Quiz 3

Name ___________________________________

Problem 1. Suppose that $X_1, \ldots, X_n$ are i.i.d., all from the probability density function

$$f(x;\mu) = \frac{1}{\mu} e^{-x/\mu} \quad x \geq 0$$

- Show that $\bar{X}$ an unbiased estimator of μ.

Solution. Note that by integrating by parts,

$$E(X_1) = \int_0^\infty x \frac{1}{\mu} e^{-x/\mu} dx = xe^{-x/\mu}\Big|_{x=0}^{\infty} - \int_0^\infty e^{-x/\mu} dx = \mu$$

Since $E(\bar{X}) = E(X_1)$ (for any distribution), we thus have $E(\bar{X}) = \mu$. □

Problem 2. Suppose that $X_1, \ldots, X_{25}$ is a random sample from a $N(\mu, 1)$ distribution. What is the length of an 80% confidence interval for μ when $\bar{X} = 19.3$?

Solution. Note that $\alpha = 0.20$ so $\alpha/2 = 0.1$. Thus we look in the table for the entry closest to 0.9 to find that $z_{0.1} = 1.28$. The interval is $\bar{X} \pm \frac{1}{\sqrt{25}} 1.28$. Thus the length of the interval is

$$2\frac{1}{5}1.28 = 0.512$$

□