

Natural Convection: Correlations and slides

- Pertinent Dimensionless Parameters

- Grashof Number:

$$Gr_L = \frac{g \beta (T_s - T_\infty) L^3}{\nu^2} \quad \square \quad \frac{\text{Buoyancy Force}}{\text{Viscous Force}}$$

$L \rightarrow$ characteristic length of surface

$\beta \rightarrow$ coefficient of thermal expansion

$$\beta = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_p$$

Perfect Gas: $\beta = 1/T \text{ (K)}$

- Rayleigh Number:

$$Ra_L = Gr_L \text{ Pr} = \frac{g \beta (T_s - T_\infty) L^3}{\nu \alpha}$$

- Mixed Convection

- A condition for which forced and free convection effects are comparable.

- Exists if $\frac{Gr_L}{Re_L^2} \sim O(1)$

- Heat Transfer Correlations for Mixed Convection:

$$Nu^n \approx Nu_{FC}^n \pm Nu_{NC}^n$$

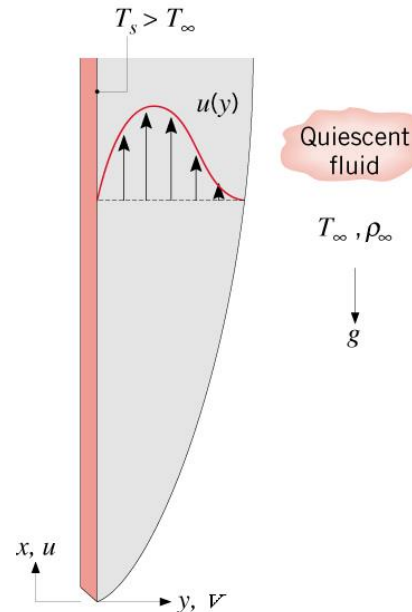
+ → assisting and transverse flows

- → opposing flows

$$n \approx 3$$

Vertical Plates

- Free Convection Boundary Layer Development on a Heated Plate:



- Ascending flow with the maximum velocity occurring in the boundary layer and zero velocity at both the surface and outer edge.
- How do conditions differ for a cooled plate ($T_s < T_\infty$)?

- Form of the x -Momentum Equation for Laminar Flow

$$\underbrace{u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}}_{\text{Net Momentum Fluxes (Inertia Forces)}} = \underbrace{g \beta (T - T_{\infty})}_{\text{Buoyancy Force}} + \underbrace{\nu \frac{\partial^2 u}{\partial y^2}}_{\text{Viscous Force}}$$

- Temperature dependence requires that solution for $u(x,y)$ be obtained concurrently with solution of the boundary layer energy equation for $T(x,y)$.

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}$$

- The equations and solutions are said to be coupled.

Derivation – Similarity Solution
out of scope for this class

➤ **Nusselt Numbers** (Nu_x and \overline{Nu}_L):

$$Nu_x = \frac{hx}{k} = - \left(\frac{Gr_x}{4} \right)^{1/4} \left. \frac{dT^*}{d\eta} \right|_{\eta=0} = \left(\frac{Gr_x}{4} \right)^{1/4} g(\text{Pr})$$

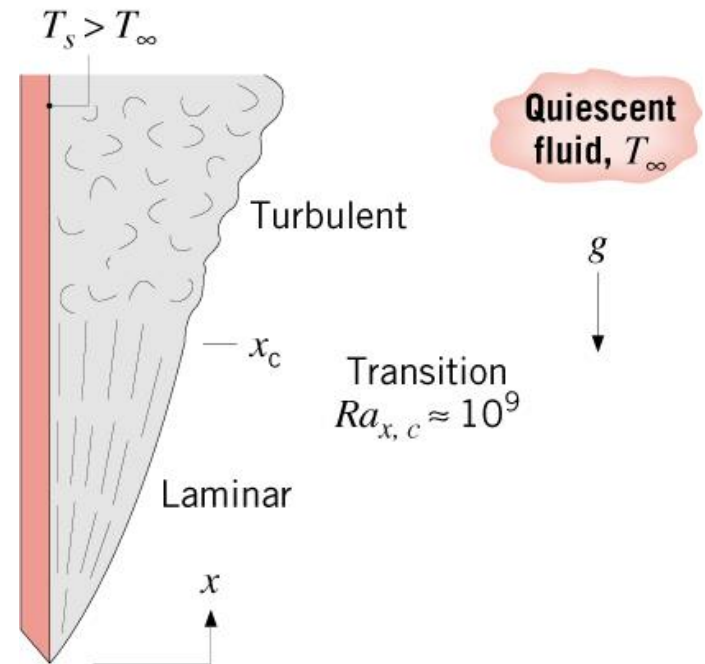
$$g(\text{Pr}) = \frac{0.75 \text{Pr}^{1/2}}{(0.609 + 1.221 \text{Pr}^{1/2} + 1.238 \text{Pr})^{1/4}} \quad (0 < \text{Pr} < \infty)$$

$$\bar{h} = \frac{1}{L} \int_0^L h dx \rightarrow \overline{Nu}_L = \frac{4}{3} Nu_L$$

- **Transition to Turbulence**

- Amplification of disturbances depends on relative magnitudes of buoyancy and viscous forces.
- Transition occurs at a critical Rayleigh Number.

$$Ra_{x,c} = Gr_{x,c} \text{Pr} = \frac{g \beta (T_s - T_\infty) x^3}{\nu \alpha} \approx 10^9$$



- Empirical Heat Transfer Correlations

➤ Laminar Flow ($Ra_L < 10^9$):

$$\overline{Nu}_L = 0.68 + \frac{0.670 Ra_L^{1/4}}{\left[1 + (0.492/Pr)^{9/16}\right]^{4/9}}$$

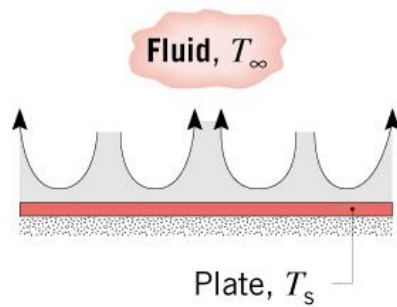
➤ Turbulent Flow (*note this is average over entire length – no need to average over laminar and turbulent lengths separately*)

$$\overline{Nu}_L = \left\{ 0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1 + (0.492/Pr)^{9/16}\right]^{4/9}} \right\}^2$$

➤ *Note that average Nu varies as $Ra_L^{1/3}$*

Horizontal Plates

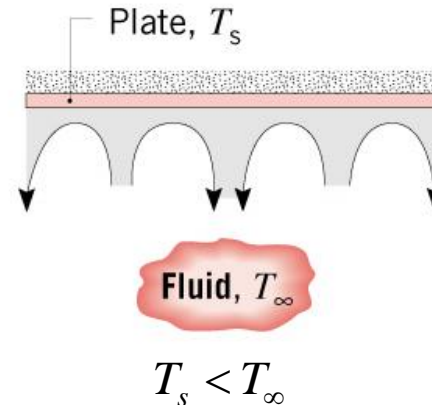
- Buoyancy force is normal, instead of parallel, to the plate.
- Flow and heat transfer depend on whether the plate is heated or cooled and whether it is facing upward or downward.
- Heated Surface Facing Upward or Cooled Surface Facing Downward



$$T_s > T_\infty$$

$$\overline{Nu}_L = 0.54 Ra_L^{1/4}$$

$$\overline{Nu}_L = 0.15 Ra_L^{1/3}$$



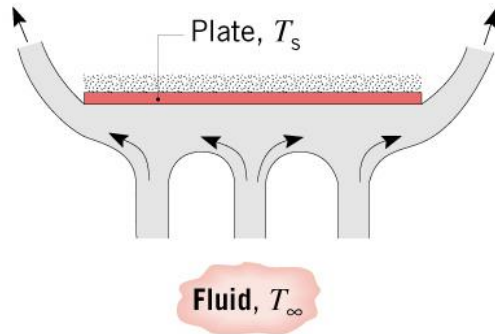
$$T_s < T_\infty$$

$$(10^4 < Ra_L < 10^7)$$

$$(10^7 < Ra_L < 10^{11})$$

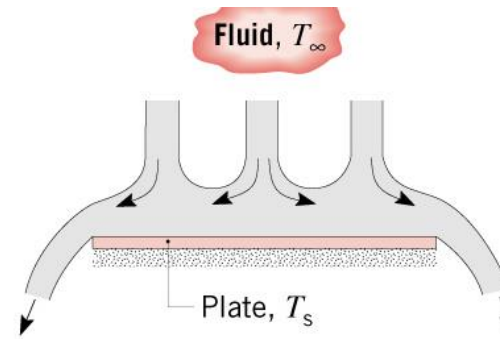
How does \bar{h} depend on L when $\overline{Nu}_L \propto Ra_L^{1/3}$?

- Heated Surface Facing Downward or Cooled Surface Facing Upward



$$T_s > T_\infty$$

$$\overline{Nu}_L = 0.27 Ra_L^{1/4}$$



$$T_s < T_\infty$$

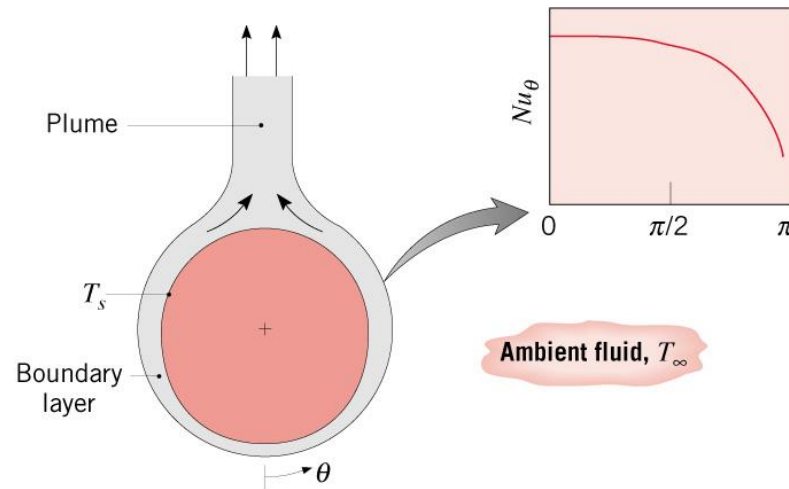
$$(10^5 < Ra_L < 10^{10})$$

Inclined Plates

- Refer to class notes

The Long Horizontal Cylinder

- Boundary Layer Development and Variation of the Local Nusselt Number for a Heated Cylinder:



- The Average Nusselt Number:

$$\overline{Nu}_D = \left\{ 0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1 + (0.559 / Pr)^{9/16} \right]^{8/27}} \right\}^2$$

$$Ra_D < 10^{12}$$

Spheres

- The Average Nusselt Number:

$$\overline{Nu}_D = 2 + \frac{0.589 Ra_D^{1/4}}{\left[1 + (0.469 / Pr)^{9/16}\right]^{4/9}}$$

Natural Convection through channels

- Refer to class notes
 - Limits – large spacing and narrow spacing
 - Concept of optimal spacing for maximum heat transfer from an array of vertical plates under forced convection
 - Correlations of Bar Cohen and Rohsenow for different boundary conditions (refer Incropera and Dewitt)