Convection Correlations

Forced Convection – External Flow

- Flat Plate

$\overline{Nu_L} = 0.664 Re_L{}^{1/2} Pr^{1/3}$ -> Laminar Flow

$Nu_x = 0.0296 Re_L{}^{4/5} Pr^{1/3}$ -> Turbulent Flow

$\overline{Nu_L} = \left(0.037 Re_L^{4/5} - 871\right) Pr^{1/3}$ -> Laminar followed by Turbulent

- Flat plate with Unheated length

$$Nu_x = \frac{Nu_x\big|_{\xi=0}}{\left[1-\left(\xi/x\right)^a\right]^b}$$

$$Nu_x\big|_{\xi=0} = C\ \mathrm{Re}_x^m\ \mathrm{Pr}^{1/3}$$

	Laminar		Turbulent	
	Isothermal	Isoflux	Isothermal	Isoflux
a	3/4	3/4	9/10	9/10
b	1/3	1/3	1/9	1/9
C	0.332	0.453	0.0296	0.0308
m	1/2	1/2	4/5	4/5

- Flow over a cylinder: Churchill and Bernstein Correlation

$$\overline{Nu}_D = 0.3 + \frac{0.62\,\mathrm{Re}_D^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1+\left(0.4/\mathrm{Pr}\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re}_D}{282{,}000}\right)^{5/8}\right]^{4/5}$$

Forced Convection – External Flow (cont.)

- Sphere

$$\overline{Nu}_D = 2 + \left(0.4\,\mathrm{Re}_D^{1/2} + 0.06\,\mathrm{Re}_D^{2/3}\right)\mathrm{Pr}^{0.4}\left(\mu/\mu_s\right)^{1/4}$$

- Isothermal Array of Cylinders

$$\overline{Nu}_D = C_2\left[C\ \mathrm{Re}_{D,\max}^{m}\ \mathrm{Pr}^{0.36}\left(\mathrm{Pr}/\mathrm{Pr}_s\right)^{1/4}\right]$$

  – *C, m* and C_2 can be found in look-up tables

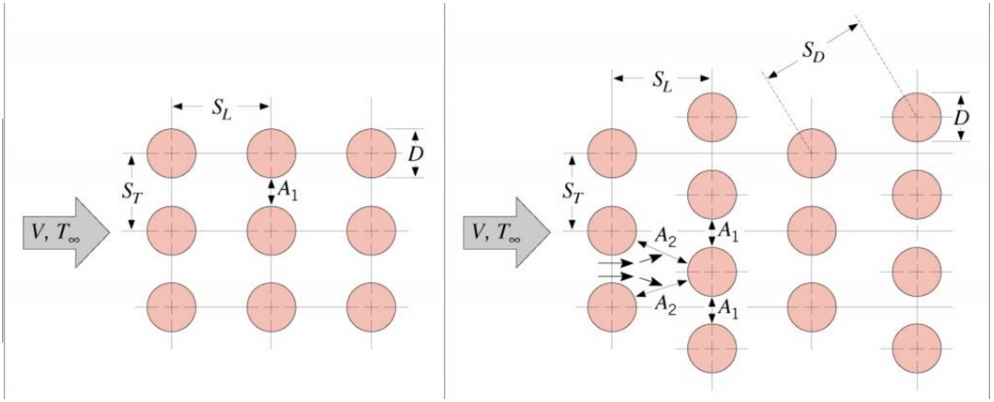
<u>In-line array</u>

$$V_{\max} = \frac{S_T}{S_T - D}V$$

<u>Staggered array</u>

$$V_{\max} = \frac{S_T}{2\left(S_D - D\right)}V \quad \text{if} \quad 2\left(S_D - D\right) \le \left(S_T - D\right)$$

$$V_{\max} = \frac{S_T}{S_T - D}V \quad \text{if} \quad 2\left(S_D - D\right) \ge \left(S_T - D\right)$$



Forced Convection – Internal Flow

- Circular pipe – Laminar Flow

$\overline{Nu_D} = 3.66$ -> Isothermal wall

$\overline{Nu_D} = 4.36$ -> Isoflux wall

- Circular pipe – Turbulent Flow

Dittus-Boelter correlation for smooth walls ($Re_D > 10{,}000$)

$\overline{Nu_D} = 0.023 {Re_L}^{4/5} Pr^n$ *[n=0.3 for heated wall; n=0.4 for cold wall]*

Gnielinski correlation for $Re_D > 3000$

$$Nu_D = \frac{(f/8)(\mathrm{Re}_D - 1000)\mathrm{Pr}}{1 + 12.7(f/8)^{1/2}\left(\mathrm{Pr}^{2/3} - 1\right)}$$

Smooth surface:

$$f = (0.790 \ln \mathrm{Re}_D - 1.64)^{-2}$$

Surface of roughness $e > 0$:

$f \rightarrow$ *Moody chart*

- Non-circular tubes
  - Use hydraulic diameter
  - Nu_D depends strongly on aspect ratio especially for laminar flow (table shown in class)
  - For turbulent flow, Nu_D for circular pipe can be used with reasonable accuracy

Forced Convection – Internal Flow (cont.)

- Circular pipe – Thermal Entry length

Laminar Flow (uniform temp)

– Combined Entry Length:

$$\left[\mathrm{Re}_D \,\mathrm{Pr}/\left(L/D\right)\right]^{1/3}\left(\mu/\mu_s\right)^{0.14} > 2:$$

$$\overline{Nu}_D = 1.86\left(\frac{\mathrm{Re}_D \,\mathrm{Pr}}{L/D}\right)^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14}$$

$$\left[\mathrm{Re}_D \,\mathrm{Pr}/\left(L/D\right)\right]^{1/3}\left(\mu/\mu_s\right)^{0.14} < 2:$$

$$\overline{Nu}_D = 3.66$$

– Thermal Entry Length:

$$\overline{Nu}_D = 3.66 + \frac{0.0668\left(D/L\right)\mathrm{Re}_D \,\mathrm{Pr}}{1 + 0.04\left[\left(D/L\right)\mathrm{Re}_D \,\mathrm{Pr}\right]^{2/3}}$$

Turbulent Flow (uniform temp)

– For long tubes $\left(L/D > 60\right)$:

$$\overline{Nu}_D \approx Nu_{D,fd}$$

– For short tubes $\left(L/D < 60\right)$:

$$\frac{\overline{Nu}_D}{Nu_{D,fd}} \approx 1 + \frac{C}{\left(L/D\right)^m}$$

$C \approx 1$
$m \approx 2/3$

Natural Convection

- Vertical Flat Plate

$$\Pr > 0.6: \quad \delta = 5x\left(\frac{Gr_x}{4}\right)^{-1/4} = 7.07\frac{x}{\left(Gr_x\right)^{1/4}} \propto x^{1/4}$$

- Empirical Heat Transfer Correlations

$$\overline{Nu}_L = 0.68 + \frac{0.670\ Ra_L^{1/4}}{\left[1+\left(0.492/\Pr\right)^{9/16}\right]^{4/9}}$$

➢ Laminar Flow $\left(Ra_L < 10^9\right)$:

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387\ Ra_L^{1/6}}{\left[1+\left(0.492/\Pr\right)^{9/16}\right]^{4/9}}\right\}^2$$

➢ Turbulent Flow $10^9 < Ra_L < 10^{12}$

- Horizontal flat plate

Facing up		Facing down	
$\overline{Nu}_L = 0.54\ Ra_L^{1/4}$	$\left(10^4 < Ra_L < 10^7\right)$	$\overline{Nu}_L = 0.27 Ra_L^{1/4}$	$\left(10^5 < Ra_L < 10^{10}\right)$
$\overline{Nu}_L = 0.15\ Ra_L^{1/3}$	$\left(10^7 < Ra_L < 10^{11}\right)$		

Natural Convection (cont.)

- Long Horizontal Cylinder

$$\overline{Nu}_D = \left\{ 0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1 + \left(0.559/\Pr\right)^{9/16}\right]^{8/27}} \right\}^2 \qquad Ra_D < 10^{12}$$

- Sphere

$$\overline{Nu}_D = 2 + \frac{0.589\ Ra_D^{1/4}}{\left[1 + \left(0.469/\Pr\right)^{9/16}\right]^{4/9}}$$

- Parallel Plates [Bar-Cohen & Rohsenow (1984)]
  - Refer to Incropera and Dewitt for correlations under different conditions