Discussion Problems **Name:** ____________________

Worksheet 8: Differential Equations II

Math 408D:

Instructor: Athil George

Problem 1. Solve the following differential equations by finding the general solution:

1. $$y' + xe^y = 0$$

2. $$y' = x^2 y - y + x^2 - 1$$

3. $$xyy' = x^2 + 1$$

Problem 2. A 1500 gallon tank initially contains 600 gallons of water with 5 lbs of salt dissolved in it. Water enters the tank at a rate of 9 gal/hr and the water entering the tank has a salt concentration of

$$C(t) = t[\frac{lb}{gal}]$$

lbs/gal. If a well-mixed solution leaves the tank at a rate of 6 gal/hr, how much salt is in the tank when it overflows?

Problem 3. Find the equation of the curve that passes through (0,2) and whose slope at (x,y) is x/y.

Problem 4. A population is modeled by the differential equation:

$$\frac{dP}{dt} = 1.2P(1 - \frac{P}{4200})$$

(a) For what values of P is the population increasing?

(b) For what values of P is the population decreasing?

(c) What are the equilibrium solutions? Hint: Equilibrium solutions denotes no change in the dependent variable.

Problem 5. Consider the set of parametric equations:

$$x = \cos(t)$$
$$y = 2\sin(t)$$

1. Find $\frac{dy}{dx}$ when $t = \frac{\pi}{2}$.
2. What curve does the parameter t trace out?

Problem 6. Consider the chemical reaction:

$$H_2 + Br_2 \rightarrow 2HBr$$

Let $x = [Hbr]$, the concentration of the product. Let a and b be the initial concentration of bromide and hydrogen,$[H_2]$ and $[Br_2]$. The law of mass action states that the rate of formation of the product is:

$$\frac{dx}{dt} = k(a-x)(b-x)^{1/2}$$

(a) Find $x(t)$ if $a = b$. Use the IC $x(0) = 0$. This means that the reaction has just started and there are no products yet.

(b) Find $t(x)$ if $a > b$. Hint: Use a substitution $u = \sqrt{b-x}$.

Problem 7. A sphere with radius 1 m has temperature 15 degrees C. It lies inside a concentric sphere of radius 2 with temperature 25 degrees C. The temperature $T(r)$ at a distance r from the common center satisfies the following differential equation.

$$\frac{d^2T}{dr^2} + \frac{2}{r}\frac{dT}{dr} = 0$$

Find the general solution to this second order differential equation. Hint: Set $S = \frac{dT}{dr}$. Solve the first-order equation. Then, solve $S = \frac{dT}{dr}$.

Problem 8. Newton's Law of Cooling states that the rate of cooling of an object is proportional to the temperature difference between the object and its surroundings.

$$\frac{dT}{dt} = -k[T - T_{amb}]$$

Answer the following questions:

1. Is this a separable ODE? Why or why not?
2. Find a general solution T(t) to the differential equation.
3. You have a cup of coffee that is at 100 degrees F. The room temperature is 70 degrees F. In 10 minutes, the coffee is 90 degrees F. How hot will the coffee be in 20 minutes? It's not half the original value!
4. What is the meaning of the constant k?
5. Suppose the rate of change of temperature with respect to time for a specific substance is 0. What is the theoretical k value for this substance?

Problem 9. Find the particular solution to the ODE modeling an RC circuit:

$$R\frac{dI}{dt} + \frac{I}{C} = 0$$

Assume $I(0) = \frac{V}{R}$. For those of you who speak physics, the circuit starts by exactly following Ohm's Law.