DATE 22/2/17 SHEET NO. B.1

B: CALIBRATION OF AN ORIFICE PLATE BY FREE JET METHOD

Aim: To determine C_d (coefficient of discharge) for a circular orifice.

Theory: Let us consider a system where water is issuing out of a reservoir through an orifice plate as shown:

As shown in figure, the constant head maintained is H such that the theoretical velocity at the orifice outlet is $V_{th} = \sqrt{2gH}$. The vena contracta is situated a little away from the orifice plate at point marked 2 and the velocity at vena contracta is defined as V_a.

So, the velocity coefficient C_v and the contraction coefficient C_c are defined as:

$$C_v = \frac{V_a}{V_{th}} \quad \text{and} \quad C_c = \frac{A_2}{A_0}$$

where A_0 is the cross-section area of orifice plate and A_2 is the area of jet at vena contracta.

The actual discharge is given by $q_a = A_2 V_a = A_0 C_c C_v V_{th} = C_d A_0 V_{th}$. Here, the discharge coefficient C_d is defined as $C_d = C_c C_v$. The discharge coefficient is experimentally found out if C_c and C_v are known.

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To obtain C_v:

With reference to Fig. 2, we can measure x_1 and y_1. We can write $x_1 = V_a t$ and $y_1 = \frac{gt^2}{2}$. Eliminating t, we obtain

$$V_a = \frac{x_1}{\sqrt{\frac{2y_1}{g}}}$$

If we measure at two points 1 and 2, we can write : $x_1 = V_a t$ $y_1 = g\frac{t_1^2}{2}$ and $x_2 = V_a t_2$ and $y_1 = \frac{g t_2^2}{2}$.

$y_2 - y_1 = \frac{1}{2}g(t_2^2 - t_1^2)$, $x_2 - x_1 = V_a(t_2 - t_1)$ and $x_2 + x_1 = V_a(t_2 + t_1)$

$$\text{Therefore } V_a = \sqrt{\frac{g}{2}\frac{(x_2^2 - x_1^2)}{(y_2 - y_1)}}$$

so,

$$C_v = \frac{V_a}{V_{th}} = \frac{x_1 / \sqrt{\frac{2y_1}{g}}}{\sqrt{2gH}}$$

$$\Rightarrow \quad C_v = \frac{V_a}{V_{th}} = \frac{\sqrt{\frac{g}{2}\frac{x_2^2 - x_1^2}{y_2 - y_1}}}{\sqrt{2gH}}$$

To Obtain C_D:

$$C_d = \frac{q_a}{q_{th}}$$

where the actual flowrate q_a is the water collected per unit time in the tank and q_{th} is the theoretically calculated flow-rate.

$$\therefore C_d = \frac{A_T \times H_T / t}{A_o \times \sqrt{2gH}}$$

where A_T and H_T are the base area and height of the tank for which the water is collected upto a time t.

- **Procedure:**

1. Studied the setup and made a line diagram.
2. Fixed the load at certain height.
3. Opened the orifice and measured the coordinates of the jet at two different points.
4. Collected the water on the volume measurement tank through the diverter for approximately 1 minute and noted the increase in height of the water level. Calculated the rate of discharge.
5. Repeated steps 2-4 for different values of the head.
6. Calculated C_d as outlined, assuming that the tank area is 903 cm^2 and the orifice diameter is 7.12 mm.
7. Plotted C_d versus the reynolds number Re
8. Estimated the uncertainity in the value of Cd.

- **Uncertainity in C_d:**

$$u_{C_d} = \sqrt{\left(\frac{A_T}{C_d}\frac{\partial C_d}{\partial A_T}u_{A_T}\right)^2 + \left(\frac{H_T}{C_d}\cdot\frac{\partial C_d}{\partial H_T}u_{H_T}\right)^2 + \left(\frac{t}{C_d}\cdot\frac{\partial C_d}{\partial t}\cdot u_t\right)^2 + \left(\frac{A_o}{C_d}\cdot\frac{\partial C_d}{\partial A_o}u_{A_o}\right)^2 + \left(\frac{H}{C_d}\frac{\partial C_d}{\partial H}u_H\right)^2}$$

Here, $\frac{A_T}{C_d}\cdot\frac{\partial C_d}{\partial A_T} = 1$; $\frac{H_T}{C_d}\cdot\frac{\partial C_d}{\partial H_T} = 1$; $\frac{t}{C_d}\cdot\frac{\partial C_d}{\partial t} = -1$; $\frac{A_o}{C_d}\cdot\frac{\partial C_d}{\partial A_o} = -1$;

$\frac{H}{C_d}\cdot\frac{\partial C_d}{\partial H} = -0.5$

$A_T = A_{T_0} \pm 0.1\%$; $H_T = H_{T_0} \pm 0.1\%$; $t = t_0 \pm 1\%$; $A_o = A_{o_0} \pm 0.1\%$;
$H = H_0 \pm 0.1\%$

$$\therefore\ u_{C_d} = \sqrt{\frac{1}{10^6} + \frac{1}{10^6} + \frac{1}{10^4} + \frac{1}{10^6} + \frac{0.25}{10^6}}$$

$$\Rightarrow \boxed{u_{C_d} = 0.0102 = 1.02\%}$$

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OBSERVATIONS:

Area of collection tank = 903 cm^2

Diameter of orifice = 7.12 mm

TABLE 1 :

	H (cm)	X_1 (cm)	Y_1 (cm)	X_2 (cm)	Y_2 (cm)	h_t (cm)	V_a (m/s)	V_{th} (m/s)	q_a (cc/s)	q_{th} (cc/s)	Cv	Cd	Reynolds no. (Re)
1	36.4	10	47.2	18	45.4	4.8 (15.1 − 10.3)	2.470	2.672	72.24	106.38	0.9244	0.6790	19760
2	53.0	9	47.6	20	45.9	6 (21.1 − 15.1)	3.033	3.224	90.3	128.36	0.9407	0.7034	24264
3	70.2	9.1	47.7	20.6	46.25	7 (8.9 − 1.9)	3.399	3.711	105.35	147.75	0.9159	0.7130	27192
4	76.2	9.15	47.7	20.6	46.35	7.2 (16.1 − 8.9)	3.518	3.866	108.36	153.92	0.9099	0.7040	28144
5	92.3	9.15	47.8	19.5	46.8	7.8 (23.9 − 16.1)	3.813	4.255	117.39	169.41	0.8961	0.6929	30504

SAMPLE CALCULATION:

For reading no. 1 —

H = 36.4 cm ; h_t = 4.8 cm , time = 60 s , $\boxed{\text{tank area} = 903\ cm^2}$

$$\therefore Q_{actual} = \frac{903 \times 4.8}{60} \frac{cm^3}{s}$$

$$= \underline{72.24\ cm^3/s}$$

$$Q_{theoretical} = A_o \times V_{theoretical}$$

$$= \frac{\pi}{4} \times (0.712)^2 \times \sqrt{2 \times 9.81 \times 0.364} \times 100\ cm^3/s$$

$$= \underline{106.38\ cm^3/s}$$

$$\therefore C_d = \frac{Q_{actual}}{Q_{theoretical}} = \frac{72.24}{106.38}$$

$$\Rightarrow \boxed{C_d = 0.6790}$$

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$$V_{theoretical} = \sqrt{2gH}$$

$$= \sqrt{2 \times 9.81 \times 0.364} \quad m/s$$

$$= \underline{2.672\ m/s}$$

$$V_{actual} = \sqrt{\frac{g}{2}\left(\frac{x_2^2 - x_1^2}{y_2 - y_1}\right)}$$

$$= \sqrt{\frac{9.81}{2} \times \left(\frac{0.18^2 - 0.1^2}{(-.454) - (-.472)}\right)}$$

$$= \underline{2.470\ m/s}$$

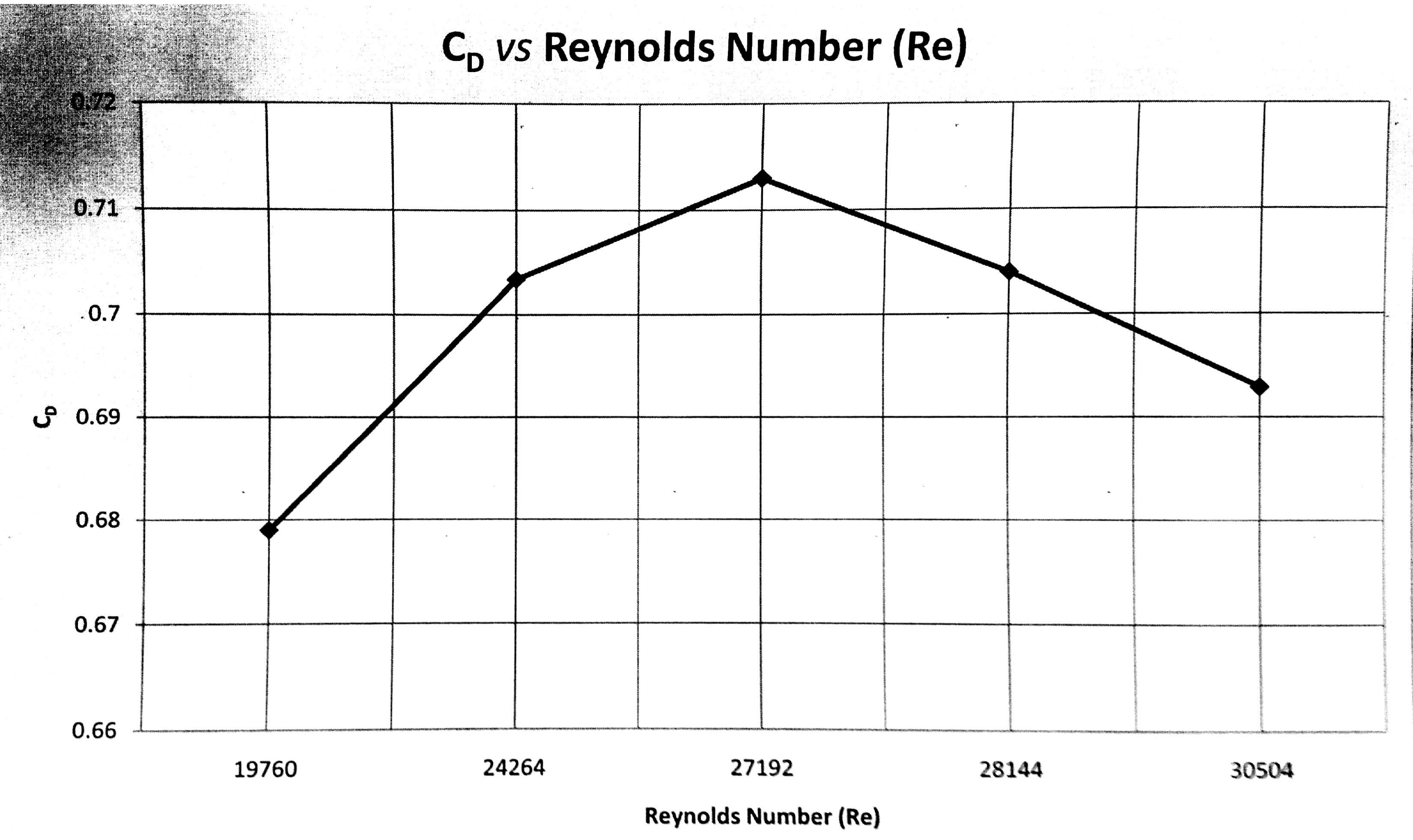
$$\therefore\ C_v = \frac{V_{actual}}{V_{theoretical}} = \frac{2.470}{2.672}$$

$$\Rightarrow \boxed{C_v = 0.9244}$$

$$Re = \frac{\rho V_a D}{\mu}$$

$$= \frac{1000 \times 2.470 \times 7.12 \times 10^{-3}}{8.9 \times 10^{-4}}$$

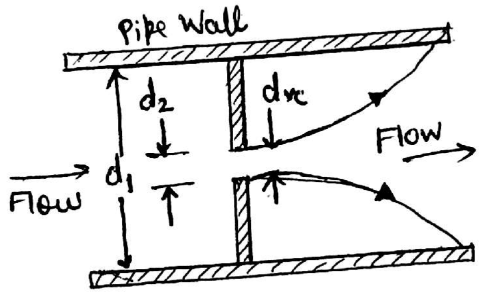
$$\boxed{Re = 19760}$$



C_D *vs* Reynolds Number (Re)
0.72
0.71
0.7
0.69
0.68
0.67
0.66
C_D
19760
24264
27192
28144
30504
Reynolds Number (Re)

● Discussions -

▶ An ORIFICE PLATE is a device used for measuring flow rate, for reducing pressure or for restricting flow (in the latter two cases it is often called a restriction plate). Either a volumetric or mass flow rate may be determined, depending on the calculation associated with the orifice plate. It uses the same principle as a Venturi Nozzle, namely Bernoulli's principle which states that there is a relation-ship between the pressure of the fluid and the velocity of the fluid. When the velocity increases, the pressure decreases and vice versa



d_1 — Pipe Diameter
d_2 — Orifice Diameter
d_{vc} — vena contracta diameter

▶ An orifice plate is a thin plate with a hole in it, which is usually placed in a pipe. When a fluid passes through the orifice, its pressure builds up slightly upstream to the orifice but as the fluid is forced to converge to pass through the hole, the velocity increases and the fluid pressure decreases. A little downstream of the orifice the flow reaches its point of maximum convergence, the vena contracta where the velocity reaches its maximum and pressure reaches its minimum. Beyond that, the flow expands, the velocity falls and the pressure increases. By measuring pressure upstream and downstream of the plate, the flow rate can be obtained.

- By assuming steady-state, incompressible, inviscid, laminar flow ~~in a horizontal pipe~~, Bernoulli's equation is given as

$$P_1 + \frac{\rho V_1^2}{2} + \rho g h_1 = P_2 + \frac{\rho V_2^2}{2} + \rho g h_2$$

- Frictional losses are not negligible and viscosity and turbulence effects are present generally. For that reason, the coefficient of discharge C_D is introduced. For rough approximations, the discharge coefficient may be assumed to be between 0.65 and 0.75

- The nature of the curve between coefficient of discharge (C_D) and Reynolds number depends on a dimensionless parameter known as β ratio.

$$\boxed{\beta = \text{Ratio of Orifice hole diameter to pipe diameter}}$$

- The nature of curve between coefficient of discharge (C_D) and Reynolds number as obtained from our experiment shows a parabolic relationship between them. The C_D increases with increasing Reynolds number, hits a maximum and then starts decreasing. This nature of the curve may be opposite for some other value of β.

- The velocity coefficient C_V which is the ratio of actual to theoretical velocity accounts for reduction in speed due to losses.