

Bayesian spatio-temporal methods for small-area estimation of HIV indicators

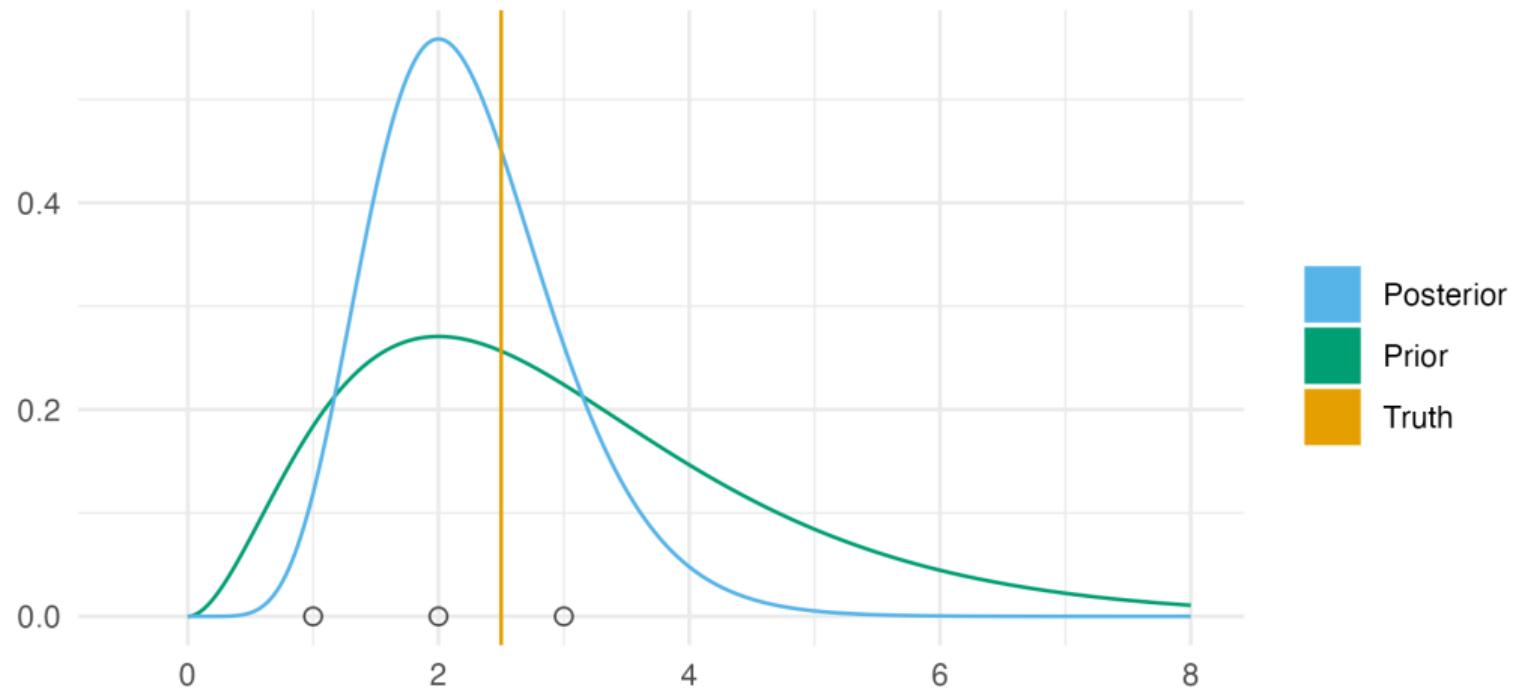
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Imperial College London

March 2023

Bayesian

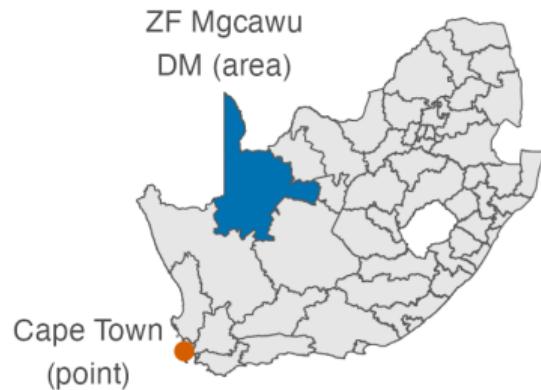
Use probability distributions for all unknowns



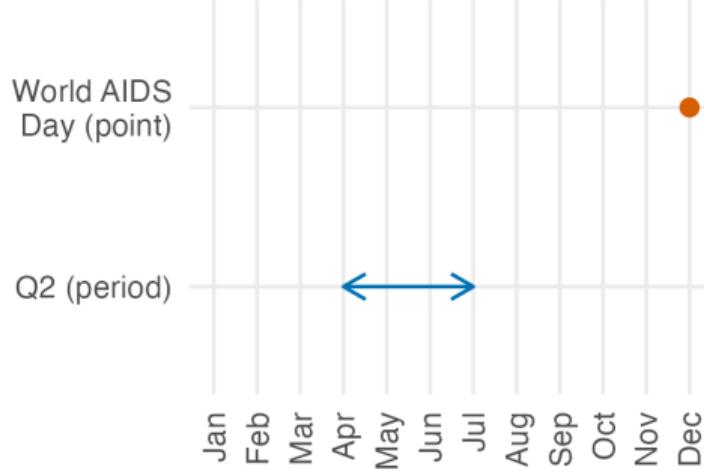
Spatio-temporal

Observed data has spatial and temporal location

A

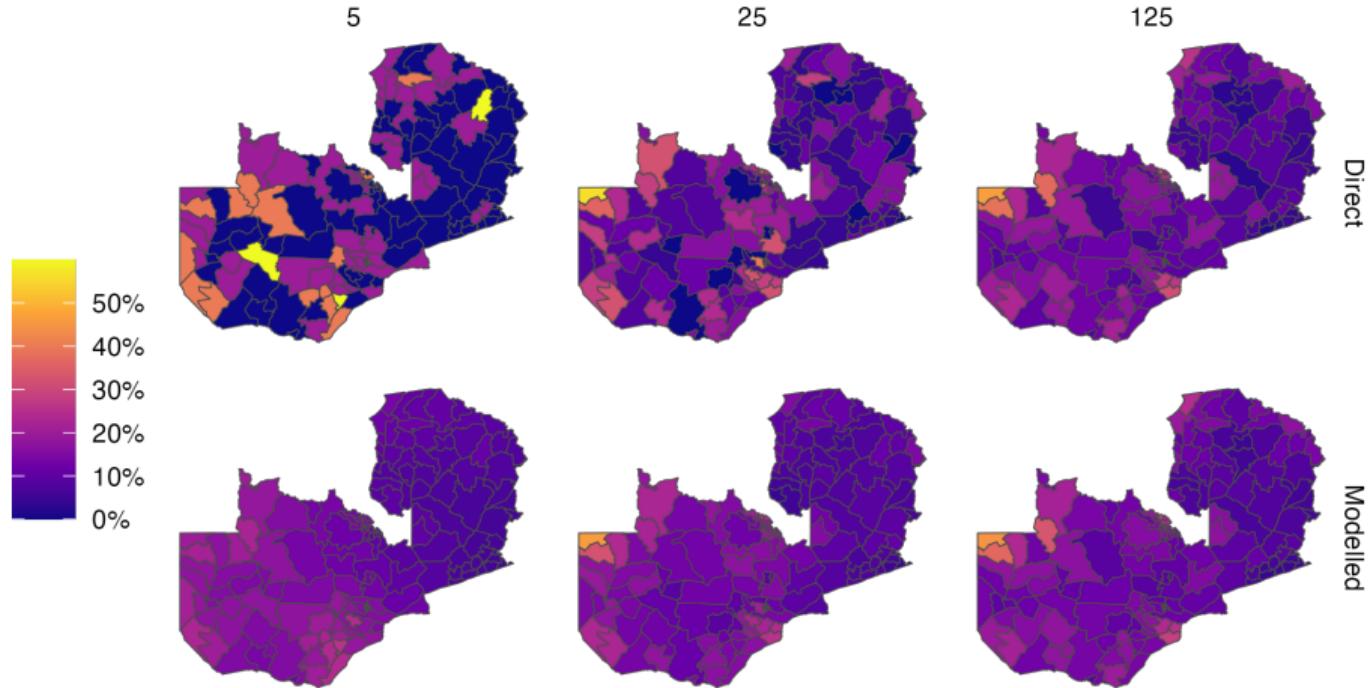


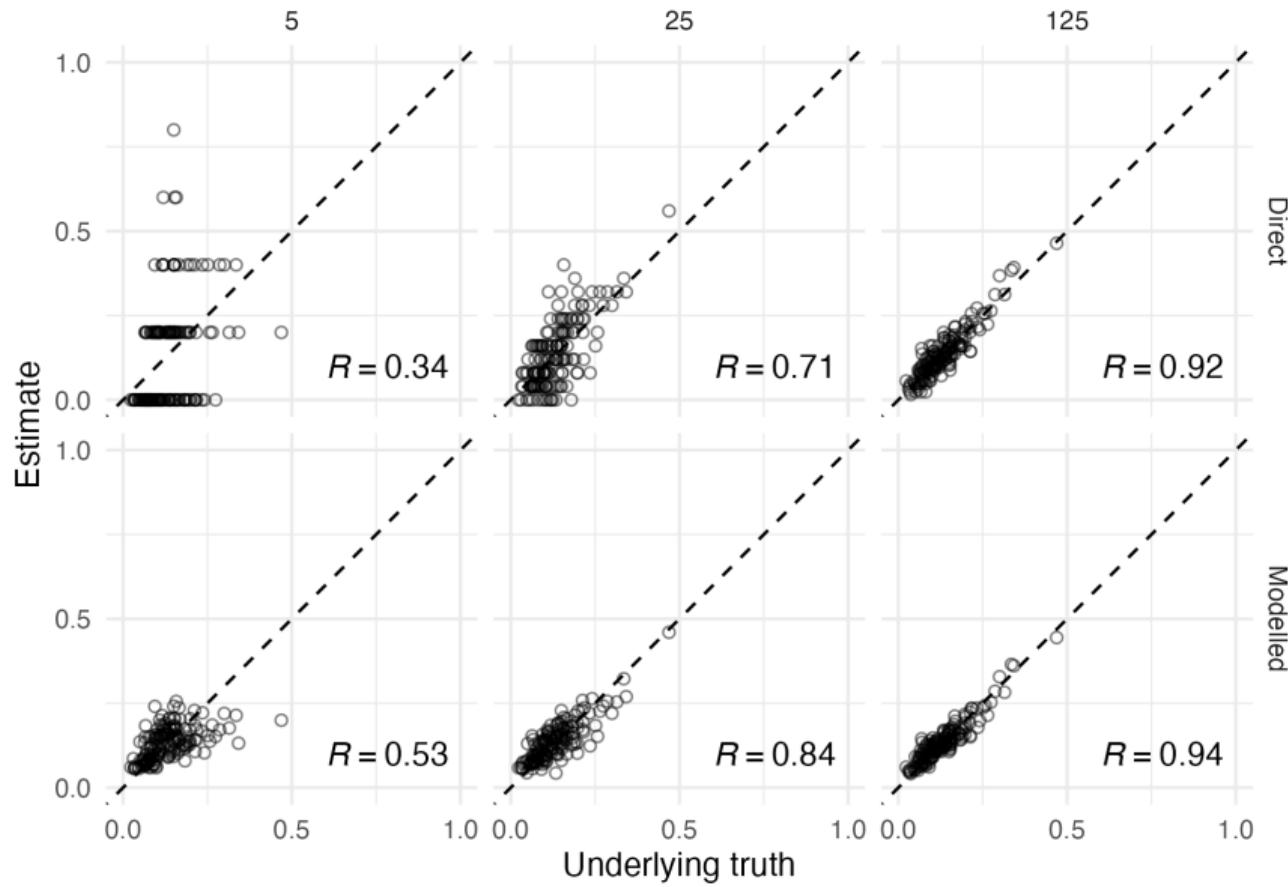
B



Small-area estimation

Sample size for demographic subgroups too low for precise direct estimates

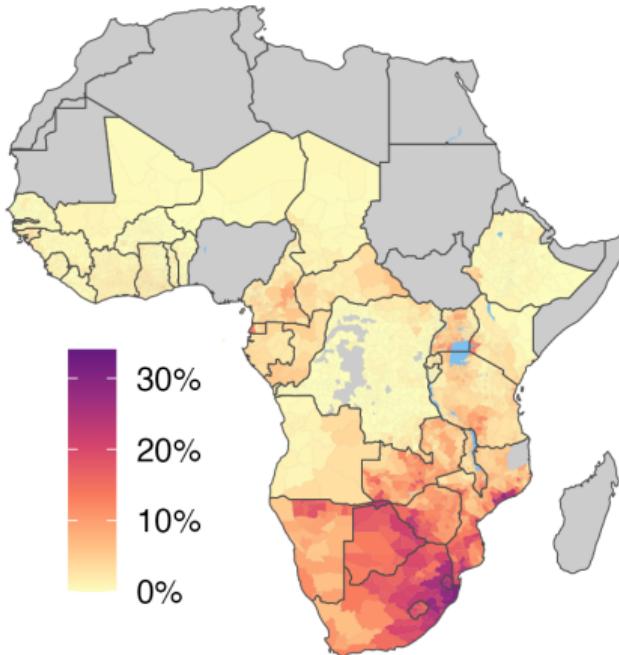
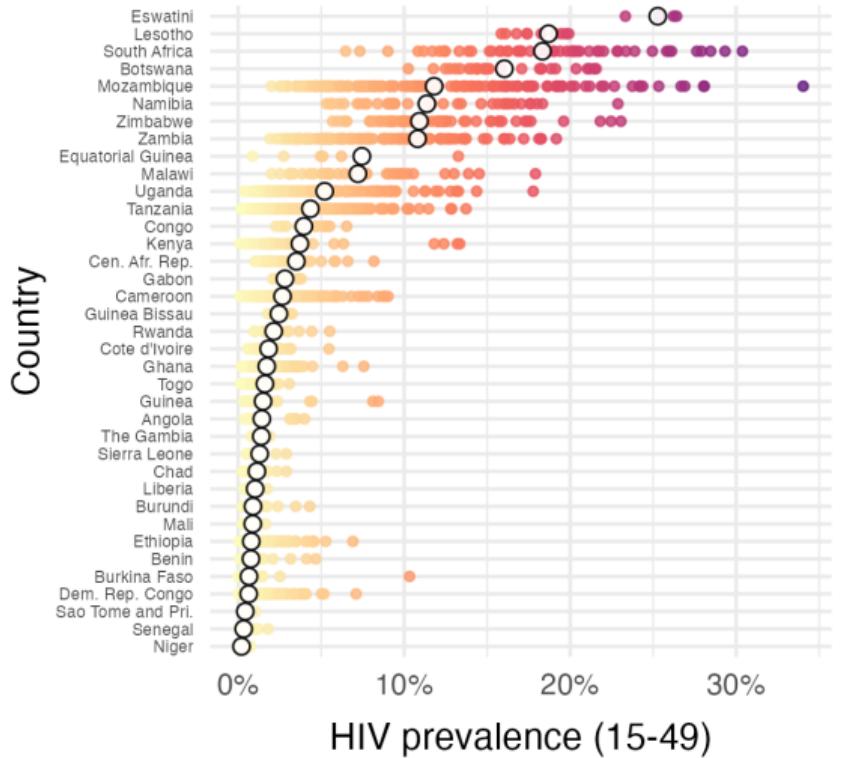




Toy example! Same principle applies to real models

e.g. Naomi¹ (Eaton et al. 2021; Esra et al. 2024)

¹See C.4 Simplified Naomi model description



Source: UNAIDS Naomi model estimates, 2023

Nearby things tend to be similar

Suppose prior correlation structure between observations!

Gaussian Markov random field model of Besag, York, and Mollié (1991)

Proportional to number of neighbours

Average of neighbours

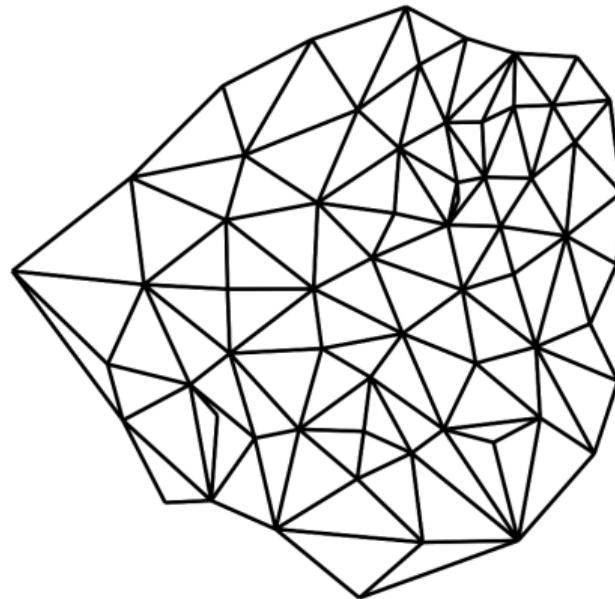
$$u_i | \mathbf{u}_{-i} \sim \mathcal{N} \left(\frac{1}{n_{\delta i}} \sum_{j:j \sim i} u_j, \frac{1}{n_{\delta i} \tau_u} \right)$$

*i*th full conditional

A



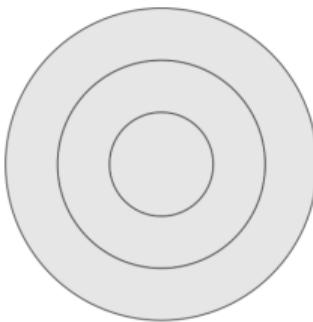
B



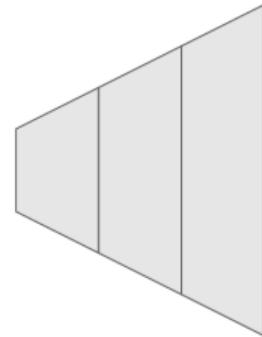
A



B



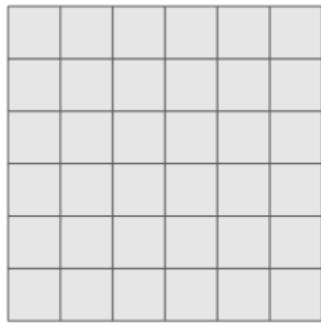
C



D



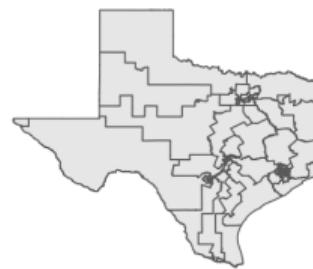
E



F

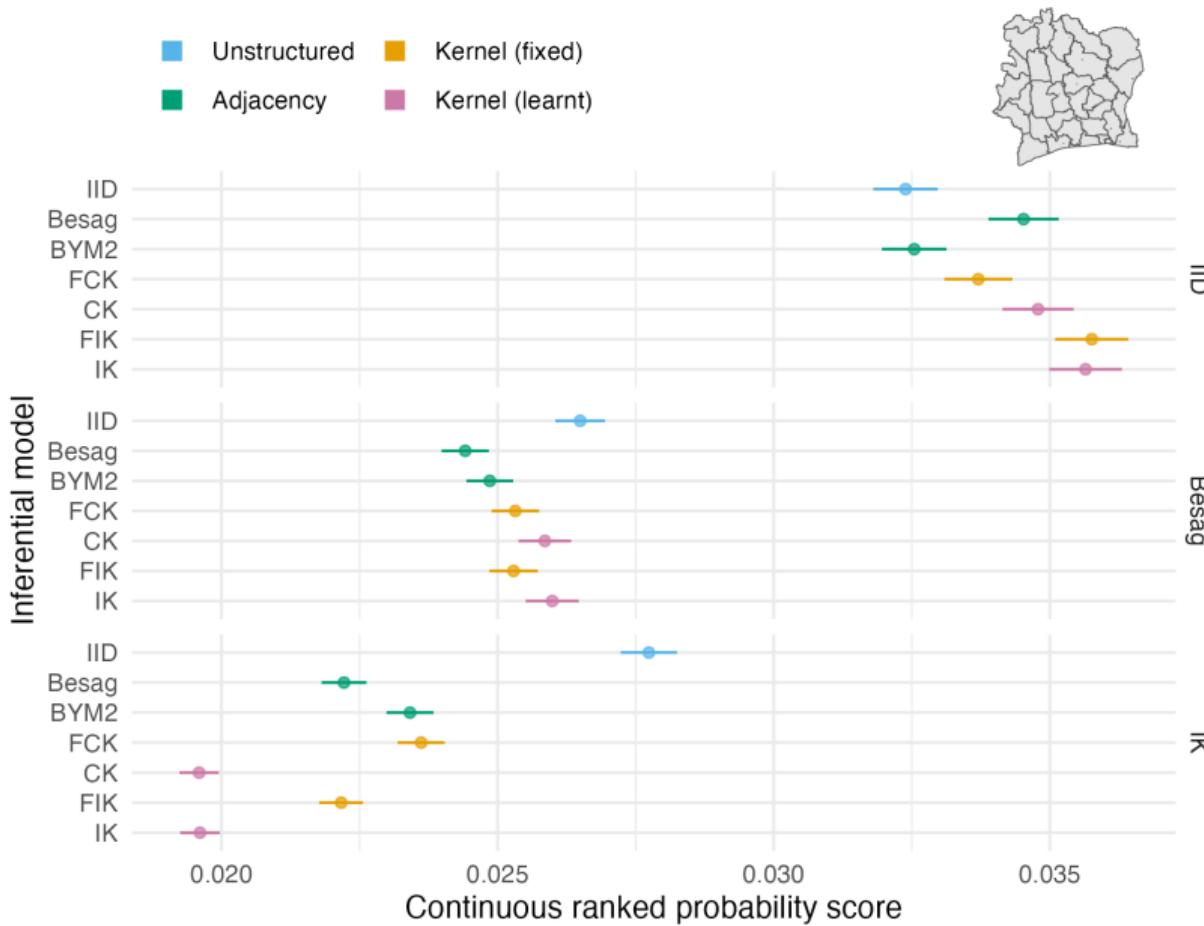


G



Measure forecast performance using strictly proper scoring rules

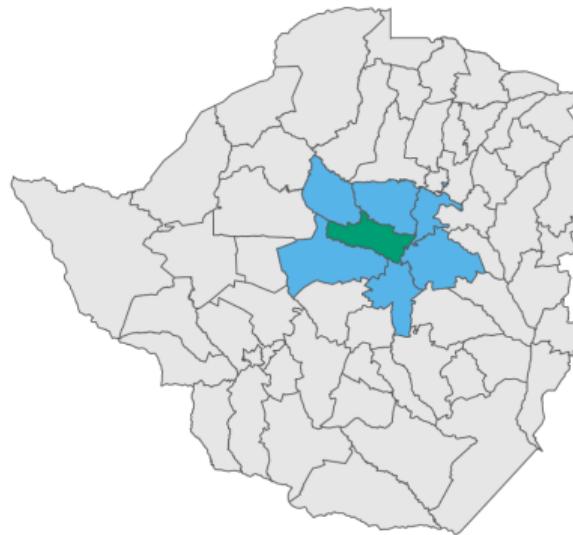
See Gneiting and Raftery (2007)



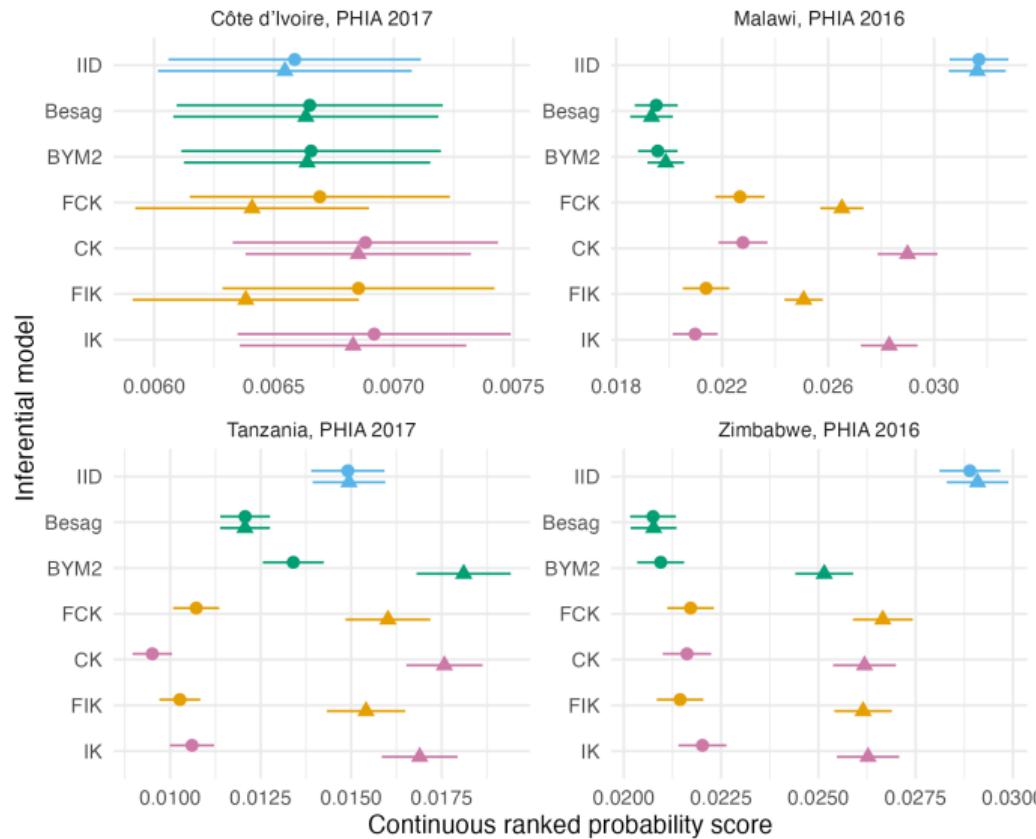
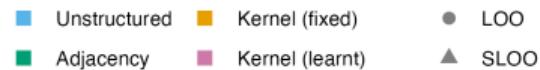
Leave-one-out (LOO)



Spatial-leave-one-out (SLOO)

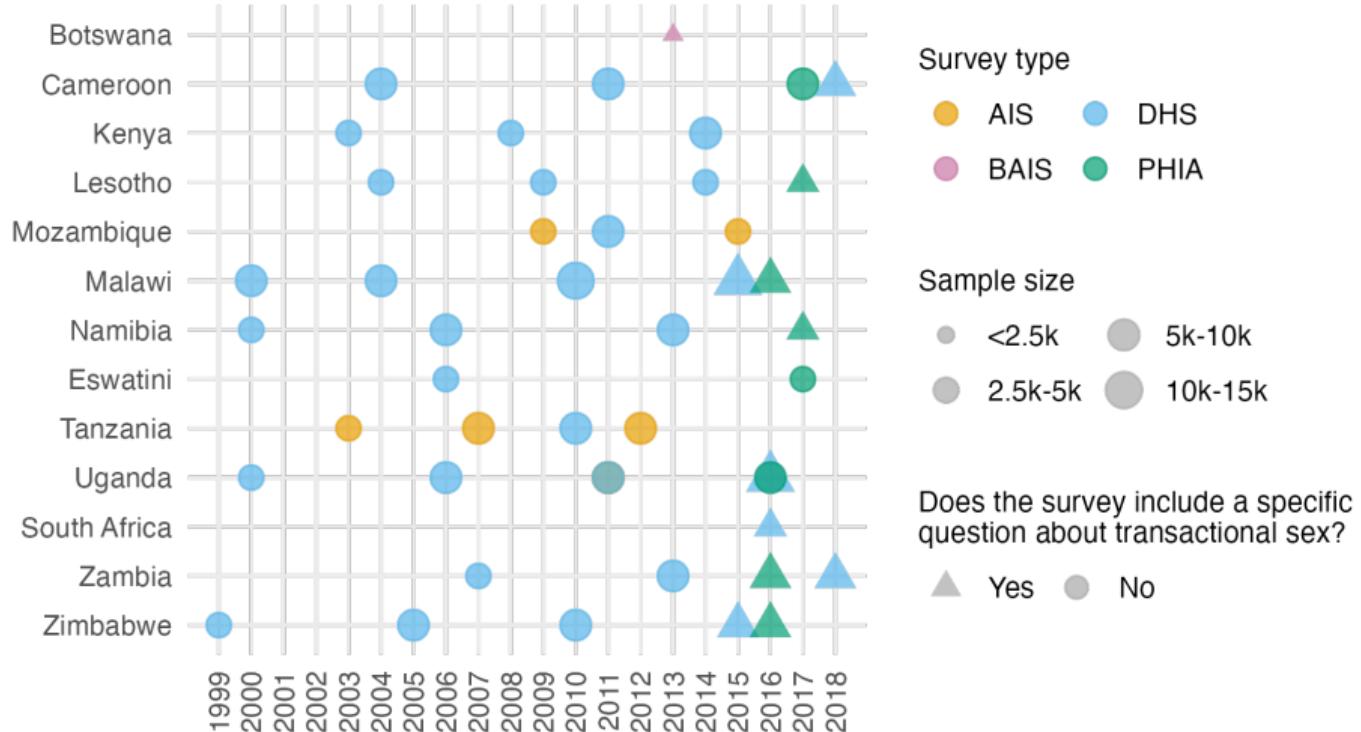


■ Training ■ Left out ■ Left out
and predicted on



GLOBAL AIDS STRATEGY 2021-2026
**END INEQUALITIES.
END AIDS.**

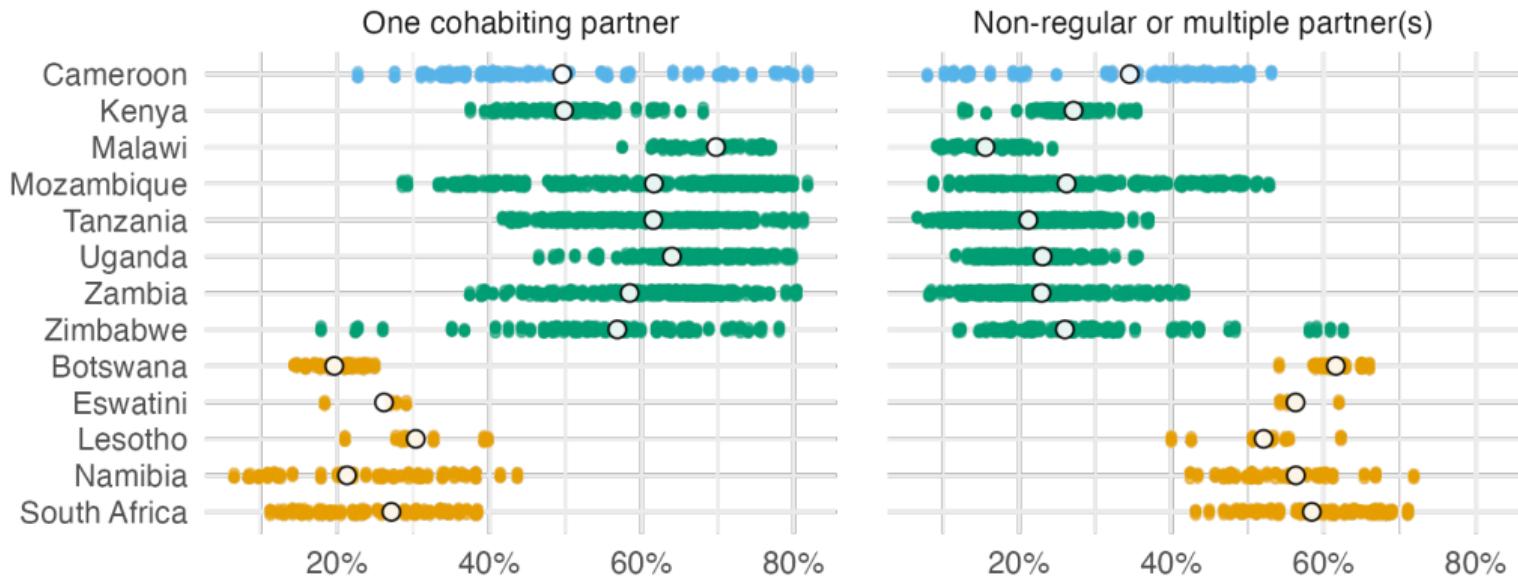




Use the multinomial-Poisson transformation of Baker (1994)

A multinomial logistic regression on $\mathbf{y} = (y_1, \dots, y_K)$ can be expressed as a Poisson regression $y_k \sim \text{Poisson}(\lambda_k)$ with observation-specific random effects with recover the sample size $m = \sum_k y_k$.

Regions of sub-Saharan Africa ● Central ● Eastern ● Southern

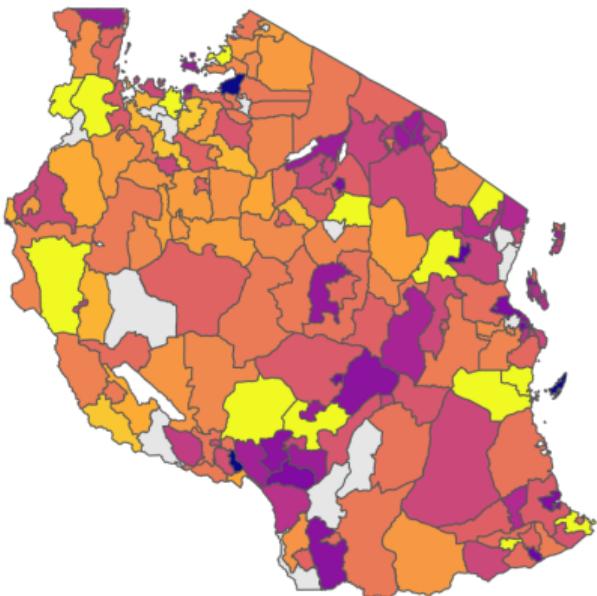


Not sexually active (not shown) + one cohabiting partner + non-regular or multiple partner(s) + FSW (not shown) = 100%

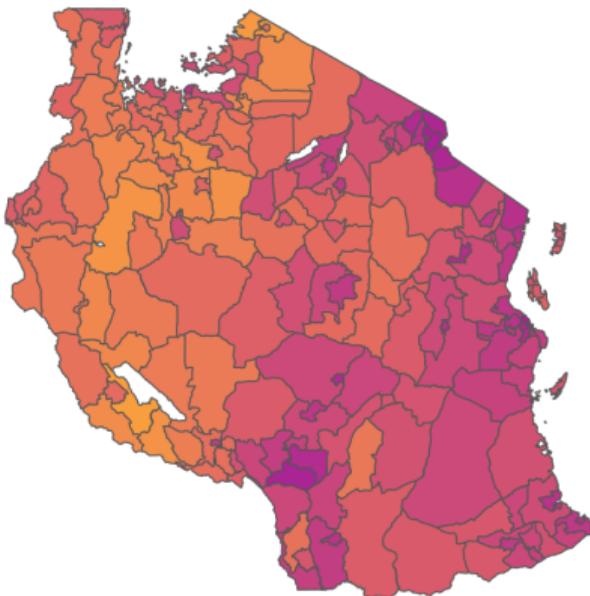
Proportion
of women
20-24
cohabiting
(2010)

100%
75%
50%
25%
0%

Direct



Modelled

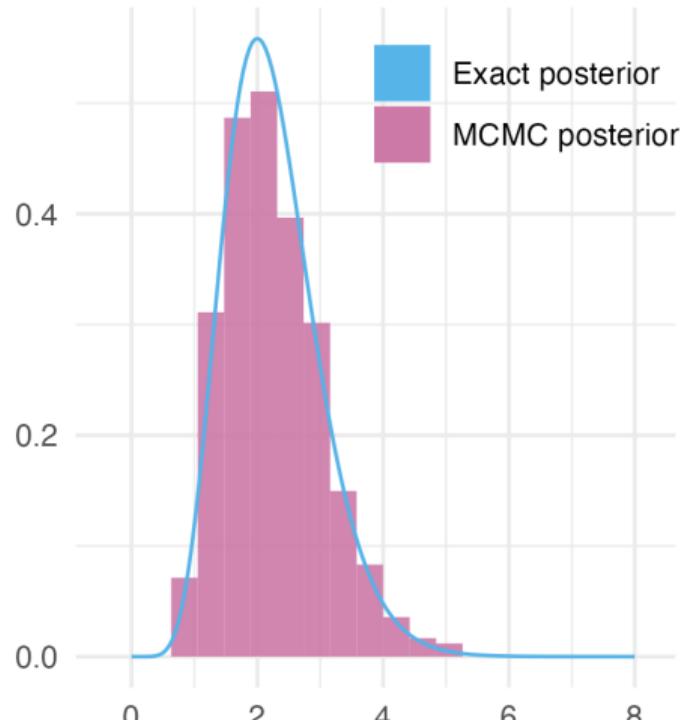
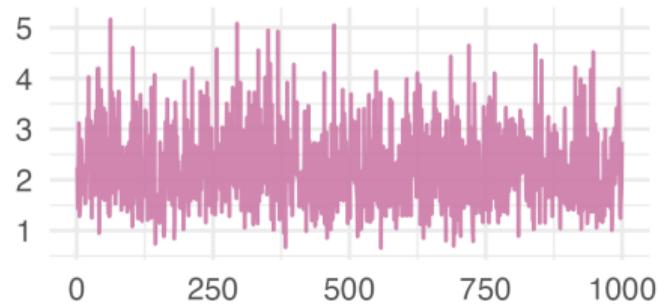
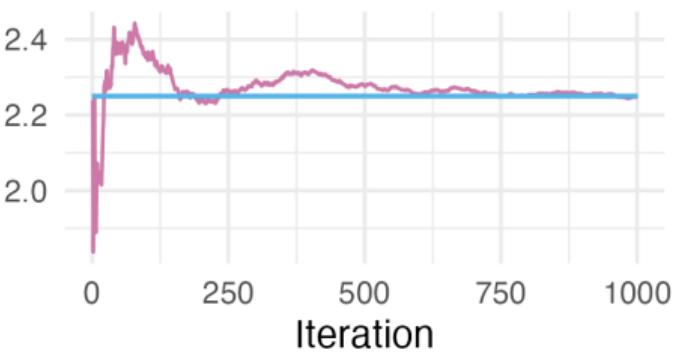


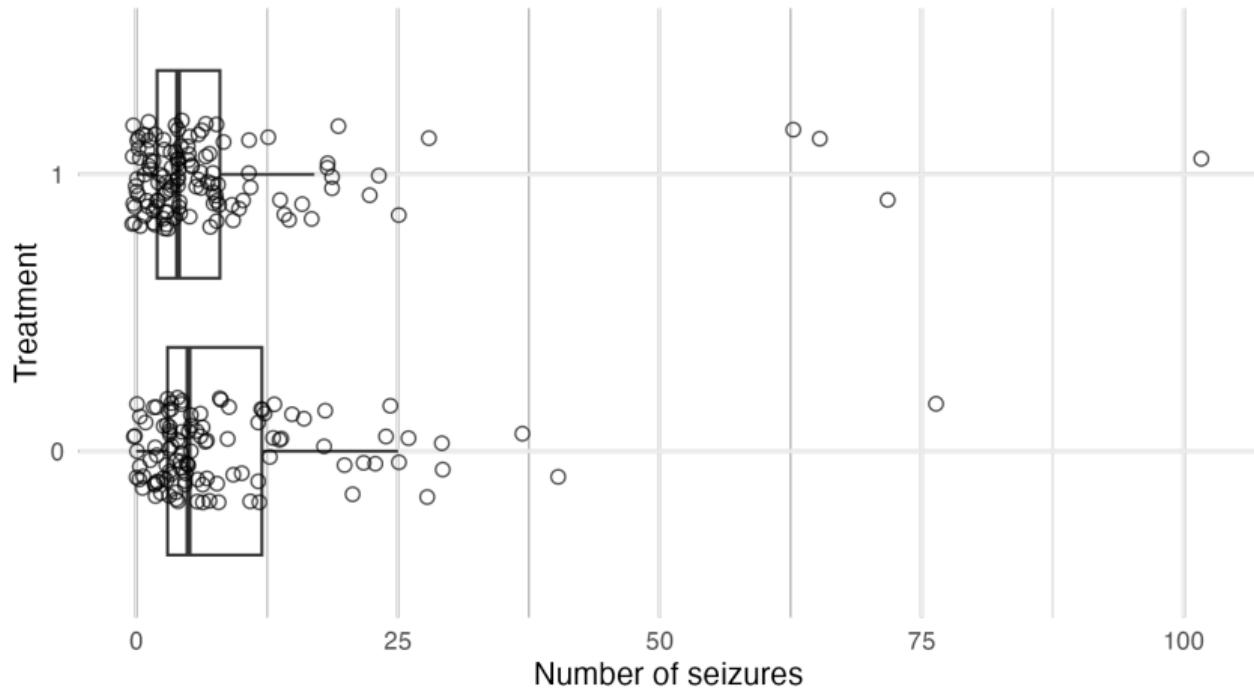
Since extended to include 1) males, 2) additional countries

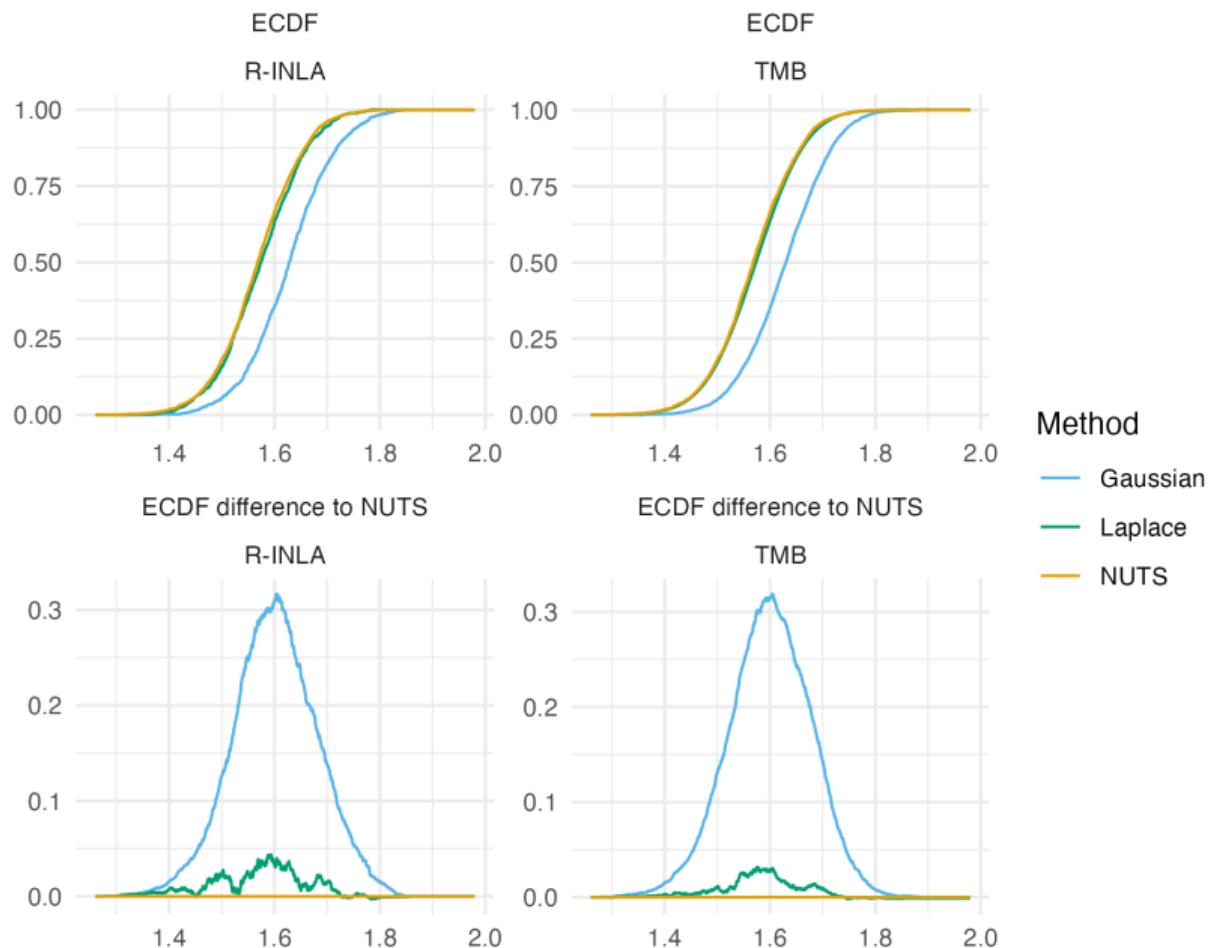
See the sub-national HIV estimates in priority populations UNAIDS tool at
hivtools.unaids.org/shipp/

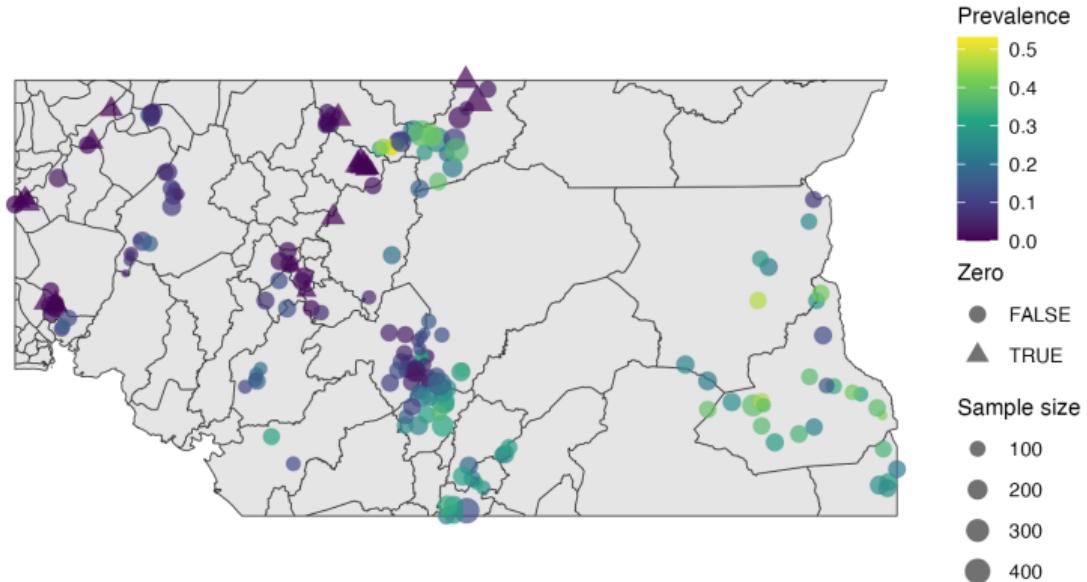
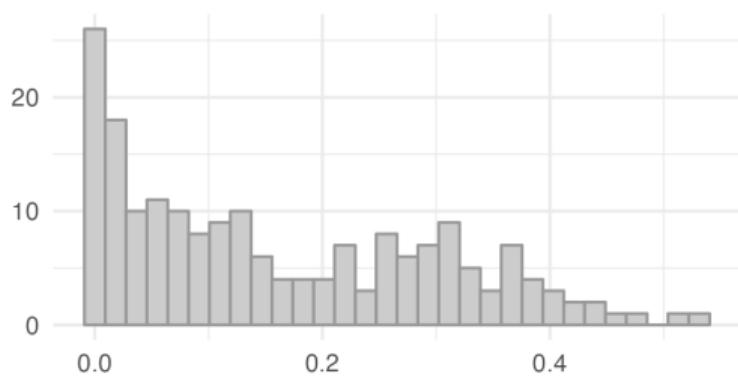
Bayesian computation amounts to solving challenging integrals

$$p(\mathbf{y}) = \int p(\mathbf{y}, \phi) d\phi$$

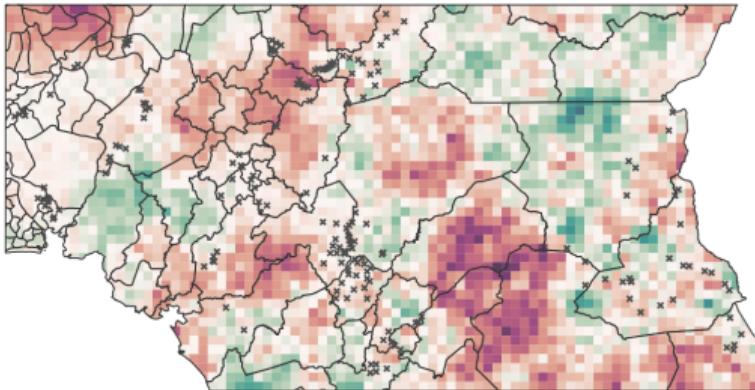
A**B****C**



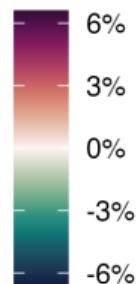


A**B**

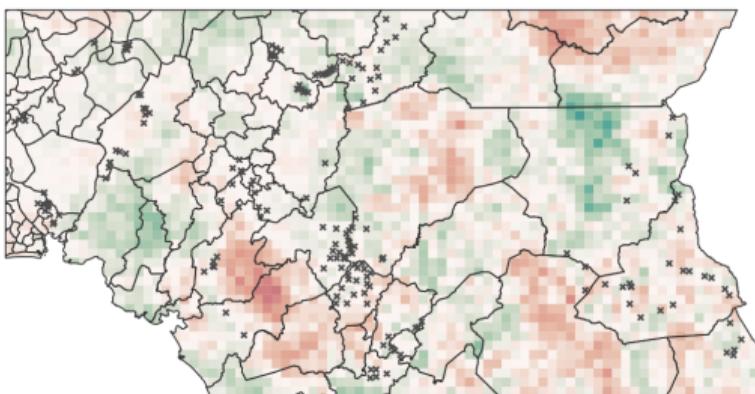
Gaussian



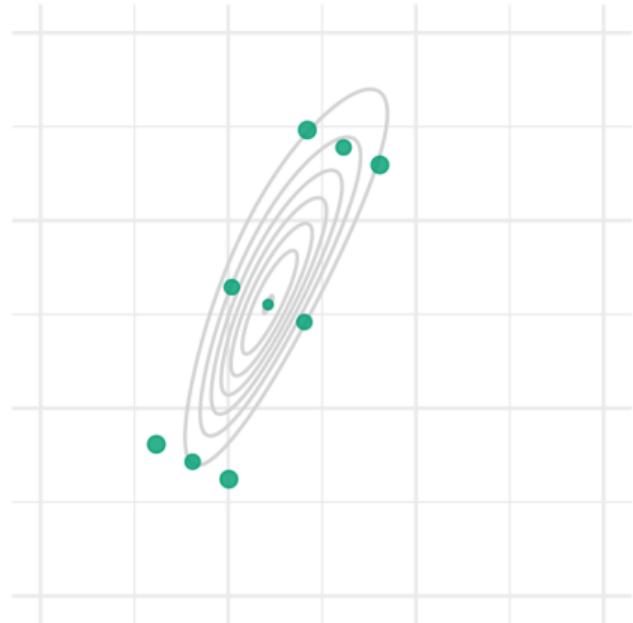
Prevalence
difference
to NUTS



Laplace

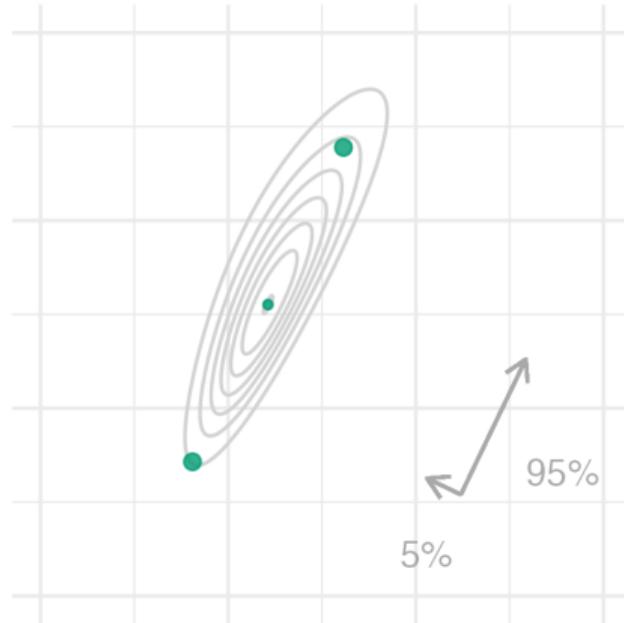


A

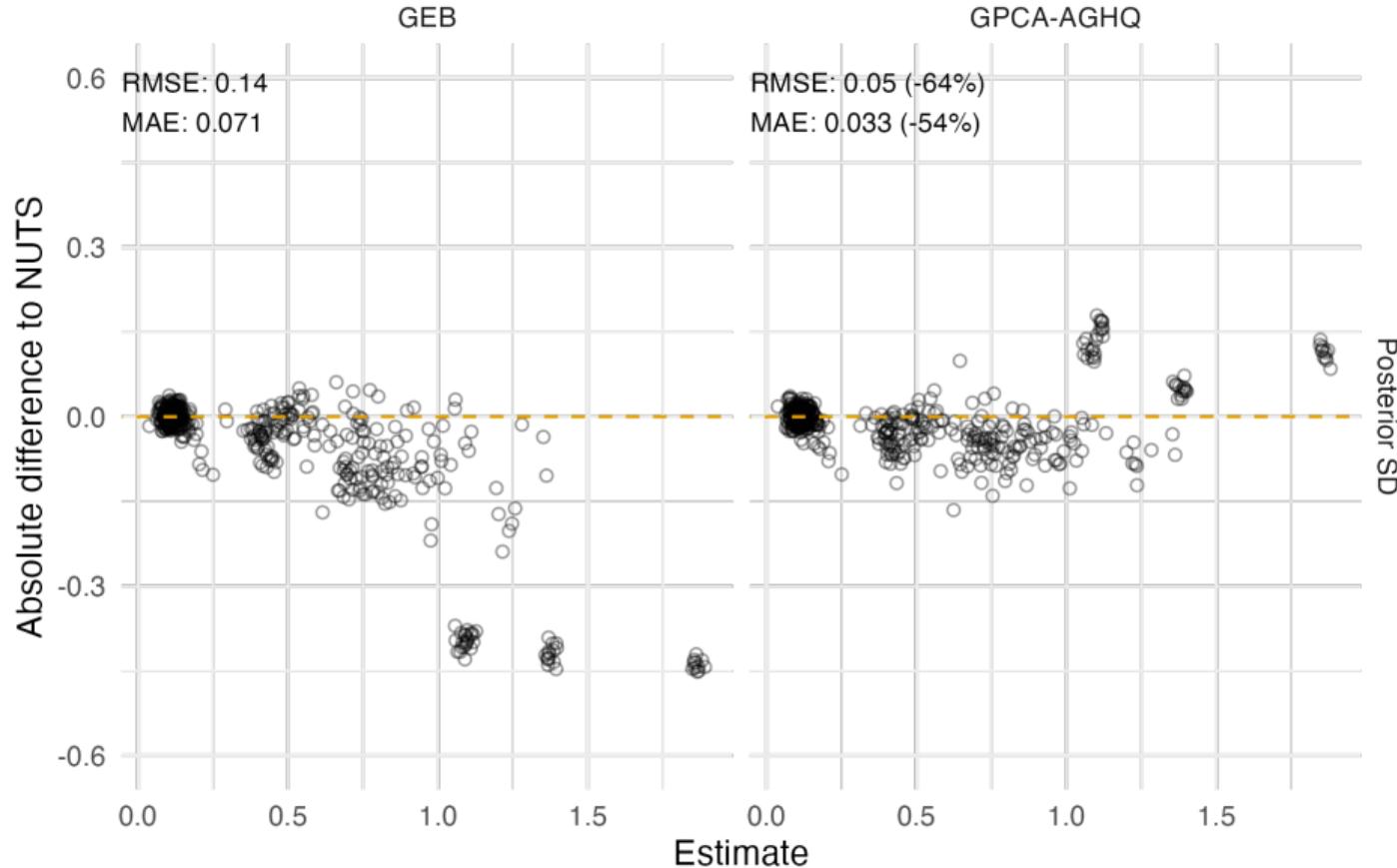


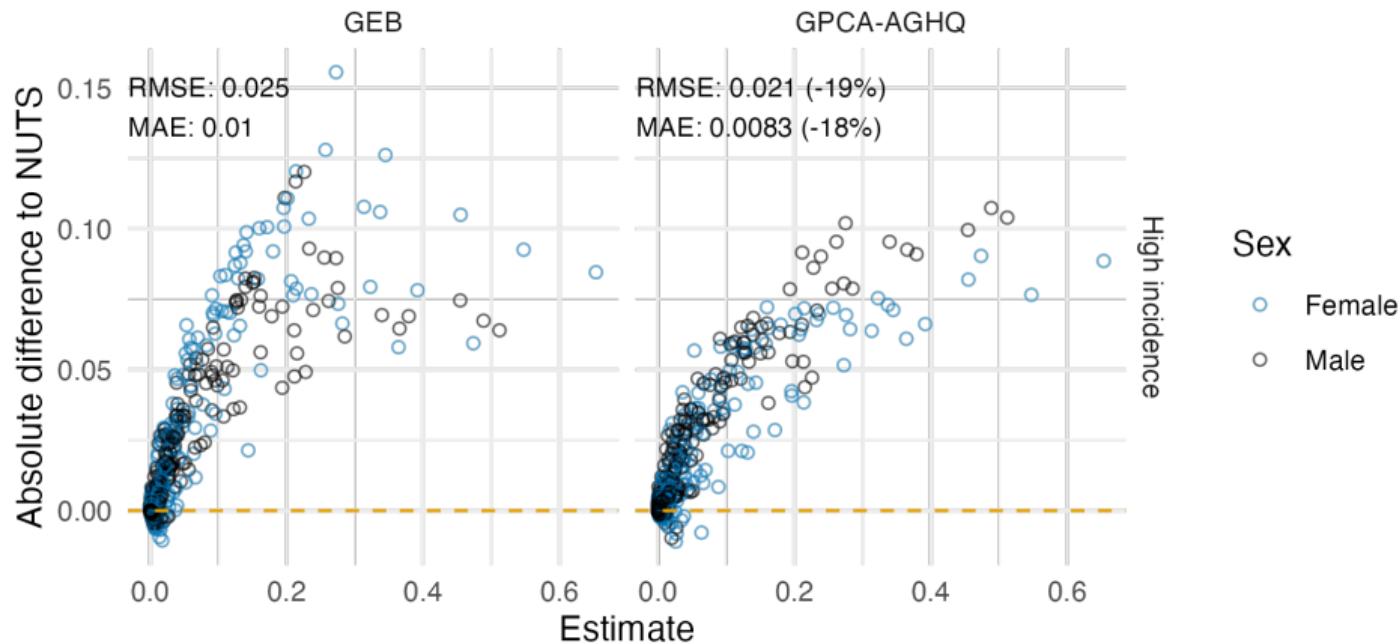
AGHQ (spectral)

B



PCA-AGHQ





Acknowledgements

Collaborator(s)	Affiliation
Jeff Eaton	Imperial, Harvard
Seth Flaxman	Oxford
Alex Stringer	Waterloo
HIV Inference Group	Imperial
Machine Learning and Global Health Network	Worldwide
StatML CDT	Imperial, Oxford

What am I up to now?

- Adaptive survey design to improve efficiency of mobile phone surveys
 - Still interested to apply this approach to household surveys e.g. for HIV
- Multilevel regression and poststratification models for district-level food security estimates

References I

- Baker, Stuart G. 1994. "The multinomial-Poisson transformation." *Journal of the Royal Statistical Society: Series D (The Statistician)* 43 (4): 495–504.
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References II

- Esra, Rachel, Mpho Mmelesi, Akeem T. Ketlogetswe, Timothy M. Wolock, Adam Howes, Tlotlo Nong, Matshelo Tina Matlhaga, Siphiwe Ratladi, Dinah Ramaabya, and Jeffrey W. Imai-Eaton. 2024. "Improved Indicators for Subnational Unmet Antiretroviral Therapy Need in the Health System: Updates to the Naomi Model in 2023." *JAIDS Journal of Acquired Immune Deficiency Syndromes* 95 (1S): e24–33.
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- Gneiting, Tilmann, and Adrian E Raftery. 2007. "Strictly proper scoring rules, prediction, and estimation." *Journal of the American Statistical Association* 102 (477): 359–78.