Discussion Problems **Name:** ____________________

Worksheet 4: Series: Integral and Comparison Tests

Math 408D:

Instructor: Athil George

Problem 1. Find the convergent value of the following geometric series:

1. $$\sum_{n=1}^{\infty} \frac{5^{n+1}}{6^n}$$

2. $$\sum_{n=0}^{\infty} \frac{4^{n+1}}{3^{2n}}$$

3. $$\sum_{n=2}^{\infty} e^{-n}$$

Problem 2. Show that the following sum converges:

$$\sum_{n=1}^{\infty} ne^{-2n^2}$$

Problem 3. Consider the series:

$$\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$$

Answer the following questions:

1. Does this series converge by integral test? What did the integral evaluate to?
2. Find the convergent value. Is this the same as the numerical result of the integral test?

Problem 4. Determine if the following series converges or diverges:

$$\sum_{n=2}^{\infty} \frac{(2n-1)(n^2-1)}{(n+1)(n^2+4)^2}$$

Problem 5. A series $\sum_{n=1}^{\infty} a_n$ is called absolutely convergent if $\sum_{n=1}^{\infty} |a_n|$ is convergent. This is not an if and only if statement, meaning its sufficient but not necessary. In other words, it does not go the other way. Is the following series absolutely convergent?

$$\sum_{n=1}^{\infty} \frac{\cos(n)}{n^2}$$

Problem 6. Prove:

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$