FINAL EXAM NOTES
MATH 461 - FALL 2016

ALEX THIES

1. Discrete Random Variables

1.1. **Summary.**

1.1.1. *Probability Mass Function.* It is important to keep in mind that with discrete random variables we compute the probability that a random variable is equal to a certain value, i.e., for X discrete, we compute $p_X(x) = P(X = x)$, this is in contrast to the continuous case. Therefore, when asked to compute something like $P(X > n)$, we must actually compute $1 - \sum_{i=0}^{n-1} p_X(i)$; or for $P(X < n)$, we actually compute $\sum_{i=0}^{n-1} p_X(i)$.

1.1.2. *Expectation.* The expected value of a discrete random variable is computed with the following formula,

$$E(X) = \sum_{x \in X} x p_X(x)$$

When asked to compute a composition of X, we need only manipulate the *weighted* x, rather than the argument of the probability mass function, e.g.,

$$\begin{aligned} E(X) &= \sum_x x p(x); \\ E(X^2) &= \sum_x x^2 p(x); \\ E(1/X) &= \sum_x x^{-1} p(x); \\ E(\log(X)) &= \sum_x \log(x) p(x). \end{aligned}$$

1.1.3. *Variance.* The variance of a discrete random variable is the expected value of the squared difference between the expectation and the result, i.e.,

$$\begin{aligned} \mathrm{Var}(X) &= E\left[(X - \mu)^2\right] \\ &= \sum_{x \in X} (x - \mu)^2 p_X(x) \\ &= \sum_x x^2 p(x) - 2\mu \sum_x x p(x) + \mu^2 \sum_x p(x) \\ &= E(X^2) - 2\mu^2 + \mu^2 \\ &= E(X^2) - \mu^2 \end{aligned}$$

$$\begin{aligned}\mathrm{Var}(X) &= E(X^2) - [E(X)]^2 \\ &= E(X^2) - E^2(X)\end{aligned}$$

Furthermore, note that

$$\mathrm{Var}(aX + b) = a^2 \,\mathrm{Var}(X)$$

1.2. **Binomial Random Variables.** A Binomial Random Variable is the classic case of an experiment in which there is a binary value assigned to success of an event which is attempted n times, with the probability of success being equal to some $p \in [0, 1]$. The classic case is tossing a coin n times, with probability of landing on heads equalling p; these values serve as the parameters of the random variable, i.e., $X \sim \mathrm{Binomial}(n, p)$.

1.2.1. *Probability Mass Function.* For $X \sim \mathrm{Binomial}(n, p)$,

$$p_X(x) = P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

1.2.2. *Expectation of Binomial.* For $X \sim \mathrm{Binomial}(n, p)$,

$$E(X) = np$$

1.2.3. *Variance of Binomial.* For $X \sim \mathrm{Binomial}(n, p)$,

$$\mathrm{Var}(X) = np(1-p)$$

1.3. **Poisson Random Variables.** A Poisson Random Variable approximates a Binomial Random Variable given large n and small p, the product, λ, of these values serve as the parameters of the random variable, i.e., $X \sim \mathrm{Poisson}(\lambda)$.

1.3.1. *Probability Mass Function.* For $X \sim \mathrm{Poisson}(\lambda)$,

$$p_X(x) = P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

1.3.2. *Expectation, and Variance of Poisson.* For $X \sim \mathrm{Poisson}(\lambda)$,

$$E(X) = \mathrm{Var}(X) = \lambda.$$

1.4. **Bernoulli Random Variables.** A Bernoulli Random Variable is the special case where a Binomial Random Variable has one trial, e.g., coin tosses, independent events with binary outcome; thus for probability of success $= p$,

$$X \sim \mathrm{Bernoulli}(p) \Leftrightarrow X \sim \mathrm{Binomial}(1, p).$$

1.4.1. *Probability Mass Function.* For $X \sim \mathrm{Bernoulli}(p)$,

$$\begin{aligned}p(0) &= P(X = 0) = 1 - p, \\ p(1) &= P(X = 1) = p.\end{aligned}$$

1.4.2. *Expectation.* For $X \sim \mathrm{Bernoulli}(p)$,

$$E(X) = p.$$

2. Continuous Random Variables

2.1. **Summary.**

2.1.1. *Probability Density Function (PDF).*

2.1.2. *Cumulative Distribution Function (CDF).*

2.1.3. *Expectation.* The expected value of a continuous random variable is computed with the following formula,

$$E(X) = \int_{-\infty}^{\infty} x f_X(x)\ dx$$

Note that this is fairly similar to the expectation of a discrete random variable, but in the language of continous functions.

2.1.4. *Variance.*

2.2. **Normal Random Variables.**

2.2.1. *PDF and CDF.*

2.2.2. *Expecatation.*

2.2.3. *Variance.*

2.2.4. *Covariance.*

2.2.5. *Normal Approximation of Binomial Random Variable.*

2.3. **Standardized Normal.**

2.3.1. *PDF and CDF.*

2.3.2. *Expecatation.*

2.3.3. *Variance.*

2.3.4. *Covariance.*

2.4. **Exponential Random Variables.**

2.4.1. *PDF and CDF.* For $X \sim \exp(\lambda)$,

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & o/w \end{cases}$$

$$F_X(x) = P(X \leq x) = \int_0^x \lambda e^{-\lambda t}\ dt$$

2.4.2. *Expecatation.*

2.4.3. *Variance.*

2.4.4. *Covariance.*

3. Jointly Distributed Random Variables

E-mail address: athies@uoregon.edu