

Eccentricity Calculation :

In[]:=* **1.5 (2)^{-0.1}**

Out[]=*
1.39955

In[]:=* **1.5 (3)^{-0.1}**

Out[]=*
1.34394

In[]:=* **1.5 (4)^{-0.1}**

Out[]=*
1.30583

In[]:=* **1.5 (5)^{-0.1}**

Out[]=*
1.27701

In[]:=* **1.5 (6)^{-0.1}**

Out[]=*
1.25394

In[]:=* **1.5 (7)^{-0.1}**

Out[]=*
1.23476

In[]:=* **1.5 (8)^{-0.1}**

Out[]=*
1.21838

In[]:=* **1.5 (9)^{-0.1}**

Out[]=*
1.20411

In[]:=* **1.5 (10)^{-0.1}**

Out[]=*
1.19149

In[]:=* **1.5 (21)^{-0.1}**

Out[]=*
1.10629

In[]:=* $\sqrt{1 - \frac{1}{(1.10629)^2}}$

Out[]=*
0.427697

In[]:=* $\sqrt{1 - \frac{1}{(1.34394)^2}}$

Out[]=*
0.66809

$$\text{In}[*]:= \sqrt{1 - \frac{1}{(1.30583)^2}}$$

Out[*]=
0.643083

$$\text{In}[*]:= \sqrt{1 - \frac{1}{(1.27701)^2}}$$

Out[*]=
0.621922

$$\text{In}[*]:= \sqrt{1 - \frac{1}{(1.25394)^2}}$$

Out[*]=
0.603337

$$\text{In}[*]:= \sqrt{1 - \frac{1}{(1.23476)^2}}$$

Out[*]=
0.586604

$$\text{In}[*]:= \sqrt{1 - \frac{1}{(1.21838)^2}}$$

Out[*]=
0.57127

$$\text{In}[*]:= \sqrt{1 - \frac{1}{(1.20411)^2}}$$

Out[*]=
0.557035

$$\text{In}[*]:= \sqrt{1 - \frac{1}{(1.19149)^2}}$$

Out[*]=
0.543691

$$\text{In}[*]:= \sqrt{1 - \frac{1}{(1.18019)^2}}$$

Out[*]=
0.531081

FR Calculation

(*Homogenous - decreasing value alpha=0.1*)

```
In[*]:= NIntegrate[ $\left[-\frac{3}{e^3} - \frac{e^2 - 3}{2e^4} \operatorname{Log}\left[\frac{1+e}{1-e}\right]\right]$ , {e, 0.783, 0.851392}]
```

```
Out[*]=  
0.0349848
```

```
In[*]:= NIntegrate[ $\left[-\frac{3}{e^3} - \frac{e^2 - 3}{2e^4} \operatorname{Log}\left[\frac{1+e}{1-e}\right]\right]$ , {e, 0.802012, 0.851392}]
```

```
Out[*]=  
0.0263178
```

```
In[*]:= NIntegrate[ $\left[-\frac{3}{e^3} - \frac{e^2 - 3}{2e^4} \operatorname{Log}\left[\frac{1+e}{1-e}\right]\right]$ , {e, 0.814356, 0.851392}]
```

```
Out[*]=  
0.0202963
```

```
In[*]:= NIntegrate[ $\left[-\frac{3}{e^3} - \frac{e^2 - 3}{2e^4} \operatorname{Log}\left[\frac{1+e}{1-e}\right]\right]$ , {e, 0.823332, 0.851392}]
```

```
Out[*]=  
0.0157023
```

```
In[*]:= NIntegrate[ $\left[-\frac{3}{e^3} - \frac{e^2 - 3}{2e^4} \operatorname{Log}\left[\frac{1+e}{1-e}\right]\right]$ , {e, 0.830308, 0.851392}]
```

```
Out[*]=  
0.0119968
```

```
In[*]:= NIntegrate[ $\left[-\frac{3}{e^3} - \frac{e^2 - 3}{2e^4} \operatorname{Log}\left[\frac{1+e}{1-e}\right]\right]$ , {e, 0.835966, 0.851392}]
```

```
Out[*]=  
0.00889942
```

```
In[*]:= NIntegrate[ $\left[-\frac{3}{e^3} - \frac{e^2 - 3}{2e^4} \operatorname{Log}\left[\frac{1+e}{1-e}\right]\right]$ , {e, 0.8407, 0.851392}]
```

```
Out[*]=  
0.00624126
```

```
In[*]:= NIntegrate[ $\left[-\frac{3}{e^3} - \frac{e^2 - 3}{2e^4} \operatorname{Log}\left[\frac{1+e}{1-e}\right]\right]$ , {e, 0.844751, 0.851392}]
```

```
Out[*]=  
0.00391637
```

$$\text{In[*]} := \text{NIntegrate}\left[\left(-\frac{3}{e^3} - \frac{e^2 - 3}{2e^4} \text{Log}\left[\frac{1+e}{1-e}\right]\right), \{e, 0.848278, 0.851392\}\right]$$

Out[*]=

0.00185304

(*Non-Homogenous - decreasing value alpha=0.1*)

$$\text{In[*]} := \text{NIntegrate}\left[\left(\frac{1}{1 + \left(\left(\frac{1}{2.25(1-e^2)}\right)^{\frac{1}{0.2}} - 1\right)^2}\right) \left(-\frac{3}{e^3} - \frac{e^2 - 3}{2e^4} \text{Log}\left[\frac{1+e}{1-e}\right]\right), \{e, 0.783, 0.851392\}\right]$$

Out[*]=

0.0045367

$$\text{In[*]} := \text{NIntegrate}\left[\left(\frac{1}{1 + \left(\left(\frac{1}{2.25(1-e^2)}\right)^{\frac{1}{0.2}} - 1\right)^2}\right) \left(-\frac{3}{e^3} - \frac{e^2 - 3}{2e^4} \text{Log}\left[\frac{1+e}{1-e}\right]\right), \{e, 0.802012, 0.851392\}\right]$$

Out[*]=

0.00165555

$$\text{In[*]} := \text{NIntegrate}\left[\left(\frac{1}{1 + \left(\left(\frac{1}{2.25(1-e^2)}\right)^{\frac{1}{0.2}} - 1\right)^2}\right) \left(-\frac{3}{e^3} - \frac{e^2 - 3}{2e^4} \text{Log}\left[\frac{1+e}{1-e}\right]\right), \{e, 0.814356, 0.851392\}\right]$$

Out[*]=

0.000785932

$$\text{In[*]} := \text{NIntegrate}\left[\left(\frac{1}{1 + \left(\left(\frac{1}{2.25(1-e^2)}\right)^{\frac{1}{0.2}} - 1\right)^2}\right) \left(-\frac{3}{e^3} - \frac{e^2 - 3}{2e^4} \text{Log}\left[\frac{1+e}{1-e}\right]\right), \{e, 0.823332, 0.851392\}\right]$$

Out[*]=

0.000429535

$$\text{In[*]} := \text{NIntegrate}\left[\left(\frac{1}{1 + \left(\left(\frac{1}{2.25(1-e^2)}\right)^{\frac{1}{0.2}} - 1\right)^2}\right) \left(-\frac{3}{e^3} - \frac{e^2 - 3}{2e^4} \text{Log}\left[\frac{1+e}{1-e}\right]\right), \{e, 0.830308, 0.851392\}\right]$$

Out[*]=

0.000252007

```
In[*]:= NIntegrate[
$$\left[ \left( \frac{1}{1 + \left( \left( \frac{1}{2.25 (1 - e^2)} \right)^{\frac{1}{0.2}} - 1 \right)^2} \right) \left( -\frac{3}{e^3} - \frac{e^2 - 3}{2 e^4} \text{Log} \left[ \frac{1 + e}{1 - e} \right] \right) \right],$$

{e, 0.835966, 0.851392}]
```

```
Out[*]=
0.000151643
```

```
In[*]:= NIntegrate[
$$\left[ \left( \frac{1}{1 + \left( \left( \frac{1}{2.25 (1 - e^2)} \right)^{\frac{1}{0.2}} - 1 \right)^2} \right) \left( -\frac{3}{e^3} - \frac{e^2 - 3}{2 e^4} \text{Log} \left[ \frac{1 + e}{1 - e} \right] \right) \right], \{e, 0.8407, 0.851392\}]$$

```

```
Out[*]=
0.0000896156
```

```
In[*]:= NIntegrate[
$$\left[ \left( \frac{1}{1 + \left( \left( \frac{1}{2.25 (1 - e^2)} \right)^{\frac{1}{0.2}} - 1 \right)^2} \right) \left( -\frac{3}{e^3} - \frac{e^2 - 3}{2 e^4} \text{Log} \left[ \frac{1 + e}{1 - e} \right] \right) \right],$$

{e, 0.844751, 0.851392}]
```

```
Out[*]=
0.0000487215
```

```
In[*]:= NIntegrate[
$$\left[ \left( \frac{1}{1 + \left( \left( \frac{1}{2.25 (1 - e^2)} \right)^{\frac{1}{0.2}} - 1 \right)^2} \right) \left( -\frac{3}{e^3} - \frac{e^2 - 3}{2 e^4} \text{Log} \left[ \frac{1 + e}{1 - e} \right] \right) \right],$$

{e, 0.848278, 0.851392}]
```

```
Out[*]=
0.0000203983
```

(*Homogenous - decreasing value alpha=-0.1*)

```
In[*]:= NIntegrate[
$$\left[ -\frac{3}{e^3} - \frac{e^2 - 3}{2 e^4} \text{Log} \left[ \frac{1 + e}{1 - e} \right] \right], \{e, 0.69962, 0.531081\}]$$

```

```
Out[*]=
-0.0414321
```

```
In[*]:= NIntegrate[
$$\left[ -\frac{3}{e^3} - \frac{e^2 - 3}{2 e^4} \text{Log} \left[ \frac{1 + e}{1 - e} \right] \right], \{e, 0.66809, 0.531081\}]$$

```

```
Out[*]=
-0.0318697
```

```
In[*]:= NIntegrate[
$$\left[ -\frac{3}{e^3} - \frac{e^2 - 3}{2 e^4} \text{Log} \left[ \frac{1 + e}{1 - e} \right] \right], \{e, 0.643083, 0.531081\}]$$

```

```
Out[*]=
-0.0249711
```

```
In[*]:= NIntegrate[ $\left(-\frac{3}{e^3} - \frac{e^2 - 3}{2 e^4} \operatorname{Log}\left[\frac{1 + e}{1 - e}\right]\right)$ , {e, 0.621922, 0.531081}]
```

```
Out[*]=  
-0.0195559
```

```
In[*]:= NIntegrate[ $\left(-\frac{3}{e^3} - \frac{e^2 - 3}{2 e^4} \operatorname{Log}\left[\frac{1 + e}{1 - e}\right]\right)$ , {e, 0.603337, 0.531081}]
```

```
Out[*]=  
-0.0150921
```

```
In[*]:= NIntegrate[ $\left(-\frac{3}{e^3} - \frac{e^2 - 3}{2 e^4} \operatorname{Log}\left[\frac{1 + e}{1 - e}\right]\right)$ , {e, 0.586604, 0.531081}]
```

```
Out[*]=  
-0.0112904
```

```
In[*]:= NIntegrate[ $\left(-\frac{3}{e^3} - \frac{e^2 - 3}{2 e^4} \operatorname{Log}\left[\frac{1 + e}{1 - e}\right]\right)$ , {e, 0.57127, 0.531081}]
```

```
Out[*]=  
-0.0079764
```

```
In[*]:= NIntegrate[ $\left(-\frac{3}{e^3} - \frac{e^2 - 3}{2 e^4} \operatorname{Log}\left[\frac{1 + e}{1 - e}\right]\right)$ , {e, 0.557035, 0.531081}]
```

```
Out[*]=  
-0.00503758
```

```
In[*]:= NIntegrate[ $\left(-\frac{3}{e^3} - \frac{e^2 - 3}{2 e^4} \operatorname{Log}\left[\frac{1 + e}{1 - e}\right]\right)$ , {e, 0.543691, 0.531081}]
```

```
Out[*]=  
-0.00239737
```

(*Non-Homogenous - decreasing value alpha=-0.1*)

```
In[*]:= NIntegrate[ $\left(\frac{1}{1 + \left(\left(\frac{1}{2.25 (1 - e^2)}\right)^{\frac{1}{-0.2}} - 1\right)^2}\right) \left(-\frac{3}{e^3} - \frac{e^2 - 3}{2 e^4} \operatorname{Log}\left[\frac{1 + e}{1 - e}\right]\right)$ , {e, 0.69962, 0.531081}]
```

```
Out[*]=  
-0.00511083
```

```
In[*]:= NIntegrate[ $\left(\frac{1}{1 + \left(\left(\frac{1}{2.25 (1 - e^2)}\right)^{\frac{1}{-0.2}} - 1\right)^2}\right) \left(-\frac{3}{e^3} - \frac{e^2 - 3}{2 e^4} \operatorname{Log}\left[\frac{1 + e}{1 - e}\right]\right)$ , {e, 0.66809, 0.531081}]
```

```
Out[*]=  
-0.00194266
```

$$\text{In[*]} := \text{NIntegrate}\left[\left(\frac{1}{1 + \left(\left(\frac{1}{2.25(1-e^2)}\right)^{\frac{1}{-0.2}} - 1\right)^2}\right) \left(-\frac{3}{e^3} - \frac{e^2 - 3}{2e^4} \text{Log}\left[\frac{1+e}{1-e}\right]\right)\right],$$

$$\{e, 0.643083, 0.531081\}]$$

Out[*]=
-0.000948136

$$\text{In[*]} := \text{NIntegrate}\left[\left(\frac{1}{1 + \left(\left(\frac{1}{2.25(1-e^2)}\right)^{\frac{1}{-0.2}} - 1\right)^2}\right) \left(-\frac{3}{e^3} - \frac{e^2 - 3}{2e^4} \text{Log}\left[\frac{1+e}{1-e}\right]\right)\right],$$

$$\{e, 0.621922, 0.531081\}]$$

Out[*]=
-0.000528456

$$\text{In[*]} := \text{NIntegrate}\left[\left(\frac{1}{1 + \left(\left(\frac{1}{2.25(1-e^2)}\right)^{\frac{1}{-0.2}} - 1\right)^2}\right) \left(-\frac{3}{e^3} - \frac{e^2 - 3}{2e^4} \text{Log}\left[\frac{1+e}{1-e}\right]\right)\right],$$

$$\{e, 0.603337, 0.531081\}]$$

Out[*]=
-0.000314737

$$\text{In[*]} := \text{NIntegrate}\left[\left(\frac{1}{1 + \left(\left(\frac{1}{2.25(1-e^2)}\right)^{\frac{1}{-0.2}} - 1\right)^2}\right) \left(-\frac{3}{e^3} - \frac{e^2 - 3}{2e^4} \text{Log}\left[\frac{1+e}{1-e}\right]\right)\right],$$

$$\{e, 0.586604, 0.531081\}]$$

Out[*]=
-0.000191606

$$\text{In[*]} := \text{NIntegrate}\left[\left(\frac{1}{1 + \left(\left(\frac{1}{2.25(1-e^2)}\right)^{\frac{1}{-0.2}} - 1\right)^2}\right) \left(-\frac{3}{e^3} - \frac{e^2 - 3}{2e^4} \text{Log}\left[\frac{1+e}{1-e}\right]\right)\right], \{e, 0.57127, 0.531081\}]$$

Out[*]=
-0.000114299

```
In[*]:= NIntegrate[
$$\left[ \left( \frac{1}{1 + \left( \left( \frac{1}{2.25(1-e^2)} \right)^{\frac{1}{-0.2}} - 1 \right)^2} \right) \left( -\frac{3}{e^3} - \frac{e^2 - 3}{2e^4} \operatorname{Log}\left[ \frac{1+e}{1-e} \right] \right) \right],$$

{e, 0.557035, 0.531081}]
```

```
Out[*]=
-0.000062619
```

```
In[*]:= NIntegrate[
$$\left[ \left( \frac{1}{1 + \left( \left( \frac{1}{2.25(1-e^2)} \right)^{\frac{1}{-0.2}} - 1 \right)^2} \right) \left( -\frac{3}{e^3} - \frac{e^2 - 3}{2e^4} \operatorname{Log}\left[ \frac{1+e}{1-e} \right] \right) \right],$$

{e, 0.543691, 0.531081}]
```

```
Out[*]=
-0.0000263849
```

FZ Calculation

(*Homogenous - decreasing value alpha=0.1*)

```
In[*]:= NIntegrate[
$$\left( \frac{e^2 - 3}{e^4} \operatorname{Log}\left[ \frac{1+e}{1-e} \right] + \frac{6}{e^3} \right), \{e, 0.783, 0.851392\}]$$

```

```
Out[*]=
-0.0699695
```

```
In[*]:= NIntegrate[
$$\left( \frac{e^2 - 3}{e^4} \operatorname{Log}\left[ \frac{1+e}{1-e} \right] + \frac{6}{e^3} \right), \{e, 0.802012, 0.851392\}]$$

```

```
Out[*]=
-0.0526355
```

```
In[*]:= NIntegrate[
$$\left( \frac{e^2 - 3}{e^4} \operatorname{Log}\left[ \frac{1+e}{1-e} \right] + \frac{6}{e^3} \right), \{e, 0.814356, 0.851392\}]$$

```

```
Out[*]=
-0.0405926
```

```
In[*]:= NIntegrate[
$$\left( \frac{e^2 - 3}{e^4} \operatorname{Log}\left[ \frac{1+e}{1-e} \right] + \frac{6}{e^3} \right), \{e, 0.823332, 0.851392\}]$$

```

```
Out[*]=
-0.0314045
```

```
In[*]:= NIntegrate[
$$\left( \frac{e^2 - 3}{e^4} \operatorname{Log}\left[ \frac{1+e}{1-e} \right] + \frac{6}{e^3} \right), \{e, 0.830308, 0.851392\}]$$

```

```
Out[*]=
-0.0239937
```


$$\text{In[*]:= NIntegrate}\left[\left(\frac{e^2 - 3}{e^4} \log\left[\frac{1 + e}{1 - e}\right] + \frac{6}{e^3}\right), \{e, 0.835966, 0.851392\}\right]$$

$$\text{Out[*]=}$$

$$-0.0177988$$

$$\text{In[*]:= NIntegrate}\left[\left(\frac{e^2 - 3}{e^4} \log\left[\frac{1 + e}{1 - e}\right] + \frac{6}{e^3}\right), \{e, 0.8407, 0.851392\}\right]$$

$$\text{Out[*]=}$$

$$-0.0124825$$

$$\text{In[*]:= NIntegrate}\left[\left(\frac{e^2 - 3}{e^4} \log\left[\frac{1 + e}{1 - e}\right] + \frac{6}{e^3}\right), \{e, 0.844751, 0.851392\}\right]$$

$$\text{Out[*]=}$$

$$-0.00783274$$

$$\text{In[*]:= NIntegrate}\left[\left(\frac{e^2 - 3}{e^4} \log\left[\frac{1 + e}{1 - e}\right] + \frac{6}{e^3}\right), \{e, 0.848278, 0.851392\}\right]$$

$$\text{Out[*]=}$$

$$-0.00370608$$

(*Non-Homogenous - decreasing value alpha=0.1*)

$$\text{In[*]:= NIntegrate}\left[\left(\frac{1}{1 + \left(\left(\frac{1}{2.25(1 - e^2)}\right)^{\frac{1}{0.2}} - 1\right)^2}\right) \left(\frac{e^2 - 3}{e^4} \log\left[\frac{1 + e}{1 - e}\right] + \frac{6}{e^3}\right), \{e, 0.783, 0.851392\}\right]$$

$$\text{Out[*]=}$$

$$-0.0090734$$

$$\text{In[*]:= NIntegrate}\left[\left(\frac{1}{1 + \left(\left(\frac{1}{2.25(1 - e^2)}\right)^{\frac{1}{0.2}} - 1\right)^2}\right) \left(\frac{e^2 - 3}{e^4} \log\left[\frac{1 + e}{1 - e}\right] + \frac{6}{e^3}\right), \{e, 0.802012, 0.851392\}\right]$$

$$\text{Out[*]=}$$

$$-0.0033111$$

$$\text{In[*]:= NIntegrate}\left[\left(\frac{1}{1 + \left(\left(\frac{1}{2.25(1 - e^2)}\right)^{\frac{1}{0.2}} - 1\right)^2}\right) \left(\frac{e^2 - 3}{e^4} \log\left[\frac{1 + e}{1 - e}\right] + \frac{6}{e^3}\right), \{e, 0.814356, 0.851392\}\right]$$

$$\text{Out[*]=}$$

$$-0.00157186$$

$$\text{In[*]:= NIntegrate}\left[\left(\frac{1}{1 + \left(\left(\frac{1}{2.25(1 - e^2)}\right)^{\frac{1}{0.2}} - 1\right)^2}\right) \left(\frac{e^2 - 3}{e^4} \log\left[\frac{1 + e}{1 - e}\right] + \frac{6}{e^3}\right), \{e, 0.823332, 0.851392\}\right]$$

$$\text{Out[*]=}$$

$$-0.00085907$$

$$\text{In[*]:= NIntegrate}\left[\frac{1}{1 + \left(\left(\frac{1}{2.25(1-e^2)}\right)^{\frac{1}{0.2}} - 1\right)^2} \left(\frac{e^2 - 3}{e^4} \text{Log}\left[\frac{1+e}{1-e}\right] + \frac{6}{e^3}\right), \{e, 0.830308, 0.851392\}\right]$$

Out[*]=
-0.000504014

$$\text{In[*]:= NIntegrate}\left[\frac{1}{1 + \left(\left(\frac{1}{2.25(1-e^2)}\right)^{\frac{1}{0.2}} - 1\right)^2} \left(\frac{e^2 - 3}{e^4} \text{Log}\left[\frac{1+e}{1-e}\right] + \frac{6}{e^3}\right), \{e, 0.835966, 0.851392\}\right]$$

Out[*]=
-0.000303286

$$\text{In[*]:= NIntegrate}\left[\frac{1}{1 + \left(\left(\frac{1}{2.25(1-e^2)}\right)^{\frac{1}{0.2}} - 1\right)^2} \left(\frac{e^2 - 3}{e^4} \text{Log}\left[\frac{1+e}{1-e}\right] + \frac{6}{e^3}\right), \{e, 0.8407, 0.851392\}\right]$$

Out[*]=
-0.000179231

$$\text{In[*]:= NIntegrate}\left[\frac{1}{1 + \left(\left(\frac{1}{2.25(1-e^2)}\right)^{\frac{1}{0.2}} - 1\right)^2} \left(\frac{e^2 - 3}{e^4} \text{Log}\left[\frac{1+e}{1-e}\right] + \frac{6}{e^3}\right), \{e, 0.844751, 0.851392\}\right]$$

Out[*]=
-0.000097443

$$\text{In[*]:= NIntegrate}\left[\frac{1}{1 + \left(\left(\frac{1}{2.25(1-e^2)}\right)^{\frac{1}{0.2}} - 1\right)^2} \left(\frac{e^2 - 3}{e^4} \text{Log}\left[\frac{1+e}{1-e}\right] + \frac{6}{e^3}\right), \{e, 0.848278, 0.851392\}\right]$$

Out[*]=
-0.0000407966

In[*]:= (*Homogenous - decreasing value alpha=-0.1*)

$$\text{NIntegrate}\left[\left(\frac{e^2 - 3}{e^4} \text{Log}\left[\frac{1+e}{1-e}\right] + \frac{6}{e^3}\right), \{e, 0.69962, 0.531081\}\right]$$

Out[*]=
0.0828641

$$\text{In[*]:= NIntegrate}\left[\left(\frac{e^2 - 3}{e^4} \text{Log}\left[\frac{1+e}{1-e}\right] + \frac{6}{e^3}\right), \{e, 0.66809, 0.531081\}\right]$$

Out[*]=
0.0637394

$$\text{In[*]} := \text{NIntegrate}\left[\left(\frac{e^2 - 3}{e^4} \log\left[\frac{1 + e}{1 - e}\right] + \frac{6}{e^3}\right), \{e, 0.643083, 0.531081\}\right]$$

Out[*]=
0.0499423

$$\text{In[*]} := \text{NIntegrate}\left[\left(\frac{e^2 - 3}{e^4} \log\left[\frac{1 + e}{1 - e}\right] + \frac{6}{e^3}\right), \{e, 0.621922, 0.531081\}\right]$$

Out[*]=
0.0391119

$$\text{In[*]} := \text{NIntegrate}\left[\left(\frac{e^2 - 3}{e^4} \log\left[\frac{1 + e}{1 - e}\right] + \frac{6}{e^3}\right), \{e, 0.603337, 0.531081\}\right]$$

Out[*]=
0.0301843

$$\text{In[*]} := \text{NIntegrate}\left[\left(\frac{e^2 - 3}{e^4} \log\left[\frac{1 + e}{1 - e}\right] + \frac{6}{e^3}\right), \{e, 0.586604, 0.531081\}\right]$$

Out[*]=
0.0225808

$$\text{In[*]} := \text{NIntegrate}\left[\left(\frac{e^2 - 3}{e^4} \log\left[\frac{1 + e}{1 - e}\right] + \frac{6}{e^3}\right), \{e, 0.57127, 0.531081\}\right]$$

Out[*]=
0.0159528

$$\text{In[*]} := \text{NIntegrate}\left[\left(\frac{e^2 - 3}{e^4} \log\left[\frac{1 + e}{1 - e}\right] + \frac{6}{e^3}\right), \{e, 0.557035, 0.531081\}\right]$$

Out[*]=
0.0100752

$$\text{In[*]} := \text{NIntegrate}\left[\left(\frac{e^2 - 3}{e^4} \log\left[\frac{1 + e}{1 - e}\right] + \frac{6}{e^3}\right), \{e, 0.543691, 0.531081\}\right]$$

Out[*]=
0.00479474

(*Non-Homogenous - decreasing value alpha=-0.1*)

$$\text{In[*]} := \text{NIntegrate}\left[\left(\frac{1}{1 + \left(\left(\frac{1}{2.25(1 - e^2)}\right)^{\frac{1}{-0.2}} - 1\right)^2}\right) \left(\frac{e^2 - 3}{e^4} \log\left[\frac{1 + e}{1 - e}\right] + \frac{6}{e^3}\right), \{e, 0.69962, 0.531081\}\right]$$

Out[*]=
0.0102217

$$\text{In[*]} := \text{NIntegrate}\left[\left(\frac{1}{1 + \left(\left(\frac{1}{2.25(1 - e^2)}\right)^{\frac{1}{-0.2}} - 1\right)^2}\right) \left(\frac{e^2 - 3}{e^4} \log\left[\frac{1 + e}{1 - e}\right] + \frac{6}{e^3}\right), \{e, 0.66809, 0.531081\}\right]$$

Out[*]=
0.00388531

$$\text{In}[*]:= \text{NIntegrate}\left[\frac{1}{1 + \left(\left(\frac{1}{2.25(1-e^2)}\right)^{\frac{1}{-0.2}} - 1\right)^2} \left(\frac{e^2 - 3}{e^4} \text{Log}\left[\frac{1+e}{1-e}\right] + \frac{6}{e^3}\right), \{e, 0.643083, 0.531081\}\right]$$

Out[*]=
0.00189627

$$\text{In}[*]:= \text{NIntegrate}\left[\frac{1}{1 + \left(\left(\frac{1}{2.25(1-e^2)}\right)^{\frac{1}{-0.2}} - 1\right)^2} \left(\frac{e^2 - 3}{e^4} \text{Log}\left[\frac{1+e}{1-e}\right] + \frac{6}{e^3}\right), \{e, 0.621922, 0.531081\}\right]$$

Out[*]=
0.00105691

$$\text{In}[*]:= \text{NIntegrate}\left[\frac{1}{1 + \left(\left(\frac{1}{2.25(1-e^2)}\right)^{\frac{1}{-0.2}} - 1\right)^2} \left(\frac{e^2 - 3}{e^4} \text{Log}\left[\frac{1+e}{1-e}\right] + \frac{6}{e^3}\right), \{e, 0.603337, 0.531081\}\right]$$

Out[*]=
0.000629475

$$\text{In}[*]:= \text{NIntegrate}\left[\frac{1}{1 + \left(\left(\frac{1}{2.25(1-e^2)}\right)^{\frac{1}{-0.2}} - 1\right)^2} \left(\frac{e^2 - 3}{e^4} \text{Log}\left[\frac{1+e}{1-e}\right] + \frac{6}{e^3}\right), \{e, 0.586604, 0.531081\}\right]$$

Out[*]=
0.000383211

$$\text{In}[*]:= \text{NIntegrate}\left[\frac{1}{1 + \left(\left(\frac{1}{2.25(1-e^2)}\right)^{\frac{1}{-0.2}} - 1\right)^2} \left(\frac{e^2 - 3}{e^4} \text{Log}\left[\frac{1+e}{1-e}\right] + \frac{6}{e^3}\right), \{e, 0.57127, 0.531081\}\right]$$

Out[*]=
0.000228597

$$\text{In}[*]:= \text{NIntegrate}\left[\frac{1}{1 + \left(\left(\frac{1}{2.25(1-e^2)}\right)^{\frac{1}{-0.2}} - 1\right)^2} \left(\frac{e^2 - 3}{e^4} \text{Log}\left[\frac{1+e}{1-e}\right] + \frac{6}{e^3}\right), \{e, 0.557035, 0.531081\}\right]$$

Out[*]=
0.000125238

$$\text{In}[*]:= \text{NIntegrate}\left[\frac{1}{1 + \left(\left(\frac{1}{2.25(1-e^2)}\right)^{\frac{1}{-0.2}} - 1\right)^2} \left(\frac{e^2 - 3}{e^4} \text{Log}\left[\frac{1+e}{1-e}\right] + \frac{6}{e^3}\right), \{e, 0.543691, 0.531081\}\right]$$

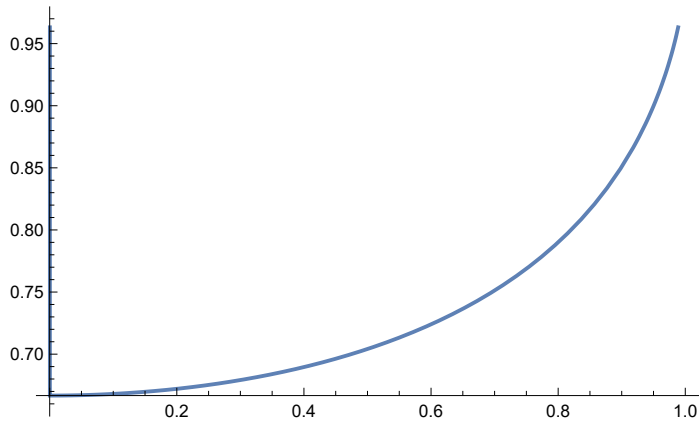
Out[*]=
0.0000527698

Plotting A1 and A3 wrt e

$$\text{In}[*]:= f[e_]:= \frac{1-e^2}{e^2} \left(\frac{1}{1-e^2} - \frac{1}{2e} \text{Log}\left[\frac{1+e}{1-e}\right] \right)$$

Plot[f[e], {e, 0, 1}]

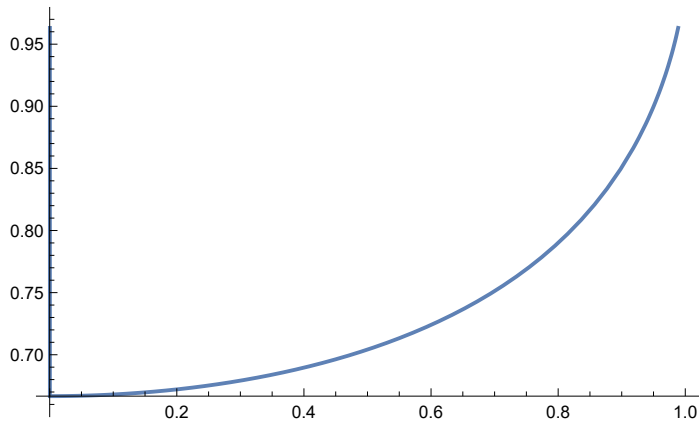
Out[*]=



$$f[x_]:= 2 \frac{1-x^2}{x^2} \left(\frac{1}{2x} \text{Log}\left[\frac{1+x}{1-x}\right] - 1 \right)$$

Plot[f[x], {x, 0, 1}]

Out[*]=



Calculation of FR Value:

$$\text{In[*]} := \text{NIntegrate}\left[\left(\frac{1}{1 + \left(\left(\frac{1}{2.25(1-e^2)}\right)^{\frac{1}{-0.2}} - 1\right)^2}\right) \left(-\frac{3}{e^3} - \frac{e^2 - 3}{2e^4} \text{Log}\left[\frac{1+e}{1-e}\right]\right), \{e, 0.69962, 0.531081\}\right]$$

Out[*]=
-0.00511083

$$\text{In[*]} := \text{NIntegrate}\left[\left(\frac{1}{1 + \left(\left(\frac{1}{2.25(1-e^2)}\right)^{\frac{1}{-0.2}} - 1\right)^2}\right) \left(-\frac{3}{e^3} - \frac{e^2 - 3}{2e^4} \text{Log}\left[\frac{1+e}{1-e}\right]\right), \{e, 0.69962, 0.519085\}\right]$$

Out[*]=
-0.00513064

$$\text{In[*]} := \text{NIntegrate}\left[\left(\frac{1}{1 + \left(\left(\frac{1}{2.25(1-e^2)}\right)^{\frac{1}{-0.2}} - 1\right)^2}\right) \left(-\frac{3}{e^3} - \frac{e^2 - 3}{2e^4} \text{Log}\left[\frac{1+e}{1-e}\right]\right), \{e, 0.69962, 0.427697\}\right]$$

Out[*]=
-0.00519885

$$\text{In[*]} := \frac{-0.00513064 - (-0.00511083)}{-0.00511083}$$

Out[*]=
0.00387608

$$\text{In[*]} := \frac{-0.00519885 - (-0.00513064)}{-0.00513064}$$

Out[*]=
0.0132946

Calculation of Fz :

$$\text{In[*]} := \text{NIntegrate}\left[\left(\frac{1}{1 + \left(\left(\frac{1}{2.25(1-e^2)}\right)^{\frac{1}{-0.2}} - 1\right)^2}\right) \left(\frac{e^2 - 3}{e^4} \text{Log}\left[\frac{1+e}{1-e}\right] + \frac{6}{e^3}\right), \{e, 0.69962, 0.531081\}\right]$$

Out[*]=
0.0102217

$$In[*]:= \text{NIntegrate}\left[\left(\frac{1}{1 + \left(\left(\frac{1}{2.25(1-e^2)}\right)^{\frac{1}{-0.2}} - 1\right)^2}\right) \left(\frac{e^2 - 3}{e^4} \text{Log}\left[\frac{1+e}{1-e}\right] + \frac{6}{e^3}\right), \{e, 0.69962, 0.519085\}\right]$$

Out[*]=
0.0102613

$$In[*]:= \text{NIntegrate}\left[\left(\frac{1}{1 + \left(\left(\frac{1}{2.25(1-e^2)}\right)^{\frac{1}{-0.2}} - 1\right)^2}\right) \left(\frac{e^2 - 3}{e^4} \text{Log}\left[\frac{1+e}{1-e}\right] + \frac{6}{e^3}\right), \{e, 0.69962, 0.427697\}\right]$$

Out[*]=
0.0103977

$$In[*]:= \frac{0.0102613 - 0.0102217}{0.0102217}$$

Out[*]=
0.00387411

$$In[*]:= \frac{0.0103977 - 0.0102613}{0.0102613}$$

Out[*]=
0.0132927

A1 and A3 Calculation at 1,3 and 8:

$$In[*]:= \frac{1 - 0.783^2}{0.783^2} \left(\frac{1}{1 - 0.783^2} - \frac{1}{2.0 \times .783} \text{Log}\left[\frac{1 + 0.783}{1 - 0.783}\right] \right)$$

Out[*]=
0.782322

$$In[*]:= \frac{1 - 0.814356^2}{0.814356^2} \left(\frac{1}{1 - 0.814356^2} - \frac{1}{2.00 \times .814356} \text{Log}\left[\frac{1 + 0.814356}{1 - 0.814356}\right] \right)$$

Out[*]=
0.797011

$$In[*]:= \frac{1 - 0.844751^2}{0.844751^2} \left(\frac{1}{1 - 0.844751^2} - \frac{1}{2.000 \times .844751} \text{Log}\left[\frac{1 + 0.844751}{1 - 0.844751}\right] \right)$$

Out[*]=
0.81339

$$In[*]:= \frac{1 - 0.69962^2}{0.69962^2} \left(\frac{1}{1 - 0.69962^2} - \frac{1}{2.000 \times .69962} \text{Log}\left[\frac{1 + 0.69962}{1 - 0.69962}\right] \right)$$

Out[*]=
0.751122

$$\text{In}[*]:= \frac{1 - 0.643083^2}{0.643083^2} \left(\frac{1}{1 - 0.643083^2} - \frac{1}{2.0000 \times .643083} \text{Log} \left[\frac{1 + 0.643083}{1 - 0.643083} \right] \right)$$

Out[*]=

0.734662

$$\text{In}[7]:= \frac{1 - 0.557035^2}{0.557035^2} \left(\frac{1}{1 - 0.557035^2} - \frac{1}{2.00000 \times .557035} \text{Log} \left[\frac{1 + 0.557035}{1 - 0.557035} \right] \right)$$

Out[7]= 0.714728

$$\text{In}[*]:= 2 \frac{1 - 0.783^2}{0.783^2} \left(\frac{1}{2.0 \times .783} \text{Log} \left[\frac{1 + 0.783}{1 - 0.783} \right] - 1 \right)$$

Out[*]=

0.435356

$$\text{In}[*]:= 2 \frac{1 - 0.814356^2}{0.814356^2} \left(\frac{1}{2.00 \times .814356} \text{Log} \left[\frac{1 + 0.814356}{1 - 0.814356} \right] - 1 \right)$$

Out[*]=

0.405979

$$\text{In}[12]:= 2 \frac{1 - 0.844751^2}{0.844751^2} \left(\frac{1}{2.000 \times .844751} \text{Log} \left[\frac{1 + 0.844751}{1 - 0.844751} \right] - 1 \right)$$

Out[12]=

0.373219

$$\text{In}[*]:= 2 \frac{1 - 0.69962^2}{0.69962^2} \left(\frac{1}{2.000 \times .69962} \text{Log} \left[\frac{1 + 0.69962}{1 - 0.69962} \right] - 1 \right)$$

Out[*]=

0.497755

$$\text{In}[*]:= 2 \frac{1 - 0.643083^2}{0.643083^2} \left(\frac{1}{2.0000 \times .643083} \text{Log} \left[\frac{1 + 0.643083}{1 - 0.643083} \right] - 1 \right)$$

Out[*]=

0.530677

$$\text{In}[13]:= 2 \frac{1 - 0.557035^2}{0.557035^2} \left(\frac{1}{2.00000 \times .557035} \text{Log} \left[\frac{1 + 0.557035}{1 - 0.557035} \right] - 1 \right)$$

Out[13]=

0.570544

Calculation:

(*FR Calculation for alpha =0.1*)

$$\text{In}[*]:= 0.0045367 / 0.782322$$

Out[*]=

0.00579902

$$\text{In}[*]:= 0.000785932 / 0.797011$$

Out[*]=

0.000986099


```
In[6]:= 0.0000487215 / 0.81339
```

```
Out[6]= 0.0000598993
```

```
(*FR Calculation for alpha =-0.1*)
```

```
In[*]:= -0.00511083 / 0.751122
```

```
Out[*]=
```

```
-0.00680426
```

```
In[*]:= -0.000948136 / 0.734662
```

```
Out[*]=
```

```
-0.00129057
```

```
In[8]:= -0.000062619 / 0.714728
```

```
Out[8]= -0.0000876124
```

```
(*Fz Calculation for alpha =0.1*)
```

```
In[*]:= -0.0090734 / 0.435356
```

```
Out[*]=
```

```
-0.0208413
```

```
In[*]:= -0.00157186 / 0.405979
```

```
Out[*]=
```

```
-0.00387178
```

```
In[14]:= -0.000097443 / 0.373219
```

```
Out[14]=
```

```
-0.000261088
```

```
(*Fz Calculation for alpha =-0.1*)
```

```
In[*]:= 0.0102217 / 0.497755
```

```
Out[*]=
```

```
0.0205356
```

```
In[*]:= 0.00189627 / 0.530677
```

```
Out[*]=
```

```
0.0035733
```

```
In[15]:= 0.000125238 / 0.570544
```

```
Out[15]=
```

```
0.000219506
```

Rho and A derivative :

```
In[*]:= \Quit
```

$$f[e_-] := \left(\frac{1}{1 + \left(\left(\frac{1}{\beta^2 (1+e^2)} \right)^{\frac{1}{2\alpha}} - 1 \right)^2} \right) \left(\frac{1-e^2}{e^2} \left(\frac{1}{1-e^2} - \frac{1}{2e} \operatorname{Log} \left[\frac{1+e}{1-e} \right] \right) \right)$$

In[*]:= D[f[e], e] \\ Simplify

Out[*]=

$$\frac{(1 - e^2) \left(\frac{2e}{(1 - e^2)^2} - \frac{(1 - e) \left(\frac{1}{1 - e} + \frac{1 + e}{(1 - e)^2} \right)}{2e(1 + e)} + \frac{\text{Log}\left[\frac{1 + e}{1 - e}\right]}{2e^2} \right)}{e^2 \left(1 + \left(-1 + \left(\frac{1}{(1 + e^2)\beta^2} \right)^{\frac{1}{2/\alpha}} \right)^2 \right)} - \frac{2 \left(\frac{1}{1 - e^2} - \frac{\text{Log}\left[\frac{1 + e}{1 - e}\right]}{2e} \right)}{e \left(1 + \left(-1 + \left(\frac{1}{(1 + e^2)\beta^2} \right)^{\frac{1}{2/\alpha}} \right)^2 \right)}$$

$$+ \frac{2(1 - e^2) \left(\frac{1}{1 - e^2} - \frac{\text{Log}\left[\frac{1 + e}{1 - e}\right]}{2e} \right)}{e^3 \left(1 + \left(-1 + \left(\frac{1}{(1 + e^2)\beta^2} \right)^{\frac{1}{2/\alpha}} \right)^2 \right)} + \frac{2(1 - e^2) \left(-1 + \left(\frac{1}{(1 + e^2)\beta^2} \right)^{\frac{1}{2/\alpha}} \right) \left(\frac{1}{(1 + e^2)\beta^2} \right)^{-1 + \frac{1}{2\alpha}} \left(\frac{1}{1 - e^2} - \frac{\text{Log}\left[\frac{1 + e}{1 - e}\right]}{2e} \right)}{e(1 + e^2)^2 \alpha \left(1 + \left(-1 + \left(\frac{1}{(1 + e^2)\beta^2} \right)^{\frac{1}{2/\alpha}} \right)^2 \right)^2 \beta^2}$$

In[*]:= f'[y]

Out[*]=

f'[y]

$$f[y_]:= \left(\frac{1}{1 + \left(\left(\frac{1}{\beta^2(1 + y^2)} \right)^{\frac{1}{2\alpha}} - 1 \right)^2} \right) \left(\frac{1 - y^2}{y^2} \left(\frac{1}{1 - y^2} - \frac{1}{2y} \text{Log}\left[\frac{1 + y}{1 - y}\right] \right) \right)$$

$$In[*]:= f[y_]:= \left(\frac{1}{1 + \left(\left(\frac{1}{2.25(1 + y^2)} \right)^{\frac{1}{0.2}} - 1 \right)^2} \right) \left(\frac{1 - y^2}{y^2} \left(\frac{1}{1 - y^2} - \frac{1}{2y} \text{Log}\left[\frac{1 + y}{1 - y}\right] \right) \right)$$

In[*]:= D[f[y], y]

Out[*]=

$$\frac{(1 - y^2) \left(\frac{2y}{(1 - y^2)^2} - \frac{(1 - y) \left(\frac{1}{1 - y} + \frac{1 + y}{(1 - y)^2} \right)}{2y(1 + y)} + \frac{\text{Log}\left[\frac{1 + y}{1 - y}\right]}{2y^2} \right)}{y^2 \left(1 + \left(-1 + 0.0173415 \left(\frac{1}{1 + y^2} \right)^{5.} \right)^2 \right)} +$$

$$\frac{0.346831 (1 - y^2) \left(\frac{1}{1 + y^2} \right)^{6.} \left(-1 + 0.0173415 \left(\frac{1}{1 + y^2} \right)^{5.} \right) \left(\frac{1}{1 - y^2} - \frac{\text{Log}\left[\frac{1 + y}{1 - y}\right]}{2y} \right)}{y \left(1 + \left(-1 + 0.0173415 \left(\frac{1}{1 + y^2} \right)^{5.} \right)^2 \right)^2} -$$

$$\frac{2 \left(\frac{1}{1 - y^2} - \frac{\text{Log}\left[\frac{1 + y}{1 - y}\right]}{2y} \right)}{y \left(1 + \left(-1 + 0.0173415 \left(\frac{1}{1 + y^2} \right)^{5.} \right)^2 \right)} - \frac{2(1 - y^2) \left(\frac{1}{1 - y^2} - \frac{\text{Log}\left[\frac{1 + y}{1 - y}\right]}{2y} \right)}{y^3 \left(1 + \left(-1 + 0.0173415 \left(\frac{1}{1 + y^2} \right)^{5.} \right)^2 \right)}$$

$$\begin{aligned}
\text{In}[*]:= & \text{NIntegrate} \left[\frac{(1-y^2) \left(\frac{2y}{(1-y^2)^2} - \frac{(1-y) \left(\frac{1}{1-y} + \frac{1+y}{(1-y)^2} \right) + \frac{\text{Log}\left[\frac{1+y}{1-y}\right]}{2y^2}}{y^2 \left(1 + \left(-1 + 0.01734152991583261 \cdot \left(\frac{1}{1+y^2} \right)^{5.} \right)^2 \right)} + \right. \\
& \left(0.3468305983166522 \cdot (1-y^2) \left(\frac{1}{1+y^2} \right)^{6.} \cdot \left(-1 + 0.01734152991583261 \cdot \left(\frac{1}{1+y^2} \right)^{5.} \right) \right. \\
& \left. \left. \left(\frac{1}{1-y^2} - \frac{\text{Log}\left[\frac{1+y}{1-y}\right]}{2y} \right) \right) \right] / \left(y \left(1 + \left(-1 + 0.01734152991583261 \cdot \left(\frac{1}{1+y^2} \right)^{5.} \right)^2 \right)^2 \right) - \\
& \frac{2 \left(\frac{1}{1-y^2} - \frac{\text{Log}\left[\frac{1+y}{1-y}\right]}{2y} \right)}{y \left(1 + \left(-1 + 0.01734152991583261 \cdot \left(\frac{1}{1+y^2} \right)^{5.} \right)^2 \right)} - \\
& \frac{2 (1-y^2) \left(\frac{1}{1-y^2} - \frac{\text{Log}\left[\frac{1+y}{1-y}\right]}{2y} \right)}{y^3 \left(1 + \left(-1 + 0.01734152991583261 \cdot \left(\frac{1}{1+y^2} \right)^{5.} \right)^2 \right)}, \{y, 0.783, 0.851392\} \Big]
\end{aligned}$$

Out[*]=

0.0173352

$$\begin{aligned}
\text{In}[*]:= & \text{NIntegrate} \left[\frac{(1-y^2) \left(\frac{2y}{(1-y^2)^2} - \frac{(1-y) \left(\frac{1}{1-y} + \frac{1+y}{(1-y)^2} \right) + \frac{\text{Log}\left[\frac{1+y}{1-y}\right]}{2y^2}}{y^2 \left(1 + \left(-1 + 0.01734152991583261 \cdot \left(\frac{1}{1+y^2} \right)^{5.} \right)^2 \right)} + \right. \\
& \left(0.3468305983166522 \cdot (1-y^2) \left(\frac{1}{1+y^2} \right)^{6.} \cdot \left(-1 + 0.01734152991583261 \cdot \left(\frac{1}{1+y^2} \right)^{5.} \right) \right. \\
& \left. \left. \left(\frac{1}{1-y^2} - \frac{\text{Log}\left[\frac{1+y}{1-y}\right]}{2y} \right) \right) \right] / \left(y \left(1 + \left(-1 + 0.01734152991583261 \cdot \left(\frac{1}{1+y^2} \right)^{5.} \right)^2 \right)^2 \right) - \\
& \frac{2 \left(\frac{1}{1-y^2} - \frac{\text{Log}\left[\frac{1+y}{1-y}\right]}{2y} \right)}{y \left(1 + \left(-1 + 0.01734152991583261 \cdot \left(\frac{1}{1+y^2} \right)^{5.} \right)^2 \right)} - \\
& \frac{2 (1-y^2) \left(\frac{1}{1-y^2} - \frac{\text{Log}\left[\frac{1+y}{1-y}\right]}{2y} \right)}{y^3 \left(1 + \left(-1 + 0.01734152991583261 \cdot \left(\frac{1}{1+y^2} \right)^{5.} \right)^2 \right)}, \{y, 0.814356, 0.851392\} \Big]
\end{aligned}$$

Out[*]=

0.0100692

$$\begin{aligned}
\text{In}[*]:= & \text{NIntegrate} \left[\frac{(1-y^2) \left(\frac{2y}{(1-y^2)^2} - \frac{(1-y) \left(\frac{1}{1-y} + \frac{1+y}{(1-y)^2} \right) + \frac{\text{Log}\left[\frac{1+y}{1-y}\right]}{2y^2}}{y^2 \left(1 + \left(-1 + 0.01734152991583261 \left(\frac{1}{1+y^2} \right)^{5.} \right)^2 \right)} + \right. \\
& \left(0.3468305983166522 \left(1-y^2 \right) \left(\frac{1}{1+y^2} \right)^{6.} \left(-1 + 0.01734152991583261 \left(\frac{1}{1+y^2} \right)^{5.} \right) \right. \\
& \left. \left(\frac{1}{1-y^2} - \frac{\text{Log}\left[\frac{1+y}{1-y}\right]}{2y} \right) \right) \Bigg] / \left(y \left(1 + \left(-1 + 0.01734152991583261 \left(\frac{1}{1+y^2} \right)^{5.} \right)^2 \right)^2 \right) - \\
& \frac{2 \left(\frac{1}{1-y^2} - \frac{\text{Log}\left[\frac{1+y}{1-y}\right]}{2y} \right)}{y \left(1 + \left(-1 + 0.01734152991583261 \left(\frac{1}{1+y^2} \right)^{5.} \right)^2 \right)} - \\
& \frac{2 (1-y^2) \left(\frac{1}{1-y^2} - \frac{\text{Log}\left[\frac{1+y}{1-y}\right]}{2y} \right)}{y^3 \left(1 + \left(-1 + 0.01734152991583261 \left(\frac{1}{1+y^2} \right)^{5.} \right)^2 \right)}, \{y, 0.844751, 0.851392\}]
\end{aligned}$$

Out[*]=

0.00194501

A1 alpha negative calculation: (wrong)

$$\text{In}[*]:= \frac{1 - 0.69962^2}{0.69962^2} \left(\frac{1}{1 - 0.69962^2} - \frac{1}{2.00 \times .69962} \text{Log} \left[\frac{1 + 0.69962}{1 - 0.69962} \right] \right)$$

Out[*]=

0.751122

$$\text{In}[*]:= \frac{1 - 0.643083^2}{0.643083^2} \left(\frac{1}{1 - 0.643083^2} - \frac{1}{2.000 \times .643083} \text{Log} \left[\frac{1 + 0.643083}{1 - 0.643083} \right] \right)$$

Out[*]=

0.734662

$$\text{In}[*]:= \frac{1 - 0.557035^2}{0.557035^2} \left(\frac{1}{1 - 0.557035^2} - \frac{1}{2.0000 \times .557035} \text{Log} \left[\frac{1 + 0.557035}{1 - 0.557035} \right] \right)$$

Out[*]=

0.714728

$$\text{In}[*]:= \mathbf{f}[\mathbf{c_}] := \left(\frac{1}{1 + \left(\left(\frac{1}{2.25 (1+c^2)} \right)^{\frac{1}{-0.2}} - 1 \right)^2} \right) \left(\frac{1-c^2}{c^2} \left(\frac{1}{1-c^2} - \frac{1}{2c} \log \left[\frac{1+c}{1-c} \right] \right) \right)$$

D[f[c], c]

Out[*]=

$$\frac{(1-c^2) \left(\frac{2c}{(1-c^2)^2} - \frac{(1-c) \left(\frac{1}{1-c} + \frac{1+c}{(1-c)^2} \right) + \frac{\log \left[\frac{1+c}{1-c} \right]}{2c^2}}{c^2 \left(1 + \left(-1 + \frac{57.665}{\left(\frac{1}{1+c^2} \right)^5} \right)^2 \right)} \right)}{c^2 \left(1 + \left(-1 + \frac{57.665}{\left(\frac{1}{1+c^2} \right)^5} \right)^2 \right)} - \frac{1153.3 (1-c^2) \left(-1 + \frac{57.665}{\left(\frac{1}{1+c^2} \right)^5} \right) \left(\frac{1}{1-c^2} - \frac{\log \left[\frac{1+c}{1-c} \right]}{2c} \right)}{c \left(\frac{1}{1+c^2} \right)^4 \cdot \left(1 + \left(-1 + \frac{57.665}{\left(\frac{1}{1+c^2} \right)^5} \right)^2 \right)^2} -$$

$$\frac{2 \left(\frac{1}{1-c^2} - \frac{\log \left[\frac{1+c}{1-c} \right]}{2c} \right)}{c \left(1 + \left(-1 + \frac{57.665}{\left(\frac{1}{1+c^2} \right)^5} \right)^2 \right)} - \frac{2 (1-c^2) \left(\frac{1}{1-c^2} - \frac{\log \left[\frac{1+c}{1-c} \right]}{2c} \right)}{c^3 \left(1 + \left(-1 + \frac{57.665}{\left(\frac{1}{1+c^2} \right)^5} \right)^2 \right)}$$

$$\text{In}[*]:= \mathbf{NIntegrate} \left[\frac{(1-c^2) \left(\frac{2c}{(1-c^2)^2} - \frac{(1-c) \left(\frac{1}{1-c} + \frac{1+c}{(1-c)^2} \right) + \frac{\log \left[\frac{1+c}{1-c} \right]}{2c^2}}{c^2 \left(1 + \left(-1 + \frac{57.665039062500014}{\left(\frac{1}{1+c^2} \right)^5} \right)^2 \right)} \right)}{c^2 \left(1 + \left(-1 + \frac{57.665039062500014}{\left(\frac{1}{1+c^2} \right)^5} \right)^2 \right)} - \right.$$

$$\frac{1153.3007812500002 (1-c^2) \left(-1 + \frac{57.665039062500014}{\left(\frac{1}{1+c^2} \right)^5} \right) \left(\frac{1}{1-c^2} - \frac{\log \left[\frac{1+c}{1-c} \right]}{2c} \right)}{c \left(\frac{1}{1+c^2} \right)^4 \cdot \left(1 + \left(-1 + \frac{57.665039062500014}{\left(\frac{1}{1+c^2} \right)^5} \right)^2 \right)^2} -$$

$$\frac{2 \left(\frac{1}{1-c^2} - \frac{\log \left[\frac{1+c}{1-c} \right]}{2c} \right)}{c \left(1 + \left(-1 + \frac{57.665039062500014}{\left(\frac{1}{1+c^2} \right)^5} \right)^2 \right)} - \frac{2 (1-c^2) \left(\frac{1}{1-c^2} - \frac{\log \left[\frac{1+c}{1-c} \right]}{2c} \right)}{c^3 \left(1 + \left(-1 + \frac{57.665039062500014}{\left(\frac{1}{1+c^2} \right)^5} \right)^2 \right)}, \{c, 0.69962, 0.531081\} \right]$$

Out[*]=

0.0000137471

$$\begin{aligned}
& \text{In}[*]:= \text{NIntegrate} \left[\frac{(1-c^2) \left(\frac{2c}{(1-c^2)^2} - \frac{(1-c) \left(\frac{1}{1-c} + \frac{1+c}{(1-c)^2} \right) + \frac{\text{Log}\left[\frac{1+c}{1-c}\right]}{2c^2}} \right)}{c^2 \left(1 + \left(-1 + \frac{57.665039062500014}{\left(\frac{1}{1+c^2} \right)^{5.}} \right)^2 \right)} - \right. \\
& \frac{1153.3007812500002 (1-c^2) \left(-1 + \frac{57.665039062500014}{\left(\frac{1}{1+c^2} \right)^{5.}} \right) \left(\frac{1}{1-c^2} - \frac{\text{Log}\left[\frac{1+c}{1-c}\right]}{2c} \right)}{c \left(\frac{1}{1+c^2} \right)^{4.} \left(1 + \left(-1 + \frac{57.665039062500014}{\left(\frac{1}{1+c^2} \right)^{5.}} \right)^2 \right)^2} - \\
& \frac{2 \left(\frac{1}{1-c^2} - \frac{\text{Log}\left[\frac{1+c}{1-c}\right]}{2c} \right)}{c \left(1 + \left(-1 + \frac{57.665039062500014}{\left(\frac{1}{1+c^2} \right)^{5.}} \right)^2 \right)} - \\
& \left. \frac{2(1-c^2) \left(\frac{1}{1-c^2} - \frac{\text{Log}\left[\frac{1+c}{1-c}\right]}{2c} \right)}{c^3 \left(1 + \left(-1 + \frac{57.665039062500014}{\left(\frac{1}{1+c^2} \right)^{5.}} \right)^2 \right)}, \{c, 0.643083, 0.531081\} \right]
\end{aligned}$$

Out[*]=

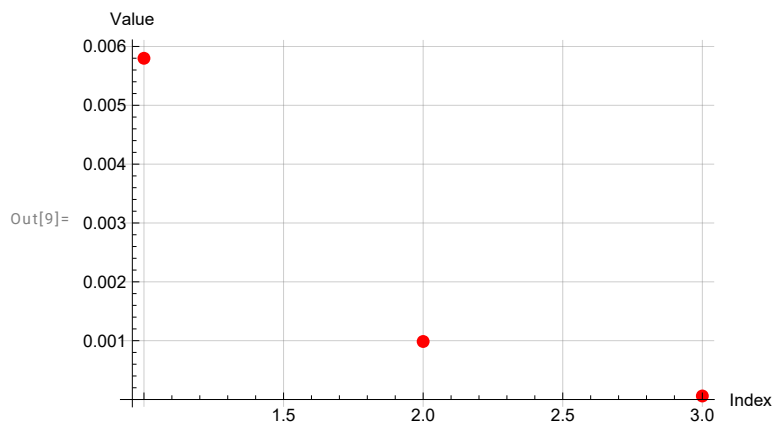
0.0000109911

$$\begin{aligned}
& \text{In}[*]:= \text{NIntegrate} \left[\frac{(1-c^2) \left(\frac{2c}{(1-c^2)^2} - \frac{(1-c) \left(\frac{1}{1-c} + \frac{1+c}{(1-c)^2} \right) + \frac{\text{Log}\left[\frac{1+c}{1-c}\right]}{2c^2}} \right)}{c^2 \left(1 + \left(-1 + \frac{57.665039062500014}{\left(\frac{1}{1+c^2} \right)^{5.}} \right)^2 \right)} - \right. \\
& \frac{1153.3007812500002 (1-c^2) \left(-1 + \frac{57.665039062500014}{\left(\frac{1}{1+c^2} \right)^{5.}} \right) \left(\frac{1}{1-c^2} - \frac{\text{Log}\left[\frac{1+c}{1-c}\right]}{2c} \right)}{c \left(\frac{1}{1+c^2} \right)^{4.} \left(1 + \left(-1 + \frac{57.665039062500014}{\left(\frac{1}{1+c^2} \right)^{5.}} \right)^2 \right)^2} - \\
& \frac{2 \left(\frac{1}{1-c^2} - \frac{\text{Log}\left[\frac{1+c}{1-c}\right]}{2c} \right)}{c \left(1 + \left(-1 + \frac{57.665039062500014}{\left(\frac{1}{1+c^2} \right)^{5.}} \right)^2 \right)} - \\
& \left. \frac{2(1-c^2) \left(\frac{1}{1-c^2} - \frac{\text{Log}\left[\frac{1+c}{1-c}\right]}{2c} \right)}{c^3 \left(1 + \left(-1 + \frac{57.665039062500014}{\left(\frac{1}{1+c^2} \right)^{5.}} \right)^2 \right)}, \{c, 0.557035, 0.531081\} \right]
\end{aligned}$$

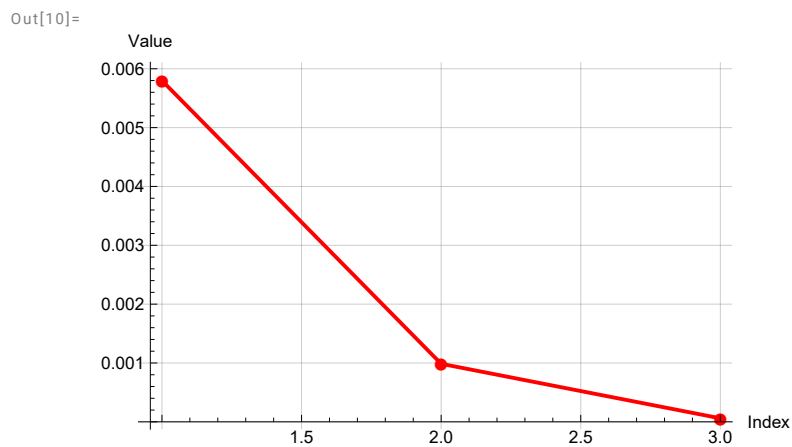
In[*]:= 3.430755385419286`*^-6 \ Simplify

Plotting

```
In[9]:= ListPlot[{{1, 0.00579902}, {2, 0.000986099}, {3, 0.0000598993}},  
  PlotStyle → {Red, PointSize[Large]}, AxesLabel → {"Index", "Value"},  
  PlotMarkers → Automatic, GridLines → Automatic]
```

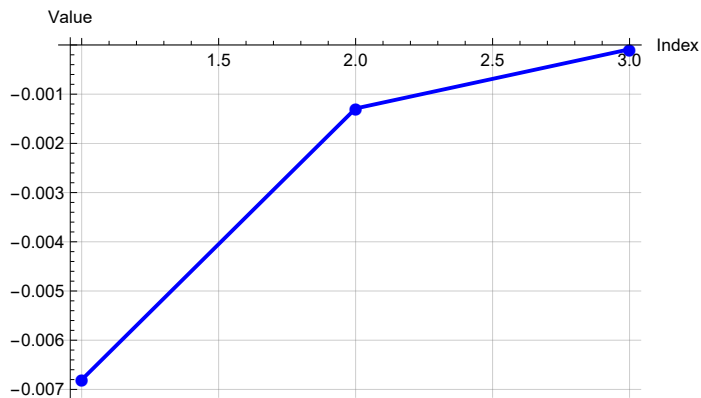


```
In[10]:= ListLinePlot[{{1, 0.00579902}, {2, 0.000986099}, {3, 0.0000598993}},  
  PlotStyle → {Red, Thick}, AxesLabel → {"Index", "Value"},  
  PlotMarkers → {"●", 10}, GridLines → Automatic]
```



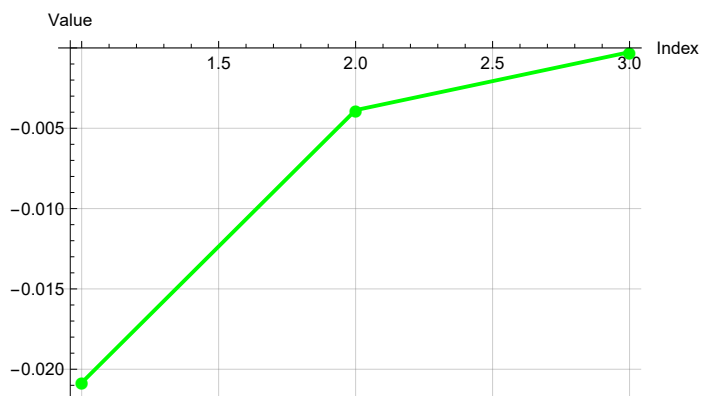
```
In[11]:= ListLinePlot[{{1, -0.00680426}, {2, -0.00129057}, {3, -0.0000876124}},  
  PlotStyle → {Blue, Thick}, AxesLabel → {"Index", "Value"},  
  PlotMarkers → {"●", 10}, GridLines → Automatic]
```

Out[11]=



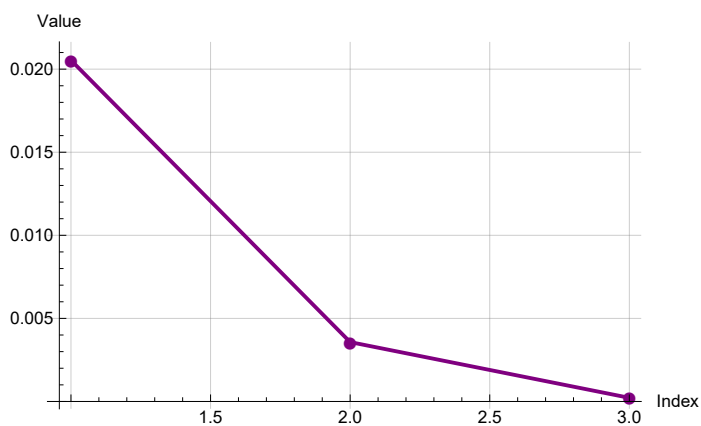
```
In[16]:= ListLinePlot[{{1, -0.0208413}, {2, -0.00387178}, {3, -0.000261088}},  
  PlotStyle → {Green, Thick}, AxesLabel → {"Index", "Value"},  
  PlotMarkers → {"●", 10}, GridLines → Automatic]
```

Out[16]=



```
In[17]:= ListLinePlot[{{1, 0.0205356}, {2, 0.0035733}, {3, 0.000219506}},  
  PlotStyle → {Purple, Thick}, AxesLabel → {"Index", "Value"},  
  PlotMarkers → {"●", 10}, GridLines → Automatic]
```

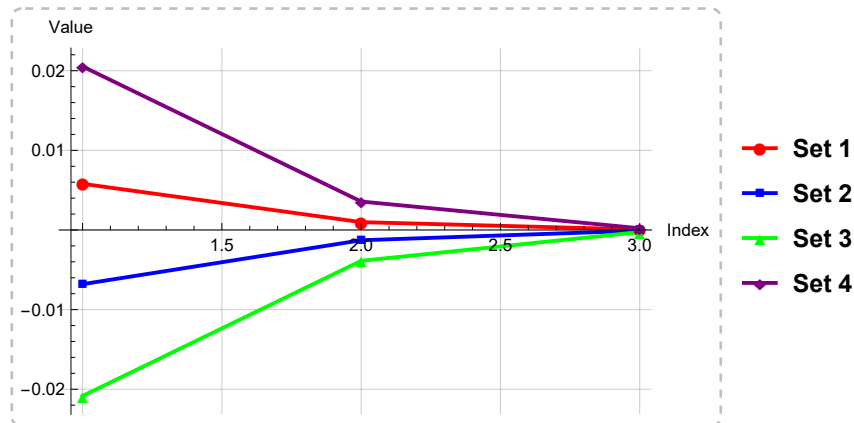
Out[17]=




```

In[19]:= ListLinePlot[{{1, 0.00579902}, {2, 0.000986099}, {3, 0.0000598993}},
  (*Set 1*) {{1, -0.00680426}, {2, -0.00129057}, {3, -0.0000876124}},
  (*Set 2*) {{1, -0.0208413}, {2, -0.00387178}, {3, -0.000261088}}, (*Set 3*)
  {{1, 0.0205356}, {2, 0.0035733}, {3, 0.000219506}} (*Set 4*)],
PlotStyle -> {Red, Blue, Green, Purple},
PlotMarkers -> {{{"●", 10}, {"■", 10}, {"▲", 10}, {"◆", 10}},
AxesLabel -> {"Index", "Value"}, GridLines -> Automatic,
PlotLegends -> {"Set 1", "Set 2", "Set 3", "Set 4"}]

```



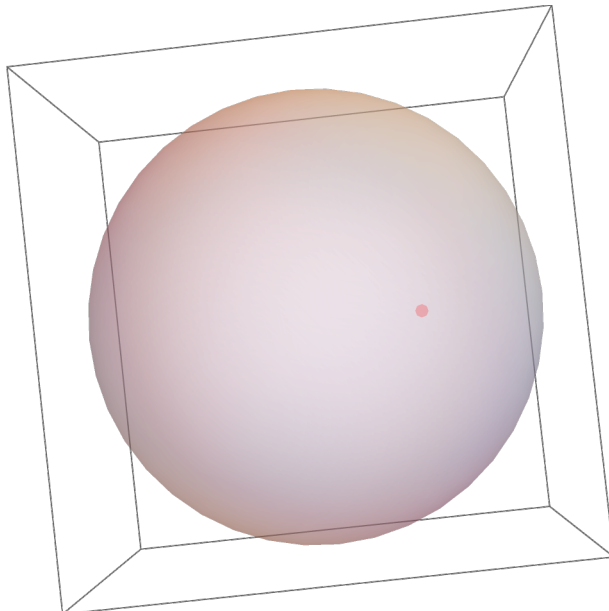
```

In[20]:= Graphics3D[{Opacity[0.4], Sphere[{0, 0, 0}, {1, 1, 2}],
  Red, PointSize[Large], Point[{0.5, 0, 0.8}]]]

```

3D Modeling :

Out[20]=



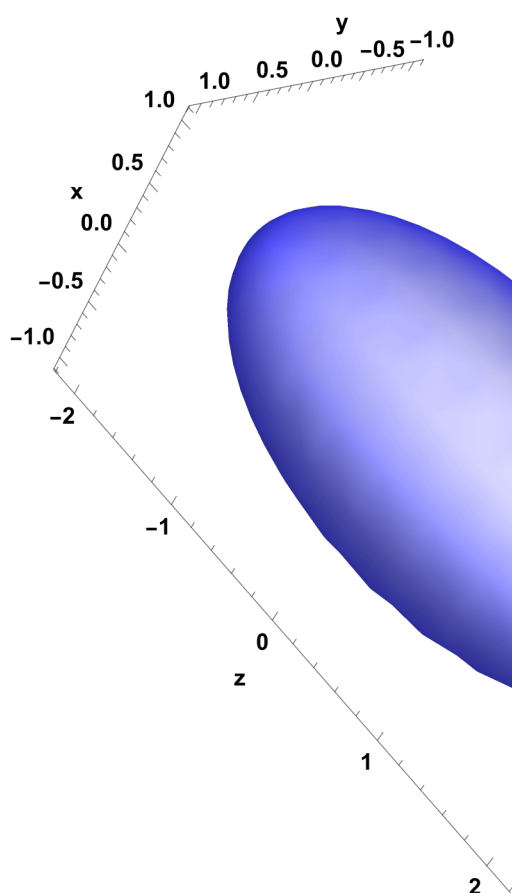
```

In[24]:= (*Parameters for a prolate spheroid*) a = 1; (*Semi-minor axis*)
c = 2; (*Semi-major axis, c>a for prolate*)

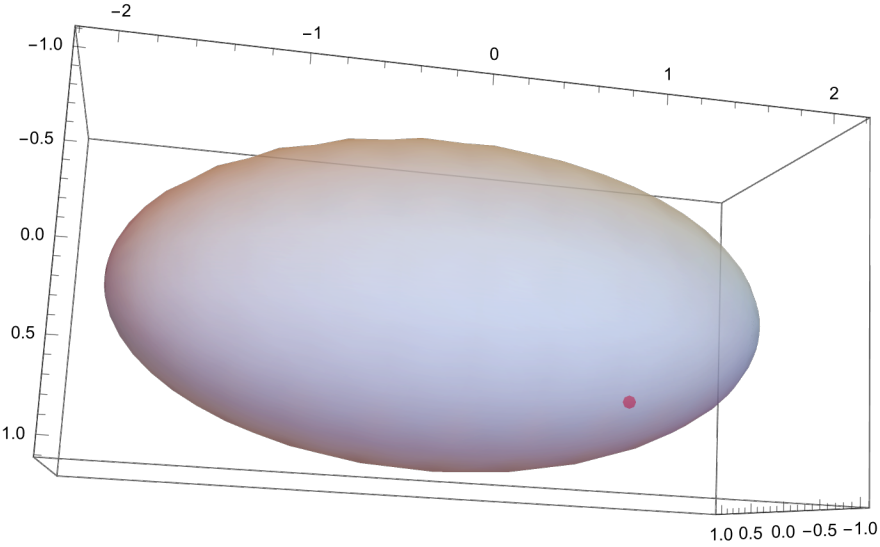
(*Parametric plot of the surface*)
ParametricPlot3D[{a Sin[ $\theta$ ] Cos[ $\varphi$ ], (*x-coordinate*)
  a Sin[ $\theta$ ] Sin[ $\varphi$ ], (*y-coordinate*) c Cos[ $\theta$ ] (*z-coordinate*)},
  { $\theta$ , 0, Pi}, { $\varphi$ , 0, 2 Pi}, PlotStyle → Directive[Opacity[0.6], Blue],
  Mesh → None, AxesLabel → {"x", "y", "z"}, Boxed → False, PlotRange → All,
  Lighting → "Neutral", LabelStyle → Directive[Bold, Medium]]
Show[ParametricPlot3D[{a Sin[ $\theta$ ] Cos[ $\varphi$ ], a Sin[ $\theta$ ] Sin[ $\varphi$ ], c Cos[ $\theta$ ]}, { $\theta$ , 0, Pi},
  { $\varphi$ , 0, 2 Pi}, PlotStyle → Directive[Opacity[0.6], LightBlue], Mesh → None], Graphics3D[
  {Red, PointSize[Large], Point[{0.5, 0, 1.2}] (*example point inside*)}]]

```

Out[26]=



Out[27]=



```
In[36]:= (*Focal distance*) f = 1;
```

```
(*Define the prolate spheroid shape with fixed foci at ±f on the z-axis*)
prolateSpheroid[a_, f_] := Module[{c, b}, c = Sqrt[a^2 + f^2];
  (*semi-major axis*) b = a;
  (*semi-minor axis*)
  ParametricPlot3D[{b * Sin[θ] * Cos[φ], b * Sin[θ] * Sin[φ], c * Cos[θ]}, {θ, 0, Pi},
    {φ, 0, 2 Pi}, Mesh → None, PlotStyle → Directive[Opacity[0.3], Blue]]
```

```
(*Generate a family of confocal spheroids by varying 'a'*)
spheroids = Table[prolateSpheroid[a, f], {a, 0.5, 1.5, 0.25}];
```

```
(*Show all together*)
Show[spheroids, AxesLabel → {"x", "y", "z"}, Boxed → False,
  PlotRange → All, Lighting → "Neutral", LabelStyle → Directive[Bold, Medium]]
```

```
Out[39]=
```

