Discussion Problems **Name:** ______________________________

Worksheet 2: Integration Techniques II

Math 408D:

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Problem 1. Evaluate the following indefinite integrals using partial fraction decomposition.

1. $$\int \frac{x^2+2x+3}{(x-6)(x^2+4)}\,dx$$

2. $$\int \frac{x^3+1}{x^3+6x^2+9x}dx$$

Problem 2. Evaluate the following integrals using partial fraction decomposition. Note that for these problems, a priori step is required to set up the partial fraction decomposition.

1. $$\int \ln(x^2 + 3x - 4)dx$$

2. $$\int \frac{1}{e^{3x} - 1}dx$$

3. $$\int_0^1 x \arctan(x)dx$$

Problem 3. Read the problem statement for Excercise 59 in Section 7.4 in the textbook. Evaluate $\int \frac{1}{-2\cos(x)-\frac{3}{2}\sin(x)}\,dx$ using the substitution proposed in the problem. Note that the function below is in the form $\mathrm{R}(\sin(x), \cos(x))$.

Problem 4. Find a general formula for the integral of the following function using integration by parts and a recurrence relation:

$$\int \cos^n(x)\,dx \tag{1}$$

Problem 5. Consider a random variable x, which denotes a sample student's score on a math exam. The lowest and highest possible scores are 2 and 5, respectively. The continuous probability distribution function (PDF) of the random variable x is:

$$P(x) = \frac{c}{x^2 - 1}, x \in [2, 5]$$

Answer the following questions:

1. The constant c is known as the normalization constant. Its purpose is to impose the constraint $\int P(x)dx = 1$. Find the constant c.
2. What is the probability that a given student received a score between 2 and 3? Hint: The probability is the area under the PDF.