F tests

Math 463, Spring 2017, University of Oregon

David A. Levin

University of Oregon

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Some housekeeping

- Next HW will be posted by Friday (due following Fri).
- After anaylsis of variance, will discuss path models (linked regression equations). Read Chapter 6 in Freedman.

- Model: random vectors $\boldsymbol{Y}, \boldsymbol{x}_1, \ldots, \boldsymbol{x}_k, \boldsymbol{\varepsilon}$, with
$$\boldsymbol{Y} = [\boldsymbol{x}_1\ \boldsymbol{x}_2 \ldots \boldsymbol{x}_k]\boldsymbol{\beta} + \boldsymbol{\varepsilon}.$$
Usual distribution of tests, etc. is *conditional* on the observed values of $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_k$.
$$\boldsymbol{\theta} := \mathbb{E}[\,\boldsymbol{Y} \mid \boldsymbol{X}\,] = \boldsymbol{X}\boldsymbol{\beta} \in \mathcal{L}(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_k)$$
- Suppose we have a sequence of subspaces $V_0 \subset V_1 \subset \cdots \subset V_r = V$. Let
$$W_k = V_k \cap V_{k-1}^{\perp},$$
so that
$$\mathbb{R}^n = V_0 \oplus \underbrace{W_1 \oplus \cdots \oplus W_r \oplus V^{\perp}}_{V_0^{\perp}}$$
- Thus the projection onto $V_0^{\perp}$ can be resolved into orthogonal pieces on each of $W_1, \ldots, W_r$ and $V^{\perp}$: Letting $\hat{\boldsymbol{Y}}_j = \Pi_{V_j}\boldsymbol{Y}$ and recall $\Pi_{W_j} = \Pi_{V_j} - \Pi_{V_{j-1}}$,
$$\begin{aligned}\|\boldsymbol{Y} - \hat{\boldsymbol{Y}}_0\|^2 &= \|\hat{\boldsymbol{Y}}_1 - \hat{\boldsymbol{Y}}_0\|^2 + \|\hat{\boldsymbol{Y}}_2 - \hat{\boldsymbol{Y}}_1\|^2 + \cdots + \|\hat{\boldsymbol{Y}} - \hat{\boldsymbol{Y}}_{r-1}\|^2 + \|\boldsymbol{Y} - \hat{\boldsymbol{Y}}\|^2 \\ &= \mathrm{SS}_{\mathrm{Reg}}(V_1 \mid V_0) + \cdots + \mathrm{SS}_{\mathrm{Reg}}(V \mid V_{r-1}) + \mathrm{SS}_{\mathrm{Res}} \\ &= (\|\boldsymbol{Y} - \hat{\boldsymbol{Y}}_0\|^2 - \|\boldsymbol{Y} - \hat{\boldsymbol{Y}}_1\|^2) + \cdots + (\|\boldsymbol{Y} - \hat{\boldsymbol{Y}}_{r-1}\|^2 - \|\boldsymbol{Y} - \hat{\boldsymbol{Y}}_r\|^2) \\ &\quad + \|\boldsymbol{Y} - \hat{\boldsymbol{Y}}_r\|^2 \\ &= \Delta\mathrm{SS}_{\mathrm{Res}}(V_1 \mid V_0) + \cdots + \Delta\mathrm{SS}_{\mathrm{Res}}(V_r \mid V_{r-1}) + \mathrm{SS}_{\mathrm{Res}}\end{aligned}$$
- The last term is noise. The other terms may possible be noise, or signal. Which?

- Each of $\mathrm{SS}_{\mathrm{Reg}}(V_i \mid V_{i-1})$ is chi-squared with noncentrality parameter $\|\Pi_{W_i}\boldsymbol{\theta}\|^2/\sigma^2$, independent of $\mathrm{SS}_{\mathrm{Res}}$. Thus the hypothesis test of

$$H_0 : \boldsymbol{\theta} \in V_{i-1} \quad \text{vs.} \quad H_1 : \boldsymbol{\theta} \in V_i \setminus V_{i-1}$$

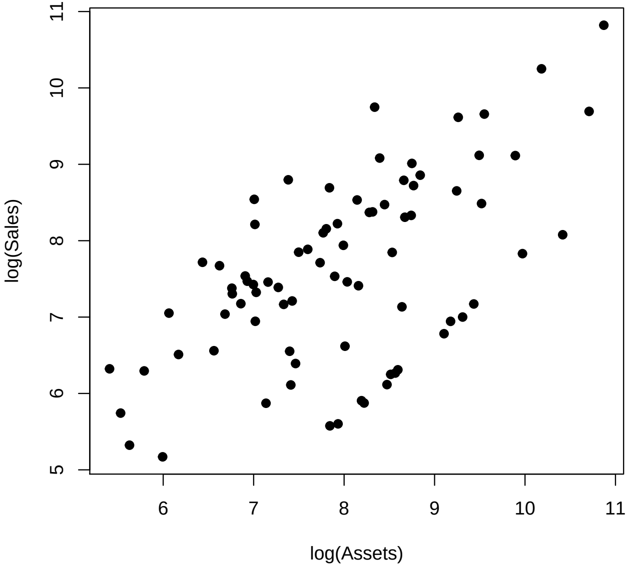
  is based on

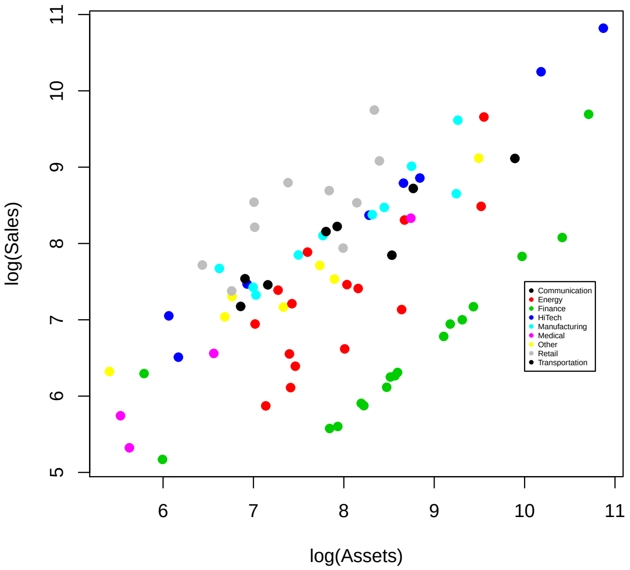
$$F = \frac{\mathrm{SS}_{\mathrm{Reg}}(V_i \mid V_{i-1})/\dim(W_i)}{\mathrm{SS}_{\mathrm{Res}}/(n - \dim(V))} .$$

- An *analysis of variance* table gives each of these tests for a sequence of nested models.

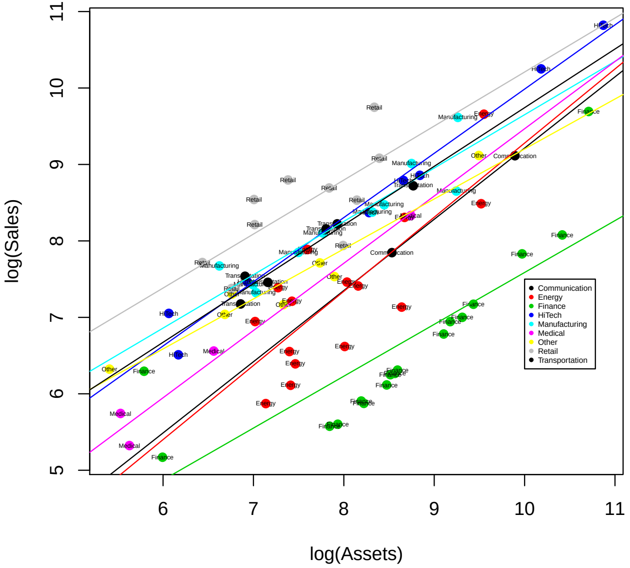
Factors, dummy variables, ANOVA

```
> comp = read.table("http://pages.uoregon.edu/dlevin/DATA/companies.txt",
+                   header=T,sep="\t")
> plot(log(Sales)~log(Assets), data=comp, col=sector, pch=19)
> legend(10,7.5,pch=19,col=1:9,legend=levels(comp$sector),cex=0.4)
> comp.lm = lm(log(Sales)~log(Assets)+sector+log(Assets):sector,
+              data=comp)
> text(log(comp$Sales)~log(comp$Assets),labels=comp$sector,cex=0.3)
> betahat = comp.lm$coef
> basebeta = comp.lm$coef[1:2]
> abline(basebeta,col=1)
> for(i in 0:8){
+ abline(basebeta+betahat[c(2+i,10+i)], col=1+i)
+ }
```


log(Sales)
log(Assets)
5
6
7
8
9
10
11
6
7
8
9
10
11



log(Sales)
log(Assets)
Communication
Energy
Finance
HiTech
Manufacturing
Medical
Other
Retail
Transportation


log(Sales)
log(Assets)
Communication
Energy
Finance
HiTech
Manufacturing
Medical
Other
Retail
Transportation

Let for $j = 2, \ldots, 9$

$$\delta_{i,j} = \begin{cases} 1 & \text{if } i \text{ data point in sector } j \\ 0 & \text{else} \end{cases}$$

Consider

$$\mathbb{E}(Y_i \mid x_i, \delta_{i,\cdot}) = \beta_0 + \beta_1 x_i + \sum_{j=2}^{9} \beta_j \delta_{i,j} + \sum_{j=2}^{9} \beta_{8+j} \delta_{i,j} x_i$$

If the i-th data point belongs to sector $k = 2, \ldots, 8$, then

$$\mathbb{E}(Y_i \mid x_i, \delta_{i,\cdot}) = (\beta_0 + \beta_k) + (\beta_1 + \beta_{8+k}) x_i$$

In other words, $\beta_0 + \beta_k$ is the intercept for the sector-k data, and $(\beta_1 + \beta_{8+k})$ is the slope for the sector-k data, where $k = 2, \ldots, 8$.

```
> model.matrix(comp.lm)[22:24,]

   (Intercept) log(Assets) sectorEnergy sectorFinance sectorHiTech
22           1    8.393442            0             0            0
23           1    8.841304            0             0            1
24           1    6.759255            0             0            0
   sectorManufacturing sectorMedical sectorOther sectorRetail
22                   0             0           0            1
23                   0             0           0            0
24                   0             0           0            1
   sectorTransportation log(Assets):sectorEnergy log(Assets):sectorFinance
22                    0                        0                         0
23                    0                        0                         0
24                    0                        0                         0
   log(Assets):sectorHiTech log(Assets):sectorManufacturing
22                 0.000000                               0
23                 8.841304                               0
24                 0.000000                               0
   log(Assets):sectorMedical log(Assets):sectorOther log(Assets):sectorRetail
22                         0                       0                 8.393442
23                         0                       0                 0.000000
24                         0                       0                 6.759255
   log(Assets):sectorTransportation
22                                0
23                                0
24                                0
```

Confidence intervals

Suppose we want to make a confidence interval for

$$E[\,Y \mid \log(\text{Assets}) = 7, \text{sector}=\text{“Energy”}]$$

Need to create a data frame with new values.

```
> newcomp = data.frame(Assets=1096, sector = "Energy")
> predict(comp.lm, newdata=newcomp, interval="confidence")

       fit      lwr      upr
1 6.370161 5.958412 6.781909
```

Want to test

$$H_0 : \beta_2 = \cdots = \beta_9 = 0$$

and (separately)

$$H_0 : \beta_{10} = \cdots = \beta_{17} = 0$$

We have nested models $V_0 \subset V_1 \subset V_2 \subset V_3$

- $V_0 : \theta \in \mathcal{L}(\mathbf{1})$. $[H_0 : \beta_1 = \cdots = \beta_{17} = 0,\ H_1 : \beta_1 \neq 0, \beta_2 = \cdots = \beta_{17} = 0]$
- $V_1 : \theta \in \mathcal{L}(\mathbf{1}, \boldsymbol{x})$, $\Delta\,\mathrm{dim} = 1$. $[H_0 : \beta_2 = \cdots = \beta_{17} = 0,$ $H_1 : \exists 1 \leq i \leq 9\,\text{s.t.}\,\beta_i \neq 0, \beta_{10} = \cdots = \beta_{17} = 0]$
- $V_2 : \theta \in \mathcal{L}(\mathbf{1}, \boldsymbol{x}, \boldsymbol{\delta}_2, \ldots, \boldsymbol{\delta}_8)$, $\Delta\,\mathrm{dim} = 8$. $[H_0 : \beta_{10} = \cdots = \beta_{17} = 0]$
- $V_3 : \theta \in \mathcal{L}(\mathbf{1}, \boldsymbol{x}, \boldsymbol{\delta}_2, \ldots, \boldsymbol{\delta}_8, \boldsymbol{x}\boldsymbol{\delta}_1, \ldots, \boldsymbol{x}\boldsymbol{\delta}_8)$, $\Delta\,\mathrm{dim} = 8$.

```
> library(xtable)
> xtable(anova(comp.lm))
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
log(Assets)	1	38.32	38.32	148.84	0.0000
sector	8	58.00	7.25	28.16	0.0000
log(Assets):sector	8	1.00	0.13	0.49	0.8611
Residuals	61	15.70	0.26		

Each line reports the distance between fitted models.
The statistic is

$$\frac{\mathrm{SS}_{\mathrm{Reg}}(V_i \mid V_{i-1})/\Delta\mathrm{df}}{\mathrm{SS}_{\mathrm{Res}}/\mathrm{df}} \sim F(\Delta\mathrm{df}, \mathrm{df}, \gamma)\,,$$

where $\gamma_i = \|\Pi_{V_i \cap V_{i-1}^{\perp}} \boldsymbol{\theta}\|^2/\sigma^2$, recalling that $\boldsymbol{\theta} = \boldsymbol{X}\boldsymbol{\beta}$. If $\boldsymbol{\theta} \in V_{i-1}$, then $\gamma_i = 0$. If $\boldsymbol{\theta} \in V_i$, then γ_i depends only on the coefficients and covariates which are in V_i.

```
> comp2.lm=lm(terms(log(Sales)~log(Assets)+log(Assets):sector+sector,keep.o
> xtable(anova(comp2.lm))
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
log(Assets)	1	38.32	38.32	148.84	0.0000
log(Assets):sector	8	57.54	7.19	27.94	0.0000
sector	8	1.46	0.18	0.71	0.6812
Residuals	61	15.70	0.26		

Note that changes the order of the subspaces does change the $\mathrm{SS}_{\mathrm{Reg}}$, which is *conditional* on the subspace below. That is, the line marked "sector" in the above table corresponds to

$$\mathrm{SS}_{\mathrm{Reg}}(\delta_2, \ldots, \delta_8 \mid 1, x, \delta_2 x, \ldots, \delta_8 x)$$

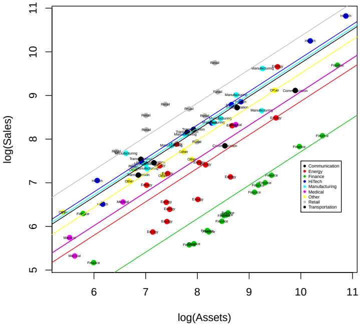
In the first table, the line labels sector corresponds to

$$\mathrm{SS}_{\mathrm{Reg}}(\delta_2, \ldots, \delta_8 \mid 1, x)$$

How to test a linear constant $\boldsymbol{T\beta} = 0$

- $H_0 : \boldsymbol{T\beta} = 0$.
- Suppose that $\boldsymbol{T}$ is a $r \times (k+1)$ matrix of rank r. H_0 specifies that $\boldsymbol{\beta}$ is orthogonal to the row space of $\boldsymbol{T}$, so $\boldsymbol{\beta}$ is contrained to lie in $k+1-r$ dimensional linear space.
- How do we translate this into a hypothesis on $\boldsymbol{\theta} = \mathbb{E}(\boldsymbol{Y} \mid \boldsymbol{X})$?
- Two points of view:
  - Theoretical description of F test.
  - Since $\mathrm{SS}_{\mathrm{Reg}}$ is the difference between $\mathrm{SS}_{\mathrm{Res}}$ for large and small models, it is enough to fit both models. How to fit model subject to the contraint in H_0?

```
> plot(log(Sales)~log(Assets), data=comp, col=sector, pch=19)
> legend(10,7.5,pch=19,col=1:9,legend=levels(comp$sector),cex=0.4)
> comp.lm.b = lm(log(Sales)~log(Assets)+sector, data=comp)
> text(log(comp$Sales)~log(comp$Assets),labels=comp$sector,cex=0.3)
> betahat = comp.lm.b$coef
> basebeta = comp.lm.b$coef[1:2]
> abline(basebeta,col=1)
> for(i in 0:7){
+ abline(basebeta+c(betahat[3+i],0), col=2+i)
+ }
```



F test with linear constraint

- Note that $\boldsymbol{\beta} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{\theta}$, so $\boldsymbol{T}\boldsymbol{\beta} = 0$ iff $\boldsymbol{T}\boldsymbol{M}^{-1}\boldsymbol{X}'\boldsymbol{\theta} = 0$.
- This is equivalent to saying that

$$\boldsymbol{\theta} \perp V_1 := \text{row space}(\boldsymbol{T}\boldsymbol{M}^{-1}\boldsymbol{X}') = \text{col space}(\underbrace{\boldsymbol{X}\boldsymbol{M}^{-1}\boldsymbol{T}'}_{\boldsymbol{B}})$$

- Since SS_{Reg} is the squared length of the projection onto V_1, we can write the numerator of the F-statistic as

$$\text{SS}_{\text{Reg}} = \|P_{V_1}\boldsymbol{Y}\|^2 = \boldsymbol{Y}'\boldsymbol{B}(\boldsymbol{B}'\boldsymbol{B})^{-1}\boldsymbol{B}'\boldsymbol{Y} = \hat{\boldsymbol{\beta}}'\boldsymbol{T}'(\boldsymbol{T}\boldsymbol{M}^{-1}\boldsymbol{T})^{-1}\boldsymbol{T}\hat{\boldsymbol{\beta}}$$

- Note that $\boldsymbol{\beta}$ is constrained to fall in the null space of $\boldsymbol{T}$ which has dimension $k+1-r$. Thus if $\boldsymbol{c}_j, j = 1, \ldots, k+1-r$ is a basis for the null space, then there is $\boldsymbol{\gamma} \in \mathbb{R}^{k+1-r}$

$$\beta = \gamma_1\boldsymbol{c}_1 + \cdots + \gamma_{n-r}\boldsymbol{c}_{k+1-r} = \boldsymbol{C}\boldsymbol{\gamma}$$

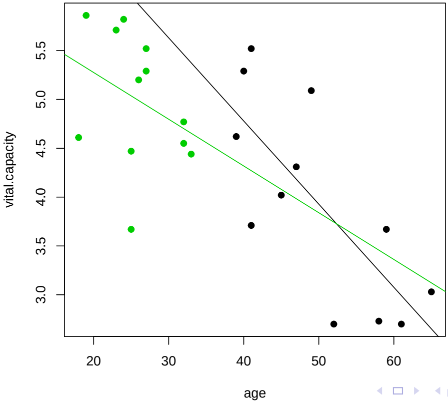
- Fitting the model subject to the constraint is equivalent to the model

$$\mathbb{E}(\boldsymbol{Y} \mid \boldsymbol{X}) = \boldsymbol{X}\boldsymbol{\beta} = \boldsymbol{X}\boldsymbol{C}\boldsymbol{\gamma}\,.$$

Thus, fitted value and RSS can be found via OLS using $\boldsymbol{X}\boldsymbol{C}$ as model matrix.

Example

```
> library(ISwR)
> plot(vital.capacity~age, data = vitcap, col=group, pch=19)
> vc.lm = lm(vital.capacity~age + age*as.factor(group), data=vitcap)
> vc.coef = vc.lm$coefficients
> abline(vc.coef[1:2])
> abline(vc.coef[1:2]+vc.coef[3:4], col=3)
```



```
> xtable(anova(vc.lm))
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
age	1	12.48	12.48	25.90	0.0001
as.factor(group)	1	0.96	0.96	2.00	0.1732
age:as.factor(group)	1	0.27	0.27	0.57	0.4603
Residuals	20	9.64	0.48		

Let us instead parameterize the model as

$$\mathbb{E}(Y_i \mid \delta, x) = \beta_0\delta_i + \eta_0(1-\delta_i) + \beta_1\delta_i x_i + \eta_1(1-\delta_i)x_i\,.$$

```
> delta = as.numeric(vitcap$group==1)
> vc.lm.2=lm(vital.capacity~delta+I(1-delta)+I(delta*age)
+            +I((1-delta)*age)-1,data=vitcap)
> xtable(summary(vc.lm.2))
```

	Estimate	Std. Error	t value	Pr(>|t|)
delta	8.1834	1.1608	7.05	0.0000
I(1 - delta)	6.2327	1.1533	5.40	0.0000
I(delta * age)	-0.0851	0.0230	-3.70	0.0014
I((1 - delta) * age)	-0.0479	0.0438	-1.09	0.2878

Note that "a difference in significance is not the same as a significant difference"!

- Want to test $H_0 : \beta_1 = \eta_1, \beta_0 = \eta_0$.
- $$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \eta_0 \\ \beta_1 \\ \eta_1 \end{bmatrix} = 0$$
- Want to determine the null space of $\boldsymbol{T}$, although it should be clear that the matrix $\boldsymbol{C}$ should add the first two columns of $\boldsymbol{X}$ together and add the last two columns of $\boldsymbol{X}$ together. The SVD can be used to determine a basis of the null space: if $\boldsymbol{T} = \boldsymbol{V}\boldsymbol{\Sigma}\boldsymbol{W}'$, then the last two columns of $\boldsymbol{W}$ are a basis for the null space of $\boldsymbol{T}$.

```
> T=matrix(c(1,-1,0,0,0,0,1,-1),nrow=2,byrow=T)
> print(svd(T,nv=4)$v)
```

```
           [,1]       [,2] [,3] [,4]
[1,]  0.7071068  0.0000000 -0.5  0.5
[2,] -0.7071068  0.0000000 -0.5  0.5
[3,]  0.0000000  0.7071068  0.5  0.5
[4,]  0.0000000 -0.7071068  0.5  0.5
```

The two columns

$$\boldsymbol{v}_1 = \frac{1}{2}\begin{bmatrix}-1\\-1\\1\\1\end{bmatrix}, \quad \boldsymbol{v}_2 = \frac{1}{2}\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$$

form a basis for the null space. Simpler to rotate the basis and use $\boldsymbol{v}_1 + \boldsymbol{v}_2$ and $\boldsymbol{v}_1 - \boldsymbol{v}_2$, i.e.

$$\boldsymbol{w}_1 = \begin{bmatrix}0\\0\\1\\1\end{bmatrix} \quad \boldsymbol{w}_2 = \begin{bmatrix}1\\1\\0\\0\end{bmatrix}$$

- $$\boldsymbol{X}\boldsymbol{C} = \boldsymbol{X}\begin{bmatrix}1 & 0\\ 1 & 0\\ 0 & 1\\ 0 & 1\end{bmatrix} = \begin{bmatrix}(\boldsymbol{x}_1 + \boldsymbol{x}_2) & (\boldsymbol{x}_3 + \boldsymbol{x}_r)\end{bmatrix}$$
- So the end result, to fit the model, regress on $\boldsymbol{x}_1 + \boldsymbol{x}_2$ and $\boldsymbol{x}_3 + \boldsymbol{x}_4$.
- ```
  > vc.lm.3=lm(vital.capacity~I(delta+I(1-delta))+I(I(delta*age)
  +             +I((1-delta)*age))-1,data=vitcap)
  > xtable(anova(vc.lm.3,vc.lm.2))
  ```

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	22	10.87				
2	20	9.64	2	1.23	1.28	0.2996

```
>
```
```

```
> X = model.matrix(vc.lm.2)
> Y = vitcap$vital.capacity
> Minv = summary(vc.lm.2)$cov.unscaled
> B = X%*%Minv%*%t(T)
> P = B%*%solve(t(B)%*%B)%*%t(B)
> SS = t(Y)%*%P%*%Y
> print(SS)
```

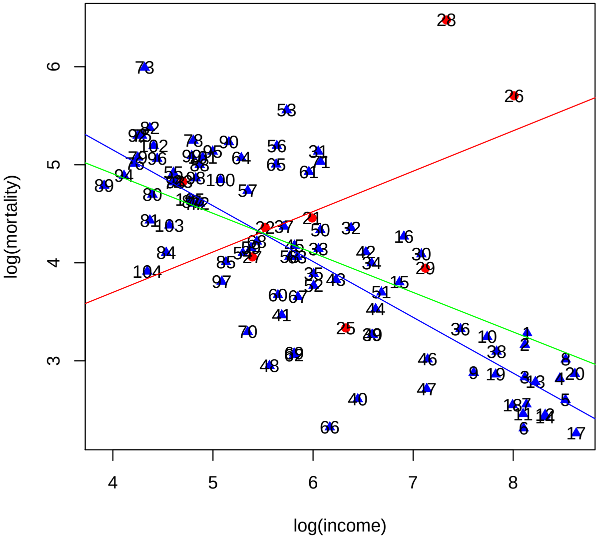
```
         [,1]
[1,] 1.234659
```

Suppose that we have a single quantitative variable X_1 and a qualitative variable X_2 taking on two possible values.

```
> inffile = url(
+   "http://pages.uoregon.edu/dlevin/DATA/infmort.txt")
> infmort = read.csv(inffile)
> infmort[1:10,]
```

```
     region income mortality              oil
1      Asia   3426      26.7 no oil exports
2    Europe   3350      23.7 no oil exports
3    Europe   3346      17.0 no oil exports
4  Americas   4751      16.8 no oil exports
5    Europe   5029      13.5 no oil exports
6    Europe   3312      10.1 no oil exports
7    Europe   3403      12.9 no oil exports
8    Europe   5040      20.4 no oil exports
9    Europe   2009      17.8 no oil exports
10   Europe   2298      25.7 no oil exports
```

```
> oilp = 1:length(infmort$oil)
> oilp[infmort$oil=="no oil exports"]=17
> oilp[infmort$oil!="no oil exports"]=19
> oilc=1:length(infmort$oil)
> oilc[infmort$oil=="no oil exports"]="blue"
> oilc[infmort$oil!="no oil exports"]="red"
> plot(log(mortality)~log(income), data=infmort,pch=oilp,col=oilc)
> attach(infmort)
> text(log(mortality)~log(income),
+      labels=as.character(row.names(infmort)))
> f1 = lm(log(mortality)~log(income),
+         data=subset(infmort,oil=="no oil exports"))
> f2 = lm(log(mortality)~log(income),
+         data=subset(infmort,oil!="no oil exports"))
> infmort2=subset(infmort,
+                 row.names(infmort)!="26"&row.names(infmort)!="28")
> abline(f1,col="blue")
> abline(f2,col="red")
> f3 = lm(log(mortality)~log(income),
+         data=subset(infmort2,oil!="no oil exports"))
> abline(f3,col="green")
```


log(mortality)
log(income)
3
4
5
6
4
5
6
7
8

```
> library(xtable)
> infmort2=na.omit(infmort2)
> f = lm(log(mortality)~log(income)*oil, data=infmort2)
> anovatab = anova(f)
> xtable(anovatab,digits=2,caption="Anova Table",label="anova")
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
log(income)	1.00	55.77	55.77	181.76	0.00
oil	1.00	0.02	0.02	0.07	0.79
log(income):oil	1.00	0.09	0.09	0.31	0.58
Residuals	95.00	29.15	0.31		

Table: Anova Table

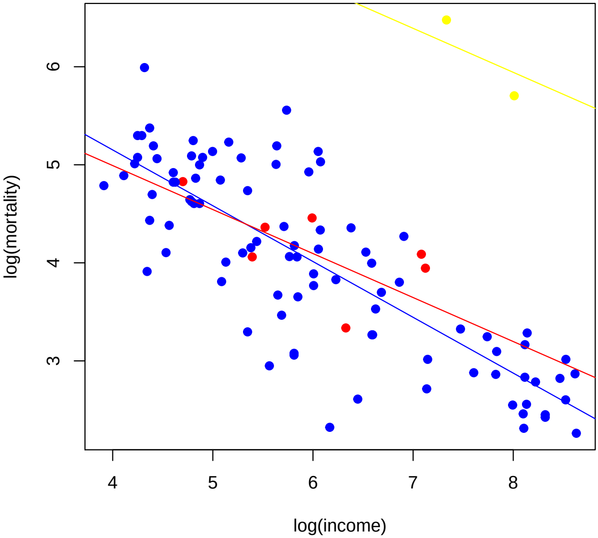
```
> g = lm(log(mortality)~log(income)+log(income):oil,data=infmort2)
> anovatab2=anova(g)
> xtable(anovatab2,digits=2,caption="Anova Table",label="anova2")
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
log(income)	1.00	55.77	55.77	183.16	0.00
log(income):oil	1.00	0.04	0.04	0.12	0.73
Residuals	96.00	29.23	0.30		

Table: Anova Table

```
> model.matrix(f)[1:10,]
   (Intercept) log(income) oiloil exports log(income):oiloil exports
1            1    8.139149              0                          0
2            1    8.116716              0                          0
3            1    8.115521              0                          0
4            1    8.466110              0                          0
5            1    8.522976              0                          0
6            1    8.105308              0                          0
7            1    8.132413              0                          0
8            1    8.525161              0                          0
9            1    7.605392              0                          0
10           1    7.739794              0                          0
```

```
> delt = (row.names(infmort)=="26"|row.names(infmort)=="28")
> infmort$delta = delt
> infmort3 = na.omit(infmort)
> h = lm(log(mortality)~log(income)*oil+delta,data=infmort3)
> hco = 1:length(infmort3$delt)
> hco[oil=="oil exports"]="red"
> hco[oil=="no oil exports"]="blue"
> hco[infmort3$delta]="yellow"
> plot(log(mortality)~log(income),data=infmort3,col=hco,pch=19)
> hc = h$coef
> ab1 = hc[c(1,2)]
> ab2 = hc[c(1,2)]+hc[c(3,5)]
> ab3 = hc[c(1,2)]+hc[c(3,5)]+c(hc[4],0)
> abline(ab1,col="blue")
> abline(ab2,col="red")
> abline(ab3,col="yellow")
```



log(mortality)
log(income)
3
4
5
6
4
5
6
7
8

```
> xtable(anova(h))
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
log(income)	1	47.08	47.08	154.44	0.0000
oil	1	4.58	4.58	15.03	0.0002
delta	1	12.78	12.78	41.93	0.0000
log(income):oil	1	0.05	0.05	0.17	0.6778
Residuals	96	29.27	0.30		