AR TUTORIAL 1 - Propositional Logic

EXERCISE ONE: represent the following sentences in propositional logic and draw truth tables.

1. Cats chase mice or birds, but not at the same time.

$(M \vee B) \wedge \neg(M \wedge B)$ where M = mice and B = birds

M	B	$M \vee B$	$\neg(M \wedge B)$	$(M \vee B) \wedge \neg(M \wedge B)$
T	T	T	F	F
T	F	T	T	T
F	T	T	T	T
F	F	F	T	F

2. If it rains, the beach will be empty.

$R \rightarrow E$ where R = rains and E = empty

R	E	$R \rightarrow E$
T	T	T
F	T	T
T	F	F
F	F	T

* $R \rightarrow E \equiv \neg R \vee E$

3. If Jane bought a piano today, she either sold her old one or took out bank loan.

$P \rightarrow S \vee B$ where P = buy piano, S = sold old one and B = bank loan

P	S	B	$P \rightarrow S \vee B$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

* $P \rightarrow S \vee B$
$\equiv \neg P \vee (S \vee B)$

EXERCISE TWO: Is the proposition $P \wedge (P \to Q)$ satisfiable? If so, give an interpretation that satisfies it. Is it valid? Why or why not?

$$P \wedge (P \to Q) \equiv P \wedge (\neg P \vee Q) \equiv (P \wedge \neg P) \vee (P \wedge Q)$$

* SATISFIABLE: if there is some interpretation which evaluates to TRUE.
  $P \wedge (P \to Q)$ is TRUE when P is TRUE and Q is TRUE.

* VALID: if all interpretations evaluate to true.
  $P \wedge (P \to Q)$ is FALSE when P is FALSE or when Q is FALSE.

EXERCISE THREE: Truth table for NAND

P	q	$P \mid q$
T	T	F
T	F	T
F	T	T
F	F	T

$$\neg(P \wedge Q) \to \neg P \vee \neg Q$$

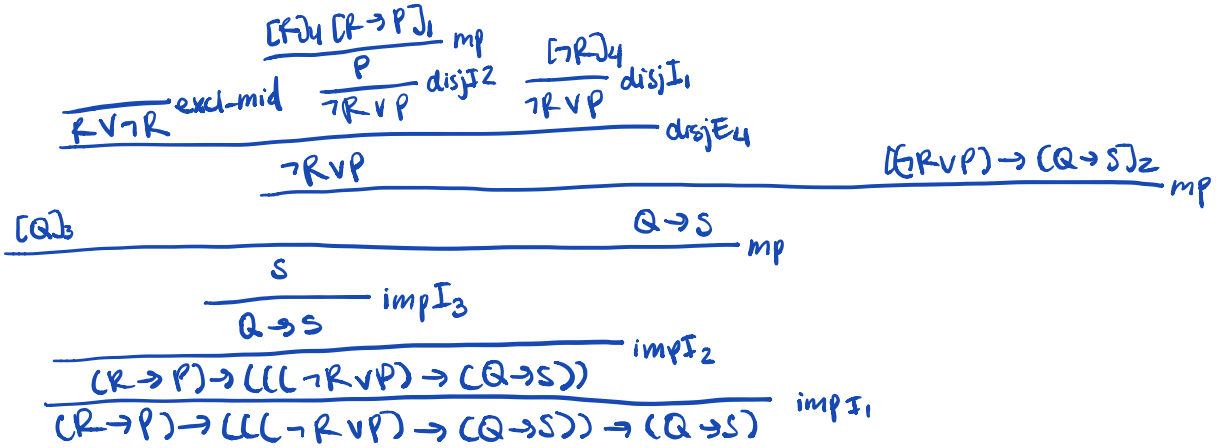
Show that NAND can be used to define $\neg$, $\wedge$, $\vee$, and $\to$

Connective	Expression using $\mid$
$\neg P$	$P \mid P$
$P \wedge Q$	$(P \mid Q) \mid (P \mid Q)$
$P \vee Q$	$(P \mid P) \mid (Q \mid Q)$
$P \to Q$	$P \mid (Q \mid Q)$

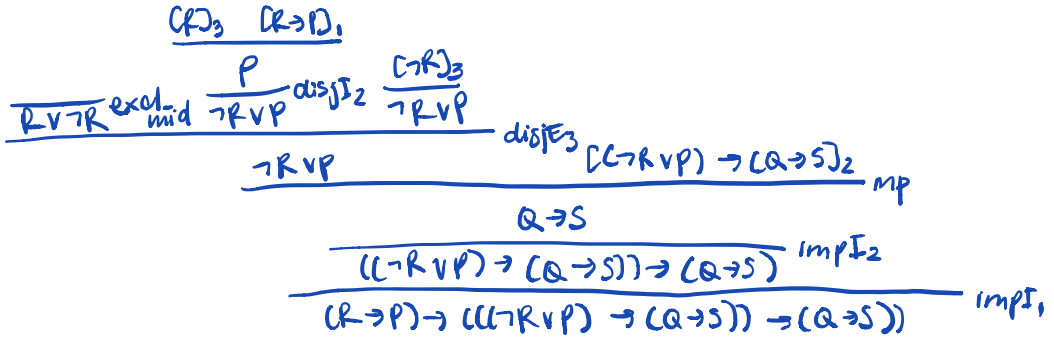
Some notes:
- $P \wedge Q \equiv \neg(P \mid Q)$
- $P \vee Q \equiv \neg(\neg P \wedge \neg Q)$
- $P \to Q \equiv \neg P \vee Q$

EXERCISE FOUR: Use Natural Deduction to give a tree representation) proof of the theorem: $(R \to P) \to (((\neg R \vee P) \to (Q \to S)) \to (Q \to S))$

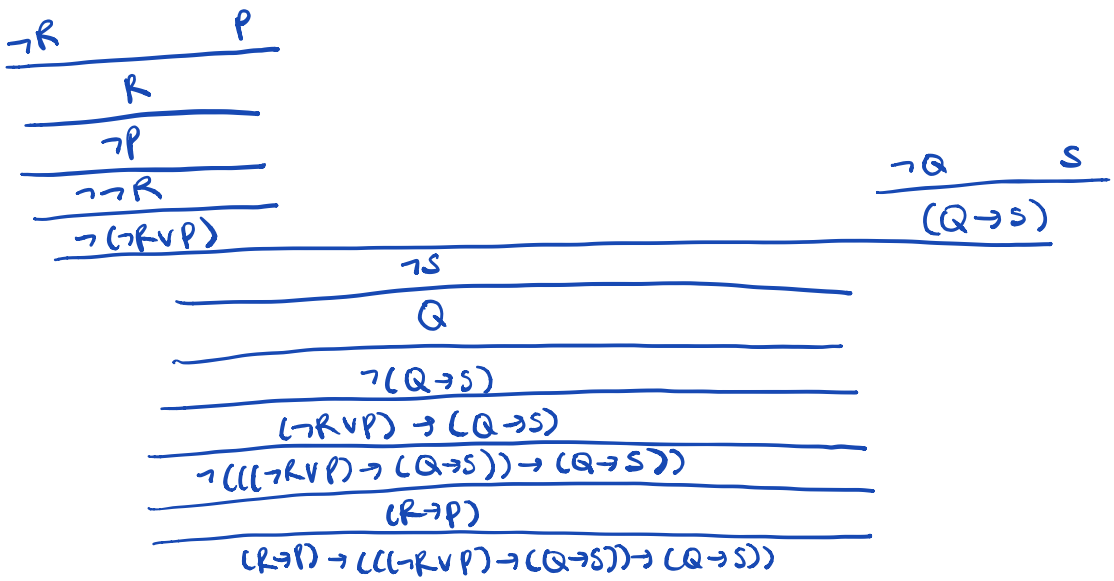
Version 1:



Version 2:

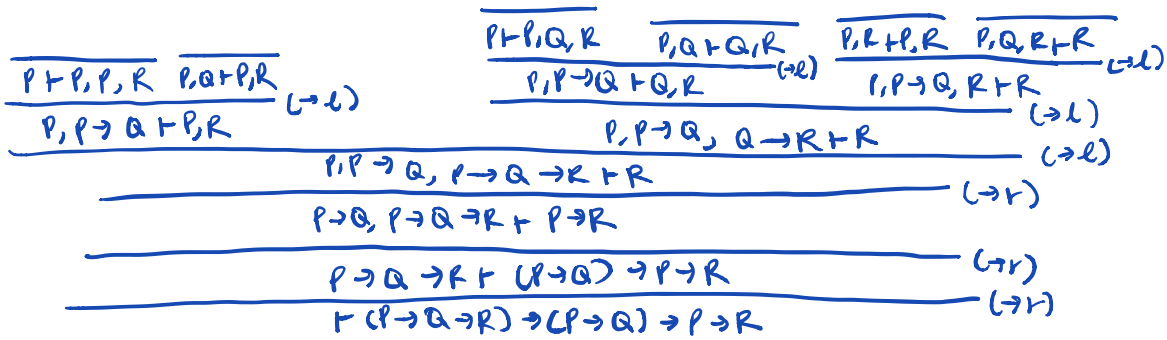


Version 3:



EXERCISE FIVE: Draw Proof trees for the following.

1. $(P \to (Q \to R)) \to ((P \to Q) \to (P \to R))$



2. $\neg\neg P \to P$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{P \vdash P}}{\vdash \neg P, P}(\neg r)}{\neg\neg P \vdash P}(\neg l)}{\vdash \neg\neg P \to P}(\to r)$$

3. $(P \to Q \wedge R) \to ((P \to Q) \wedge (P \to R))$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{P \vdash P, Q}\quad\dfrac{\dfrac{}{P, Q, R \vdash Q}}{P, Q \wedge R \vdash Q}(\wedge l)}{P, P \to Q \wedge R \vdash Q}(\to l)}{P \to Q \wedge R \vdash P \to Q}(\to r)\quad\dfrac{\dfrac{\dfrac{}{P \vdash P, R}\quad\dfrac{\dfrac{}{P, Q, R \vdash R}}{P, Q \wedge R \vdash R}(\wedge l)}{P, P \to Q \wedge R \vdash R}(\to l)}{P \to Q \wedge R \vdash P \to R}(\to r)}{P \to Q \wedge R \vdash (P \to Q) \wedge (P \to R)}(\wedge r)}{\vdash (P \to Q \wedge R) \to (P \to Q) \wedge (P \to R)}(\to r)$$

4. $(\neg P \to Q) \to (\neg Q \to P)$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P, \neg Q \vdash P}}{\neg Q \vdash \neg P, P}(\neg r)}{\vdash \neg P, \neg Q \to P}(\to r)\quad\dfrac{\dfrac{\dfrac{}{Q \vdash Q, P}}{\neg Q, Q \vdash P}(\neg l)}{Q \vdash \neg Q \to P}(\to r)}{\neg P \to Q \vdash \neg Q \to P}(\to l)}{\vdash (\neg P \to Q) \to \neg Q \to P}(\to r)$$

5. $P \vee \neg P$

$$\dfrac{\dfrac{\dfrac{}{P \vdash P}}{\vdash P, \neg P}(\neg r)}{\vdash P \vee \neg P}(\vee r)$$