AR TUTORIAL 3 - Representation and Reasoning

EXERCISE ONE: Express on logic the following statements. Use the relations on (x, ℓ) to denote that a point x is on line ℓ or, equivalently, that line ℓ is on point x.

1. On any two distinct points there is always a line.

$$\forall a, b \ (a \neq b) \wedge [\exists \ell . \text{on}(a, \ell) \wedge \text{on}(b, \ell)] \quad \text{or}$$
$$\forall a, b . \ a \neq b \rightarrow \exists \ell . (\text{on}(a, \ell) \wedge \text{on}(b, \ell))$$

2. On any two distinct points there is not more than one line.

$$\forall \ell, m . \ \forall a, b . \ a \neq b \wedge \text{on}(a, \ell) \wedge \text{on}(b, \ell) \wedge \text{on}(a, m) \wedge \text{on}(b, m) \Rightarrow \ell = m$$

3. Every line has at least two distinct points.

$$\forall \ell . \ \exists a, b . \ a \neq b \wedge \text{on}(a, \ell) \wedge \text{on}(b, \ell)$$

4. There exist at least three points not lying on the same line.

$$\exists a, b, c . \ a \neq b \wedge b \neq c \wedge a \neq c \wedge \neg(\exists \ell . \text{on}(a, \ell) \wedge \text{on}(b, \ell) \wedge \text{on}(c, \ell))$$

EXERCISE 2: Formalize exercise 1 in Isabelle.

EXERCISE 3: Formalize and prove the following in Isabelle.

1. Not all points lie on the same line.

Contradiction: there exists a point not on the line. (a, b, c)

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lemma exists-a-point_not_on_line: "∃x. ¬on x ℓ"
    proof -
        obtain a b c where line: "¬(on a ℓ ∧ on b ℓ ∧ on c ℓ)"
            using three-points-not-on-line by blast
        thus ?thesis by blast
qed
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* For the rest, check theory files