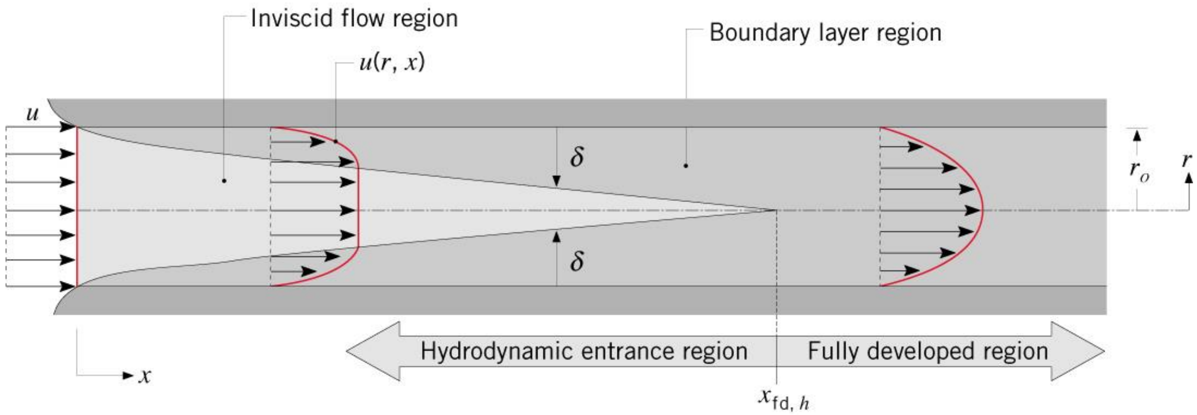
Internal Flow:
Forced Convection

Entrance Conditions

- Must distinguish between entrance and fully developed regions.
- Hydrodynamic Effects: Assume laminar flow with uniform velocity profile at inlet of a circular tube.

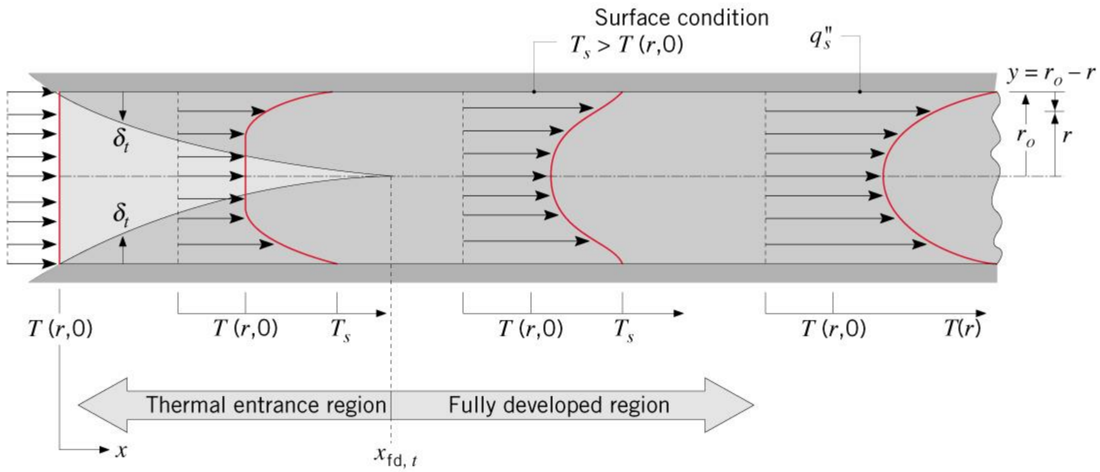


 – Velocity boundary layer develops on surface of tube and thickens with increasing *x*.

 – Inviscid region of uniform velocity shrinks as boundary layer grows.

 – Subsequent to boundary layer merger at the centerline, the velocity profile becomes parabolic and invariant with *x*. The flow is then said to be <u>hydrodynamically fully developed</u>.

- Thermal Effects: Assume laminar flow with uniform temperature, $T(r,0)=T_i$, at inlet of circular tube with uniform surface temperature, $T_s \neq T_i$, or heat flux, q_s''.



  – Thermal boundary layer develops on surface of tube and thickens with increasing x.

  – Isothermal core shrinks as boundary layer grows.

  – Subsequent to boundary layer merger, dimensionless forms of the temperature profile (for T_s and q_s'') become independent of x. Conditions are then said to be thermally fully developed.

Derivations

- Refer to class notes for derivations
  - Laminar flow through a circular tube

Fully Developed Flow

- Laminar Flow in a Circular Tube:

The local Nusselt number is a constant throughout the fully developed region, but its value depends on the surface thermal condition.

– Uniform Surface Heat Flux (q_s''):

$$Nu_D = \frac{hD}{k} = 4.36$$

– Uniform Surface Temperature (T_s):

$$Nu_D = \frac{hD}{k} = 3.66$$

- Turbulent Flow in a Circular Tube:

– For a smooth surface and fully turbulent conditions $\left(\mathrm{Re}_D > 10{,}000\right)$, the Dittus – Boelter equation may be used as a first approximation:

$$Nu_D = 0.023\,\mathrm{Re}_D^{4/5}\,\mathrm{Pr}^n \qquad \begin{cases} n = 0.3 & \left(T_s < T_m\right) \\ n = 0.4 & \left(T_s > T_m\right) \end{cases}$$

– The effects of wall roughness and transitional flow conditions $\left(\mathrm{Re}_D > 3000\right)$ may be considered by using the Gnielinski correlation:

$$Nu_D = \frac{\left(f/8\right)\left(\mathrm{Re}_D - 1000\right)\mathrm{Pr}}{1 + 12.7\left(f/8\right)^{1/2}\left(\mathrm{Pr}^{2/3} - 1\right)}$$

Smooth surface:

$$f = (0.790\ \ln \mathrm{Re}_D - 1.64)^{-2}$$

Surface of roughness $e > 0$:

$$f \rightarrow \textit{Moody chart}$$

- Noncircular Tubes:
  - Use of hydraulic diameter as characteristic length:

$$D_h \equiv \frac{4A_c}{P}$$

  - Since the local convection coefficient varies around the periphery of a tube, approaching zero at its corners, correlations for the fully developed region are associated with convection coefficients averaged over the periphery of the tube.

  - Laminar Flow:

    The local Nusselt number is a constant whose value depends on the surface thermal condition $(T_s \ or \ q''_s)$ and the duct aspect ratio.

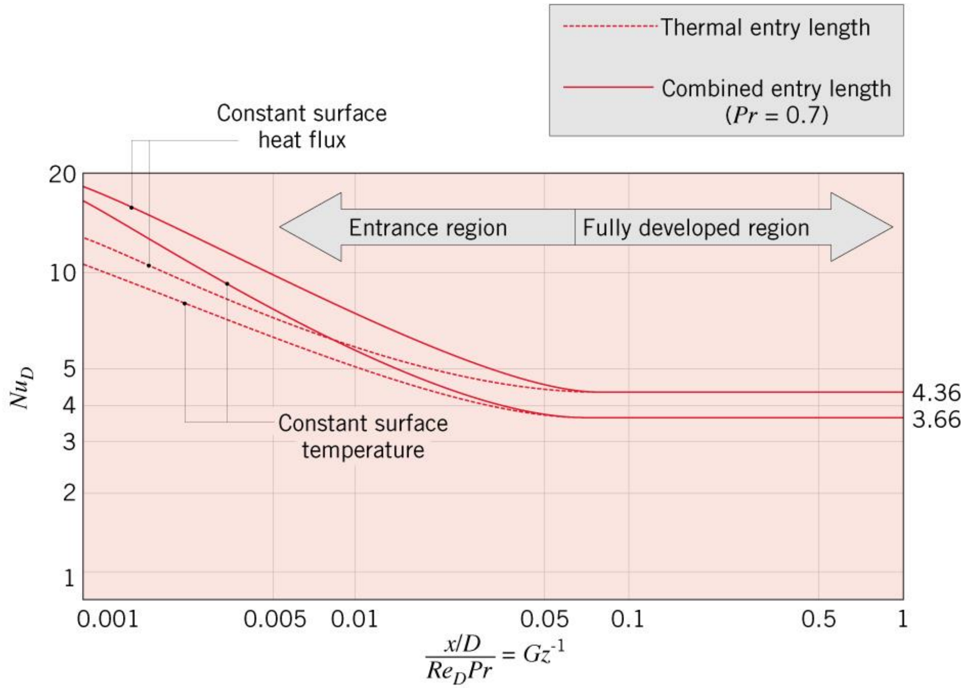
  - Turbulent Flow:

    As a first approximation, the Dittus-Boelter or Gnielinski correlation may be used with the hydraulic diameter, irrespective of the surface thermal condition.

Effect of the Entry Region

- The manner in which the Nusselt decays from inlet to fully developed conditions for laminar flow depends on the nature of thermal and velocity boundary layer development in the entry region, as well as the surface thermal condition.

Laminar flow in a circular tube.



– Combined Entry Length:

Thermal and velocity boundary layers develop concurrently from uniform profiles at the inlet.

- Thermal Entry Length:

  Velocity profile is fully developed at the inlet, and boundary layer development in the entry region is restricted to thermal effects. Such a condition may also be assumed to be a good approximation for a uniform inlet velocity profile if $\Pr >> 1$.

- Average Nusselt Number for Laminar Flow in a Circular Tube with Uniform Surface Temperature:
  - Combined Entry Length:

$$\left[\mathrm{Re}_D \Pr/\left(L/D\right)\right]^{1/3}\left(\mu/\mu_s\right)^{0.14} > 2:$$

$$\overline{Nu}_D = 1.86\left(\frac{\mathrm{Re}_D \Pr}{L/D}\right)^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14}$$

$$\left[\mathrm{Re}_D \Pr/\left(L/D\right)\right]^{1/3}\left(\mu/\mu_s\right)^{0.14} < 2:$$

$$\overline{Nu}_D = 3.66$$

  - Thermal Entry Length:

$$\overline{Nu}_D = 3.66 + \frac{0.0668\left(D/L\right)\mathrm{Re}_D \Pr}{1 + 0.04\left[\left(D/L\right)\mathrm{Re}_D \Pr\right]^{2/3}}$$

- Average Nusselt Number for Turbulent Flow in a Circular Tube :
  - Effects of entry and surface thermal conditions are less pronounced for turbulent flow and can be neglected.
  - For long tubes $\left(L/D > 60\right)$:

$$\overline{Nu}_D \approx Nu_{D,\,fd}$$

  - For short tubes $\left(L/D < 60\right)$:

$$\frac{\overline{Nu}_D}{Nu_{D,\,fd}} \approx 1 + \frac{C}{\left(L/D\right)^m}$$

$$C \approx 1$$
$$m \approx 2/3$$

- Noncircular Tubes:
  - Laminar Flow:

    $\overline{Nu}_{D_h}$ depends strongly on aspect ratio, as well as entry region and surface thermal conditions. Refer to table shown in class

- Turbulent Flow:

  As a first approximation, correlations for a circular tube may be used with D replaced by D_h.

- When determining $\overline{Nu}_D$ for any tube geometry or flow condition, all properties are to be evaluated at

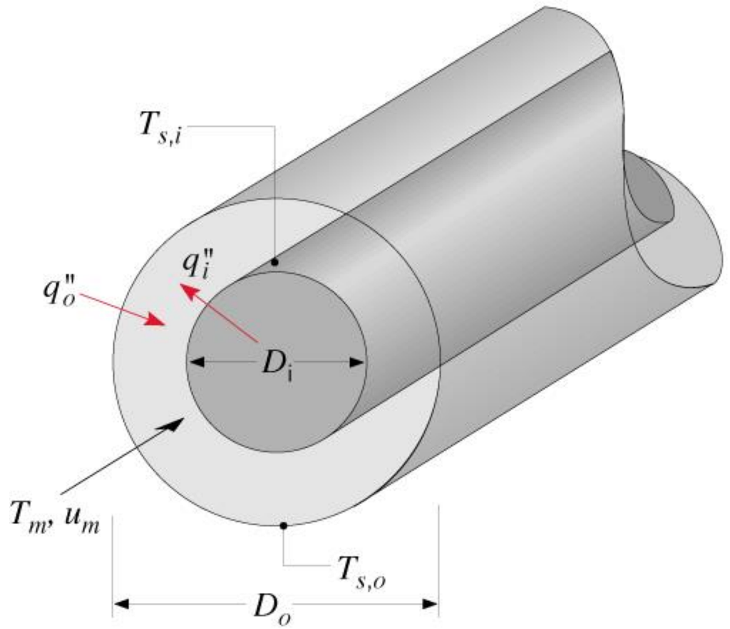
$$\overline{T}_m \equiv \left(T_{m,i} + T_{m,o}\right)/2$$

Refer to the problem solved in class that involved iteration for exit temperature

The Concentric Tube Annulus

- Fluid flow through region formed by concentric tubes.

- Convection heat transfer may be from or to inner surface of outer tube and outer surface of inner tube.



- Surface thermal conditions may be characterized by uniform temperature $\left(T_{s,i}, T_{s,o}\right)$ or uniform heat flux $\left(q''_i, q''_o\right)$.

- Convection coefficients are associated with each surface, where

$$q''_i = h_i\left(T_{s,i} - T_m\right)$$

$$q''_o = h_o\left(T_{s,o} - T_m\right)$$

Annulus (cont)

$$Nu_i \equiv \frac{h_i D_h}{k} \qquad Nu_o \equiv \frac{h_o D_h}{k}$$

$$D_h = D_o - D_i$$

- Fully Developed Laminar Flow

  Nusselt numbers depend on D_i / D_o and surface thermal conditions (standard tables available)

- Fully Developed Turbulent Flow

  Correlations for a circular tube may be used with D replaced by D_h .