Discussion Problems **Name:** ____________________

Worksheet 1: Absolute and Conditional Convergence

Math 408D:

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Problem 1. Determine if the following series is absolutely or conditionally convergent:

$$\sum_{n=1}^{\infty} \frac{\sin(\frac{\pi}{2} + \pi n)\sqrt{n}}{n+1}$$

Problem 2. Determine if the following series converges or diverges. Hint: Apply logarithm rules.

$$\sum_{n=1}^{\infty} n^{\frac{1}{n}}$$

Problem 3. Consider the following function:

$$f(x) = \sqrt{16 - 2x}$$

(a) Express the function as an infinite series.

(b) Write out the first 3 terms of the expansion.

(c) State the radius of convergence.

(d) State the interval of convergence.

Problem 4. We will see that the main use of a power series is that it provides a way to represent some of the most important functions that arise in mathematics, physics, and chemistry. In particular, a type of power series is called a Bessel function, after the German astronomer Friedrich Bessel (1784–1846). The 0th order Bessel Function is defined as:

$$J_n(x) = \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}$$

These functions first arose when Bessel solved Kepler's equation for describing planetary motion. Since that time, these functions have been applied in many different physical situations, including the temperature distribution in a circular plate and the shape of a vibrating drumhead. Find the domain of the 0th order Bessel Function. In other words, find the values of x for which the series defined above converges.