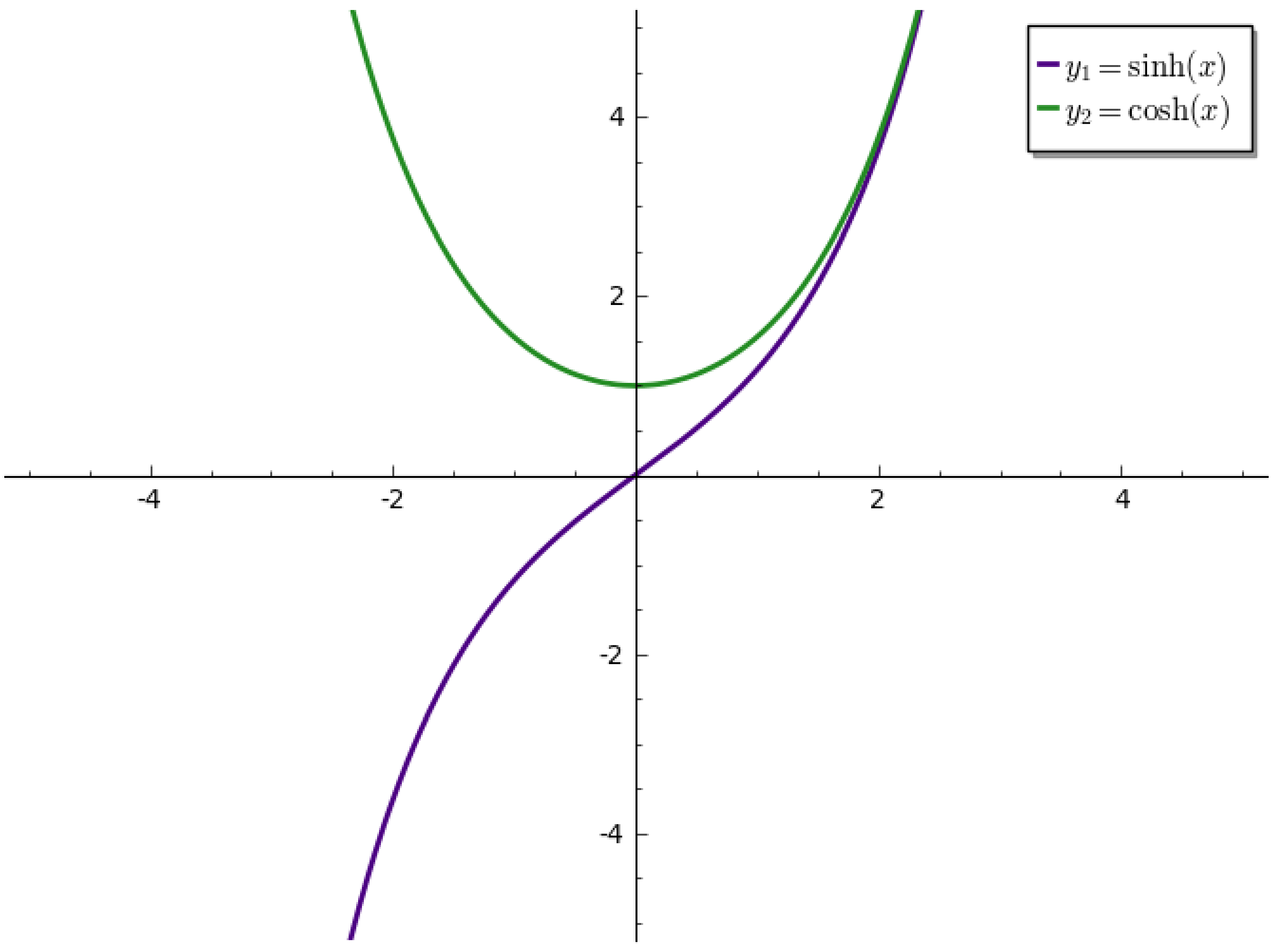
397_homework3

Problem 3.b

Plot graphs of sinh(x) and cosh(x).

```
x = var('x')

P1 = plot(sinh(x), -5, 5, ymin=-5, ymax=5, color='indigo', thickness=2,
legend_label='$y_{1} = \sinh(x)$')
P2 = plot(cosh(x), -5, 5, ymin=-5, ymax=5, color='forestgreen',
thickness=2, legend_label='$y_{2} = \cosh(x)$')

show(P1+P2, figsize=7)
```

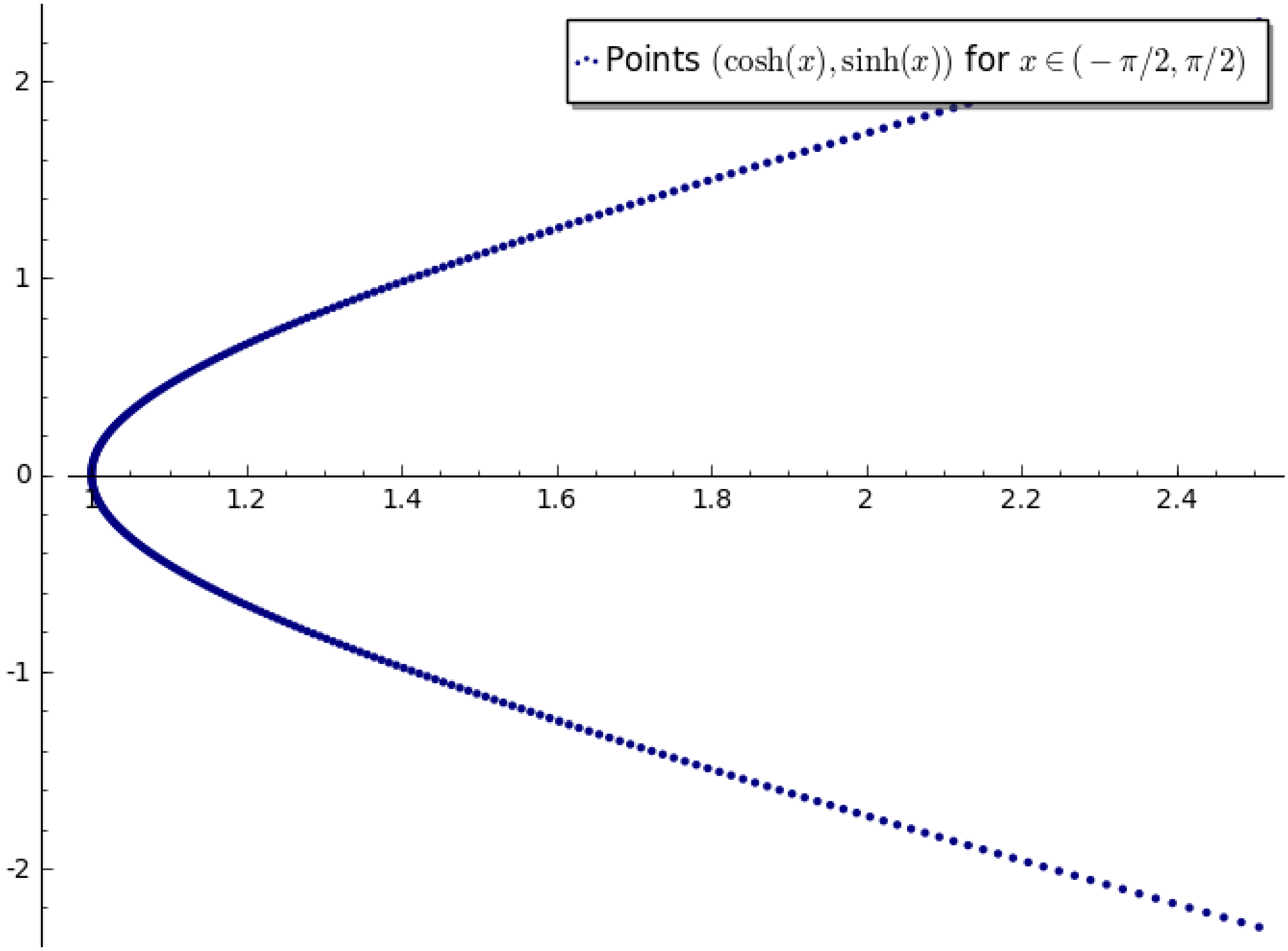


Problem 3.c

Plot points (cosh(x), sinh(x)) for a large range of x-values.

I went with x-values ranging from approximately -pi/2 to pi/2, with a step size of 0.01.

```
pointList1 = [ (cosh(x), sinh(x)) for x in [-1.57, -1.56, .., 1.57] ]
PL1 = points(pointList1, color='navy', legend_label=('Points $(\cosh(x),
\sinh(x))$ for $x \in (-\pi/2, \pi/2)$'))
show(PL1, figsize=7)
```



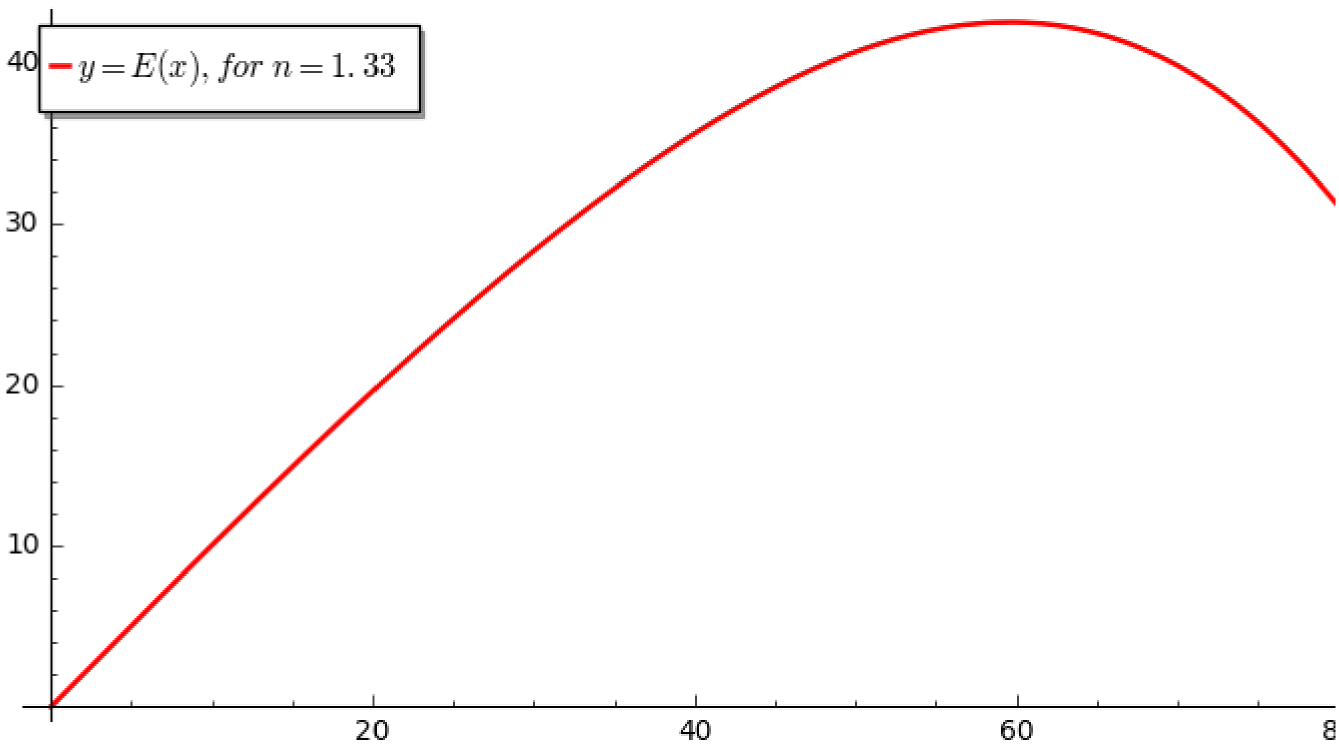
Problem 4.b

Use Sage to graph the E(α) function when n = 1.33 (red light) and to estimate the maximum value to two decimal places.

We define the various refraction indices, functions for each index, and plots for each function, finally, we show the red function.

```
n_red = 1.33
n_orange = 1.333
n_green = 1.336
n_blue = 1.34

k = pi/180

f_r(x) = (4*arcsin(sin(k*x)/n_red) - 2*k*x) / k
f_o(x) = (4*arcsin(sin(k*x)/n_orange) - 2*k*x) / k
```

```
f_g(x) = (4*arcsin(sin(k*x)/n_green) - 2*k*x) / k
f_b(x) = (4*arcsin(sin(k*x)/n_blue) - 2*k*x) / k

RED = plot(f_r(x), 0, 90, color='red', thickness=1.8, legend_label='$y =
E(x), for \ n = 1.33$')
ORANGE = plot(f_o(x), 0, 90, color='orange', thickness=1.8,
legend_label='$y = E(x), for \ n = 1.333$')
GREEN = plot(f_g(x), 0, 90, color='green', thickness=1.8,
legend_label='$y = E(x), for \ n = 1.336$')
BLUE = plot(f_b(x), 0, 90, color='blue', thickness=1.8, legend_label='$y
= E(x), for \ n = 1.34$')

show(RED, aspect_ratio=1, figsize=8)
```

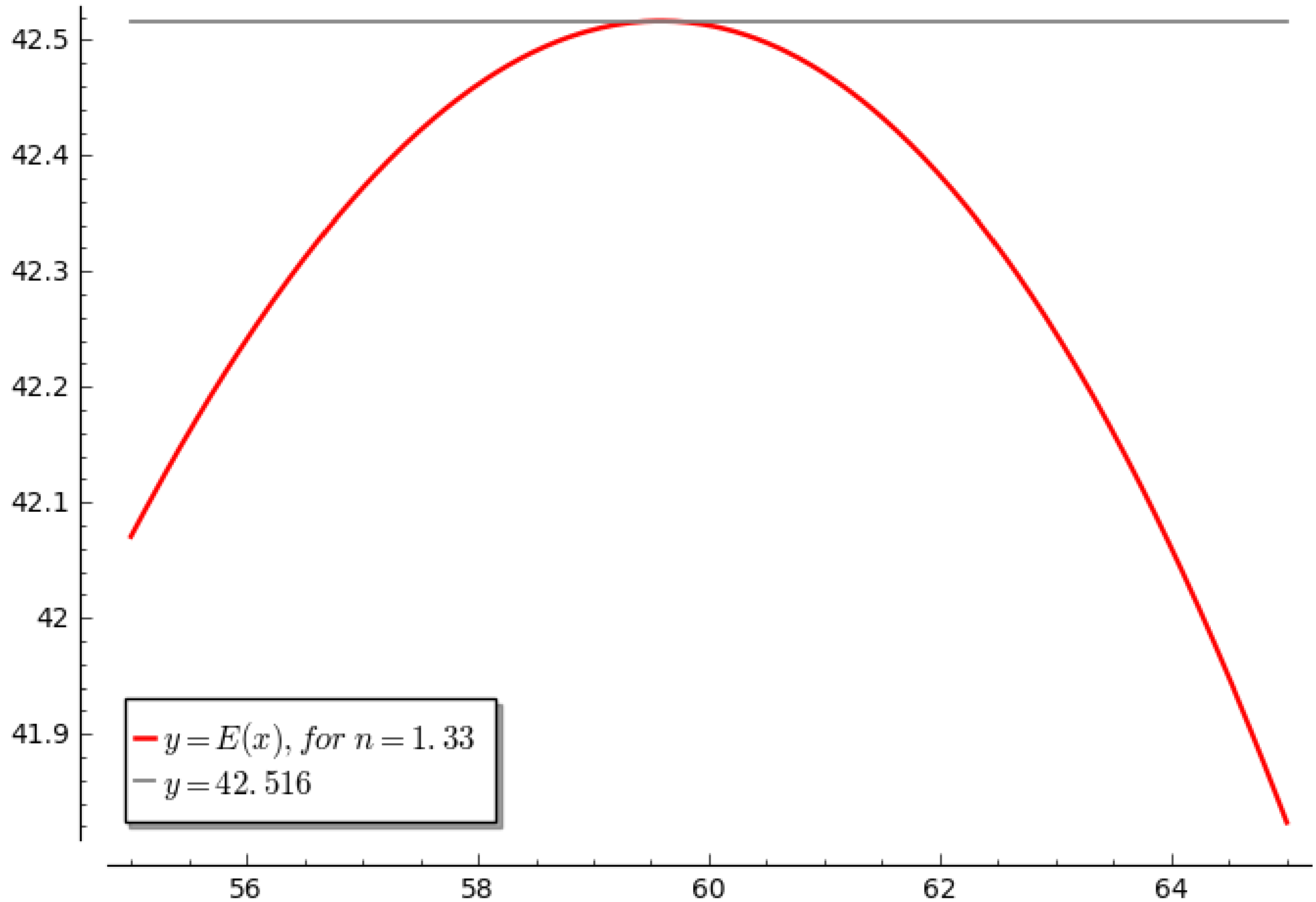


By trial and error, I whittled the estimate down to 42.516; from there we use the find_root command to approximate the maximum.

```
max_Degrees = 42.516

g(x) = max_Degrees
max_Line = plot(g(x), 55, 65, color='grey', thickness=1.3,
legend_label='$y = 42.516$')
max_RED = plot(f_r(x), 55, 65, color='red', thickness=1.8,
legend_label='$y = E(x), for \ n = 1.33$')

show(max_RED+max_Line, aspect_ratio=10, figsize=7)

RED_hat = f_r(x) - max_Degrees
RED_max = find_root(RED_hat, 55,65)
```

```
print 'Estimated maximum for Red =', N(max_Degrees, digits=5), 'at', 'x
=', N(RED_max, digits=5), 'degrees'
```

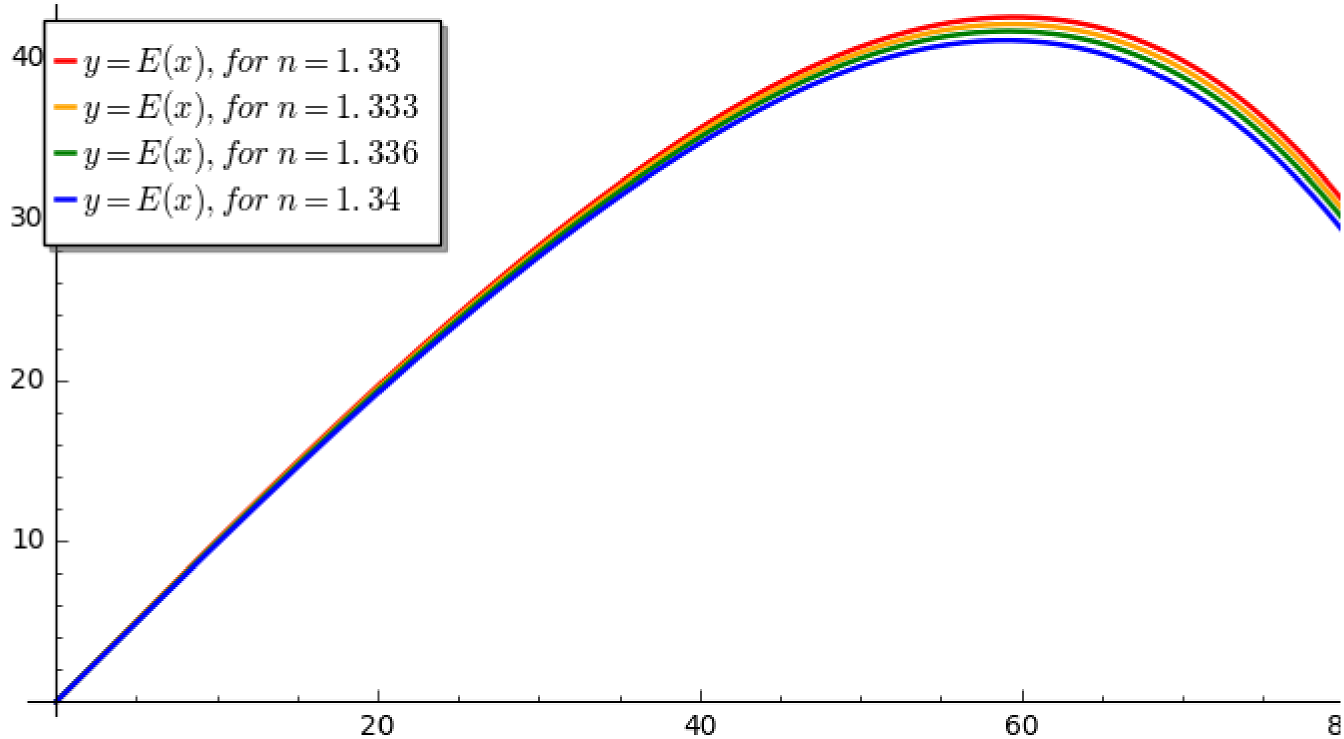
```
Estimated maximum for Red = 42.516 at x = 59.716 degrees
```

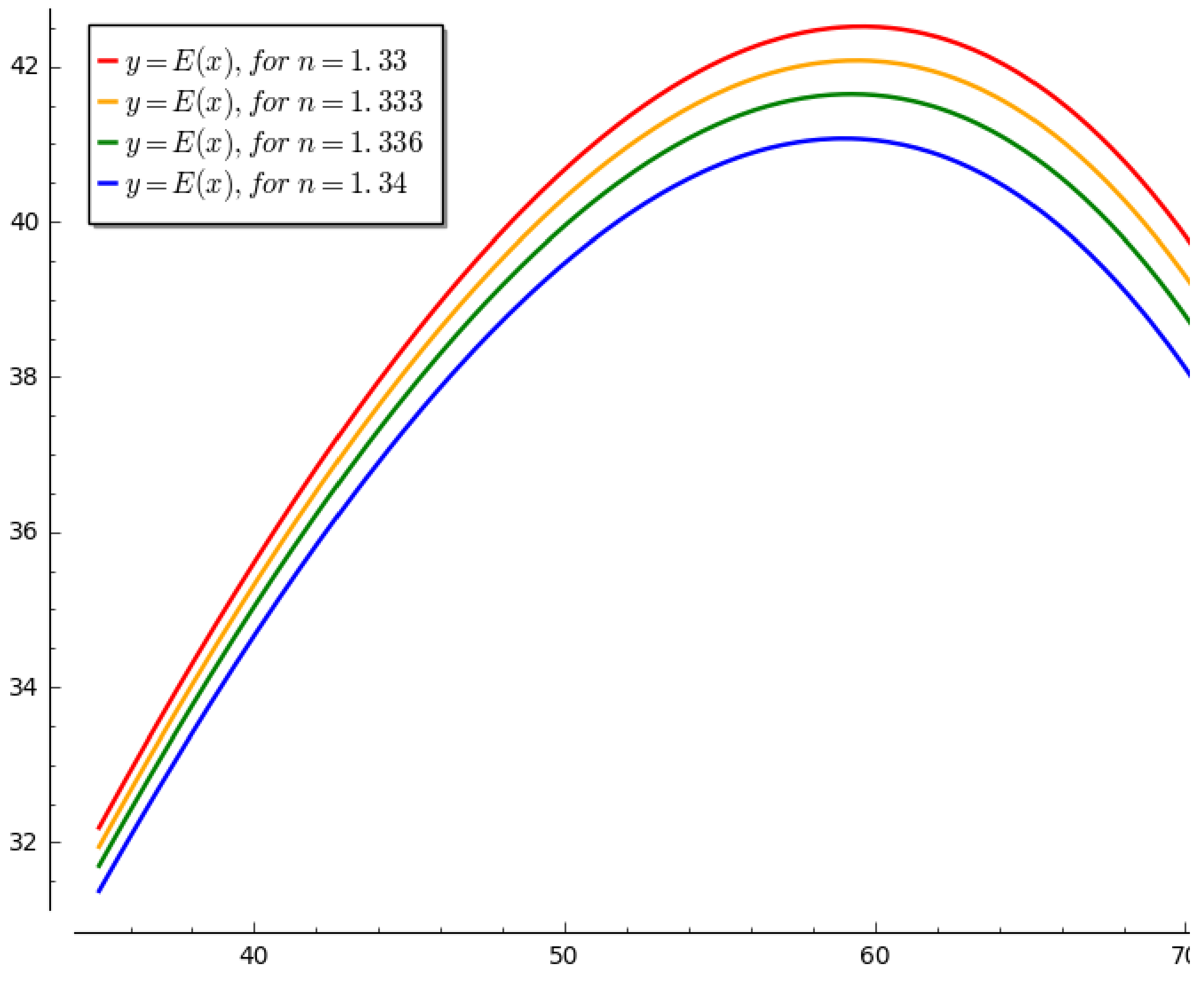


Problem 4.c

Use Sage to plot these four functions on a single graph, and use matching colors.

```
RED_2 = plot(f_r(x), 35, 75, color='red', thickness=1.8,
legend_label='$y = E(x), for \ n = 1.33$')
ORANGE_2 = plot(f_o(x), 35, 75, color='orange', thickness=1.8,
legend_label='$y = E(x), for \ n = 1.333$')
GREEN_2 = plot(f_g(x), 35, 75, color='green', thickness=1.8,
legend_label='$y = E(x), for \ n = 1.336$')
BLUE_2 = plot(f_b(x), 35, 75, color='blue', thickness=1.8,
legend_label='$y = E(x), for \ n = 1.34$')

show(RED+ORANGE+GREEN+BLUE, aspect_ratio=1, figsize=8)
show(RED_2+ORANGE_2+GREEN_2+BLUE_2, aspect_ratio=2.5, figsize=8)
```

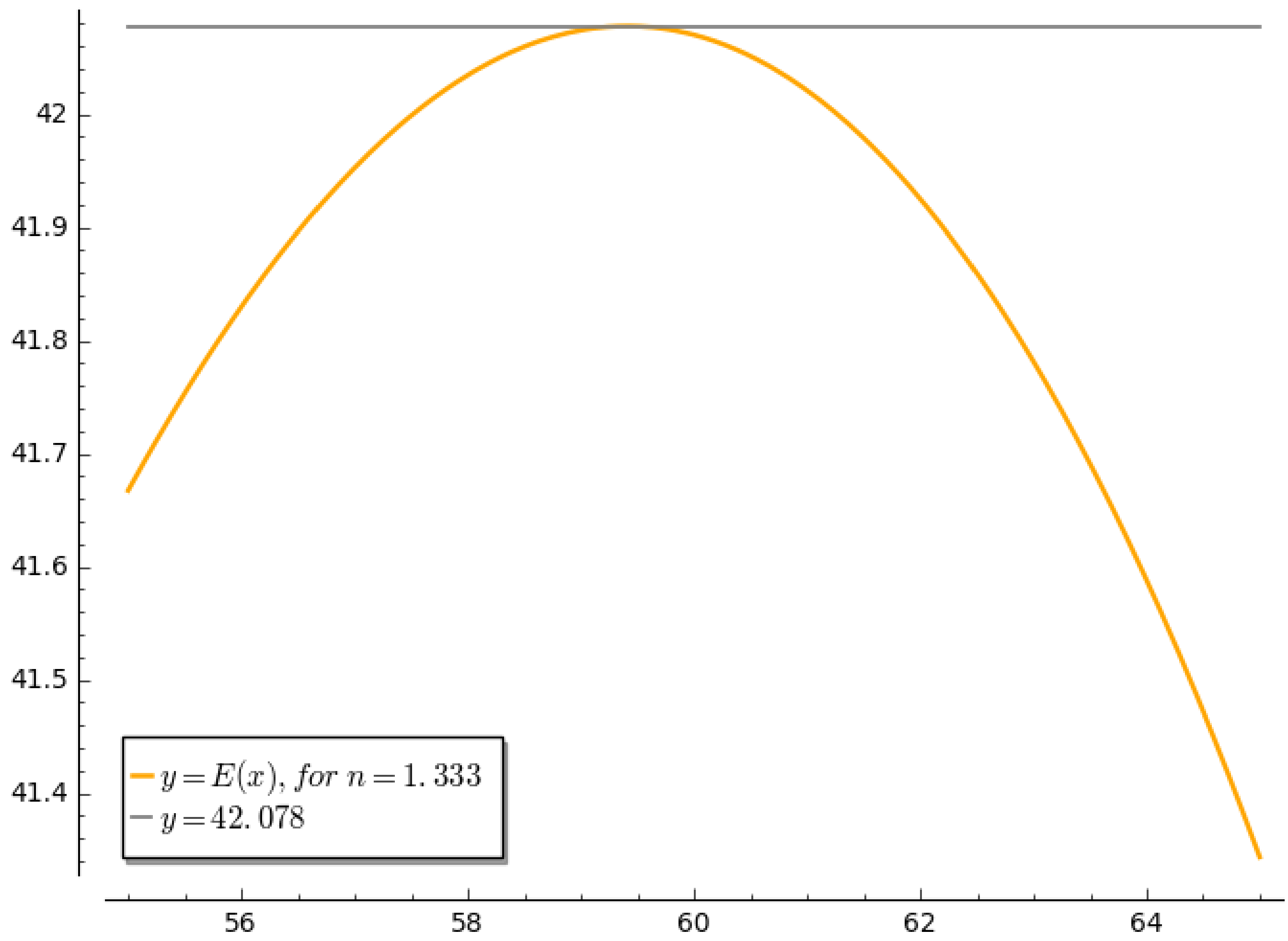

$y=E(x)$, for $n=1.33$
$y=E(x)$, for $n=1.333$
$y=E(x)$, for $n=1.336$
$y=E(x)$, for $n=1.34$
40
30
20
10
20
40
60
8


$y=E(x)$, for $n=1.33$
$y=E(x)$, for $n=1.333$
$y=E(x)$, for $n=1.336$
$y=E(x)$, for $n=1.34$
42
40
38
36
34
32
40
50
60
7(

Problem 4.d

```
max_Degrees = 42.078

g(x) = max_Degrees
max_Line = plot(g(x), 55, 65, color='grey', thickness=1.3,
legend_label='$y = 42.078$')
max_ORANGE = plot(f_o(x), 55, 65, color='orange', thickness=1.8,
legend_label='$y = E(x), for \ n = 1.333$')

show(max_ORANGE+max_Line, aspect_ratio=10, figsize=7)

ORANGE_hat = f_o(x) - max_Degrees
ORANGE_max = find_root(ORANGE_hat, 55,65)
print 'Estimated maximum for Orange =', N(max_Degrees, digits=5), 'at',
'x =', N(ORANGE_max, digits=5), 'degrees'
```

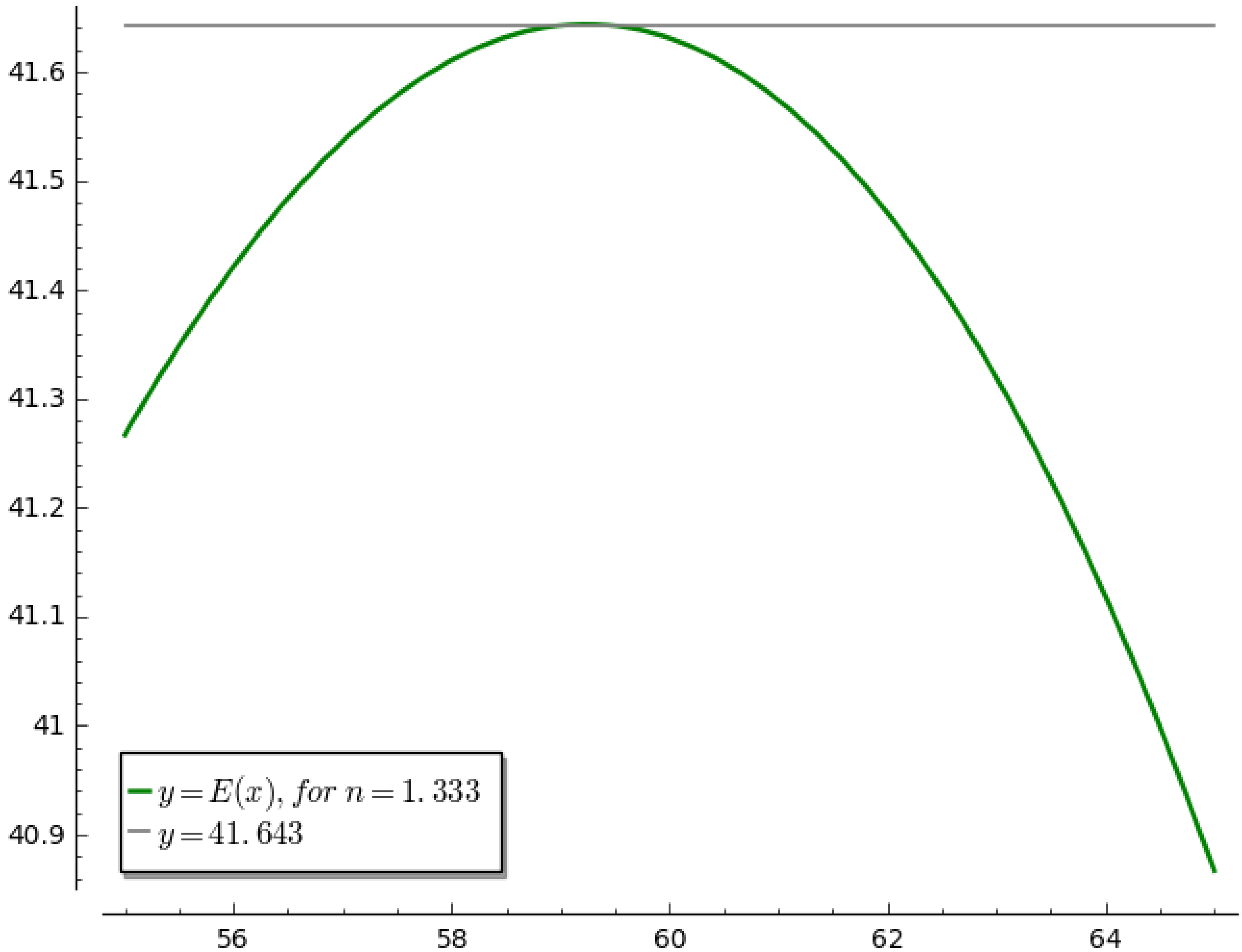
```
Estimated maximum for Orange = 42.078 at x = 59.480 degrees
```




```
max_Degrees = 41.643

g(x) = max_Degrees
max_Line = plot(g(x), 55, 65, color='grey', thickness=1.3,
legend_label='$y = 41.643$')
max_GREEN = plot(f_g(x), 55, 65, color='green', thickness=1.8,
```

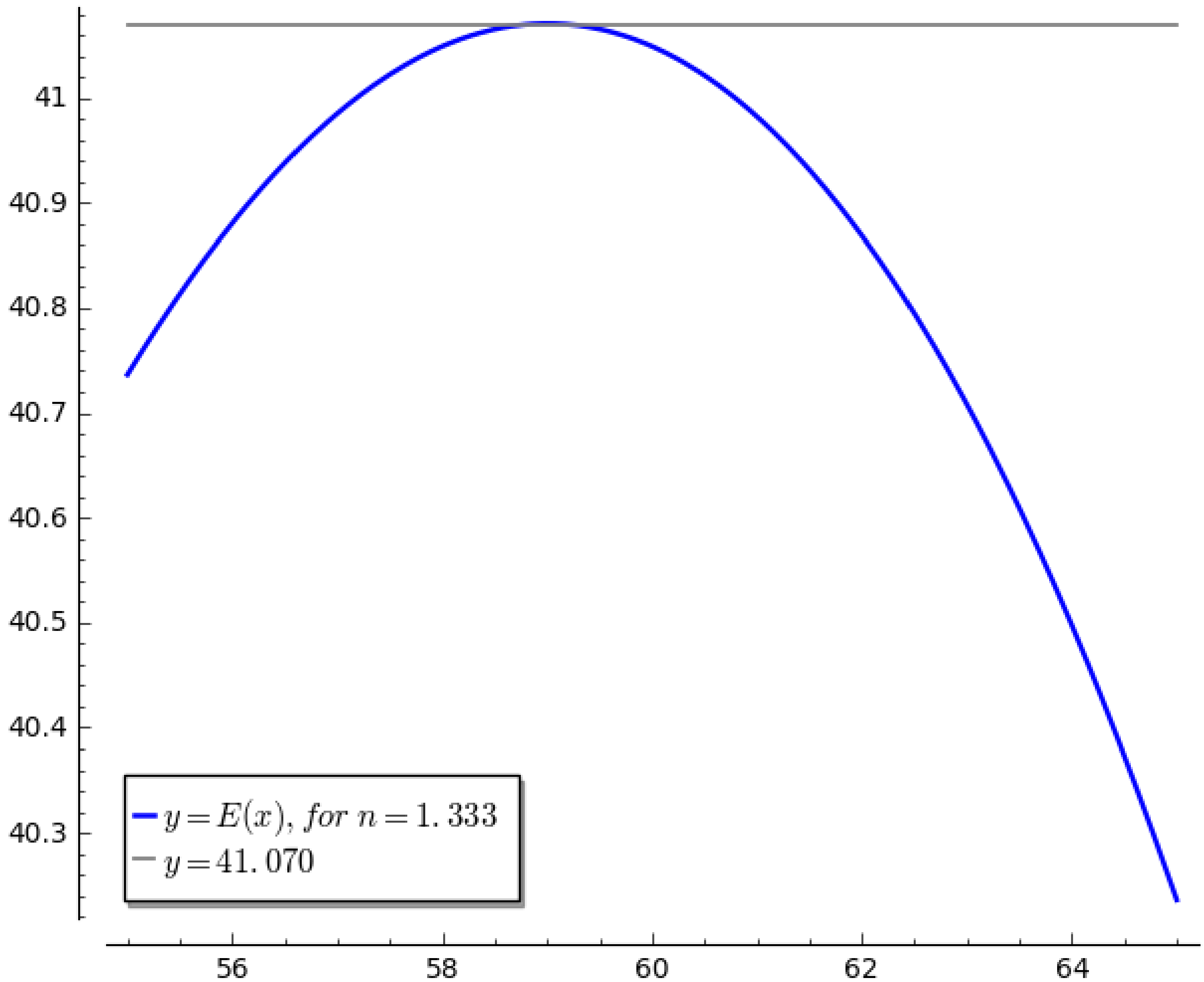
```
legend_label='$y = E(x), for \ n = 1.333$')

show(max_GREEN+max_Line, aspect_ratio=10, figsize=7)

GREEN_hat = f_g(x) - max_Degrees
GREEN_max = find_root(GREEN_hat, 55,65)
print 'Estimated maximum for Green =', N(max_Degrees, digits=5), 'at',
'x =', N(GREEN_max, digits=5), 'degrees'
```

```
Estimated maximum for Green = 41.643 at x = 59.432 degrees
```




```
max_Degrees = 41.0705

g(x) = max_Degrees
max_Line = plot(g(x), 55, 65, color='grey', thickness=1.3,
legend_label='$y = 41.070$')
max_BLUE = plot(f_b(x), 55, 65, color='blue', thickness=1.8,
legend_label='$y = E(x), for \ n = 1.333$')

show(max_BLUE+max_Line, aspect_ratio=10, figsize=7)

BLUE_hat = f_b(x) - max_Degrees
BLUE_max = find_root(BLUE_hat, 55,65)
print 'Estimated maximum for Blue =', N(max_Degrees, digits=5), 'at', 'x
=', N(BLUE_max, digits=5), 'degrees'
```

```
Estimated maximum for Blue = 41.070 at x = 59.153 degrees
```

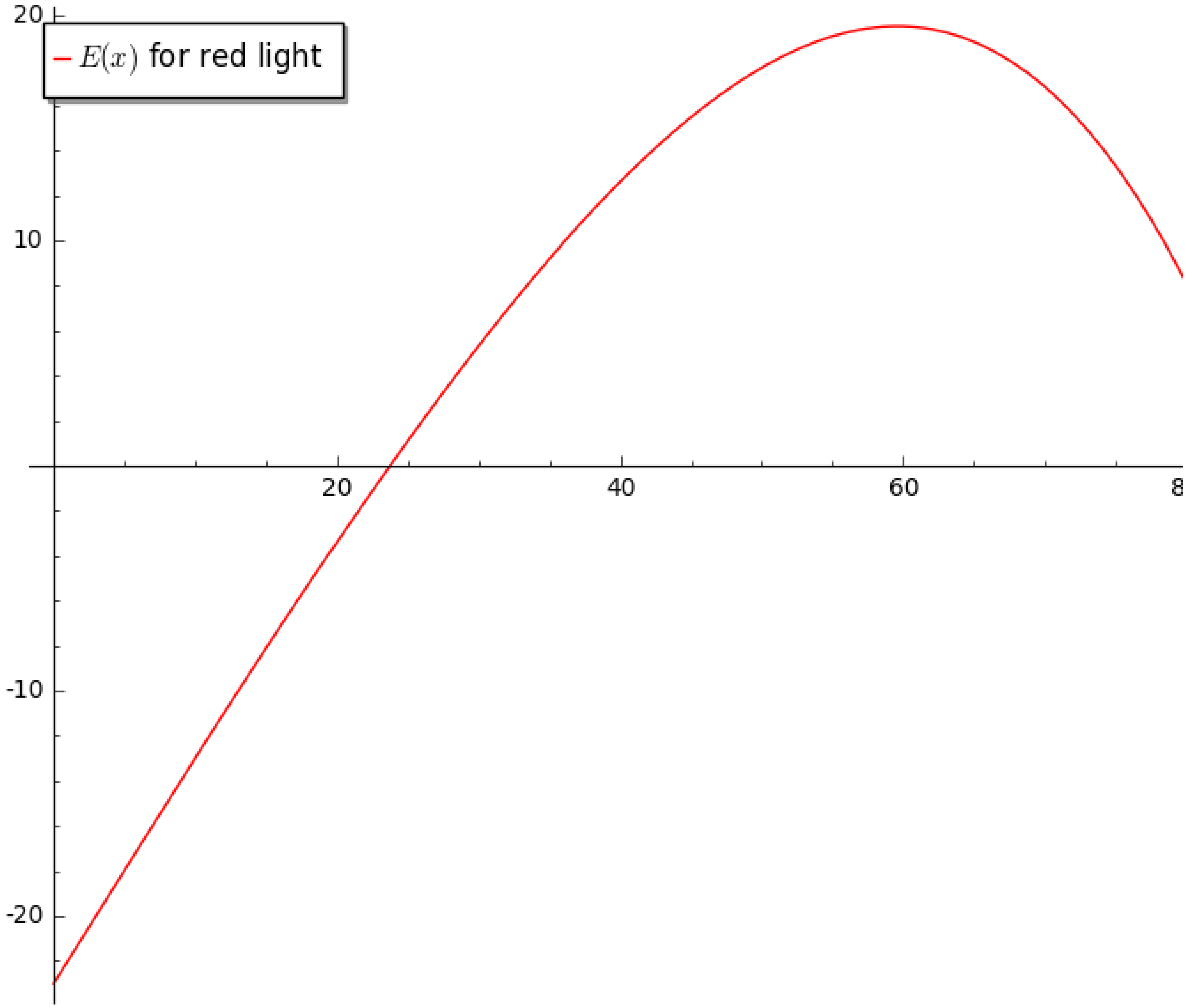


Problem 4.e

For red light, there are two incoming angles x_1 and x_2 for which the reflected ray has E = 23. Find x_1 and x_2, giving them in degrees. (You will probably need to use Sage for this part).

```
R1 = f_r(x) - 23
R1_plot = plot(R1, 0,90, color='red', legend_label='$E(x)$ for red light')
show(R1_plot)

x_1 = N(find_root(R1, 20,30), digits=5)
x_2 = N(find_root(R1, 80,90), digits=5)

print 'The two incoming angles for which the reflected ray has an angle of 23 degrees are', 'x_1 =', x_1, 'degrees, and x_2 =', x_2, 'degrees.'
```

```
The two incoming angles for which the reflected ray has an angle of 23 degrees are x_1 = 23.742 degrees, and x_2 = 85.627 degrees.
```


20
10
-10
-20
20
40
60
8
$E(x)$ for red light