HOMEWORK 6
MATH 463 - SPRING 2017

ALEX THIES

1. Assignment

1.1. Problem 7.A.12. Suppose X, Y, Z are independent normal random variables, each having variance 1. The means are $\alpha + \beta$, $\alpha + 2\beta$, $2\alpha + \beta$, respectively: α, β are parameters to be estimated. Show that maximum likelihood and OLS give the same estimates. Note: this wont usually be truethe result depends on the normality assumption.

Solution. □

1.2. Problem 7.D.2. Conversely, if U is uniform on $[0, 1]$, show that $F^{-1}(U)$ has distribution function F. (This idea is often used to simulate IID picks from F.)

Solution. □

1.3. Problem 7.D.7.
(a) If $\mathbb{P}(Y_i = 1 | X) = \Lambda(X_i\beta)$, show that logit $\mathbb{P}(Y_i = 1 | X) = X_i\beta$.
(b) If logit $\mathbb{P}(Y_i = 1 | X) = X_i\beta$, show that $\mathbb{P}(Y_i = 1 | X) = \Lambda(X_i\beta)$.

Solution.

(a)
(b)

□

1.4. Problem 7.D.13. Show that the log likelihood for the probit model is concave, and strictly concave if X has full rank. Hint: this is like exercise 11.

Solution. □

1.5. Problem 7.E.6. Student # 77 is Presbyterian, went to public school, and graduated. What does this subject contribute to the likelihood function? Write your answer using φ in equation (15).

Solution. □

1.6. Problem 7.6. Powers and Rock (1999) consider a two-equation model for the effect of coaching on SAT scores:

$$X_i = 1 \text{ if } U_i\alpha + \delta_i > 0, \text{ else } X_i = 0 \qquad \text{(assignment)}$$
$$Y_i = cX_i + V_i\beta + \sigma\,\varepsilon_i \qquad \qquad \text{(response)}$$

Here, $X_i = 1$ if subject i is coached, else $X_i = 0$. The response variable Y_i is subject i's SAT score; U_i and V_i are vectors of personal characteristics for subject i, treated as data. The latent variables $(\delta_i, \varepsilon_i)$ are I.I.D. bivariate normal with mean

0, variance 1, and correlation ρ; they are independent of the U's and V's. (In this problem, U and V are observable, δ and ε are latent.)

(1) Which parameter measures the effect of coaching? How would you estimate it?
(2) State the assumptions carefully (including a response schedule, if one is needed). Do you find the assumptions plausible?
(3) Why do Powers and Rock need two equations, and why do they need ρ?
(4) Why can they assume that the disturbance terms have variance 1? Hint: look at sections 7.2 and 7.4.

Solution.

(a)
(b)
(c)
(d)

$\square$

1.7. **Problem 7.7.** Shaw (1999) uses a regression model to study the effect of TV ads and candidate appearances on votes in the presidential elections of 1988, 1992, and 1996. With three elections and 51 states (DC counts for this purpose), there are 153 data points, i.e., pairs of years and states. Each variable in the model is determined at all 153 points. In a given year and state, the volume TV of television ads is measured in 100s of GRPs (gross rating points). Rep.TV , for example, is the volume of TV ads placed by the Republicans. AP is the number of campaign appearances by a presidential candidate. UN is the percent undecided according to tracking polls. PE is Perots support, also from tracking polls. (Ross Perot was a maverick candidate.) RS is the historical average Republican share of the vote. There is a dummy variable D1992, which is 1 in 1992 and 0 in the other years. There is another dummy D1996 for 1996. A regression equation is fitted by OLS, and the Republican share of the vote is

$$-0.326 - 2.324 \times \mathrm{D}_{1992} - 5.001 \times \mathrm{D}_{1996}$$

$$+ 0.430 \times (\mathrm{Rep.TV} - \mathrm{Dem.TV}) + 0.766 \times (\mathrm{Rep.AP} - \mathrm{Dem.AP})$$

$$+ 0.066 \times (\mathrm{Rep.TV} - \mathrm{Dem.TV}) \times (\mathrm{Rep.AP} - \mathrm{Dem.AP})$$

$$+ 0.032 \times (\mathrm{Rep.TV} - \mathrm{Dem.TV}) \times \mathrm{UN} + 0.089 \times (\mathrm{Rep.AP} - \mathrm{Dem.AP}) \times \mathrm{UN}$$

$$+ 0.006 \times (\mathrm{Rep.TV} - \mathrm{Dem.TV}) \times \mathrm{RS} + 0.017 \times (\mathrm{Rep.AP} - \mathrm{Dem.AP}) \times \mathrm{RS}$$

$$+ 0.009 \times \mathrm{UN} + 0.002 \times \mathrm{PE} + 0.014 \times \mathrm{RS} + \mathrm{error}.$$

(a) What are dummy variables, and why might D1992 be included in the equation?
(b) According to the model, if the Republicans buy another 500 GRPs in a state, other things being equal, will that increase their share of the vote in that state by $0.430 \times 5 \doteq 2.2$ percentage points? Answer yes or no, and discuss briefly. (The 0.430 is the coefficient of Rep. TV Dem. TV in the second line of the equation.)

Solution.

(a)
(b)
(c)

$\square$

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