

HOMEWORK 4

Problem 1. Let $\mathbf{x}_1 = (1, 1, 1, 1, 1)'$, $\mathbf{x}_2 = (3, -1, 4, 6, 3, 3)'$, $\mathbf{x}_3 = (7, 3, 2, 0, 3, 3)'$, $\mathbf{x}_4 = (8, 4, 9, -5, 4, 4)'$, $\mathbf{Y} = (4, 36, 44, 12, 16, 8)'$, $V = \mathcal{L}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4)$. Suppose we wish to test $H_0 : \beta_4 = 0, \beta_2 = \beta_3$.

- (a) Find two matrix \mathbf{A} such that H_0 is equivalent to $\mathbf{A}\boldsymbol{\beta} = 0$.
- (b) Find $\hat{\boldsymbol{\beta}}$, $\hat{\mathbf{Y}} = \mathbf{X}\hat{\boldsymbol{\beta}}$, and $\mathbf{Z} = \mathbf{A}\hat{\boldsymbol{\beta}}$ for one of your choices of \mathbf{A} .
- (c) Define V_0 so that $\boldsymbol{\theta} := \mathbb{E}[\mathbf{Y} | \mathbf{X}] \in V_0$ if and only if $\mathbf{A}\boldsymbol{\beta} = 0$. Find $\hat{\mathbf{Y}}_0 = \Pi_{V_0} \mathbf{Y}$, $\mathbf{Y} - \hat{\mathbf{Y}}$ and $\hat{\mathbf{Y}}_1 = \hat{\mathbf{Y}} - \hat{\mathbf{Y}}_0$.
- (d) Determine $\text{SS}_{\text{Res}} = \|\mathbf{Y} - \hat{\mathbf{Y}}\|^2$, $\text{SS}_{\text{Res}}(V_0) = \|\mathbf{Y} - \hat{\mathbf{Y}}_0\|^2$, and the F -statistic.
- (e) Verify that $\|\hat{\mathbf{Y}} - \hat{\mathbf{Y}}_0\|^2 = \mathbf{Z}'[\mathbf{A}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{A}']^{-1}\mathbf{Z}$.
- (f) Find \mathbf{c} and \mathbf{a} so that $\|\hat{\mathbf{Y}} - \hat{\mathbf{Y}}_0\|^2 = \langle \mathbf{a}, \mathbf{Y} \rangle^2 / [S^2 \|\mathbf{a}\|^2]$

Problem 2. Consider the data in the dataset `teengamb` in the package `faraway`:

```
install.packages("faraway")
library(faraway)
data(teengamb)
```

The last line should bring up a description of the variables.

Is there a difference between males and females as relates to gambling behavior? Fit any appropriate model(s) and carry out any appropriate test(s).

Problem 3. Suppose that 11 plots of land are plotted with three varieties of corn. The following lists the yields for the three varieties:

I	II	III
52	64	53
56	57	55
60	62	58
56		50

- (a) Test the hypothesis that the three varieties all have the same expected yield .
- (b) Suppose that for the corn yield the true means were 70, 75, 95 and that $\sigma = 20$. Find the power of the $\alpha = 0.05$ level test for equal means.
- (c) How large should n_0 , the number of observations per treatment (number of plots per treatment) be in order to have power at least 0.90 for the parameters in (a)?