# MT 30001

## Materials Engineering (3-0-0)

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# Difference between Crystal & Lattice

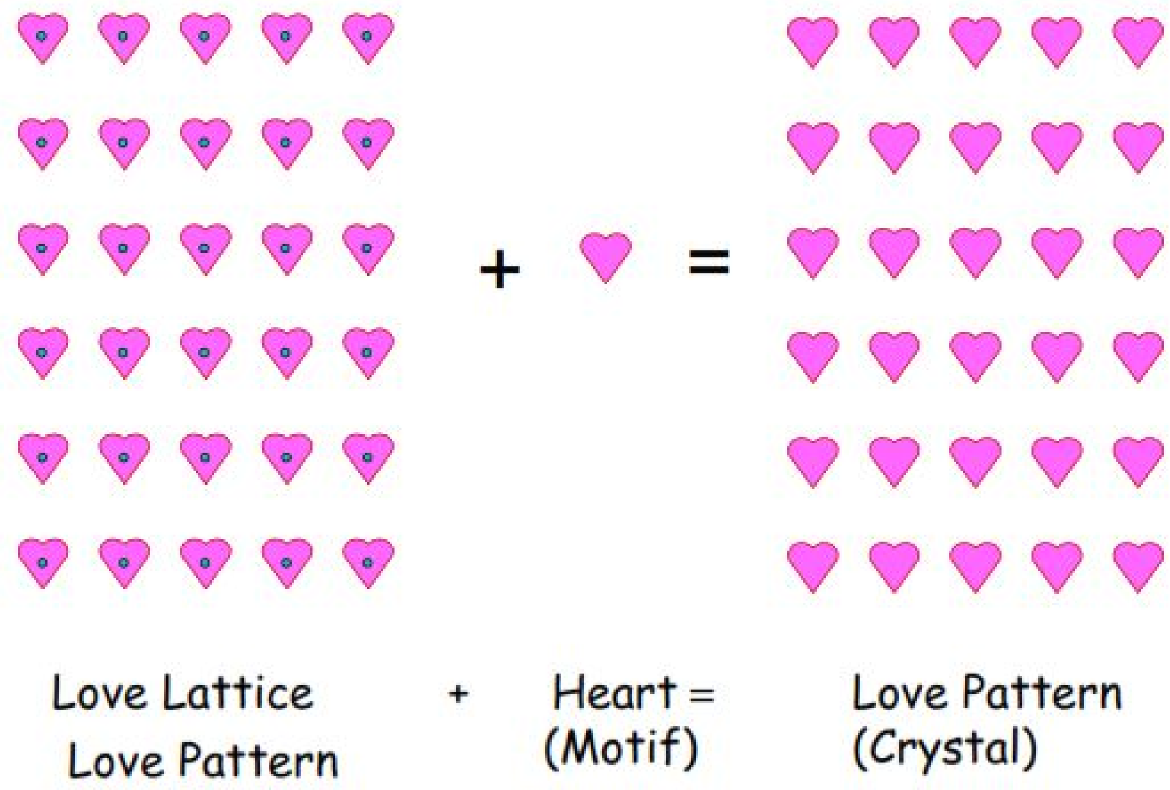
| **Crystal** | **Lattice** |
| --- | --- |
| A 3D translationally periodic arrangement of **atoms** | A 3D translationally periodic arrangement of **points** |

What is the relation between the two?

Crystal = Lattice + Motif

# Difference between Crystal & Lattice

Lattice + Motif = Crystal

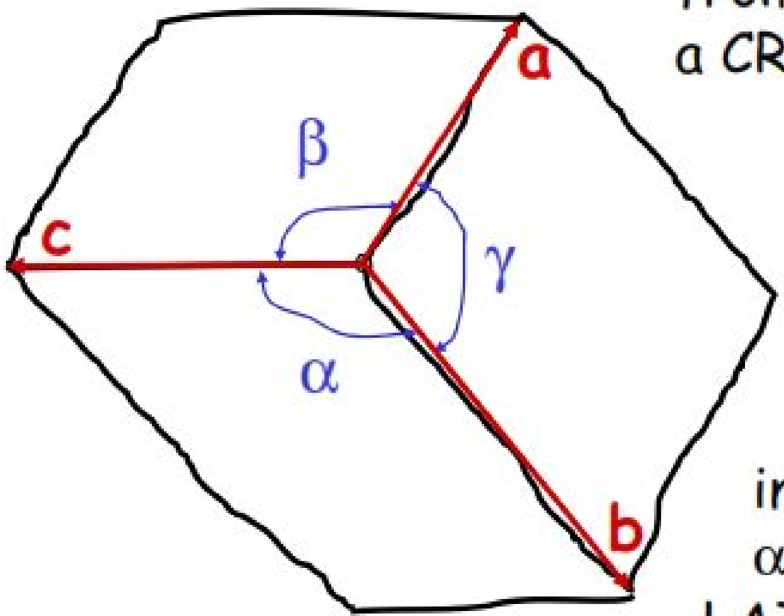


# Unit Cell

## Size and shape of the unit cell:

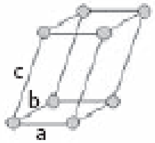
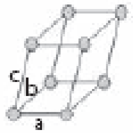
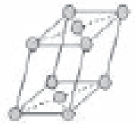
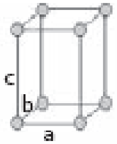
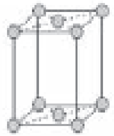
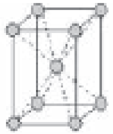
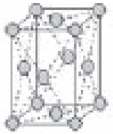
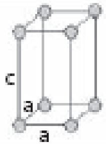
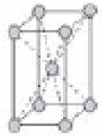
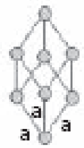
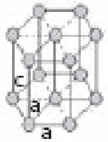
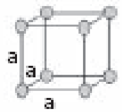
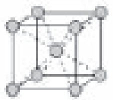
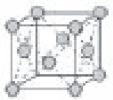
1. A corner as origin

2. Three edge vectors {**a**, **b**, **c**} from the origin define a CRSYTALLOGRAPHIC COORDINATE SYSTEM




3. The three lengths *a*, *b*, *c* and the three interaxial angles $\alpha$, $\beta$, $\gamma$ are called the LATTICE PARAMETERS

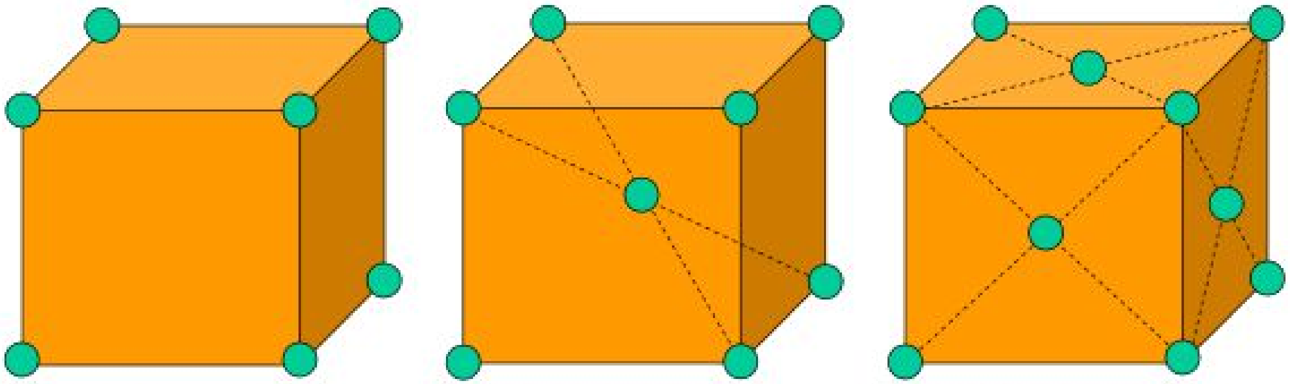
# Crystal system and Bravais Lattice

| Name | Conditions | Primitive | Base centered | Body centered | Face centered |
|---|---|---|---|---|---|
| Triclinic | $a \neq b \neq c$<br>$\alpha \neq \beta \neq \gamma$ |  | | | |
| Monoclinic | $a \neq b \neq c$<br>$\alpha = \gamma = 90° \neq \beta$ |  |  | | |
| Orthomombic | $a \neq b \neq c$<br>$\alpha = \beta = \gamma = 90°$ |  |  |  |  |
| Tetragonal | $a = b \neq c$<br>$\alpha = \beta = \gamma = 90°$ |  = | |  | |
| Rhombohedral (trigonal) | $a = b = c$<br>$\alpha = \beta = \gamma \neq 90°$ |  | | | |
| Hexagonal | $a = b \neq c$<br>$\alpha = \beta = 90°, \gamma = 120°$ |  | | | |
| Cubic | $a = b = c$<br>$\alpha = \beta = \gamma = 90°$ |  | |  |  |

# Cubic Bravais Lattice

## The three cubic Bravais lattices

| Crystal system | Bravais lattices |
|---|---|
| 1. Cubic | P I F |



Simple cubic
Primitive cubic

Body-centred cubic
Cubic I

Face-centred cubic
Cubic F

# Lattice Defects

- ✓ **Real crystals deviate from the perfect periodicity**

- ✓ **While the concept of the perfect lattice is adequate for explaining the *structure-insensitive properties* of metal, it is necessary to consider a number of types of lattice defects to explain *structure-sensitive properties***

| Structure-insensitive | Structure-sensitive |
|---|---|
| Elastic constants | Electrical Conductivity |
| Melting point | Semiconductor properties |
| Density | Yield stress |
| Specific heat | Fracture strength |
| Coefficient of thermal expansion | Creep strength |

# Different Types of Lattice Defects

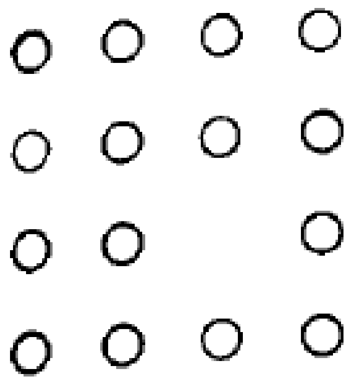
**Point defects**

**Line defects**

**Surface defects**

**Volume defects**

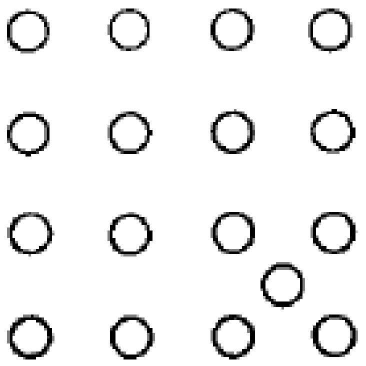
# Point Defects



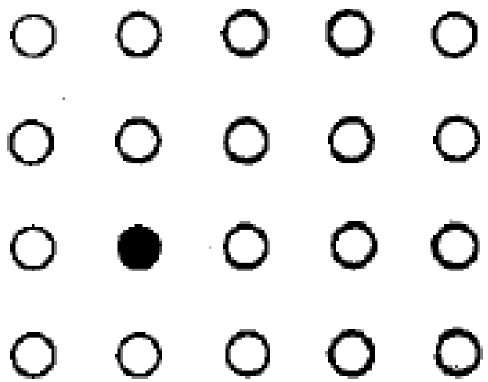
Vacancy

## Number of vacant sites (n)

$$\frac{n}{N} = e^{-E_s/kT}$$

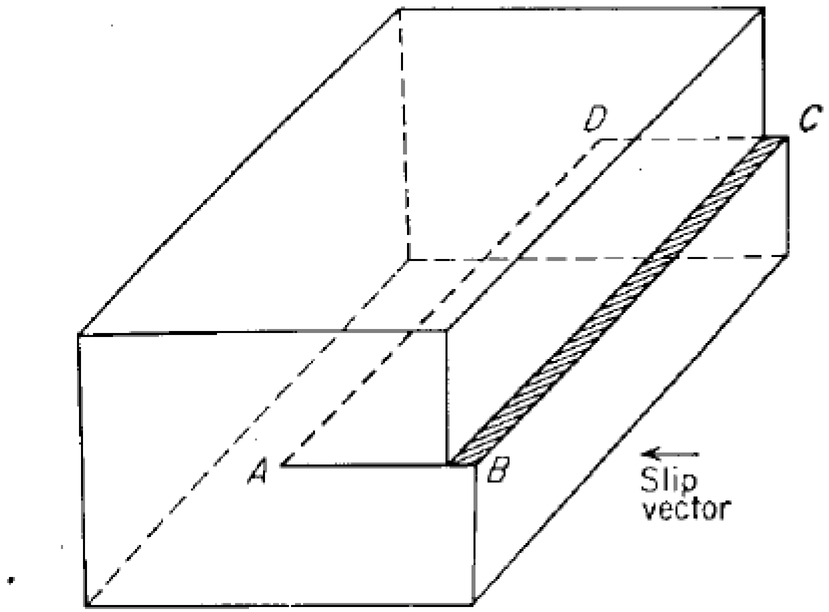


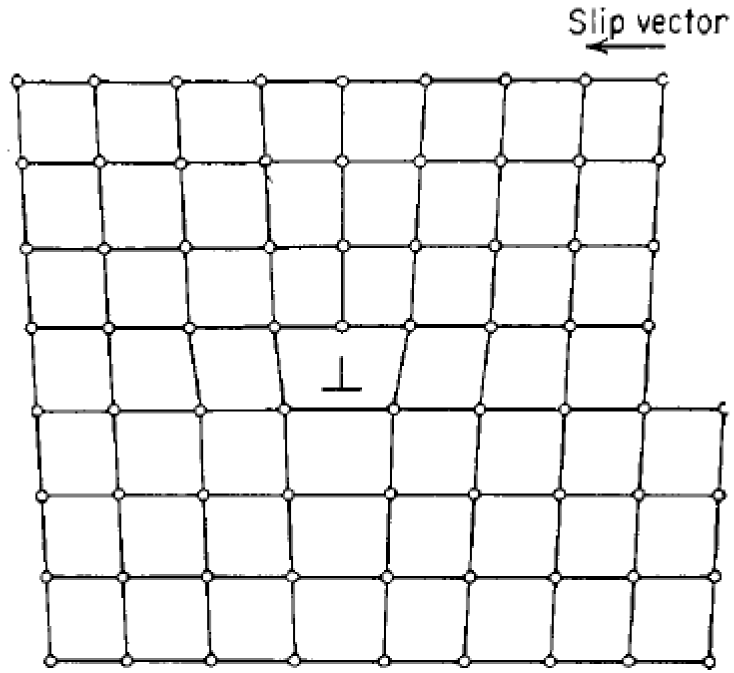
Interstitial



Impurity

# Line Defects - Dislocations

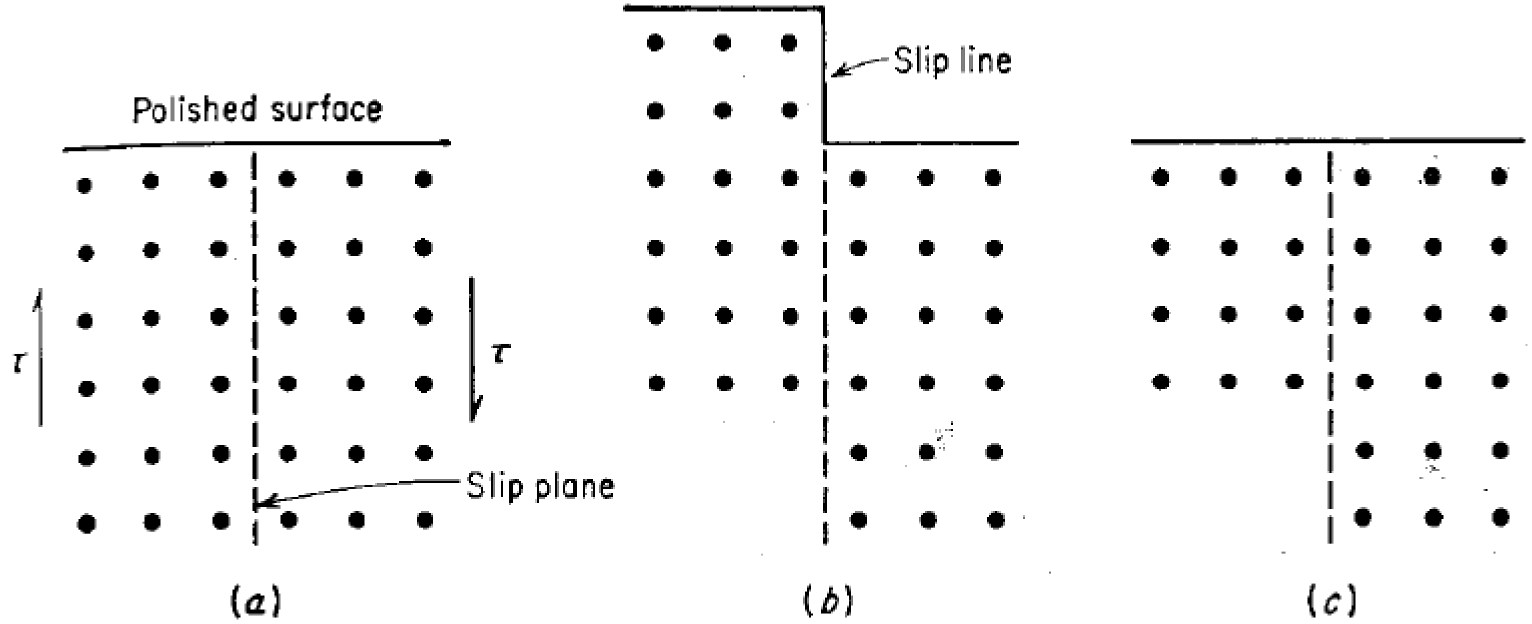



**Edge dislocation produced by slip in cubic system**

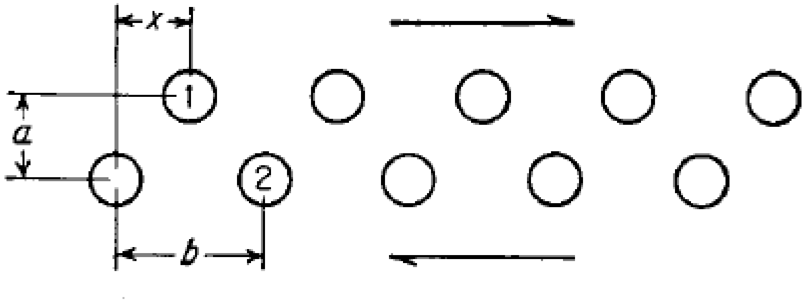
**Atomic arrangement near edge dislocation**

# Deformation by slip

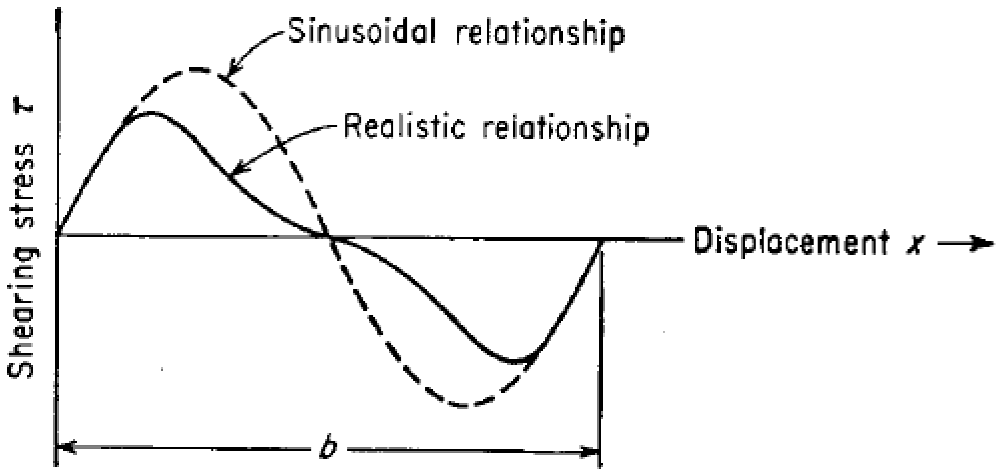



**Schematic drawing of classical idea of slip**

# Slip in perfect lattice




**Shear displacement of plane of atoms over another**




**Variation of shearing stress with displacement in slip direction**

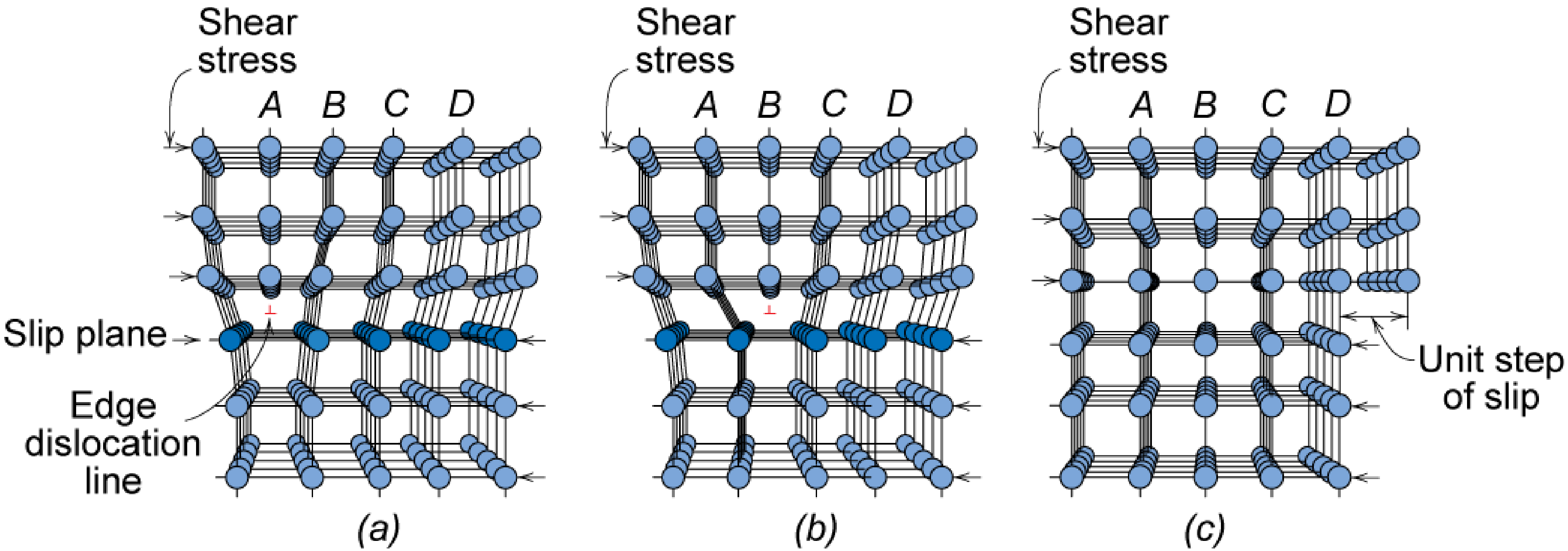
**Relation between shear stress and displacement**

$$\tau = \tau_m \sin \frac{2\pi x}{b}$$

**The max. stress at which slip should occur**

$$\tau_m = \frac{G}{2\pi}\frac{b}{a}$$

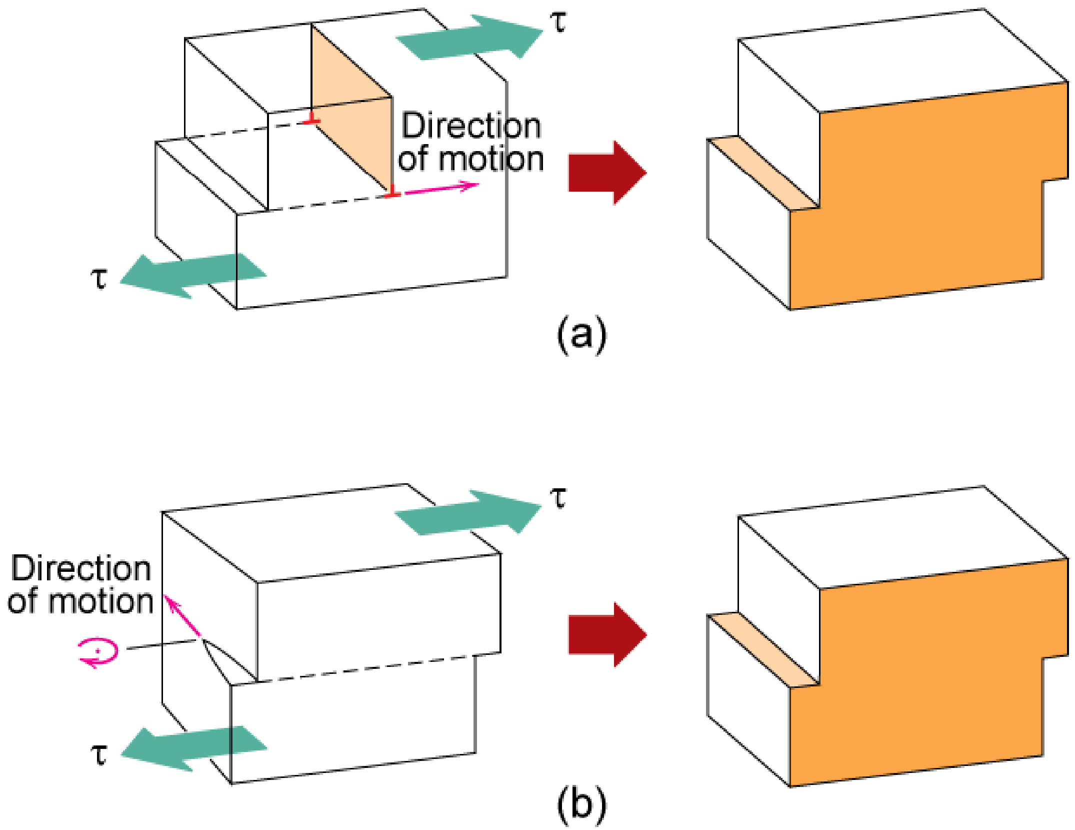
# Slip by dislocation movements




- If dislocations don't move,
  deformation doesn't occur!

# Dislocation Motion

- Dislocation moves along slip plane in slip direction perpendicular to dislocation line
- Slip direction same direction as Burgers vector



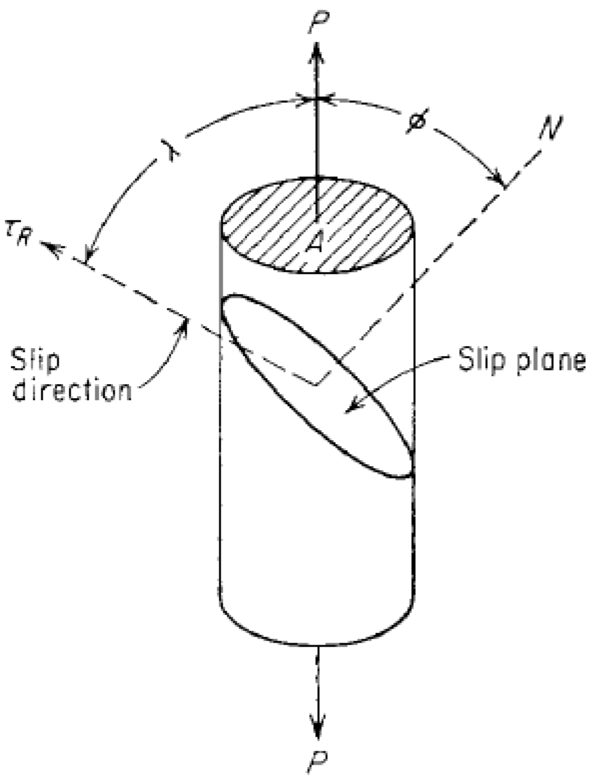

**Edge dislocation**

**Screw dislocation**

From Fig. 10.2 *Callister's Materials Science and Engineering, Adapted Version.*

# Critical Resolved Shear Stress for Slip

**Slip begins when the shearing stress on the slip plane in the slip direction reaches a threshold value – called as <u>critical resolved shear stress (CRSS)</u>**

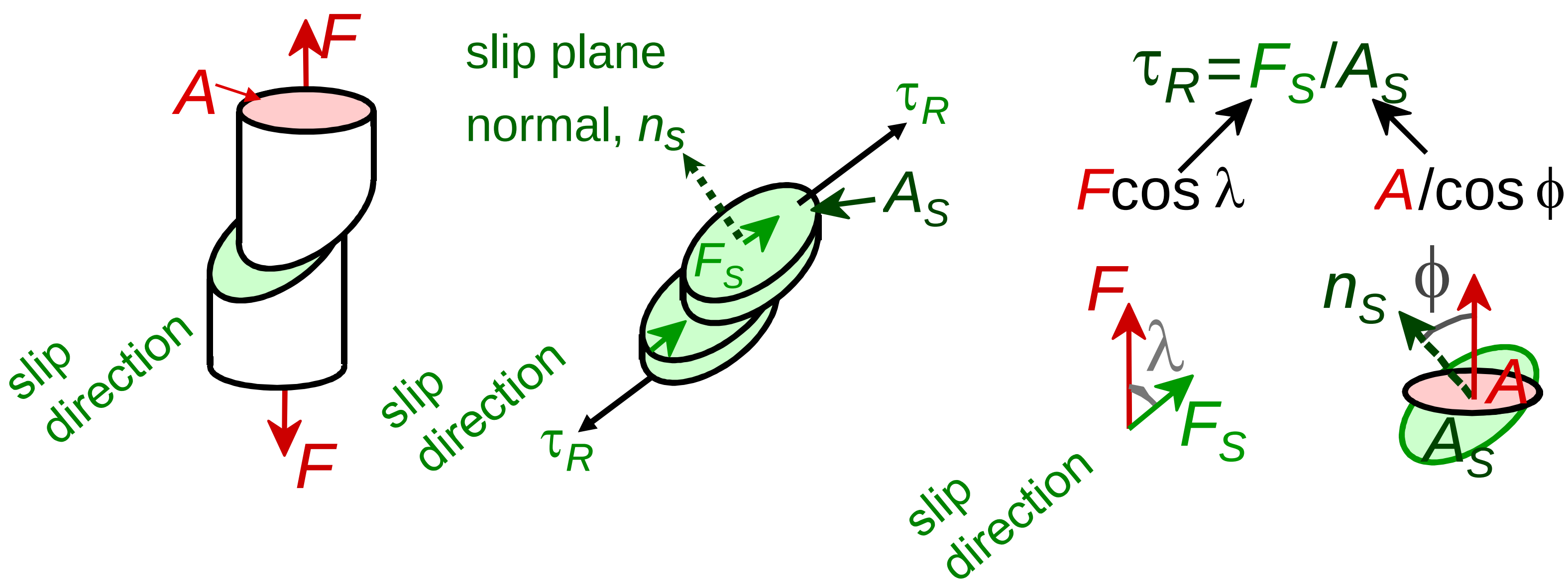



$$\tau_R = \frac{P\cos\lambda}{A/\cos\phi} = \frac{P}{A}\cos\phi\cos\lambda$$

# Stress and Dislocation Motion

- Crystals slip due to a resolved shear stress, $\tau_R$.
- Applied tension can produce such a stress.

| Applied tensile stress: $\sigma = F/A$ | Resolved shear stress: $\tau_R = F_S/A_S$ | Relation between $\sigma$ and $\tau_R$ |
|---|---|---|




$$\tau_R = \sigma \cos\lambda \cos\phi$$

# Critical Resolved Shear Stress

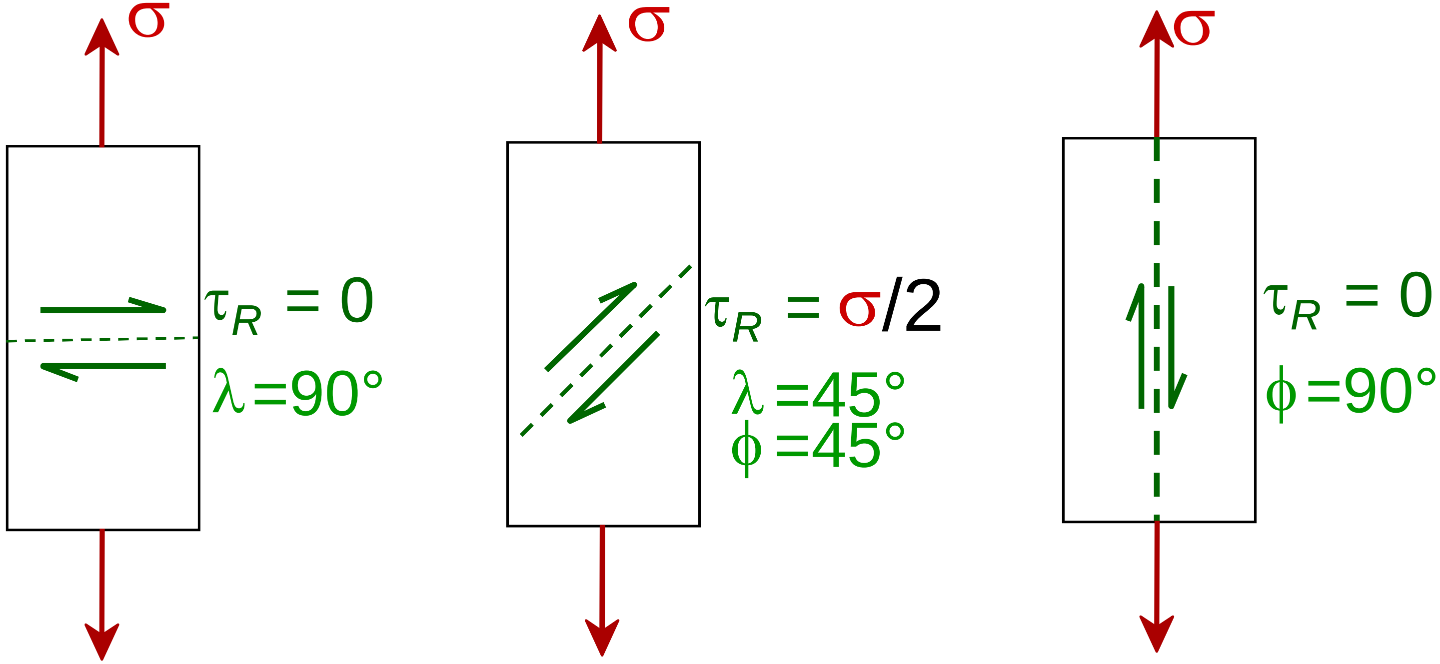
- Condition for dislocation motion: $\tau_R > \tau_{CRSS}$

typically

$10^{-4}$ GPa to $10^{-2}$ GPa

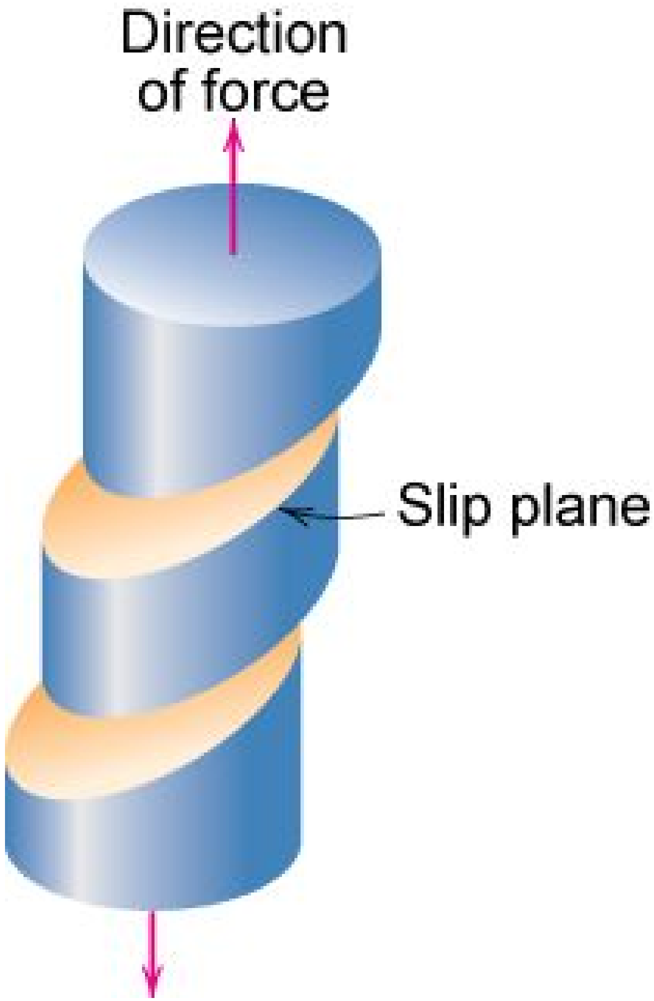
- Crystal orientation can make it easy or hard to move dislocation

$$\tau_R = \sigma \cos\lambda \cos\phi$$




$\tau$ maximum at $\lambda = \phi = 45^{\circ}$

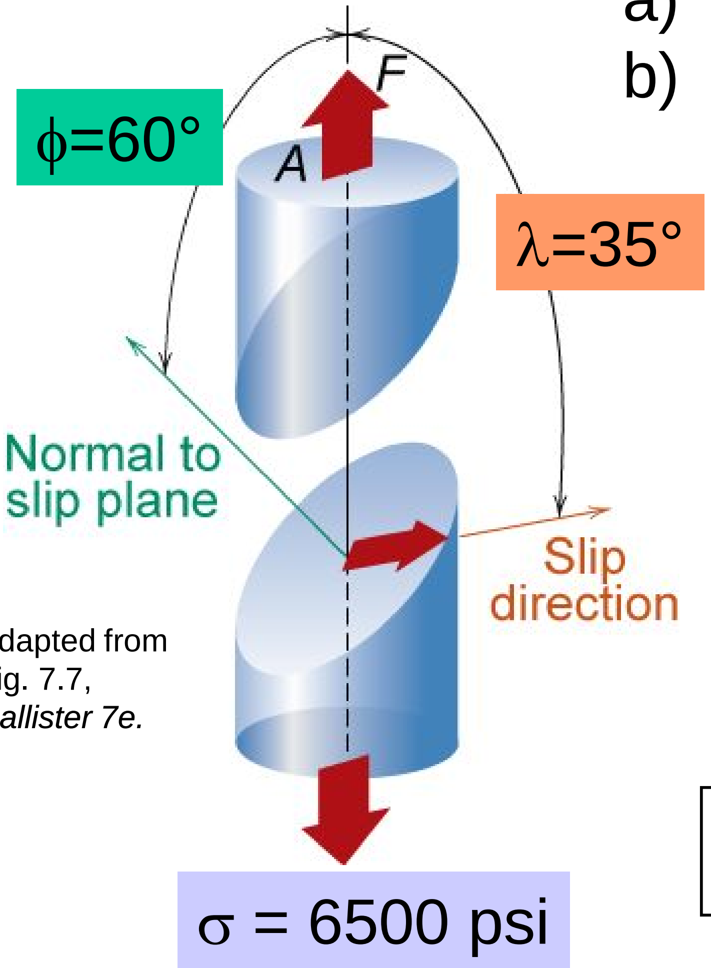
# Single Crystal Slip




From Fig. 10.8
*Callister's Materials Science and Engineering, Adapted Version.*

From Fig. 10.9,
*Callister's Materials Science and Engineering, Adapted Version.*

# Ex: Deformation of single crystal




Adapted from Fig. 7.7, *Callister 7e.*

a) Will the single crystal yield?
b) If not, what stress is needed?

$$\tau_{crss} = 3000 \text{ psi}$$

$$\tau = \sigma \cos\lambda \cos\phi$$

$$\sigma = 6500 \text{ psi}$$

$$\tau = (6500 \text{ psi})\ (\cos 35^\circ)(\cos 60^\circ)$$

$$= (6500 \text{ psi})\ (0.41)$$

$$\boxed{\tau = 2662 \text{ psi} \quad < \tau_{crss} = 3000 \text{ psi}}$$

So the applied stress of 6500 psi will not cause the crystal to yield.

# Ex: Deformation of single crystal

What stress *is* necessary (i.e., what is the yield stress, $\sigma_y$)?

$$\tau_{crss} = 3000 \text{ psi} = \sigma_y \cos\lambda \cos\phi = \sigma_y (0.41)$$

$$\therefore \sigma_y = \frac{\tau_{crss}}{\cos\lambda \cos\phi} = \frac{3000 \text{ psi}}{0.41} = \underline{\underline{7325 \text{ psi}}}$$

So for deformation to occur the applied stress must be greater than or equal to the yield stress

$$\boxed{\sigma \geq \sigma_y = 7325 \text{ psi}}$$

# Assignment

Determine the tensile stress that need to be applied along the [1-10] axis of a silver crystal to cause slip on the (1-1-1)[0-11] system. The CRSS is 6 MPa.

The angle between tensile axis $[1\bar{1}0]$ and normal to $(1\bar{1}\bar{1})$ is

$$\cos\phi = \frac{(1)(1) + (-1)(-1) + (0)(-1)}{\sqrt{(1)^2 + (-1)^2 + (0)^2}\sqrt{(1)^2 + (-1)^2 + (-1)^2}} = \frac{2}{\sqrt{2}\sqrt{3}} = \frac{2}{\sqrt{6}}$$

The angle between tensile axis $[1\bar{1}0]$ and slip direction $[0\bar{1}\bar{1}]$ is

$$\cos\lambda = \frac{(1)(0) + (-1)(-1) + (0)(-1)}{\sqrt{2}\sqrt{(0)^2 + (-1)^2 + (-1)^2}} = \frac{1}{\sqrt{2}\sqrt{2}} = \frac{1}{2}$$

$$\sigma = \frac{P}{A} = \frac{\tau_R}{\cos\phi\cos\lambda} = \frac{6}{2/\sqrt{6} \times \frac{1}{2}} = 6\sqrt{6} = 14.7\ \text{MPa}$$

# Slip Motion in Polycrystals

- Stronger - grain boundaries pin deformations
- Slip planes & directions ($\lambda$, $\phi$) change from one crystal to another.
- $\tau_R$ will vary from one crystal to another.
- The crystal with the largest $\tau_R$ yields first.
- Other (less favorably oriented) crystals yield later.


From Fig. 10.10, *Callister's Materials Science and Engineering, Adapted Version.*

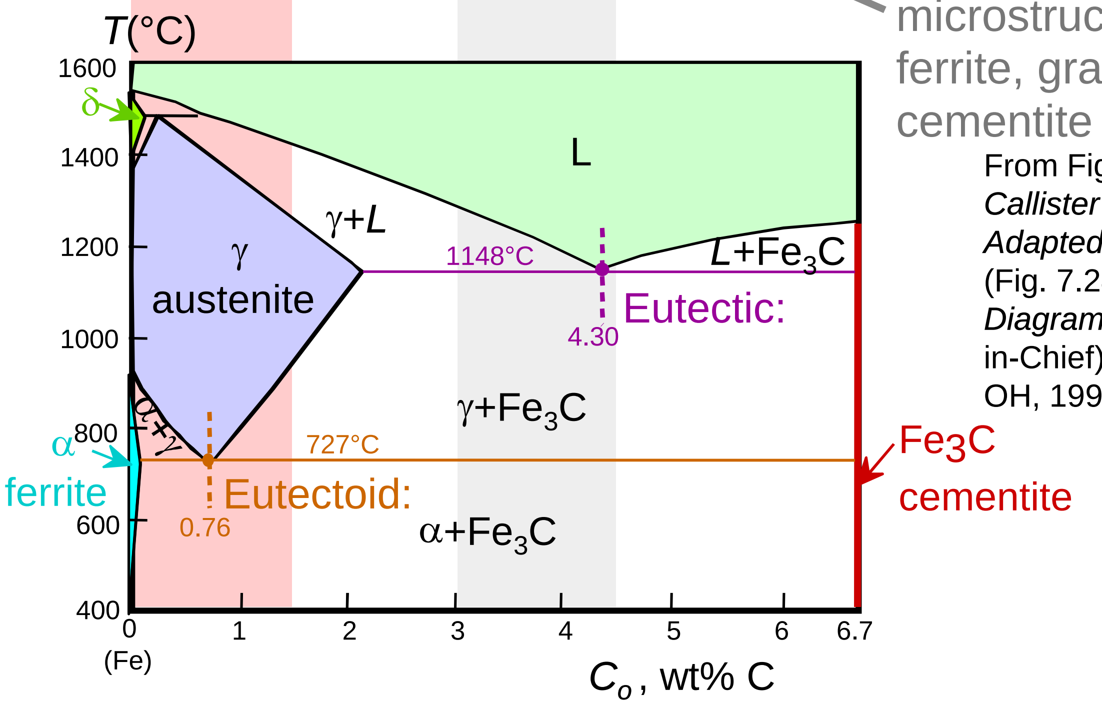
(Fig. 10.10 is courtesy of C. Brady, National Bureau of Standards [now the National Institute of Standards and Technology, Gaithersburg, MD].)

# Taxonomy of Metals

- Metal Alloys
  - Ferrous
    - Steels <1.4 wt% C
    - Cast Irons 3-4.5 wt% C
  - Nonferrous
    - Cu
    - Al
    - Mg
    - Ti

microstructure: ferrite, graphite cementite

From Fig. 9.1
*Callister's Materials Science and Engineering, Adapted Version.*




From Fig. 7.24
*Callister's Materials Science and Engineering, Adapted Version.*
(Fig. 7.24 adapted from *Binary Alloy Phase Diagrams*, 2nd ed., Vol. 1, T.B. Massalski (Ed.-in-Chief), ASM International, Materials Park, OH, 1990.)

# Iron-Carbon (Fe-C) Phase Diagram

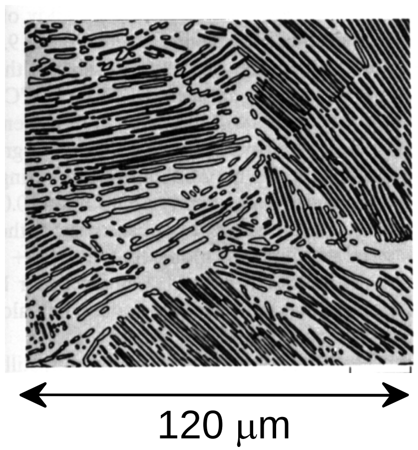
- 2 important points

-Eutectic (A):

$$L \Rightarrow \gamma + Fe_3C$$

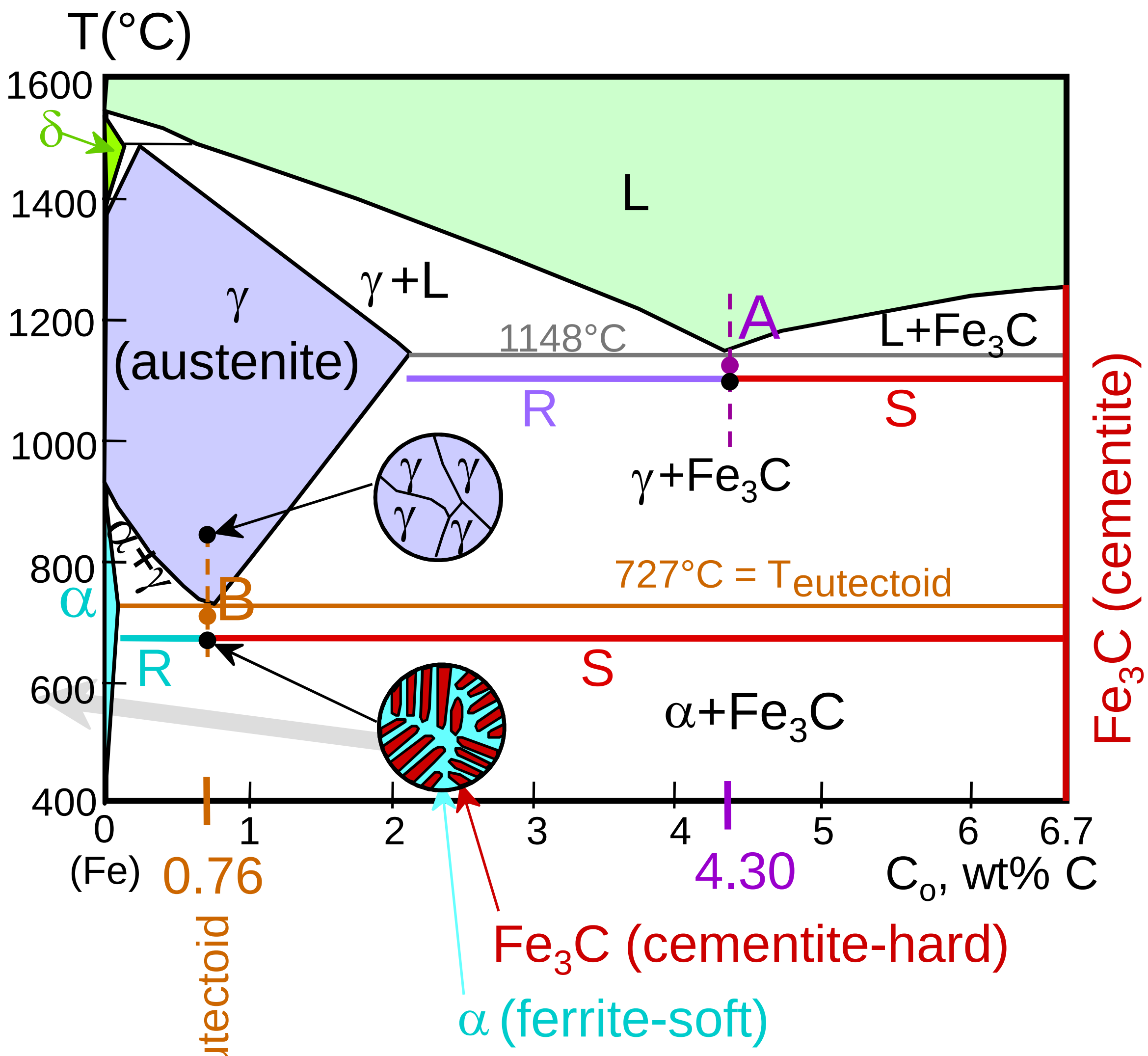
-Eutectoid (B):

$$\gamma \Rightarrow \alpha + Fe_3C$$



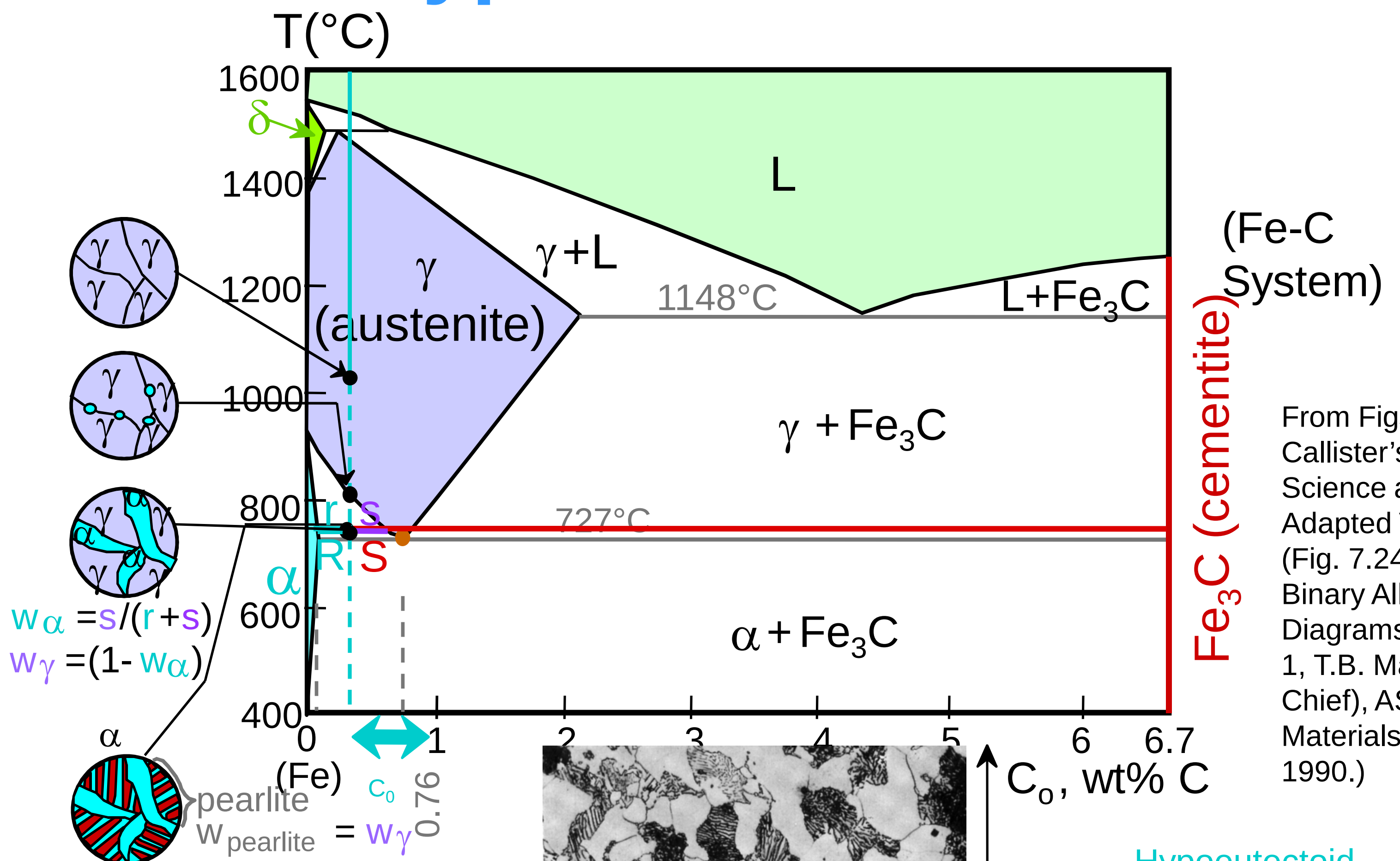

Result: Pearlite = alternating layers of $\alpha$ and $Fe_3C$ phases

(From Fig. 7.27, Callister Adapted Version.)

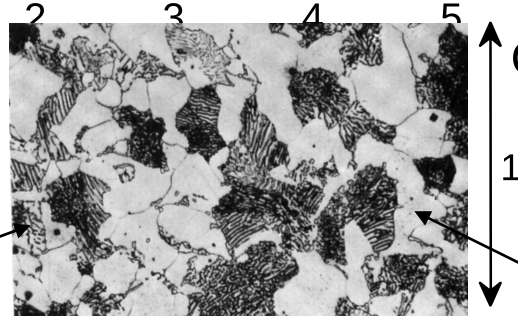



From Fig. 7.24, Callister Adapted Version.

# Hypoeutectoid Steel

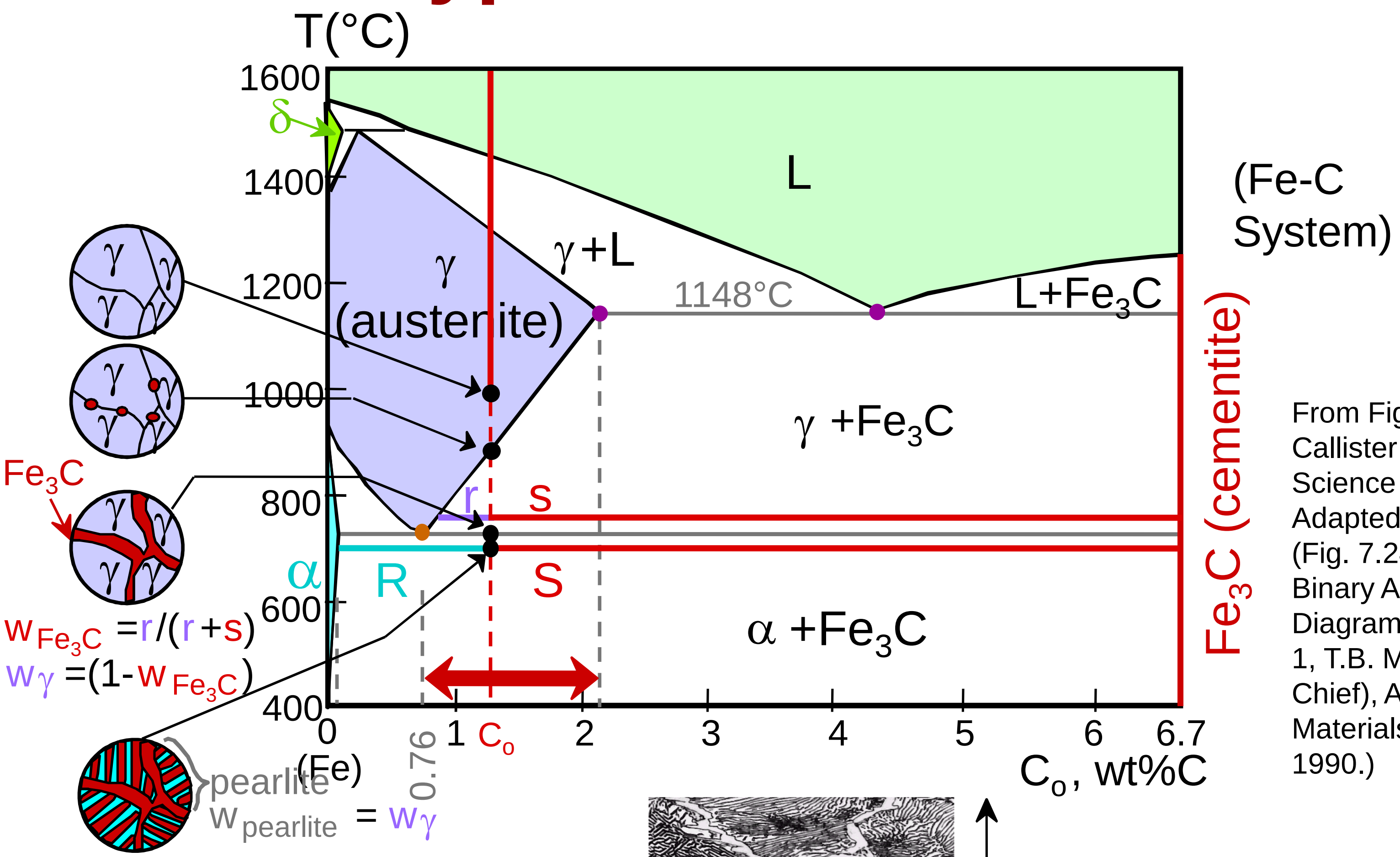



From Figs. 7.24 and 7.29 Callister's Materials Science and Engineering, Adapted Version. (Fig. 7.24 adapted from Binary Alloy Phase Diagrams, 2nd ed., Vol. 1, T.B. Massalski (Ed.-in-Chief), ASM International, Materials Park, OH, 1990.)




From Fig. 7.30 Callister's Materials Science and Engineering, Adapted Version.

# Hypereutectoid Steel



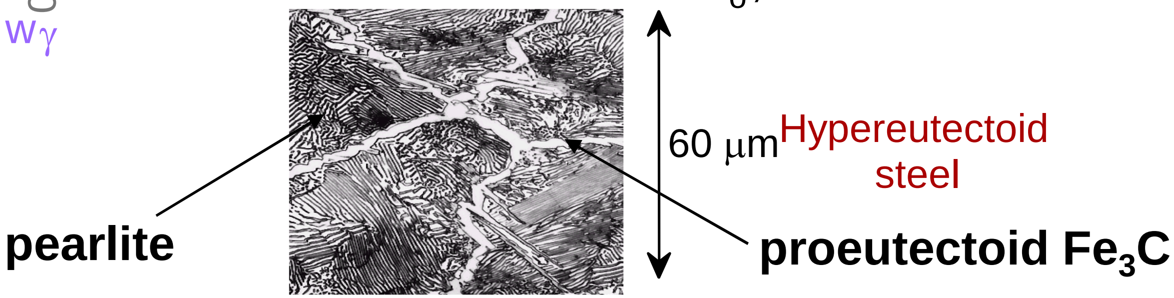

$w_{Fe_3C} = r/(r+s)$

$w_\gamma = (1-w_{Fe_3C})$

$w_\alpha = S/(R+S)$

$w_{Fe_3C} = (1-w_\alpha)$

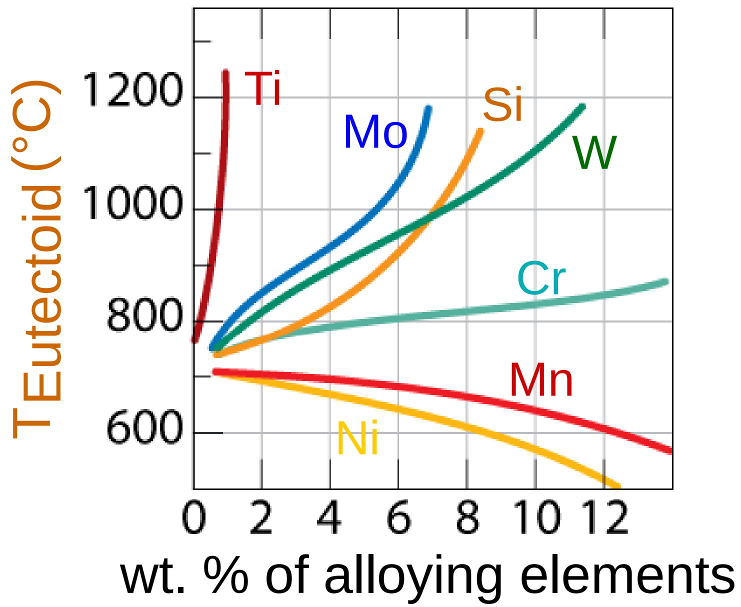
From Figs. 7.24 and 7.32, Callister's Materials Science and Engineering, Adapted Version. (Fig. 7.24 adapted from Binary Alloy Phase Diagrams, 2nd ed., Vol. 1, T.B. Massalski (Ed.-in-Chief), ASM International, Materials Park, OH, 1990.)




From Fig. 7.33, Callister's Materials Science and Engineering, Adapted Version.

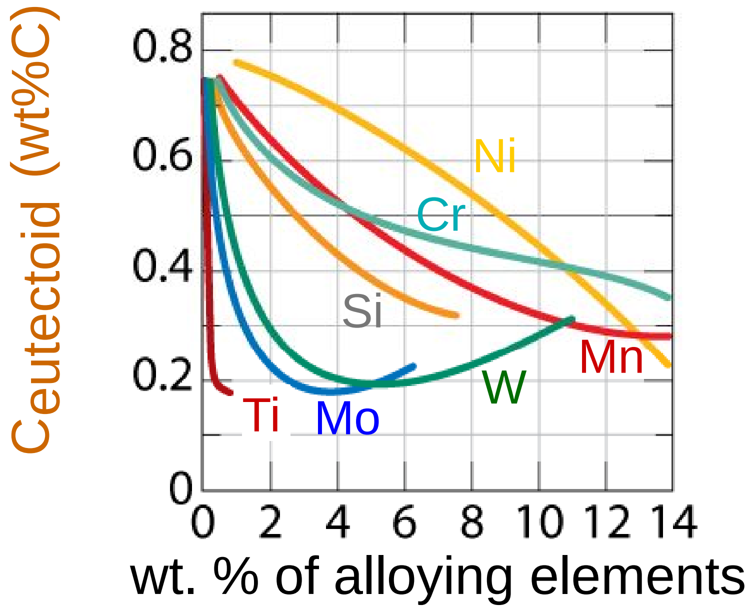
# Alloying Steel with More Elements

- $T_{eutectoid}$ changes:




From Fig. 7.34
Callister's Materials Science and Engineering, Adapted Version.
(Fig. 7.34 from Edgar C. Bain, Functions of the Alloying Elements in Steel, American Society for Metals, 1939, p. 127.)

- $C_{eutectoid}$ changes:




From Fig. 7.35
Callister's Materials Science and Engineering, Adapted Version.
(Fig. 7.35 from Edgar C. Bain, Functions of the Alloying Elements in Steel, American Society for Metals, 1939, p. 127.)

# Steels

Low Alloy | High Alloy

low carbon <0.25 wt% C | Med carbon 0.25-0.6 wt% C | high carbon 0.6-1.4 wt% C

| Name | plain | HSLA | plain | heat treatable | plain | tool | austenitic stainless |
|---|---|---|---|---|---|---|---|
| Additions | none | Cr,V Ni, Mo | none | Cr, Ni Mo | none | Cr, V, Mo, W | Cr, Ni, Mo |
| Example | 1010 | 4310 | 1040 | 4340 | 1095 | 4190 | 304 |
| Hardenability | 0 | + | + | ++ | ++ | +++ | 0 |
| TS | - | 0 | + | ++ | + | ++ | 0 |
| EL | + | + | 0 | - | - | -- | ++ |
| Uses | auto struc. sheet | bridges towers press. vessels | crank shafts bolts hammers blades | pistons gears wear applic. | wear applic. | drills saws dies | high T applic. turbines furnaces V. corros. resistant |

increasing strength, cost, decreasing ductility



Based on data provided in Tables 9.1(b), 9.2(b), 9.3, and 9.4, *Callister's Materials Science and Engineering, Adapted Version.*

# Ferrous Alloys

Iron containing – Steels - cast irons

Nomenclature AISI & SAE

10xx Plain Carbon Steels

11xx Plain Carbon Steels (resulfurized for machinability)

15xx Mn (10 ~ 20%)

40xx Mo (0.20 ~ 0.30%)

43xx Ni (1.65 - 2.00%), Cr (0.4 - 0.90%), Mo (0.2 - 0.3%)

44xx Mo (0.5%)

where xx is wt% C x 100

example: 1060 steel – plain carbon steel with 0.60 wt% C

Stainless Steel -- >11% Cr

# Cast Iron

- Ferrous alloys with > 2.1 wt% C
  - more commonly 3 - 4.5 wt%C
- low melting (also brittle) so easiest to cast

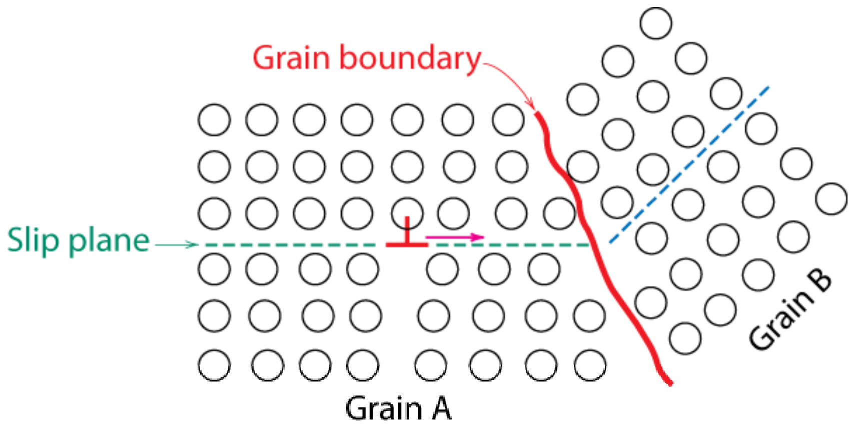
- Cementite decomposes to ferrite + graphite

  $Fe_3C \rightarrow 3\ Fe\ (\alpha) + C\ (\text{graphite})$

  - generally a slow process

# Strategies for Strengthening: 1: Reduce Grain Size

- Grain boundaries are barriers to slip.
- Barrier "strength" increases with Increasing angle of misorientation.
- Smaller grain size: more barriers to slip.




From Fig. 10.14,
*Callister's Materials Science and Engineering, Adapted Version.*

(Fig. 10.14 is from *A Textbook of Materials Technology*, by Van Vlack, Pearson Education, Inc., Upper Saddle River, NJ.)

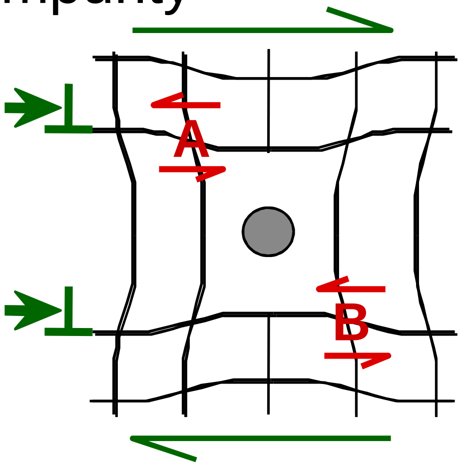
- Hall-Petch Equation:

$$\sigma_{yield} = \sigma_o + k_y d^{-1/2}$$

# 4 Strategies for Strengthening: 2: Solid Solutions

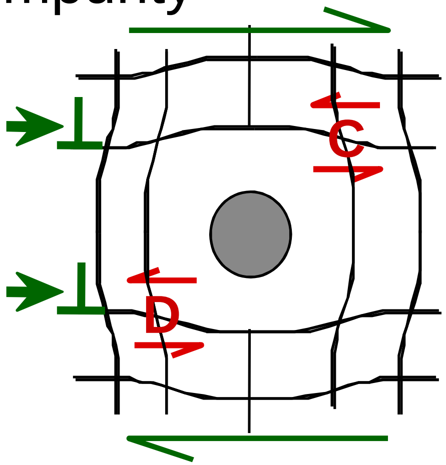
- Impurity atoms distort the lattice & generate stress.
- Stress can produce a barrier to dislocation motion.

- Smaller substitutional impurity



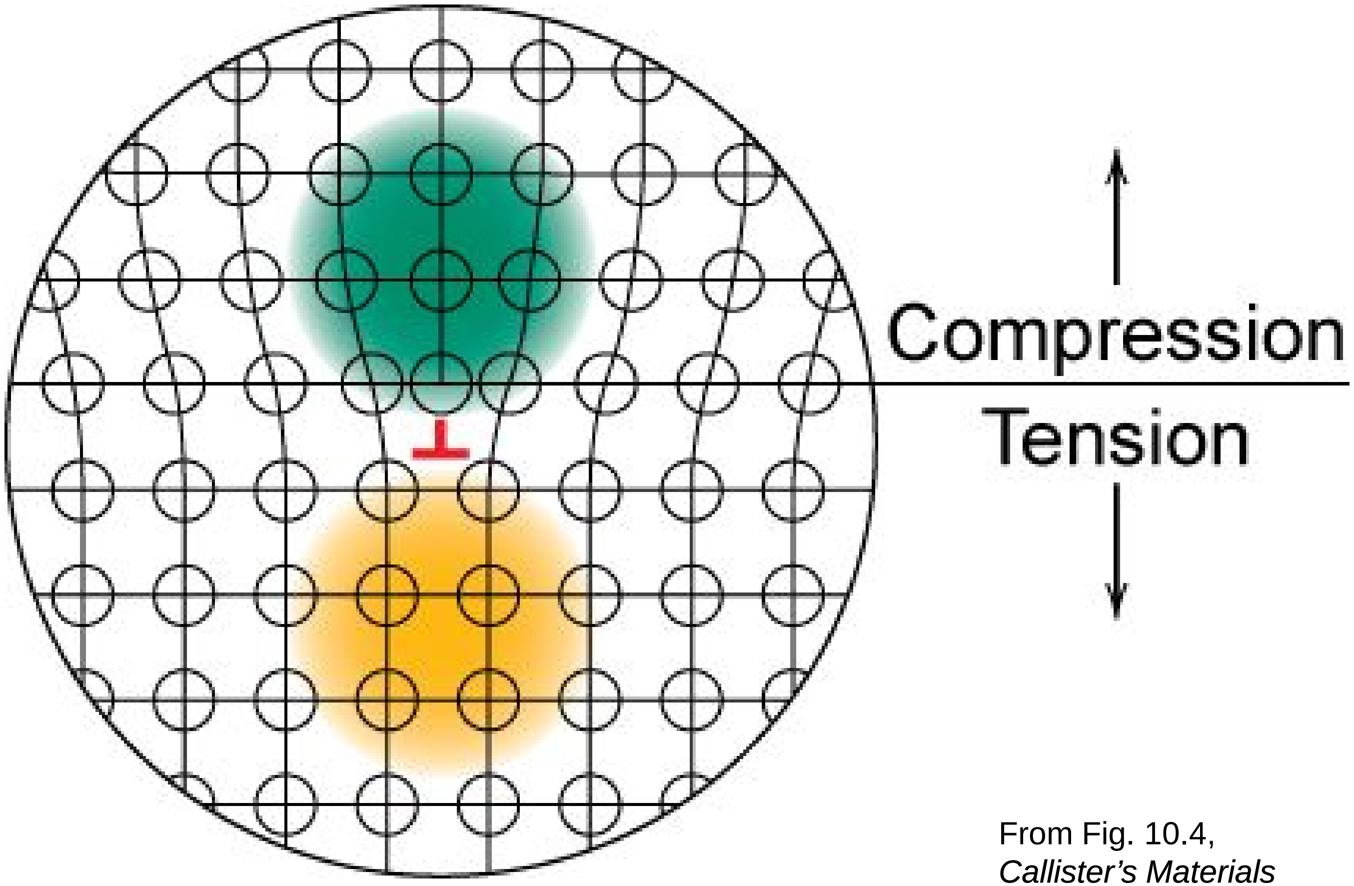

Impurity generates local stress at **A** and **B** that opposes dislocation motion to the right.

- Larger substitutional impurity




Impurity generates local stress at **C** and **D** that opposes dislocation motion to the right.

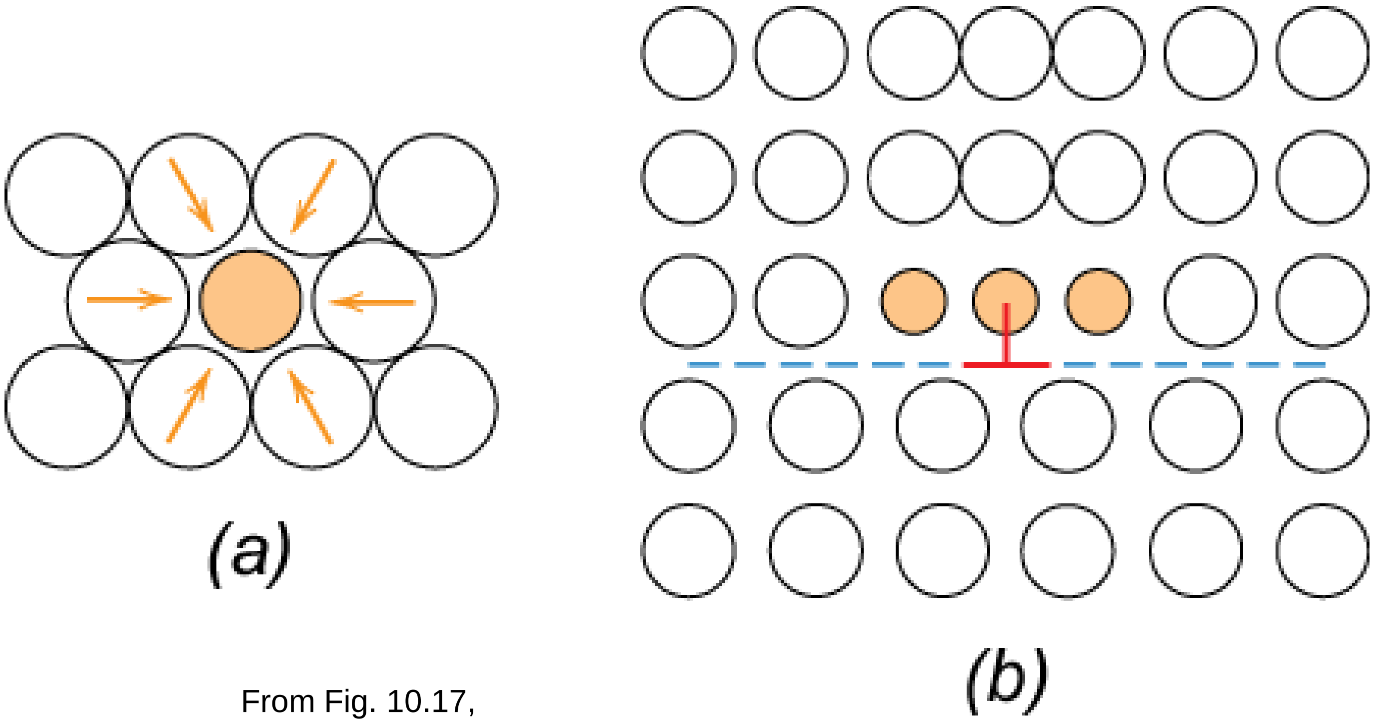
# Stress Concentration at Dislocations




From Fig. 10.4,
*Callister's Materials Science and Engineering, Adapted Version.*

# Strengthening by Alloying

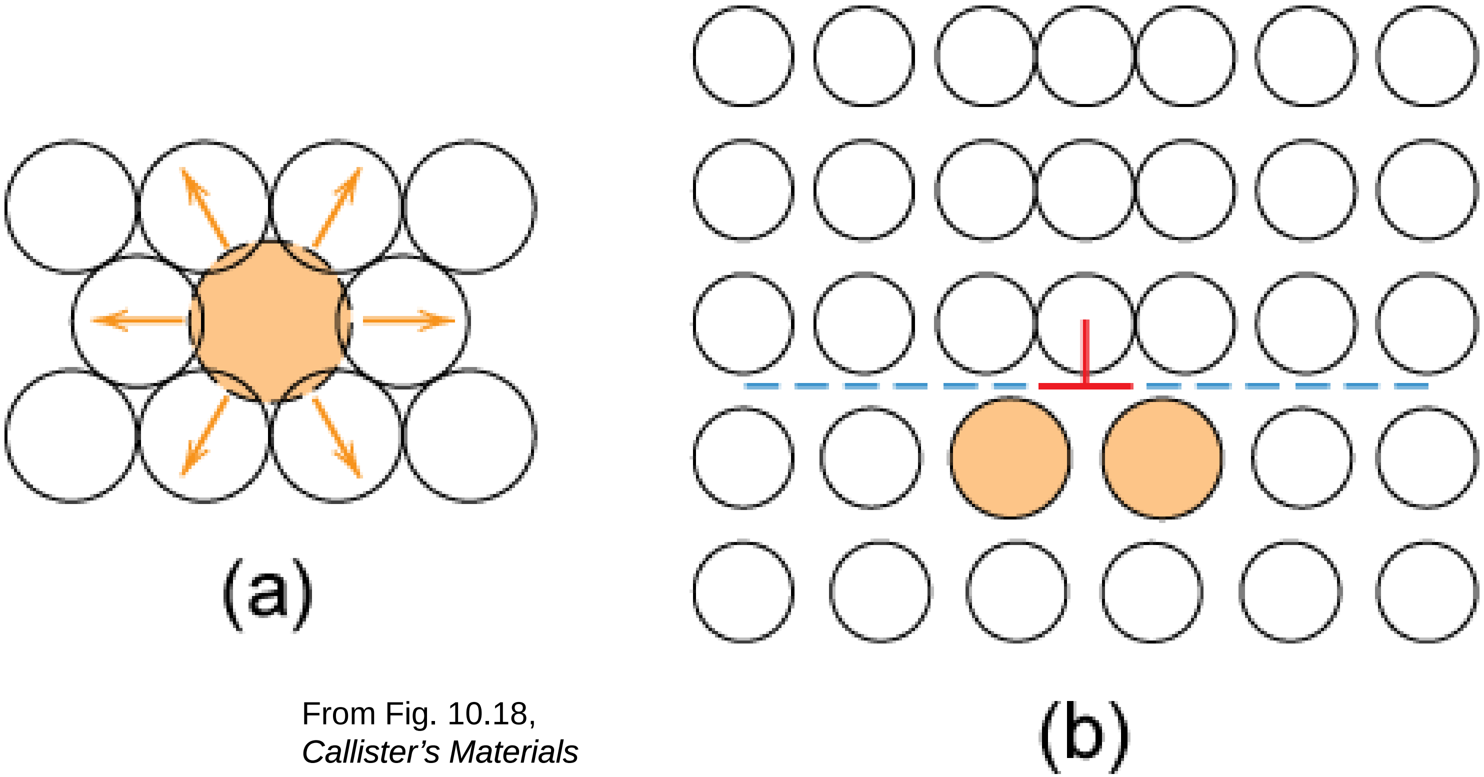
- small impurities tend to concentrate at dislocations
- reduce mobility of dislocation ∴ increase strength




From Fig. 10.17, *Callister's Materials Science and Engineering, Adapted Version.*

# Strengthening by alloying

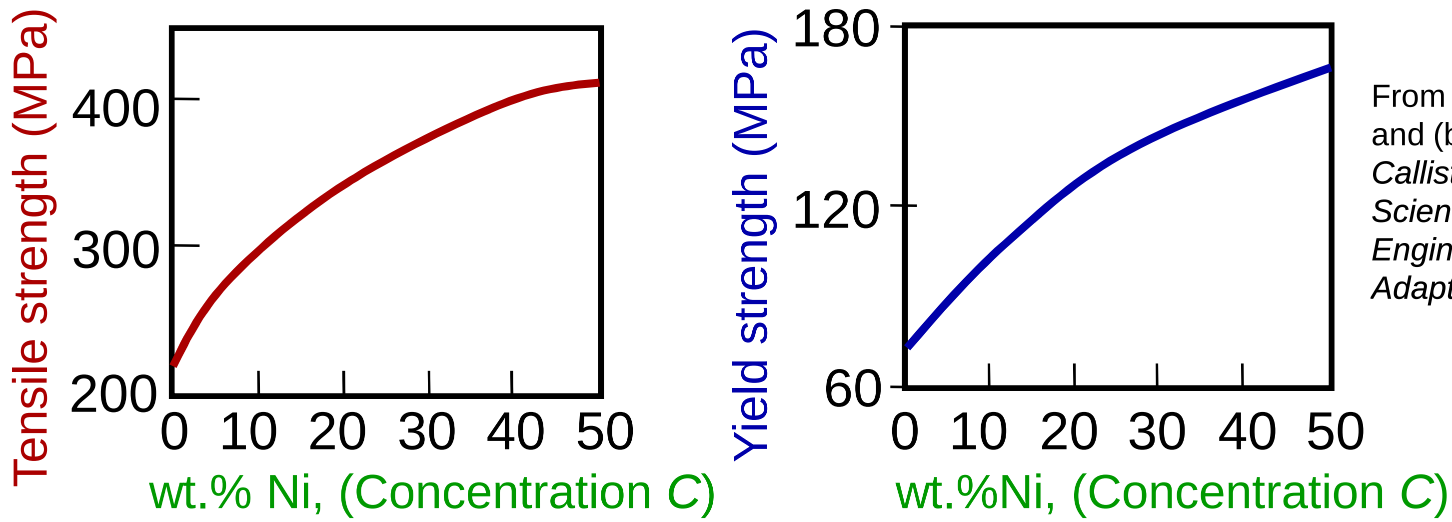
- large impurities concentrate at dislocations on low density side




From Fig. 10.18, *Callister's Materials Science and Engineering, Adapted Version.*

# Ex: Solid Solution Strengthening in Copper

- Tensile strength & yield strength increase with wt% Ni.



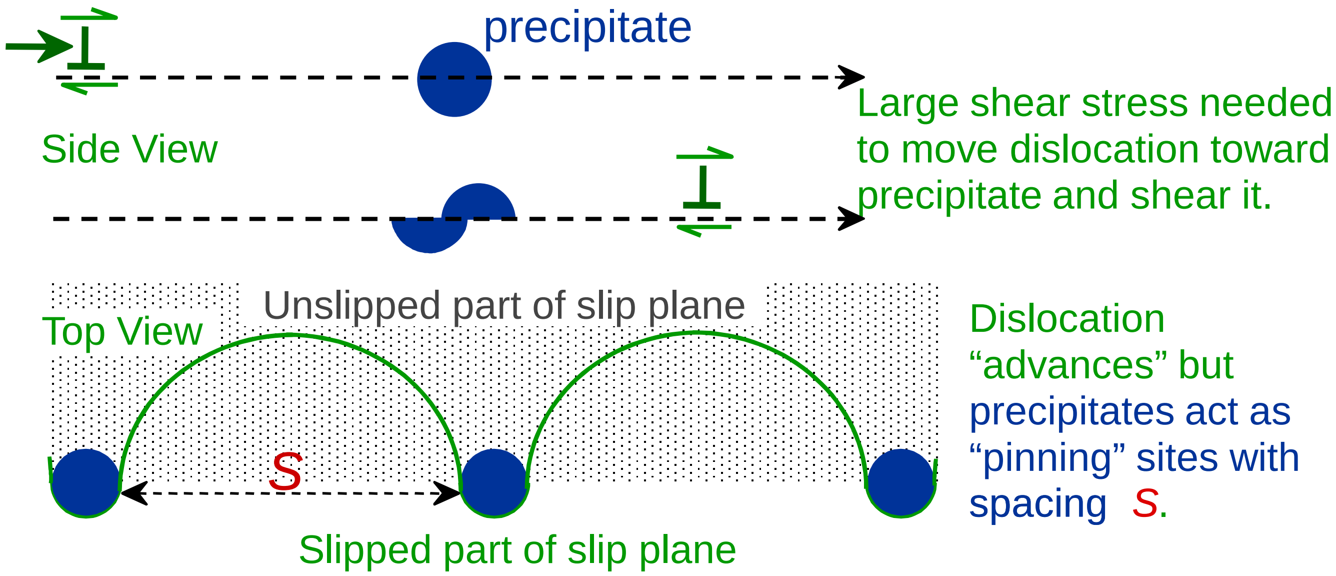

From Fig. 10.16 (a) and (b), *Callister's Materials Science and Engineering, Adapted Version.*

- Empirical relation: $\sigma_y \sim C^{1/2}$

- Alloying increases $\sigma_y$ and *TS*.

# 4 Strategies for Strengthening: 3: Precipitation Strengthening

- Hard precipitates are difficult to shear.
  Ex: Ceramics in metals (SiC in Iron or Aluminum).



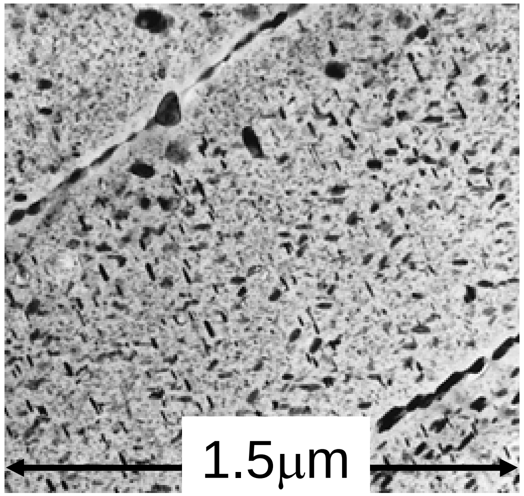

- Result: $\sigma_y \sim \dfrac{1}{S}$

# Application: Precipitation Strengthening

- Internal wing structure on Boeing 767


From chapter-opening photograph, Chapter 11, *Callister 5e.* (courtesy of G.H. Narayanan and A.G. Miller, Boeing Commercial Airplane Company.)

- Aluminum is strengthened with precipitates formed by alloying.

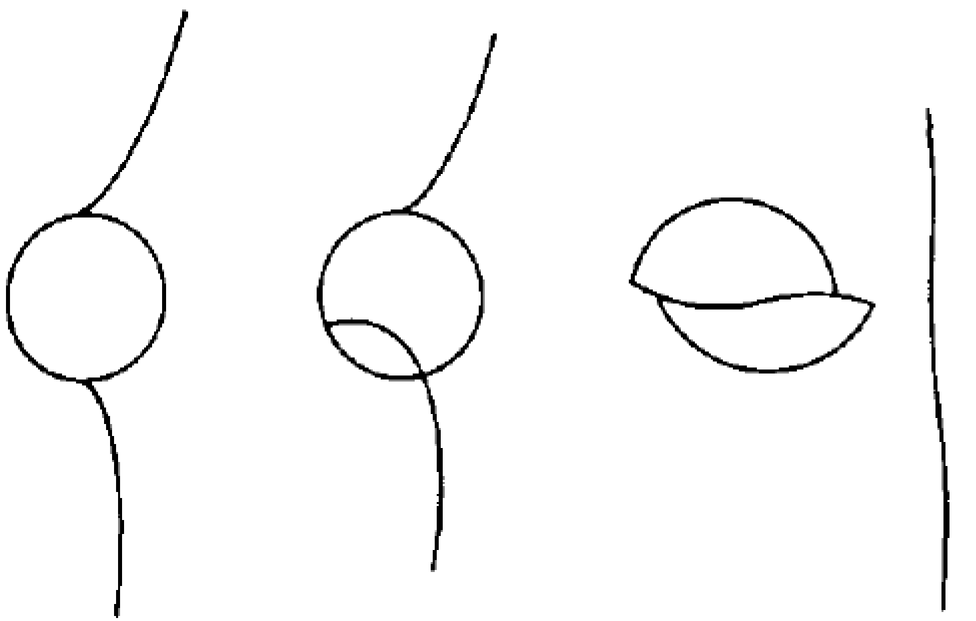



From Fig. 10.31, *Callister's Materials Science and Engineering, Adapted Version.*

(Fig. 10.31 is courtesy of G.H. Narayanan and A.G. Miller, Boeing Commercial Airplane Company.)

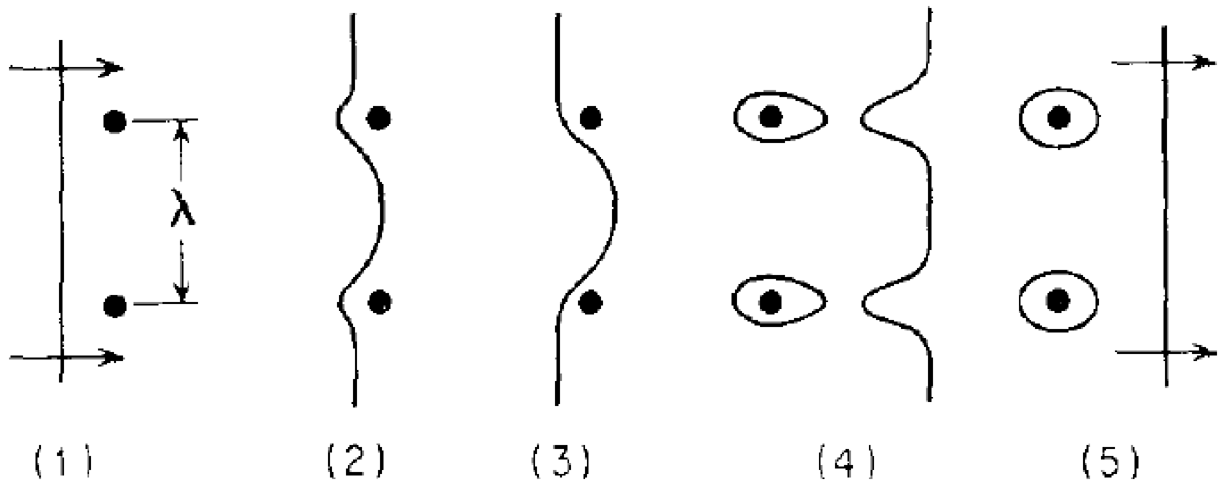
# Interaction between precipitates and dislocation

When the particles are small and/or soft, dislocations can cut and deform the particles as shown in the Figure



# Interaction between precipitates and dislocation

For the case of averaged non-coherent precipitates Orowan proposed the mechanism illustrated in the Fig. below. The yield stress is determined by the shear stress required to bow a dislocation line between two particles separated by a distance $\lambda$, where $\lambda$>R.




Schematic drawing, of stages in passage of a dislocation between widely separated obstacles, based on Orowan's mechanism of dispersion hardening

Stage 1: A straight dislocation line approaching two particles.

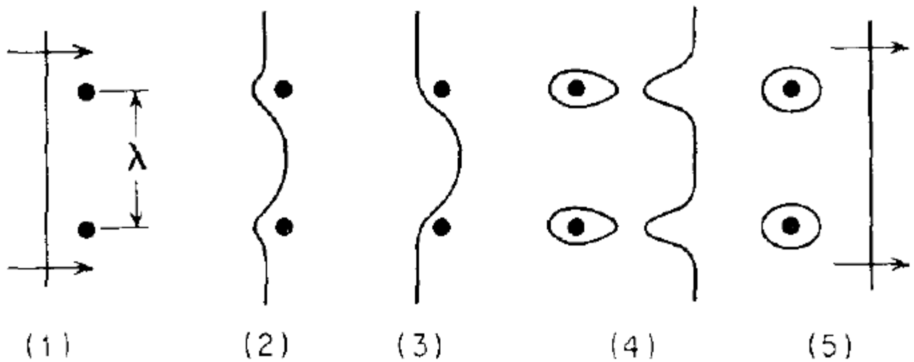
Stage 2: The line is beginning to bend.

Stage 3: It has reached the critical curvature. The dislocation can then move forward without further decreasing its radius of curvature (R).

$$R = Gb/2\tau_0$$

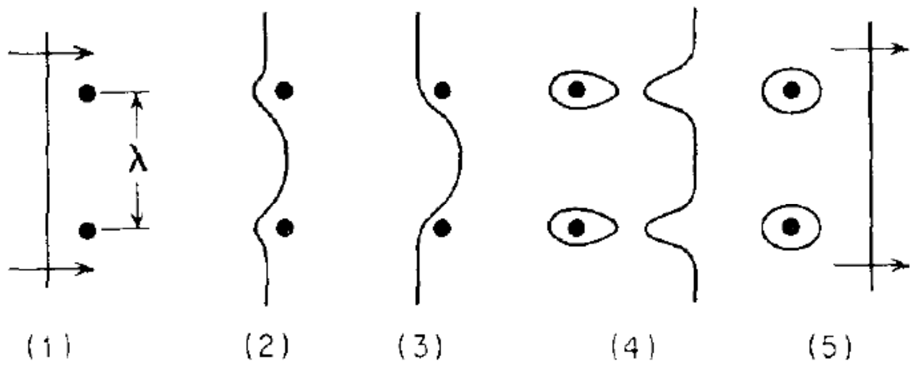
and $\lambda = 2R$, so that the shear stress required to force the dislocation between the obstacles is:

$$\tau_0 = \frac{Gb}{\lambda}$$



Stage 4: Since the segments of dislocation that meet on the other side of the particle are of opposite sign. they can annihilate each other over part of their length, leaving a dislocation loop around each particle

Stage 5: The original dislocation is then free to move on




Every dislocation gliding over the slip plane adds one loop around the particle. These loops exert a back stress on dislocation sources which must be overcome for additional slip to take place. This requires an increase in shear stress, with the result that dispersed non-coherent particles cause the matrix to strain-harden rapidly

# Exercise

An alurninum-4% copper alloy has a yield stress of 600 MPa. Estimate the particle spacing in this alloy. Given G ~ 27.6 GPa; b~0.25 nm.

At this strength level we are dealing with a precipitation-hardening alloy that has been aged beyond the maximum strength. The strengthening mechanism is dislocation bypassing of particles.

$$\lambda = Gb/\tau_0$$

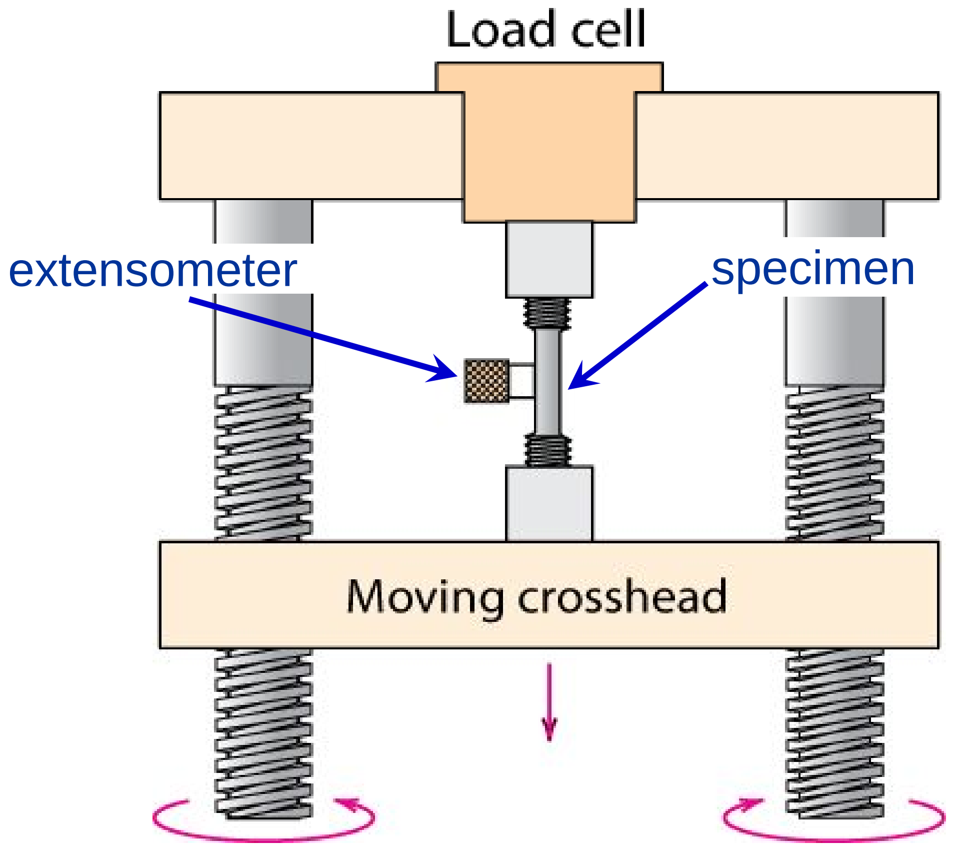
G ~ 27.6 GPa; b~0.25 nm; **$\tau_0$= 600/2 = 300 MPa**

$$\lambda = \text{interparticle spacing} = \frac{(27.6 \times 10^9 \text{ Pa}) \times (2.5 \times 10^{-10} \text{ m})}{3 \times 10^8 \text{ Pa}}$$

$$= 2.3 \times 10^{-8} \text{ m} = 0.023\ \mu\text{m} = 23 \text{ nm}$$

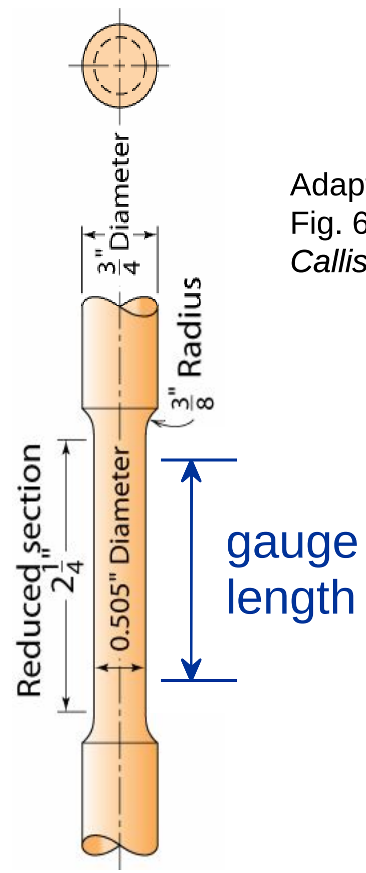
# Stress-Strain Testing

- **Typical tensile test machine**




From Fig. 9.9, *Callister's Materials Science and Engineering, Adapted Version.*
(Fig. 9.9 is taken from H.W. Hayden, W.G. Moffatt, and J. Wulff, *The Structure and Properties of Materials*, Vol. III, *Mechanical Behavior*, p. 2, John Wiley and Sons, New York, 1965.)

- **Typical tensile specimen**



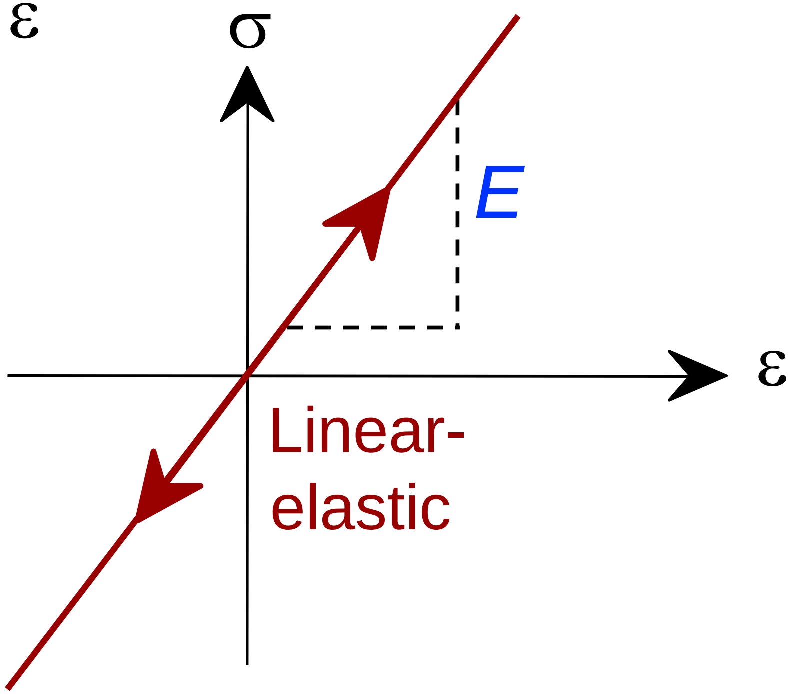

Adapted from Fig. 6.2, *Callister 7e.*

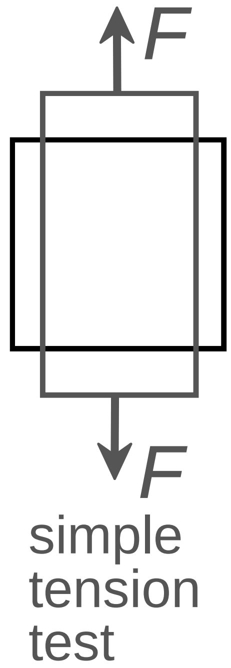
# Linear Elastic Properties

- **Modulus of Elasticity, *E*:**
  **(also known as Young's modulus)**

- **Hooke's Law:**

$$\sigma = E\,\varepsilon$$

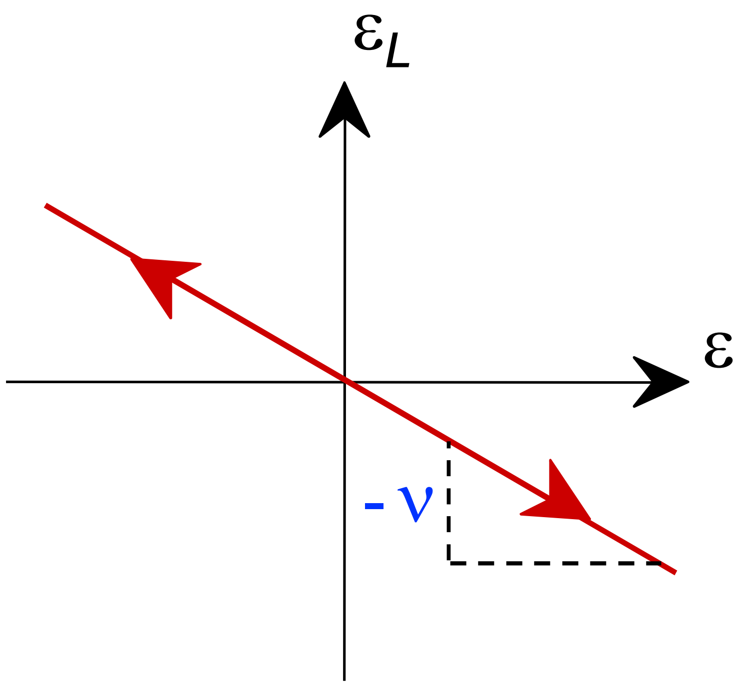


# Poisson's ratio, $\nu$

- **Poisson's ratio, $\nu$:**

$$\nu = -\frac{\varepsilon_L}{\varepsilon}$$



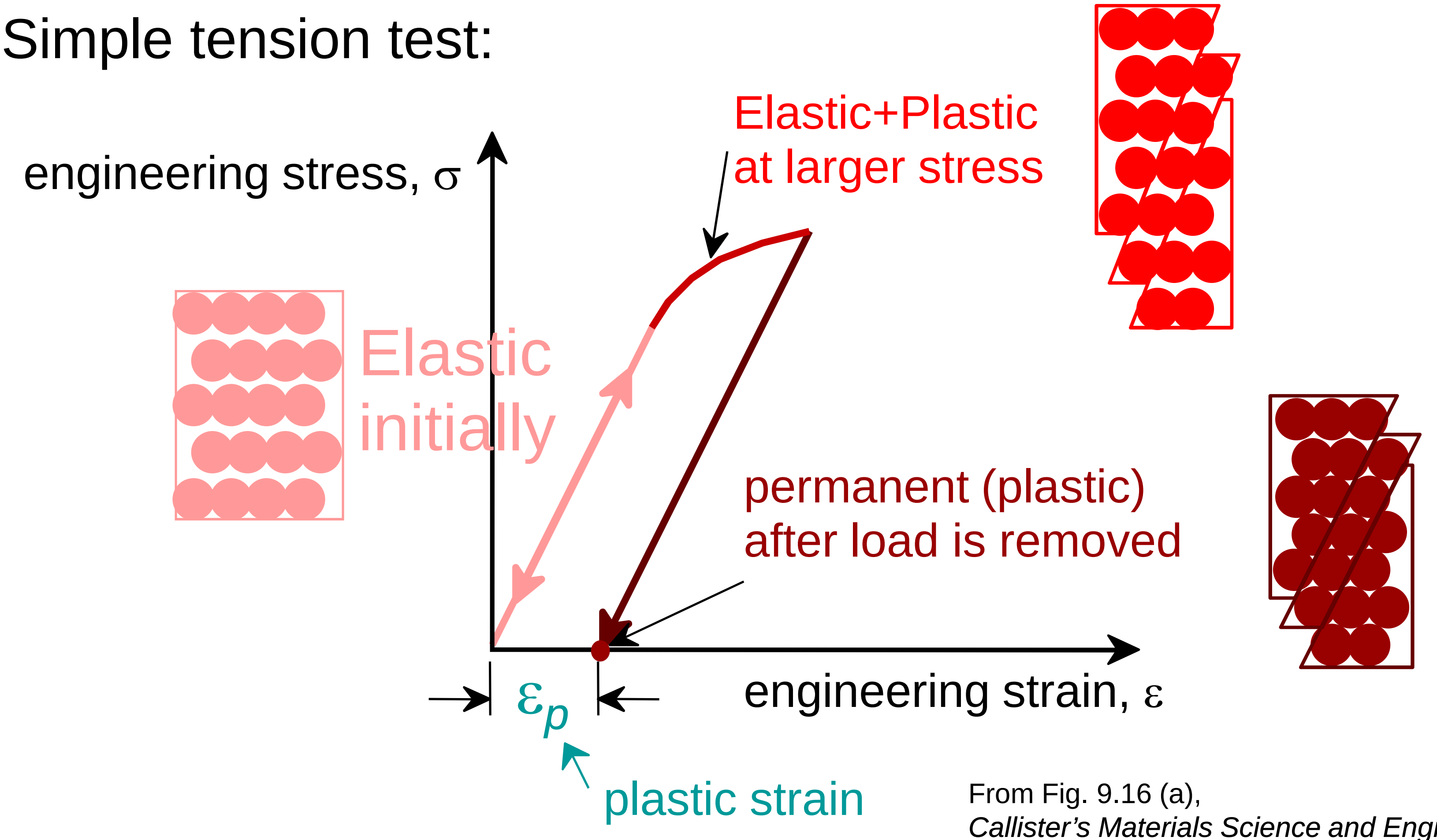

metals: $\nu$ ~ 0.33
ceramics: $\nu$ ~ 0.25
polymers: $\nu$ ~ 0.40

Units:
$E$: [GPa] or [psi]
$\nu$: dimensionless

# Plastic (Permanent) Deformation

(at lower temperatures, i.e. $T < T_{melt}/3$)

- Simple tension test:

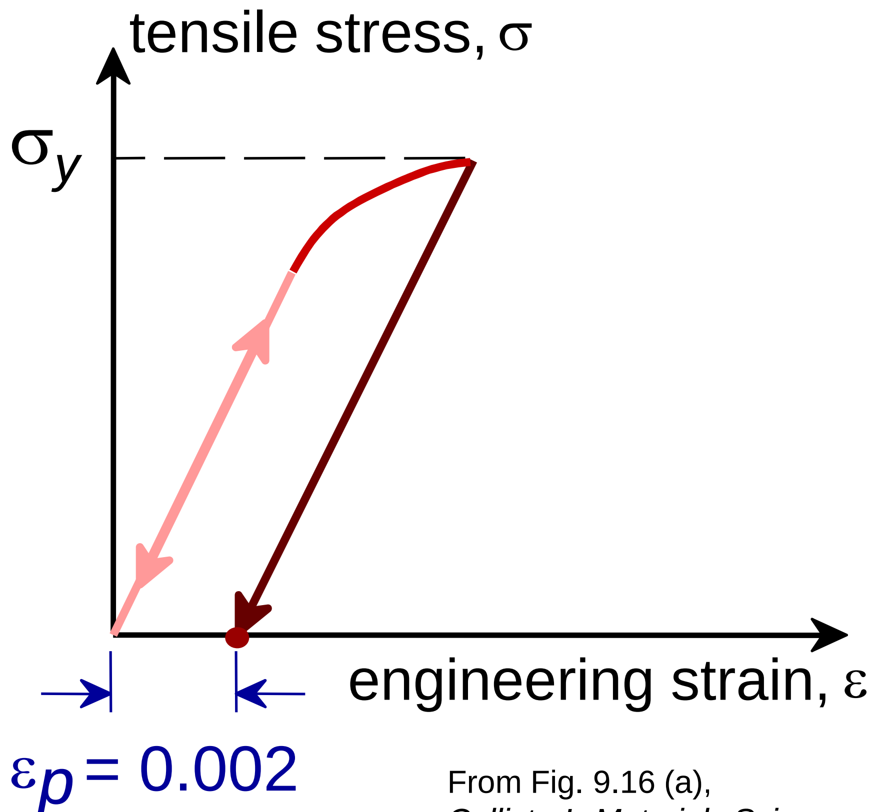



From Fig. 9.16 (a),
*Callister's Materials Science and Engineering, Adapted Version.*

# Yield Strength, $\sigma_y$

- **Stress at which *noticeable* plastic deformation has occurred.**

when $\varepsilon_p = 0.002$




$\sigma_y$ = yield strength

Note: for 2 inch sample

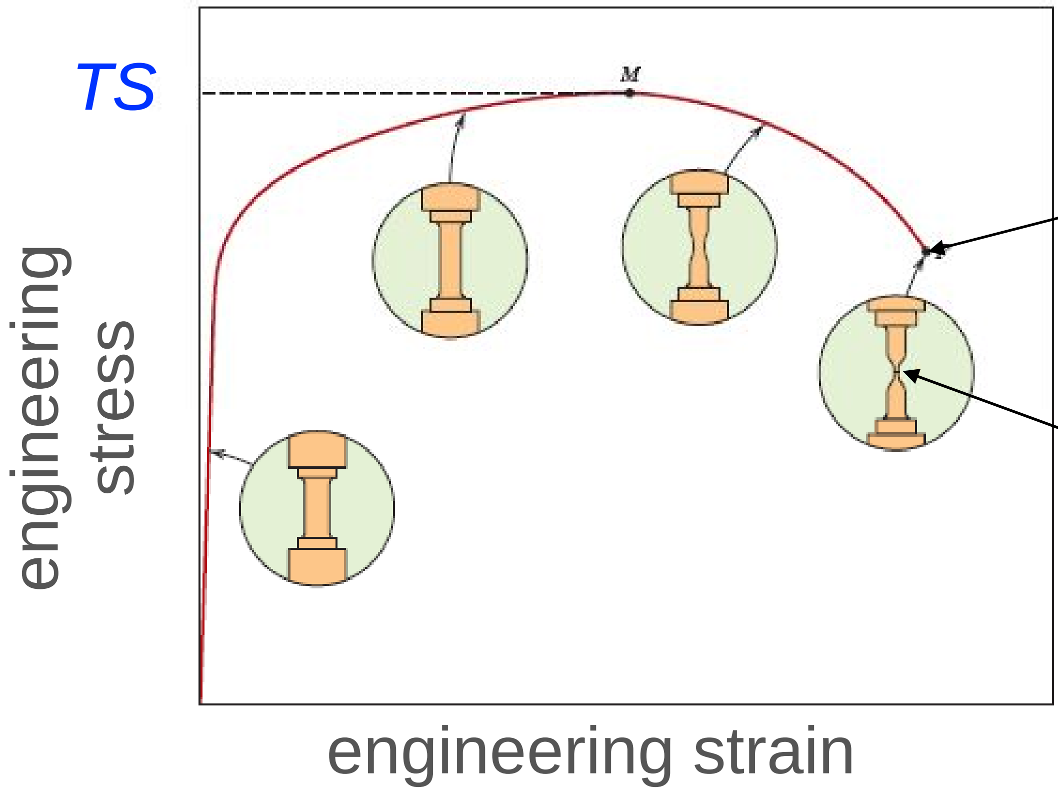
$\varepsilon = 0.002 = \Delta z/z$

$\therefore\ \Delta z = 0.004$ in

From Fig. 9.16 (a),
*Callister's Materials Science and Engineering Adapted Version.*

# Tensile Strength, TS

- Maximum stress on engineering stress-strain curve.




From Fig. 9.17 *Callister's Materials Science and Engineering, Adapted Version.*

*F* = fracture or ultimate strength

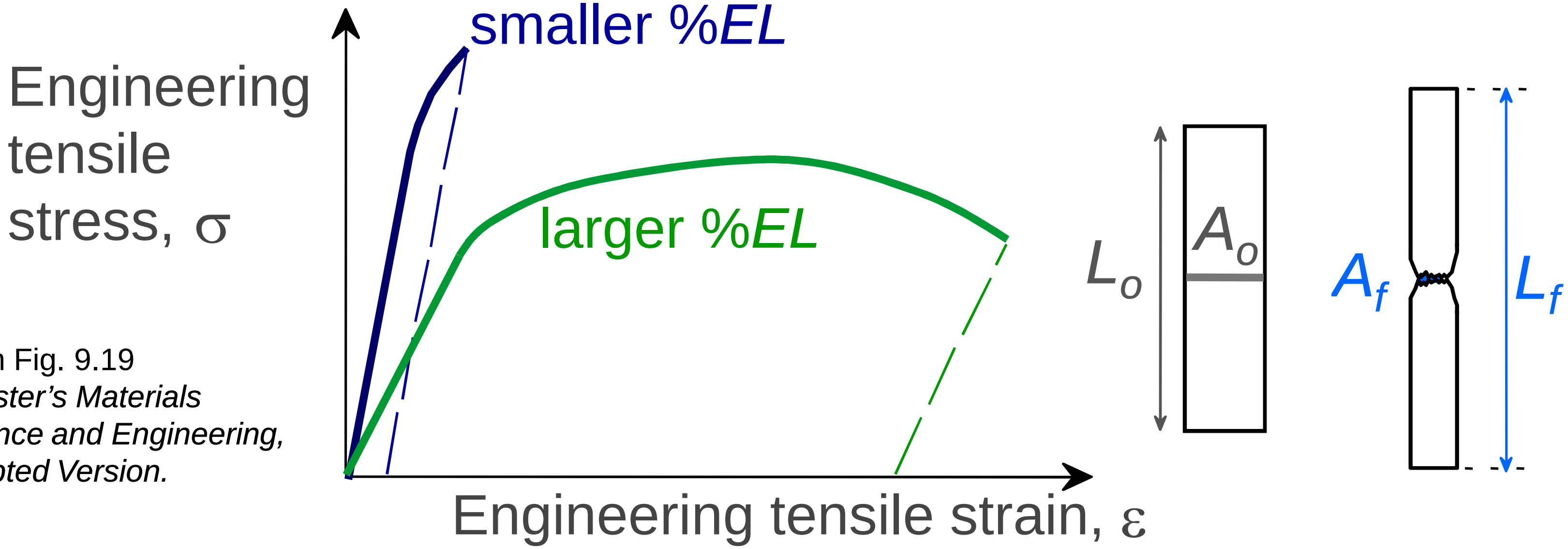
Neck – acts as stress concentrator

- Metals: occurs when noticeable necking starts.
- Polymers: occurs when polymer backbone chains are aligned and about to break.

# Ductility

- Plastic tensile strain at failure:

$$\%EL = \frac{L_f - L_o}{L_o} \text{ x } 100$$



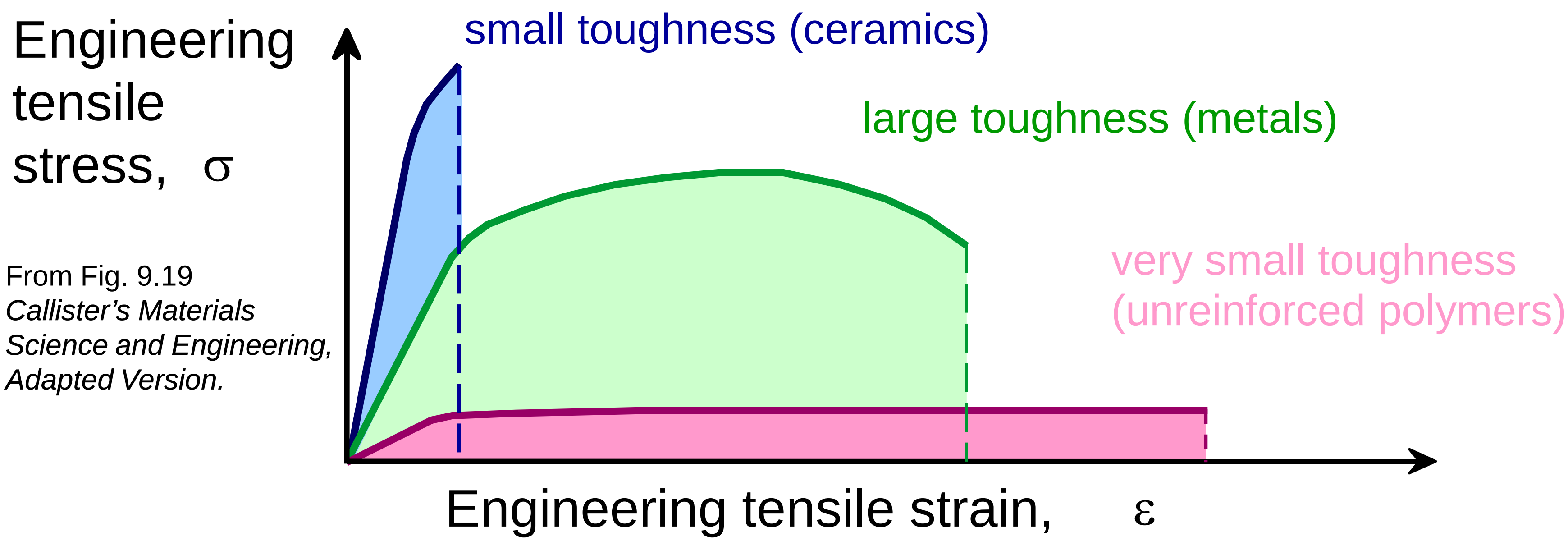

From Fig. 9.19
*Callister's Materials Science and Engineering, Adapted Version.*
.

- Another ductility measure:

$$\%RA = \frac{A_o - A_f}{A_o} \text{ x } 100$$

# Toughness

- Energy to break a unit volume of material
- Approximate by the area under the stress-strain curve.

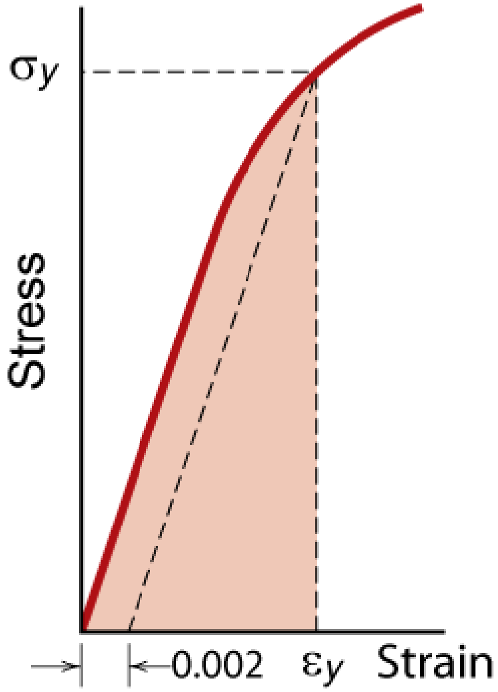



From Fig. 9.19
*Callister's Materials Science and Engineering, Adapted Version.*

Brittle fracture: elastic energy
Ductile fracture: elastic + plastic energy

# Resilience, $U_r$

- Ability of a material to store energy
  - Energy stored best in elastic region



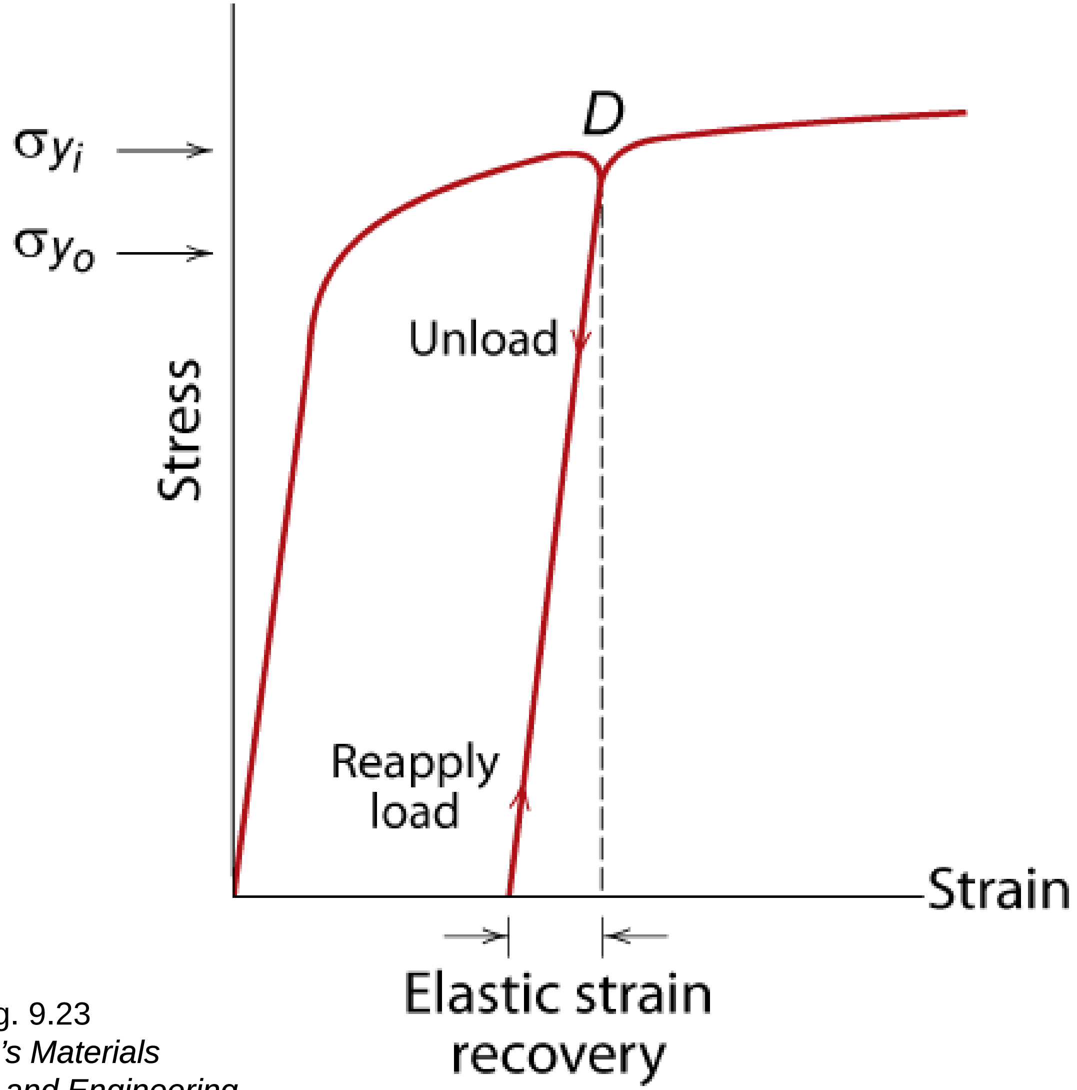

From Fig. 9.21
*Callister's Materials Science and Engineering, Adapted Version.*

$$U_r = \int_0^{\varepsilon_y} \sigma d\varepsilon$$

If we assume a linear stress-strain curve this simplifies to

$$U_r \cong \frac{1}{2}\sigma_y \varepsilon_y$$

# Elastic Strain Recovery




From Fig. 9.23
*Callister's Materials Science and Engineering, Adapted Version.*
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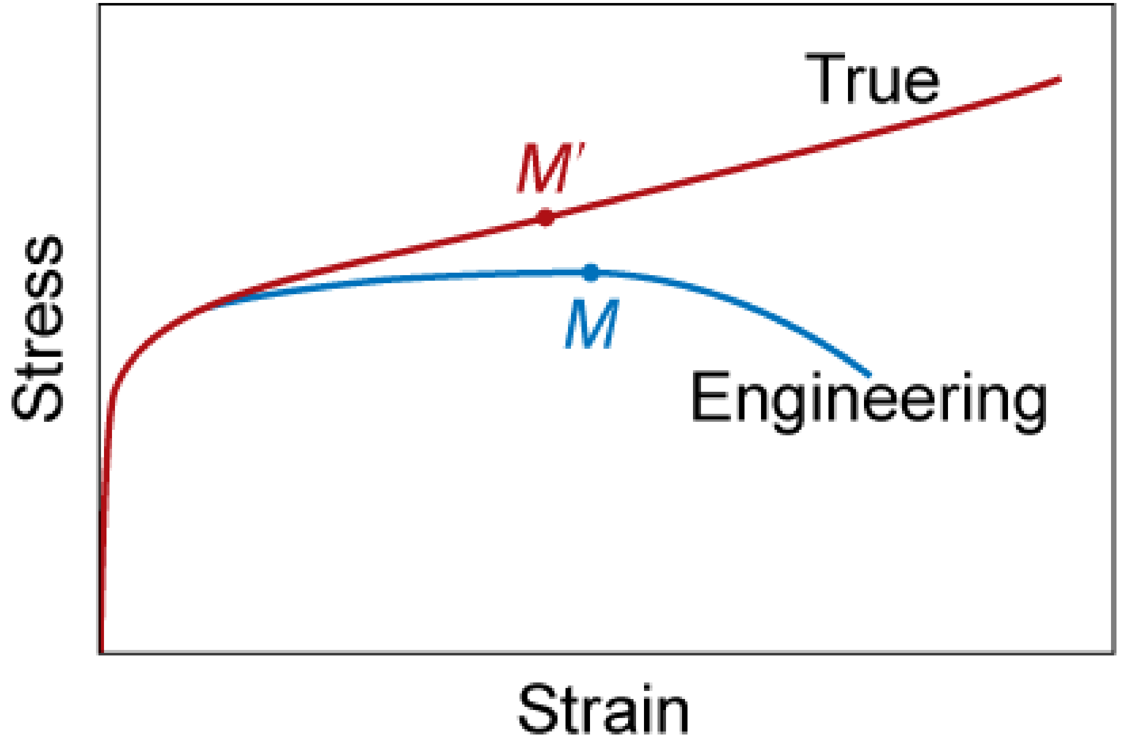
# True Stress & Strain

Note: specimen area changes when sample stretched

- True stress $\sigma_T = F/A_i$
- True Strain $\varepsilon_T = \ln(\ell_i/\ell_o)$

$$\boxed{\begin{aligned}\sigma_T &= \sigma(1+\varepsilon)\\ \varepsilon_T &= \ln(1+\varepsilon)\end{aligned}}$$




From Fig. 9.22
*Callister's Materials Science and Engineering, Adapted Version.*

# Assignment

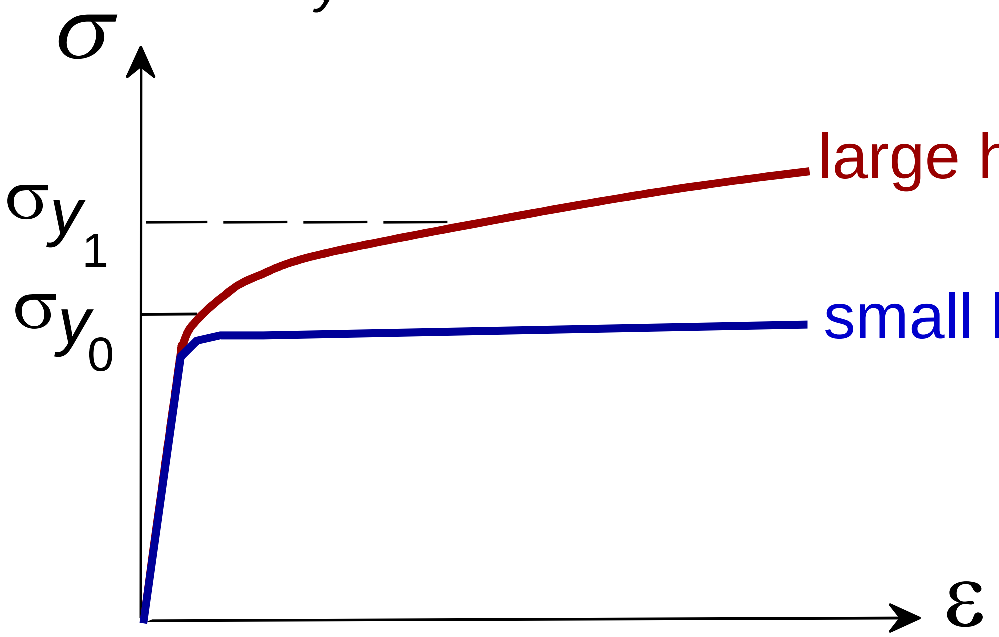
A cylindrical specimen of steel having an original diameter of 12.8 mm (0.505 in.) is tensile tested to fracture and found to have an engineering fracture strength $\sigma_f$ of 460 MPa (67,000 psi). If its cross-sectional diameter at fracture is 10.7 mm (0.422 in.), determine:

**(a)** The ductility in terms of percent reduction in area.

**(b)** The true stress at fracture.

# Hardening

- An increase in $\sigma_y$ due to plastic deformation.




- Curve fit to the stress-strain response:

$$\sigma_T = K\left(\varepsilon_T\right)^n$$

"true" stress ($F/A$)

"true" strain: $\ln(L/L_o)$

hardening exponent:
$n$ = 0.15 (some steels)
to $n$ = 0.5 (some coppers)

# Assignment

Compute the strain-hardening exponent (n) for an alloy in which a true stress of 415 MPa produces true strain of 0.10. Assume a value of 1035 MPa for K.

# Instability in Tension

- ✓ Necking generally begins at maximum load during the tensile deformation of a ductile metal.
- ✓ An ideal plastic material in which no strain hardening occurs would become unstable in tension and begin to neck just as soon as yielding took place.
- ✓However, a real metal undergoes strain hardening, which tends to increase the load-carrying capacity of the specimen as deformation increases. This effect is opposed by the gradual decrease in the cross-sectional area of the specimen as it elongates.

# Instability in Tension - Necking

- ✓Necking or localized deformation begins at maximum load, where the increase in stress due to decrease in the cross-sectional area of the specimen becomes greater than the increase in the load-carrying ability of the metal due to strain hardening.

- ✓This condition of instability leading to localized deformation is defined by the condition $dP = 0$.

$$P = \sigma A$$

$$dP = \sigma\, dA + A d\sigma = 0$$

This leads to the following relationship,

$$-\frac{dA}{A} = \frac{d\sigma}{\sigma} \qquad \text{..........(1)}$$

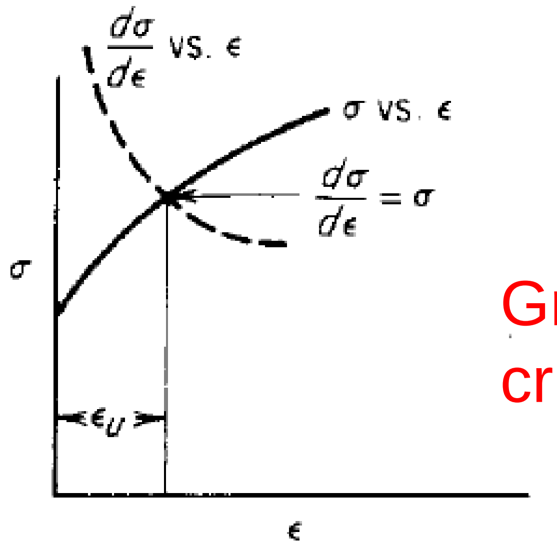
# Instability in Tension - Necking

From the constancy-of-volume relationship,

$$\frac{dL}{L} = -\frac{dA}{A} = d\varepsilon \qquad \text{..........(2)}$$

*So that at a point of tensile instability (combining Equation 1 and Equation 2)*

$$\frac{d\sigma}{d\varepsilon} = \sigma$$




Graphical interpretation of necking criterion

# Exercise

If the true-stress-true-strain curve is given by the relationship: $\sigma = 1400\varepsilon^{0.33}$ where stress is in MPa, what is the ultimate tensile strength of the material?

# Hints:

$$\sigma_T = K(\varepsilon_T)^n$$

hardening exponent:
$n$ = 0.15 (some steels)
to $n$ = 0.5 (some coppers)

"true" stress ($F/A$)

"true" strain: $\ln(L/L_o)$

$\varepsilon_T = \mathrm{n}$ ……….How you get the relationship??

$$\sigma_T = \sigma(1+\varepsilon)$$

$$\varepsilon_T = \ln(1+\varepsilon)$$

**Answer: 698 MPa**

# Strain Rate

Strain rate is defined as $\dot{\varepsilon} = {}^{d\varepsilon}\!/_{dt}$

## Engineering Strain rate

$$\dot{\varepsilon} = {}^{d\varepsilon}\!/_{dt} = \frac{d(L - L_0)/L_0}{dt} = \frac{1}{L_0}\frac{dL}{dt} = \frac{v}{L_0}$$

## True Strain rate

$$\dot{\varepsilon}_T = {}^{d\varepsilon_T}\!/_{dt} = \frac{d[\ln({}^{L}\!/_{L_0})]}{dt} = \frac{1}{L}\frac{dL}{dt} = \frac{v}{L}$$

**Here *v* is crosshead velocity of the machine**

# Relation between Stress and Strain Rate

A general relationship exists between tensile stress and strain rate, at constant strain and temperature:

$$\sigma_T = C(\dot{\varepsilon}_T)^m$$

where *m is known as the strain-rate sensitivity*

# Exercise

**The parameters obtained from tensile tests of a commercially pure aluminum are as follows at a true strain of 0.25.**

| | 294 K | 713 K |
|---|---|---|
| $C$: | 70.3 MPa | 14.5 MPa |
| $m$: | 0.066 | 0.211 |

**Determine the change in flow stress for a two order of magnitude change (say 1 to 100 $s^{-1}$) in strain rate at each of the temperatures.**

At 294 K $\quad \sigma_a = C(\dot{\varepsilon})^m = 70.3(1)^{0.066} = 70.3 \text{ MPa}$

$\sigma_b = 70.3(100)^{0.066} = 95.3 \text{ MPa} \qquad \sigma_b/\sigma_a = 1.35$

At 713 K $\quad \sigma_a = 14.5(1)^{0.211} = 14.5 \text{ MPa}$

$\sigma_b = 14.5(100)^{0.211} = 38.3 \text{ MPa} \qquad \sigma_b/\sigma_a = 2.64$

# Variability in Material Properties

- Elastic modulus is material property
- Critical properties depend largely on sample flaws (defects, etc.). Large sample to sample variability.
- Statistics
  - Mean

$$\bar{x} = \frac{\sum^{n} x_n}{n}$$

  - Standard Deviation

$$s = \left[ \frac{\sum^{n} (x_i - \bar{x})^2}{n - 1} \right]^{\frac{1}{2}}$$

where $n$ is the number of data points

# Design or Safety Factors

- Design uncertainties mean we do not push the limit.
- Factor of safety, *N*

$$\sigma_{working} = \frac{\sigma_y}{N}$$

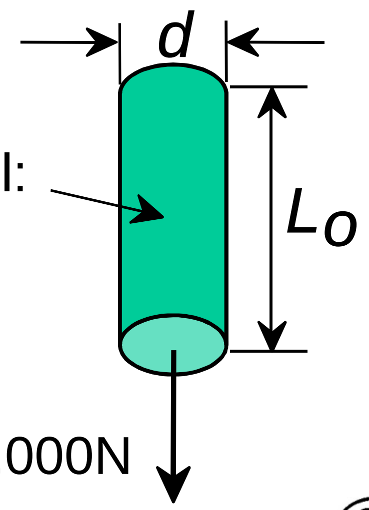
Often *N* is between 1.2 and 4

- Example: Calculate a diameter, *d*, to ensure that yield does not occur in the 1045 carbon steel rod below. Use a factor of safety of 5.

$$\sigma_{working} = \frac{\sigma_y}{N}$$

$$\frac{220{,}000N}{\pi\left(d^2/4\right)} = \frac{\sigma_y}{5}$$

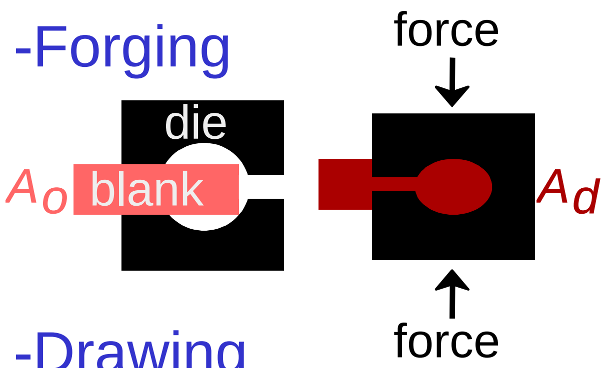
1045 plain carbon steel:
$\sigma_y$ = 310 MPa
*TS* = 565 MPa

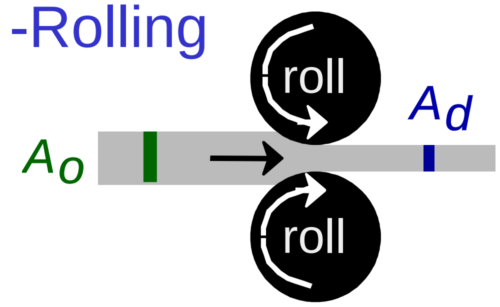



$\boxed{d = 0.067 \text{ m} = 6.7 \text{ cm}}$

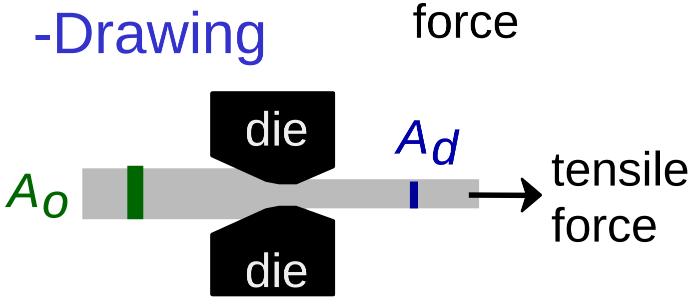
# 4 Strategies for Strengthening: 4: Cold Work (%*CW*)

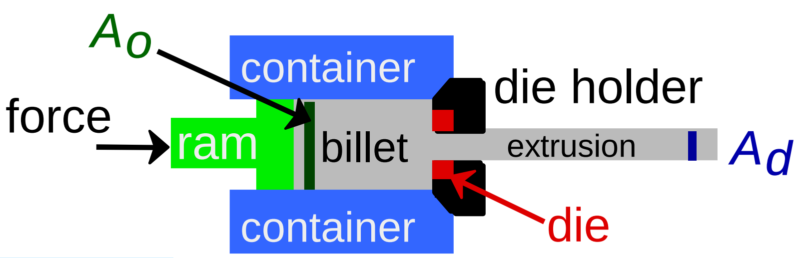
- Room temperature deformation.
- Common forming operations change the cross sectional area:




Adapted from Fig. 11.8, *Callister 7e.*

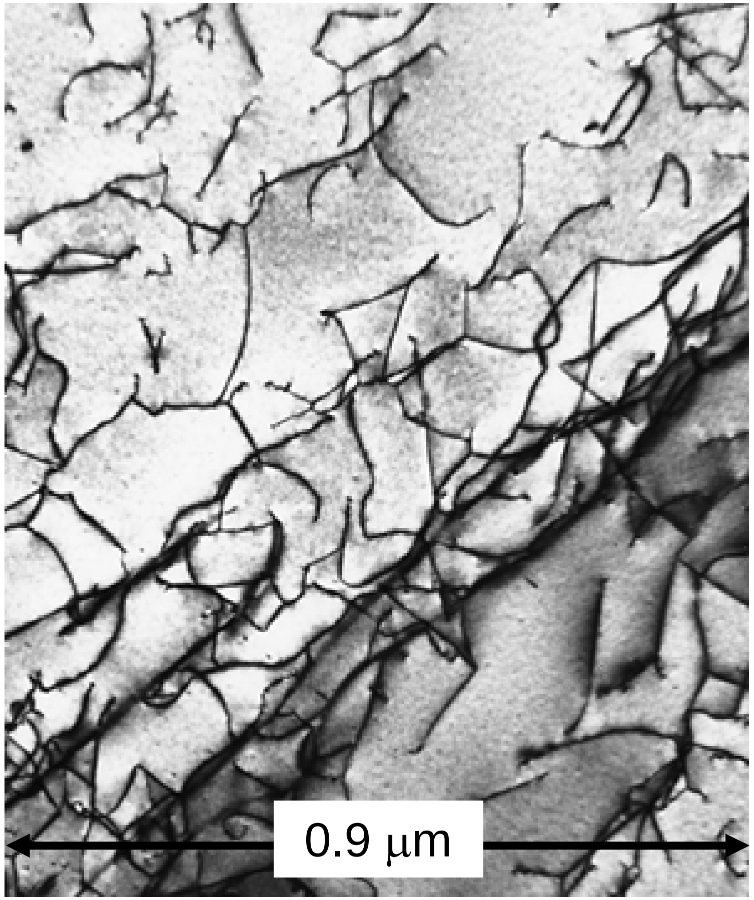



$$\%CW = \frac{A_o - A_d}{A_o} \text{ x } 100$$

# Dislocations During Cold Work

- Ti alloy after cold working:




- Dislocations entangle with one another during cold work.
- Dislocation motion becomes more difficult.

From Fig. 5.10, *Callister's Materials Science and Engineering, Adapted Version.*

(Fig. 5.10 is courtesy of M.R. Plichta, Michigan Technological University.)

# Result of Cold Work

$$\text{Dislocation density} = \frac{\text{total dislocation length}}{\text{unit volume}}$$

– Carefully grown single crystal

→ ca. $10^3$ mm$^{-2}$

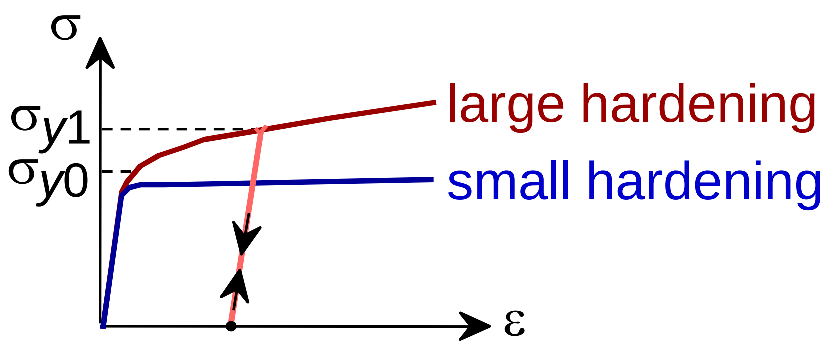
– Deforming sample increases density

→ $10^9$-$10^{10}$ mm$^{-2}$

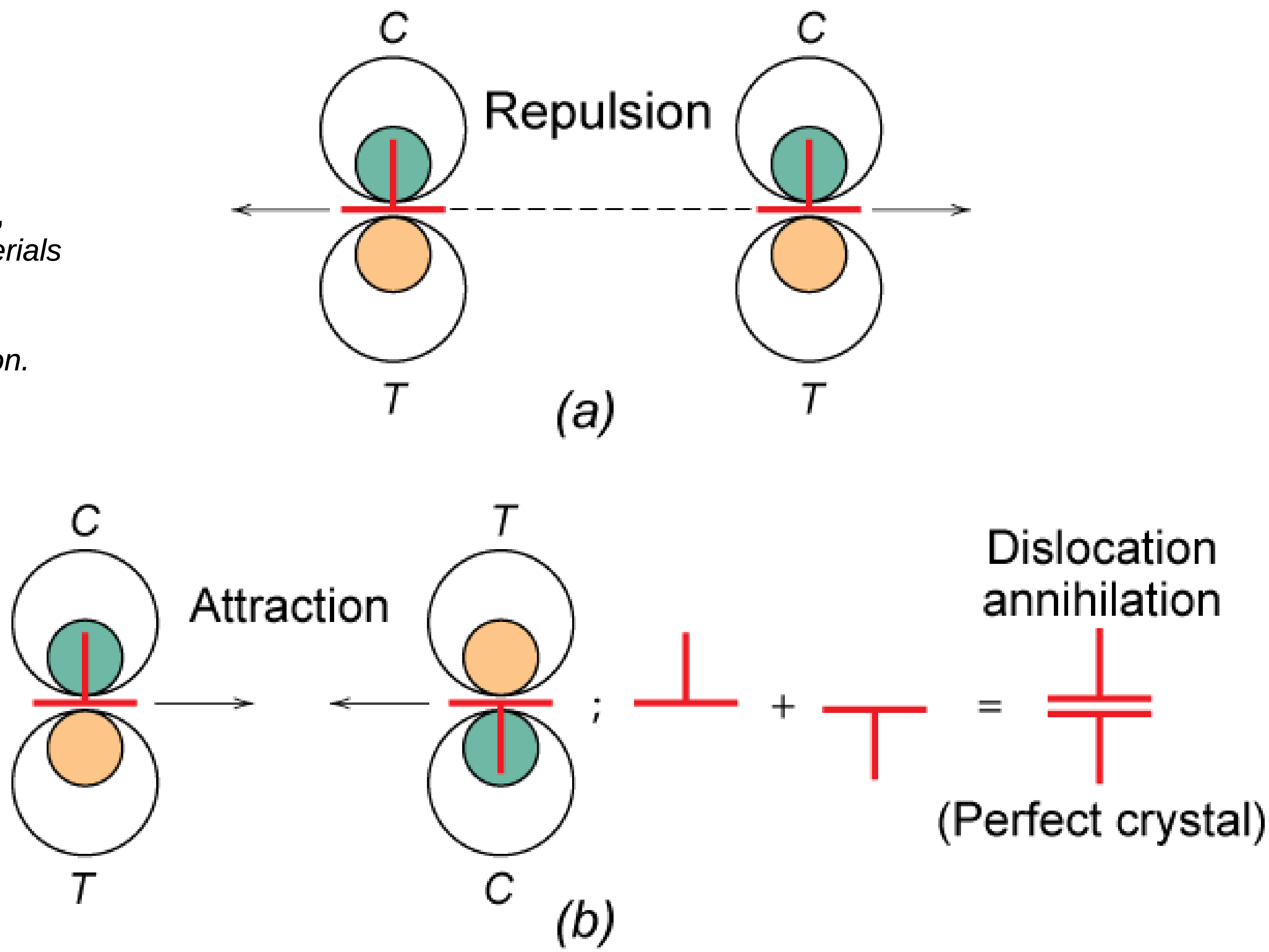
– Heat treatment reduces density

→ $10^5$-$10^6$ mm$^{-2}$

- Yield stress increases as $\rho_d$ increases:



# Effects of Stress at Dislocations

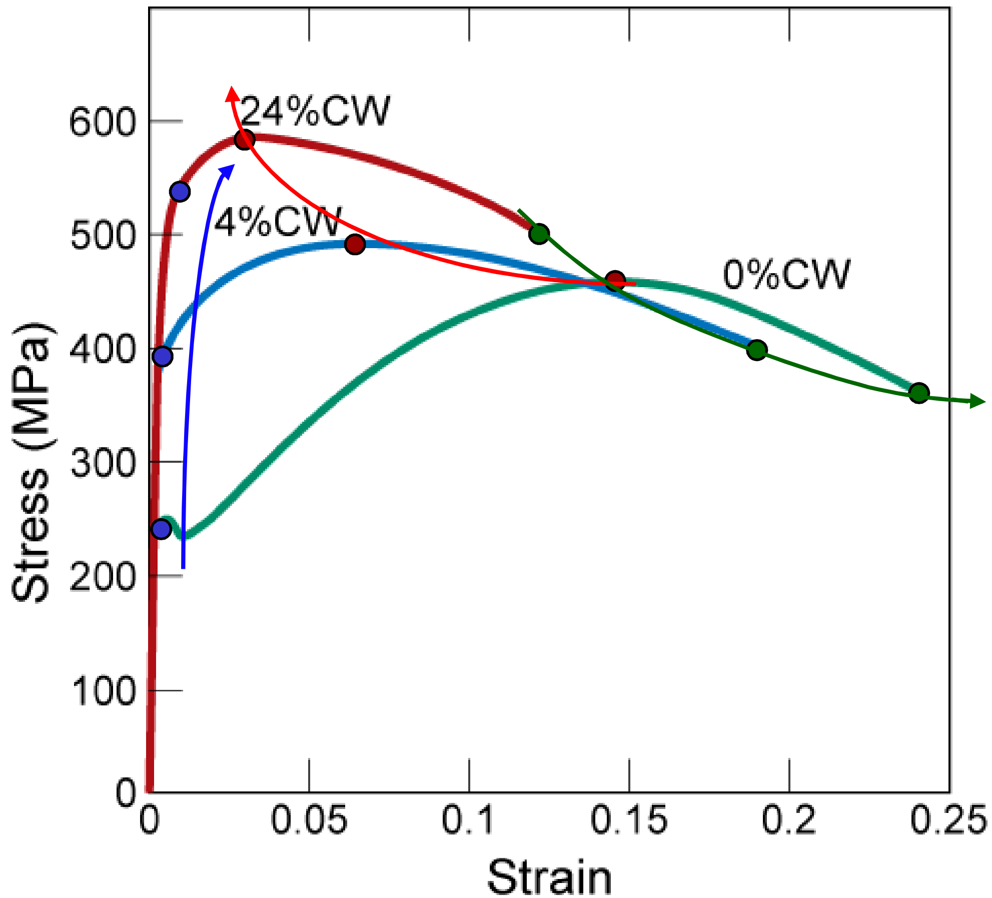



From Fig. 10.5, *Callister's Materials Science and Engineering, Adapted Version.*

# Impact of Cold Work

As cold work is increased

- Yield strength ($\sigma_y$) increases.
- Tensile strength (*TS*) increases.
- Ductility (%*EL* or %*AR*) decreases.

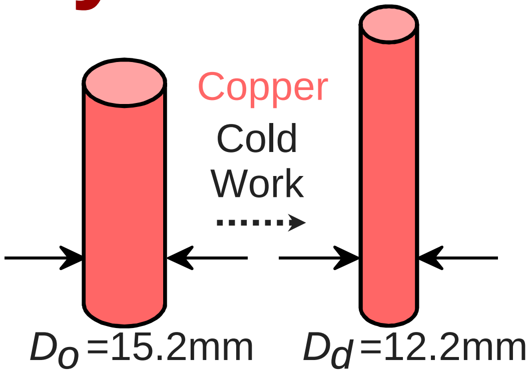


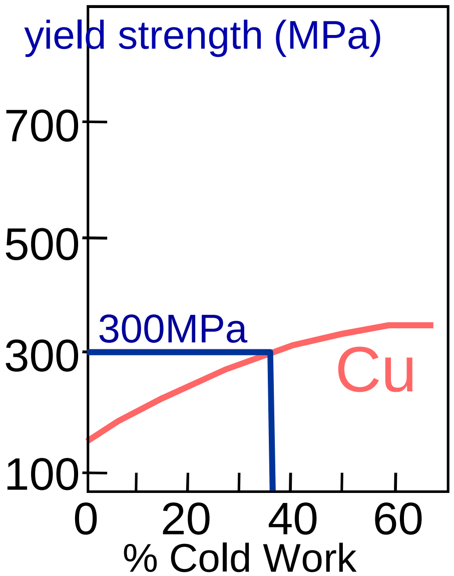

From Fig. 10.20
*Callister's Materials Science and Engineering, Adapted Version.*

# Cold Work Analysis

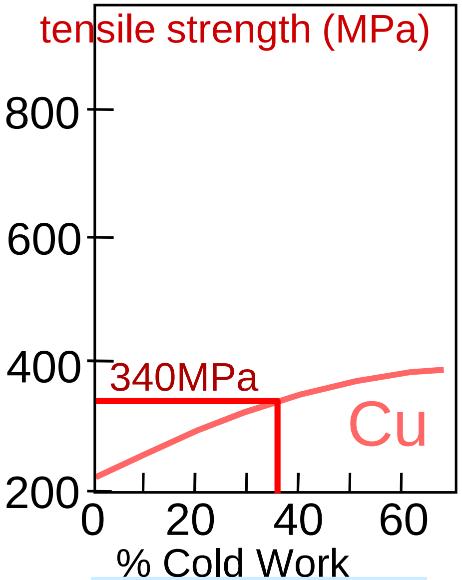
- What is the tensile strength & ductility after cold working?

$$\%CW = \frac{\pi r_o^2 - \pi r_d^2}{\pi r_o^2} \times 100 = 35.6\%$$

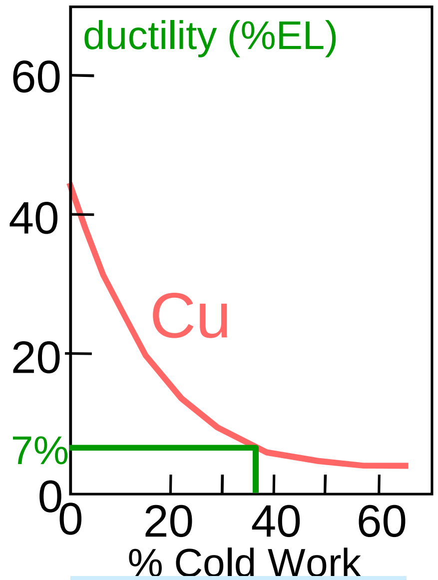



$\sigma_y = 300\text{MPa}$




$TS = 340\text{MPa}$

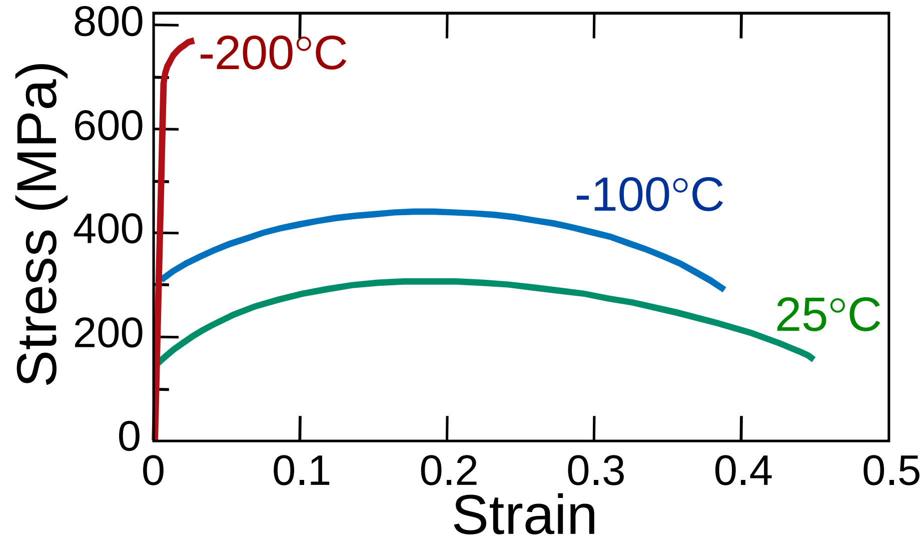



$\%EL = 7\%$

From Fig. 10.19, *Callister's Materials Science and Engineering, Adapted Version.* (Fig. 10.19 is adapted from *Metals Handbook: Properties and Selection: Iron and Steels*, Vol. 1, 9th ed., B. Bardes (Ed.), American Society for Metals, 1978, p. 226; and *Metals Handbook: Properties and Selection: Nonferrous Alloys and Pure Metals*, Vol. 2, 9th ed., H. Baker (Managing Ed.), American Society for Metals, 1979, p. 276 and 327.)

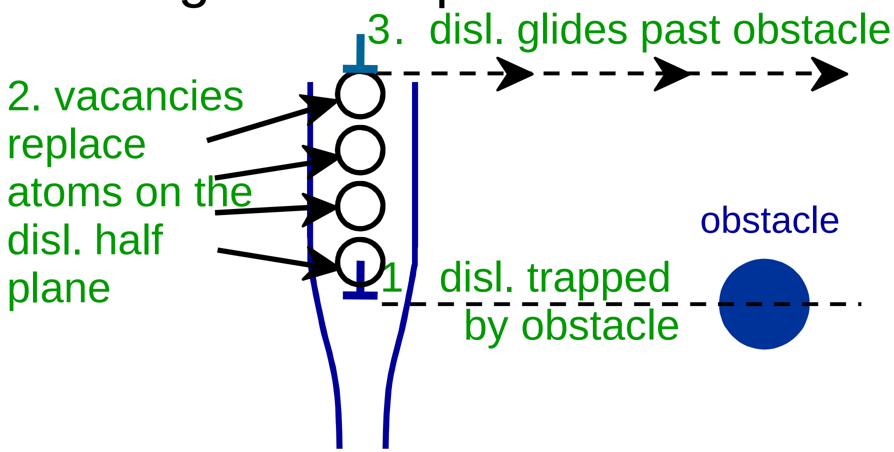
# $\sigma$-$\varepsilon$ Behavior vs. Temperature

- Results for polycrystalline iron:




From Fig. 9.20 *Callister's Materials Science and Engineering, Adapted Version.*

- $\sigma_y$ and *TS* *decrease* with increasing test temperature.
- %*EL* *increases* with increasing test temperature.
- Why? Vacancies help dislocations move past obstacles.



## Effect of Heating (Annealing) After *Cold Working*

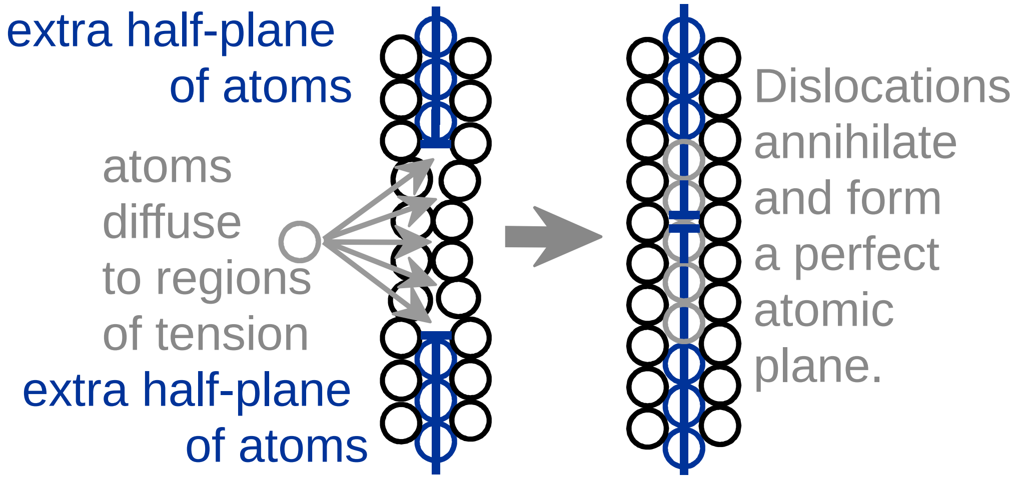
- **Annealing** of the cold worked structure at high temperature ***softens the metal*** and reverts to a strain-free condition.
- Annealing restores the ductility to a metal that has been severely strain hardened.
- Annealing can be divided into ***three distinct processes:***
  - ✓ **Recovery**
  - ✓ **Recrystallization**
  - ✓ **Grain growth**

# Recovery

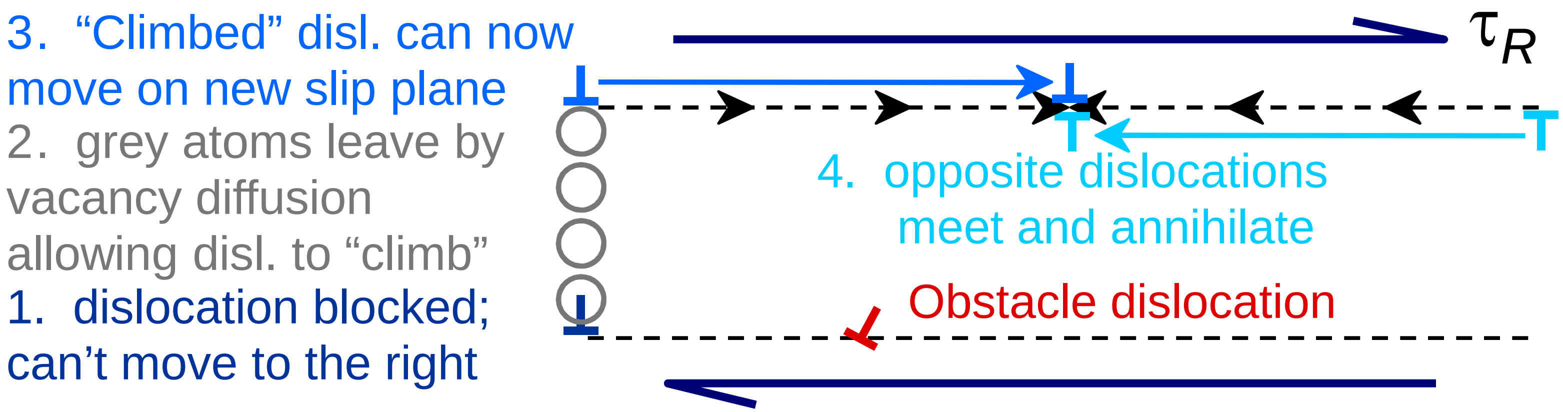
Annihilation reduces dislocation density.

- Scenario 1

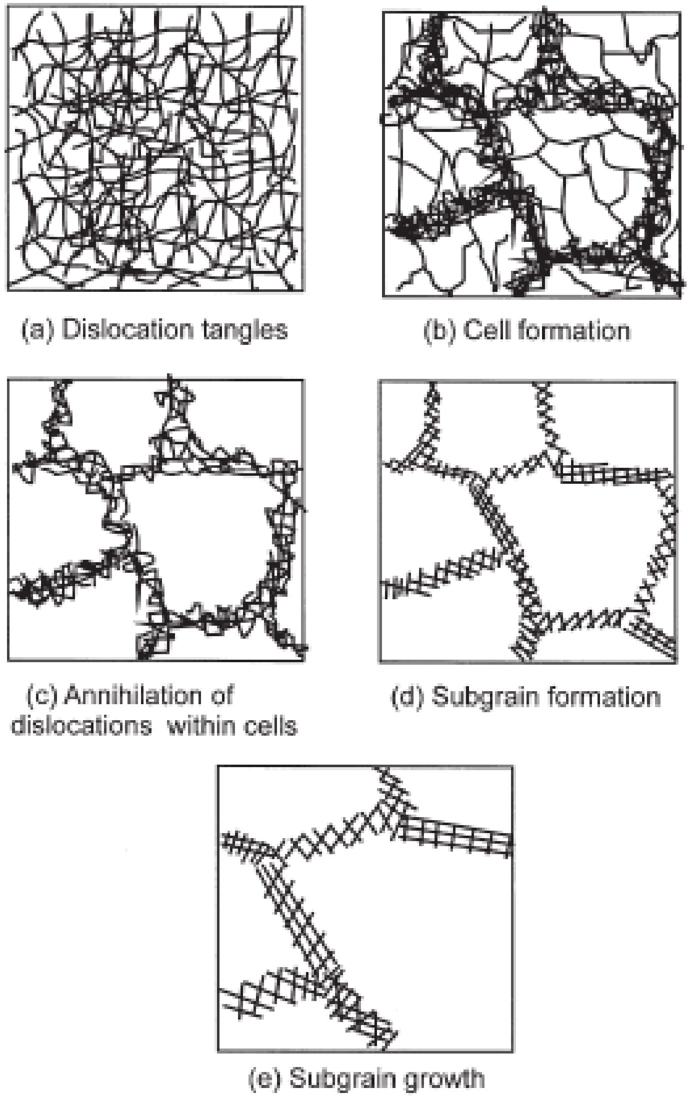
  Results from diffusion




- Scenario 2

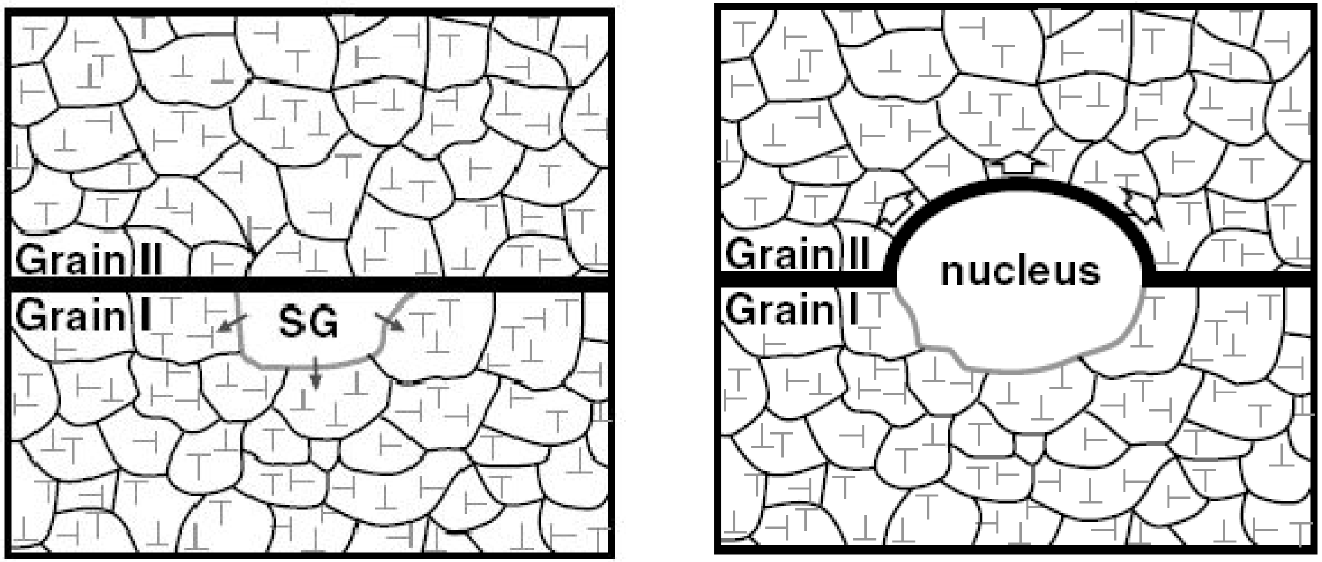


# Recovery




Various stages in the recovery of a plastically deformed material

# Recrystallization

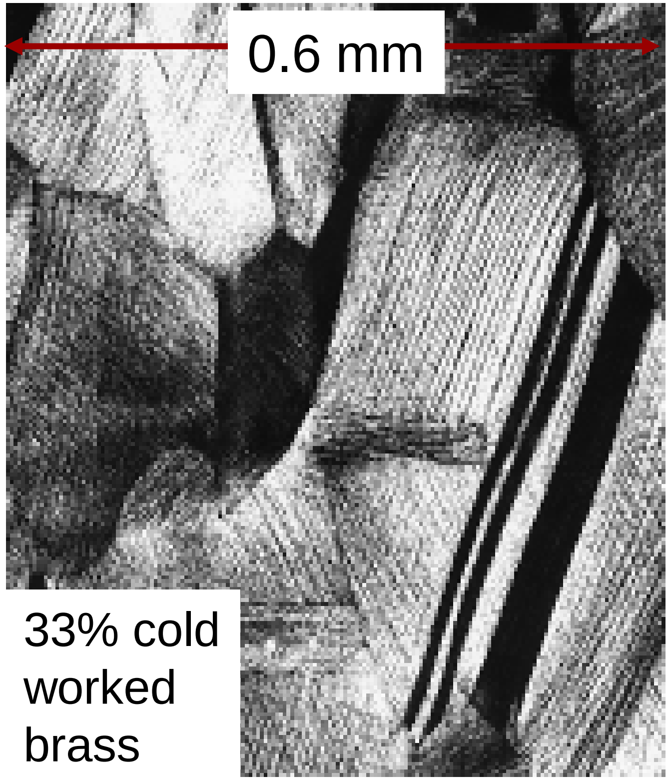



(a) (b)

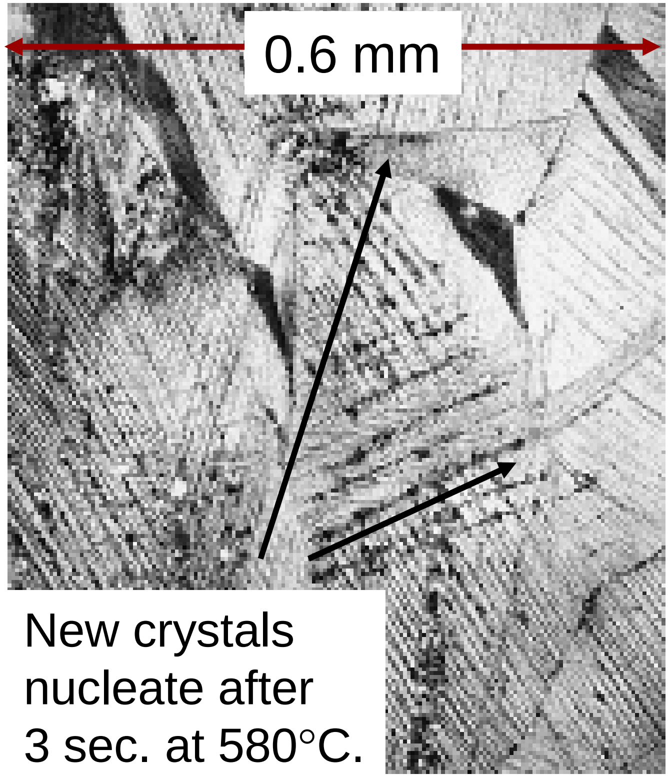
Schematic illustration of nucleation event: (a) subgrain (SG) initially grows within Grain I and reaches the critical size which allows it to overcome the capillary force; (b) subsequently it can bulge into Grain II as a new strain free grain

# Recrystallization

- New grains are formed that:
  - -- have a small dislocation density
  - -- are small
  - -- consume cold-worked grains.




33% cold worked brass

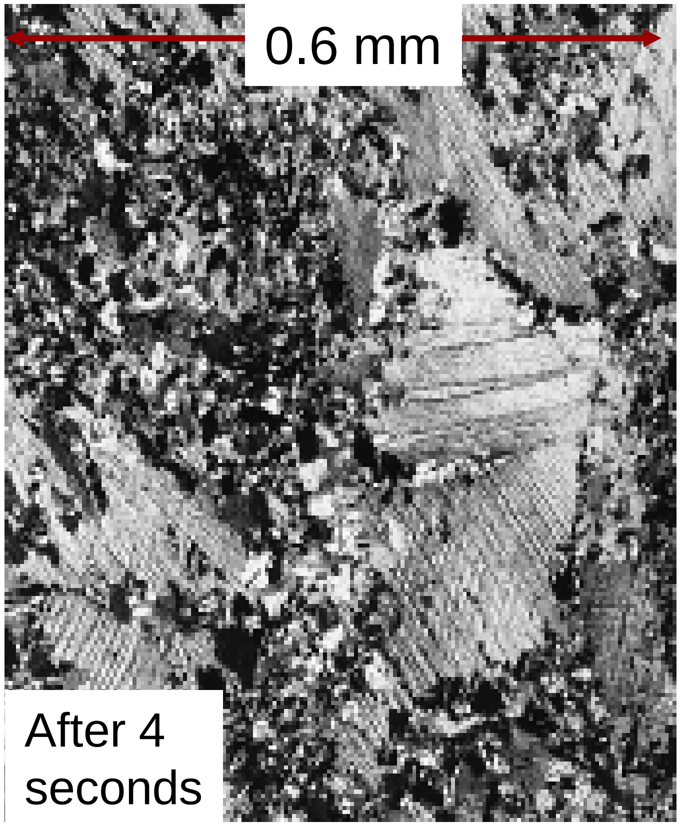


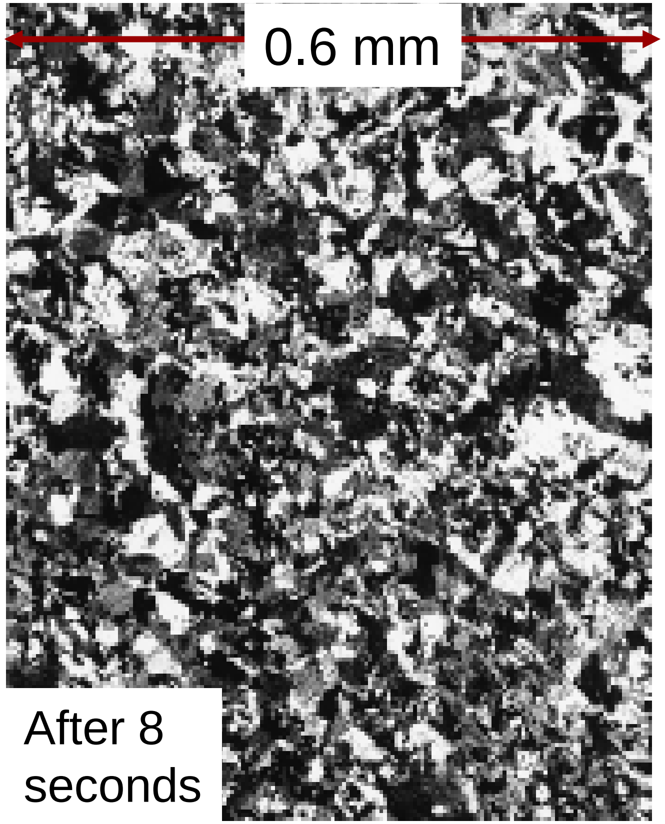

New crystals nucleate after 3 sec. at 580°C.

From Fig. 10.21 (a),(b), *Callister's Materials Science and Engineering, Adapted Version.* (Fig. 10.21 (a),(b) are courtesy of J.E. Burke, General Electric Company.)

# Further Recrystallization

- All cold-worked grains are consumed.

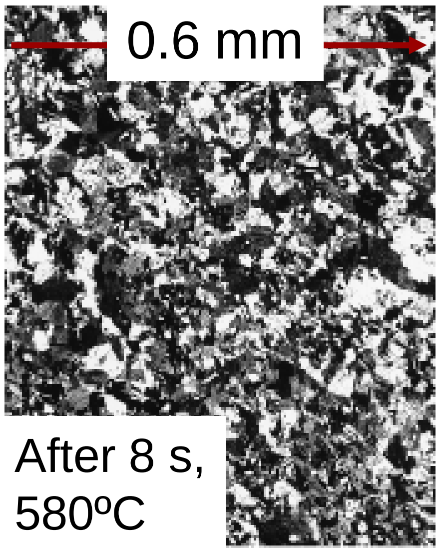


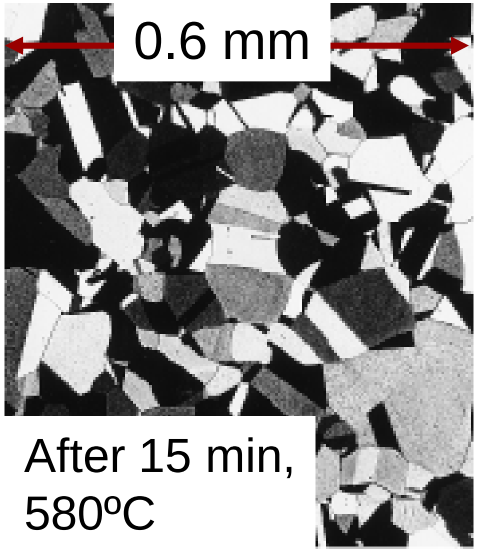




From Fig. 10.21 (c),(d), *Callister's Materials Science and Engineering, Adapted Version.* (Fig. 10.21 (c),(d) are courtesy of J.E. Burke, General Electric Company.)

# Grain Growth

- At longer times, larger grains consume smaller ones.
- Why? Grain boundary area (and therefore energy) is reduced.




From Fig. 10.21 (d),(e) *Callister's Materials Science and Engineering, Adapted Version.* (Fig. 10.21 (d),(e) are courtesy of J.E. Burke, General Electric Company.)

- Empirical Relation:

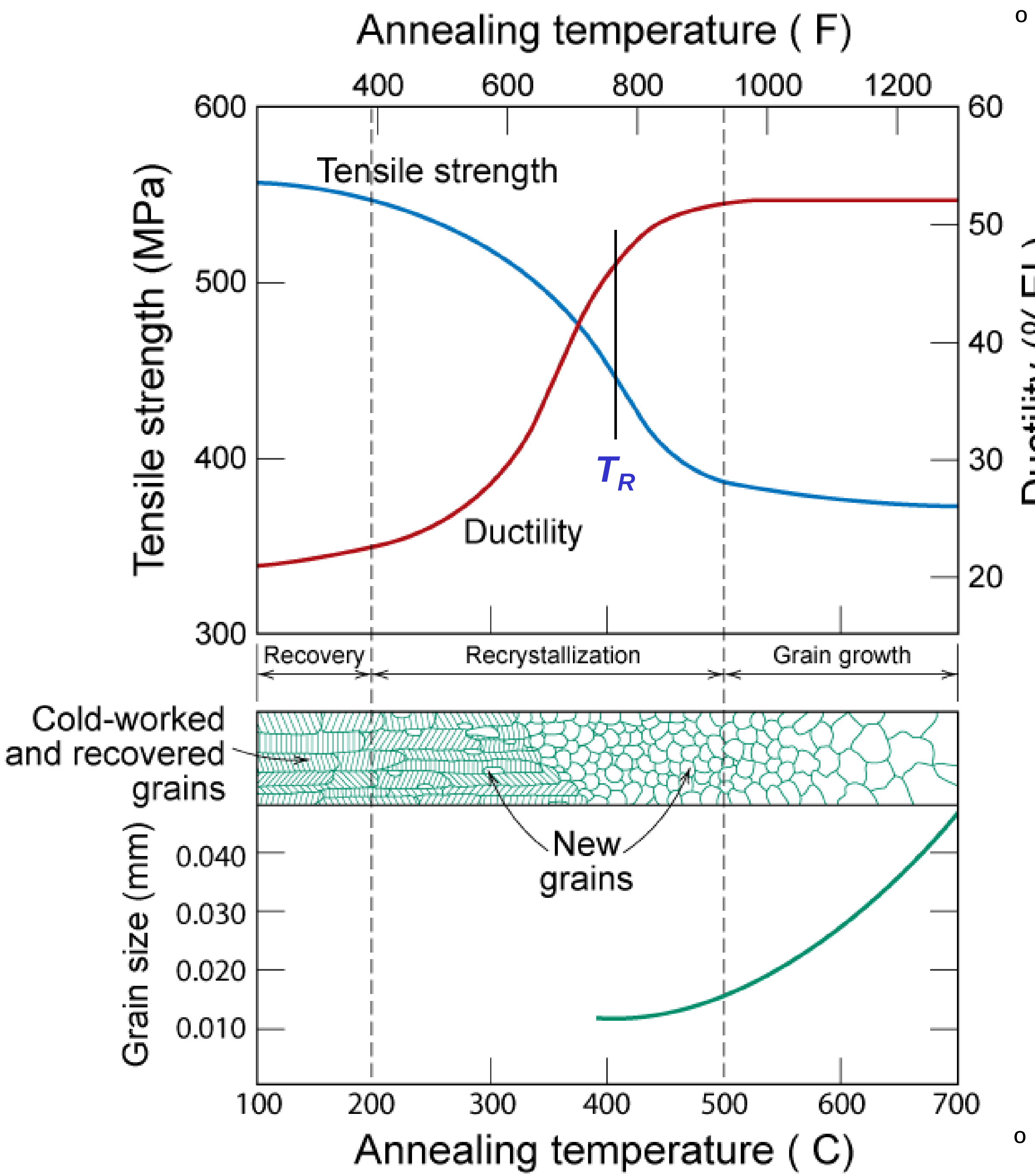
exponent typ. ~ 2

grain diam.
at time t.

$$d^n - d_o^n = Kt$$

coefficient dependent on material and *T*.

elapsed time




$T_R$ = recrystallization temperature

From Fig. 10.22, *Callister's Materials Science and Engineering, Adapted Version.*

# Recrystallization Temperature, $T_R$

$T_R$ = recrystallization temperature = point of highest rate of property change

1. $T_m$ => $T_R \approx$ 0.3-0.6 $T_m$ (K)
2. Due to diffusion → annealing time→ $T_R = f(t)$
   shorter annealing time => higher $T_R$
3. Higher %$CW$ => lower $T_R$
4. Pure metals lower $T_R$ due to dislocation movements
   - Dislocation can move easily in pure metals => lower $T_R$

# Summary

• **Recovery** : The restoration of the physical properties of the cold worked metal without any observable change in microstructure. *Strength is not affected*.

• **Recrystallization** : The cold worked structure is replaced by a new set of strain-free grains due to migration of high angle grain boundaries. *Hardness and strength decrease but ductility increases.*

• **Grain growth** : Occurs at higher temperature where some of the recrystallized fine grains start to grow rapidly.