Discussion Problems **Name:** ______________________________

Challenge

Math 408D:

Instructor: Athil George

Problem 1. Consider the Gamma function as an analog to the factorial function for non-integers:

$$\Gamma(n) = \int_0^\infty t^{n-1}e^{-t}dt$$

Show that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$. Take the following steps:

1. Use the substitution $u = \sqrt{t}$.
2. You should have an integral involving e^{-u^2}. Recognize the symmetry of this function and change the lower bound of the integral to -∞.
3. To evaluate the integral of the squared exponential from $(-\infty, \infty)$, use the following root-mean square identity.

$$\sqrt{(\int_{-\infty}^{\infty} e^{-u^2}du)^2} = \sqrt{\int_{-\infty}^{\infty} e^{-x^2}dx \int_{-\infty}^{\infty} e^{-y^2}dy} = \sqrt{\iint_A e^{-x^2-y^2}dA}$$

4. Use the polar coordinate transformation to solve the double integral ($dA = rdrd\theta$).