QUIZ 3 WARM-UP: ANOVA TABLES

Consider the following graph of length versus age for mussels found in two locations. Location is indicated by color.

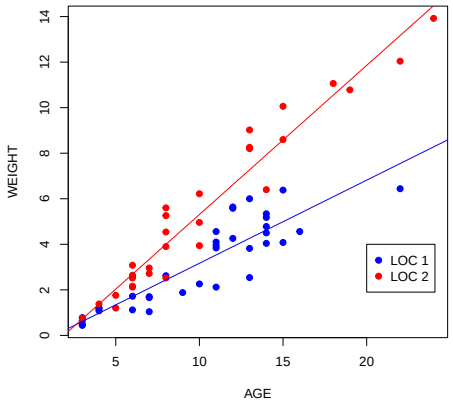

FIGURE 1. Weight vs. age.

In the table below, the each row (except the last) in the column marked "Sum Sq" gives the difference in residual sum-of-squares between two models. For each row in the table, answer the following:

- What are the two models being compared?
- What is the hypothesis being tested?
- What is the test statistic? Compute it, and state your conclusion about the null hypothesis.

Overall, what do conclude about the data?

	Df	Sum Sq
AGE	1	428.11
LOCATION	1	75.57
AGE:LOCATION	1	32.77
Residuals	61	48.25

Discussion. Let $\boldsymbol{Y}$ be the weight, $\boldsymbol{x}$ be the age, and $\boldsymbol{w}$ be the location. Let $\delta_{i,2}$ be the indicator that observation i is in location 2. We have the following sequence of

models:

$$\begin{aligned}
\mathbb{E}[\boldsymbol{Y} \mid \boldsymbol{x}, \boldsymbol{w}] &= \beta_0 \mathbf{1} && \text{model 0} \\
&= \beta_0 \mathbf{1} + \beta_1 \boldsymbol{x} && \text{model 1} \\
&= \beta_0 \mathbf{1} + \beta_1 \boldsymbol{x} + \beta_2 \boldsymbol{\delta}_2 && \text{model 2} \\
&= \beta_0 \mathbf{1} + \beta_1 \boldsymbol{x} + \beta_2 \boldsymbol{\delta}_2 + \beta_3 \boldsymbol{x}\boldsymbol{\delta}_2 && \text{model 3}
\end{aligned}$$

For row 1, we are testing

$$H_0 : \beta_1 = \beta_2 = \beta_3 = 0 \quad \text{vs} \quad H_1 : \beta_1 \neq 0, \beta_2 = \beta_3 = 0,$$

that is, comparing models 1 and 0, using the statistic

$$F = \frac{428.11}{48.25/61} = 541.2375$$

The associated p-value is near 0, when we reject H_0. For row 2, we are testing

$$H_0 : \beta_2 = \beta_3 = 0 \quad \text{vs} \quad H_1 : \beta_2 \neq 0, \beta_3 = 0,$$

that is, comparing models 1 and 2. The F-statistic is

$$F = \frac{75.57}{48.25/61} = 95.54$$

which is again significant. For row 3, we are testing

$$H_0 : \beta_3 = 0 \quad \text{vs} \quad H_1 : \beta_3 \neq 0,$$

comparing models 2 and 3, using

$$F = \frac{32.27}{48.25/61} = 41.43,$$

also signficant.

These tests suggest that the largest model (model 3) is correct.

□