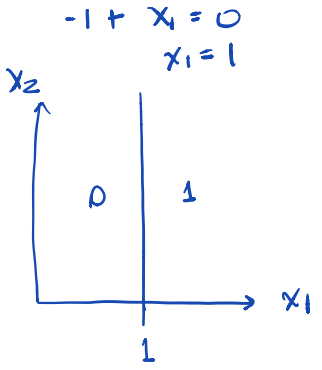
IAML Tutorial 3

1. Consider using logistic regression for a two-class classification problem in two dimensions:

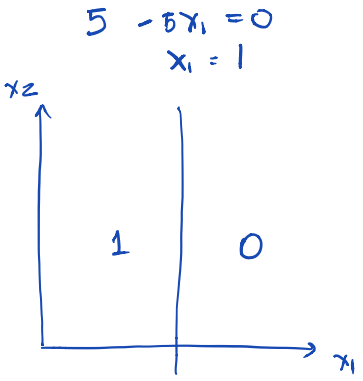
$$p(y=1|\underline{x}) = \sigma(w_0 + w_1 x_1 + w_2 x_2)$$

Here σ denotes the logistic/sigmoid function $\sigma(z) = \frac{1}{1+\exp(-z)}$, y is the target which takes on values 0 or 1, $\underline{x} = (x_1, x_2)$ is a vector in 2D input space, and $\underline{w} = (w_0, w_1, w_2)$ are the parameters of the logistic regressor.

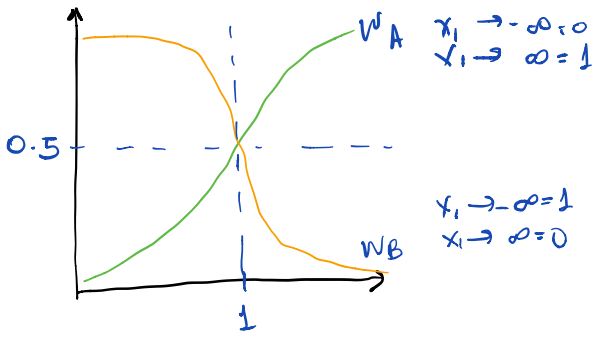
a. Consider a weight vector $w_A = (-1, 1, 0)$. Sketch DB, mark regions 0 and 1.



b. $w_B = (5, -5, 0)$. Sketch DB, indicate regions 0 and 1.



c. Plot $p(y=1|x)$ as a function of x_i for x_A and x_B; comment on the differences between the two.



$m_{w_B} > m_{w_A}$ when $x_1 = 1$

2. Consider the logistic regression setup in the previous questions, but with a new weight vector $w_A = (0, -1, 1)$. Consider the following dataset:

Instance	x_1	x_2	Class
0	0.5	-0.35	−
1	-0.1	0.1	−
2	-1.2	1.0	+

Compute the gradient of the log likelihood of the logistic regression model for this dataset.

Suppose we take a single gradient step with $\eta = 1.0$; what is the new parameter setting? Do the new parameters do a better job in classifying the training data?

Instance	x_1	x_2	$\sigma(w^T x)$	$y_i - \sigma(w^T x)$
0	0.5	-0.35	0.30	-0.30
1	-0.1	0.1	0.55	-0.55
2	-1.2	1.0	0.90	0.1

$$0 \to \sigma\left([0\ -1\ 1]\begin{bmatrix}1\\0.5\\-0.35\end{bmatrix}\right) \to \sigma(-0.85) = \frac{1}{1+e^{0.85}} = 0.299 \approx 0.30$$

$$1 \to \sigma\left([0\ -1\ 1]\begin{bmatrix}1\\-0.1\\0.1\end{bmatrix}\right) \to \sigma(0.2) = \frac{1}{1+e^{-0.2}} = 0.549 \approx 0.55$$

$$2 \to \sigma\left([0\ -1\ 1]\begin{bmatrix}1\\-1.2\\1.0\end{bmatrix}\right) \to \sigma(2.2) = \frac{1}{1+e^{-2.2}} = 0.9$$

$\partial L/\partial w_0 = 1 \times (-0.30) + 1 \times (-0.55) + 1 \times (0.1) = -0.75$

$\partial L/\partial w_1 = 0.5 \times (-0.30) + (-0.1 \times -0.55) + (-1.2 \times 0.1) = -0.215$

$\partial L/\partial w_2 = -0.35 \times (-0.30) + (0.1 \times -0.55) + (1.0 \times 0.1) = 0.15$

$w' = w + \eta g$

$= \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} + 1 \begin{bmatrix} -0.75 \\ -0.215 \\ 0.15 \end{bmatrix} = \begin{bmatrix} -0.75 \\ -1.215 \\ 1.15 \end{bmatrix}$

New instance classification

$0 \rightarrow \sigma\left[\begin{bmatrix} -0.75 & -1.215 & 1.15 \end{bmatrix} \begin{bmatrix} 1 \\ 0.5 \\ -0.35 \end{bmatrix}\right] = \sigma(-1.76) = 0.15$

$1 \rightarrow \sigma\left[\begin{bmatrix} -0.75 & -1.215 & 1.15 \end{bmatrix} \begin{bmatrix} 1 \\ -0.1 \\ 0.1 \end{bmatrix}\right] = \sigma(-0.5135) = 0.37$

$2 \rightarrow \sigma\left[\begin{bmatrix} -0.75 & -1.215 & 1.15 \end{bmatrix} \begin{bmatrix} 1 \\ -1.2 \\ 1.0 \end{bmatrix}\right] = \sigma(1.858) = 0.87$

instance	x_1	x_2	$P(y=1 \mid x_i)$	
0	0.5	-0.35	0.15	} improve
1	-0.1	0.1	0.37	
2	-1.2	1.0	0.87	- slightly worse

* will be set right with further gradient steps ♡.