Quiz Jan 31

Name _____

Problem 1. Suppose that $X_1, \ldots, X_n$ are i.i.d. $N(0, \sigma^2)$. Show that $\frac{1}{n}\sum_{i=1}^n X_i^2$ is a consistent estimator of σ^2.

Solution. Note that $E(X_i^2) = \sigma^2$. The Law of Large Numbers implies that

$$\frac{1}{n}\sum_{i=1}^n X_i^2 \xrightarrow{Pr} E(X_i^2) = \sigma^2,$$

which means that $n^{-1}\sum_{i=1}^n X_i^2$ is a consistent estimator of σ^2. □