Discussion Problems **Name:** ____________________

Worksheet 7: Implicit Differentiation and Applications II

Math 408C:

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Problem 1. Find the derivative of $y\sec(x) = x\tan(y)$ with x being the dependent variable. That is let x(y).

Problem 2. Two aircraft pass the same geographical coordinates at some point in time, but are prescribed two different altitudes. Plane A is flying at 417 mph at a heading of 225 degrees with an altitude of 21,000 ft, and Plane B is flying at 582 mph with a easterly heading with an altitude of 20,000 ft. This is known as a separation heading. Consider Figures 1 and 2 below.
Find the magnitude of the speed of separation when Plane A and Plane B are 7.6 and 5.7 mi from their geographic intersection. In other words, how fast is the distance between them increasing? Or decreasing? Hopefully not the latter! This is called a collision course. Here are some reminders/hints:

- Use Law of Cosine to find the horizontal separation between the planes.
- 5.7 mi = 30,000 ft, 7.6 mi = 40,000 ft
- 1 mi = 5280 ft
- Assume that the planes don't change altitude and heading. Their trajectory is a straight line at all points in time.
- Keep track of units!

Problem 3.

1. What is the 1000th derivative of $y = cos(x)$
2. What is the nth derivative of $y = xe^x$
3. What is the 1000th derivative of $y = sin(x)$
4. Find an x(t) such that $\frac{dx(t)}{dt} = x(t)$

Problem 4. An approach path for an aircraft landing is shown in the figure and satisfies the following conditions:

- The cruising altitude is h when descent starts at a horizontal distance l, from touchdown at the origin. Assume that the flight starts and ends with zero vertical velocity.
- The pilot must maintain a constant horizontal speed v throughout descent.
- The vertical acceleration should not exceed a constant k (which is much less than the acceleration due to gravity).

A figure is shown below to guide you.

1. Find a cubic polynomial $P(x) = ax^3 + bx^2 + cx + d$ that satisfies the first condition by imposing suitable conditions on P(x) and P'(x) at the start of descent and at touchdown.
2. Show that $\frac{6hv^2}{l^2} \geq k$ if the second and third conditions are imposed. The original function is in the form y=P(x). However, remember that acceleration and speeds are derivatives with respect to **time**. So our function is really in the form P(x(t)), since the horizontal distance is also a function of time. Taking the derivative of P(x(t)) using the chain rule:

$$\frac{dP}{dt}(x) = \frac{dP}{dx}\frac{dx}{dt} = P'(x)v$$

   The quantity $\frac{dP}{dt}(x)$ is the rate of change of the height with respect to t, but parametrized as a function of x.
3. Suppose that an airline decides not to allow vertical acceleration of a plane to exceed $k = 860mi/hr^2$. If the cruising altitude of a plane is 35,000 ft and the horizontal speed is 300 mi/hr (This is a very fast landing for any plane, but let's assume worst case. Ideally, the pilot will have to slow the plane down tremendously), how far away from the airport should the pilot start descent?

Figures

Problem 2

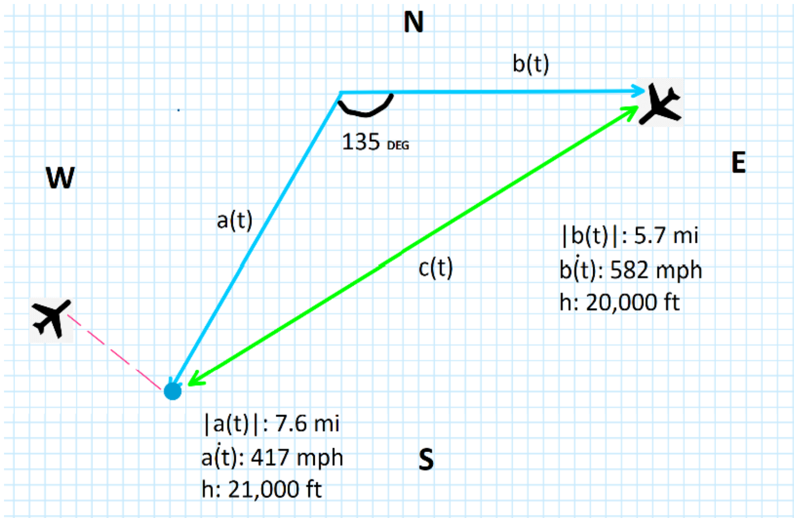

Figure 1: Bird's Eye View of the Planes

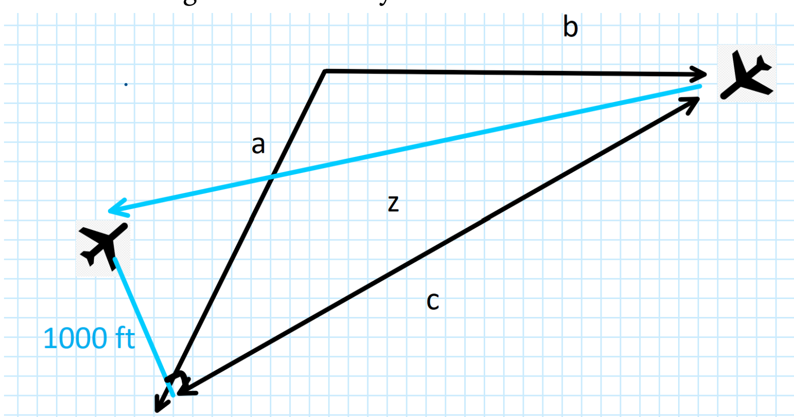

Figure 2: 3D view of Planes

Problem 4

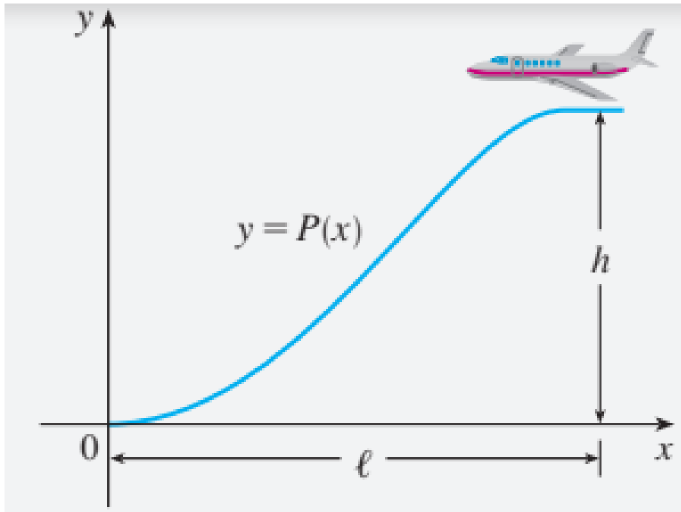

Figure 3: Descent Profile y=P(x)