Quiz 4

Name _____

Problem 1. Suppose that $X_1, \ldots, X_n$ are i.i.d. with $E(X_1) = \mu$ and $\mathrm{Var}(X_1) = \sigma^2$.

- Show that both X_2 and $\bar{X}$ are unbiased estimators of μ.
- Show that $\bar{X}$ is a better estimator than X_2 of μ.

Solution. Note that $E(X_2) = E(X_1) = \mu$ since the $X_1, \ldots, X_n$ are i.i.d. Also,

$$E(\bar{X}) = E(X_1) = \mu\,.$$

We have

$$\mathrm{Var}(\bar{X}) = \frac{\sigma^2}{n} < \sigma^2 = \mathrm{Var}(X_1)$$

so $\bar{X}$ is more efficient that X_2. □