Math 465/565, Winter 2011, Prof. Sinclair

Final Exam

March 18, 2011

Instructions:

- 1. Undergraduates: Do 4 of the 5 questions, indicating clearly which question you do not want graded. Graduate students are required to do all of the problems.
- 2. Read all questions carefully. If you are confused, ask me!
- 3. You should have 10 pages including this page. Make sure you have the right number of pages.
- 4. Make sure your name is clearly printed on the first page and your initials are on all subsequent pages.
- 5. If necessary you may use the backs of pages.
- 6. Box your answers when appropriate.
- 7. Calculators and other electronic devices are not allowed.

Name:		
UO ID:		

Page:	2	3	4	5	6	7	8	9	10	Total
Points:	20	10	20	10	20	10	20	10	30	150
Score:										

Page 2 of 10 Initials:

1. There are 2 white marbles in box A and 3 red marbles in box B. At each step in the process a marble is selected from each box and the 2 marbles are interchanged. (Thus box A always contains two marbles and box B always contains three marbles). The system may be described by three states s_0 , s_1 , s_2 which denote the number of red marbles in box A.

[10 pts]

(a) Find the transition matrix of the system.

[10 pts] (b) Find the probability that there 2 red marbles in box A after three steps.

Page 3 of 10 Initials:

[10 pts] (c) Find the stationary distribution.

Page 4 of 10 Initials:

2. Patients arrive at a doctor's office according to a Poisson process with rate $\lambda = 1/10$ (on average one every 10 minutes). The doctor will not see a patient until at least three patients are in the waiting room.

[10 pts]

(a) What is the expected waiting time until the first patient is admitted to see the doctor?

[10 pts] (b) What is the probability that nobody is admitted to see the doctor in the first hour?

Page 5 of 10 Initials:

[5 pts]

(c) Each patient pays either a \$10 or \$20 copay when they arrive (whether or not they actually see the doctor). If the probability of paying \$10 is 1/3 and the probability of paying \$20 is 2/3, what is the expected amount of money received at the end of 2 hours?

[5 pts] (d) What is the variance in the amount of money received at the end of 2 hours?

3. Suppose $\{B_t: t \in [0,\infty)\}$ is a standard Brownian motion and set $X_t = B_t^2 - t$.

[10 pts]

(a) What is an interpretation of X_t ? (That is, what is X_t telling you about B_t ?).

[10 pts] (b) Prove that X_t satisfies the martingale property with respect to B_t , that is show

$$E[X_t|B_r, r \le s] = X_s.$$

Page 7 of 10 Initials:

[10 pts]

(c) Suppose a < 0 < b, and let T be the first time either $X_t = a$ or $X_t = b$. Find the probability that $X_T = a$.

4. Suppose X_1, X_2, \ldots are independent (not necessarily identically distributed) random variables with (finite) expectations μ_1, μ_2, \ldots Let

$$S_n = X_1 + \dots + X_n - (\mu_1 + \dots + \mu_n).$$

[10 pts] (a) Prove or find a counterexample: S_n is a Martingale.

[10 pts] (b) Suppose T is the first time that $|S_n| > 10$. Is T a stopping time? Why or why not?

Page 9 of 10 Initials:

[10 pts] (c) Prove or find a counterexample: $P\{T < \infty\} = 1$.

Page 10 of 10

Initials:

5. Give the (axiomatic) definition of the following processes making sure to define all relevant terms.

[15 pts]

(a) Brownian motion (Hint: There are five axioms).

[15 pts] (b) Poisson process (Hint: There are three axioms).