Math 397
Guide for final paper

For the "final paper" in this course, you will choose a calculus topic that either involves some kind of real-world application or lends itself to numerical experimentation using a computer. You will discuss the mathematics behind the topic and ideally do some modeling or experimentation using Sage. The main difference from the midterm is that I want about 50% of the paper to be new material that you haven't seen already in the course. Fifty percent of the *concepts* don't necessarily have to be new—I'm not asking that you go learn a lot of new material you haven't seen before—but 50% of the paper should be spent on new examples or new explorations that you haven't seen before in this course.

For resources, you are allowed to use anything: class notes, the readings posted on the course website, books, other things you find online. And you can come talk to me at any time either about ideas or questions. However, what you write must be in your own words and you should put some thought into how you present things. Some important things to keep in mind:

- Tell a story. Mathematics is not just a collection of facts, there should always be an underlying narrative.
- Do not write with me in mind as your intended audience. Imagine that you are writing for someone else in the class who does not understand the material as well as you do.
- Because I want 50% of the paper to be new stuff, you will most likely need to choose a topic different from your midterm topic. However, if you worked on a particularly rich topic, you might be able to continue with it by adding more examples or exploring other uses of the same idea.

The project will be due at noon on Tuesday, June 12 (finals week).

Guidelines

The paper must be written in LaTeX, and be 8–12 pages single-spaced (the 12 page limit is not absolute, but if you get really excited about your topic try not to go much over 12).

You don't have to use the midterm template, but the file shows how to set up your title, sections, subsections, Remarks, Theorems, Examples, and so forth. You can use this file as your base and just add your own text, or simply copy the pieces you need for your paper.

Your paper must include each of the following:

- At least one displayed math environment;
- At least one itemize or enumerate environment;
- At least three diagrams or graphs (not drawn in by hand);

- At least one numbered equation and a reference to it;
- At least one use of the Definition environment;
- At least one use of a Remark/Example/Theorem environment;
- A bibliography (potentially very small) and a citation in the body of the paper.

Grading rubric:

Grades will be on a 30-point scale, organized as follows:

depth and correctness	8
meets guidelines	3
typesetting style	4
storytelling	5
modeling/experimentation	5
innovation	5

You should expect tougher grading than the midterm. Here's what I mean by each of these:

depth and correctness: Is what you wrote mathematically correct? Does it go into a reasonable amount of depth? Does it explain enough about the problem so that people can follow and understand?

meets guidelines: Does your paper satisfy the LaTeX guidelines?

typesetting style: Does the typesetting look good and conform to preferred mathematical conventions? Have the LaTeX commands that you should know at this point in the course been used where appropriate? See the LaTeX resources provide for the midterm for help.

storytelling : Did you convey a narrative rather than just a collection of facts? Could a student with the assumed background follow your steps and understand what you have written?

modeling/experimentation : To what extent did you use Sage (or other mathematical software) to shed light on the concepts of the paper? How complex were the examples that you explored? Do they demonstrate only basic information, or do they explore deeper material?

innovation : If everything you write is something that I did in class, or you saw on a HW assignment, or is in a course reading, that's zero innovation. For a 5 score in this category, around 50% of the material in your paper should feel new to me when I read it.

Note that these things are closely tied together. It is unlikely that someone could get a 5 in storytelling if they also get a 0 in depth and correctness. It is very hard to tell a good story based on incorrect knowledge.

Here are some possible topics for the paper. These are not meant to be exhaustive, just ideas to get you started. If you have another topic you would like to use it is probably fine, just consult with me to make sure.

(1) Talk about the general theory of Taylor series and radii of convergence. Explore what Taylor approximations look like in Sage, and how many terms it takes for certain Taylor series to give a decent approximation to a function. As part of the same or a similar project, you could explore how fast it takes some infinite series to converge. There are various infinite series for computing π, and you could explore which give better approximations.

(2) Solving and modeling basic physics problems involving calculus. We've done some of these in class this quarter, but there are tons of others where one combines pendulums, springs, and moving objects in various ways.

You could also use Sage to see what solutions to physics problems look like even when they can't be solved exactly. For example, what if there were a universe where pendulums behaved according to the equation $\theta'' = -\sin^2\theta$? What would the motion of these pendulums look like?

Another way to go with this project is to explore Lagrangian mechanics and how physics problems are solved using the calculus of variations.

(3) Explore the theory of Riemann sums, the method of exhaustion, and similar ideas. How could you use Sage to implement these computations? What visual aids can you produce to help understand these ideas?

(4) You could probably write a whole paper on the hyperbolic trig functions cosh and sinh. Where do they come from, what are they good for, what kinds of problems are they used to solve?

(5) You could probably write a paper on the history of e^x and $\ln x$. When did they first arise? How a were they defined? Why were they defined that way?

(6) Investigate the Euler method for finding approximate solutions to differential equations and how this compares to the Runge-Kutta method in practice. Explore situations where the Euler method doesn't work very well, compare the approximate values of the Euler method to an analytical solution, etc.

(7) Explore the Kepler problem for motion under a central force (like planetary motion). There are many things you can do here. Figure out how to have Sage find numerical solutions and experiment with what the orbits look like under different initial conditions. Instead of 2-body problems, try to analyze 3-body problems. Instead of a $\frac{1}{r^2}$ force law, figure out what trajectories look like for a $\frac{1}{r}$ law or a $\frac{1}{r^3}$ law.

(8) Curves. There are all kinds of curves and questions about them. The cycloid, the Archmidean spiral, the deloid...try looking up "List of curves" in Wikipedia for a rather expansive selection. For every curve there are interesting questions about its arclength, area enclosed by it, etc. Here's a question we didn't explore about the Brachistochrone problem: how much faster is it for a bead to fall down the brachistochrone curve than to go down a straight line? What about compared to a parabolic curve, or to an exponential curve? These questions can be answered with Sage.