Discussion Problems **Name:** ______________________________

Worksheet 3: Improper Integrals

Math 408D:

Instructor: Athil George

Problem 1. Find if each of the type I improper integrals converges or diverges. If it converges, what value does it converge to? For 3.1, assume $p > 1$.

1. $$\int_0^\infty e^{-x}\sin(x)dx$$

2. $$\int_e^\infty \frac{1}{x\ln(x)}dx$$

3. $$\int_a^\infty \frac{c}{x^p}dx$$

4. $$\int_{-\infty}^1 \frac{1}{x^2+x}dx$$

Problem 2. Determine if the following sequences converge or diverge. If it converges, what is the value? Otherwise, give an explanation as to why you determined it diverges.

1. $$a_n = \arctan(\ln(n))$$

2. $$a_n = \sin(\frac{n\pi}{n^2 - 1})$$

3. $$a_n = n^5 e^{-n}$$

4. $$a_n = \cos(\ln(n))$$

Problem 3. Consider the region bounded by the curves:

$$y = \frac{c}{x^2}$$
$$x = a$$
$$x = 0$$

What is the volume of the solid that is formed when the region is revolved about the x-axis?

Problem 4. Evaluate the following type II improper integral.

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \tan(x)dx$$

Problem 5. Evaluate the following integral (Hint: To solve this problem, you need to evaluate both a type I and type II integral. This can be achieved by splitting the integral into two parts using the basic integral rule ($\int_a^b = \int_a^c + \int_c^b$, where $a < c < b$)

$$\int_0^\infty \frac{1}{\sqrt{x}(x-1)} dx$$

Problem 6. Recall that the Fibonnaci sequence is defined as a sequence of numbers with the following recursion scheme:

$$F_{n+1} = F_n + F_{n-1}$$

The golden ratio is the ratio of the current and previous term as the sequence goes to ∞. Find the golden ratio.