MATH 441 CRN 33477 Quiz 1

April 16th, 2018

(1) Prove that the subset $\{(x_1, x_2, x_3) \in \mathbb{C}^3 : x_1 x_2 = 0\}$ is not a subspace of the vector space $\mathbb{C}^3$.

Proof. Let $U = \{(x_1, x_2, x_3) \in \mathbb{C}^3 : x_1 x_2 = 0\}$.
Then $(1, 0, 0) \in U$ and $(0, 1, 0) \in U$, but the sum of these two vectors, which equals $(1, 1, 0)$, is not in U. Thus U is not closed under addition. Thus U is not a subspace of $\mathbb{C}^3$.

(2) Suppose v_1, v_2, v_3 spans V. Prove that the list $v_1, v_2 + v_3, -v_3$ also spans V.

Proof. Let $v \in V$.

Because v_1, v_2, v_3 spans V, there exist $b_1, b_2, b_3 \in \mathbb{F}$ such that $v = b_1 v_1 + b_2 v_2 + b_3 v_3$.

Then $v = b_1 v_1 + b_2 v_2 + (b_2 v_3 - b_2 v_3) + b_3 v_3 = b_1 v_1 + (b_2 v_2 + b_2 v_3) + (-b_2 v_3 + b_3 v_3)$

$= b_1 v_1 + b_2 (v_2 + v_3) + (b_2 - b_3)(-v_3)$.

Thus $v = a_1 v_1 + a_2 (v_2 + v_3) + a_3 (-v_3)$ for $a_1 = b_1,\ a_2 = b_2,\ a_3 = b_2 - b_3$.

Therefore $v_1, v_2 + v_3, -v_3$ spans V by definition.