CHAPTER 9: HIGH-DIMENSIONAL KNOT THEORY

The theory of knots in R^3 naturally generalizes to a study of knotting in R^n, with $n > 3$, and many new and fascinating aspects of knot theory appear in this high-dimensional setting. What is perhaps most surprising is that many problems that are intractable in the classical case have been solved for high-dimensional knots. There is also a strong interplay between knot theory in different dimensions, and this interplay leads to an array of new topics at the border of the classical and high-dimensional settings.

The definitions of polygonal knot and of deformation of knots generalizes immediately to R^4, (or for that matter R^n); one can simply consider sequences of points in 4-space instead of R^3. Knots formed in this way are called 1-dimensional knots in 4-space, or, more briefly, 1-knots. It turns out though that there is really no interesting theory of such 1-knots; all such knots in 4-space are equivalent.

The appropriate generalization increases the dimension of the knot as well as the dimension of the ambient space. The definition of surface given in Chapter 4 easily generalizes to yield a definition of surfaces in 4-space. A 2-knot is a surface in R^4 that is homeomorphic to S^2, the standard sphere in 3-space. Figure 9.1 is a schematic illustration of such a knot. Section 1 discusses some of the details of the definitions, as well as a new subtlety that arises at the foundation of the subject.

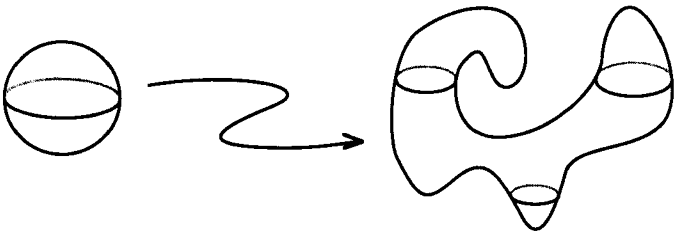


Figure 9.1

One of the pleasures of studying high-dimensional knot theory is the discovery that it is possible to visualize, and sketch, knots in higher dimensions. Sections 2 and 3 demonstrate this, and should help the reader gain intuition about R^4 and the knotting phenomena that occur there.

Section 4 describes a property of classical knots in R^3, called *sliceness*, that is defined in terms of knotted 2-spheres in R^4. It is here that the interplay between dimensions 3 and 4 comes out. The notion of slice knot can be used to define an equivalence relation called *concordance* on the set of classical knots, and it turns out that there is a natural (abelian) group structure on the set of concordance classes of knots. Very little is known about this group, and what is known is summarized in Section 5.

1 Defining Knots in Higher Dimensions

The definition of *knot* given in Chapter 2 ruled out the possibility of infinite knotting, as illustrated in Figure 2.2. In higher dimensions the definitions must also rule out such pathologies. As in the classical case, this can be done using either polygonal knots or smooth knots. Unlike the classical case, the two theories that arise can be distinct. For instance, it is true that every smooth knot can be closely approximated by a polygonal knot, and that if two different approximations are chosen, the two polygonal knots are equivalent. However, when two inequivalent smooth knots are approximated by polygonal knots, it is possible that the resulting knots may be polygonally equivalent.

The study of the distinctions between the theories lies at the foundations of topology and is beyond the scope of this chapter. Only the smooth theory, probably the easiest to describe, is summarized. The discussion begins with the definition of the k-sphere.

□ **DEFINITION.** *The k-sphere, S^k, is the set of unit vectors in R^{k+1}; that is,*

$$S^k = \{(x_1,\ldots,x_{k+1}) \in R^{k+1} \mid x_1^2 + \cdots + x_{k+1}^2 = 1\}.$$

Two important examples are S^1, which is the unit circle in the plane, and S^2, which is the 2-sphere illustrated in Figure 9.1. (The convention of calling these spheres k-spheres rather than $(k+1)$-spheres is based on the fact that intrinsically S^k is k-dimensional, it is only a subset of a $(k+1)$-dimensional space.)

□ **DEFINITION.** *A smooth knotted k-sphere in R^n, K, is a subset of R^n of the form $F(S^k)$, where F is a one-to-one differentiable function from S^k to R^n with everywhere nonsingular derivative.*

(Recall that the derivative of such a function assigns to each point p in S^k a linear map, $D_p(F)$, from the set of tangent vectors to S^k at p to R^n. The linear map is nonsingular if its image is k-dimensional. Also, note that this definition corresponds to the definition of smooth knot given in Chapter 1 in the case of $k = 1$ and $n = 3$. The nonsingularity condition eliminates the type of knotting that was illustrated in the second figure of Chapter 2.)

The definition of equivalence is harder. Suppose that K_0 and K_1 are smooth k-knots in R^n. They are considered smoothly equivalent if there is a family of differentiable functions, F_t, $0 \leq t \leq 1$, from S^k to R^n such that:

(1) for all t, F_t is one-to-one with everywhere nonsingular derivative,

(2) $F_0(S^k) = K_0$ and $F_1(S^k) = K_1$, and

(3) the function G from $S^k \times [0,1]$ to R^n defined by $G(p,t) = F_t(p)$ is differentiable.

Roughly stated, two knots are equivalent if one can be smoothly deformed into the other through a sequence of smooth knots.

EXERCISES

1.1. Describe the 0-sphere, S^0. Explain why all 0-knots in R^2 are equivalent.

1.2. (a) Give a definition of a high-dimensional link. Your definition should include the possibility of having components of different dimension.

(b) Define equivalence of high-dimensional links.

2 Three Dimensions from a 2-dimensional Perspective

In Edwin A. Abbott's novel *Flatland* the narrator tries to describe a 2-sphere to the inhabitants of a plane. As Abbot intended, that description leads to an understanding of how 4-dimensional phenomena can be studied from a 3-dimensional perspective.

When a plane parallel to the x–y plane is lowered through space, its intersections with a 2-sphere give a series of 2-dimensional cross-sections of the sphere. The first non-trivial cross-section consists of a single point. The point then opens up into a circle which grows until its radius is that of the sphere, and then it shrinks down to a point and disappears. This sequence of cross-sections forms the frames of a Flatlandian movie of a sphere in 3-space.

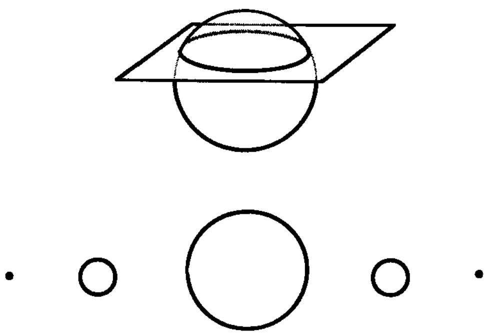


Figure 9.2

Flatlandian movies could be made that would illustrate other surfaces in 3-space. For instance, Figure 9.3 shows some frames from a description of a surface homeomorphic to a sphere and Figure 9.4 illustrates a Flatlandian film of a torus.

Links could also be shown to Flatlanders as a series of 2-dimensional cross-sections. For instance, the first non-trivial cross-section of a link might consist of two points in the plane. Each of those points immediately splits into a pair of points. The center two points then rotate

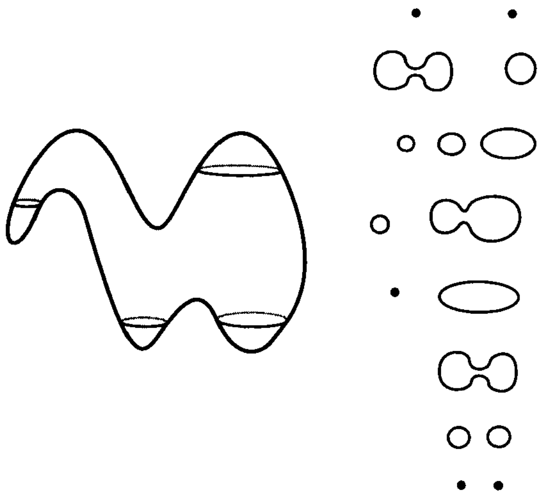


Figure 9.3

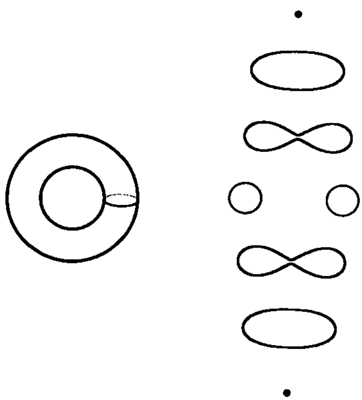


Figure 9.4

about one another. Finally, pairs of points rejoin and then disappear. This is in Figure 9.5.

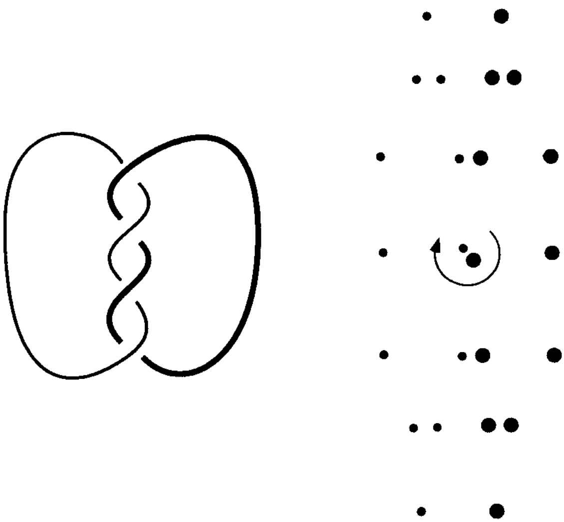


Figure 9.5

The exercises ask you to describe other knots and links to Flatlanders.

EXERCISES

2.1. Draw a series of frames illustrating cross-sections of the sphere illustrated in Figure 9.6.

2.2. Draw a series of figures illustrating the cross-sections of a pair of nested spheres.

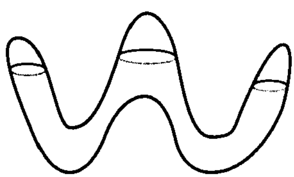


Figure 9.6

2.3. Find an algorithm for deciding if a surface described via a series of cross sections is connected.

2.4. Describe the cross sections of the Figure-8 knot and the Borromean rings.

2.5. The (p,q)-torus knot can be placed so that its cross-sections are extremely simple to describe. Find that description.

2.6. Draw 2-dimensional cross-sections of the knotted torus illustrated in Figure 9.7.

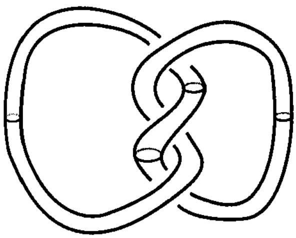


Figure 9.7

3 Three-dimensional Cross-sections of a 4-dimensional Knot

Just as 3-space can be swept out by a plane, 4-space can also be swept out by 3-dimensional hyperplanes. Often the fourth coordinate is viewed as the time, or t, coordinate and the hyperplanes are viewed as parameterized by time. To be precise, let $H_\tau = \{(x,y,z,t) \in R^4 \mid t = \tau\}$. A 2-knot in 4 space, K, can be described via a sequence of 3-dimensional cross-sections of the form $H_t \cap K$; many of these intersections might be classical knots or links in H_t, which is naturally identified with R^3. The simplest such sequence begins with a point in R^3 which immediately opens up into an unknotted circle. The circle grows for a while and then shrinks back into a point and disappears. This is similar to the sequence illustrated in Figure

9.2. Corresponding to Figure 9.3 there is another picture of a 2-sphere in 4-space which begins with 2 points. Both of the 2-knots described by these sequences are in fact trivial, where a trivial 2-knot in R^4 is a knot which can be deformed into the standard S^2 in $H_0 = R^3$.

Of much greater interest is the sequence of cross-sections drawn in Figure 9.8. Here only the first half of the series is illustrated.

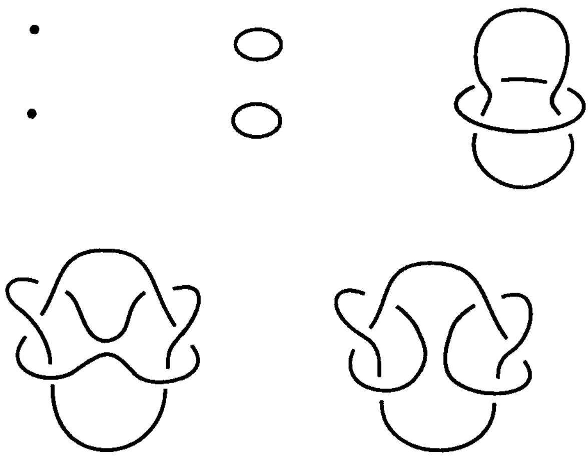


Figure 9.8

The second half appears as the first in reverse order. The 2-knot that this describes is called the *spun trefoil*, and was among the first examples of nontrivial 2-knots. It was discovered by Artin.

Proving that the spun trefoil is nontrivial requires algebraic topology, and in particular a careful study of its fundamental group. There actually is a method of showing that it is knotted which is based on colorings, but a proof of the validity of this approach depends on a careful study of the fundamental group in any case. Unfortunately, there is no simple generalization of Reidemeister moves in dimension 4. A description of colorings will be given later in the section.

Other examples of 2-knots are easily constructed in a similar manner. The exercises ask you to form a few. An interesting family of examples, described by Zeeman, can be formed by slightly modifying the sequence of cross-sections just described. The first half of the sequence is the same as for the spun trefoil. Now, before reversing the sequence to construct the "bottom half" of the knot, one of the two trefoils can first be twisted about its axis as illustrated in Figure 9.9. If it is twisted k times, the resulting knot is called the k-twist spin of the trefoil. One of Zeeman's remarkable discoveries was that the 1-twist spin of the trefoil (or any other knot for that matter) is actually unknotted. An immediate consequence of this is that the unknotted 2-sphere in 4-space can be deformed so that some of its cross-sections are nontrivial knots in 3-space! (Stallings first constructed an example of this phenomena prior to Zeeman's work.)

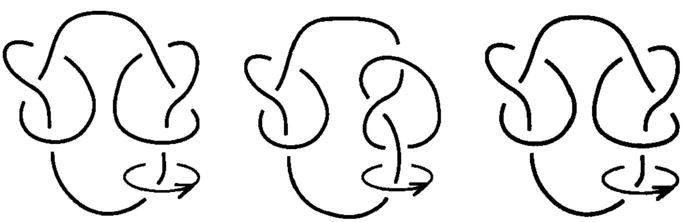


Figure 9.9

One useful fact in studying knotted 2-spheres is that any 2-knot in R^4 can be slightly deformed so that all but a finite number of cross-sections are either classical links or empty. The finite set of exceptions correspond to transitions where components appear or disappear, or else components band together or split apart. Operations of this second type are called *band moves*. These two types of transitions are illustrated in Figure 9.10. The descriptions of 2-knots already presented had this property, and for the rest of the chapter all knots will be assumed to have such cross-sections.

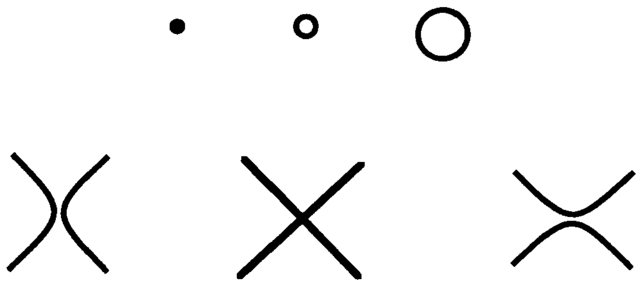


Figure 9.10

COLORING 2-KNOTS

A proof that if a classical knot or link diagram is colorable then every diagram for that link is colorable was given in Chapter 3, and was based on the Reidemeister moves. The proof actually showed more: once a coloring is picked for a diagram, then there is a *canonical* choice of coloring for all other diagrams. To see this, observe that when a Reidemeister move is performed on a colored diagram there is a natural choice of coloring for the new diagram.

A 2-knot is called colorable if every cross-section can be colored so that nearby cross-sections are colored in a consistent manner, and at least two colors appear. When new components appear there is no restriction on how they are colored, but when components join together, the colorings have to be the same at the point that they meet. A nontrivial coloring of the spun trefoil is illustrated in Figure 9.11.

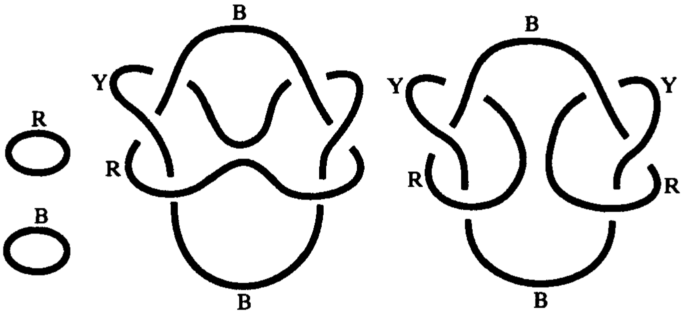


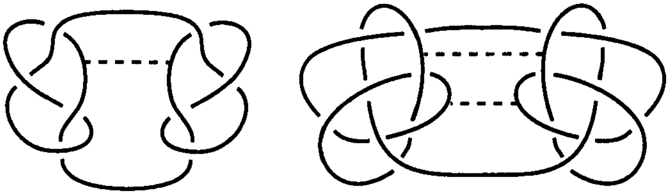
Figure 9.11

It is left to the reader to check that if the trivial components in the first frame are colored red and blue, then the coloring of the second frame must be as illustrated. More examples are presented in the exercises.

EXERCISES

3.1. Illustrate the cross-sections of a knotted 2-sphere for which the middle cross-section is as illustrated in Figure 9.12a. The dotted line provides a hint.

3.2. Repeat Exercise 1 for the knot illustrated in Figure 9.12b. Why is the surface you construct a 2-sphere?



(a) (b)

Figure 9.12

3.3. Repeat Exercise 1 for the knots illustrated in Figure 9.13.

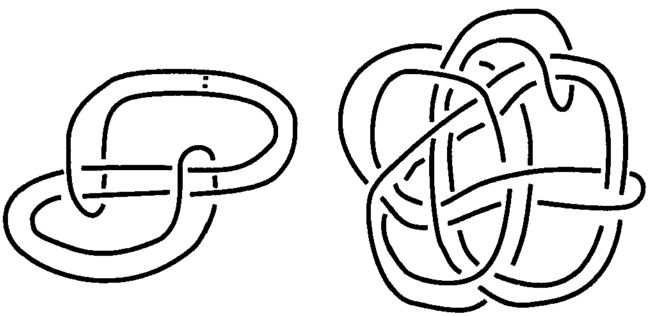


Figure 9.13

3.4. Find a nontrivial coloring or show that one does not exist for each of the knots constructed in the previous exercises.

3.5. For any knot K, the knot $K\#K^{rm}$, as illustrated in Figure 9.12, occurs as the middle cross-section of a knotted

2-sphere in R^4, called the *spin* of K. Describe the general construction of the 2-knot. Show that if K is colorable, then so is the spin of K.

3.6. The sequence of drawings in Figure 9.14 illustrate a surface in 4-space. The surface is not a sphere. What is it?

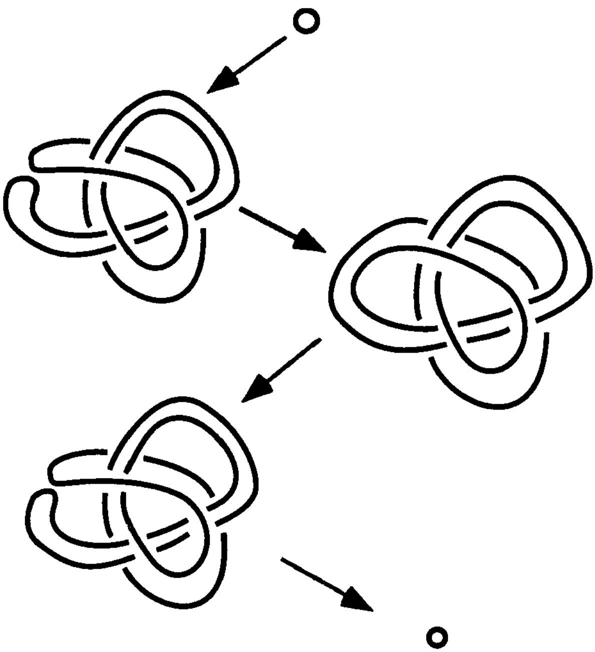


Figure 9.14

3.7. The sequence of diagrams in Figure 9.15 (taken from [Fox]) illustrate the cross-sections of a knotted surface in 4-space for which one cross-section is a nontrivial link. Verify that the surface is a 2-sphere. (Epstein showed that this 2-knot is in fact trivial.)

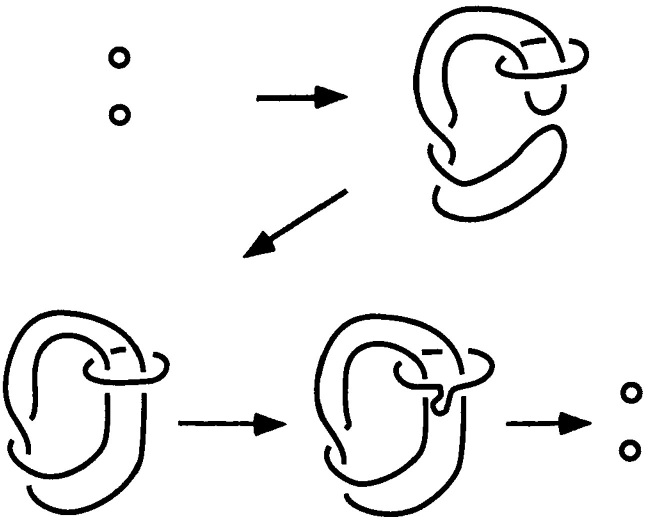


Figure 9.15

3.8. The theory of mod(p) labelings applies to 2-knots also. Use this to show that the knots constructed in Exercise 1 are nontrivial.

4 Slice Knots

The last section presented several examples of classical knots in R^3 that arise as cross-sections (or slices) of 2-knots in R^4; these were illustrated in Figures 9.8, 9.12 and 9.13. Such knots are called *slice knots*. Determining if a knot is slice is a fascinating problem at the border between classical knot theory and the high-dimensional theory.

There is a useful alternative definition of slice knot. Note that the x–y plane in 3-space is the boundary of upper half space, $R^3_+ = \{(x,y,z) \mid z \geq 0\}$. Similarly, 3-space, R^3, can be identified with the x–y–z hyperplane (H_0) in upper 4-space, $R^4_+ = \{(x,y,z,t) \mid t \geq 0\}$. A classical knot in R^3 is slice if it is the boundary of a *smooth disk* in R^4_+, and the disk it bounds is called its *slice disk*. A slice disk is illustrated schematically in Figure 9.16.

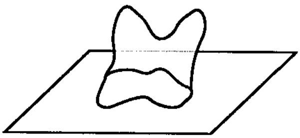


Figure 9.16

The equivalence of the two definitions of slice knot is fairly simple to describe. If a knot is the cross-section of a knotted 2-sphere, K, then the portion of K lying above the cross-section forms a slice disk. On the other hand, if a classical knot bounds a slice disk, then the union of the disk and the mirror image of that disk in lower 4-space, R^4_-, forms a knotted 2-sphere.

It was seen in the previous section, Exercise 3.5, that the connected sum of a knot and its mirror image is always slice. (Actually, some care with orientation is needed here; the correct statement is that $K \# K^{rm}$ is slice.) After describing a few more examples, methods of proving that a knot cannot be sliced will be described. A knot is called

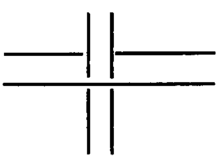


Figure 9.17

a ***ribbon knot*** if it bounds a disk with self intersections only of the type illustrated in Figure 9.17. Such a disk is called a *ribbon disk.*

Two examples of ribbon knots were drawn in Figure 9.13, the first is actually the square knot, which was the first example of a slice knot in the previous section. The diagrams make it clear how the name ribbon arises.

□ **THEOREM 1.** *Every ribbon knot is slice.*

PROOF

A series of cross-sections of a slice disk is easy to construct. Near the ribbon intersections of the ribbon disk the knot can be pinched to divide it up into several components. That collection of components forms an unlink and each one can be shrunk to a point. This is illustrated in Figure 9.18 below. Usually this algorithm is excessive in that a slice disk could be found without introducing so many components. □

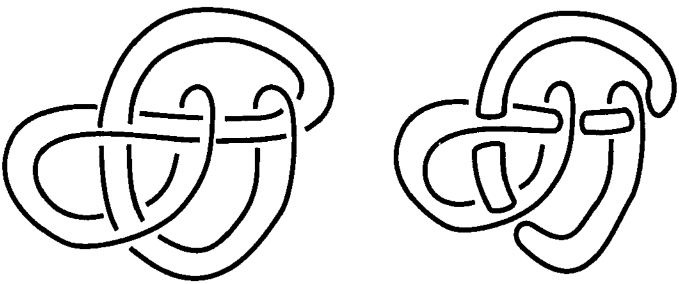


Figure 9.18

□ **CONJECTURE.** *Every slice knot is ribbon.*

This conjecture has been known since the early 1960's and remains one of the most challenging problems in the field.

CONDITIONS ON SLICE KNOTS

If a knot is slice there are strong restrictions on its possible Seifert matrices. These in turn place restrictions on the Alexander polynomial and signatures of slice knots. The proof of the main theorem is beyond what can be presented here, but the corollaries follow fairly easily.

□ **THEOREM 2.** *If a knot K is slice and V is any Seifert matrix arising from a Seifert surface of genus g, then there is an invertible (determinant 1) integer matrix M such that MVM^t is of the form*

$$\begin{pmatrix} 0 & B \\ C & D \end{pmatrix}$$

where B, C, and D are $g \times g$ matrices with $B - C = \pm I_g$, where I_g is the $g \times g$ identity matrix.

□ **COROLLARY 3.** *The Alexander polynomial of a slice knot can be factored as $\pm t^k f(t) f(t^{-1})$ for some integer polynomial f and integer k.*

PROOF

The Alexander polynomial is given by

$$\begin{aligned} \det(V - tV^t) &= \det(M(V - tV^t)M^t) \\ &= \det(MVM^t - tMVM^t). \end{aligned}$$

This last matrix is of the form

$$\begin{pmatrix} 0 & B - tC^t \\ C - tB^t & D - tD^t \end{pmatrix}$$

and $f(t)$ can be taken to be $\det(B - tC^t)$. □

EXAMPLE
The trefoil knot is not slice, as its Alexander polynomial is irreducible.

This result is quite useful, but it fails for a knot as simple as the granny, the connected sum of the trefoil with itself.

□ **COROLLARY 4.** *If a knot is slice, then its signature and all its ω-signatures (for ω not a root of $\Delta_K(t)$) are* 0.

PROOF
Only the real signature will be discussed; a proof for the complex signatures is similar.

The matrix $V + V^t$ which must be diagonalized can be put in the form

$$\begin{pmatrix} 0 & S \\ S^t & R \end{pmatrix}$$

and since $V + V^t$ is invertible over the reals, the matrix S is invertible. Hence, simultaneous row and column operations can be used to put this matrix into the form

$$\begin{pmatrix} 0 & I_g \\ I_g & R \end{pmatrix}.$$

Further row and column operations can be used to eliminate the bottom right-hand block. Finally, it is a straightforward calculation that the signature of the matrix

$$\begin{pmatrix} 0 & I_g \\ I_g & 0 \end{pmatrix}$$

is 0. □

EXAMPLE

As the signature of the trefoil is 2, the granny has signature 4, and is not slice.

If a knot has a Seifert form which is similar to a matrix of the form

$$\begin{pmatrix} 0 & B \\ C & D \end{pmatrix}$$

as above, then it is called *algebraically slice*. Theorem 2 thus states that every slice knot is algebraically slice. In higher dimensions there are corresponding notions of slice knots and algebraically slice, and results of Kervaire and Levine imply that in higher dimensions a converse result holds; a high-dimensional knot is slice if and only if it is algebraically slice. The surgery theoretic methods used in their proofs fail for classical knots, and Casson and Gordon proved that there are algebraically slice knots in R^3 that are not slice. This is described further in Exercise 4.5.

EXERCISES

4.1. Prove that if a knot can be reduced to an unlink of $n+1$ components by performing n band moves, then it is a ribbon knot. (It should be clear that it is slice.) Hence, the solution of Exercise 2.5 shows that knots of the form $K\#K^{rm}$ are actually ribbon.

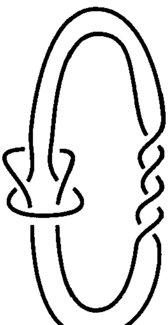


Figure 9.19

4.2. Show that the knot in Figure 9.19 is a ribbon knot. Also, argue that if the knot has n twists instead of 4 as illustrated, then for n odd it is genus 1 and hence is a prime slice knot. Finally show that it is the cross-section of a colorable 2-knot for all n.

4.3. Which knots with 7 or fewer crossings have polynomials satisfying the polynomial condition on slice knots given by Corollary 3.

4.4. The n-twisted double of the unknot, illustrated in Figure 9.20, with $n = 2$ has Seifert matrix V given by

$$\begin{pmatrix} -1 & 1 \\ 0 & n \end{pmatrix}.$$

For what values of n does the polynomial satisfy the conditions of Corollary 3? Show that for these values of n the knot is actually algebraically slice. (Hint: If a quadratic polynomial factors as desired, it has rational roots.)

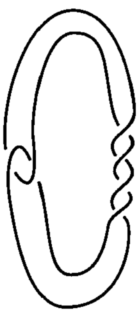


Figure 9.20

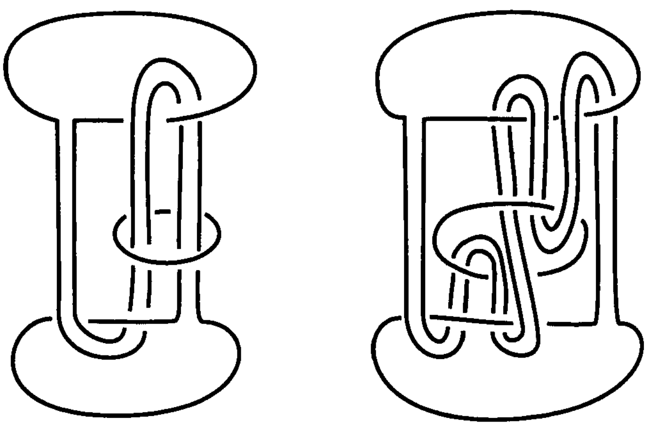
4.5. The doubled knots of the previous exercise are slice only when $n = 0$ or 2, as proved by Casson and Gordon. Show that for $n = 2$ the knot is ribbon. (Hint: consider the examples in Exercise 4.2.)

4.6. Not every unknotting number 1 knot is slice, as was seen in the previous exercises. However, every unknotting number 1 knot is the boundary of a genus 1 surface in R^4_+. Show this by finding a pair of band moves that changes an unknotting number 1 knot into a two component link and then into the unknot. The corresponding surface, which is completed by letting the unknot shrink to a point, is of genus 1. Why? (The bands can all be drawn near the crossing that needs to be changed.)

4.7. A ribbon knot is always the slice of a 2-knot, as described in the proof of Theorem 1. Show that if that 2-knot is colorable, then at the middle cross-section parallel arcs of the ribbon in the diagram are colored the same color.

4.8. (a) A link of two components in R^4 is called *splittable* if it can be deformed so that the components lie on opposite sides of the (y,z,t)-hyperplane. Show that the number of colorings of a split link, including trivial colorings, is the product of the number of colorings for each component.

(b) Figure 9.21a illustrates a link of two components which is the cross-section of a link of two 2-knots in R^4. Show that it is not splittable by counting colorings.



(a) (b)

Figure 9.21

(c) Repeat part b for Figure 9.21b. This example is more interesting, as each component taken individually is trivial. (Can you see why?)

5 The Knot Concordance Group

Using the notion of sliceness, an equivalence relation called *concordance* can be placed on the set of classical knots. The operation of connected sum induces a group structure on the set of concordance classes of knots, and understanding the structure of this group is one of the outstanding problems in knot theory.

□ **DEFINITION.** *Knots K and J are called concordant if $K\#J^{rm}$ is slice.*

□ **THEOREM 5.** *Concordance forms an equivalence relation on the set of knots.*

PROOF
That the relation is reflexive follows from the earlier observation that $K\#K^{rm}$ is always slice. That it is symmetric is automatically satisfied. Transitivity is quickly reduced to showing that if knots K and $K\#J$ are slice, then so is J; the proof of this statement calls for a geometric construction which, although not too difficult, is not presented here. □

□ **LEMMA 6.** *If K_1 is concordant to K_2 and J_1 is concordant to J_2, then $K_1\#J_1$ is concordant to $K_2\#J_2$.*

PROOF
The proof calls for a geometric fact which has not been proved in the text; if knots K and J are slice, then so is $K\#J$. The reader should be able to sketch the argument. The rest of the proof is formal. One needs to show that

$$(K_1\#J_1)\#(K_2\#J_2)^{rm}$$

is slice. However, this knot is the same as

$$(K_1\#K_2^{rm})\#(J_1\#J_2^{rm}),$$

which is the connected sum of two slice knots. □

This lemma implies that the connected sum operation induces a well-defined operation on the set of concordance classes of knots.

□ **THEOREM 7.** *With respect to the operation induced by connected sum, the set of concordance classes of knots forms an abelian group, denoted C_1^3.*

PROOF
Associativity follows from the fact that connected sum of knots is an associative operation. Similarly for commutativity. The identity element is given by the concordance class of the unknot, U, since $K\#U = K$. (The concordance class of the unknot consists of all slice knots.) The inverse of the concordance class of K is given by the concordance class of K^{rm}, since $K\#K^{rm}$ is slice. □

THE STRUCTURE OF C_1^3
As mentioned earlier, understanding the structure of

this group is one of the outstanding problems of low-dimensional topology. The few facts that are known about the group can be easily summarized here.

Fact 1: The concordance group is countable.

Every knot can be deformed so that all its vertices have rational coordinates. It follows that there are only a countable number of knot types, so certainly there are only a countable number of concordance classes.

Fact 2: The function that sends K to $\sigma(K)/2$ is a homomorphism from the concordance group onto Z, and hence the concordance group is infinite. (Here $\sigma(K)$ is the signature function defined in Chapter 6.)

That $\sigma(K)/2$ is a homomorphism follows from the observation that signature adds under connected sum (Exercise 3.6 of Chapter 6) and Corollary 4. (It is easily seen that $\sigma(K)$ is always even; see Exercise 3.4 of Chapter 6.) The trefoil has signature 2, and surjectivity follows.

Fact 3: There are elements of order 2 in the concordance group.

The figure-8 knot, K, provides one such example. As its Alexander polynomial is irreducible, it is not slice. However, it is negative amphicheiral so that $K = K^{rm}$ and hence $K \# K$ is slice.

Fact 4: C_1^3 maps homomorphically onto Z^∞.

An infinite collection of homomorphisms to Z can be defined using ω-signatures, and these can be pieced together to define the desired homomorphism. Levine found examples demonstrating surjectivity.

Beyond these few observations little more is known; for instance, whether or not there are elements of finite order other than order 2 is unknown. Recent advances in low-dimensional topology only indicate that this problem is even more complicated than anticipated.

It is possible to define concordance groups of higher-dimensional knots, and surprisingly the structure of these groups is well understood. Letting C_n^{n+2} denote the concordance group of n-knots in R^{n+2}, it is known that C_n^{n+2} is trivial for n even, and is isomorphic to

$$(Z/2Z)^\infty \oplus (Z/4Z)^\infty \oplus (Z)^\infty$$

for n odd.

EXERCISES

5.1. Use the result that whenever the knots K and $K\#J$ are slice then J is also slice to prove that concordance is transitive.

5.2. Use Alexander polynomials to prove that the trefoil and the (2,5)-torus knots are not concordant.

5.3. The $(2,p)$-torus knot has Alexander polynomial $(t^p+1)/(t+1)$, which is irreducible for p prime. Use this to prove that for p prime the $(2,p)$-torus knot is not concordant to a knot of genus less than $(p-1)/2$.

5.4. (Casson) Although, by Exercise 4.6, unknotting number 1 knots always bound genus 1 surfaces in R^4_+, there are unknotting number 1 knots that are not concordant to genus 1 knots. Find an example of this.