Discussion Problems **Name:** ________________________

Worksheet 11: Optimization

Math 408C:

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Problem 1. We have two people running lines relative to a reference with displacement

$$s_1(t) = t^2 - 10t$$
$$s_2(t) = t(t-5)(t-10)$$

Clearly, both runners come back to their original positions after 10 seconds.

(a) Find the $\lim_{t\to 10} \frac{s_1(t)}{s_2(t)}$

(b) What is the significance of this limit?

(c) Compare/Discuss the existence of this limit in this example, and in a general example where $s_1(t)$ and $s_2(t)$ are replaced generically with f(x) and g(x) . Note: Do problem 5 in 4.4 first before answering this part.

Problem 2. Small birds like finches alternate between flapping their wings and keeping them folded while gliding. In this project, we analyze this phenomenon and try to determine how frequently a bird should flap his wings. Some of the principles are the same as for fixed wing aircraft and so we begin by considering how required power and energy depend on the speed of airplanes.

1. The power required to propel an airplane forward at velocity, v, is:

$$P = Av^3 + \frac{BL^2}{v}$$

   Note that A and B are constants specific to the aircraft and L is the lift, the upward force supporting the weight of the plane. Find the speed that minimizes the required energy.

2. The speed found in Problem 1 minimizes power but a faster speed might use less fuel. The energy needed to propel the airplane a unit distance is $E = P/v$. At what speed is energy minimized?

3. How much faster is the speed for minimum energy than the speed for minimum power?

4. In applying the equation for Problem 1 to bird flight we split the term Av^3 into two parts: $A_b v^3$ for the bird's body, and $A_w v^3$ for the bird's wing. Let x be the fraction of time in flapping mode. If m is the bird's mass and all the lift occurs during flapping, then the lift is mg/x and so the power needed during flapping is:

$$P_{flap} = (A_w + A_b)v^3 + \frac{B(mg/x)^2}{v}$$

   The power while wings are folded is $P_{fold} = A_b v^3$. Show that the average power over an entire flight cycle is:

$$\bar{P} = xP_{flap} + (1-x)P_{fold} = A_b v^3 + xA_w v^3 + \frac{Bm^2g^2}{xv}$$

5. For what value of x is the power a minimum? What can you conclude if the bird flies slowly?What can you conclude if the bird flies faster and faster?

6. The average energy over a cycle is $\bar{E} = \bar{P}/v$. What is the value of x that minimizes $\bar{E}$?

Problem 3. Consider a projectile being shot with some velocity v_0 at an angle of θ. The height is given as $h(t) = y(t) = v_0 \sin(\theta) - 0.5gt^2$. We also are given that $x(t) = v_0 \cos(\theta)t$. Show that the projectile's range is maximized at $\theta = \frac{\pi}{4}$ or $\theta = \frac{3\pi}{4}$. Ignore air resistance. You do not need to know the value of g.