

Quiz 2

Name _____

Problem 1. Suppose that X_1, X_2, X_3 are independent and identically distributed. The common probability mass function is

$$p(k; \theta) = \theta(1 - \theta)^{k-1}, \quad k = 1, 2, \dots$$

If $X_1 = 4, X_2 = 2, X_3 = 6$, find the MLE of θ .

Solution. We have

$$L(\theta; 4, 2, 3) = \theta(1 - \theta)^{4-1} \theta(1 - \theta)^{2-1} \theta(1 - \theta)^{6-1} = \theta^3(1 - \theta)^9$$

$$\ell(\theta; 4, 2, 3) = \log L(\theta; 4, 2, 3) = 3 \log \theta + 9 \log(1 - \theta)$$

$$\begin{aligned} \frac{\ell'(\theta)}{\partial \theta} &= \frac{3}{\theta} - \frac{9}{1 - \theta} \\ &= \frac{3(1 - \theta) - 9\theta}{\theta(1 - \theta)} \\ &= \frac{3 - 12\theta}{\theta(1 - \theta)} \end{aligned}$$

Thus, $\ell'(\theta) = 0$ if and only if

$$\begin{aligned} 0 &= 3 - 12\theta \\ \theta &= \frac{3}{12} = \frac{1}{4}. \end{aligned}$$

Note that $\ell'(\theta) > 0$ for $\theta < 1/4$ and $\ell'(\theta) < 0$ for $\theta > 1/4$, whence $1/4$ is a global maximum. Thus $\hat{\theta} = 1/4$ □