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# From Play to Thoughtful Learning: A Design Strategy to Engage Children With Mathematical Representations

KAMRAN SEDIG
*The University of Western Ontario, Middlesex College*
*Canada*
sedig@uwo.ca

Many children do not like learning mathematics. They do not find mathematics enjoyable and engaging, and they think it is difficult to learn. Computer-based games have the potential and possibility of addressing this problem. The purpose of this research is to investigate whether and how games can be designed to help children learn mathematics in an enjoyable and motivating way. To address this problem, a design strategy is proposed. The strategy is to engage children with mathematical representations using a game as the learning context. This game is designed in such a way that it can initially motivate children to engage with the environment in a playful fashion and then the design promotes a gradual shift of the children's attention towards the representations of mathematical concepts. In this strategy, representations of mathematical concepts are used to mediate between game-playing and learning. As the game progresses, the representations become a prominent part of the game. To demonstrate the feasibility and effectiveness of the strategy, a game-based learning environment, Super Tangrams, was designed and implemented. This game takes children from almost no knowledge of transformation geometry to some nontrivial knowledge, involving composite reflections and complex rotations. A study evaluating the effectiveness of the game (i.e., the strategy) is presented. The results of the study suggest that, despite the explicitness and difficulty of the mathematical concepts involved, children found the learning process fun and engaging. Furthermore, children exhibited significant improvement in their knowledge of transformation geometry concepts. A number of conclusions with regard to children, design, and learning mathematics are drawn from this research.

*"I don't like math because it's boring."* (Grade-6 student)

*"Math is difficult. It's not fun. I like the arts."* (Grade-6 student)

*"I don't like math. We have to sit down and do it on a piece of paper. [It] doesn't have a meaning or a goal at the end. All you do is exercises. It is really really really boring. I don't like doing it that way."* (Grade-6 student)

Motivation and learning context play important roles in the learning of mathematics (NCTM, 2000; Ma & Kishor, 1997). These quotes from three grade-6 students typify the attitude of a large number of middle-school students towards mathematics. The attitudes expressed are not ones that can be understood or analyzed through a "simple single-cause/single-effect linear progression" (Martinez & Martinez, 1996, p. 2). These attitudes encompass feelings towards mathematics, both motivational and cognitive, which weave together in a chain reaction. These feelings originate from and are influenced by one's memories of past failures and successes, by one's peers, teachers, and parents, by the methods of teaching to which one has been subjected, by the type of mathematics to which one has been exposed, and by the learning environments in which one has encountered mathematics (Martinez & Martinez, 1996).

The National Council of Teachers of Mathematics (NCTM, 2000) recommends the use of computer software in children's mathematics education. Additionally, it suggests and emphasizes the importance of using learning contexts that are relevant and memorable to children and engage them with mathematical ideas. One of the contexts in which children like to learn is computer-based games (Mumtaz, 2001; Rieber, 1996). However, design of effective and engaging game-based learning environments is not straightforward (Alessi & Trollip, 2001). The purpose of this research is to investigate whether and how games can be designed to help children learn mathematics in an enjoyable and motivating way. This article synthesizes ideas in game design, interaction design, and mathematics learning to propose a design strategy to engage children with representations of mathematical concepts. The design strategy uses a game as the learning context to initially motivate children to engage with the environment in a playful fashion and to gradually shift children's attention towards nontrivial mathematical concepts in a deep and thoughtful fashion. To demonstrate the feasibility and effectiveness of the strategy, a game-based learning environment has been designed, implemented, and evaluated. This article describes the game and its design, and reports the results of its evaluation both in terms of children's learning as well as whether children enjoy learning mathematics in this context.

In the next section, a brief conceptual and terminological background is provided. It discusses the role of play and computer games in learning, the different types of existing educational games, the role of representations in mathematical learning, and the role of interaction with representations in reasoning and learning. Afterwards, the game-based learning environment embodying the proposed strategy is described. This is followed by a report of the results of the study that evaluated this software. Finally, conclusions from the study are discussed and a general design strategy is proposed that may prove effective in engaging children with mathematical concepts in an enjoyable and thoughtful fashion.

## BACKGROUND

Playing games as a means of facilitating learning is not new in education. It has been studied extensively as a potential and valuable educational tool (Avedon & Sutton-Smith, 1971; Shears & Bower, 1974; Piaget, 1951, 1952). One of the primary advantages and uses of games is that they can provide intrinsically motivating learning environments (Alessi & Trollip, 2001). Motivation and context are central to learning, particularly mathematics learning (Tennyson, 1996; NCTM, 2000; Ma & Kishor, 1997; Skemp, 1986). There is suggestive evidence that games and play have more positive effect on motivation and retention of knowledge than conventional instruction (Jonnavithula & Kinshuk, 2005). However, it is important to note that motivation alone is not sufficient for effective learning.

Games are goal-oriented and usually have a number of characteristics such as rules, competition, points, and some combination of skill versus challenge. In recent times, educational computer games, often called edutainment, have been used extensively for elementary and middle school children. There are three prevalent types of such games: drills, simulations, and hypermedia (Alessi & Trollip, 2001). Most educational games are of the drill variety and are repetitive in nature. They are meant to allow children to practice and reinforce what they already know. A smaller number of educational software, simulations, has an underlying model which children explore. Although simulations provide vicarious learning experiences and can be effective for learning, they are not usually goal-oriented, and hence may not be perceived as fun and play for children. The hypermedia environments are not real games, as they often do not have rules and winning. Most hypermedia environments are resource-based activities, with many of them incorporating click-on actions which are supposed to be motivating. It is highly questionable whether these contribute positively to the learning process (Mayer, 2001).

How to design game-based interactive learning environments that engage children in deep learning of mathematical concepts is still an open research question. However, there are a number of issues that must be taken into account if we are to design such environments. These issues are discussed next.

Learning is a process that is mediated by thinking and mental engagement (Jonassen, Peck, & Wilson, 1999). Therefore, learning improves as the quality and depth of mental elaboration and engagement increases and declines as the quality of processing and engagement decreases—in other words, thoughtfulness increases learning (Hannafin & Hooper, 1993; NRC, 2000). Thinking and mental processes themselves are mediated by external notations and representations, artifacts, interactions, and activities (Beynon, Nehaniv, & Dautenhahn, 2001; Sedig & Sumner, 2006; Sedig & Liang, 2006). These mediators influence (e.g., guide, canalize, extend, and even constrain) learners' cognitive processes, and hence their learning (Sedig & Liang, 2006). As such, well-orchestrated design strategies facilitate thoughtful reasoning and improved learning (Hannafin & Hooper, 1993; Norman, 1993).

Most mathematical learning in the early years of education, and indeed later on, is through observation of and interaction with external information, either in the form of real objects, physical manipulatives, or representations of objects and concepts. Arguably, all computer-based learning environments are made of representations of information with which learners interact. There are two broad types of representations: textual and visual (Anderson, Meyer, & Olivier, 2002), also known as descriptive and depictive (Schnotz, 2002). Textual representations are lexical and language-like, and their meanings are usually conveyed through rules, conventions, and training (Tversky & Lee, 1999). Visual representations are more analogical in nature. Visual representations have more specificity, as opposed to textual notations which are semantically dense and potentially harder to interpret (Blackwell, 2002). Therefore, an important consideration in the design of mathematical environments should be the choice of the representations used

(Cuoco & Curcio, 2001; NCTM, 2000). As textual notations are dense and lack concreteness, one of the factors that contribute towards children not engaging with mathematical concepts is that such concepts are represented using textual, algebraic symbols. Hence, visual representations are becoming increasingly more important in mathematical learning (Rowhani & Sedig, 2005; Yerushalmy & Shternberg, 2001; Arcavi & Hadas, 2000; Barwise & Etchemendy, 1998; Diezmann & English, 2001; English, 1997; Healy & Hoyles, 2001; NCTM; Hitt, 2002).

One of the most important factors in the effectiveness of today's educational software is their interactiveness—that is, the provision of opportunities for learners to interact with the embedded representations of information. This adds a temporal dimension to the static representations, making them dynamic and allowing learners to more easily explore their latent meanings (Sedig & Sumner, 2006; Sedig & Liang, 2006; Yerushalmy, 2005). As the meaning of representations can be deeply encoded in them, interaction should be designed such that it allows learners to decode and elaborate this meaning. Recent research investigating the role of interaction in reasoning and learning with representations suggests that different designs engage learners in different forms of reasoning and cognitive processes and result in different qualities of learning (Sedig, Rowhani, Morey, & Liang 2003; Sedig, Schenk, & Liang, 2006). Hence, the design of interaction with the representations plays an important role in children's engagement with the mathematical concepts. There is still much to be learned about how to make representations interactive so as to facilitate thought processes conducive to meaningful learning.

Many existing educational games do not seem to take full advantage of the computer medium to engage children in deep, thoughtful learning of mathematics, as most of them assume that children are already familiar with the educational content and are more about communication of facts and engaging children in further practice. A desirable learning environment should be designed such that children can enter the environment with no or very little knowledge of the embedded mathematical concepts. The environment should then engage them with the mathematical concepts in such a way that they be able to gradually build up their knowledge and skills as they navigate through the environment, enjoying the learning process and being motivated through the game.

The next section describes Super Tangrams, a game-based learning environment whose objective is to engage children in thoughtful learning of nontrivial transformation geometry concepts in an enjoyable way.

## GAME DESCRIPTION

Super Tangrams (ST), the game-based learning environment described here, was designed and implemented to demonstrate the feasibility and effectiveness of the strategy previously described. ST is an interactive game designed to initially motivate children (grade 6+) to engage with the environment in a playful fashion and to gradually shift their attention towards nontrivial aspects of two-dimensional transformation geometry concepts through thoughtful engagement with visual representations of these concepts.

### Goals of the Game:

The primary design goals of ST are:

1. make the learning of transformation geometry fun and engaging; and
2. provide interactive visual representations to help children reason about transformation geometry concepts.

The mathematics education goals of ST are to help children:

1. appreciate that a rotation is not merely turning a shape around its center, but involves setting an angle of rotation as well as setting a center of rotation anywhere on a two-dimensional plane;
3. realize that rotating a shape both turns and translates it;
4. understand that translation is not simply sliding a shape on a plane, but moving a shape along a straight line, in a certain direction, and by a certain amount;
5. understand that a translation arrow (vector) represents the distance and direction in which an object moves;
6. explore the relationships among different transformations and their equivalences;
7. understand the effect of reflection on symmetric and asymmetric shapes;
8. develop a sense of which transformation is more effective to use in a given situation;
9. develop a sense of visualization both in terms of how to put shapes together and in terms of what transformation or combination of transformations to apply to a shape to move it to a desired position; and

10. realize that composite reflections are sufficient for performing all transformations, that is, that the effect of any transformation can be achieved by an appropriate sequence of reflections.

### Game Activity

ST is based on the Chinese Tangrams puzzle activity (Read, 1965). The tangrams activity provides a game situation and context in which transformation of the puzzle pieces takes place, and, as a result, lends itself naturally to the learning of transformation geometry (Rahim & Sawada, 1989). This activity is usually presented as a set of seven two-dimensional geometric figures (two small triangles, a medium triangle, two large triangles, a square, and a parallelogram) that must be assembled, by moving the pieces together, into a larger shape. The pieces can be cut from a single large square. Figure 1 depicts this square and its seven cut pieces. The angles in all these pieces have only three values: 45, 90, or 135 degrees. In this game, a puzzle is presented by providing an outline and seven geometric pieces to fit the outline. Figure 2 shows an example of a puzzle in ST. To solve a puzzle, all the puzzle pieces must be moved and arranged to fit into the outline. Puzzles may have from one up to more than 128 possible solutions. The version of ST reported here is comprised of a set of sequentially-presented puzzles organized in three levels of difficulties.

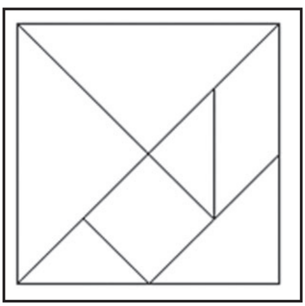


**Figure 1.** Square from which tangrams pieces are created

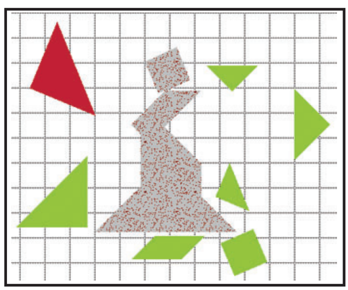


**Figure 2.** An example of a tangrams puzzle

### Interaction With Puzzle Pieces

A central feature of ST is the operationalization of the game activity. In ST, each puzzle is presented as a target outline with the geometric shapes (pieces) placed around it, as in Figure 2. The speckled area represents the outline. The dark triangle, in contrast to the lighter shapes, is the currently selected piece. Clicking on a piece selects it. Once a piece is selected, a transformation operation has to be performed on the shape to move it to a desired location—either translation, rotation, or reflection. Rather than interacting with the pieces directly, ST uses an indirect mediating mechanism, that is, formal visual representations of transformation geometry concepts. Learners interact with these representations to effect a change in the position of the pieces. In this type of interaction the locus of attention is on the visual representation of the transformation function, and learners indirectly interact with the selected piece through the intermediary of the transformation representation (Sedig & Liang, 2006). Selecting a transformation function results in the visual representation of the transformation to appear on the screen, as well as a "ghost" image of where the piece will move under the chosen transformation. Figure 3 shows a screen capture of ST in which the triangle is the selected piece, and reflection is the selected transformation function.[1] Transformation parameters are adjusted by interacting with the handles (controls) on the visual representation (labeled in the figure). One handle adjusts the angle of reflection, and the other handle adjusts the distance of the line of reflection from the piece. The line of reflection moves perpendicular to the line connecting the selected piece and its ghost. Any interaction with these handles provides immediate visual feedback in terms of the position of the line of reflection, its angle, and the position of the ghost image. Once a transformation's parameters are adjusted, clicking on the GO button (seen in the right panel) will cause the application of the transformation. An animation showing the movement of the selected piece into the position of the ghost image occurs.

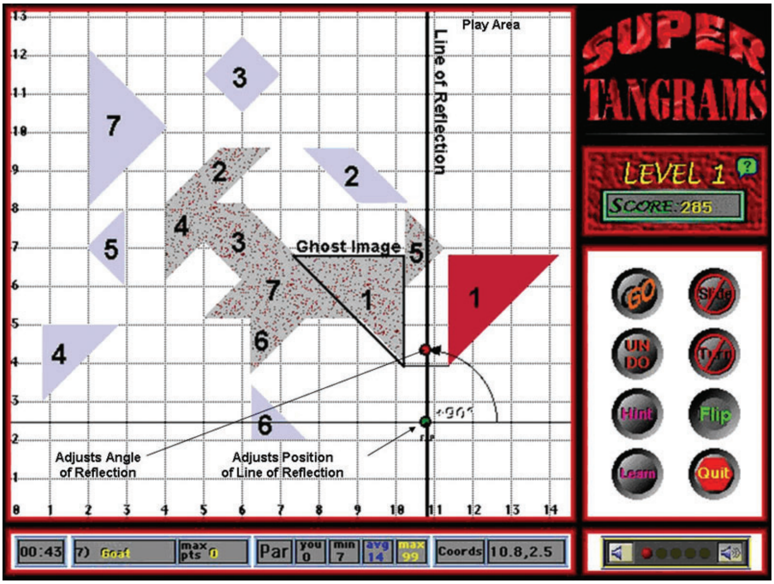



**Figure 3.** An example of how a piece can be moved to another location using the reflection function

## Design Strategies to Promote Thoughtful Learning

ST is an experiential learning environment. A design goal of ST is to engage children in a "praxis" (Kolb, 1984) of action-reflection on transformation geometry—that is, an interplay between experiential action in a game environment and reflection upon the objects of learning, namely the transformation geometry concepts. The program is designed such that this action-reflection is in a spiral fashion, with the ratio of thoughtful reasoning to action increasing constantly. As children gain expertise in how to use a transformation and gain a deeper understanding of it, they are required to think harder to be able to progress through the game. This creates a sense of challenge, motion, and accomplishment, an essential and attractive feature for children. Several strategies are used to encourage children to think deeply about their actions. These are presented next:

### Strategy 1: Scaffolding the Mediating Mechanism

A central strategy to encourage thoughtful processing of the concepts is to scaffold the visual representations that mediate the game playing process. This scaffolding process takes place at the different levels of the game. The scaffolding of the representation of rotation is presented next.

Figure 4 shows the representation of rotation in Level 1 of ST. The arc of rotation is attached to the selected piece (i.e., dark triangle). To rotate the selected shape, as in the case of translation, the learner can only interact with an arc of rotation that mediates between children's intentions and the selected piece. Both the angle of rotation and the center of rotation have to be adjusted. The angle of rotation can be changed by clicking on the handle at the tip of the arc and dragging it in a circular motion either clockwise or counter-clockwise. Meanwhile, a number is displayed next to the center depicting the value of the changing angle. The center of rotation can be dragged to any position on the Play Area. As the arc is adjusted, the position of the ghost image is dynamically updated to show the destination that would result if the current rotation were applied.

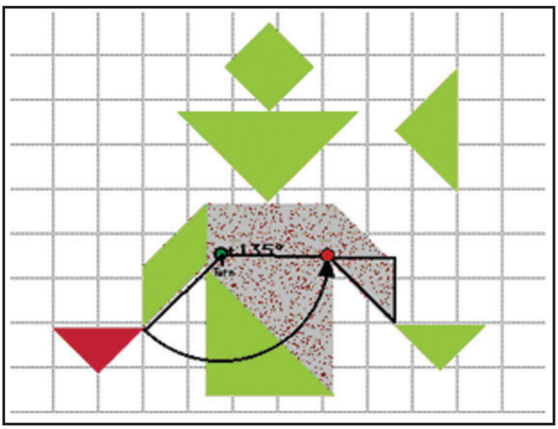



**Figure 4.** An example of how a piece can be rotated to another location in Level 1

In Level 1, children encounter this representation for the first time. If they have never seen the representation of the arc, they are not expected to grasp the significance of the angle or center of rotation and usually do not pay much attention to them or the deeper meanings of the representation.

Often, children change the angle so that the ghost image has the same orientation as the target area, and then adjust the center so that the ghost image covers that area. However, Level 1 provides children with the opportunity to experience how to express themselves through the arc of rotation, start thinking about rotation in terms of an arc, and get a sense of how the mechanism operates.

Level 1 allows children to get into the game and use transformation operations without *any prior knowledge* of the concepts. In Level 1, children are mainly focused on solving the tangrams puzzles. As they enter the environment trying to play the game, they notice that the only parts of the Play Area which are active are the control points of the representations. They quickly learn, through experience, that to do anything in the game, they need to interact with these mechanisms and know how they work.

In Level 2 the ghost image is not displayed anymore—that is, the visual scaffold is removed. In reality, the ghost image is not an inherent component of the transformation representations. In ST, the ghost image is used as a strategy to facilitate children's entry into the game. Figure 5 shows an example of a puzzle in Level 2. The target area is the speckled square at the top of the statue-like outline. As can be seen, at this Level, children cannot rely on the visual cue provided by the ghost image to know where the selected piece will move. The angle of rotation needs to be determined correctly; otherwise, moving a piece to a target area can take prolonged effort, sometimes requiring dozens of transformations to get a piece to a desired target location, leading to deducted points (see below for further description of the point system). Determining the correct angle is not an easy task for the majority of children, especially since many middle-school children have a vague notion of how to calculate such angles. Determining the angle requires mindfulness and reflective thought. Once a correct angle is determined and the arc is adjusted for that angle, children need to determine where the center of rotation should be. This is usually easier to determine than the angle. Having completed Level 1, many children would know that after a rotation the point attached to the tail of the arc would be transformed to the point at the head of the arc. This would mean that they can easily adjust the center of rotation so that the head of the arrow is positioned at the point of the target area where they want it to go. In most cases, they can do this without understanding the deeper concepts related to how the position of the center of rotation is determined. Therefore, the cognitively challenging task is setting the angle of rotation. Figure 5 shows three possible angle settings for moving the selected square to the target area. These angle settings may result in different settings for the center of rotation, as seen in the figure.

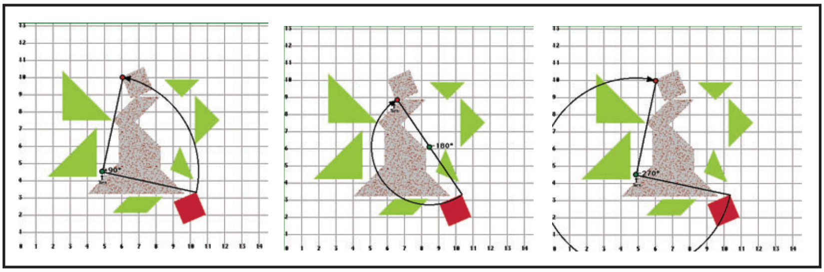


**Figure 5.** Rotating the selected square to the target area using three different angle settings in Level 2 (Left to right: +90, -180, and –270 degrees)

In Level 3, the arc is also removed and children have to learn to determine the center of rotation without it. Figure 6 shows an example of rotation in Level 3. Two pieces of information still remain: the current angle of the arc (in this case +315 degrees) and the radius of the arc. The position of the handle for adjusting the angle of rotation is moved close to the center in order not to reveal where the head of the arc of rotation is. The reader is at this point encouraged to try to determine (a) where the selected shape is supposed to move, (b) what the angle of rotation should be, and (c) where the center should be placed[2]. As can be seen, this is not trivial and requires a great deal of mathematical knowledge and reflective thought. Figure 7 shows the correct settings for the arc (-135 degrees). To facilitate the visualization of this action, in this article, the target area is enclosed as a triangular area on the outline. Additionally, the corner of the selected piece attached to the arc and the corner of the target area where it will move after the transformation are marked with black dots. To perform an accurate transformation, children need to either use external artifacts to make the calculations or visualize the arc in their mind.

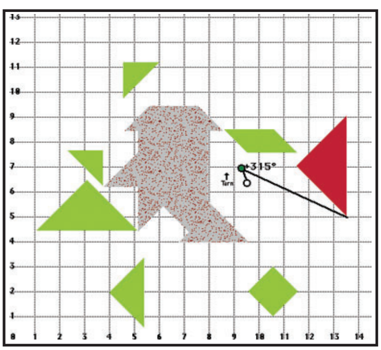



**Figure 6.** An example of the representation of the arc of rotation in Level 3

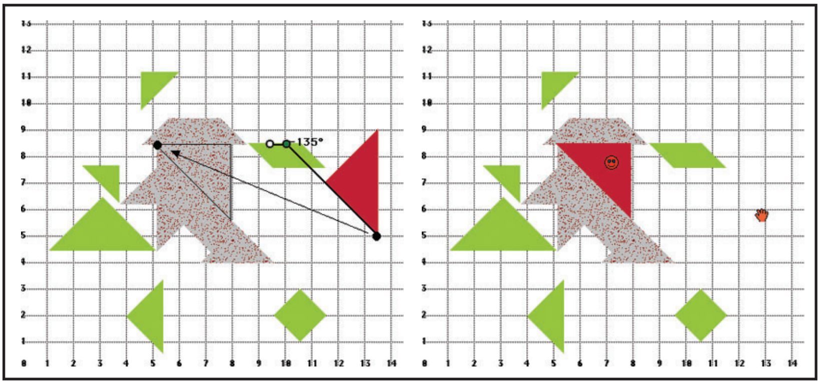



**Figure 7.** Adjusting the settings of the arc of rotation to move a selected triangle to the target area in Level 3, and the resulting successful transformation

The same principle of three scaffolded levels with reduced features and increased difficulty is also employed for the translation and reflection transformations. Figure 8 shows the scaffolded representations for all the three transformation functions. In the case of translation, in Level 2, the ghost image disappears and the focus of play shifts from the ghost image to the vector—that is, placing its tail at a corner of a shape and adjusting its head to where that corner should move. In Level 3, the tail of the vector is not interactive any more and is fixated at a coordinate location on the screen. Hence, the focus of play shifts to its relationship using the coordinate grid on the screen and how the length and direction of the vector should be calculated. In the case of reflection, in Level 2, the ghost image disappears and the focus of play shifts from the ghost image to the line attached to the selected piece which is perpendicular to the line of reflection. Based on experience from Level 1, the end of this line signifies the location to which the attached corner of the selected piece will move. In Level 3, the perpendicular line disappears. Hence, the focus of play shifts to the relationship between the positioning and angle of the line of reflection and the coordinate grid and where each corner of the selected piece will end up given the placement of the line of reflection and its angular settings.

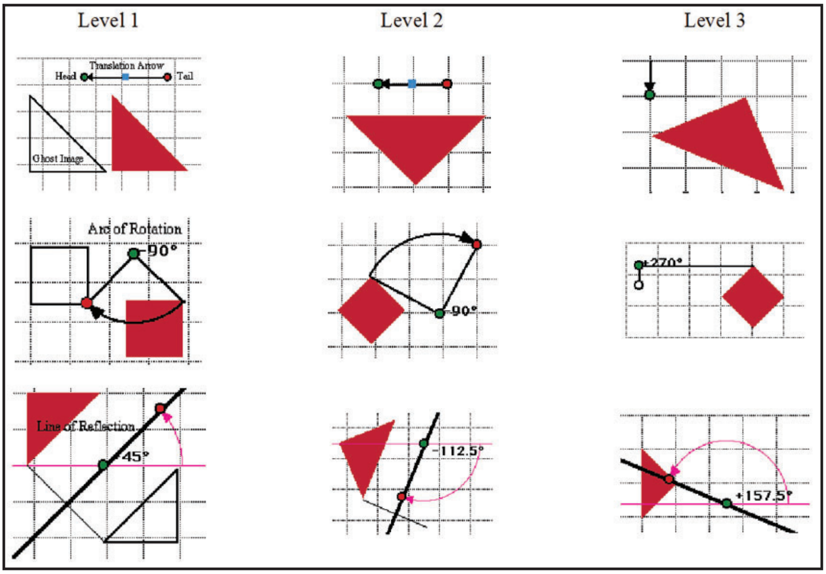



**Figure 8.** Visual scaffolds for translation, rotation, and reflection

## Strategy 2: Constraints

Another feature designed to foster deeper mathematical thinking is that children are required to solve some of the puzzles using only one of the transformations. Figure 3 is an example of a puzzle in which children can only use the reflection function. As can be seen, puzzle pieces have been placed such that, once a player figures out where they should move, they can be moved to the desired target using one reflection. A more difficult situation is when application of a single reflection is not sufficient to move each piece to the target position. For instance, at the end of Level 1, children need to solve two puzzles using only the reflection transformation. Figure 9 shows a puzzle in Level 1, called "whale," which children refer to as "killer whale," suggesting the difficulty of solving the puzzle. In this puzzle, children only have access to the reflection operation. The minimum number of operations for solving the puzzle is 15—two reflections for the six symmetric polygons and three for the asymmetric one, the parallelogram. This is quite a challenging puzzle for two main reasons: it takes a while for children to realize that any transformation can be achieved using a composite set of reflections, and children need to visualize or determine one or two intermediary target positions that would take a piece to its final destination within the outline.

Another example of a constrained puzzle is one in which children can only use rotation. In such puzzles, children need to realize that the rotation transformation not only turns a shape around its own center but also translates it.

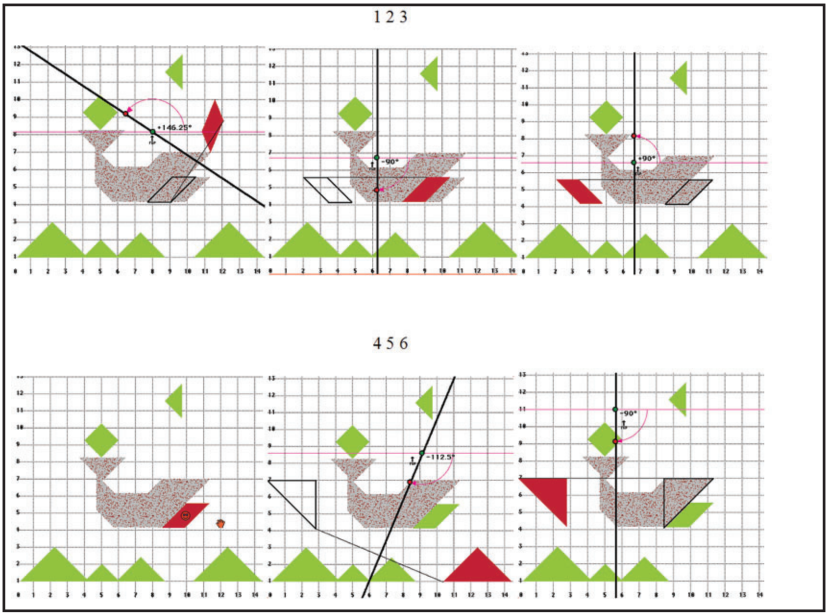



**Figure 9.** An example of a constrained puzzle and composite reflections to solve the puzzle—three reflections to move the parallelogram to the target location and two to move the triangle

### Strategy 3: Par and Scoring System

ST has a “par” and scoring system. Each puzzle can be solved using a minimum, average, or maximum number of moves (i.e., transformations), and children’s scores depend on the number of moves they make; this feature is meant to encourage a reasoned and thoughtful playing strategy for solving the puzzles, rather than one based purely on trial and error.

## IMPLEMENTATION ISSUES

### Making the Visual Representations Interactive

The most difficult and crucial part in the implementation of ST, was the operationalization of the mediating mechanism—that is, how the visual representations of the transformation concepts should work, both to support thoughtful learning as well as game play. A process of incremental implementation, testing, observation, analysis, and modification of the design was required to discover the fine-grain details of how the interface should work. Through a series of pilot studies it became clear that very small changes in the interface could affect how children used the system, resulting in cognitive and motivational problems. The smallest details of the interface not only could hinder and constrain children's understanding of the concepts, but could make them frustrated and lead to disliking the software. To illustrate, consider Figure 10, which shows the ST representation for the easiest transformation, translation. The dark triangle is the piece to be moved, the speckled area is the outline, and the transparent triangle, the ghost, shows the destination that would result if the current translation vector were applied. The vector has three sensitive handles (controls) at the tail, midpoint, and head. The length and direction of the vector can be changed by moving (dragging) either the head or the tail, and these actions move the ghost image. The entire vector can be moved by dragging the midpoint, but this does not change the position of the ghost image.

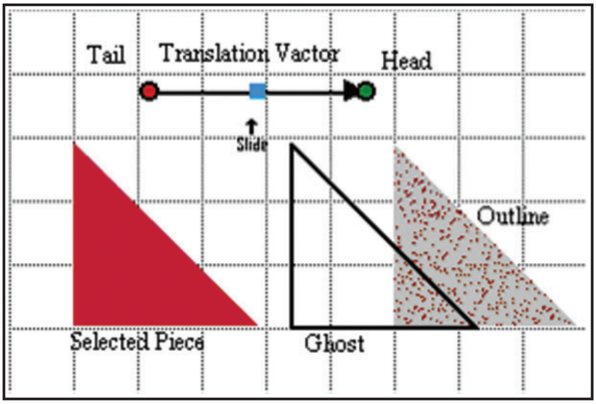



**Figure 10.** An example of how a piece can be transformed to another location

There are many possible ways of designing the interaction with this transformation (Sedig & Sumner, 2006; Sedig & Liang, 2006); three are presented here.

The easiest method for children would be to select a shape, select the slide transformation, and then click and drag out the translation arrow (matching the ghost image to the target area). Translation would happen when the arrow were released. While this is effective in enabling children to move the piece, in practice it can be used with very little mathematical thought or reasoning. A child would click on, say, the centre of the piece, drag to the centre of the target (watching the ghost image) and release. This is fast, easy to learn, and intuitive, but children would merely be focusing on the activity and would not learn about the properties of the translation arrow (i.e., the independence of length and direction of translation from the location of the arrow, what happens if the tail is dragged, the relationship between all the points of the selected shape and the ghost image, and so on).

Another method would be to select a shape and select the slide transformation; then clicking in the transformation area would cause a random translation vector to appear at the mouse location. The learner could then adjust this vector, and, after adjustment, click on a GO button to make the translation happen. The transformation area can remain active so that a click off of the arrow would cause another (random) arrow to appear at the mouse location. This option would make the learner attend to the translation vector, but it causes other problems. Pilot studies showed that children had an intuitive notion that translation was the same as dragging and were not consciously aware that translation implied movement along a straight line. They would click on the selected shape and drag. The arrow appeared but did not change, and the piece did not move. In trials, one student said it felt as if there was a "brake" on the mouse. If they tried clicking at another location, the same thing happened, only this time the arrow was placed somewhere else on the screen. Finally, they would get frustrated and complained why ST was not similar to other programs in which one simply selected and dragged an object.

An underlying source of some of these problems is that children may come to a learning environment expecting objects to behave the same as other noneducational tools that they have used. Children expect to have easy entry into the learning environment, and have little patience for accessing "help." Therefore, any solution to this problem must feel easy and natural to them, while maintaining the integrity of the learning process.

The difficulty here is that children focus on moving the piece rather than creating a translation vector. They need to shift from a physical model of movement to a logical or representational model (represented by the vector). Since the educational objective of the game is engaging children with the transformation concepts, the interaction had to be designed to attract children's attention to the representation rather than the shape (Sedig, Klawe, & Westrom, 2001), hence the mediating mechanism.

In the final implementation, children select a shape and choose the slide transformation. At this point a randomly placed translation arrow appears on the screen, along with the ghost image of the selected shape. Initially, the head, tail, and midpoint of the arrow are flashing to attract attention to the arrow. The only points inside the Play Area that are mouse-sensitive are the three handles on the arrow, implicitly drawing children's attention to the mediating mechanism. Once the arrow is adjusted, children click the GO button. At first use, it normally takes students from 10 seconds to a minute to learn how to control the arrow, but they immediately realize that the arrow must be the focus of their attention. It is this shift in the focus of interaction which results in a more focused exploration of the meaning of translation.

This same process was used to implement the rotation and reflection transformations. In both these cases, the only mouse-sensitive portions of the screen are the control points on the visual representations of rotation and reflection.

## Embellishing the Game

An important aspect of the implementation of ST involved adding features that would make the game more fun for children. Most children like sharp and bright colors, so colors and patterns were used to create a sense of motion and change as well as beauty. As a player moves from one puzzle to the next, the colors and patterns of puzzle pieces change. Each level employs different colors to give a different sense to the activity. Level 3, for example, uses fractal patterns and has a mystical feel. Different puzzles have different music playing in the background. Extrinsic encouraging comments such as "Well done!" with graphics and cartoons are presented each time a puzzle is completed. Comments of strong praise like "You are a genius!" result only when puzzles are completed with minimum number of transformations.

## EVALUATION METHODOLOGY

The design strategy proposed in this article is to engage children with mathematical representations using a game as the learning context. This game should be designed in such a way that it can initially motivate children to engage with the environment in a playful fashion and then the design promotes a gradual shift of the children's attention towards the representations of mathematical concepts. These representations are intended to mediate the game-playing process, as a result of which children are motivated to learn mathematics in a thoughtful fashion.

This section presents the methodology that was used to evaluate whether the design strategy proposed in this article and implemented through a game called Super Tangrams was effective in helping children learn nontrivial mathematical concepts in a deep and thoughtful fashion. A muti-method research design was used. ST was evaluated quantitatively and qualitatively to determine its effectiveness—both in terms of learning as well as motivation. The qualitative portion of this phase was intended to capture children's voices, to provide explanations for some of the "hows" and "whys" of the experimental findings, and to cross-validate the quantitative findings.

### Subjects

The subjects who participated in the evaluation of ST all came from an upper-middle-class school in Canada. This school was selected because of the largeness of the students in grade 6, and because of the willingness of all grade-6 teachers to allow their students to take part in this study. Three whole classes participated in the study, comprised of 58 grade-6 students. Two of the classes had mixed-grade students: grades 5/6 and 6/7. Each class had 15 grade-6 students. All grade-6 students participated in the study. The third class was comprised of 29 grade-6 students. All participated in the study. None of these students had seen, heard of, or used ST before.

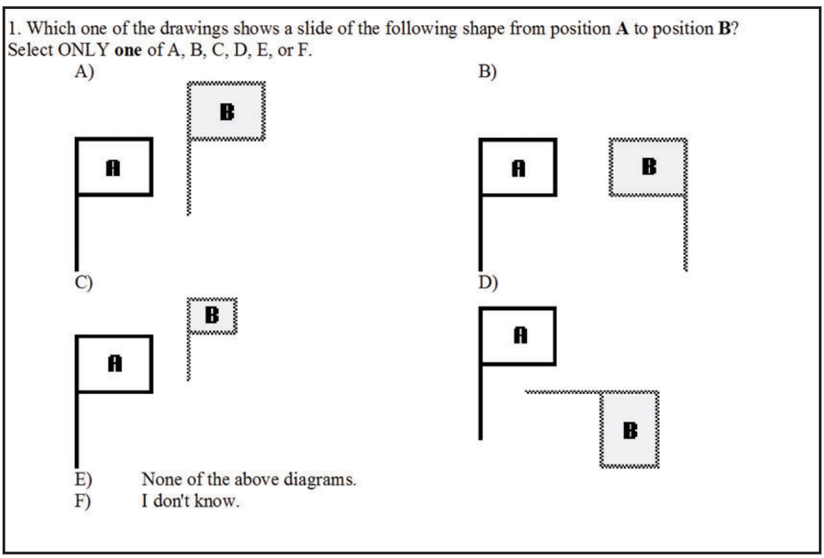
To gauge the stability of the test (described later) that would be administered, and to determine if a repetition of measures (pre and posttest) would influence children's knowledge of the mathematical concepts, a control group was included. Since the first school did not have any more grade-6 students, the control group came from another school, most of whose students were from diverse national backgrounds. Once again, the whole class, whose size was 20, participated in the study.

## Sources of Data

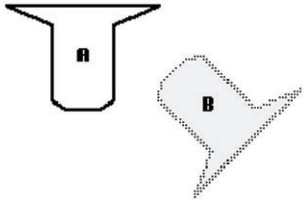
Two sources of data were used to study the effectiveness of ST and its design: an achievement test and a design questionnaire.

## Achievement Test (AT)

A paper-and-pencil transformation geometry test was constructed to (a) provide a measure of students' overall understanding of transformation geometry, and (b) to provide measures of students' finer-grain understanding of some of the specific concepts, such as translation, rotation, reflection, and composite transformations. AT was more difficult than government-administered standardized tests, as it was needed to find out whether ST would help students tackle more difficult questions. AT contained 51 questions with varying degrees of difficulties. The questions were conceptually divided among four categories: (a) translation, (b) rotation and angle, (c) reflection, and (d) composite transformations. Three sample AT questions with different degrees of difficulty are presented in Figure 11, question one being easy, two being intermediate, and three being difficult relative to the age group participating in the study.



2. I want to move this shape from position **A** to position **B** as shown here.




A) Can I do this by turning it 45 degrees clockwise? Yes No I don't know ☐☐☐

B) Can I do this by turning it 135 degrees clockwise? Yes No I don't know ☐☐☐

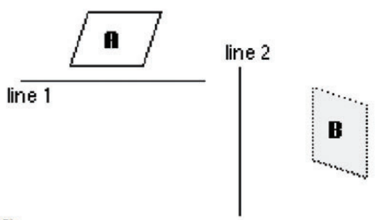
C) Can I do this by turning it 225 degrees counter-clockwise? Yes No I don't know ☐☐☐

D) Can I do this by turning it 135 degrees counter-clockwise? Yes No I don't know ☐☐☐

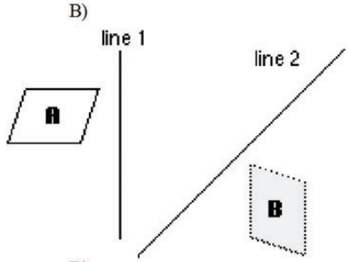
E) None of the above.
F) I don't know.

3. In the drawings below, which one of the combination of flips correctly moves the shape from position **A** to position **B**? (Line 1 is the first flip line, and line 2 is the second flip line.)
Select ONLY **one** of A, B, C, D, E, or F.

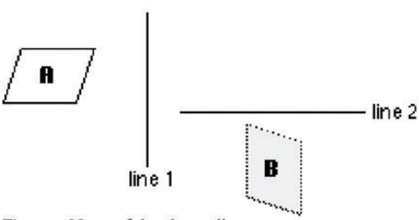
A)



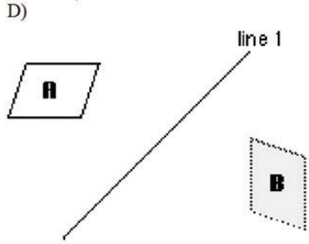

B)




C)



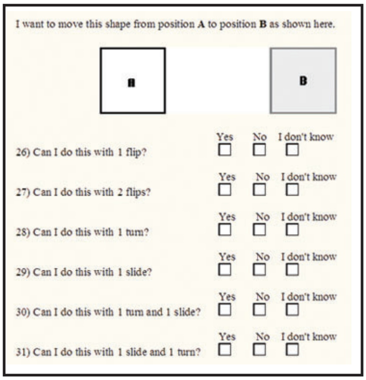

D)




E) None of the above diagrams.
F) I don't know.

**Figure 11.** Sample AT questions

Some sets of questions in AT were grouped together to provide indicators of students' understanding of the relationships among and equivalences between the different transformations. The following (Figure 12) is an example of a grouped set of questions. The logic for grouping questions was that a correct answer to *all* of them would provide a better indicator of students' understanding of the relationship among the different transformations.


I want to move this shape from position **A** to position **B** as shown here.

A B

26) Can I do this with 1 flip? Yes No I don't know

27) Can I do this with 2 flips? Yes No I don't know

28) Can I do this with 1 turn? Yes No I don't know

29) Can I do this with 1 slide? Yes No I don't know

30) Can I do this with 1 turn and 1 slide? Yes No I don't know

31) Can I do this with 1 slide and 1 turn? Yes No I don't know

**Figure 12.** Example of a grouped set of questions

To assess AT's reliability, a coefficient of stability was calculated by applying the Pearson product-moment correlation coefficient to the pretest and posttest scores of the control group. AT obtained a high reliability coefficient, with $r = 0.88$.

### Design Questionnaire (DQ)

DQ consisted of Likert scale questions to assess children's responses towards the learning environment and their perception of it. Many scaled questions ended with an open-ended prompt, "Explain Why," to add more qualitative depth to children's quantitative responses. A sample question is:
Compared to other ways of learning math, how much fun was learning math through Super Tangrams?
A) much more fun than other ways
B) somewhat more fun than other ways
C) just as fun as other ways
D) somewhat less fun than other ways
E) much less fun than other ways
Explain why:

## Design

A quasi-experimental nonequivalent pretest-posttest group design was chosen, which is prevalent in this type of research. Intact classes of students were used as the groups. Intact classes were used to make it easier for teachers to send a whole class during allocated hours to participate in the study, hence not disrupting their school schedule. As such, the design consisted of four groups of students: 29, 15, 15, and 20. Each group was given a different treatment. One treatment consisted in using ST, along with adult mediation. This would be a typical treatment when educational software is used at schools. Another treatment consisted of using ST, as previously described, without any adult mediation. Another treatment consisted of using ST, without its embellishments and without any adult mediation. This was to see whether the design strategy was effective even without embellishments. Finally, the control group did not use ST. The purpose of the first three treatments was to investigate whether the design strategy is effective under the three differing conditions.

*G1*. *N*=29. Students used ST, and there was an adult available to answer questions and help them with the game. Adult mediation was provided in case children would find the game too challenging.

*G2*. *N*=15. Students used ST on their own without any adult mediation.

*G3*. *N*=15. Students had a reduced version of ST without embellishments such as background music, colorful patterns, scoring, and sound effects, and they used the program without any adult mediation.

*G4*. *N*=20. This formed the control group. Students did not use ST, but did write the pre and post tests.

Hence, the design and order of the study was as follows, where AT was used as pre and posttest:

*G1*: AT, using ST + mediation, AT, filling out DQ
*G2*: AT, using ST, AT, filling out DQ
*G3*: AT, using ST – embellishments, AT, filling out DQ
*G4*: AT, no treatment during the period in which other groups used ST, AT

### Setting

This study was conducted in a small room at the school. Eight computers were arranged on three long tables, with two or three computers on each table. No more than 16 students were in the computer room at any given time. All sessions of the study for groups G1, G2, and G3 were conducted in this room. Since group G1 was large, it was divided into two smaller groups of 15 and 14 so they would fit in the room.

### Procedure

Permission was obtained from the principal and the teachers to conduct this study at their school. Additionally, students were given forms describing the research project and the software. Students were told that they were not obliged to participate in the investigation, and that they could decide to drop out of the study any time that they felt they did not want to participate in it anymore. Once consent forms were obtained from the students, all participating teachers (including that of the control group) agreed not to teach any topics related to geometry or transformation geometry for the duration of the study. Students and teachers were told that different groups would receive different treatments. To prevent possible biases toward the versions, neither teachers nor students were informed about how their treatments differed.

A researcher visited each classroom and, in the presence of the teacher, asked the subjects to write a pretest, that is, AT, giving them 40 minutes to complete the task. Subjects were told that the results of the test would not affect their mathematics grade at school. Subjects were explicitly asked not to guess answers to questions and were told that the purpose of the study was to assist researchers understand how to design better educational software.

After the pretests, groups G1, G2, and G3 spent 10 sessions, approximately 40 minutes in length, using the software in pairs, where subjects were paired by the teachers according to any criteria they saw fit. These sessions were held during school hours when children were scheduled to study mathematics. Subjects would leave their classes and come to the room in which the computers and the software were set up. Each pair of students was assigned a computer to work on. At the beginning of the first session, a researcher gave a brief overview of the goal and rules of the program and how to operate it. During the subsequent sessions, since the program kept a log of the last puzzle the subjects had solved, subjects would continue from the last puzzle in the game that they had solved. During all sessions, a researcher was always present in the room, but no teachers were present. Subjects in group G1 who needed help would raise their hand to receive assistance; the researcher would answer procedural questions related to the operation of the program, but would not provide direct instruction on test-related content. Subjects in groups G2 and G3, however, had to figure things out on their own and were advised that answers to their questions could be found in the program. Group G4 did not know anything about ST and simply continued with their normal school work during the course of the study.

Immediately after the 10 treatment sessions, all groups completed AT again, this time as a posttest. Once again, this test was administered in their classroom in the presence of their teachers. Additionally, groups G1, G2, and G3 completed DQ.

## RESULTS

The purpose of this section is to analyze and report the results of the evaluative phase of this research. The main goal of this section is not to demonstrate whether ST is a good game or not. Rather, the purpose is to determine: (a) whether it is possible for children to learn nontrivial mathematics in the context of game-based learning environments, (b) whether the design strategy that was used in ST makes this learning possible, and (c) whether children enjoy learning mathematics in such contexts. As such, the results are presented in two subsections: achievement results and motivation results.

### Achievement Results

This subsection is divided into: analysis of the subjects' overall achievements, analysis of the subjects' answers to the grouped questions, and, finally, the subjects' perception of their own learning.

### Overall Achievement Results

Statistical data analysis was performed to show that each one of the three treatment groups (mediated, without mediation, and without mediation and embellishments) exhibited improvement after interaction with the learning environment. The statistical method used to establish improvements from pre to posttest scores are paired samples *t*-tests.

Before assessing the effectiveness of the learning environment, a paired samples *t*-test (two-tailed) was used to compare the pre and posttest scores for the control group. This was to determine if repeating AT influenced students' knowledge of transformation geometry on the test. No statistically significant differences were found ($t$ (19) = 0.62; $p > 0.05$). Since the control group sample came from another school, to avoid confounding effects the control group was not used in comparisons with the other treatment conditions.

At a descriptive level, Table 1 shows the pre and posttest AT scores for the three treatment groups G1, G2, and G3. As can be seen, mean pretest scores were at about the same level for groups G1 and G2, and a little higher for group G3. Different teachers may account for the small differences on the pretest. The results are approximately equal for the posttest scores. All three groups show improvement from the pretest scores. The results indicate that whereas the distribution of pretest scores were relatively normal (i.e., mean ≈ median), the distribution of posttest scores for groups G1 and G2 were negatively skewed (i.e., mean < median); that is, in these groups, a majority of the students showed superior knowledge rather than a few students improving dramatically.

**Table 1**

Measures of Central Tendency and Variability by Group for Pre-Posttest and Pre-Posttest Change Scores

| | | Tests (pre-post change) | Measures | | | |
|---|---|---|---|---|---|---|
| | | | Central Tendency | | Variability | |
| Group | N | | Mean% | Median% | SD | Range |
| | | pretest | 21.8 | 22.6 | 13.5 | 0.0 - 57.1 |
| G1 | 29 | posttest | 75.6 | 85.8 | 19.5 | 40.7 - 100.0 |
| | | (Δ) | 53.8 | 63.2 | 19.2 | 19.8 - 91.0 |
| | | pretest | 25.0 | 25.2 | 15.0 | 1.1 - 59.8 |
| G2 | 15 | posttest | 76.1 | 86.2 | 20.6 | 41.2 - 100.0 |
| | | (Δ) | 51.1 | 61.0 | 23.1 | 14.2 - 89.6 |
| | | pretest | 37.1 | 37.9 | 14.7 | 7.9 - 70.5 |
| G3 | 15 | posttest | 74.9 | 71.2 | 14.6 | 54.8 - 96.0 |
| | | (Δ) | 37.8 | 33.3 | 17.5 | 2.4 - 70.2 |

At an inferential level, paired sampled *t*-tests (two-tailed) were performed. All three groups exhibited significant improvement in their achievements between pre and posttest scores ($t_{G1}(28) = 15.08$, $p$=0.0001; $t_{G2}(14)$=8.56, $p$=0.0001; $t_{G3}(14)$=8.36, $p$=0.0001).

### Analysis of Grouped Questions

In addition to that analysis, frequency counts of the number of subjects who answered individual pre and posttest questions revealed more fine-grained patterns in the data that support the overall evaluation of the of the children's learning. For instance, Table 2 shows an analysis of the answers to the grouped questions presented in the previous section. If a child answered all questions in a grouped set correctly, he/she received a score of 1. If any of the answers to any of the questions were wrong, the child received a score of 0. The results show that of 59 children, no one managed to answer the set of grouped questions correctly on the pretest. However, there is marked improvement on the posttest. This pattern was repeated for all other sets of grouped questions.

**Table 2**

Children Getting all the Answers to a Set of Grouped Questions Correct

| Group | *N* | Pretest | Posttest | (Δ/n)% |
| --- | --- | --- | --- | --- |
| G1 | 29 | 0 | 17 | 59 |
| G2 | 15 | 0 | 12 | 80 |
| G3 | 15 | 0 | 6 | 40 |

### Subjects' Perception of Their Learning

Children' responses to questions in DQ dealing with their perception of how much they thought they had learned after playing the game were analyzed. Table 3 displays the means for individual questions, and an overall mean for the questions shown in the table. Responses were rated on a 5-point Likert scale in which "A" responses on the scale were assigned a value of 5, and "E" responses were assigned a value of 1.

**Table 3**

Different Groups' Perception of Their Learning After Playing Super Tangrams

| | *Question Number (see below)* | | | | |
| --- | --- | --- | --- | --- | --- |
| **Group** | **1** | **2** | **3** | **4** | **Mean[1]** |
| G1 | 4.6 | 4.4 | 4.5 | 4.6 | 4.5 |
| G2 | 4.6 | 4.3 | 4.2 | 4.1 | 4.2 |
| G3 | 3.8 | 4.2 | 4.1 | 4.0 | 4.1 |

**1.** How new were the math concepts in Super Tangrams for you?
A) everything was new to me; B) most things were new and some things were review; C) half was new and half was review; D) some things were new and most things were review; E) nothing was new to me.

**2.** Compared to what you knew about **turn** before playing Super Tangrams, how much have you learned about turn now that you have played the game?
A) I have learned so much that I can't believe it; B) I have learned quite a bit; C) I have learned some; D) I have learned very little; E) I have not learned anything at all.

**3.** Compared to what you knew about **flip** before playing Super Tangrams, how much have you learned about flip now that you have played the game?
A) I have learned so much that I can't believe it; B) I have learned quite a bit; C) I have learned some; D) I have learned very little; E) I have not learned anything at all.

**4.** Compared to what you knew about **slide** before playing Super Tangrams, how much have you learned about slide now that you have played the game?
A) I have learned so much that I can't believe it; B) I have learned quite a bit; C) I have learned some; D) I have learned very little; E) I have not learned anything at all.

[1] Mean score for Questions 2, 3, and 4.

Mean scores show that children perceived that they learned "quite a bit" (all mean scores are greater than 4). It is noteworthy that the children's perception of their own learning coincides with their achievement results.

## Motivation Results

This subsection analyzes the data to determine whether the children felt motivated to learn mathematics in the context in which it was presented to them. Nine questions from DQ were used to construct a motivation index. Table 4 displays the means for each individual question as well as an overall, mean motivation index for each group. Responses were rated on a 5-point scale.

**Table 4**
Indices of the Different Groups' Overall Motivation Response

| | *Question Number* | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| ***Group*** | ***1*** | ***2*** | ***3*** | ***4*** | ***5*** | ***6*** | ***7*** | ***8*** | ***9*** | ***Mean Motivation Index*** |
| G1 | 4.8 | 4.5 | 4.6 | 4.6 | 4.4 | 4.7 | 4.7 | 4.8 | 4.4 | 4.6 |
| G2 | 4.5 | 4.3 | 4.1 | 4.3 | 4.0 | 4.5 | 4.3 | 4.3 | 4.2 | 4.3 |
| G3 | 4.6 | 4.1 | 4.1 | 3.9 | 4.6 | 4.6 | 3.4 | N/A | N/A | 4.2 |

[1] I like to learn math from computer games like Super Tangrams.
A) strongly agree, B) agree, C) undecided, D) disagree, E) strongly disagree.

[2] I like the way motion geometry was presented in Super Tangrams.
A) strongly agree, B) agree, C) undecided, D) disagree, E) strongly disagree.

[3] Compared to other educational math programs you have used, what do you think about Super Tangrams?
A) it is the best, B) it is better than most, C) it is neither better nor worse, D) it is worse than most, E) it is the worst.

[4] Compared to other educational games you have played, how much did you like playing Super Tangrams?
A) loved it, B) liked it, C) so-so, D) disliked it, E) hated it.

[5] How much has Super Tangrams helped you like motion geometry?
A) has made me love motion geometry, B) has made me like motion geometry, C) hasn't done anything for me, D) has made me dislike motion geometry, E) has made me hate motion geometry.

[6] Compared to other ways of learning math, how much fun was learning math through Super Tangrams?
A) much more fun than other ways, B) somewhat more fun than other ways, C) just as fun as other ways, D) somewhat less fun than other ways, E) much less fun than other ways.

[7] How much did you like the colors and patterns of the puzzle pieces in Super Tangrams?
A) loved it, B) liked it, C) so-so, D) disliked it, E) hated it.

[8] How much did you like having background music in Super Tangrams?
A) loved it, B) liked it, C) so-so, D) disliked it, E) hated it.

[9] How much did you like having sound effects in Super Tangrams?
A) loved it, B) liked it, C) so-so, D) disliked it, E) hated it.

The mean motivation indices (4.6, 4.3, and 4.2 out of 5) suggest a strong positive response towards the learning context. Table 5 displays mean indices for each group, as well as each group's perception of how much mathematics they thought was involved in interacting with ST.

**Table 5**
Perception of How Much Mathematics Was Involved in ST by Group

| *Group* | *Question Number* *1* | *Mean Motivation Index* |
|---|---|---|
| G1 | 4.4 | 4.6 |
| G2 | 4.2 | 4.3 |
| G3 | 4.3 | 4.2 |

[1] I though that Super Tangrams was full of math.
A) strongly agree; B) agree; C) undecided; D) disagree; E) strongly disagree.

The indices suggest that the children perceived ST to be "full of math." These results suggest that being perceived as "full of math" did not have a negative motivational effect on the children.

Most of the questions in Table 6 terminated with open-ended questions that asked the children to "explain why" they had responded to the questions as they did. Table 8 presents a sample of the written comments of the children. To prepare this table, all children's responses to DQ were collected, categorized according to their stem questions, and read in their entirety. Within each category, children's responses were further classified based on their commonality (e.g., satisfaction from learning, enjoyment of music, and comparison with textbooks). All the reported comments in the table are verbatim.

**Table 6**

Written Comments of Children From Different Groups

| Sex | Comment |
|---|---|
| M | It's so fun! Why do boring Math!? |
| F | The presentation of Super Tangrams was good because every thing was easy at first and then it got harder. |
| M | You could learn or/and master motion geometry and you could learn a lot, and also it is fun. |
| F | There was music and every thing was different. |
| F | It was much easier to picture flips, turns, or slides especially since I was learning as I went along. |
| M | It taught me so much. |
| F | It got challenging, good graphics, rad music, was not confusing, and I learned alot. |
| M | Most [other games] don't really focus on the concept or the skill it's trying to teach. |
| F | It taught me lots and was fun. |
| M | You get to show your ability to do math. |
| F | I don't know but it just make me feel I love it. |
| M | It presents the fun side of math. |
| F | The way it is made, makes it fun and interesting. |
| M | It showed it [motion geometry] clearly, and it was easy to learn from. It showed how they moved. |
| F | They gradually made it harder so that when you begun you could learn. If they always did a ghost image, it would be too easy and if you always did it like on level 3 it would be too hard. |
| M | [I learned most in Level 3 because] you have to figure everything out for yourself. |
| F | Super Tangrams is fun and it teaches you in small steps and is better than book work. |

**Table 6 Continued**

| | |
|---|---|
| F | [In Level 3] you have to figure everything out for yourselve. |
| M | It really made you think. it seems to work your way up to a challenge. |
| F | I don't know. I just enjoyed it. |
| F | I first thought it was boring and hard but once I played it, it became fun. |
| M | It's more fun playing game of math then to write pages of questions. |

Finally, Table 7 lists the comments and responses of one of the weakest students in G1. (According to her teacher, this student had Attention Deficit Disorder). This student's pretest score was 1.1% (nearly one and a half standard deviations below the mean score of her group), and her posttest score was 49.6% (one and a third standard deviations below the mean for her group).

**Table 7**

A Weak Student's Perception of Super Tangrams

| *Stem Question* | *Student Response* |
|---|---|
| I [strongly agree] that I like to learn math from computer games like Super Tangrams | because it Makes Math More interesting. |
| I [agree] that I liked the way motion geometry was presented in Super Tangrams | I found it hard to understand but it was fun. |
| Compared to other educational math programs I have used, I think Super Tangrams is [the best] | because it's Much Moor interesting. |
| Compared to other educational games I have played, I [liked] Super Tangrams because | it was hard to understand but fun. |
| Super Tangrams has helped me like motion geometry because | Motion geomitry is fun. |
| Compared to other ways of learning math, Super Tangrams was [much more fun] because | it had interesting ways of teaching. |
| I [liked] Level 1 | it was the easiest. |
| I learned [most] in Level 3 | it was a challenge becase there was No goset imeges. |

## SUMMARY AND CONCLUSIONS

Many children do not enjoy mathematics and are not motivated to learn mathematics. One of the reasons for this negativity is their initial encounters with the subject and how it is presented to them. Learning mathematical topics requires a number of conditions to be present, among which are the following: (a) the topic needs to be situated in an appropriate activity or context, (b) the activity should motivate children to learn mathematics, (c) the activity should actively engage children with the topic, (d) the activity should engage children with the topic in a thoughtful manner, promoting mindful learning of the embedded concepts. Game-based learning environments have the promise and potential to facilitate the above conditions. However, promotion and facilitation of these conditions depends on the design strategy of the games.

The purpose of this research has been to investigate whether and how games can be designed to help children learn mathematics in as enjoyable and motivating way. To address this problem, a design strategy has been proposed. The strategy is to engage children with mathematical representations using a game as the learning context. This game is designed in such a way that it can initially motivate children to engage with the environment in a playful fashion and then the design promotes a gradual shift of the children's attention towards the representations of mathematical concepts. These representations are intended to mediate the game-playing process, as a result of which children are motivated to learn mathematics in a thoughtful fashion. To demonstrate the feasibility and effectiveness of the strategy, a game-based learning environment, Super Tangrams, was designed, implemented, and evaluated. This article has described ST. This game takes children from almost no knowledge of transformation geometry to some nontrivial knowledge involving composite reflections and complex rotations.

Three treatments were used to study the strategy: using the game as described in a previous section, using the game without embellishments, and using the game with adult mediation. A multi-method empirical study was conducted to evaluate whether games, such as ST whose design incorporates this strategy, are effective at engaging children with mathematical topics, and consequently helping them learn some nontrivial mathematical concepts related to transformation geometry. The results of the study indicate that, under all three conditions, children exhibited significant improvement in their knowledge of transformation geometry concepts. Furthermore, the results suggest that, despite the explicitness and difficulty of the mathematical concepts involved, children found the learning process to be fun and enjoyable.

This study and, subsequently, its results, have certain limitations. First, the study explored only one implementation of the strategy, namely ST. Second, the mathematical concepts were only of a specific kind, namely transformation geometry. Third, the subjects worked in pairs, which may have influenced how they engaged with the game. Fourth, although the subjects who used ST all came from the same school, they came from three different classes and with different teachers. Despite these limitations, the research presented in this article, some general conclusions can be drawn from it; furthermore, it has implications for the design of effective learning environments for children.

Some immediate, broad conclusions include: (a) game-based learning environments can be designed such that they help children learn nontrivial mathematical concepts and positively contribute towards their motivation and enjoyment of learning mathematics; (b) games can be designed in such a way as to take children from little or no knowledge of mathematical concepts to higher degrees of knowledge. This type of game is different from the ones in which children's skills are simply sharpened and reinforced through repetition and practice; (c) design plays a central role in how children engage with the mathematical concepts; (d) if a learning environment be designed properly, even though the embedded concepts may be beyond what children do at school, there may be no need for adult mediation; this is an important point, as some children may like to work independently and test their own abilities; (e) embellishments, such as sound effects and color, even though they add to the richness of an environment and help children like the game, play a secondary role and are not as important as the operationalization of the activity and how children are engaged with the game; and f) a game can be designed to require children to "really" think hard and be "full of math," but still be regarded as "fun" by children, without a negative effect on their attitude towards the environment.

A specific conclusion that can be drawn from this study, and one that requires further research, is how a game should be designed to invite children into a learning situation. Super Tangrams implements the strategy as follows. When children initially enter the environment, Super Tangrams presents itself as a fun game, requiring no knowledge of transformation geometry. However, to play the game, children need to interact with the visual representations of the transformation concepts. That is, the representations mediate between the children and the game. At this stage, the representations are transparent, and children do not need to be thoughtful and conscious of their presence. In other words, the children have no need of understanding the mediating mechanism in order to play the game—playing the game being the children's primary interest and concern. As children progress through the game, the interface is designed such that children must become more conscious and thoughtful of the representations in order to be successful at the game and solve the puzzles. In other words, the mediating mechanism between the children and the game becomes less transparent, requiring attention and elaboration. By Level 3 of the game, the activity is receded to the background and is not the primary focus of the attention of the children. Instead, what has become prominent and has come to the foreground is a mental game with the representations. That is, expression of action requires more thinking about the mediating mechanism than the game itself, which mechanism embodies the mathematical representations of the concepts to be learned.

As was stated in the Background section, much of mathematics learning involves making sense of and understanding the representations of mathematical concepts. Therefore, an important research question that emerges from this study is the following: How can scaffolded representations of mathematical concepts be incorporated in game-based learning environments in such a way that children start with play and end up with conscious, thoughtful engagement with the representations? This research has been a step in addressing this question with regard to how such a strategy can be operationalized for transformation geometry concepts. Future research needs to focus on: (a) operationalizing the strategy by designing, implementing, and evaluating other games that engage children with mathematical concepts, and (b) figuring out how to effectively incorporate mathematical representations in these games such that the representations mediate between game-playing and learning.

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## Notes

[1] For the purposes of this article, puzzle pieces and where they need to be placed in the outline have been numbered from 1 through 7.

[2] Hint: bisection of angles will help.