HOMEWORK 5 – MATH 441
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ALEX THIES
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Assignment: 3.B - 6, 16, 21; 3.C - 4, 10, 14; 3.D - TBA;

Section 3.B

Problem 6. Prove that there does not exist a linear map $T : \mathbb{R}^5 \to \mathbb{R}^5$ such that

$$\text{range } T = \text{null } T.$$

Proof. □

Problem 16. Suppose there exists a linear map on V whose null space and range are both finite-dimensional. Prove that V is finite-dimensional.

Proof. □

Problem 21. Suppose V is finite-dimensional and $T \in \mathscr{L}(V, W)$. Prove that T is surjective if and only if there exists $S \in \mathscr{L}(V, W)$ such that TS is the identity map on W.

Proof. □

Section 3.C

Problem 4. Suppose $v_1, \ldots, v_m$ is a basis of V and W is finite-dimensional. Suppose $T \in \mathscr{L}(V, W)$. Prove that there exists a basis $w_1, \ldots, w_m$ of W such that all the entries in the first column of $\mathcal{M}(T)$ (with respect to the bases $v_1, \ldots, v_m$ and $w_1, \ldots, w_m$) are 0 except for possibly a 1 in the first row, first column.

Proof. □

Problem 10. Suppose A is an m-by-n matrix and C is an n-by-p matrix. Prove that

$$(AC)_{j,\cdot}$$

In other words, show that row j of AC equals (row j of A) times C.

Proof. □

Problem 14. Prove that matrix multiplication is associative. In other words, suppose A, B, and C are matrices whose sizes are such that $(AB)C$ makes sense. Prove that $A(BC)$ makes sense and that $(AB)C = A(BC)$.

Proof. □

Section 3.D