Discussion Problems **Name:** ____________________

Worksheet 6: Taylor and Maclaurin Series

Math 408D:

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Problem 1. Find an infinite power series representation for f(x), where:

$$f(x) = x^3 \arctan(x^2)$$

Problem 2. Use a Taylor expansion of order 3 (T_3) to estimate:

$$H = \sin(0.1)$$

Hint: Since the 0.1 is close to 0, center your expansion around $x = 0$. This special expansion is called a Maclaurin Expansion. What is the error from the true value of H?

Problem 3. Consider:

$$I(x) = \int_{-0.1}^{0.1} e^{-x^2} dx$$

Approximate the integral using the following Taylor Expansions. What would the center of this expansion be? Compare to the error, where $I(x) = 0.1933$.

1. T_1
2. T_2

Problem 4. Find the interval of convergence of the following series:

$$\sum_{n=0}^{\infty} (-1)^n \frac{(n+2)!}{4^n} (2x+6)^n$$

Problem 5. It turns out that some differential equations can be solved using Taylor Series! Solve the differential equation with the following initial condition:

$$\frac{dy}{dx} = x + y, y(0) = 0$$

Problem 6. Consider the function:

$$f(x) = \frac{x}{1 - x - x^2}$$

Show that this function can be written as $\sum_{n=0}^{\infty} F_n x^n$, where F_n are the terms in the Fibonacci sequence. Start by setting f(x) equal to the power series $\sum_{n=0}^{\infty} a_n x^n$ write out the first two terms to find a pattern.

Problem 7. A differential equation of the form $\dot{x} = f(x)$ is called a dynamical system. Let's say that $f(x) = \cos(x)$. Let's say that the system is at steady state at $x = \pi$. Use a first order Taylor approximation to approximate the dynamics in steady state.

Problem 8. Show that the Maclaurin expansion of:

$$f(x) = \cos(x)$$

converges for all x.

Problem 9. The period of a pendulum is the time it takes for a pendulum to make one complete back-and-forth swing. For a pendulum with length L that makes a maximum angle θ_{max} with the vertical, its period T is given by:

$$T = 4\sqrt{\frac{L}{g}}\int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{1-k^2\sin^2(\theta)}}d\theta$$

Note that g is the acceleration due to gravity and$k = \sin(\frac{\theta_{max}}{2})$. Figure 1 shows a diagram. Note that this formula for the period arises from a non-linearized model of a pendulum. In some cases, for simplification, a linearized model (n = 1) is used and $\sin(\theta)$ is approximated by θ. Use the binomial series to estimate the period of this pendulum. Perform both a first and second order approximation and compare your results.

Figures

Problem 9

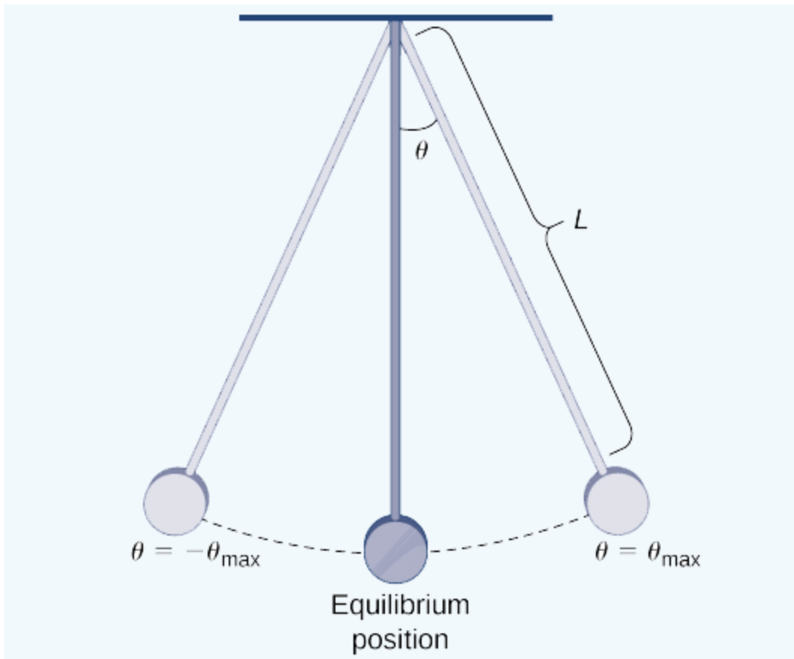

Figure 1: Diagram of Pendulum Parameters