UNIVERSITY OF WASHINGTON

AMATH 482 A WI 20: COMPUTATIONAL METHODS FOR DATA ANALYSIS

Homework 3

Abstract

The goal of this project is to analyze and filter noise for different movies.
The videos show an oscillation of an object of which position has to be found with different method. With given position matrix we can use mathematical methods to solve the motion equation.
Some of the videos have added difficulties which can be seen as noise and the task is to add methods to filter out these added movements.

Ather Ahmad

Instructor: Craig GIN

1 Introduction and Overview

To get the position of the moving mass is the most important task. This can be achieved with looking at the matrix for each picture. To make it easier the method of edge detection can be used. To get rid of noise movement we can look at points which we can assume being stationary and look at the relative movement of the mass of interest. To analyze the movement itself we can use the method of Principal component analysis (PCA).

2 Theoretical Background

2.1 Motion equation

To solve the motion equation theoretically is an important part of the task since the solution depends on a few parameters which will be varied until the difference between both theoretically and measured solution is minimized. These parameters are the real physical properties of the whole system.

2.1.1 Differential equation

The given video shows an example of a spiral pendulum, which is an type of harmonic oscillator. The motion equation can be described by

$$\vec{F} = -D \cdot \vec{y} \tag{1}$$

with D: spring constant.

Using Newton's laws of motion and assuming the mass is constant $m(t) = m$ then we know that

$$\vec{F} = m \cdot \vec{a} = m \cdot \ddot{\vec{y}}. \tag{2}$$

This way we get our ansatz

$$m \cdot \ddot{\vec{y}} = -D \cdot \vec{y}. \tag{3}$$

The solution for this problem can simply be solved with an sinus function with $\omega = \sqrt{\frac{D}{m}}$

$$y(t) = sin(\omega t). \tag{4}$$

Using this knowledge we can figure out the ratio $\frac{D}{m}$ of the real system.

2.2 Principal component analysis

PCA is used to structure, simplify and illustrate extensive data sets by approximating a large number of statistical variables with a smaller number of meaningful linear combinations.

$$\vec{a} = (a_1\ a_2\ ...\ a_n) \quad \text{M-mean (average)} \tag{5}$$

$$\sigma^2 = \frac{1}{n}\sum_{k=1}^{n}(a_k - M)^2 \tag{6}$$

$$\text{If we subtract M first } \vec{a} = (a_1\ a_2\ ...\ a_n)\begin{pmatrix} a_1 \\ a_2 \\ . \\ . \\ . \\ a_n \end{pmatrix}$$

$$\sigma^2 = 1/n \sum a_k^2 = \frac{1}{n}\vec{a}\vec{a}^T \quad \text{pop var} \tag{7}$$

$$\sigma^2 = \frac{1}{n-1}\vec{a}\vec{a}^T \tag{8}$$

unbiased estimater

$$\vec{a} = (a_1\ a_2\ ...\ a_n) \tag{9}$$

$$\vec{b} = (b_1\ b_2\ ...\ b_n) \tag{10}$$

$$\sigma_a^2 = \frac{1}{n-1}\vec{a}\vec{a}^T \sigma_b^2 = \frac{1}{n-1}\vec{b}\vec{b}^T \tag{11}$$

2.2.1 Covarience

$$\sigma_{ab}^2 = \frac{1}{n-1}\vec{a}\vec{b}^T \tag{12}$$

$$\sigma^2 = 0 \tag{13}$$

$$\text{uncorrelated} \tag{14}$$

$$\text{(statistically independent)} \tag{15}$$

3 Algorithm Implementation and Development

4 Computational Results

5 Summary and Conclusions

6 Appendix A: MATLAB functions used and brief implementation explanation

7 Appendix B: MATLAB codes

load('cam1_1.mat')
implay(vidFrames1_1)
with the following code.
numFrames = size(vidFrames1_1,4);
for j = 1:numFrames
X = vidFrames1_1(:,:,:,j);
imshow(X); drawnow
end