EE 523 Homework 4

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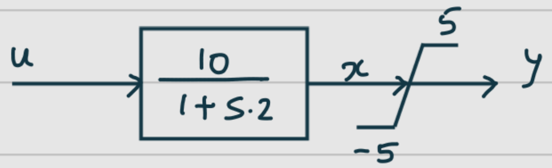
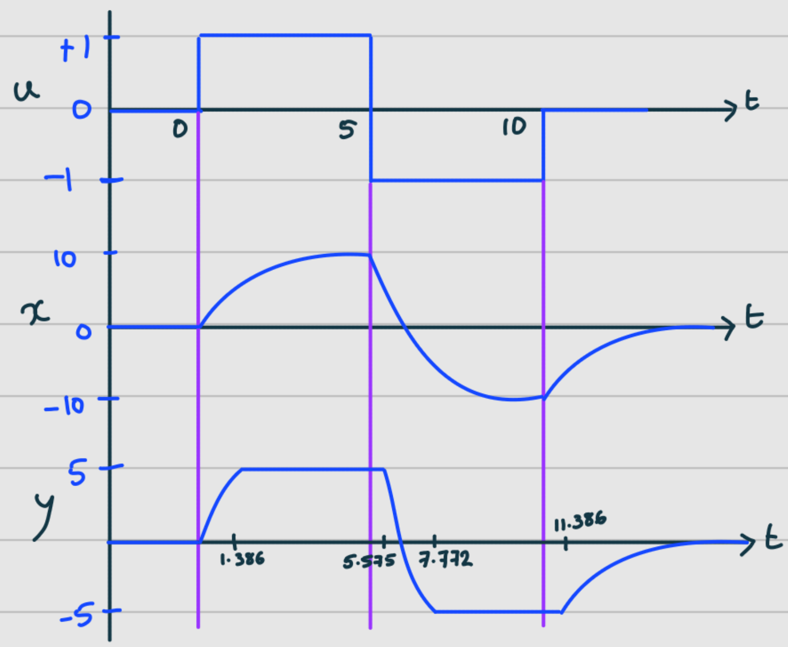
Consider Amplifier A :-



For input (a)



$0 \le t \le 5$

$$Y(s) = \frac{10}{1+2s} \cdot \frac{1}{s} = 10\left(\frac{1/2}{s+1/2} \cdot \frac{1}{s}\right)$$

$$Y(s) = 10\left(\frac{1}{s} - \frac{1}{s+1/2}\right)$$

$$y(t) = 10\left[1 - e^{-t/2}\right]$$

for $y(t) = 5$

$$5 = 10\left(1 - e^{-t/2}\right)$$

$$\frac{1}{2} = 1 - e^{-t/2}$$

$$e^{-t/2} = \frac{1}{2}$$

$$t/2 = \ln 2$$

$$t = 1.386 \text{ sec}$$

$5 \le t \le 10$

$$Y(s) = \frac{10}{1+2s} \cdot \left(\frac{-2}{s}\right) = 20\left(\frac{1}{s+1/2} - \frac{1}{s}\right)$$ without DC shift & time shift

$$y(t) = 10 - 20\left[1 - e^{-(t-5)/2}\right]$$ with DC shift & time shift

for $y(t) = 5$

$$5 = 10 - 20\left[1 - e^{-(t-5)/2}\right]$$

$$e^{-(t-5)/2} = 3/4$$

$$t - 5 = 0.575$$

$$t = 5 + 0.575$$

$$t = 5.575 \text{ sec}$$

for $y(t) = -5$

$$-5 = 10 - 20\left(1 - e^{-(t-5)/2}\right)$$

$$e^{-(t-5)/2} = 1/4$$

$$t - 5 = 2.772$$

$$t = 5 + 2.772$$

$$t = 7.772 \text{ sec}$$

$t \geq 10$

$$y(t) = -10e^{-(t-10)/2}$$

For $y(t) = -5$

$$-5 = -10e^{-(t-10)/2}$$

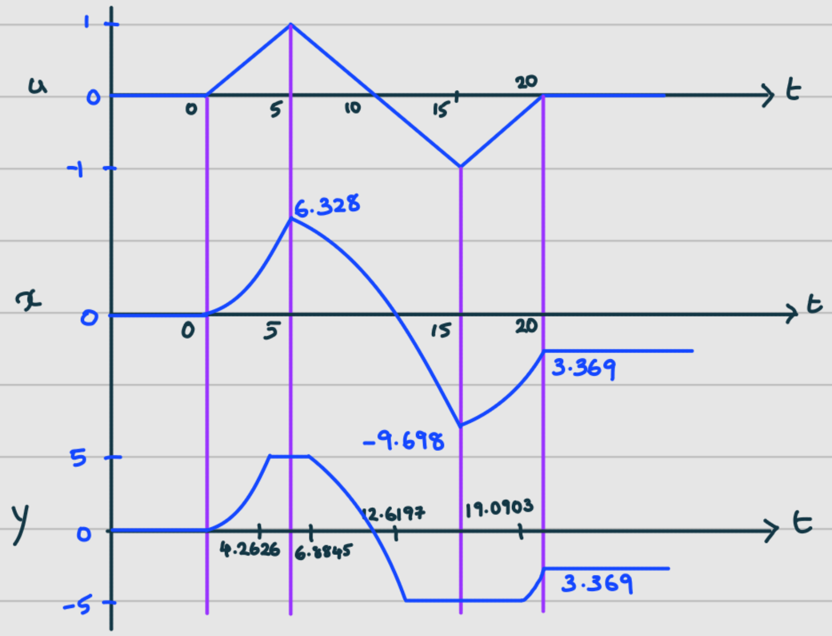
$$e^{t-10/2} = 2$$

$$t - 10 = 1.386$$

$$t = 10 + 1.386$$

$$\underline{\underline{t = 11.386 \text{ sec}}}$$

For input (b)



$0 \leq t \leq 5$

$$Y(s) = \frac{10}{1+2s} \cdot \frac{0.2}{s^2} = 2\left(\frac{1}{s^2} \frac{1/2}{s+1/2}\right) = 2\left[\frac{1}{s^2} - \frac{2}{s} + \frac{2}{s+1/2}\right]$$

$$y(t) = 2\left[t - 2 + 2e^{-t/2}\right]$$

at $t = 5$

$$y(5) = 10\left[5 - 2 + 2e^{-5/2}\right] = \underline{\underline{6.328}}$$

For $y(t) = 5$

$$5 = 2\left(t - 2 + 2e^{-t/2}\right)$$

Solving exponential eqn using Matlab

$$\underline{\underline{t = 4.2626 \text{ sec}}}$$

$5 \leq t \leq 15$

$$Y(s) = \frac{10}{1+2s} \cdot \frac{-0.2}{s^2}$$

$$y(t) = -2\left[t-2+2e^{-t/2}\right]$$ without DC shift & time shift

$$y(t) = 6.328 - 2\left[t-5-2+2e^{-(t-5)/2}\right]$$ with DC shift & time shift

$$y(t) = 6.328 - 2\left[t-7+2e^{-(t-5)/2}\right]$$

at $t=15$, $y(15) = 6.328 - 2\left[15-7+2e^{-10/2}\right] = \underline{\underline{-9.698}}$

for $y(t)=5 \Rightarrow 5 = 6.328 - 2\left[t-7+2e^{-(t-5)/2}\right]$

On solving $\underline{\underline{t = 6.8845}}$ sec

for $y(t) = -5 \Rightarrow -5 = 6.328 - 2\left[t-7+2e^{-(t-5)/2}\right]$

On solving $\underline{\underline{t = 12.6197}}$ sec

$15 \leq t \leq 20$

$$y(s) = \frac{10}{1+2s} \cdot \frac{0.2}{s^2}$$

$$y(t) = 2\left[t-2+2e^{-t/2}\right]$$ without DC shift & time shift

$$y(t) = -9.698 + 2\left[t-15-2+2e^{-(t-15)/2}\right]$$ with DC shift & time shift

$$y(t) = -9.698 + 2\left[t-17+2e^{-(t-15)/2}\right]$$

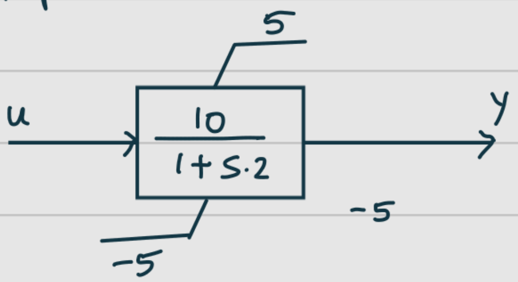
at $t = 20$

$$y(20) = -9.698 + 2\left[20-17+2e^{-5/2}\right] = \underline{\underline{-3.369}}$$

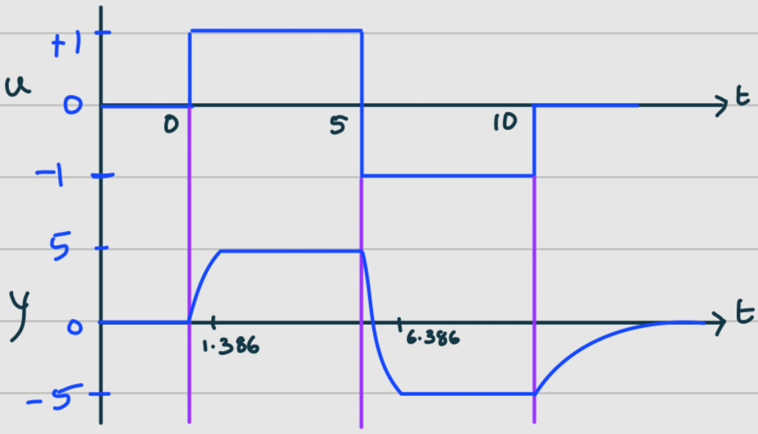
For $y = -5 \Rightarrow -5 = -9.698 + 2\left[t-17+2e^{-(t-15)/2}\right]$

On solving, $t = \underline{\underline{19.0903}}$ sec

Consider Amplifier B :-



For input (a)



$5 \leq t \leq 10$

$y(t) = 5 - 20\left[1 - e^{-(t-5)/2}\right]$

for $y(t) = -5$

$-5 = 5 - 20\left[1 - e^{-(t-5)/2}\right]$

$\frac{1}{2} = 1 - e^{-(t-5)/2}$

$\frac{1}{2} = e^{-(t-5)/2}$

$t - 5 = 2 \ln 2 = 1.386$

$\underline{\underline{t = 6.386}}$

ydot = $-y + 10u$

At $t = 5^{+}$sec

ydot $= -5 + 10(-1) = -15 < 0$

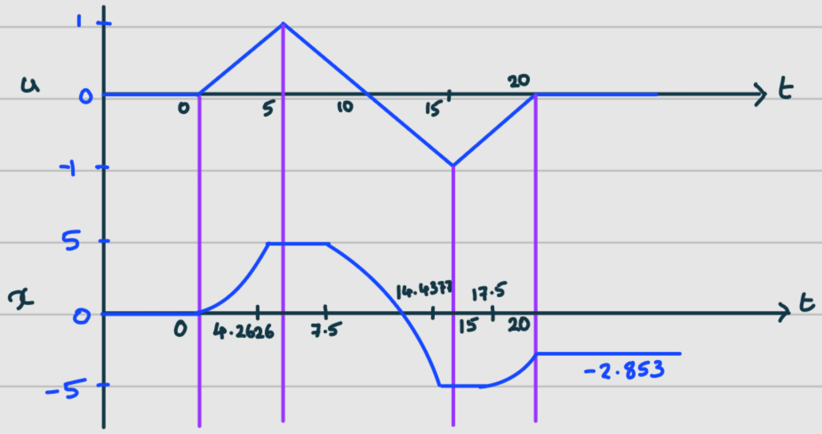
$\therefore$ Y decreases

At $t = 10^{+}$sec

ydot $= -(-5) + 10(0) = 5 > 0$

$\therefore$ Y increases

For input (b)



Ydot = $-Y + 10u$

At $t = 5^{+}$ sec

$\dot{y} = -5 + 10(1) = 5 > 0$

$\therefore$ y will not change

$-5 + 10(1 - 0.2(t-5)) = 0$

$1 - 0.2(t-5) = \frac{1}{2}$

$t - 5 = 2.5$

$\underline{\underline{t = 7.5\,\text{sec}}}$

$7.5 \le t \le 15$

$y(t) = 5 - 2\left[t - 7.5 - 2 + 2e^{-(t-7.5)/2}\right]$

$y(t) = 5 - 2\left[t - 9.5 + 2e^{-(t-7.5)/2}\right]$

For $y = -5$

$-5 = 5 - 2\left[t - 9.5 + 2e^{-(t-7.5)/2}\right]$

On solving

$\underline{\underline{t = 14.4377\,\text{sec}}}$

From $t = 7.5^{+}$, ydot $< 0 \Rightarrow$ y decreases

At $t = 15^{+}$ sec

$\dot{y} = -(-5) + 10(-1) = -5 < 0$

$\therefore$ y will not change

$-(-5) + 10(-1 + 0.2(t-15)) = 0$

$-1 + 0.2(t-15) = -\frac{1}{2}$

$t - 15 = 2.5$

$\underline{\underline{t = 17.5\,\text{sec}}}$

$17.5 \le t \le 20$

$y(t) = -5 + 2\left[t - 17.5 - 2 + 2e^{-(t-17.5)/2}\right]$

$y(t) = -5 + 2\left[t - 19.5 + 2e^{-(t-17.5)/2}\right]$

at $t = 20$

$y(20) = -5 + 2\left[20 - 19.5 + 2e^{-2.5/2}\right]$

$= \underline{\underline{-2.853}}$

From $t = 17.5^{+}$, y dot $> 0 \Rightarrow$ y increases