EE 523 Homework-3

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Type 1 Model

Bus 1 is assumed to be slack bus

Dynamic Equations for G2

$$\dot{\theta}_2 = (\omega_2 - 1)377$$

$$2 \times 58.5 \dot{\omega}_2 = P_{m_2} - P_{e_2}$$

$$117 \dot{\omega}_2 = P_{m_2} - P_{e_2}$$

$\omega_s = 2\pi \times 60 = 377$ rad/sec

$H_2 = 6.5 \times \frac{900}{100} = 58.5$ sec

$K_D = 0$

$$P_{e_2} = P_{G_2} + 2.7 \times 10^{-4}(I_{d_2}^2 + I_{q_2}^2)$$

$$P_{G_2} = V_{d_2} I_{d_2} + V_{q_2} I_{q_2}$$

$$Q_{G_2} = V_{q_2} I_{d_2} - V_{d_2} I_{q_2}$$

$R_S = 0.0025 \times \frac{100}{900} = 2.7 \times 10^{-4}$

$$V_{d_2} = V_2 \sin(\theta_2 - \delta_2)$$

$$V_{q_2} = V_2 \cos(\theta_2 - \delta_2)$$

$$I_{d_2} = (E'_{q_2} - V_{q_2})/0.033$$

$$I_{q_2} = (E'_{d_2} - V_{d_2})/-0.061$$

$X'_{d_2} = 0.3 \times \frac{100}{900} = 0.033$

$X'_{q_2} = 0.55 \times \frac{100}{900} = 0.061$

$$72 \dot{E}'_{q_2} = -E'_{q_2} - (0.2 - 0.033) I_{d_2} + E_{fd_2}$$

$$72 \dot{E}'_{q_2} = -E'_{q_2} - 0.167 I_{d_2} + E_{fd_2}$$

$T'_{do_2} = 8 \times \frac{900}{100} = 72$ sec

$X_{d_2} = 1.8 \times \frac{100}{900} = 0.2$

$$3.6 \dot{E}'_{d_2} = -E'_{d_2} + (0.188 - 0.061) I_{q_2}$$

$$3.6 \dot{E}'_{d_2} = -E'_{d_2} - 0.127 I_{q_2}$$

$T'_{q_{o2}} = 0.4 \times \frac{900}{100} = 3.6$ sec

$X_{q_2} = 1.7 \times \frac{100}{900} = 0.188$

Power Flow:-

$$0 = P_{G_2} - P_{L_2} - V_2 V_6 (59.98)\cos(\delta_2 - \delta_6 - 90^\circ)$$
$$0 = Q_{G_2} - Q_{L_2} - V_2 V_6 (59.98)\sin(\delta_2 - \delta_6 - 90)$$

$Y_{26} = 59.98$

$\theta_{26} = 90^\circ$

Dynamic Equations for G3

$$\dot{\theta}_3 = (\omega_3 - 1)\,377$$

$\omega_s = 2\pi \times 60 = 377$ rad/sec

$$2 \times 55.575\, \dot{\omega}_3 = P_{m_3} - P_{e_3}$$

$H_3 = 6.175 \times \frac{900}{100} = 55.575$ sec

$$111.15\, \dot{\omega}_3 = P_{m_3} - P_{e_3}$$

$K_D = 0$

$$P_{e_3} = P_{G_3} + 2.7\times10^{-4}\left(Id_3^2 + Iq_3^2\right)$$

$R_S = 0.0025 \times \frac{100}{900} = 2.7\times10^{-4}$

$$P_{G_3} = Vd_3 Id_3 + Vq_3 Iq_3$$
$$Q_{G_3} = Vq_3 Id_3 - Vd_3 Iq_3$$

$$Vd_3 = V_3 \sin(\theta_3 - \delta_3)$$
$$Vq_3 = V_3 \cos(\theta_3 - \delta_3)$$

$$Id_3 = (E'q_3 - Vq_3)/0.033$$
$$Iq_3 = (E'd_3 - Vd_3)/-0.061$$

$X'd_3 = 0.3 \times \frac{100}{900} = 0.033$

$X'q_3 = 0.55 \times \frac{100}{900} = 0.061$

$$72\, \dot{E}'q_3 = -E'q_3 - (0.2 - 0.033) Id_3 + Efd_3$$

$T'd_{03} = 8 \times \frac{900}{100} = 72$ sec

$Xd_3 = 1.8 \times \frac{100}{900} = 0.2$

$$72\, \dot{E}'q_3 = -E'q_3 - 0.167\, Id_3 + Efd_3$$

$$3.6\, \dot{E}'d_3 = -E'd_3 + (0.188 - 0.061) Iq_3$$

$T'q_{03} = 0.4 \times \frac{900}{100} = 3.6$ sec

$Xq_3 = 1.7 \times \frac{100}{900} = 0.188$

$$3.6\, \dot{E}'d_3 = -E'd_3 - 0.127\, Iq_3$$

Power flow:-

$$0 = P_{G_3} - P_{L_3} - V_3 V_{11} (59.98)\cos(\delta_3 - \delta_{11} - 90^\circ)$$
$$0 = Q_{G_3} - Q_{L_3} - V_3 V_{11} (59.98)\sin(\delta_3 - \delta_{11} - 90^\circ)$$

$Y_{3\,11} = 59.98$

$\theta_{3\,11} = 90^\circ$

Dynamic Equations for G4

$$\dot{\theta}_4 = (\omega_4 - 1)\,377$$

$$2 \times 55.575\,\dot{\omega}_4 = P_{m_4} - P_{e_4}$$

$$11115\,\dot{\omega}_4 = P_{m_4} - P_{e_4}$$

$\omega_s = 2\pi \times 60 = 377$ rad/sec

$H_4 = 6.175 \times \frac{900}{100} = 55.575$ sec

$K_D = 0$

$$P_{e_4} = P_{a_4} + 2.7\times10^{-4}\left(Id_4^2 + Iq_4^2\right)$$

$$P_{a_4} = Vd_4\,Id_4 + Vq_4\,Iq_4$$

$$Q_{a_4} = Vq_4\,Id_4 - Vd_4\,Iq_4$$

$R_S = 0.0025 \times \frac{100}{900} = 2.7\times10^{-4}$

$$Vd_4 = V_4 \sin(\theta_4 - \delta_4)$$

$$Vq_4 = V_4 \cos(\theta_4 - \delta_4)$$

$$Id_4 = (E'q_4 - Vq_4)/0.033$$

$$Iq_4 = (E'd_4 - Vd_4)/-0.061$$

$X'd_4 = 0.3 \times \frac{100}{900} = 0.033$

$X'q_4 = 0.55 \times \frac{100}{900} = 0.061$

$$72\,\dot{E}'q_4 = -Eq'_4 - (0.2 - 0.033)Id_4 + Efd_4$$

$$72\,\dot{E}'q_4 = -Eq'_4 - 0.167\,Id_4 + Efd_4$$

$T'd_{04} = 8 \times \frac{900}{100} = 72$ sec

$X_{d_4} = 1.8 \times \frac{100}{900} = 0.2$

$$3.6\,\dot{E}'d_4 = -Ed'_4 + (0.188 - 0.061)Iq_4$$

$$3.6\,\dot{E}'d_4 = -E'd_4 - 0.127\,Iq_4$$

$T'q_{04} = 0.4 \times \frac{900}{100} = 3.6$ sec

$Xq_4 = 1.7 \times \frac{100}{900} = 0.188$

Power flow:-

$$0 = P_{G4} - P_{L4} - V_4 V_{10}(59.98)\cos(\delta_4 - \delta_{10} - 90^\circ)$$

$$0 = Q_{G4} - Q_{L4} - V_4 V_{10}(59.98)\sin(\delta_4 - \delta_{10} - 90^\circ)$$

$Y_{410} = 59.98$

$\theta_{410} = 90^\circ$

Bus 4

$$P_4 = \sum_{j=1}^{11} Y_{4j} V_4 V_j \cos(\delta_4 - \delta_j - \theta_{4j})$$

$$Q_4 = \sum_{j=1}^{11} Y_{4j} V_4 V_j \sin(\delta_4 - \delta_j - \theta_{4j})$$

Bus 5

$$P_5 = \sum_{j=1}^{11} Y_{5j} V_5 V_j \cos(\delta_5 - \delta_j - \theta_{5j})$$

$$Q_5 = \sum_{j=1}^{11} Y_{5j} V_5 V_j \sin(\delta_5 - \delta_j - \theta_{5j})$$

Bus 6

$$P_6 = \sum_{j=1}^{11} Y_{6j} V_6 V_j \cos(\delta_6 - \delta_j - \theta_{6j})$$

$$Q_6 = \sum_{j=1}^{11} Y_{6j} V_6 V_j \sin(\delta_6 - \delta_j - \theta_{6j})$$

Bus 7

$$P_7 = \sum_{j=1}^{11} Y_{7j} V_7 V_j \cos(\delta_7 - \delta_j - \theta_{7j})$$

$$Q_7 = \sum_{j=1}^{11} Y_{7j} V_7 V_j \sin(\delta_7 - \delta_j - \theta_{7j})$$

Bus 8

$$P_8 = \sum_{j=1}^{11} Y_{8j} V_8 V_j \cos(\delta_8 - \delta_j - \theta_{8j})$$

$$Q_8 = \sum_{j=1}^{11} Y_{8j} V_8 V_j \sin(\delta_8 - \delta_j - \theta_{8j})$$

Bus 9

$$P_9 = \sum_{j=1}^{11} Y_{9j} V_9 V_j \cos(\delta_9 - \delta_j - \theta_{9j})$$

$$Q_9 = \sum_{j=1}^{11} Y_{9j} V_9 V_j \sin(\delta_9 - \delta_j - \theta_{9j})$$

Bus 10

$$P_{10} = \sum_{j=1}^{11} Y_{10j} V_{10} V_j \cos(\delta_{10} - \delta_j - \theta_{10j})$$

$$Q_{10} = \sum_{j=1}^{11} Y_{10j} V_{10} V_j \sin(\delta_{10} - \delta_j - \theta_{10j})$$

Bus 11

$$P_{11} = \sum_{j=1}^{11} Y_{11j} V_{11} V_j \cos(\delta_{11} - \delta_j - \theta_{11j})$$

$$Q_{11} = \sum_{j=1}^{11} Y_{11j} V_{11} V_j \sin(\delta_{11} - \delta_j - \theta_{11j})$$

Type 2 Model

Bus 1 is assumed to be slack bus

Dynamic Equations for G2

$$\dot{\theta}_2 = (\omega_2 - 1)\,377$$

$$2 \times 58.5\,\dot{\omega}_2 = P_{m_2} - P_{e_2}$$

$$117\,\dot{\omega}_2 = P_{m_2} - P_{e_2}$$

$\omega_s = 2\pi \times 60 = 377$ rad/sec

$H_2 = 6.5 \times \frac{900}{100} = 58.5$ sec

$K_D = 0$

$$P_{e_2} = \sum_{j=1}^{4} Y_{a_{2j}} E_2' E_j' \cos(r_2 - r_j - \theta_{a_{2j}})$$

$$72\,\dot{E}_{q_2}' = -E_{q_2}' - (0.2 - 0.033) I_{d_2} + E_{fd_2}$$

$$72\,\dot{E}_{q_2}' = -E_{q_2}' - 0.167\,I_{d_2} + E_{fd_2}$$

$T_{do_2}' = 8 \times \frac{900}{100} = 72$ sec

$X_{d_2} = 1.8 \times \frac{100}{900} = 0.2$

$X_{d_2}' = 0.3 \times \frac{100}{900} = 0.033$

$$3.6\,\dot{E}_{d_2}' = -E_{d_2}' + (0.188 - 0.061) I_{q_2}$$

$$3.6\,\dot{E}_{d_2}' = -E_{d_2}' - 0.127\,I_{q_2}$$

$T_{qo_2}' = 0.4 \times \frac{900}{100} = 3.6$ sec

$X_{q_2} = 1.7 \times \frac{100}{900} = 0.188$

$X_{q_2}' = 0.55 \times \frac{100}{900} = 0.061$

$$I_{d_2} = \sum_{j=1}^{4} Y_{a_{2j}} E_j' \sin(\theta_2 - r_j - \theta_{a_{2j}})$$

$$I_{q_2} = \sum_{j=1}^{4} Y_{a_{2j}} E_j' \cos(\theta_2 - r_j - \theta_{a_{2j}})$$

Dynamic Equations for G3

$$\dot{\theta}_3 = (\omega_3 - 1)\,377$$

$$2 \times 55.575\,\dot{\omega}_3 = P_{m_3} - P_{e_3}$$

$$111.15\,\dot{\omega}_3 = P_{m_3} - P_{e_3}$$

$\omega_s = 2\pi \times 60 = 377$ rad/sec

$H_3 = 6.175 \times \frac{900}{100} = 55.575$ sec

$K_D = 0$

$$P_{e_3} = \sum_{j=1}^{4} Y_{a_{3j}} E_3' E_j' \cos(r_3 - r_j - \theta_{a_{3j}})$$

$$72\,\dot{E}'_{q_3} = -E'_{q_3} - (0.2-0.033)I_{d_3} + E_{fd_3}$$

$$T'_{do_3} = 8 \times \frac{900}{100} = 72 \text{ sec}$$

$$X_{d_3} = 1.8 \times \frac{100}{900} = 0.2$$

$$X'_{d_3} = 0.3 \times \frac{100}{900} = 0.033$$

$$\boxed{72\,\dot{E}'_{q_3} = -E'_{q_3} - 0.167\,I_{d_3} + E_{fd_3}}$$

$$3.6\,\dot{E}'_{d_3} = -E'_{d_3} + (0.188-0.061)I_{q_3}$$

$$T'_{qo_3} = 0.4 \times \frac{900}{100} = 3.6 \text{ sec}$$

$$X_{q_3} = 1.7 \times \frac{100}{900} = 0.188$$

$$X'_{q_3} = 0.55 \times \frac{100}{900} = 0.061$$

$$\boxed{3.6\,\dot{E}'_{d_3} = -E'_{d_3} - 0.127\,I_{q_3}}$$

$$\boxed{I_{d_3} = \sum_{j=1}^{4} Y_{a_{3j}} E'_j \sin(\theta_3 - r_j - \theta_{a_{3j}})}$$

$$\boxed{I_{q_3} = \sum_{j=1}^{4} Y_{a_{3j}} E'_j \cos(\theta_3 - r_j - \theta_{a_{3j}})}$$

Dynamic Equations for G4

$$\boxed{\dot{\theta}_4 = (\omega_4 - 1)\,377}$$

$$\omega_s = 2\pi \times 60 = 377 \text{ rad/sec}$$

$$2 \times 55.575\,\dot{\omega}_4 = P_{m_4} - P_{e_4}$$

$$H_4 = 6.175 \times \frac{900}{100} = 55.575 \text{ sec}$$

$$\boxed{111.15\,\dot{\omega}_4 = P_{m_4} - P_{e_4}}$$

$$K_D = 0$$

$$\boxed{P_{e_4} = \sum_{j=1}^{4} Y_{a_{4j}} E'_4 E'_j \cos(r_4 - r_j - \theta_{a_{4j}})}$$

$$72\,\dot{E}'_{q_4} = -E'_{q_4} - (0.2-0.033)I_{d_4} + E_{fd_4}$$

$$T'_{do_4} = 8 \times \frac{900}{100} = 72 \text{ sec}$$

$$X_{d_4} = 1.8 \times \frac{100}{900} = 0.2$$

$$X'_{d_4} = 0.3 \times \frac{100}{900} = 0.033$$

$$\boxed{72\,\dot{E}'_{q_4} = -E'_{q_4} - 0.167\,I_{d_4} + E_{fd_4}}$$

$$3.6\,\dot{E}'_{d_4} = -E'_{d_4} + (0.188-0.061)I_{q_4}$$

$$T'_{qo_4} = 0.4 \times \frac{900}{100} = 3.6 \text{ sec}$$

$$X_{q_4} = 1.7 \times \frac{100}{900} = 0.188$$

$$X'_{q_4} = 0.55 \times \frac{100}{900} = 0.061$$

$$\boxed{3.6\,\dot{E}'_{d_4} = -E'_{d_4} - 0.127\,I_{q_4}}$$

$$Id_4 = \sum_{j=1}^{4} Y_{G_{4j}} E_j' \sin(\theta_4 - r_j - \theta_{G_{4j}})$$

$$Iq_4 = \sum_{j=1}^{4} Y_{G_{4j}} E_j' \cos(\theta_4 - r_j - \theta_{G_{4j}})$$

From Matlab, Y_{Net} =

```
Columns 1 through 4

         0 -     59.988i           0 +       0i           0 +       0i           0 +       0i
         0 +          0i           0 -  21.177i           0 +       0i           0 +       0i
         0 +          0i           0 +       0i           0 -  21.177i           0 +       0i
         0 +          0i           0 +       0i           0 +       0i           0 -  21.177i
         0 +     59.988i           0 +       0i           0 +       0i           0 +       0i
         0 +          0i           0 +       0i           0 +       0i           0 +       0i
         0 +          0i           0 +       0i           0 +       0i           0 +       0i
         0 +          0i           0 +       0i           0 +       0i           0 +       0i
         0 +          0i           0 +       0i           0 +       0i           0 +       0i
         0 +          0i           0 +       0i           0 +       0i           0 +       0i
         0 +          0i           0 +       0i           0 +       0i           0 +       0i
         0 +          0i           0 +  21.177i           0 +       0i           0 +       0i
         0 +          0i           0 +       0i           0 +  21.177i           0 +       0i
         0 +          0i           0 +       0i           0 +       0i           0 +  21.177i

Columns 5 through 8

         0 +     59.988i           0 +       0i           0 +       0i           0 +       0i
         0 +          0i           0 +       0i           0 +       0i           0 +       0i
         0 +          0i           0 +       0i           0 +       0i           0 +       0i
         0 +          0i           0 +       0i           0 +       0i           0 +       0i
    3.9604 -      99.57i      -3.9604 +  39.604i           0 +       0i           0 +       0i
   -3.9604 +     39.604i       23.762 -  297.57i     -19.802 +  198.02i           0 +       0i
         0 +          0i      -19.802 +  198.02i      32.906 -  232.64i     -3.6004 +  36.004i
         0 +          0i           0 +       0i     -3.6004 +  36.004i      7.2007 -  71.237i
         0 +          0i           0 +       0i           0 +       0i     -3.6004 +  36.004i
         0 +          0i           0 +       0i           0 +       0i           0 +       0i
         0 +          0i           0 +       0i           0 +       0i           0 +       0i
         0 +          0i           0 +  59.988i           0 +       0i           0 +       0i
         0 +          0i           0 +       0i           0 +       0i           0 +       0i
         0 +          0i           0 +       0i           0 +       0i           0 +       0i

Columns 9 through 12

         0 +          0i           0 +       0i           0 +       0i           0 +       0i
         0 +          0i           0 +       0i           0 +       0i           0 +  21.177i
         0 +          0i           0 +       0i           0 +       0i           0 +       0i
         0 +          0i           0 +       0i           0 +       0i           0 +       0i
         0 +          0i           0 +       0i           0 +       0i           0 +       0i
         0 +          0i           0 +       0i           0 +       0i           0 +  59.988i
         0 +          0i           0 +       0i           0 +       0i           0 +       0i
    -3.6004 +     36.004i           0 +       0i           0 +       0i           0 +       0i
     40.567 -     231.19i      -19.802 +  198.02i           0 +       0i           0 +       0i
   -19.802 +     198.02i       23.762 -  297.57i     -3.9604 +  39.604i           0 +       0i
         0 +          0i      -3.9604 +  39.604i      3.9604 -   99.57i           0 +       0i
         0 +          0i           0 +       0i           0 +       0i           0 -  81.165i
         0 +          0i           0 +       0i           0 +  59.988i           0 +       0i
         0 +          0i           0 +  59.988i           0 +       0i           0 +       0i

Columns 13 through 14

         0 +          0i           0 +       0i
         0 +          0i           0 +       0i
         0 +     21.177i           0 +       0i
         0 +          0i           0 +  21.177i
         0 +          0i           0 +       0i
         0 +          0i           0 +       0i
         0 +          0i           0 +       0i
         0 +          0i           0 +       0i
         0 +          0i           0 +       0i
         0 +          0i           0 +  59.988i
         0 +     59.988i           0 +       0i
         0 +          0i           0 +       0i
         0 -     81.165i           0 +       0i
         0 +          0i           0 -  81.165i
```

From Matlab, Y_{gen} =

3.1519 - 12.697i	1.5578 + 7.2405i	1.0648 + 1.5522i	1.5415 + 2.1172i
1.5578 + 7.2405i	1.299 - 10.992i	0.75503 + 0.97052i	1.088 + 1.3202i
1.0648 + 1.5522i	0.75503 + 0.97052i	1.5317 - 8.3645i	1.8025 + 3.9505i
1.5415 + 2.1172i	1.088 + 1.3202i	1.8025 + 3.9505i	2.6612 - 10.228i

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$$P_{e_2} = \sum_{j=1}^{4} Y_{G_{2j}} E_2' E_j' \cos(\theta_2 - \theta_j - \theta_{G_{2j}})$$

Dynamic Equations for G3

$$\dot{\theta}_3 = (\omega_3 - 1)\,377$$

$$2 \times 55.575\,\dot{\omega}_3 = P_{m_3} - P_{e_3}$$

$$111.15\,\dot{\omega}_3 = P_{m_3} - P_{e_3}$$

$\omega_s = 2\pi \times 60 = 377$ rad/sec

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$$P_{e_3} = \sum_{j=1}^{4} Y_{G_{3j}} E_3' E_j' \cos(\theta_3 - \theta_j - \theta_{G_{3j}})$$

Dynamic Equations for G4

$$\dot{\theta}_4 = (\omega_4 - 1)\,377$$

$$2 \times 55.575\,\dot{\omega}_4 = P_{m_4} - P_{e_4}$$

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$\omega_s = 2\pi \times 60 = 377$ rad/sec

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