# AR TUTORIAL 5 - Unification and Rewrite Rules

EXERCISE ONE: Apply one way unification (matching), outline behaviour, determine success/failure. If success, determine substitution.

1. Pattern: $x = x$ Target: $2 = 2$

$x = x \quad 2 = 2$

$\downarrow$ decompose

$(x \equiv 2) \wedge (x \equiv 2)$

$\downarrow$ eliminate

$(2 \equiv 2) \wedge (x \equiv 2)$

$\downarrow$ delete

$x \equiv 2$

Success; substitution: $x = 2$

2. Pattern: $x = x$ Target: $2+2=4$

$x = x \quad 2+2=4$

$\downarrow$ decompose

$(x = 2+2) \wedge (x = 4)$

Fail.

3. Pattern: $p(f(x,y), y)$ Target: $p(f(a, g(b)), g(b))$

$p(f(x,y), y) \quad p(f(a, g(b)), g(b))$

$\downarrow$ decompose

$x \equiv a \wedge y \equiv g(b) \wedge y \equiv g(b)$

$\downarrow$ decompose and eliminate

$x \equiv a \wedge g(b) \equiv g(b) \wedge y \equiv g(b)$

$\downarrow$ delete

$x \equiv a \wedge y \equiv g(b)$

Success; substitution: $x = a$ and $y = g(b)$

4. Pattern: $x = b$ Target: $a = y$

Fail; target contains a variable $\rightarrow a = y$

EXERCISE TWO: Apply two-way unification, outline behaviour, determine success/failure. If success, determine substitution.

1. Pattern: $X=b$ Target: $a=Y$

$X=b \quad a=Y$

$\downarrow$ decompose

$(X \equiv a) \wedge (b \equiv Y)$

$\downarrow$ switch

$(X \equiv a) \wedge (Y \equiv b)$ $\longrightarrow$ you can only match variables on LHS!

success; substitution: $X=a$ and $Y=b$

2. Pattern: $X=b$ Target: $Y=a$

$X=b \quad Y=a$

$\downarrow$

$(X \equiv Y) \wedge (b \equiv a)$

Fails; can't match variable with variable $(X \not\equiv Y)$

3. Pattern: $p(X,a)$ Target: $p(f(Y),Y)$

$p(X,a) \quad p(f(Y),Y)$

$\downarrow$ decompose

$(X \equiv f(Y)) \wedge (a \equiv Y)$

$\downarrow$ switch

$(X \equiv f(Y)) \wedge (Y \equiv a)$

$\downarrow$ eliminate

$(X \equiv f(a)) \wedge (Y \equiv a)$

success; substitution: $X=f(a)$ and $Y=a$

4. Pattern: $p(X,g(X))$ Target: $p(f(Y),Y)$

$p(X,g(X)) \quad p(f(Y),Y)$

$\downarrow$ decompose

$(X \equiv f(Y)) \wedge (g(X) \equiv Y)$

$\downarrow$ switch

$(X \equiv f(Y)) \wedge (Y \equiv g(X))$

$\downarrow$ eliminate

$(X \equiv f(Y)) \wedge (Y \equiv g(f(Y)))$

Fails; occurs check $\rightarrow$ causes a unification of a variable V and a structure S to fail if S contains V.

$\rightarrow$ happens if variables are on both sides.

e.g. $X = X + a$

5. Pattern: $(a+X)+b$ Target: $a+Y$

$$(a+X)+b \qquad a+Y$$
$$\downarrow \quad \text{decompose}$$
$$((a+X)\equiv a) \wedge (b\equiv Y)$$
$$\downarrow \quad \text{switch}$$
$$(a\equiv(a+X)) \wedge (Y\equiv b)$$

Fails; conflict.

EXERCISE THREE: Consider the following pair of terms, where $f, g, h$ are function symbols, $a$ is a constant, and $X$ and $Y$ are variables.

$$h(g(X, f(Y,a)), f(a,Y)) \text{ and } h(f(a,a), X)$$

What non-trivial property $R$ should the function $g$ have so that $R$ can be built into the unification algorithm to enable the two terms to unify?

$$h(g(X, f(Y,a)), f(a,Y)) \qquad h(f(a,a), X)$$
$$\downarrow \quad \text{decompose}$$
$$g(X,f(Y,a)) \equiv f(a,a) \wedge f(a,Y) \equiv X$$
$$\downarrow \quad \text{switch}$$
$$g(X,f(Y,a)) \equiv f(a,a) \wedge X \equiv f(a,Y)$$
$$\downarrow \quad \text{eliminate}$$
$$g(f(a,Y), f(Y,a)) \equiv f(a,a) \wedge X \equiv f(a,Y)$$
$$\downarrow \quad g \text{ property}$$
$$f(a,Y) \equiv f(a,a) \wedge X \equiv f(a,Y)$$
$$\downarrow \quad \text{delete}$$
$$g(X, f(Y,a)) \equiv X$$
$$\downarrow \quad g \text{ property}$$
$$g(X,Y) \equiv X$$

* A suitable property is that $g(X,Y) = X$, for all $X$.

Adding this to the unification algorithm means that the two terms given can unify with the following substitutions:

$$X = f(a,a) \qquad Y = a$$

this can be shown by performing the substitutions on both terms and applying the property of $g$.

**EXERCISE FOUR:** Consider the following rewrite rules.

(1) $\neg\neg A \Rightarrow A$
(2) $\neg(A \wedge B) \Rightarrow \neg A \vee \neg B$
(3) $\neg(A \vee B) \Rightarrow \neg A \wedge \neg B$

Find one normal form of the following formula by applying rewrite rules to it until no more apply, i.e. show one complete branch of the search space.

$$\neg(\neg p \wedge (q \vee \neg r))$$

$$\begin{aligned} \neg(\neg p \wedge (q \vee \neg r)) &= \neg\neg p \vee \neg(q \vee \neg r) && \text{rule 2} \\ &= p \vee \neg(q \vee \neg r) && \text{rule 1} \\ &= p \vee \neg q \vee \neg\neg r && \text{rule 3} \\ &= p \vee \neg q \vee r && \text{rule 1} \end{aligned}$$

**EXERCISE FIVE:** Show that the application of the rule

$$X \star Y + X \star Z \Rightarrow X \star (Y + Z)$$

will terminate.

To show that the rule terminates, we need some decreasing measure, such as the following:

- no. of arithmetic operations decreases
- no of terms decreases
- depth of tree decreases