Discussion Problems **Name:** ______________________

Worksheet 7: Differential Equations I

Math 408D:

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Problem 1. Consider a family of functions:

$$f(x) = \frac{1 + ce^x}{1 - ce^x}$$

1. Show that all of these functions, where c is a constant, satisfy the following first-order differential equation:

$$y' = \frac{1}{2}(y^2 - 1)$$

2. Find a unique solution (a value for c) if f(x) passes through f(0) = 2

Problem 2.

1. For what values of r does the function $y = e^{rx}$ satisfy the differential equation:

$$2y'' + y' - y = 0$$

2. For two values of r ($r_1$and r_2) in the range of r, show that $y = ae^{r_1 x} + be^{r_2 x}$ is also a solution.

Problem 3. Find the power series representation solution of the first-order differential equation:

$$y' + 2xy = 0$$

Hint: Start by assuming y is analytic, which means it can be expressed as an infinite sum.

Problem 4.

1. Consider the 2nd order differential equation:

$$\frac{d^2y}{dx^2} + \omega^2 y = 0$$

   Assume ω is a constant. Show that a general solution to this differential equation is in the form:

$$y(x) = A\cos(\omega x) + b\sin(\omega x)$$

2. The simplified equation of motion (EOM) for a pendulum with small swing angles can be modeled with a similar differential equation:

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta$$

   Find the general solution. You answer should be $\theta(t)$.

3. Based on your answer for the previous part, how many constants appear in the general solution for an nth order differential equation?

4. Assume $\theta(0) = 0$ and $\theta(\pi/\omega^2) = \pi/20$. **In this case, we know two values of the angle at different times.**What is the particular solution? A problem with these structure of initial conditions is called an boundary value problem (BVP)

5. Assume $\theta(0) = \pi/20$ and $\frac{d\theta}{dt}(0) = 0$. **In this case, we know the angle and the angular velocity at the same time** $t = 0$**.** What is the particular solution? A problem with these structure of initial conditions is called an initial value problem (IVP)

Problem 5.

1. Consider the 2nd order differential equation:

$$\frac{d^2y}{dx^2} + \omega^2 y = 0$$

Assume ω is a constant. Show that a general solution to this differential equation is in the form:

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2. The simplified equation of motion (EOM) for a pendulum with small swing angles can be modeled with a similar differential equation:

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Find the general solution. You answer should be $\theta(t)$.

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5. Assume $\theta(0) = \pi/20$ and $\frac{d\theta}{dt}(0) = 0$. **In this case, we know the angle and the angular velocity at the same time $t = 0$.** What is the particular solution? A problem with these structure of initial conditions is called an initial value problem (IVP)