Math 410: Knot Theory

Reading assignment 2

1. *Read* problems 3.5 and 3.6 on page 20. Read sections 2.4 and 2.5 in the textbook. The proof of theorem 3 is vague, despite the result being intuitive. Read it over a few times, try your best to make sense of it, or at least to figure out what parts are unclear. Don't spend hours trying to understand it if you don't want to.

2. What parts of the reading did you find challenging or unclear? What words did you have to look-up elsewhere? What parts would you like to understand better? What arguments were you unable to follow?

3. Find two knots that are **not** equivalent but have the same projection onto some plane. (Remember that a knot diagram is a projection with an indication of over-and under-crossings. A projection is just the projected points, no indication of what goes over or under). You don't need to determine a sequence of points to name your knot, a couple of thoughtful diagrams is enough here.

4. Consider the trefoil model we discussed in class:

   $$(0,1,1),(1/2,-\sqrt{3}/2,-1),(-\sqrt{3}/2,-1/2,1),(1/2,\sqrt{3}/2,-1),(\sqrt{3}/2,-1/2,1),(-1,0,-1)$$

   When we drew this on the board, we had projected it onto the xy-plane.

   - Justify the claim that our projection was a regular projection. (You shouldn't go so deep into this that you are writing down equations, just quickly verify the definition of "regular" based on the diagram).
   - There are lots of directions we might have used to project our knot onto a plane (e.g. the xz-plane, the yz-plane, the plane perpendicular to the vector (1,2,3), etc). Determine a plane of projection through the origin so that the resulting diagram has a triple point (i.e. three **or more** different points on the knot that are projected down to the same single point on the diagram. This, of course, results in a non-regular diagram.) It is often easiest to label a plane by telling your audience a vector[1] that it is perpendicular to.
   - Determine a plane of projection that is different from the previous one such that the resulting diagram has a vertex of the knot being projected to the same point on the diagram as another point of the knot.

5. Show (without writing down any elementary deformations) that an oriented trefoil knot is reversible. You may use the results of problem 3.6 (though this is not necessary).

[1] If you want to make sure such a vector is unique, one way to do this is to require all such vectors to be unit length, and have non-negative z coordinate.