Log Transformation Interpretations

When you are in the case where you have log transformed your x variable, your regression equation will look like the following:

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 \times log(x)$$

Intercept

The intercept estimate ($\hat{\beta}_0$) represents the estimated mean value of y when the explanatory variable is 0. However, in a $log(x)$ scenario, you cannot evaluate this when $x = 0$, since $log(0) = \infty$. So, we need to evaluate the equation when $log(x) = 0$, which occurs when $x = 1$.

Thus, the intercept is interpreted as the expected mean value of y when $x = 1$.

Slope

For the slope interpretation, we rely on the properties of logarithms. We are interested in increasing x and seeing what the associated change in y is. Typically, in a linear regression we increase x by 1 unit and $\hat{\beta}_1$ is the expected change in the mean of y.

However, when we have $log(x)$ we need to increase x slightly differently. This is because $log(x + 1) \neq log(x) + log(1)$. So, we need to figure out a way to increase x that allows for us to separate $log(x)$ from the change in y.

From properties of logarithms, we know $log(2x) = log(2) \times log(x)$. Thus, if we double x we get the following:

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 \times log(2x)$$
$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 \times \Big(log(2) \times log(x)\Big)$$
$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 \times log(2) + \hat{\beta}_1 \times log(x)$$

Notice that our original equation was $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 \times log(x)$. So, by doubling x we saw a $\hat{\beta}_1 \times log(2)$ change in the mean of y.