

# Investigate the exponential distribution in R and compare it with the Central Limit Theorem

Athirah K.

April 20, 2018

## Overview

The purpose of this data analysis is to investigate the exponential distribution in R and compare it with the Central Limit Theorem. The exponential distribution can be simulated in R with `rexp(n, lambda)` where `lambda` is the rate parameter. The mean of exponential distribution is  $1/\lambda$  and the standard deviation is also  $1/\lambda$ . Set `lambda = 0.2` for all of the simulations. You will investigate the distribution of averages of 40 exponentials. Note that you will need to do a thousand simulations.

## Simulations

Set the simulation variables `lambda`, `exponentials`, and `seed`.

```
ECHO=TRUE
set.seed(1337)
lambda = 0.2
exponentials = 40
```

## Run Simulations with variables

```
simMeans = NULL
for (i in 1 : 1000) simMeans = c(simMeans, mean(rexp(exponentials, lambda)))
```

## Sample Mean versus Theoretical Mean

### Sample Mean

Calculating the mean from the simulations with give the sample mean.

```
mean(simMeans)
```

```
## [1] 5.055995
```

### Theoretical Mean

The theoretical mean of an exponential distribution is  $\lambda^{-1}$ .

```
lambda^-1
```

```
## [1] 5
```

### Comparison

There is only a slight difference between the simulations sample mean and the exponential distribution theoretical mean.

```
abs(mean(simMeans)-lambda^-1)
```

```
## [1] 0.05599526
```

### Sample Variance versus Theoretical Variance

#### Sample Variance

Calculating the variance from the simulation means with give the sample variance.

```
var(simMeans)
```

```
## [1] 0.6543703
```

#### Theoretical Variance

The theoretical variance of an exponential distribution is  $(\lambda * \sqrt{n})^{-2}$ .

```
(lambda * sqrt(exponentials))^-2
```

```
## [1] 0.625
```

### Comparison

There is only a slight difference between the simulations sample variance and the exponential distribution theoretical variance.

```
abs(var(simMeans)-(lambda * sqrt(exponentials))^-2)
```

```
## [1] 0.0293703
```

### Distribution

This is a density histogram of the 1000 simulations. There is an overlay with a normal distribution that has a mean of  $\lambda^{-1}$  and standard deviation of  $(\lambda * \sqrt{n})^{-1}$ , the theoretical normal distribution for the simulations.

```
library(ggplot2)
ggplot(data.frame(y=simMeans), aes(x=y)) +
  geom_histogram(aes(y=..density..), binwidth=0.2, fill="#0072B2",
    color="black") +
  stat_function(fun=dnorm, arg=list(mean=lambda^-1,
    sd=(lambda*sqrt(exponentials))^-1),
    size=2) +
  labs(title="Plot of the Simulations", x="Simulation Mean")
```

