means as well as square the distances between the individual observations and the group means and squared values are always positive.

Calculating an F-statistic in `R`

Calculating these quantities by hand would be terrible! Instead, we will use `R` to output these values.

The `aov()` function in `R` stands for **a**nalysis **o**f **v**ariance. Why they didn't call it `anova()` is beyond me!

The `aov()` function takes two inputs, the first is a "formula" similar to what you've seen in the `favstats()` function. The response variable comes first, then the explanatory variable. The second input is the dataset that should be used.

Let's give the code and the output a look!

term	df	sumsq	meansq	statistic	p.value
Genre	4	6.446	1.611	0.969	0.426
Residuals	167	277.679	1.663	NA	NA

17. What is the sum of squares for `Genre`?

The sum of squares for Genre is 6.446 (SSG).

18. What is the sum of squares for the errors?

The sum of squares for error is 277.679 (SSE).

19. How was the mean squares for `Genre` found?

The mean squares for Genre (MSG) is found by diving the SSG by 4 (the degrees of freedom for Genre).

20. How was the mean squares for the errors found?

The mean squares for the errors (MSE) is found by diving the SSE by 167 (the degrees of freedom for the errors).

21. What is the resulting F-statistic?

F = 0.969

22. Why is there an NA in the `statistic` column for the `Residuals`?

The Residuals column aligns with the errors, we are testing the difference between groups; therefore the Residuals MSE is our denominator for our test to compare Genres.

Inference for an ANOVA

23. Based on the p-value associated with the F-statistic you found in #21, do you think this is a small F-statistic or a large F-statistic?

We have a p-value of 0.426; therefore the F-statistic is a small F-statistic since there is a 42% chance of seeing an F-statistic of 0.969 or larger on an F-distribution with 4 numerator degrees of freedom and 167 denominator degrees of freedom.

24. Do you believe this statistic is likely to occur if the null hypothesis is true?

Yes, since our p-value of $0.426 > 0.05$, we would fail to reject the null and say there is a 42.6% chance of seeing an F-statistic of 0.969 or larger if the null hypothesis is true (if there was no difference in the mean movie rating between the genres).