Signal Behaviour Simulation of RCL Circuit

ATHIRA KRISHNA K A, ADHITHYANATH, ADARSH N, ADVAITH KRISHNA

October 2025

1 Abstract

This project explores the application of Laplace and Inverse Laplace Transforms in the field of signal processing, specifically through the virtual simulation of an RLC circuit. The RLC circuit, consisting of a resistor (R), inductor (L), and capacitor (C), is modeled using a second-order differential equation that describes its voltage-current relationship. By applying the Laplace Transform, this complex differential equation is converted into a simpler algebraic form in the frequency (s) domain, allowing for easier analysis of system behavior under various damping conditions. The Inverse Laplace Transform is then used to obtain the time-domain response, revealing how the circuit reacts to different inputs and damping levels—underdamped, critically damped, and overdamped.

2 Introduction

In the field of electrical and electronics engineering, signal processing plays a vital role in understanding and controlling the behavior of dynamic systems. Signals, such as voltage and current, often vary with time and are governed by differential equations. Solving these equations directly in the time domain can be mathematically challenging, especially when analyzing complex or higher-order systems. To simplify the process, engineers use mathematical tools such as the Laplace Transform and its inverse, which convert time-domain problems into the frequency (s) domain, where analysis becomes more intuitive and algebraically straightforward.

The Laplace Transform converts differential equations into algebraic equations, making it possible to analyze a system's behavior, determine stability, and predict both transient and steady-state responses. Once the system's response is determined in the s-domain, the Inverse Laplace Transform is applied to retrieve the corresponding time-domain behavior, which can then be visualized and interpreted. This method forms the foundation of modern signal processing, control systems, and circuit analysis.

In this project, an RLC circuit—comprising a resistor (R), inductor (L), and capacitor (C)—is used as a model system to study signal behavior using Laplace and Inverse Laplace Transforms. The RLC circuit is chosen because it represents a second-order linear system, capable of exhibiting different types of damping: underdamped, critically damped, and overdamped responses. These damping conditions influence how the circuit responds to changes in input signals, reflecting key concepts of oscillation, stability, and energy dissipation.

By simulating the circuit mathematically in Python (SymPy and Matplotlib) or MATLAB, the system's response is obtained without using any physical hardware. The simulation provides graphical representations of input and output signals under various damping conditions, highlighting how system parameters affect performance. Through this study, the project demonstrates how Laplace Transform methods serve as powerful tools for analyzing and solving engineering problems involving time-dependent signals and systems.

3 Methodology

The methodology of this project focuses on analyzing the response of a series **RLC circuit** using **Laplace and Inverse Laplace Transforms**. The process is divided into several steps as follows:

3.1 Step 1: Define Circuit Parameters

The circuit components include a **Resistor (R)**, **Inductor (L)**, and **Capacitor (C)**. Appropriate values are selected for simulation, and the input voltage $V(t)$ is chosen as a step, impulse, or sinusoidal signal. This step establishes the foundation for modeling.

3.2 Step 2: Formulate the Governing Differential Equation

Applying Kirchhoff's Voltage Law (KVL) to the series RLC circuit gives the differential equation:

$$L\frac{d^2i(t)}{dt^2} + R\frac{di(t)}{dt} + \frac{i(t)}{C} = V(t)$$

This second-order linear equation captures the transient and steady-state response of the circuit.

3.3 Step 3: Apply Laplace Transform

The time-domain differential equation is converted to the frequency domain (s-domain) using the Laplace Transform:

$$L[s^2I(s) - si(0) - i'(0)] + R[sI(s) - i(0)] + \frac{I(s)}{C} = V(s)$$

Assuming zero initial conditions ($i(0) = 0, i'(0) = 0$), it simplifies to:

$$I(s) = \frac{V(s)}{Ls^2 + Rs + \frac{1}{C}}$$

$I(s)$ represents the Laplace Transform of the current $i(t)$, also known as the transfer function.

3.4 Step 4: Analyze Damping Conditions

The damping condition is determined from:

$$\Delta = R^2 - 4\frac{L}{C}$$

- **Underdamped** ($\Delta < 0$): Oscillatory response with decaying amplitude.
- **Critically damped** ($\Delta = 0$): Fastest response without oscillation.
- **Overdamped** ($\Delta > 0$): Slow response without oscillation.

3.5 Step 5: Compute Inverse Laplace Transform

The inverse Laplace Transform converts $I(s)$ back to the time-domain response $i(t)$, showing transient and steady-state behavior. This step can be automated using **Python (SymPy)** or **MATLAB**.

3.6 Step 6: Simulation Using Python or MATLAB

The computational implementation involves:

1. Defining circuit parameters and input voltage.
2. Formulating the Laplace-domain equation.
3. Calculating the Inverse Laplace Transform to obtain $i(t)$.
4. Plotting current and voltage versus time for different damping cases.

3.7 Step 7: Visualize and Interpret Results

Graphs of input and output signals are plotted for underdamped, critically damped, and overdamped cases. Key observations include:

- Peak overshoot in underdamped response.
- Fast response in critically damped case.
- Slow settling in overdamped case.

3.8 Step 8: Validation and Discussion

Simulation results are validated against theoretical predictions, and any discrepancies are analyzed. This ensures accuracy and understanding of system behavior under different conditions.

4 Results

4.1 Symbolic Transfer Function

The symbolic transfer function of the series RLC circuit is given by:

$$H(s) = \frac{1}{LCs^2 + RCs + 1}$$

This represents the relationship between the capacitor voltage $V_c(s)$ and the input voltage $V_{in}(s)$ in the s-domain. Using the Laplace transform allows analysis of the circuit for various damping conditions without solving differential equations directly in the time domain.

4.2 Natural Frequency and Critical Resistance

For $L = 1$ mH and $C = 1$ μF:

$$\omega_0 = \frac{1}{\sqrt{LC}} = 31622.78\ rad/s \approx 5032.92\ Hz$$

$$R_{crit} = 2\sqrt{\frac{L}{C}} = 63.2456\ \Omega$$

The damping conditions are:

- **Underdamped:** $R = 10\ \Omega$
- **Critically damped:** $R = R_{crit}$
- **Overdamped:** $R = 200\ \Omega$

4.3 Step Response

The step response of the capacitor voltage $V_c(t)$ shows distinct behaviors for different damping conditions:

- **Underdamped (R = 10):** Exhibits oscillatory behavior with decaying amplitude.
- **Critically damped (R = 63.24):** Fast rise to steady-state without overshoot.
- **Overdamped (R = 200):** Slow rise to steady-state without overshoot.

4.4 Forced Sinusoidal Response

For an input $V_{in} = \sin(\omega_0 t)$:

- **Underdamped:** Resonance-like oscillations and higher amplitude peaks.
- **Critically damped:** Smooth sinusoidal tracking with moderate amplitude.
- **Overdamped:** Low amplitude response, slow tracking of the input signal.

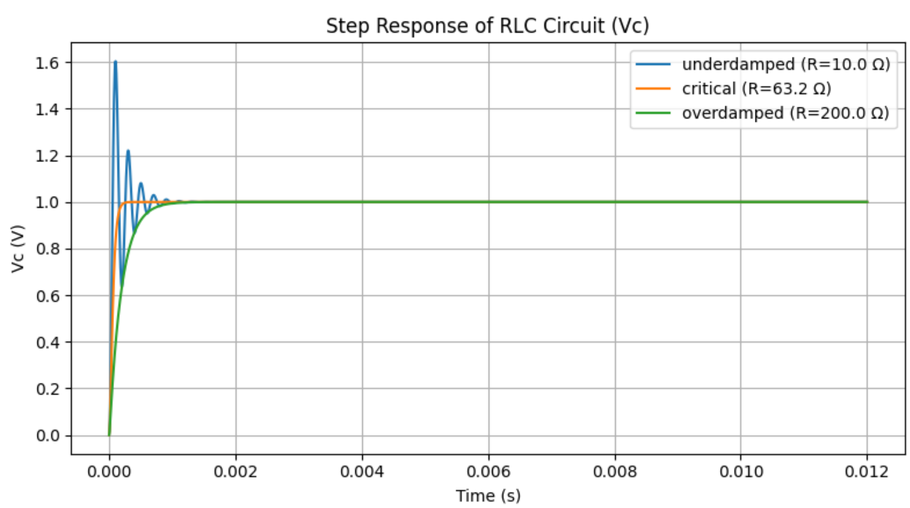
4.5 Frequency Response

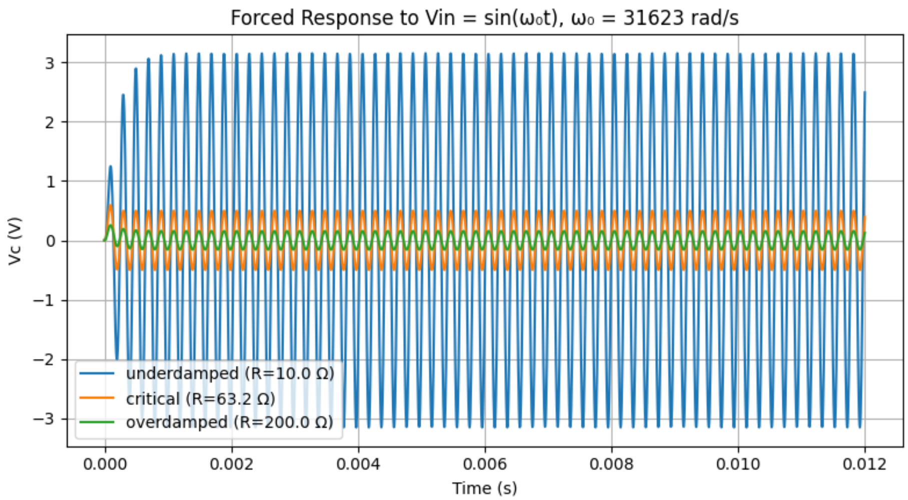
The frequency response magnitude $|H(j\omega)|$ demonstrates the following for different damping conditions:

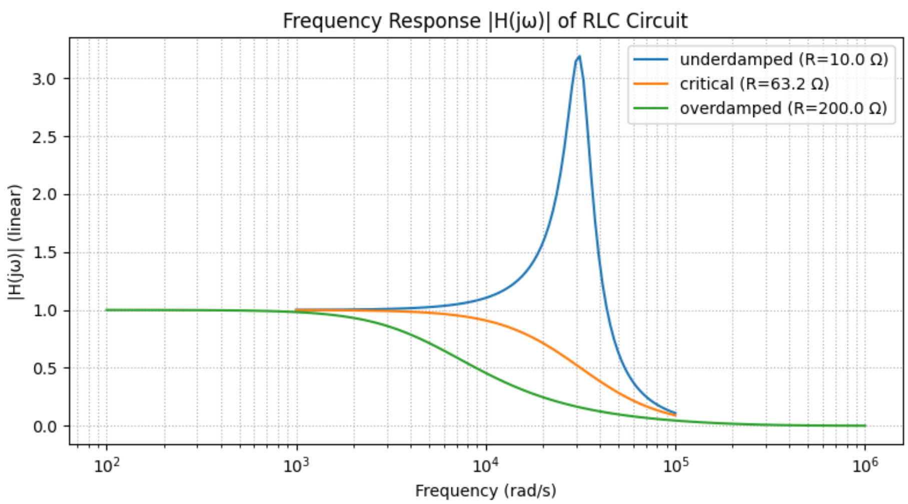
- **Underdamped:** Peak near natural frequency ω_0, showing resonance behavior.
- **Critically damped:** Flatter response near ω_0, no sharp peak.
- **Overdamped:** Broad, low-magnitude response with minimal resonance.

4.6 Observations

- Damping resistance significantly affects transient and steady-state behavior.
- Underdamped circuits show oscillatory responses and overshoot.
- Critically damped circuits settle fastest without overshoot.
- Overdamped circuits respond slowly without oscillations.
- The resonance peak in the frequency response is most pronounced in the underdamped case.







5 Limitation

While the simulation of an RLC circuit using Laplace and Inverse Laplace Transforms provides valuable insights into the system's behavior, there are certain limitations to this approach:

- **Ideal Assumptions:** The analysis assumes ideal components (resistors, inductors, and capacitors) with no parasitic effects. In real-world scenarios, non-ideal behaviors such as component tolerances, stray capacitances, and inductive coupling can affect the results.

- **Zero Initial Conditions:** Most simulations assume zero initial energy in the system (i.e., no initial current or voltage). This may not reflect actual startup conditions in physical circuits.
- **Limited to Linear Systems:** Laplace transform techniques are only valid for linear, time-invariant (LTI) systems. Nonlinear effects (e.g., saturation in inductors) cannot be modeled using this method.
- **No Noise Consideration:** The simulation does not account for electrical noise or disturbances that are commonly present in practical circuits.
- **Time-Domain Complexity:** While Laplace simplifies solving differential equations, the resulting inverse Laplace expressions for time-domain behavior can become algebraically complex and hard to interpret manually.
- **No Real-Time Dynamics:** This method provides a theoretical analysis of behavior, but does not simulate real-time dynamic changes or allow interactive testing as a hardware setup or real-time simulator would.

Despite these limitations, the Laplace-based simulation remains a powerful analytical tool for understanding and predicting the fundamental behavior of RLC circuits.

6 Acknowledgement

I would like to express my sincere gratitude to all those who have supported me in the completion of this project titled *"Signal Processing using Laplace and Inverse Laplace Transform"*.

I am especially thankful to my instructor/guide Siju Swamy Sir, for their valuable guidance, constructive feedback, and encouragement throughout the course of this work.

I also appreciate the support of my peers and colleagues who provided useful discussions and insights, which helped shape the direction and clarity of this project.

Lastly, I acknowledge the resources and tools that greatly assisted in the simulation and analysis process, particularly the Python `SymPy` library and MATLAB, which were essential for performing the symbolic and graphical computations.

This project has been a great learning experience in understanding the application of Laplace Transforms in electrical signal analysis, and I am grateful for the opportunity to explore this topic in depth.

7 Conclusion

This project successfully demonstrated the application of Laplace and Inverse Laplace Transforms in analyzing the behavior of an RLC circuit without the need for physical hardware. By transforming the system's differential equations into the s-domain, we simplified the mathematical analysis and obtained the transfer function of the circuit. The inverse transformation allowed us to retrieve time-domain responses, which clearly showed how the system reacts under different damping conditions.

The results revealed key characteristics of the system:

- **Underdamped systems** exhibit oscillations before reaching steady-state.
- **Critically damped systems** return to steady-state fastest without oscillation.
- **Overdamped systems** settle slowly without oscillations.

These findings highlight the effectiveness of Laplace transform techniques in electrical engineering and signal processing. This method not only simplifies complex calculations but also provides a clear framework for understanding dynamic system behavior.

Overall, this simulation-based approach reinforces foundational concepts in circuit theory and transforms, and it serves as a powerful tool for designing and analyzing systems in both academic and practical engineering contexts.

8 References

B. P. Lathi, *Signal Processing and Linear Systems*, Oxford University Press, 1998.

Wasyl Wasylkiwskyj, *Signals and Transforms in Linear Systems Analysis*, Springer, 2013.

Phil Dyke, *An Introduction to Laplace Transforms and Fourier Series*, Springer, 2nd ed., 2014.

J. Lighthill, *Fourier and Laplace Transforms*, Cambridge University Press, 1958.

S. Palani, *Signals and Systems*, Springer, 2022.

Michael Corinthios, *Signals, Systems, Transforms, and Digital Signal Processing with MATLAB*, CRC Press, 2018.

Edward Layer and Krzysztof Tomczyk, *Signal Transforms in Dynamic Measurements*, Springer, 2015.

A. Chakrabarti, *Circuit Theory (Analysis and Synthesis)*, Dhanpat Rai & Co., 7th Edition, 2018.

A. Bruce Carlson, *Circuits: Engineering Concepts and Analysis of Linear Electric Circuits*, Cengage Learning, 2009.

W. H. Hayt and J. E. Kemmerly, *Engineering Circuit Analysis*, McGraw-Hill Education, 8th Edition, 2012.