READING ASSIGNMENT 1 – MATH 410
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ALEX THIES
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Part 2

I found the majority of the reading fairly easy to follow, albeit sometimes I'd need to re-read a sentence more than once in order to parse its meaning. Something that occured to me while reading is that this might be the first time a textbook/course is 'building a definition' so that we get an object with certain desired properties. Obviously, this is a pretty common practice in mathematics, but at the undergraduate level definitions tend to be presented without context (at least in my experience), this is an exception.

I had to look-up a few terms, notably **polygonal curve** and **unit velocity**, but for different reasons. The concept of a polygonal curve isn't too difficult for me to grasp after two quarters of geometry courses, but nonetheless I didn't know a firm definition for them, so I wanted to find one. This contrasts with having no earthly idea what unit velocity meant, other than a vague idea that unit always means '1' and velocity is related to derivatives. In the case of a polygonal curve I was able to find a suitable definition, unfortunately I have yet to figure out a search query containing any of the terms 'unit', 'velocity', 'knot', 'theory' without either finding information about the nautical unit of speed, or getting lost in the weeds of physics.

The most difficult part of the reading for me was the section on deformations, mostly because the definition is pretty 'thick.' This was one of those cases where I re-read one sentence several times. Nonetheless, this section lead to one of those fun[1] moments in mathematics where you read one sentence over and over, wave your hands around a bit while trying to think about a geometric concept, and suddenly it makes sense.

Part 3

Upon searching the internet for a continuous parameterization of a trefoil, I found this:

$$x = \sin t + 2\sin 2t,$$
$$y = \cos t - 2\cos 2t,$$
$$z = -\sin 3t.$$

[1]Well, fun-ish

Unfortunately, this parameterization seems to fail every condition that we want, other than it being smooth. The first problem is that its domain is $t \in [0, 2\pi]$ and upon restricting the domain to $[0, 1]$ we no longer have a closed curve. The second problem is that we clearly do not have $f(0) = f(1)$, or any of the conditions that we want when $f(x) = f(y)$. However, it should be obvious that this curve is in fact smooth, as each of its components are infinitely-differentiable. Given that I'm still hazy on what unit velocity means, I can't speak to whether or not this curve has unit velocity. Despite a fair amount of looking, I was unable to find another parameterization of the trefoil.

Part 4

To come up with an explicit simple closed polygonal curve in $\mathbb{R}^3$ I roughly copied down Figure 2.4(a) from the text, made it a right-handed trefoil instead of a left-handed one, and tried to come up with some ordered pairs that made sense for the endpoints of the line segments. I'm reasonably convinced that the minimal number of vertices for the trefoil is 6 (i.e., 5 line segments), but a few of my peers have been unconvinced, so I'm reserving a definitive answer for now. Given this, the curve I made uses 10 endpoints (i.e., 9 segments). Additionally, the curve is projected into $\mathbb{R}^2$ for the sake of simplicity. To be completely honest, I don't like the curve that I came up with, its fairly arbitrary and seemingly not very symmetric, but at this point I don't know how to come up with a better one. Here it is:

$$\begin{aligned} P_0 = P_{10} &= (0,0), \\ P_1 &= (1,0), \\ P_2 &= (2,1), \\ P_3 &= (1/4,2), \\ P_4 &= (1/3,1/2), \\ P_5 &= (2/3,1/2), \\ P_6 &= (3/2,3), \\ P_7 &= (2/3,2), \\ P_8 &= (1/2,3/2), \\ P_9 &= (-1/4,3/2). \end{aligned}$$

Note that (P_5, P_6) overlaps (P_2, P_3) when projected onto $\mathbb{R}^2$, just as (P_2, P_3) overlaps (P_7, P_8), and (P_8, P_9) overlaps (P_3, P_4).

Part 5

I forgot to do this part until I sat down to review this assignment before sending it to you, so I'll have to leave it unanswered for now.