ASSIGNMENT

Athira BJ4
Roll no: 4
S1 MCA

MODULE 3

1. Show that every simple graph on n vertices is a subgraph of k_n

Answer:

Let G be a simple graph with n vertices that means there was no loop or parallel edges. where $G(k_n)$ be a graph with complete graph with n vertices. Hence the number of vertices are same so we can write it as

$$V(G) \subseteq V(k_n)$$

In a complete graph with n vertices has nC_2 edges

ie, $\frac{(n-1)}{2}$ edges

Simple graph has also have nC_2 edges so, edges set are also subset of complete graph.

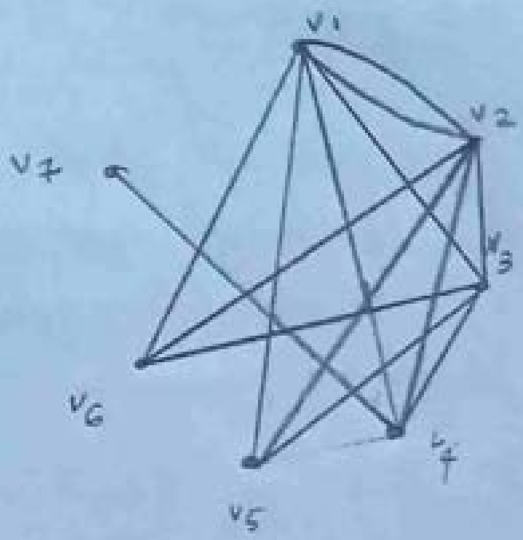
$$E(G) \subseteq E(k_n)$$

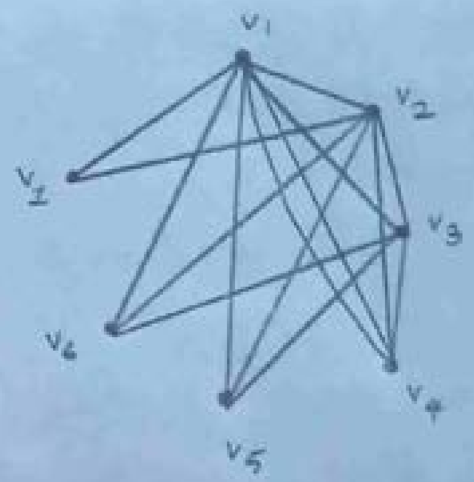
2. Show that every subgraph of a bipartite graph is bipartite.

Answer:

Where G is a bipartite graph with its vertex V can be partitioned two disjoint set V_1 and V_2. The subgraph H of G is the graph whose some of them are in V_2 and V_1 are considered every edges of H each of the end vertices in V_1 and other in V_2 so H is also bipartite graph is bipartite.

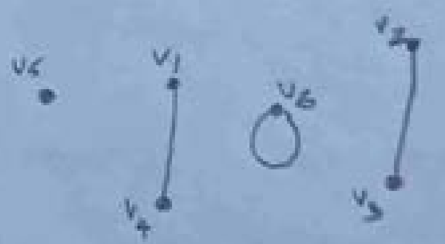
3. Verify whether the integer sequences (7, 6, 5, 4, 3, 3, 2) and (6, 6, 5, 4, 3, 3, 1) are graphical





4. Let G be a graph in which there is no pair of adjacent edges since there are no adjacent edges are the degree of each vertex is either zero or one or two

Eg:



d(v1) = 1
d(v2) = 1
d(v3) = 1
d(v4) = 1
d(v5) = 0
d(v6) = 2

5. Prove that it is impossible to have a group of nine people at a party such that each knows exactly five of others in the group.

Answer:

each vertex represent a person.

If two persons knows each other there is an edge between them since each knows 5 others

Degree of vertices = 5

Sum of degree of vertices $= 9 \times 5 = 45$

from first degree of graph theory

$$\sum d(v) = 2e$$

Here $45 \neq 2e$ because $2e$ is an even number,

45 is not even.

It is impossible to have a group of nine people at a party that each one knows exactly 5 of the others

6 Let G be a graph with n vertices t of which have degree k, and the others have degree k+1 prove that $t = (k+1)n - 2e$,

Answer:

where e is the no. of edges in G

By first theorem of graph theory

$$\sum d(v) = 2e$$

t of which have degree k.

ie, tk.

other have degree k+1

$(n-t)(k+1)$

$$\Rightarrow \sum d(v) = tk + (n-t)(k+1)$$

$$= tk + nk - tk + n - t$$

$$= nk + n - t$$

$$2e = (k+1)n - t$$

$$t = (k+1)n - 2e$$

7. let G be a regular graph, where k is an odd number prove that the no. of edges in G is a multiple of k?

Let G be a k regular graph with n vertices

Let e be the no. of edges in G

$d(v) = k$ for each vertex of G

$\Sigma d(v) = nk$ —— ①

By first theorem of graph theory

$\Sigma d(v) = 2e$ —— ②

from ① and ② $\Rightarrow 2e = nk$

$$e = \frac{nk}{2}$$

$$= \left(\frac{n}{2}\right)k$$

no. of edges of G is a multiple of k.

8. Let G be a graph with n vertices and exactly n-1 edges prove that G has either vertex of degree 1 or an isolated vertex

n = no. of vertex

$n-1$ = No. of edges

suppose degree of each vertex in $G \geq 2$

$d(v) \geq 2$

$\Sigma d(v) \geq 2n$

$\Rightarrow 2(n-1) \geq 2n$

$\Rightarrow n-1 \geq n$

9 what is the smallest integer n s the complete graph K_n has atleast 500 edges?

No. of edges of $K_n = {}^nC_2$

$${}^nC_2 \geq 500$$

$$\frac{n(n-1)}{2} \geq 500$$

$$n(n-1) \geq 1000$$

$$n^2 - n \geq 1000$$

$$n^2 - n - 1000 \geq 0$$

$$n = 32.1265, \ -31.1265$$

$$\therefore n \geq 32, \ n \geq 33$$

10 Prove that there is no simple graph with 7 vertex one of which has degree 2. two have degree 3, three have degree 4 and the remaining vertex has degree 5

Assume that such a graph exist.

by first theorem in graph theory.

$$\sum d(v) = 2e \qquad v = 6.$$

$$= 1(2) + 2(3) + 3(4) + 1(5)$$

$$= 2 + 6 + 12 + 5$$

$$= 25$$

$$2e \neq 25$$

$\therefore$ There is no such a graph exist The assumption is contradition

MODULE 5

1. calculate the karl pearson's algorithm of correlation from the following data

x	6	8	12	15	18	20	24	28	31
y	10	12	15	15	18	25	22	26	28

x	y	xy	x^2	y^2
6	10	60	36	100
8	12	96	64	144
12	15	180	144	225
15	15	225	225	225
18	18	324	324	324
20	25	500	400	625
24	22	528	576	484
28	26	728	784	676
31	28	868	961	784
162	171	3509	3614	3587

$\bar{x} = \frac{1}{n}\Sigma x$

$= \frac{1}{9}\,162$

$= \underline{\underline{18}}$

$\bar{y} = \frac{1}{n}\Sigma y$

$= \frac{1}{9}\,171 = \underline{19}$

$$xy = \frac{\frac{1}{n}\Sigma xy - \bar{x}\bar{y}}{\sqrt{\left(\frac{1}{n}\Sigma x^2 - \bar{x}^2\right)\left(\frac{1}{n}\Sigma y^2 - \bar{y}^2\right)}}$$

$$= \frac{\frac{1}{9}\,3509 - (18 \times 19)}{\sqrt{\left(\frac{3614}{9} - (18)^2\right)\left(\frac{3587}{9} - (19)^2\right)}}$$

$$= \frac{47.8889}{49.9535} = \underline{\underline{0.9586}}$$

2 compute karl pearsons coefficient correlation in the following serves relating the cost of living and arges.

x	y	x^2	y^2	xy
100	98	1000	9604	9800
101	99	10201	9801	9999
103	99 97	10609	9801	10197
102	95	10404 1000	9409	9894
100	92	9801	9025	9500
99	95	9409	8464	9108
97	94	9604	9025	9215
98	90	9216	8836	9212
			8100	8640
96	91	9025	8281	8645
95				
991	950	98269	90346	94210

$$\bar{x} = \frac{\Sigma x}{n} = \frac{991}{10} = 9.91 \;, \bar{y} = \frac{\Sigma y}{n} = 95$$

$$r(x,y) = \frac{\frac{1}{n}\Sigma xy - \bar{x}\bar{y}}{\sqrt{(\frac{1}{n}\Sigma x^2 - \bar{x}^2)(\frac{1}{n}\Sigma y^2 - \bar{y}^2)}}$$

$$= \frac{\frac{1}{10} \times 94210 - 99.1 \times 95}{\sqrt{(\frac{1}{10} \times 98269 - 99.1^2)(\frac{1}{10} \times 90346 - 95^2)}}$$

$$= \frac{9421 - 9414.5}{\sqrt{(98269 - 9820.81)(9034.6 - 9025)}}$$

$$= \frac{6.5}{7.64} = 0.8507$$

3 The following data relate to the marks obtained by 10 student of a class in statistics and costing.

Marks in statistics	30	38	28	27	28	23	30	33	28	35
Marks in costing:	29	27	22	29	20	29	18	21	27	22

X	Y	Rx	Ry	D=Rx-Ry	D^2
30	29	4.5	2	2.5	6.25
38	27	1	4.5	3.5	12.25
28	22	7	6.5	0.5	0.25
27	29	9	2	7	49
28	20	7	9	2	4
23	29	10	2	8	64
30	18	4.5	10	5.5	30.25
33	21	3	8	5	25
28	27	7	4.5	2.5	6.25
35	22	2	6.5	4.5	20.25
					$\Sigma D^2 = 217.5$

correlation factor $= \Sigma m(m^2-1)$

$$CF = 2(2^2-1) + 3(3^2-1) + 3(3^2-1) + 2(2^2-1) + 2(2^2-1)$$

$$= \underline{\underline{66}}$$

rank correlation coefficient

$$r = 1 - \frac{6\left(\Sigma D^2 + \frac{1}{12} CF\right)}{n(n^2-1)}$$

$$= 1 - \frac{6\left(217.5 + \frac{66}{12}\right)}{10(10^2-1)} = 1 - \frac{1338}{990}$$

$$= \underline{\underline{-0.35}}$$

4 Find the coefficient of rank correlation between the marks obtained in Mathematics (x) and thus in statistics by 10 students of certain class out of a total of 50 mark in each subject

X	Y	R_x	R_y	D	D^2
12	16	10	8.5	1.5	2.25
18	15	8.5	10	-1.5	2.25
32	28	3	3.5	-0.5	0.25
18	16	8.5	8.5	0	0
25	24	4.5	5	-0.5	0.25
24	22	6	6	0	0
25	28	4.5	3.5	1	1
40	36	1	1	0	0
38	34	2	2	0	0
22	19	7	7	0	0
					6

$$CF = 2(2^2-1) + 2(2^2-1) + 2(2^2-1) + 2(2^2-1)$$
$$= \underline{24}.$$

Rank correlation coefficient is given by

$$r = 1 - \frac{6(\Sigma D^2 + \frac{1}{12} CF)}{n(n^2-1)}$$

$$= 1 - \frac{6(6 + \frac{24}{12})}{10(10^2-1)}$$

$$= 1 - \frac{6 \times 8}{990} = 1 - \frac{48}{990} = \underline{\underline{0.95}}$$

5 obtain the equation of the two lines of regression for following data

x	43	44	46	40	44	42	45	42	38	40	42	57
y	29	31	19	18	19	27	27	29	41	30	26	10

Hence obtain the value of correlation efficiency b/w x and y.

x	y	x^2	y^2	xy
43	29	1849	841	1247
44	31	1936	961	1364
46	19	2116	361	874
40	18	1600	324	720
44	19	1936	361	836
42	27	1764	729	1134
45	27	2025	729	1215
42	29	1764	841	1218
38	41	1444	1681	1558
40	30	1600	900	1200
42	25	1764	676	1092
57	10	3249	100	530
528	306	23047	8504	13028

Line regression of y on x is given by

$$y-\bar{y} = b_{yx}(x-\bar{x})$$

$$b_{yx} = \frac{cov(x,y)}{\sigma x^2}$$

$$cov(x,y) = \frac{1}{n}\Sigma xy - \bar{x}\bar{y}$$

$$= \frac{1}{12} \times 13028 - 43.58 \times 25.5 \quad \left(\bar{x} = \frac{523}{12},\ \bar{y} = \frac{306}{12}\right)$$

$$= -25.63$$

$$\sigma x^2 = \frac{\Sigma x^2}{n} - \bar{x}^2 = \frac{23047}{12} - 43.6^2$$

$$= 1920.58 - 1892.25$$

$$= \cancel{-24.63} \; -21.37$$

$$\text{ie, } b_{yx} = \frac{-25.63}{21.37} = -1.199$$

ie, y on x $= y - 25.5 = -1.199(x - 43.5)$.

$$y = -1.199x + 1.199 \times 43.58 + 25.5$$

$$y = -1.199x + 77.752$$

Line of regression of x on y is given by,

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

where $b_{xy} = \frac{\operatorname{Cov}(x,y)}{\sigma y^2}$

~~Cov(x,y)~~

$$\sigma y^2 = \frac{\Sigma y^2}{1} - \bar{y}^2$$

$$= \frac{8504}{12} - 25.5^2$$

$$= 58.75$$

$$\text{ie, } b_{xy} = \frac{-25.63}{58.75}$$

$$= -0.436$$

ie, x on y $= x - \bar{x} = b_{xy}(y - \bar{y})$

$$= x - 43.58 = -0.436(y - 25.5)$$

$$= x = -0.436y + 0.436 \times 25.5 + 43.58$$

$$= -0.436y + 54.698$$

The correlation coefficient is,

$$r(x,y) = \frac{\frac{1}{n}\Sigma xy - \bar{x}\bar{y}}{\sqrt{\left(\frac{1}{n}\Sigma x^2 - \bar{x}^2\right)\left(\frac{1}{n}\Sigma y^2 - \bar{y}^2\right)}}$$

$$= \frac{\frac{1}{12} \times 13028 - 43.58 \times 25.5}{\sqrt{\left(\frac{1}{12} \times 23047 - 43.58^2\right)\left(\frac{1}{12} \times 8509 - 25.5^2\right)}}$$

$$= -0.725$$

8. Find the regression equation of y on x where y and x are the marks obtained by 10 students as given below.

x : 20 45 65 40 55 35 15 80 25 50

y 20 60 55 45 75 35 25 90 10 50

x	y	x^2	y^2	xy
20	20	400	400	400
45	60	2025	3600	2700
65	55	4225	3025	3575
40	45	1600	2025	1800
55	75	3025	5625	4125
35	35	1225	1225	1225
15	25	225	225	375
80	90	6400	6400	7200
25	10	625	1025	250
50	50	2500	2500	2500
430	465	22250	27225	24150

Line of regression of y on x is given by.

$$y - \bar{y}\ b_{yx} (x - \bar{x}).$$

$$b_{yx} = \frac{\text{cov}(x, y)}{\sigma x^2}$$

$$\text{cov}(x,y) = \frac{1}{n} \Sigma xy - \bar{x}\bar{y}$$

$$\text{cov}(x,y) = \frac{1}{10} \times 24150 - 46.5 \times 43$$

$$= 2415 - 1999.5$$

$$= 415.5$$

$$\sigma x^2 = \frac{1}{n}\Sigma x^2 - \bar{x}^2$$

$$= \frac{1}{10} \times 22250 - 43^2$$

$$= 2225 - 1849$$

$$= 376$$

$$b_{yx} = \frac{415.5}{376} = \underline{\underline{1.105}}$$

Now y on x is given by,

$$y - 46.5 = 1.105(x - 43)$$

$$y = 1.105x - 1.105 \times 43 + 46.5$$

$$y = 1.105x - 1.015$$

7 By method of least squares, find the straight line of the form $y = ax + b$ that best fits the following data

x	1	2	3	4	5
y	12	19	28	39	57

let $y = ax + b$ — ①

normal equations are given by

$$\Sigma y = a\Sigma x + nb \text{ — ②}$$

$$\Sigma xy = a\Sigma x^2 + b\Sigma x \text{ — ③}$$

x	y	xy	x^2
1	12	12	1
2	19	38	4
3	28	84	9
4	39	156	16
5	57	285	25
15	155	575	55

substituting values of $\Sigma x, \Sigma y, \Sigma xy$ and Σx^2 in ② & ③

$155 = 15a + 5b$ — ④

$575 = 55a + 15b$ — ⑤

solving ④ and ⑤ we get

④ × 3 ⇒ $465 = 45a + 15b$ — ⑥

$575 = 55a + 15b$ — ⑦

⑤ − ⑥ ⇒ $110 = 10a$

⇒ $a = 11$ and $b = -2$

using in ① line of best fit is $y = 11x - 2$

8. If y is the pull required to lift a load x by means of pulley block, find a linear law of the form $y = ax + b$ connecting x and y using the following data.

x	10	14	20	24
y	40	50	60	80

Value of y when $x = 110$

Answer:

Linear law is given by $y = ax + b$ — ①

$\Sigma y = a\Sigma x + nb$ — ②

$\Sigma xy = a\Sigma x^2 + b\Sigma x$ — ③

y	x	xy	x^2
10	40	400	1600
14	50	700	2500
20	60	1200	3600
24	80	1920	6400
68	230	4220	14100

sustituting values in ② and ③

$$68 = 230a + 4b$$

$$4220 = 14100a + 230b$$

solving we get $a = 0.354$ and $b = -3.371$

using these values in ①

$$y = 0.354x + -3.371$$

Now substituting given value of x

$$y = 0.354 \times 110 + -3.371$$

$$y = \underline{\underline{35.564}}$$

9. Fit a second degree parabola of the form $y = ax^2 + bx + c$ to the following data.

x	0.5	1.0	1.5	2.0	2.5	3.0	3.5
y	1.2	1.4	1.7	2.1	2.9	3.2	4.6

let the parabola of best fit be given by,

$$y = ax^2 + bx + c \quad \text{—} ①$$

Normal equations are given by.

$$\Sigma y = a\Sigma x^2 + b\Sigma x + nc \quad \text{—} ②$$

$$\Sigma xy = a\Sigma x^3 + b\Sigma x^2 + c\Sigma x \quad \text{—} ③$$

$$\Sigma x^2 y = a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2 \quad \text{—} ④$$

x	y	xy	x^2	x^2y	x^3	x^4
0.5	1.2	0.6	0.25	0.3	0.125	0.0625
1.0	1.4	1.4	1	1.4	1	1
1.5	1.7	2.55	2.25	3.825	3.375	5.0625
2.0	2.1	4.2	4	8.4	8	16
2.5	2.9	7.25	6.25	18.125	15.625	39.0625
3.0	3.2	9.6	9	28.8	27	81
3.5	4.6	16.1	12.25	56.35	42.875	150.0625
14	17.1	41.7	35	117.2	98	292.25

substituting values in (2), (3) and (4)

$171 = 35a + 14b + 7c$ — (5)

$417 = 98a + 35b + 14c$ — (6)

$117.2 = 292.25a + 98b + 35c$ — (7)

solving (5), (6) and (7) we get

$a = 0.3238,\ b = -0.2238,\ c = 1.2714$

substituting in (1), parabola of best fit is,

$$y = 0.3238x^2 - 0.2238x + 1.2714.$$

10. the following table represents the observational values of an experiment fit a second degree parabola of the form $V = a + bR + cR^2$ to the data?

V	20	30	40	50	60	70	80
R	2.7	2.2	1.97	1.65	1.43	1.11	0.93

Answer: the parabola of $V = a + bR + cR^2$ — (1)

Normal equations are given by.

$\Sigma V = c\Sigma R^2 + b\Sigma R + na$ — (2)

$\Sigma VR = c\Sigma R^3 + b\Sigma R^2 + a\Sigma R$ — (3)

$\Sigma VR^2 = c\Sigma R^4 + b\Sigma R^3 + a\Sigma R^2$ — (4)

V	R	RV	R^2	R^2V	R^3	R^4
20	2.7	54	7.29	145.8	19.68	53.14
30	2.2	66	4.84	145.2	10.64	23.42
40	1.97	78.8	3.88	155.236	7.64	15.06
50	1.65	82.5	2.72	136.125	4.49	7.41
60	1.43	85.8	2.04	122.69	2.92	4.16
70	1.11	77.7	1.23	86.24	1.36	1.51
80	0.93	74.4	0.86	69.192	0.80	0.74
350	11.99	519.2	22.86	860.494	47.53	105.46

$$350 = 7a + 11.99b + 22.86c \quad \text{— (5)}$$

$$519.2 = 11.99a + 22.86b + 47.53c \quad \text{— (6)}$$

$$860.49 = 22.86a + 47.53b + 105.46c \quad \text{— (7)}$$

solving above equations we get,

$a = 142.15$

$b = -75.39$

$c = 11.32$

parabola of best fit is

$$v = 142.15 - 75.39R + 11.32R^2$$