AR TUTORIAL 2 - First Order Logic

EXERCISE ONE: Give an interpretation that satisfies $\forall x. \forall y. (p(x) \to p(y))$.

Any interpretation with a domain containing a single member (i.e. a singleton set) will satisfy the formula.
e.g. if the domain is $\{a\}$ then either $p(a)$ is true or $p(a)$ is false.
In either case, the statement is true.

EXERCISE TWO: Isabelle exercises.

EXERCISE THREE: Prove the following first order statements.

1. $(\forall x. Px \to Q) \to (\exists x. Px \to Q)$

$Pa \to Q \vdash Pa \to Q$	assumption
$\forall x. Px \to Q \vdash Pa \to Q$	allE
$\forall x. Px \to Q \vdash \exists x. Px \to Q$	exI
$\vdash (\forall x. Px \to Q) \to (\exists x. Px \to Q)$	allI

2. $\forall x. \neg Px$, assuming that $\neg \exists x. Px$

$Px_0 \vdash Px_0$	assumption
$Px_0 \vdash \exists x. Px$	exI
$\neg \exists x. Px, Px_0 \vdash \bot$	notE
$\neg \exists x. Px \vdash \neg P x_0$	notI
$\neg \exists x. Px \vdash \forall x. \neg Px$	allI

3. $\exists x. \neg Px$, assuming that $\neg \forall x. Px$ is true

$\neg Px_0 \vdash \neg Px_0$	assumption
$\neg Px_0 \vdash \exists x. \neg Px$	exI
$\neg \exists x. \neg Px, \neg Px_0 \vdash \bot$	notE
$\neg \exists x. \neg Px \vdash Px_0$	ccontr
$\neg \exists x. \neg Px \vdash \forall x. Px$	allI
$\neg \forall x. Px, \neg \exists x. \neg Px \vdash \bot$	notE
$\exists x. \neg Px \vdash \neg \forall x. Px$	ccontr