HOMEWORK 2 – MATH 441
April 16, 2018

ALEX THIES
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Assignment: 1.C - 21, 23; 2.A - 5, 6, 9, 10, 12, 14, 15

Section 1.C

Problem 21. Suppose

$$U = \{(x, y, x + y, x - y, 2x) \in \mathbb{F}^5 : x, y \in \mathbb{F}\}$$

Find a subspace W of $\mathbb{F}^5$ such that $\mathbb{F}^5 = U \oplus W$.

Solution. □

Problem 23. Prove or give a counterexample: if U_1, U_2, W are subspaces of V such that

$$V = U_1 \oplus W \quad \text{and} \quad V = U_2 \oplus W$$

then $U_1 = U_2$.

Solution. □

Section 2.A

Problem 5.

(a) Show that if we think of $\mathbb{C}$ as a vector space over $\mathbb{R}$, then the list $(1 + i, 1 - i)$ is linearly independent.
(b) Show that if we think of $\mathbb{C}$ as a vector space over $\mathbb{C}$, then the list $(1 + i, 1 - i)$ is linearly dependent.

Solution. □

Problem 6. Suppose v_1, v_2, v_3, v_4 is linearly independent in V. Prove that the list

$$v_1 - v_2, v_2 - v_3, v_3 - v_4, v_4$$

is also linearly independent.

Proof. □

Problem 9. Prove or give a counterexample: If $v_1, \ldots, v_m$ and $w_1, \ldots, w_m$ are linearly independent lists of vectors in V, then $v_1 + w_1, \ldots, v_m + w_m$ is linearly independent.

Proof. □

Problem 10. Suppose $v_1, \ldots, v_m$ is linearly independent in V and $w \in V$. Prove that if $v_1 + w, \ldots, v_m + w$ is linearly dependent, then $w \in \operatorname{Span}(v_1, \ldots, v_m)$.

Proof. $\square$

Problem 12. Explain why there does not exist a list of six polynomials that is linearly independent in $\mathcal{P}_4(\mathbb{F})$.

Proof. $\square$

Problem 14. Prove that V is infinite-dimensional if and only if there is a sequence $v_1, v_2, \ldots$ of vectors in V such that $v_1, \ldots, v_m$ is linearly independent for every positive integer m.

Proof. $\square$

Problem 15. Prove that $\mathbb{F}^\infty$ is infinite-dimensional.

Proof. $\square$