

Discussion Problems

Name: \_\_\_\_\_

**Worksheet 1: Integration Techniques**

Math 408D:

Instructor: Athil George

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**Problem 1.** Solve the following indefinite integrals using integration by parts (Hint: Use u-substitution first for the first one). Reminder: Don't forget '+C' if you are solving an indefinite integral!

**Problem 2.** Evaluate the following integrals using a trigonometric substitution.

1.  $\int \frac{1}{\sqrt{x^2+16}} dx$

2.  $\int_0^a \frac{1}{(a^2+x^2)^{\frac{3}{2}}} dx, a > 0$

**Problem 3.** Evaluate the following trigonometric integrals using either a trigonometric substitution or identity.

1.  $\int \csc(x) \, dx$
2. Recall the strategy you learned in Lecture 2 to evaluate integrals of the form  $\int \sin^m(x) \cos^n(x) \, dx$ . This can also be found in the blue boxed text in page 481 of your textbook. Using this strategy, evaluate  $\int \sin^3(x) \cos^4(x) \, dx$
3. Recall the strategy you learned in Lecture 2 to evaluate integrals of the form  $\int \tan^m(x) \sec^n(x) \, dx$ . This can also be found in the blue boxed text in page 482 of your textbook. Using this strategy, evaluate  $\int \tan^2(x) \sec^4(x) \, dx$
4.  $\int_0^\pi \cos^4(t) \, dt$

**Problem 4.** Find a formula for the area of an ellipse with the equation  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ . Don't worry about value  $c$  in Figure 1. This denotes the location of the focus of the ellipse ( $c^2 = a^2 - b^2$ ). Hint: Solve for  $y$  first! The area,  $A$ , should be in terms of  $a$  and  $b$ ,  $A = A(a, b)$ .

**Problem 5.** Find the area of the lune shown in Figure 2.

**Problem 6.** Evaluate the following indefinite integral. There are many steps to this problem.

$$\int x^2 \sqrt{x^2 + 2x + 5} \, dx$$

## Figures

### Problem 4

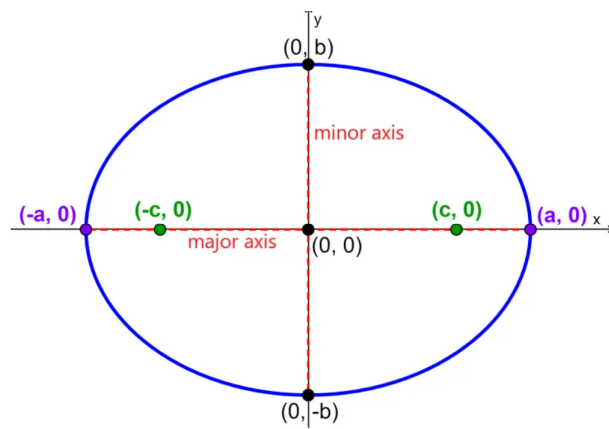


Figure 1: Graph of Ellipse

### Problem 5

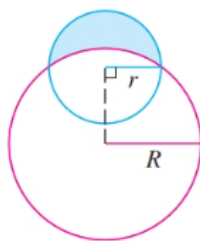


Figure 2: Graph of Intersecting Circles Forming Lune