

# Limits and Continuity

Joshua Cole

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## 1 Easy Limits

Some limits are easier to solve than others. Take for example the equation,

$$\lim_{x \rightarrow 4} \frac{x+3}{x^2+1}$$

Its numerator can never be zero and in the case of  $x = 4$  deducing the solution is trivial.

## 2 Derivatives Are Always Harder

The equation for a derivative of  $f(x)$  is,

$$f'(x) = \lim_{x \rightarrow x_0} \frac{f(x_0) - f(x)}{x_0 - x}$$

Unlike an easy limit this denominator is always zero, or at least close enough to it as to make little difference. Until the delta can be removed from the denominator we are stuck.

## 3 Left and Right Hand Limits

A left hand limit is written as,

$$\lim_{x \rightarrow x_0^-}$$

It means that  $x$  is less than  $x_0$ . A right hand limit is written as,

$$\lim_{x \rightarrow x_0^+}$$

It means that  $x$  is greater than  $x_0$ .

As an example of this let's consider the function,

$$f(x) = \begin{cases} x+1 & x > 0 \\ -x+2 & x < 0 \end{cases}$$

When given a left-handed limit the equation would come out to 2. However, with the right-handed limit the function would come out to 1.

## 4 Continuity

A function is continuous at  $f(x_0)$  if the

$$\lim_{x \rightarrow x_0} f(x) - f(x_0) = 0$$

. This gives it nice properties like being the same on either side of the point as it approaches the delta.