MATH 441 CRN 33477 Quiz 4

June 4th, 2018

(1) Suppose p is the polynomial defined by $p(x) = x^2 + 3x - 1$. If $T \in \mathcal{L}(\mathbb{R}^2)$ has the matrix $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ with respect to some basis of $\mathbb{R}^2$, find the matrix of $p(T)$ with respect to the same basis.

$$\mathcal{M}(p(T))$$
$$= \mathcal{M}(T^2 + 3T - I) = \mathcal{M}(T^2) + \mathcal{M}(3T) - \mathcal{M}(I) = \mathcal{M}(T)\mathcal{M}(T) + 3\mathcal{M}(T) - \mathcal{M}(I)$$

$$= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} + 3 \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 3 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 0 & 3 \end{bmatrix}.$$

(2) Suppose $T \in \mathcal{L}(\mathbb{F}^3)$ is not injective. Prove that $\dim E(2, T) \leqslant 2$.

Proof. Otherwise, $\dim E(2, T) = 3$ and $E(2, T) = \mathbb{F}^3$. Then $\operatorname{null}(T - 3I) = \mathbb{F}^3$ and $T - 3I = 0$. Therefore $T = 3I$, which contradicts that T is not injective.