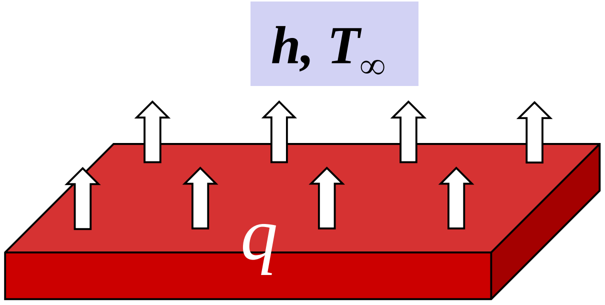
Conduction: Theory of Extended Surfaces

Why extended surface?



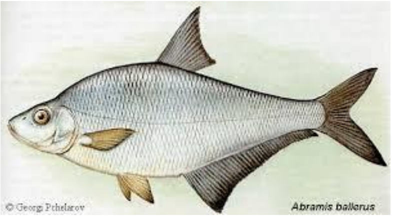
$$q = hA(T_s - T_\infty)$$

Increasing *h*

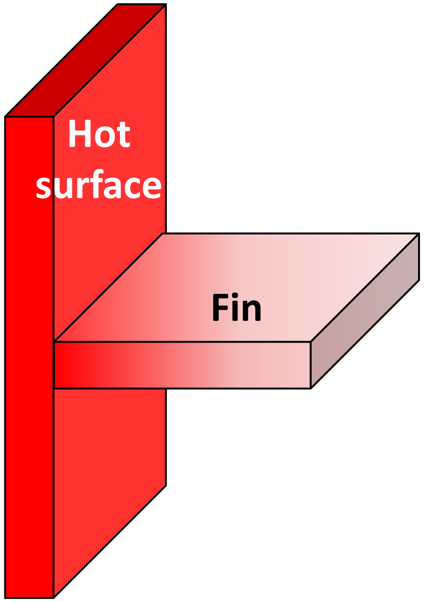
Increasing *A*

Fins as extended surfaces

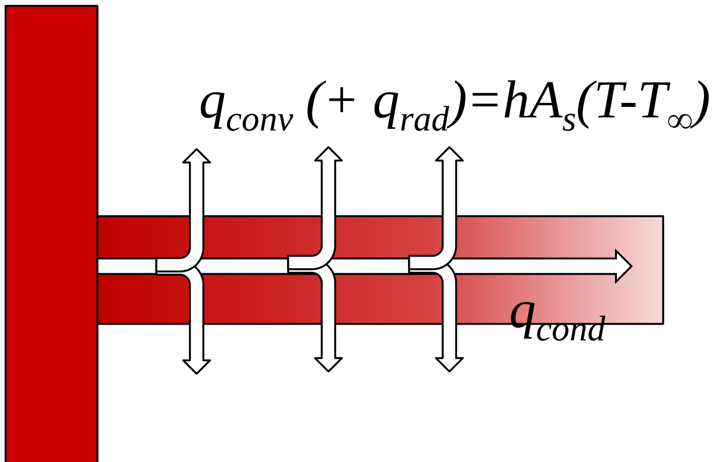
A **fin** is a thin component or appendage attached to a larger body or structure



In the context of heat transfer also, these are components protruding out of a heated (or cold) surface



What happens in a fin?

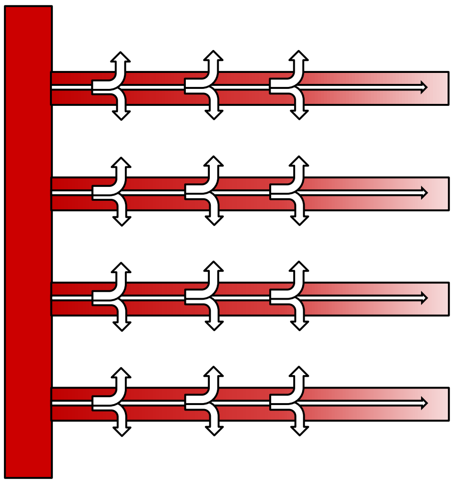


$$q_{conv} = hA_s(T-T_\infty)$$

$$q_{cond} = -kA_c dT/dx$$

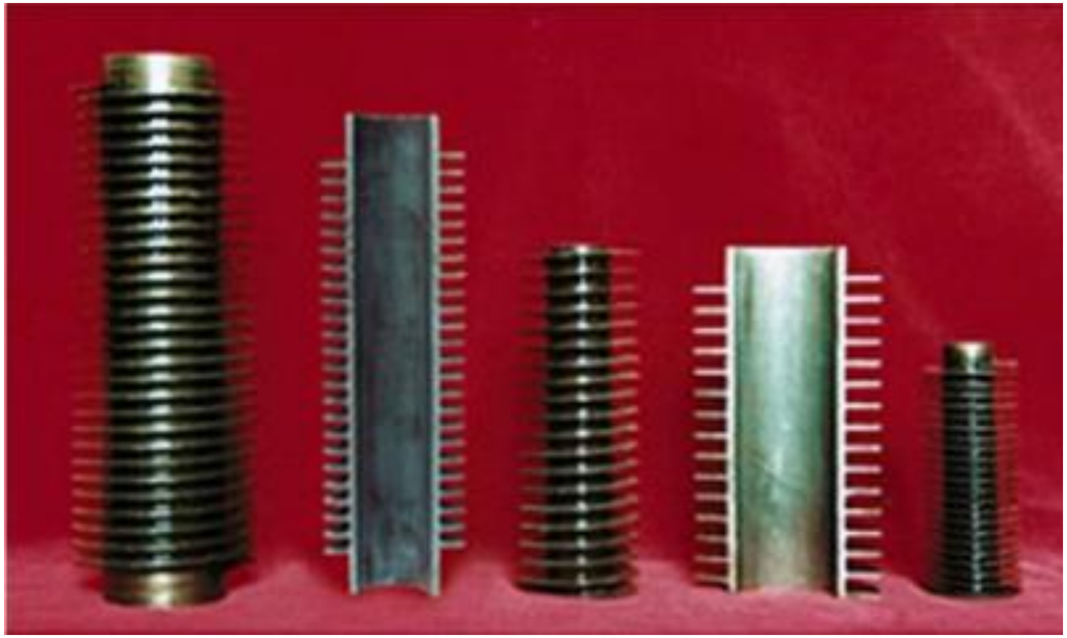
An extended surface (combined conduction-convection system) is a solid within which heat transfer by conduction is *assumed* to be one dimensional, while heat is also transferred by convection (and/or radiation) from the surface in a direction transverse to that of conduction.

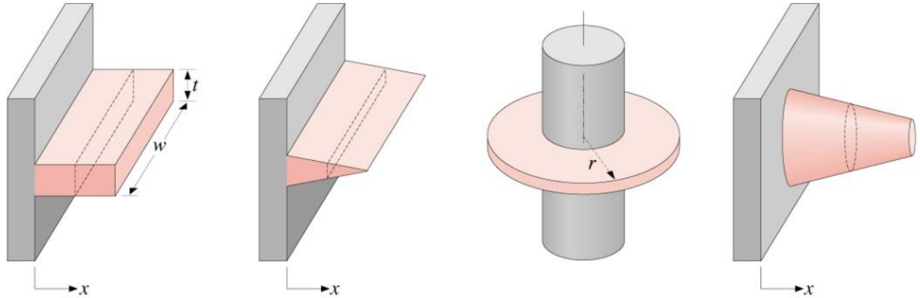
What happens in a fin?



An extended surface (combined conduction-convection system) is a solid within which heat transfer by conduction is *assumed* to be one dimensional, while heat is also transferred by convection (and/or radiation) from the surface in a direction transverse to that of conduction.

Examples of Fins



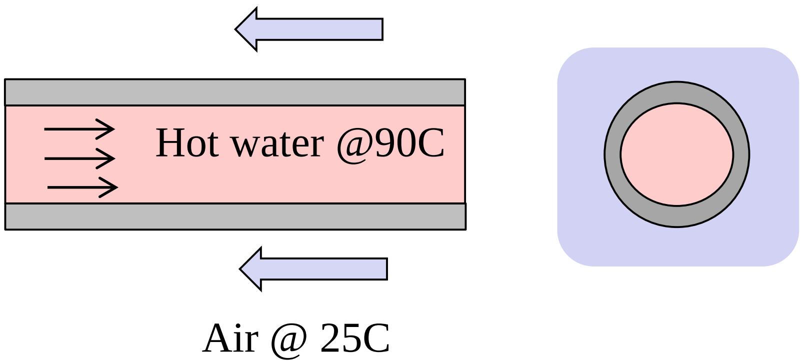


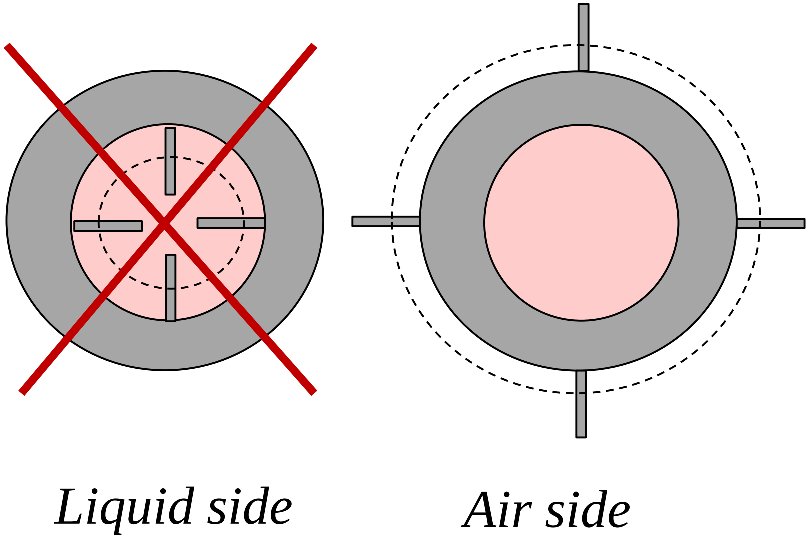
Question time

- Heat is transferred from hot water flowing inside a tube to cooling air flowing over the tube. To enhance heat transfer rate, **which side** should the **fins be installed**?

- Fins are most **beneficial** where ***h* is low**

- Fin *dimensions* and *k* are **critical** design parameters





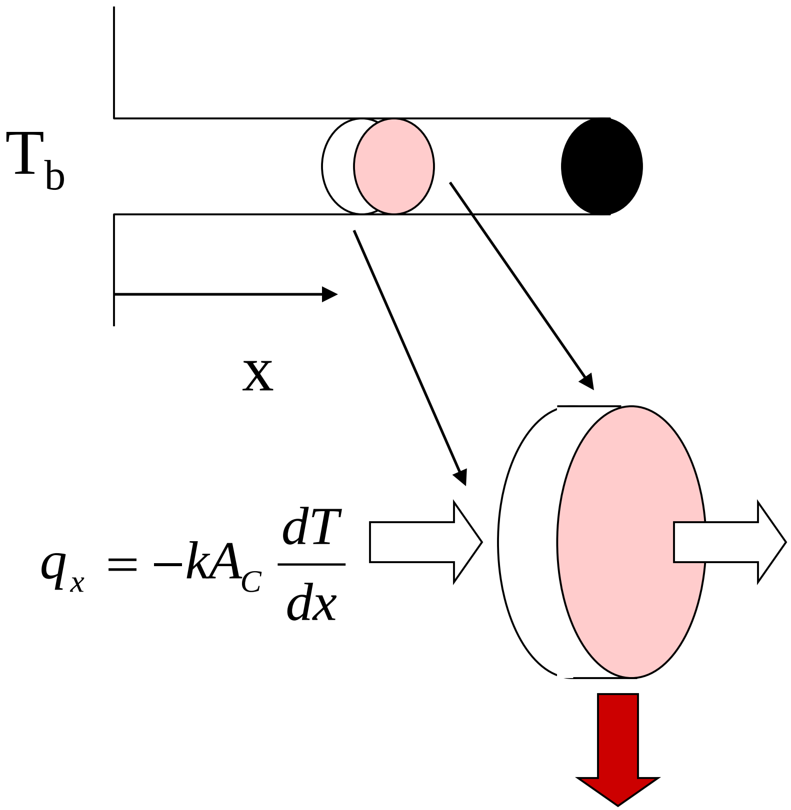
Summary

- Extended surfaces help in enhancing heat dissipation
  - Increases surface area for heat exchange

- Fins: most common embodiment of extended surface
  - Can be of varied shape and forms

- Fin peformance = *f (h, material, size)*

Fin Analysis



P: the fin perimeter

A_c: the fin cross-sectional area

$$q_x = -kA_C \frac{dT}{dx}$$

$$q_{x+dx} = q_x + \frac{dq_x}{dx} dx$$

$dq_{conv} = h(dA_S)(T - T_\infty)$, where dA_S is the surface area of the element

Energy balance: $$q_x = q_{x+dx} + dq_{conv} = q_x + \frac{dq_x}{dx} + hP(T - T_\infty)$$

Fin Analysis (cont.)

$$\frac{d}{dx}(kA_c\frac{dT}{dx}) - hP(T - T_\infty) = 0 \qquad A_c = A_c(x)$$

$$\theta = T - T_\infty \quad \rightarrow \quad \frac{d}{dx}(A_c\frac{d\theta}{dx}) - \frac{hP}{k}\theta = 0$$

For a constant cross-section A_c

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0, \quad m^2 = \frac{hP}{kA_c} \quad \longrightarrow \quad \boxed{\theta(x) = C_1 e^{mx} + C_2 e^{-mx}}$$

Need two boundary conditions

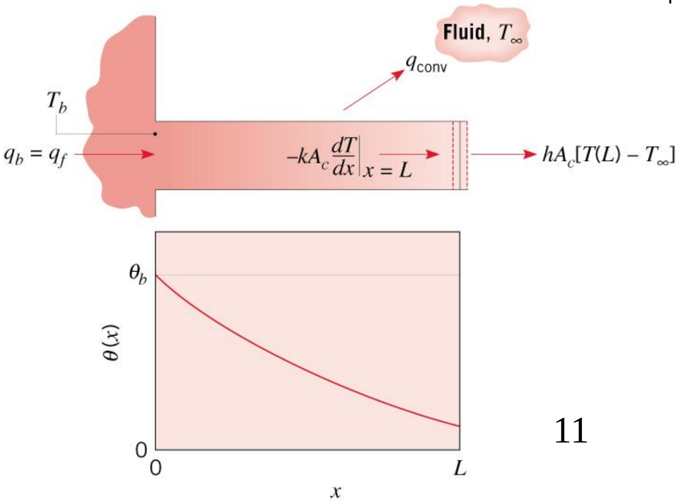
- *Base:* $\theta = \theta_b \text{ at } x = 0$
- *Tip: 4 scenarios at x = L*

Temperature profiles

Case	Tip Condition	Temp. Distribution	Fin heat transfer
A	Convection heat transfer: $h\theta(L)=-k(d\theta/dx)_{x=L}$	$\dfrac{\cosh m(L-x)+(h/mk)\sinh m(L-x)}{\cosh mL+(h/mk)\sinh mL}$	$M\theta_b \dfrac{\sinh mL+(h/mk)\cosh mL}{\cosh mL+(h/mk)\sinh mL}$
B	Adiabatic $(d\theta/dx)_{x=L}=0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$	$M\theta_b \tanh mL$
C	Given temperature: $\theta(L)=\theta_L$	$\dfrac{(\theta_L/\theta_b)\sinh m(L-x)+\sinh m(L-x)}{\sinh mL}$	$M\theta_b \dfrac{(\cosh mL-\theta_L/\theta_b)}{\sinh mL}$
D	Infinitely long fin $\theta(L)=0$	e^{-mx}	$M\ \theta_b$

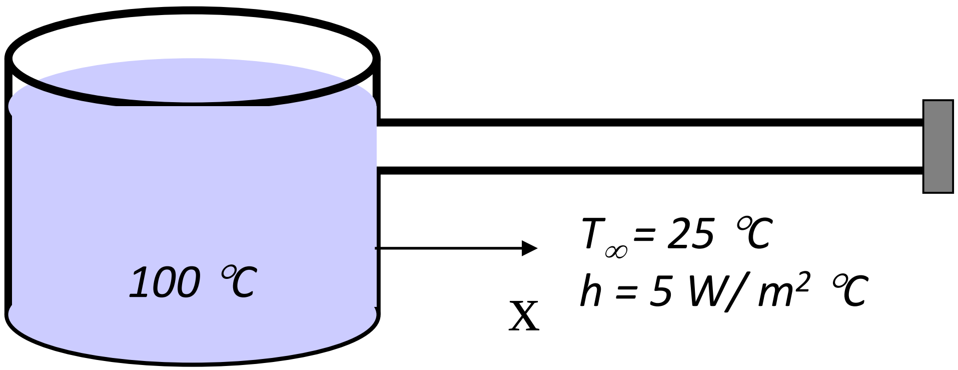
$$\theta \equiv T - T_\infty, \qquad m^2 \equiv \frac{hP}{kA_C}$$

$$\theta_b = \theta(0) = T_b - T_\infty, \qquad M = \sqrt{hPkA_C}\,\theta_b$$



Example Problem

An Aluminum pot is used to boil water as shown below. The handle of the pot is *20-cm long, 3-cm wide*, and *0.5-cm thick*. The pot is exposed to room air at 25°C, and the convection coefficient is *5 W/m²-K*. Assume no heat transfer at the end of the handle. Question: can you touch the handle when the water is boiling? (*k* for Al = 237 W/m-K)

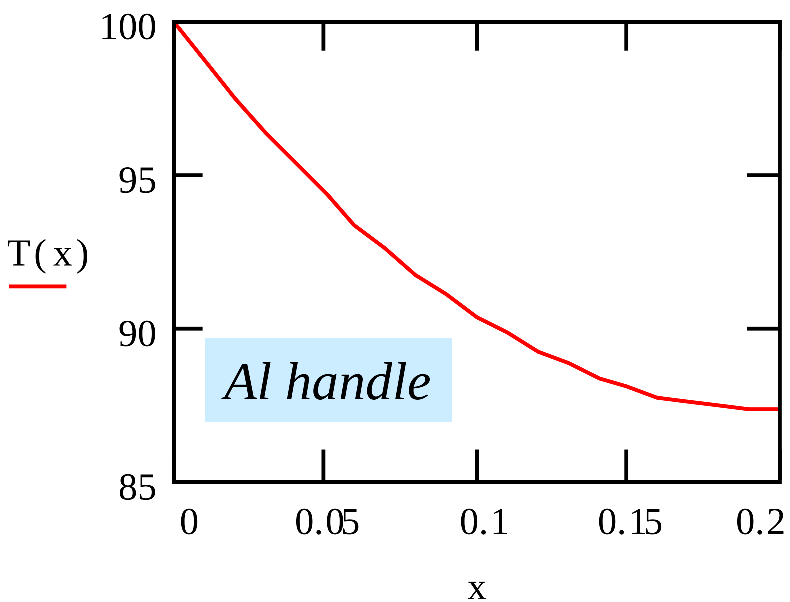


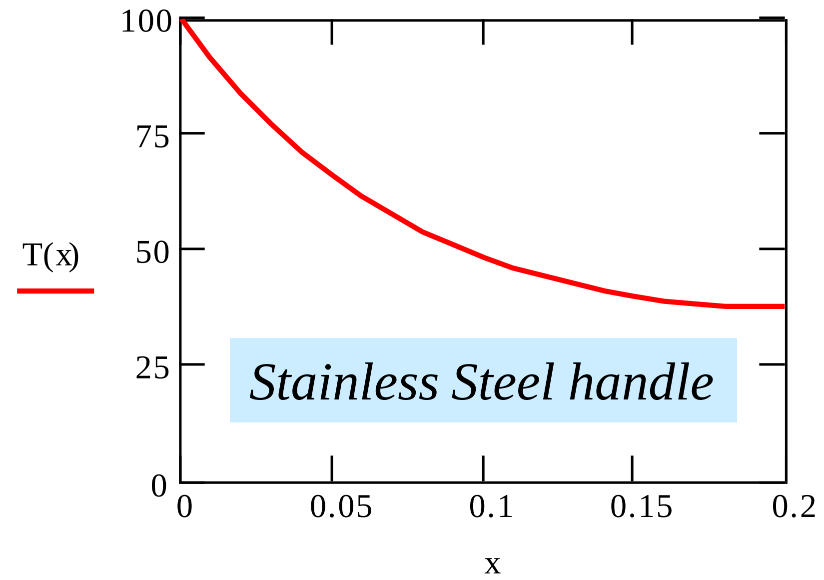
<u>Steps</u>

- Treat handle as fin
- Identify tip condition
  - *Adiabatic*
- Calculate P, m, M
- Get temp. profile
- Calculate T at x=L= 20 cm

Example (contd…)

Temperature distribution along the pot handle





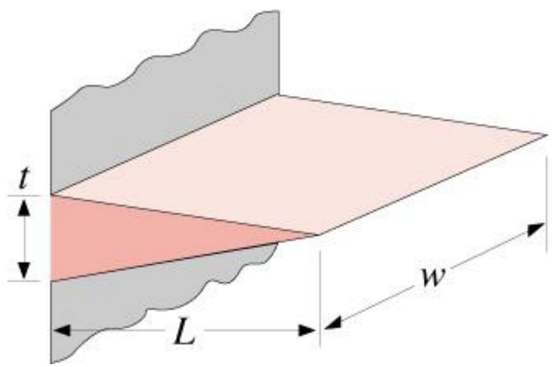
Temperature at the tip = *87.3 ℃* → **Not safe** to touch

Why?

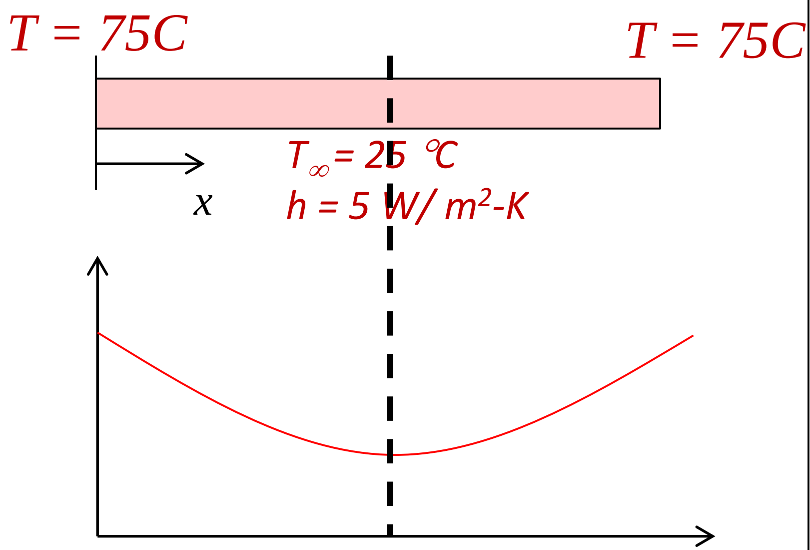
What can you do?

Question time?

Triangular fin



How can we get the temp profile?



1. What are the boundary conditions?
2. How will the temp. profile look like?
3. Any other way we can think of?

Fin parameters: Effectiveness (ε_f)

Fin effectiveness (ε_f): ***how effective*** is the fin

• *Ratio of heat transferred in presence of fin to in its absence*

$$\boxed{\varepsilon_f \equiv \frac{q_f}{hA_{c,b}\theta_b}} \longrightarrow$$

>1 → fin is effective

<1 → should not include fin

$$\varepsilon_f \uparrow \text{ with } \downarrow h, \uparrow k \text{ and} \downarrow A_c / P$$

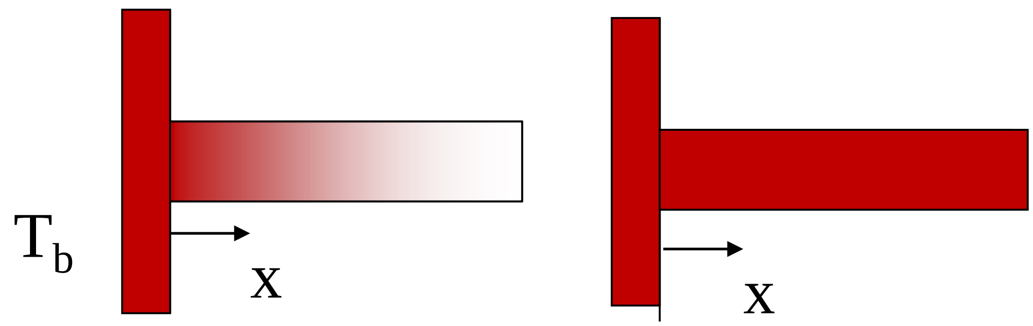
Remember we wanted fins on the air side!!

Fin parameters: Efficiency (η_f)

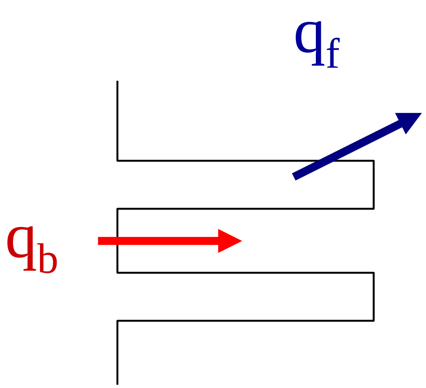
Fin efficiency (η_f): ***how close to ideal scenario*** is the fin

- *Ratio of heat transferred to that if entire fin were at base temp*

$$\boxed{\eta_f \equiv \frac{q_f}{q_{f,\max}} = \frac{q_f}{hA_f\theta_b}}$$



Real situation ***Ideal situation***



For a **fin array** with *N fins*,

A_B: total base area

A_b, A_t: base and tip area of fin

A_f: surface area of a single fin (excluding tip)

$$\boxed{q_f = h(A_s - NA_b + N\eta_f(A_f + A_t))\theta_b}$$

Fin array in heat sinks

Parallel fins

Radial fins

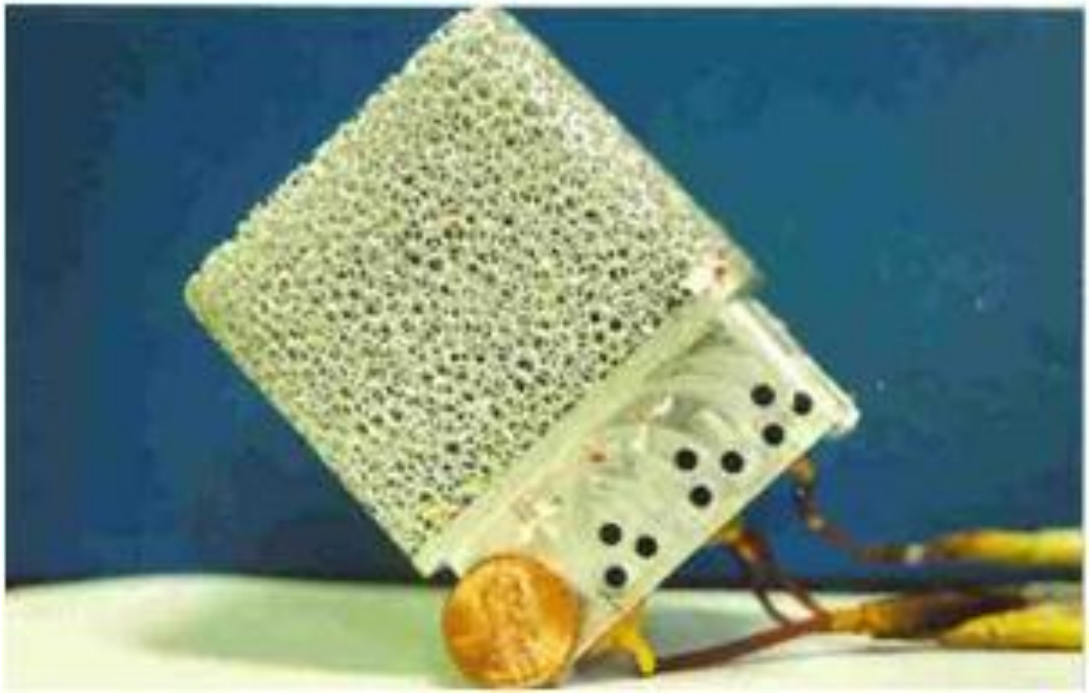
Pin fins

Circular heat sink

Dovetail fins

Summary

- Solved ODE for 1-D heat conduction to get temp profile
  - 4 different tip conditions
- Fin performance metrics – Effectiveness (ε_f) & Efficiency (η_f)
- Fin arrays used to design heat sinks
  - Commonly used in computing products



Thank you!!