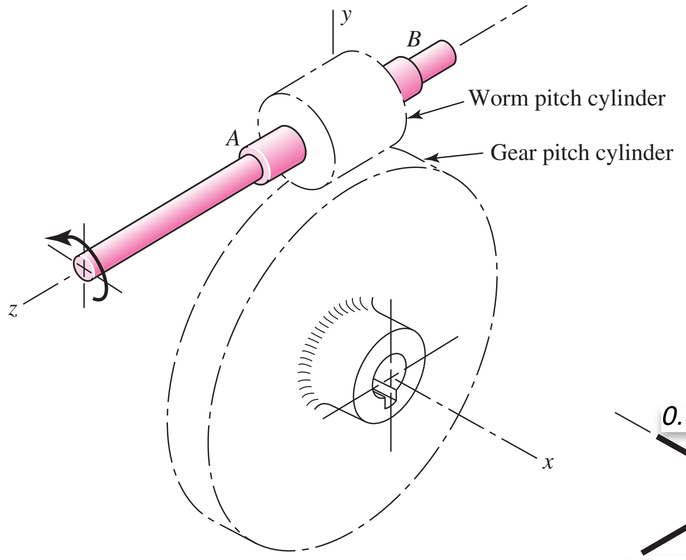
Assignment/Tutorial 8
(Rolling Contact Bearings)

April 13, 2017

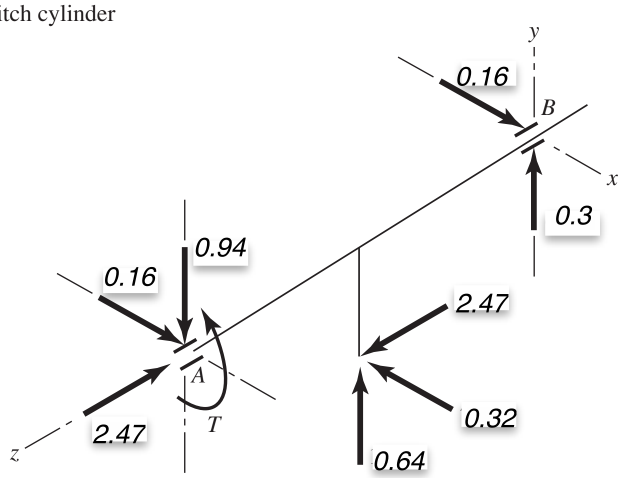
ME-30602, 2016-17 Spring Semester

1. Problems 11.1, 11.2, 11.3, 11.5, 11-6, 11-7, 11-12, 11-16, 11-17, 11-19, 11-20, 11-21, 11-24, 11-25, 11-27
   From Chapter 11 of Shigley's Mechanical Engineering Design book. (On Rolling Contact Bearings).

Problem: The worm shaft shown in the figure (a) below transmits 1000W at 600rpm. A static force analysis (assuming the bearing at A takes thrust load) is shown in figure (b). The loads are in kN. The desired life is 25kh and the application factor is 1.3.



(a)



(b)

(a) For the above case select a 02-series angular-contact bearing at A and a 02-series straight roller bearing at B. Combined reliability is 0.99. Since the axial thrust is significantly larger than the radial loads and bearing at A is taking the thrust. The chance of failure of bearing B is much less. Thus choose a reliability of 0.99 for bearing at A and reliability of 1 for bearing B. The Weibull parameters for the ball bearings are $x_0 = 0.02, \theta - x_0 = 4.139, b = 1.483.$

Assignment/Tutorial 8
(Shafts)

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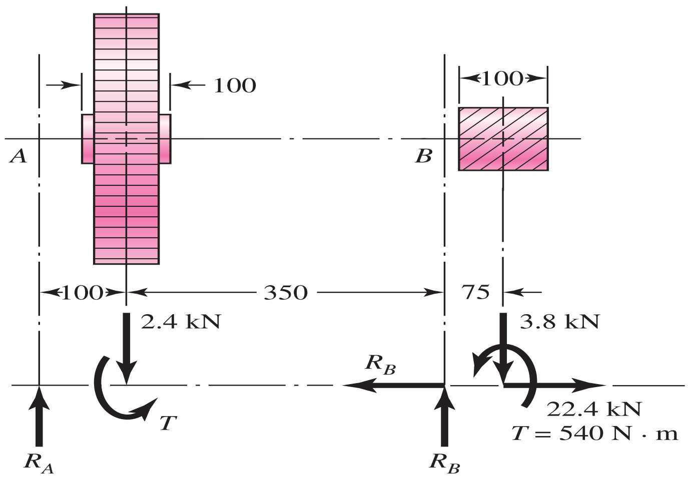
1. Problems 7.1(d), 7.3, 7.9, 7.10, 7.19, 7.20 (For fatigue failure use M-G criterion) From Chapter 7 of Shigley's Mechanical Engineering Design book. (Shafts).

Note: The problems are picked from 10th edition of the book. In case you have a different edition the problem numbers may change. You should check this.

<u>Problem:</u> An AISI 1020 CD steel shaft is to be designed to support the spur gear and overhanging worm as shown below. The shaft speed is 300rpm. The bearing at A takes pure radial load and the bearing at B can take axial thrust for rotation of worm in either direction. The radial loads on the gear and the worm are in the same plane. The torque transfer between gears and the shaft takes place through rectangular keys. Design the shaft by following the steps below.



(a) Draw the diagrams corresponding to bending moment, torque and axial thrust.

(b) Identify the critical locations and determine the minimum shaft diameter at those locations based on a factor of safety of 2 for the shaft. Use static yielding and modified Goodman criteria. Maintain a diameter ratio *(D/d)* of 1.2 and fillet-radius to smaller diameter ratio *(r/d)* of 0.05 at each bearing location. The fatigue stress concentration factors at the keyways are 5.0 for bending and 3 for torsion.

(c) Check for the failure of keys for both the spur gear and the worm. The factor of safety for key should be 1.8. Key material is AISI 1006 HR steel.

Tutorial 8 — Solution (Shaft problem - second page)

Part-a

$\Sigma F_y = 0$:

$\boxed{R_A + R_B = 6.2\ kN}$

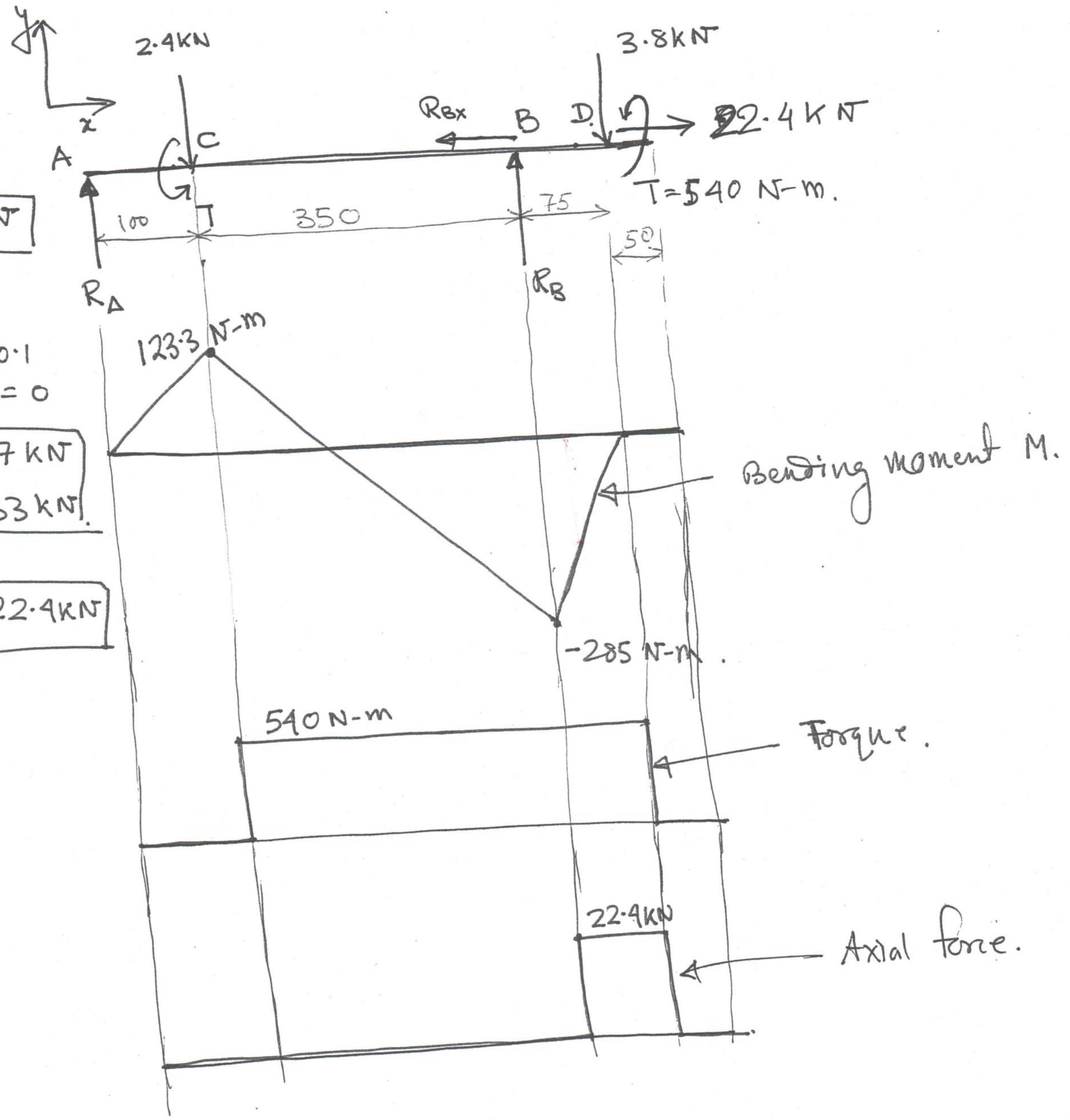
$\Sigma M_A = 0$:

$+R_{Bx} \times 0.45 - 2.4 \times 0.1 - 3.8 \times 0.525 = 0$

$\Rightarrow \boxed{\begin{array}{l} R_B = 4.967\ kN \\ R_A = 1.233\ kN \end{array}}$

$\Sigma F_x = 0$:

$\boxed{R_{Bx} = 22.4\ kN}$



Part-b

Most critical point: Just right of point B:

$$\boxed{\begin{array}{l} M = 285\ N\text{-}m \\ T = 540\ N\text{-}m. \\ F_{axial} = 22.4\ kN \end{array}}$$

$D/d = 1.2$ at B, $r/d = 0.05$ $\Rightarrow$

$$\boxed{\begin{array}{lll} K_t = 2.0 & \text{(Axial)} \\ K_t = 1.95 & \text{(bending)} \\ K_{ts} = 1.6 & \text{(torsion)} \end{array}}$$ (A-15-7 — A-15-9)

Static stress concentration factors.

Notch Sensitivity Factor: q_i depends on fillet radius which we do not know, yet.

We will take guess values of K_f and K_{fs} to be same as the static stress concentration factors.

<u>From A-20</u>: AISI 1020 CD Steel:

$$S_{ut} = 470\ \text{MPa}, \quad S_y = 390\ \text{MPa}.$$

$$(\underline{\text{Eq 6-19}}) \longrightarrow K_a = a\,S_{ut}^{\,b} = 4.51(470)^{-0.251}$$

$$= 0.883$$

$$K_b = 0.9 \quad (\text{guess}).$$

$$K_c = K_d = K_e = 1.$$

$$\Rightarrow \quad S_e = (0.883)(0.9)(1)(1)(1)(0.5)(470)$$

$$= \underline{186\ \text{MPa}}$$

Bending moment is completely reversing.
Axial load and Torque are constants.

$$\Rightarrow \quad M_a = 285\ \text{N-m}, \quad T_a = 0, \quad F_a = 0$$

$$M_m = 0, \quad T_m = 546\ \text{N-m}, \quad F_m = 22.4\ \text{kN}$$

<u>Ignore axial load first:</u>

Von-Mises stresses. eq 7-5, 7-6

$$\sigma_a' = \left\{\left(\frac{32 K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16 K_{fs} T_a}{\pi d^3}\right)^2\right\}^{1/2}$$

$$\sigma_m' = \left\{\left(\frac{32 K_f M_m}{\pi d^3}\right)^2 + 3\left(\frac{16 K_{fs} T_m}{\pi d^3}\right)^2\right\}^{1/2}.$$

$K_b = K_t = 1.95$?

$K_{ft} = \cancel{0.62}$ 1.95

$K_{ts} = K_{t3}$

$K_{fs} = 1.6$

$$\Rightarrow \quad \sigma_a' = \frac{32 K_f M_a}{\pi d^3} = \frac{5661}{d^3}$$

$$\sigma_m' = \sqrt{3}\,.\,\frac{16 K_{fs} T_m}{\pi d^3} = \frac{7621.57}{d^3}.$$

<u>Modified Goodman criteria</u>: $\dfrac{1}{n_f} = \dfrac{\sigma_a'}{S_e} + \dfrac{\sigma_m'}{S_{ut}}$

$$\Rightarrow \quad \frac{1}{2} = \frac{5661}{186\times10^6\, d^3} + \frac{7621.57}{470\times10^6\, d^3} \Rightarrow \boxed{d = 45.4\ \text{mm}}$$

Since we started in a conservative manner, We can choose the standard size to be

Table A-17 $\boxed{d = 45\ mm}$ instead of next value d=50mm

Now we do a through calculation with this starting dia d=45mm. (at right of b).

For the fillet: $r = 0.05 \times 45\ mm = 2.25\ mm$.

$$\Rightarrow q \simeq 0.78, \quad q_{shear} \cong 0.81 \quad \left(\text{Fig } \begin{matrix} 6\text{-}20 \\ 6\text{-}21 \end{matrix}\right)$$

(Eq 6-32)

$$\Rightarrow \begin{cases} K_{f\,axial} = 1 + q\,(K_{t\,axial} - 1) \\ \qquad = 1 + 0.78\,(2-1) = 1.78 \\ K_{f\,bending} = 1 + 0.78(1.95-1) = 1.741 \\ K_{f\,shear} = 1 + 0.81\,(1.6-1) = 1.486. \end{cases}$$

$\Rightarrow$ k_a remains the same $\Rightarrow$ $k_a = 0.883$

(Eq 6-20) $\leftarrow$ $k_b = \left(\dfrac{d}{7.62}\right)^{-0.107} = \left(\dfrac{45}{7.62}\right)^{-0.107} = 0.827.$

$$k_c = k_d = k_e = 1.$$

$$\Rightarrow \boxed{S_e = 171.6\ MPa}$$

$$\sigma_a\big|_{bending} = \frac{32\,K_f\,M_a}{\pi d^3} = \underline{55.5\ MPa}$$

$$\sigma_a\big|_{axial} = \tau_a = 0$$

$$\sigma_m\,_{bending} = 0, \qquad \sigma_m\big|_{axial} = K_{fs} \cdot \frac{22.4 \times 10^3}{\pi d^2/4} = \underline{25.07\ MPa}$$

$$\tau_m = K_{fs} \cdot \frac{16\,T_m}{\pi d^3} = \underline{44.9\ MPa}$$

$$\Rightarrow \boxed{\sigma_a' = 55.5\ MPa} \quad \sigma_m' = \left(\left(\sigma_{m\,axial}\right)^2 + 3\tau_m^2 \right)^{1/2} \cong 83.17\ MPa$$

May be less.

81.2

Eq - 655, 6-56.

$$\Rightarrow \boxed{n_f = \frac{1}{\sigma_a'/S_e + \sigma_m'/S_{ut}} = 2.} \text{ (okay).}$$

Static yield: $\sigma'_{max} = \sigma'_a + \sigma'_m = 138.7$ MPa.

$\Rightarrow n_y = S_y / \sigma'_{max} = 2.812$ (okay).

$\Rightarrow$ $\boxed{d = 45 \text{ mm at the right of B is okay}}$

At the left of B: $\frac{D}{d} = 1.2 \Rightarrow$ $\boxed{D = 54 \text{ mm}}$

Again this is not a standard size. Since major part of the shaft will be of diameter D. We should choose standard value for this

Let $\boxed{D = 60 \text{ mm}, \Rightarrow d = 50 \text{ mm.}}$ (Table A-17).

Next critical location is the spur gear, because of key way.

D = 60 mm.

Load: $M_a = 1233$ N-m., $M_m = 0$ $\boxed{K_f = 5.0}$

$T_m = 540$ N-m, $T_a = 0$, $\boxed{K_{fs} = 3.0}$

No axial load.

$$\Rightarrow \sigma_a' = K_f \cdot \frac{32 M_a}{\pi D^3} = 29.07 \text{ MPa}$$

$$\sigma_m' = \sqrt{3} \cdot K_{fs} \cdot \frac{T_m \times 16}{\pi D^3} = 66.16 \text{ MPa.}$$

$$\Rightarrow n_f = \left(29.07/171.6 + \frac{66.16}{470}\right)^{-1} = 3.22 \text{ (okay)}$$

$\boxed{\text{So, the sizes mentioned above are okay.}}$

Shaft size: Between bearings A and B: $\boxed{D = 60 \text{ mm.}}$

For bearing A and Bearing B and the remaining: $\boxed{d = 50 \text{ mm}}$

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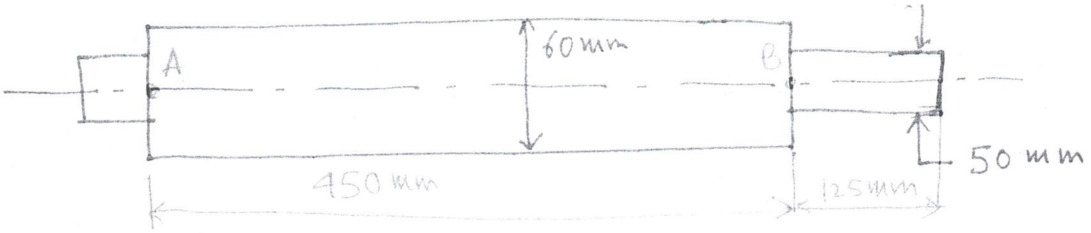


Part-C : Design of rectangular key.

Material — AISI 1006 HR steel. $\Rightarrow$ $\boxed{S_y = 170\text{ MPa},\quad S_{sy} = 0.577 S_y = 98.1\text{ MPa}}$

Key for spur gear :

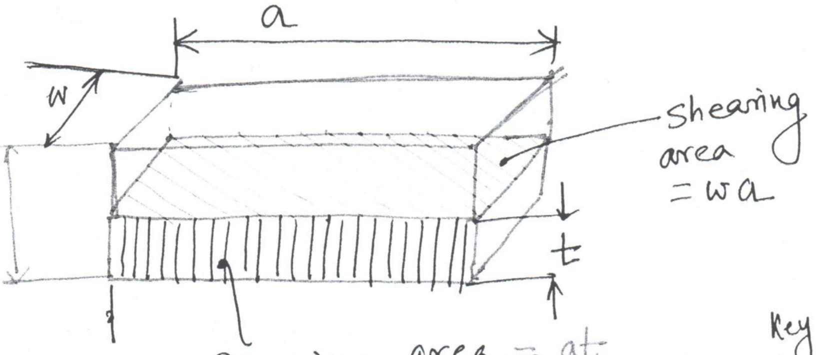
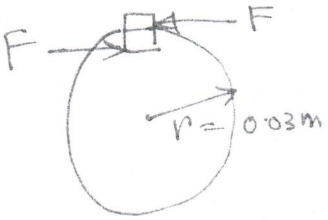


Table 7-6 :
Key size for shaft (dia = 60 mm)

w = 16 mm
h = 12 mm
t = 5.5 mm

Key way depth.

Failure due to shearing : $T = 540$ N-m $\Rightarrow$ $F = 540/0.03 = 18$ kN



$$\frac{S_{sy}}{n} = \frac{F}{wa} \Rightarrow \frac{98.1\times10^6}{1.8} = \frac{18\times10^3}{16\times10^{-3}\times a}$$

$$\Rightarrow \boxed{a = 20.64\text{ mm}}$$

Failure due to bearing: $\frac{S_y}{n} = \frac{F}{at} \Rightarrow \frac{170\times10^6}{1.8} = \frac{18\times10^3}{5.5\times a}$

$$\Rightarrow \boxed{a = 34.65\text{ mm}}$$

$\Rightarrow$ Key length should be larger than 34.65 mm.

Key for worm :

Table 7-6 : Key size (shaft dia = 50mm)

$\boxed{w = 12\text{ mm},\quad h = 10\text{ mm},\quad t = 5\text{ mm}}$

Shearing : $a = \frac{20.64}{(50/60)} \times \frac{16}{12}$ mm $= 27.52$ mm.

Bearing: $a = 34.65 \times \frac{60}{50} \times \frac{5.5}{5}$ mm $= 45.74$ mm.

$\Rightarrow$ Key length $a \geq 45.74$ mm (for worm).

Since both keys should have $a \simeq 100$ mm (width of worm & gear hub) – both are safe.
... are safe.