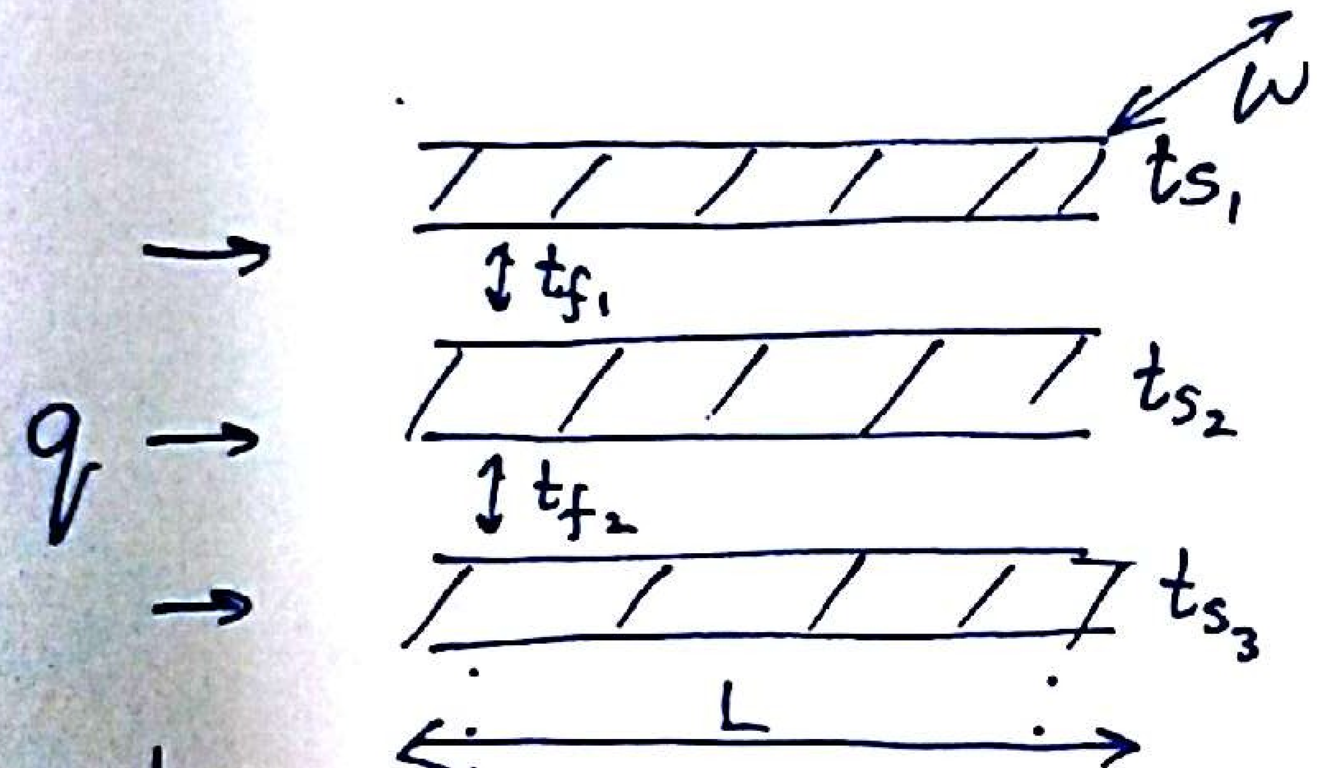
Heat Transfer

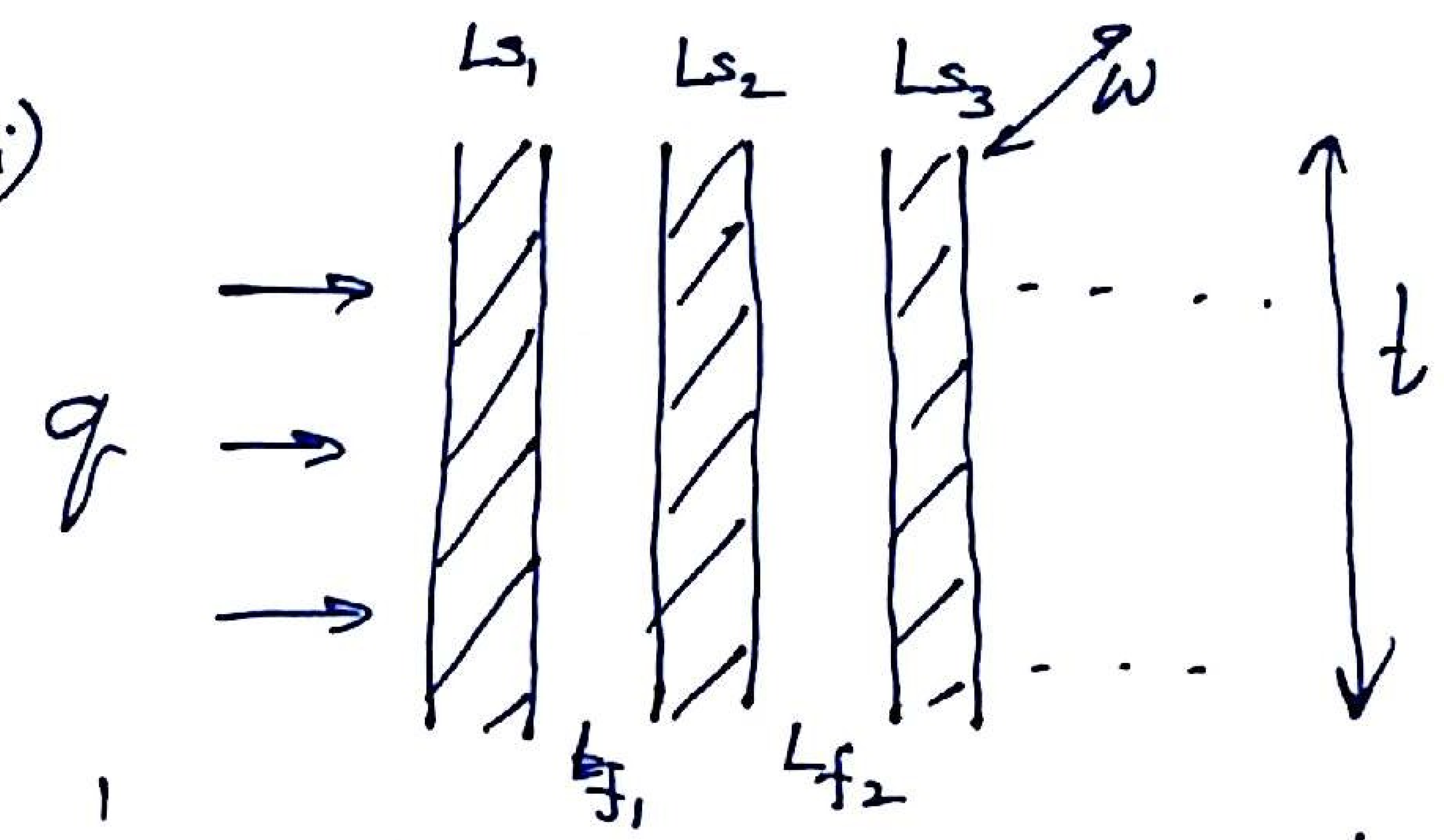
Answers to End-Sem 2016

1. In a porous medium, we get two extreme scenarios
(i) when solid & fluid phases are parallel to each other w.r.t. direction of heat flow resulting in <u>lowest thermal resistance</u>
(ii) when solid & fluid phases are in series w.r.t. the direction of heat flow resulting in <u>highest thermal resistance</u>

(i)



(ii)



(i)

$$\frac{1}{R_{th}\big|_i} = \frac{k_s(t_{s_1}W)}{L} + \frac{k_f(t_{f_1}W)}{L} + \cdots + \frac{k_s(t_{s_2}W)}{L} + \cdots$$

or $$\frac{k_e(tW)}{L} = \frac{W}{L}\sum_{i=1}^{\infty}\left(k_s t_{s_i} + k_f t_{f_i}\right)$$

or $$k_e\big|_{max} = k_s \sum_{i=1}^{\infty}\frac{t_{si}}{t} + k_f \sum_{i=1}^{\infty}\frac{t_{fi}}{t}$$

or $$k_e\big|_{max} = (1-\varepsilon)k_s + \varepsilon k_f$$

(ii)

$$R_{th}\big|_{ii} = \frac{L_{s_1}}{k_s(Wt)} + \frac{L_{f_1}}{k_f(Wt)} + \frac{L_{s_2}}{k_s(Wt)} + \cdots$$

or $$\frac{L}{k_e(Wt)} = \frac{1}{Wt}\sum_{i=1}^{\infty}\left(\frac{L_{si}}{k_s} + \frac{L_{fi}}{k_f}\right)$$

or $$\frac{1}{k_e\big|_{min}} = \frac{1}{k_s}\sum_{i=1}^{\infty}\frac{L_{si}}{L} + \frac{1}{k_f}\sum_{i=1}^{\infty}\frac{L_{fi}}{L} = \frac{(1-\varepsilon)}{k_s} + \frac{\varepsilon}{k_f}$$

or $$k_e\big|_{min} = \left(\frac{1-\varepsilon}{k_s} + \frac{\varepsilon}{k_f}\right)^{-1}$$

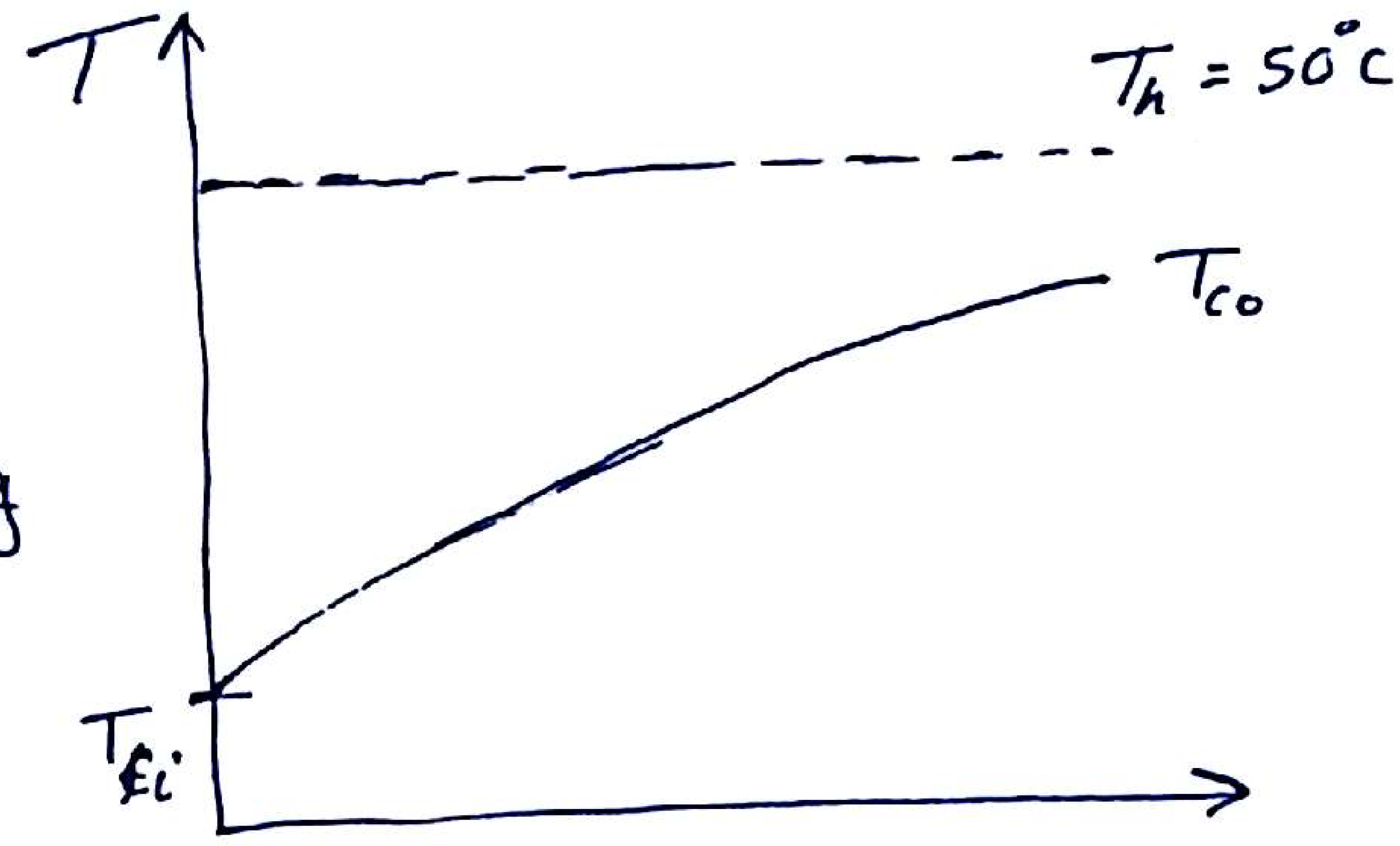
$$\therefore \boxed{\left(\frac{1-\varepsilon}{k_s} + \frac{\varepsilon}{k_f}\right)^{-1} \leq k_e \leq (1-\varepsilon)k_s + \varepsilon k_f}$$

5. $q = 200$ kW

$U = 1000\ W/m^2-K$

Since ammonia is condensing

T_h = constant



a) $\dot{m}_c = 2.39$ kg/s

$T_{ci} = 20°C$

$\varepsilon = 0.6$

For this scenario, $\dot{m}_h C_{p_h} \to \infty$ & $(\dot{m}C_p)_s = \dot{m}_c C_{p_c}$

$\Rightarrow \varepsilon = 1 - e^{-N}$ $\qquad N \to NTU = \dfrac{UA}{(\dot{m}C_p)_s}$

$\Rightarrow N = -\ln(1-0.6) = 0.916$

$\therefore A = \dfrac{(\dot{m}C_p)_s \cdot N}{U} = \dfrac{2.39 \times 4.18 \times 10^3}{10^3} \times 0.916\ m^2$

or $\boxed{A = 9.16\ m^2}$

b) If $\dot{m}_c$ is reduced to half, both q and T_{cout} will change. ~~NTU~~ =

$N = \dfrac{UA}{(\dot{m}C_p)_s} = \dfrac{1000 \times 9.16}{\frac{2.39}{2} \times 4.18 \times 10^3} = 1.832$

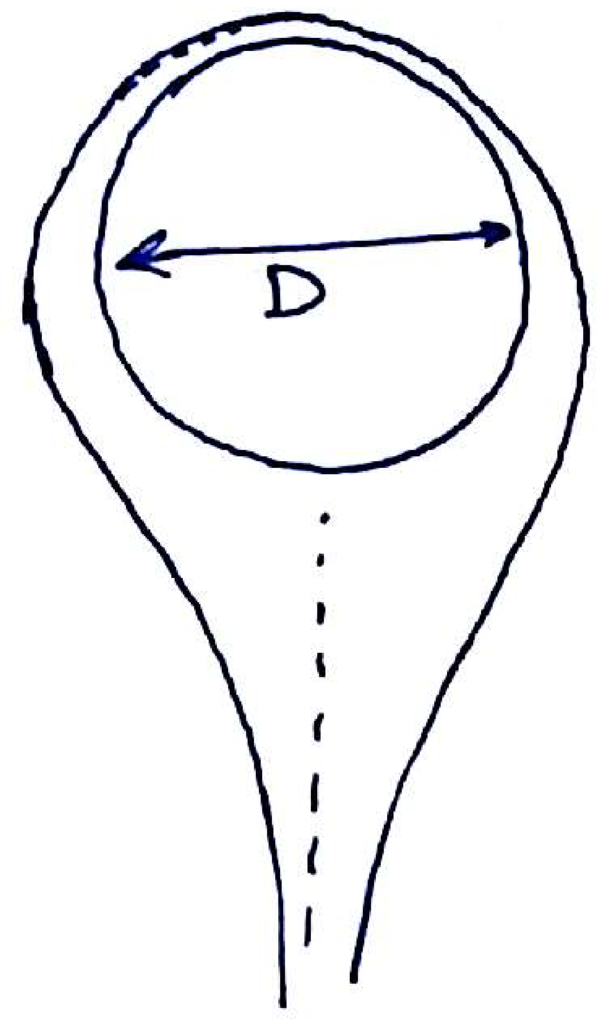
$\therefore \varepsilon = 1 - e^{-N} = 0.84$

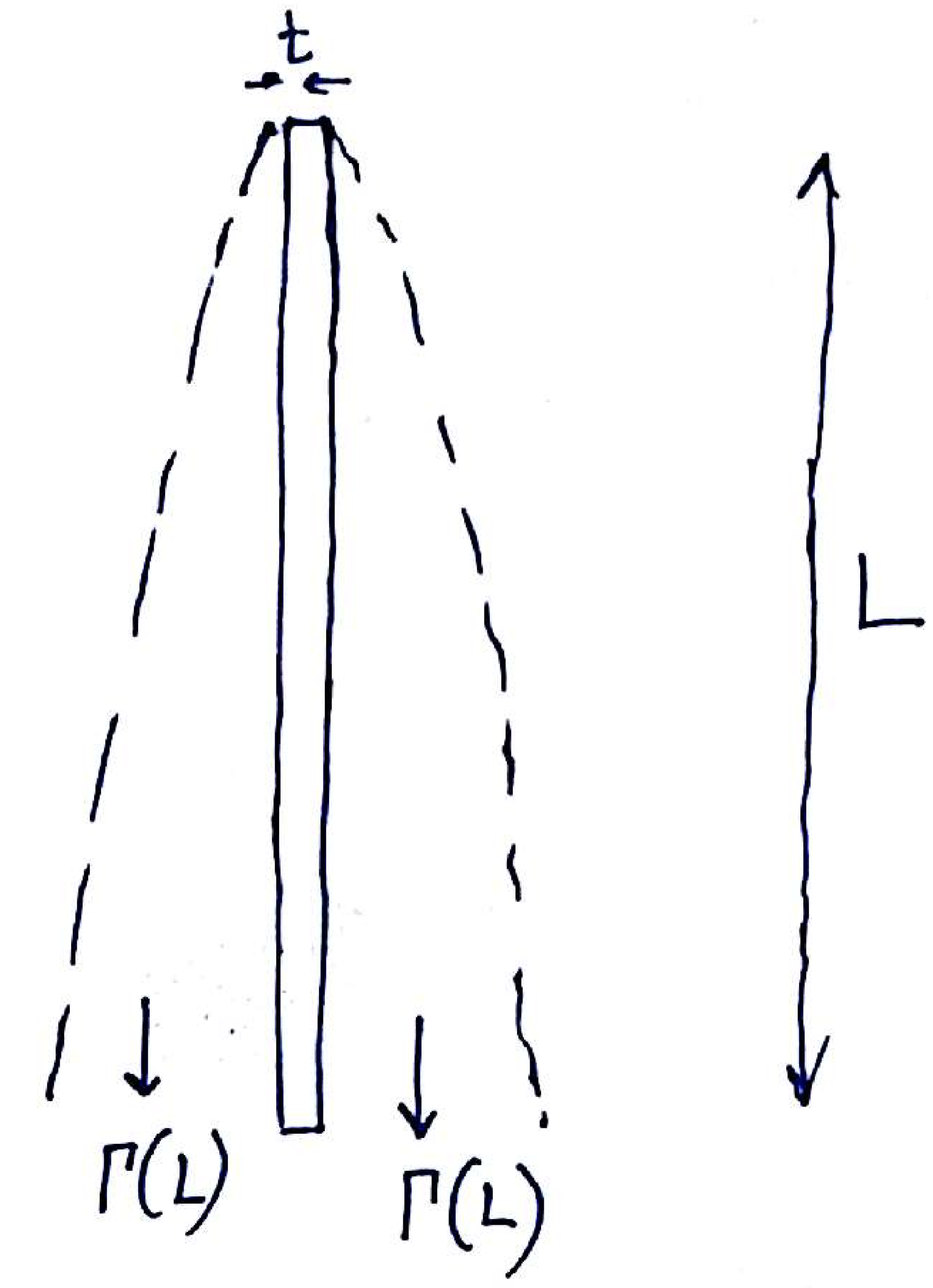
$\dfrac{q_{new}}{q_{old}} = \dfrac{(\dot{m}_c C_{p_c})_{new}(T_{co} - T_{ci})_{new}}{(\dot{m}_c C_{p_c})_{old}(T_{co} - T_{ci})_{old}} = \dfrac{(\dot{m}_c C_{p_c})_{new}}{(\dot{m}_c C_{p_c})_{old}} \times \dfrac{\varepsilon_{new}}{\varepsilon_{old}}$

or $q_{new} = 200 \times 0.5 \times \dfrac{0.84}{0.6}$ kW $= 140$ kW

or $\boxed{q_{new} = 140\ kW \Rightarrow 30\%\ \text{reduction}}$

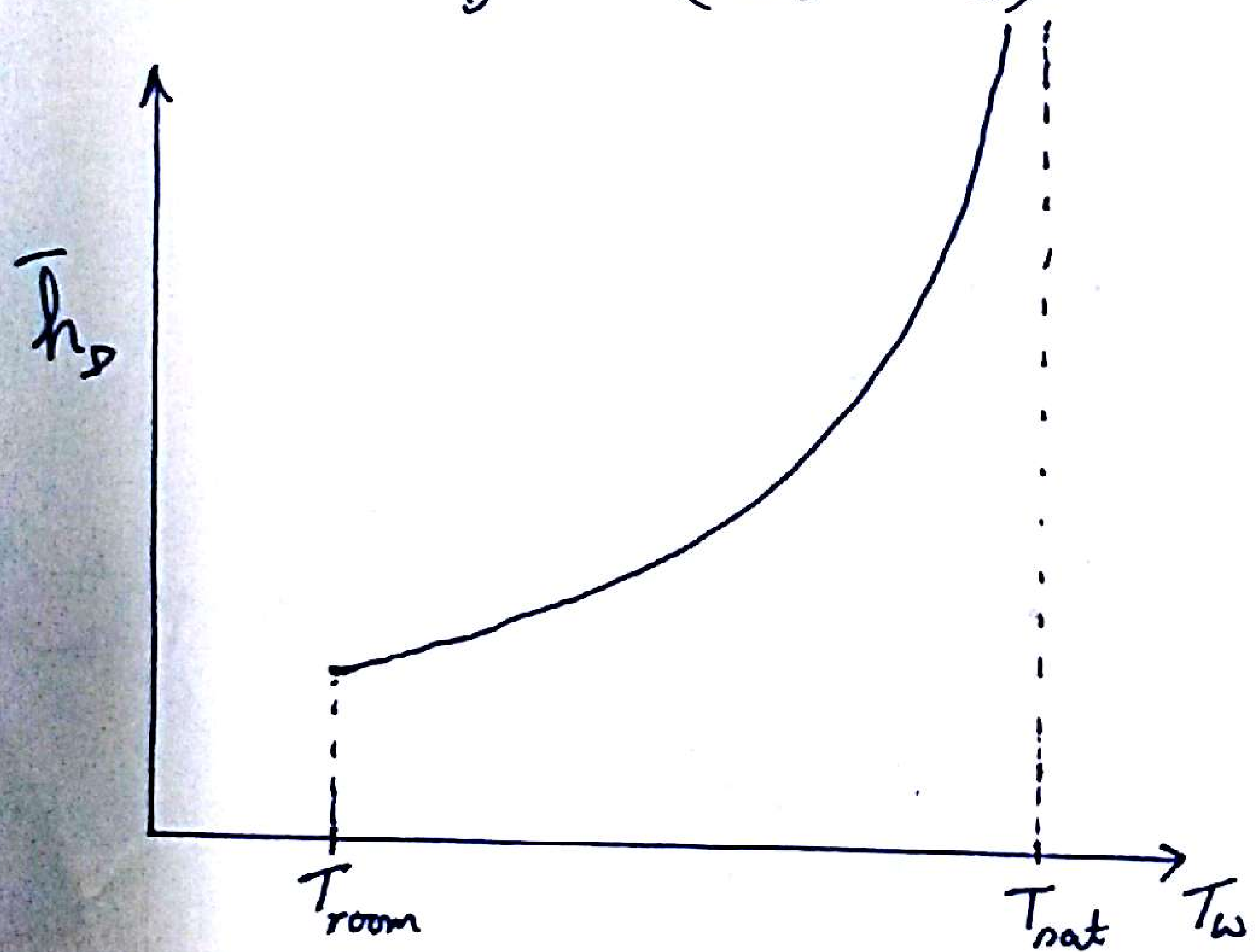
6.





i) $\bar{h}_D = Nu_D \cdot \frac{k_\ell}{D} = 0.729 \left[\frac{\rho_\ell g (\rho_\ell - \rho_v) h'_{fg} k_\ell^3}{\mu_\ell (T_{sat} - T_w) D} \right]^{1/4}$

or $\bar{h}_D \propto (T_{sat} - T_w)^{-1/4}$



As T_w increases from room temp. to T_{sat}

a) $\bar{h}_D$ increases following the dependence of $(T_{sat} - T_w)^{-1/4}$

b) ~~see~~ $\bar{h}_D \to \infty$ as $T_w \to T_{sat}$

→ reaches an asymptotic value of ∞ as T_w approaches T_{sat}

~~However as~~ This is because as $T_w \to T_{sat}$, there is no condensate film and hence any thermal resistance across it.

c) However, even though $\bar{h}_D \to \infty$,

$$q_s = \bar{h}_D A_s (T_{sat} - T_w) \propto (T_{sat} - T_w)^{3/4} \to 0$$

This is expected since there is no condensation if $T_w = T_{sat}$

b) If the tube is flattened, the total perimeter remains the same.

$$\Rightarrow \pi D = 2(L+t)$$

$$\simeq 2L \quad \text{assuming } t \ll L$$

$$\Rightarrow L = \frac{\pi D}{2}$$

If Γ is the condensation rate per unit width defined by $\Gamma = \frac{\dot{m}_l}{w\, h'_{fg}}$ where w = width/depth

$$\Gamma(D) = \frac{\bar{h}_D (T_{sat} - T_w)\, \pi D w}{w\, h'_{fg}}$$

$$= \overline{Nu}_D \frac{k_l}{D} \frac{\pi D}{h'_{fg}} (T_{sat} - T_w)$$

$$= \frac{\pi k_l \Delta T}{h'_{fg}} \overline{Nu}_D \quad \text{where } \Delta T = T_{sat} - T_w$$

$$\Gamma(L) = \bar{h}_L \Delta T . \frac{L w}{w\, h'_{fg}}$$

$$= \frac{k_l \Delta T}{h'_{fg}} \overline{Nu}_L$$

both sides of the plate is now available for condensation

$\therefore$ %age increase in condensation rate $= \frac{2\Gamma(L) - \Gamma(D)}{\Gamma(D)}$

$$= \frac{2\overline{Nu}_L - \pi \overline{Nu}_D}{\pi \overline{Nu}_D}$$

$$= \frac{2}{\pi} \frac{\overline{Nu}_L}{\overline{Nu}_D} - 1$$

$$= \frac{2}{\pi} \frac{0.943\, L^{3/4}}{0.729\, D^{3/4}} - 1$$

$$= \frac{2 \times 0.943}{\pi \times 0.729} \times \left(\frac{\pi D}{2D}\right)^{3/4} - 1 \quad \left[\because L = \frac{\pi D}{2}\right]$$

$$= 0.155$$

$\therefore$ Condensation rate increases by 15.5% (Ans)

Q.2

Given, Mass flow rate, $W = 0.06$ kg/s

$\rho_{air} = 1.13$ kg/m^3, $k_{air} = 0.03$ W/m-K, $C_p = 1005$ J/kg-K,

$\nu_{air} = 1.7 \times 10^{-5}$ m^2/s, $d = 5 \times 10^{-2}$ m

$$W = \rho A u$$

$$\therefore u = \frac{W}{\rho A}$$

$$\therefore Re = \frac{\rho u D}{\mu} = \frac{uD}{\nu} = \frac{W}{\rho A}\frac{D}{\nu} = \frac{0.06 \times 4}{1.13 \times \pi \times (5 \times 10^{-2})^2} \times \frac{5 \times 10^{-2}}{1.7 \times 10^{-5}}$$

$Re = 79536$ [Turbulent flow].

Circular and square tube have the same cross-sectional area.

$$\therefore \frac{\pi}{4} d^2 = a^2$$

$$\therefore a = \frac{d\sqrt{\pi}}{2} = \frac{5 \times 10^{-2}\sqrt{\pi}}{2} = 0.04431 \text{ m}.$$

$\therefore$ For square tube

$$Re = \frac{u_1 D_1}{\nu} = \frac{W}{\rho a^2} \times \frac{4a^2}{4a} \times \frac{1}{\nu} = \frac{0.06}{1.13 \times 0.04431} \times \frac{1}{1.7 \times 10^{-5}}$$

$Re \approx 70489$. [So, turbulent flow]

Dittus - Boelter correlation

$$\overline{Nu}_D = 0.023\, Re_L^{4/5}\, Pr^{0.3} \quad \text{(for heating)}$$

For circular tube

$$\overline{Nu} = 0.023\,(79536)^{4/5}\, Pr^{0.3} = 191.5\, Pr^{0.3}$$

For square tube

$$\overline{Nu} = 0.023\,(70489)^{0.8}\, Pr^{0.3} = 173.87\, Pr^{0.3}$$

Convective heat transfer co-efficients for circular and square tube are denoted by h_c and h_s respectively.

$$\therefore \frac{h_c}{h_s} = \frac{\left(\frac{h_c D}{k} \frac{k}{D}\right)}{\left(\frac{h_s D_h}{k} \frac{k}{D_h}\right)} = \frac{\overline{Nu_c}}{\overline{Nu_s}} \frac{a}{D}.$$

$$\therefore \frac{q_c}{q_s} = \frac{h_c \pi D}{h_s 4a} = \frac{\overline{Nu_c}}{\overline{Nu_s}} \frac{a}{D} \frac{\pi D}{4a} = \frac{\pi}{4} \frac{0.023 Re_c^{0.8} Pr^n}{0.023 Re_s^{0.8} Pr^n}$$

$$= \frac{\pi}{4}\left(\frac{Re_c}{Re_s}\right)^{0.8} = \frac{\pi}{4}\left(\frac{79536}{70489}\right)^{0.8} = 0.865$$

$$\therefore q_c = 0.865 q_s.$$

Q.3

Given,

$T_\infty = 4°C$, $P = 1\ atm$, $V = 0.3\ m/s$, $T_i = 20°C$,

$k_a = 0.03\ W/m\text{-}K$, $D = 5\times10^{-3}\ m$, $k_f = 0.8\ W/mK$

$$\therefore Re = \frac{VD}{\nu} = \frac{0.3\times5\times10^{-3}}{1.7\times10^{-5}} = 88.24$$

$$\therefore h = 5k\frac{Re^{1/3}}{D} = 5\times0.03\frac{(88.24)^{1/3}}{5\times10^{-3}} = 133.56$$

(a) $\therefore q = hA_s(T_i - T_\infty) = 133.56\times4\pi\left(\frac{5\times10^{-3}}{2}\right)^2(20-4)$

$= 0.168$ watt.

(b) At the fruit surface

$q_{cond} = q_{conv}$

$$\therefore -k_f\left.\frac{dT}{dr}\right|_{r=R} = h(T_s - T_\infty)$$

$$\therefore \left.\frac{dT}{dr}\right|_{r=R} = -\frac{h}{k_f}(20-4) = -\frac{133.56}{0.8}\times16 = -2671.2\ K/m.$$

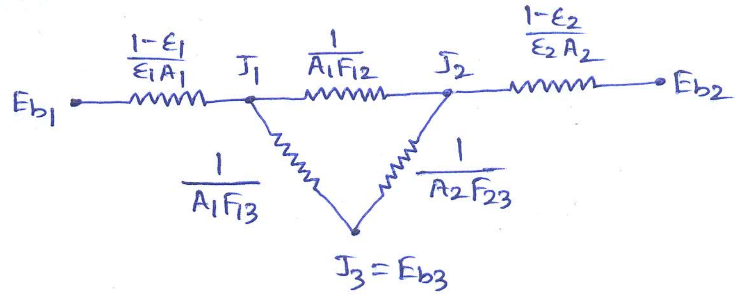
Characteristic length $(L_c) = \dfrac{\frac{4}{3}\pi r^3}{4\pi r^2} = \dfrac{r}{3} = \dfrac{D}{6}$

$\therefore \overline{Nu} = \dfrac{hL_c}{k_a} = \dfrac{133.56\times5\times10^{-3}}{6\times0.03} = 3.71$ (Based on characteristic length)

$\overline{Nu} = \dfrac{133.56\times5\times10^{-3}}{0.03} = 22.26$ $\left(\overline{Nu} = \dfrac{hD}{k}\right)$

Q.4

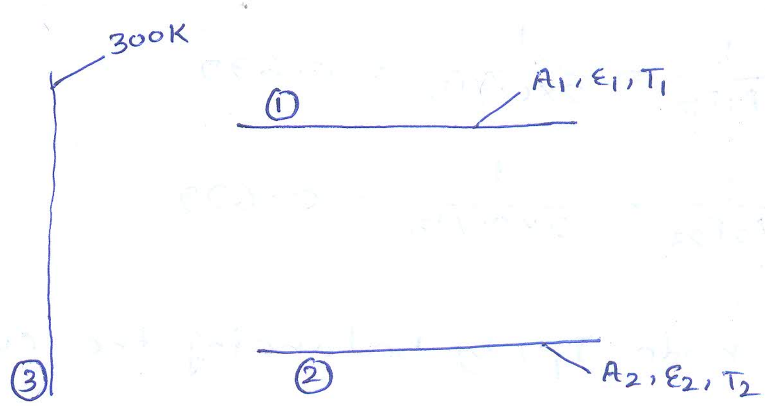
(a) Equivalent network



(b) calculation of shape factors

As the area of the room A_3 is very large, the resistance $\frac{1-\varepsilon_3}{\varepsilon_3 A_3}$ may be taken as zero.

Thus, $E_{b3} = J_3$



For plate (1), by summation rule

$$F_{11} + F_{12} + F_{13} = 1$$

$$F_{11} = 0$$

$$\therefore F_{13} = 1 - F_{12} = 0.715.$$

For plate (2),

$$F_{22} + F_{23} + F_{21} = 0$$

$$F_{22} = 0$$

$$\therefore F_{23} = 1 - F_{21} = 0.715$$

(c) Blackbody emissive power

$$E_{b1} = \sigma T_1^4 = 5.67 \times 10^{-8} \times (1500)^4 = 287.04 \text{ kW/m}^2$$

$$E_{b2} = \sigma T_2^4 = 5.67\times10^{-8}(1000)^4 = 56.7 \text{ kW/m}^2$$

$$E_{b3} = \sigma T_3^4 = 5.67\times10^{-8}(300)^4 = 0.46 \text{ kW/m}^2$$

Radiosity

$$J_3 = E_{b3} = 0.46 \text{ kW/m}^2$$

$$\frac{1-\varepsilon_1}{\varepsilon_1 A_1} = \frac{1-0.3}{0.3\times2} = 1.167$$

$$\frac{1-\varepsilon_2}{\varepsilon_2 A_2} = \frac{1-0.7}{0.7\times2} = 0.214$$

$$\frac{1}{A_1F_{12}} = \frac{1}{2\times0.285} = 1.754$$

$$\frac{1}{A_1F_{13}} = \frac{1}{2\times0.715} = 0.699$$

$$\frac{1}{A_2F_{23}} = \frac{1}{2\times0.715} = 0.699$$

At node J_1, by balancing the currents

$$\frac{E_{b1}-J_1}{1.167} = \frac{J_1-J_2}{1.754} + \frac{J_1-E_{b3}}{0.699} \quad \text{— (i)}$$

At node J_2,

$$\frac{E_{b2}-J_2}{0.214} = \frac{J_2-J_1}{1.754} + \frac{J_2-E_{b3}}{0.699} \quad \text{— (ii)}$$

E_{b1}, E_{b2} and E_{b3} are known.

Solving eqn. (i) and (ii) simultaneously for J_1 and J_2,

$$J_1 = 96 \text{ kW/m}^2$$

$$J_2 = 47.4 \text{ kW/m}^2$$

(d) Heat lost by the plate at 1500K

$$\dot{Q}_1 = \frac{E_{b1}-J_1}{\left(\frac{1-\varepsilon_1}{\varepsilon_1 A_1}\right)} = \frac{287.04-96}{1.167} = 163.7 \text{ kW.}$$