CORRELATIONS FOR CONVECTIVE HEAT TRANSFER

I. CORRELATIONS FOR FORECD CONVECTION

1. Forced convection from flat plate

Flow regime	Range of application	Correlation
Laminar, local	$T_w = \text{const}$, $\text{Re}_x < 5\times10^5$, $0.6 < \text{Pr} < 50$	$\text{Nu}_x = 0.322\,\text{Re}_x^{1/2}\,\text{Pr}^{1/3}$
Laminar, local	$T_w = \text{const}$, $\text{Re}_x < 5\times10^5$, $\text{Re}_x\,\text{Pr} > 100$	$\text{Nu}_x = \dfrac{0.3387\,\text{Re}_x^{1/2}\,\text{Pr}^{1/3}}{\left[1+\left(\dfrac{0.0468}{\text{Pr}}\right)^{2/3}\right]^{1/4}}$
Laminar, local	$q_w = \text{const}$, $\text{Re}_x < 5\times10^5$, $0.6 < \text{Pr} < 50$	$\text{Nu}_x = 0.453\,\text{Re}_x^{1/2}\,\text{Pr}^{1/3}$
Laminar, local	$q_w = \text{const}$, $\text{Re}_x < 5\times10^5$	$\text{Nu}_x = \dfrac{0.4637\,\text{Re}_x^{1/2}\,\text{Pr}^{1/3}}{\left[1+\left(\dfrac{0.0207}{\text{Pr}}\right)^{2/3}\right]^{1/4}}$
Laminar, average	$\text{Re}_L < 5\times10^5$, $T_w = \text{const}$	$\overline{\text{Nu}}_L = 2\,\text{Nu}_{x=L} = 0.664\,\text{Re}_L^{1/2}\,\text{Pr}^{1/3}$
Laminar, local	$T_w = \text{const}$, $\text{Re}_x < 5\times10^5$, $\text{Pr} \ll 1$ (liquid metals)	$\text{Nu}_x = 0.564\,\text{Re}_x^{1/3}\,\text{Pr}^{1/3}$
Laminar, local	$T_w = \text{const}$, starting at $x = x_0$, $Re_x < 5\times10^5$, $0.6 < \text{Pr} < 50$	$\text{Nu}_x = 0.332\,\text{Re}_x^{1/2}\,\text{Pr}^{1/3}\left[1-\left(\dfrac{x_0}{x}\right)^{3/4}\right]^{-1/3}$
Turbulent, local	$T_w = \text{const}$, $5\times10^5 < \text{Re}_x < 10^7$	$\text{St}_x\,\text{Pr}^{2/3} = 0.0296\,\text{Re}_x^{-0.2}$
Turbulent, local	$T_w = \text{const}$, $10^7 < \text{Re}_x < 10^9$	$\text{St}_x\,\text{Pr}^{2/3} = 0.185\left(\log \text{Re}_x\right)^{-2.584}$
Turbulent, local	$q_w = \text{const}$, $5\times10^5 < \text{Re}_x < 10^7$	$\text{Nu}_x = 1.04\,\text{Nu}_{x,T_w=\text{const}}$
Laminar-turbulent, average	$T_w = \text{const}$, $\text{Re}_x < 10^7$, $\text{Re}_{crit} = 5\times10^5$	$\overline{\text{St}}\,\text{Pr}^{2/3} = 0.037\,\text{Re}_L^{-0.2} - 871\,\text{Re}_L^{-1}$, $\overline{\text{Nu}}_L = \text{Pr}^{1/3}\left(0.037\,\text{Re}_L^{0.8} - 871\right)$
Laminar-turbulent, average	$T_w = \text{const}$, $\text{Re}_x < 10^7$, liquids, μ_∞ at T_∞ and μ_w at T_w	$\overline{\text{Nu}}_L = 0.036\,\text{Pr}^{0.43}\left(Re_L^{0.8} - 9200\right)\left(\dfrac{\mu_\infty}{\mu_w}\right)^{1/4}$
High-speed flow	$T_w = \text{const}$, $q = hA\left(T_w - T_\infty\right)$	Same as for low-speed flow with properties evaluated at $T^* = T_\infty + 0.5\left(T_w - T_\infty\right) + 0.22\left(T_{aw} - T_\infty\right)$

[Note: All properties are evaluated at $T_f = \left(T_\infty + T_w\right)/2$]

2. Boundary layer thickness correlations over flat plate

Flow regime	Range of application	Correlation
Laminar	$\mathrm{Re_x} < 5\times10^5$	$\dfrac{\delta}{x} = 5.0\,\mathrm{Re_x^{-1/2}}$
Turbulent	$\mathrm{Re_x} < 10^7$, $\delta = 0$ at $x = 0$	$\dfrac{\delta}{x} = 0.381\,\mathrm{Re_x^{-1/5}}$
Turbulent	$5\times10^5 < \mathrm{Re_x} < 10^7$, $\mathrm{Re_{crit}} = 5\times10^5$, $\delta = \delta_{lam}$ at $\mathrm{Re_{crita}}$	$\dfrac{\delta}{x} = 0.381\,\mathrm{Re_x^{-1/5}} - 10256\,\mathrm{Re_x^{-1}}$

3. Friction coefficient correlations over flat plate

Flow regime	Range of application	Correlation
Laminar, local	$\mathrm{Re_x} < 5\times10^5$	$C_{fx} = 0.332\,\mathrm{Re_x^{1/2}}$
Turbulent, local	$5\times10^5 < \mathrm{Re_x} < 10^7$	$C_{fx} = 0.0592\,\mathrm{Re_x^{-1/5}}$
Turbulent, local	$10^7 < \mathrm{Re_x} < 10^9$	$C_{fx} = 0.37\left(\log \mathrm{Re_x}\right)^{-2.584}$
Turbulent, average	$10^9 < \mathrm{Re_x} < \mathrm{Re_{crit}}$	$\bar{C}_{fx} = \dfrac{0.455}{\left(\log \mathrm{Re_L}\right)^{2.854}} - \dfrac{A}{\mathrm{Re_L}}$

where A can be obtained from the following table

$\mathrm{Re_{crit}}$	3×10^5	5×10^5	10^6	3×10^6
A	1055	1742	3340	8940

4. Nusselt number correlations in forced convection heat transfer over flat plate with unheated starting length

$$\mathrm{Nu_x} = \frac{\mathrm{Nu_x}\big|_{x_0=0}}{\left[1-\left(\dfrac{x_0}{x}\right)^a\right]^b}, \text{ where } \mathrm{Nu_x}\big|_{x_0=0} = C\,\mathrm{Re}_x^m\,\mathrm{Pr}^{1/3}$$

Values of a, b, C and m can be obtained from the following table

	Laminar, local		Turbulent, local	
	$T_w = \text{const}$	$q_w = \text{const}$	$T_w = \text{const}$	$q_w = \text{const}$
a	3/4	3/4	9/10	9/10
b	1/3	1/3	1/9	1/9
C	0.332	0.453	0.0296	0.0308
m	1/2	1/2	4/5	4/5

5. Nusselt number correlations in forced convection heat transfer over sphere and cylinder

Average Nusselt number for forced convection over an isothermal sphere:

$$\overline{\mathrm{Nu}}_{\mathrm{D}} = 2 + \left(0.4\mathrm{Re}_{\mathrm{D}}^{1/2} + 0.06\mathrm{Re}_{\mathrm{D}}^{2/3}\right)\mathrm{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_S}\right)^{1/4},$$

which is valid for $3.5 \le \mathrm{Re}_{\mathrm{D}} \le 80000$ and $0.7 \le \mathrm{Pr} \le 380$. The fluid properties in this case are evaluated at the free-stream temperature T_∞, except for μ_S which is evaluated at the surface temperature T_S.

Average Nusselt number for cross flow over an isothermal cylinder:

$$\overline{\mathrm{Nu}}_{\mathrm{D}} = 0.3 + \frac{0.62\,\mathrm{Re}_{\mathrm{D}}^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1+\left(\frac{0.4}{\mathrm{Pr}}\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re}_{\mathrm{D}}}{282000}\right)^{5/8}\right]^{4/5},$$

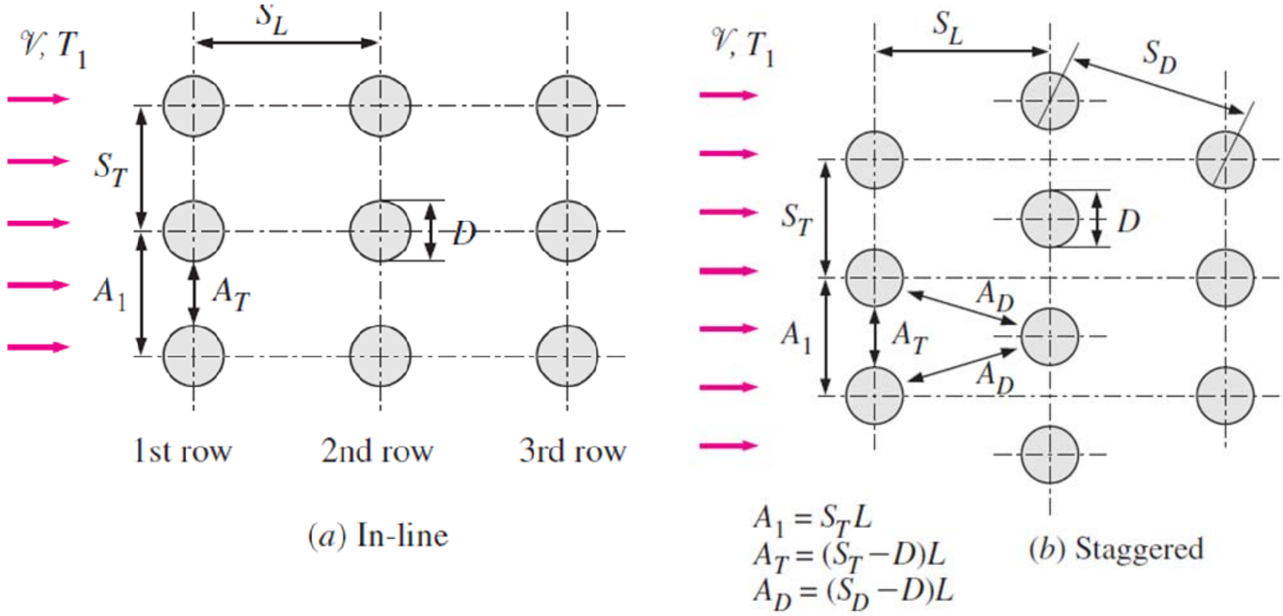
which is valid for $\mathrm{Re}_{\mathrm{D}}\,\mathrm{Pr} > 0.2$. The fluid properties are evaluated at the film temperature $T_f = \left(T_\infty - T_S\right)/2$.

Average Nusselt number for cross flow over isothermal tube banks:

$$\overline{\mathrm{Nu}}_{\mathrm{D}} = C\,\mathrm{Re}_{\mathrm{D}}^{\mathrm{m}}\,\mathrm{Pr}^{n}\left(\frac{\mathrm{Pr}}{\mathrm{Pr}_s}\right)^{1/4},$$

which is valid for $0 < \mathrm{Re}_{\mathrm{D}} < 2\times10^{6}$ and $0.7 < \mathrm{Pr} < 500$. The fluid properties in this case are evaluated at the free-stream temperature T_∞, except for Pr_s which is evaluated at temperature $T_m = (T_i + T_e)/2$, where T_i and T_e are temperature of the fluid at the inlet and outlet of the tube bank, respectively.

Typical arrangement of tubes in a tube bank is depicted in the schematic below.



Schematic representation of the arrangement of the tubes in in-line and staggered tube banks (A_1, A_T, and A_D are flow areas at indicated locations, and L is the length of the tubes).

Values of C, m and n can be obtained from the following table (valid when number of tubes is greater than 16)

Arrangement	**Range of Re_D**	**Correlation**
In-line	$0-100$	$\overline{Nu}_D = 0.9\, Re_D^{0.4}\, Pr^{0.36}\left(\frac{Pr}{Pr_s}\right)^{1/4}$
	$100-1000$	$\overline{Nu}_D = 0.52\, Re_D^{0.5}\, Pr^{0.36}\left(\frac{Pr}{Pr_s}\right)^{1/4}$
	$1000-2\times10^5$	$\overline{Nu}_D = 0.27\, Re_D^{0.63}\, Pr^{0.36}\left(\frac{Pr}{Pr_s}\right)^{1/4}$
	$2\times10^5-2\times10^6$	$\overline{Nu}_D = 0.033\, Re_D^{0.8}\, Pr^{0.4}\left(\frac{Pr}{Pr_s}\right)^{1/4}$
Staggered	$0-500$	$\overline{Nu}_D = 1.04\, Re_D^{0.4}\, Pr^{0.36}\left(\frac{Pr}{Pr_s}\right)^{1/4}$
	$500-1000$	$\overline{Nu}_D = 0.71\, Re_D^{0.5}\, Pr^{0.36}\left(\frac{Pr}{Pr_s}\right)^{1/4}$
	$1000-2\times10^5$	$\overline{Nu}_D = 0.35\left(\frac{S_T}{S_L}\right)^{0.2} Re_D^{0.6}\, Pr^{0.36}\left(\frac{Pr}{Pr_s}\right)^{1/4}$
	$2\times10^5-2\times10^6$	$\overline{Nu}_D = 0.031\left(\frac{S_T}{S_L}\right)^{0.2} Re_D^{0.8}\, Pr^{0.36}\left(\frac{Pr}{Pr_s}\right)^{1/4}$

6. Nusselt number correlations in forced convection heat transfer inside circular pipes: case of hydrodynamically and thermally fully developed flow

Laminar flow with isothermal wall: $\overline{Nu}_D = 3.66$

Laminar flow with isoflux wall: $\overline{Nu}_D = 4.36$

For every other geometry, separate analysis to be made but $\overline{\mathrm{Nu}}_{\mathrm{D}} = \mathrm{const}$.

Turbulent flow:

(i) Dittus-Boelter correlation for smooth wall $\left(\mathrm{Re}_{\mathrm{D}} > 10000\right)$: $\overline{\mathrm{Nu}}_{\mathrm{D}} = 0.023\,\mathrm{Re}_{\mathrm{L}}^{4/5}\mathrm{Pr}^{\mathrm{n}}$ ($n = 0.3$ for heated wall; $n = 0.4$ for cold wall),

(ii) Gnielinski correlation $\left(\mathrm{Re}_D > 3000\right)$: $\overline{\mathrm{Nu}}_{\mathrm{D}} = \dfrac{(f/8)\left(\mathrm{Re}_D - 1000\right)\mathrm{Pr}}{1 + 12.7(f/8)^{1/2}\left(\mathrm{Pr}^{2/3} - 1\right)}$ (f for smooth surface can be obtained as $f = \left(0.790\,(\log \mathrm{Re}_D) - 1.64\right)^{-2}$, while for rough surface look into Moody chart)

Above results may be used for other geometries by replacing diameter by hydraulic diameter.

7. Nusselt number correlations in forced convection heat transfer inside circular pipes: Developing region

Laminar flow (isothermal wall):

(i) Combined entry length:

$$\overline{Nu}_D = 1.86\left(\frac{\mathrm{Re}_D\,\mathrm{Pr}}{L/D}\right)^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14} \quad \text{for } \left[\mathrm{Re}_D\,\mathrm{Pr}/(L/D)\right]^{1/3}(\mu/\mu_s)^{0.14} > 2,$$

and $\overline{Nu}_D = 3.66$ for $\left[\mathrm{Re}_D\,\mathrm{Pr}/(L/D)\right]^{1/3}(\mu/\mu_s)^{0.14} < 2$

(ii) Thermal entry length: $\overline{Nu}_D = 3.66 + \dfrac{0.0668(D/L)\mathrm{Re}_D\,\mathrm{Pr}}{1 + 0.04\left[(D/L)\mathrm{Re}_D\,\mathrm{Pr}\right]^{2/3}}$

Turbulent flow (isothermal wall):

(i) Long tubes with $\dfrac{L}{D} > 60$: $\overline{Nu}_D \approx Nu_{D,fd}$

(ii) Short tubes with $\dfrac{L}{D} < 60$: $\dfrac{\overline{Nu}_D}{Nu_{D,fd}} \approx 1 + \dfrac{C}{(L/D)^m}$ where $C \approx 1$ and $m \approx 2/3$

8. Nusselt numbers for fully developed laminar flow in concentric tube annuals

Nusselt number for fully developed laminar flow in a circular tube annulus (of inner diameter D_i and outer diameter D_o) with one surface insulated and the other at constant temperature

D_i/D_o	$\overline{\mathrm{Nu}}_{\mathrm{Di}}$	$\overline{\mathrm{Nu}}_{\mathrm{Do}}$
0	-	3.66
0.05	17.46	4.06
0.1	11.56	4.11
0.258	7.37	4.23
0.5	5.74	4.43
≈ 1	4.86	4.86

II. CORRELATIONS FOR NATURAL CONVECTION

1. Nusselt number correlations in natural convection heat transfer over isothermally heated flat plate

Vertical flat plate:

For $\Pr > 0.6$, $\Pr > 0.6$: $\delta = 5x\left(\dfrac{Gr_x}{4}\right)^{-1/4} = 7.07\dfrac{x}{\left(Gr_x\right)^{1/4}} \propto x^{1/4}$

(i) Laminar flow $\left(Ra_L < 10^9\right)$: $\overline{Nu}_L = 0.68 + \dfrac{0.670\ Ra_L^{1/4}}{\left[1+\left(0.492/\Pr\right)^{9/16}\right]^{4/9}}$

(ii) Turbulent flow $\left(10^9 < Ra_L < 10^{12}\right)$: $\overline{Nu}_L = \left\{0.825 + \dfrac{0.387\ Ra_L^{1/6}}{\left[1+\left(0.492/\Pr\right)^{9/16}\right]^{4/9}}\right\}^2$

Vertical flat plate:

(i) Facing up:

$$\overline{Nu}_L = 0.54\, Ra_L^{1/4} \text{ for } 10^4 < Ra_L < 10^7$$

$$\overline{Nu}_L = 0.15\, Ra_L^{1/3} \text{ for } 10^7 < Ra_L < 10^{11}$$

(ii) Facing down:

$$\overline{Nu}_L = 0.27\, Ra_L^{1/4} \text{ for } 10^5 < Ra_L < 10^{10}$$

2. Nusselt number correlations in natural convection heat transfer over isothermally heated sphere and cylinder

Sphere: $$\overline{Nu}_D = 2 + \frac{0.589\ Ra_D^{1/4}}{\left[1 + \left(0.469/\Pr\right)^{9/16}\right]^{4/9}}$$

Long cylinder: $$\overline{Nu}_D = \left\{0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1 + \left(0.559/\Pr\right)^{9/16}\right]^{8/27}}\right\}^2 \quad \text{for} \quad Ra_D < 10^{12}$$

Over the years it has been found that average Nusselt number can be represented in the following functional form for a variety of circumstances:

$$\overline{Nu} = C\left(Gr_f \Pr_f\right)^m = C\ Ra_f^m ,$$

where the subscript f indicates that the properties in the dimensionless groups are evaluated at the film temperature $T_f = \left(T_\infty + T_w\right)/2$.

Values of C and m can be obtained from the following table

Geometry	$Gr_f\ Pr_f$	C	m
Vertical planes and cylinders	$10^{-1}-10^{4}$	Use Fig. 1	Use Fig. 1
	$10^{4}-10^{9}$	0.59	1/4
	$10^{9}-10^{13}$	0.021	2/5
	$10^{9}-10^{13}$	0.1	1/3
Horizontal cylinders	$0-10^{-5}$	0.4	0
	$10^{-5}-10^{4}$	Use Fig. 2	Use Fig. 2
	$10^{4}-10^{9}$	0.53	1/4
	$10^{9}-10^{12}$	0.13	1/3
	$10^{-10}-10^{-2}$	0.675	0.058
	$10^{-2}-10^{2}$	1.02	0.148
	$10^{2}-10^{4}$	0.850	0.188
	$10^{4}-10^{7}$	0.480	1/4
	$10^{7}-10^{12}$	0.125	1/3

Fig. 1:

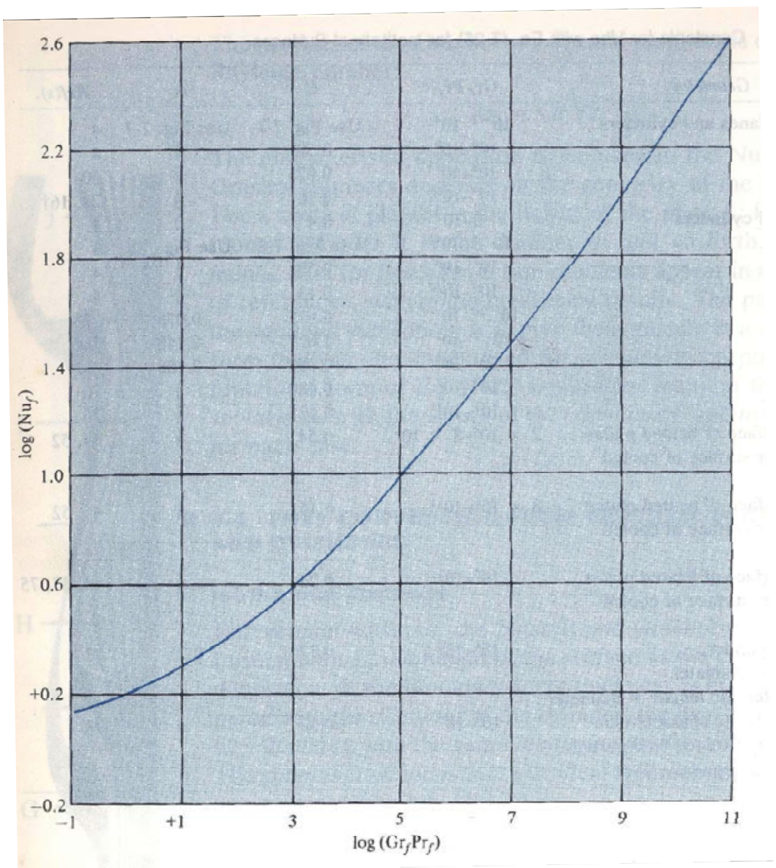
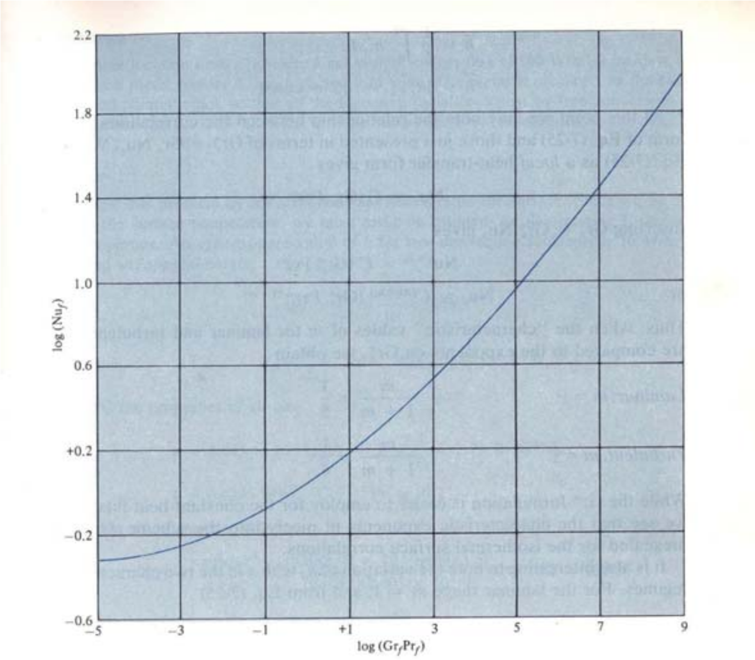


Fig. 2:



3. Nusselt number correlations in natural convection heat transfer inside enclosures

Average Nusselt number for a horizontal cavity (two of the opposing walls are maintained at different temperatures T_1 and T_2, while other walls are thermally insulated from surroundings) when heated from below:

$$\overline{Nu}_L = 0.069 Ra_L^{1/3} \Pr^{0.074} \quad \text{for} \quad 3\times10^5 \le Ra_L \le 7\times10^9,$$

where all properties are evaluated at the average temperature $T_{av} = \left(T_1 + T_2\right)/2$.