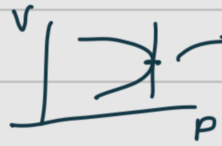
01/09/24

Power System Stability

491
521 Voltage stability — Power flow (SS concept)



→ Static limit

Stability under disturbances
↳ small
↳ large

Small signal stability → Linearization
- Ability to damp out small perturbations
- Oscillations

$\Delta\dot{x} = A\Delta x$ eigen values → real < 0
img
↳ modes
↓
suff. damp.

+ eigen vectors

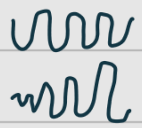
Transient Stability — Faults, Gen drop → Non linear models
- Recovery from large disturbances
- Islanding? Voltage Collapse?

Stability Concepts

- Small signal stability
  - Load fluctuations
  - Gen Changes
  - Oscillatory modes
    - Well damped?

load, renewables } damp out

unstable



sustained

-ve damping

+ve damp.

Small signal stable

Power world → Oscillations simulation

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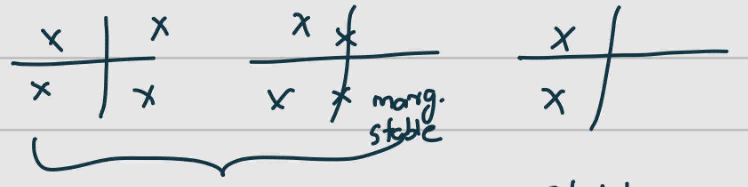
- Small signal stability
  - Local Stability
  - Stability in the sense of Lypnov
- Large disturbance stability
  - Transient stability

Linearization

$\dot{x} = f(x) \Rightarrow \Delta\dot{x} = A\Delta x$

$\hookrightarrow$ eigen values

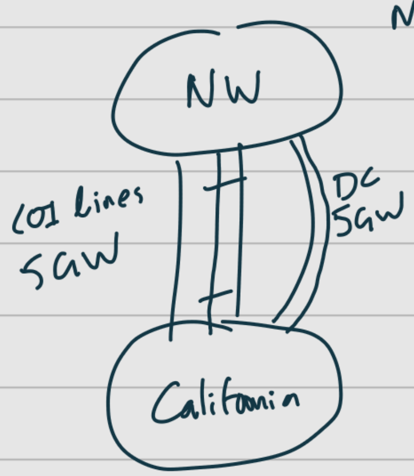


SS unstable

Stable

Aug 10, 96 event

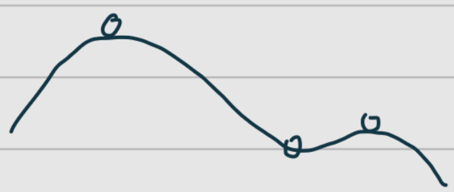
McNary - Umatilla



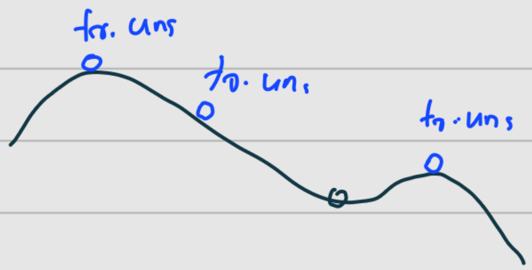
- 9GW
UFLS operated (not enough)
many islands

unstable 0.25Hz

Inter-area mode



SS stable



transient stable
Loss of sync., Islanding

24X7 Power grid - (N-1) security

NERC - NA (North american) Electricity Reliability Corp

FERC - Federal Energy Regulatory Commission

N-1 Contingencies
- Loss of TL
- Loss of gen

Numerical Integration

⇒ Euler   order 1 } explicit

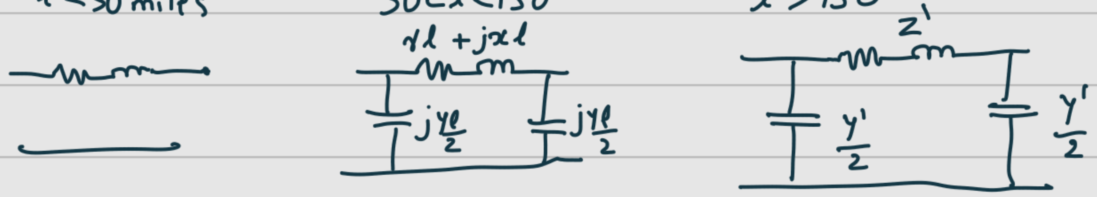
⇒ RK4 (Runge-Kutta 4) } explicit

⇒ Trapezoidal Rule } Implicit (stable)

01/18/24 EE 521 ⟶ SS Analysis ⟶ Phasors

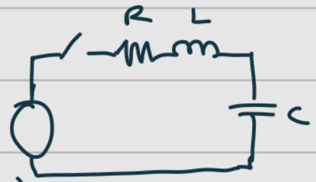
TL — π ckt

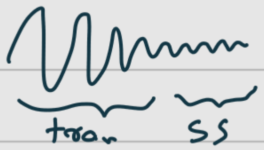
Short	Medium	long
$l < 50$ miles	$50 < l < 150$	$l > 150$



Phasors ⟶ Balanced ⟶ Positive Sequence Analysis

same freq $3\phi \Rightarrow$ single phasor





$V\sin(\omega t+\phi)$

$I\sin(\omega t+\theta)$

$$I\angle\theta = \frac{V\angle\phi}{R + j\omega L + \frac{1}{j\omega C}} \longleftarrow \text{Impedance}$$

Phasors can be used for circuit analysis for the <u>Steady State Analysis</u>

EE 523 < SS Stability, Tr. St. } dynamics — ~~Phasor~~

$$C\frac{dv}{dt} = i$$

$$L\frac{di}{dt} = v(t) - Ri_L - v_c \qquad \text{Transient Analysis}$$

Distributed Parameter Circuit

— Unit length : second order DE

— Whole length

  └ Wave equations : $\frac{\Delta v}{\Delta x}, \frac{\Delta i}{\Delta x}$ } PDE

EMTP :

Generators & loads across the tr. N/W

— As simple as possible

Dynamics (upto 2Hz) ⟶ Quasi Stationary Assumption (Quasi static Assumption) ⟶ Not valid if inverter comes into picture

20Hz, 30Hz

↓

Adv. P.E.

↓

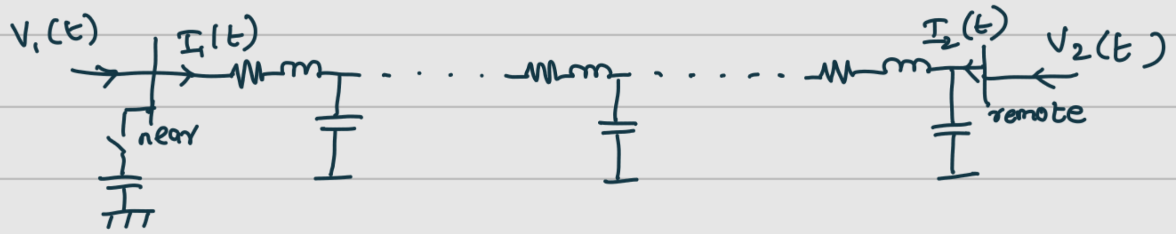
Slow w.r.t. 60Hz

∴ Use phasor (there will be small error)

US Gen (2022)			
	Coal	~ 20%	87% sync. m/c
	Natural Gas	~ 40%	
	Nuclear	~ 20%	
	Hydro	~ 7%	
	Renewables	~ 13%	IBR

$\bar{I}_{BUS} = \bar{Y}_{BUS} \bar{V}_{BUS}$ — each qty will be time varying

$\bar{I}_{BUS}(t) = \bar{Y}_{BUS} \bar{V}_{BUS}(t)$



Dynamic Effect : Equations

↳ reflected at remote end after a delay (Electromagnetic Wave Effect)

— speed of light

for 100 mile line ⇒ one way time delay

$$\Rightarrow \tau = \frac{100 \times 1.6 \times 10^3}{3 \times 10^8} = 0.5 \times 10^{-3}\,\text{sec} = 0.5\,\text{msec}$$

After 2 msec ⇒ $v(t), i(t)$ reached SS

⇒ Quasistationary ⇒ Dynamics slow w.r.to. 60Hz

$$P_i(t) = P_{Gi}(t) - P_{Li}(t) = \sum Y_{ij} V_i(t) V_j(t) \cos[\delta_i(t) - \delta_j(t) - \theta_{ij}]$$

$$Q_i(t) = Q_{Gi}(t) - Q_{Li}(t) = \sum Y_{ij} V_i(t) V_j(t) \sin[\delta_i(t) - \delta_j(t) - \theta_{ij}]$$

Tr. N/W is instantaneous

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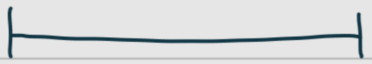
Phasors ⟶ Quasi Stationary Assumption

– Dynamics much slower than 60Hz

$V_i(t)\angle\delta_i(t)$ $V_j(t)\angle\delta_j(t)$



– TL is rep. in PDE (Wave eqns)

100 mile TL ⟶ $\tau = 0.5$ msec

Typical disturbance, wave dynamics damp out in ~2 msec

$r << \omega L$ Resistance ⟶ damps out

↳ losses

– Increase surface area ⟶ bundling

Sync. M/c dynamics time >> TL dynamics

⇒ TL dynamics – Instantaneous

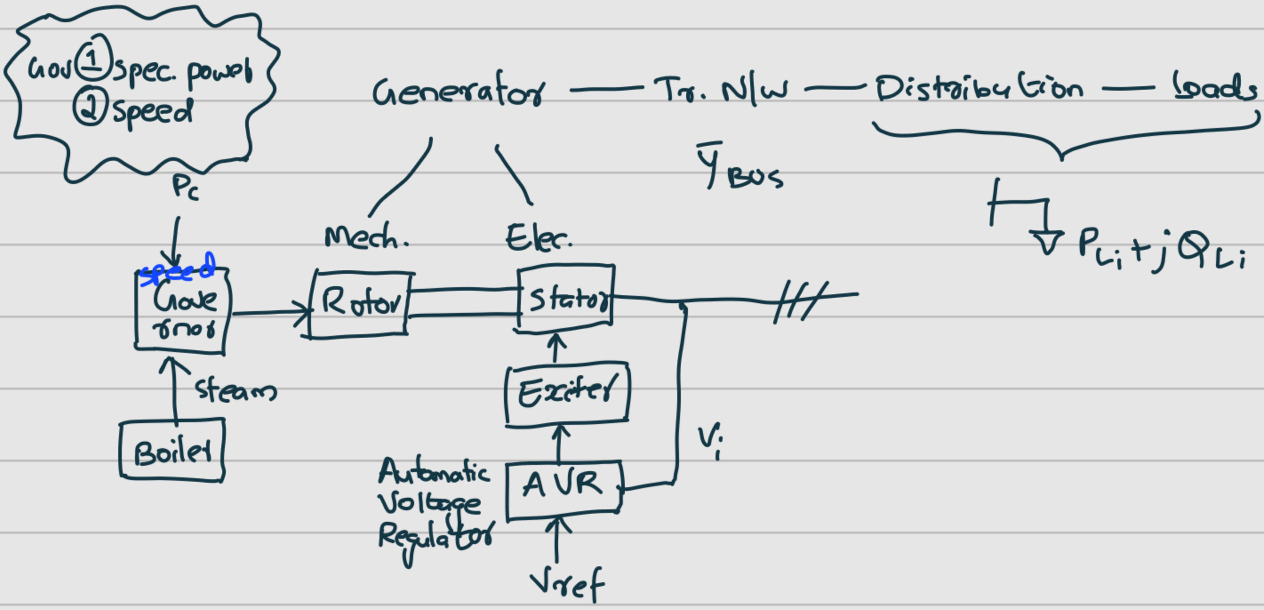
DAE form (Differential Algebraic Eqn)

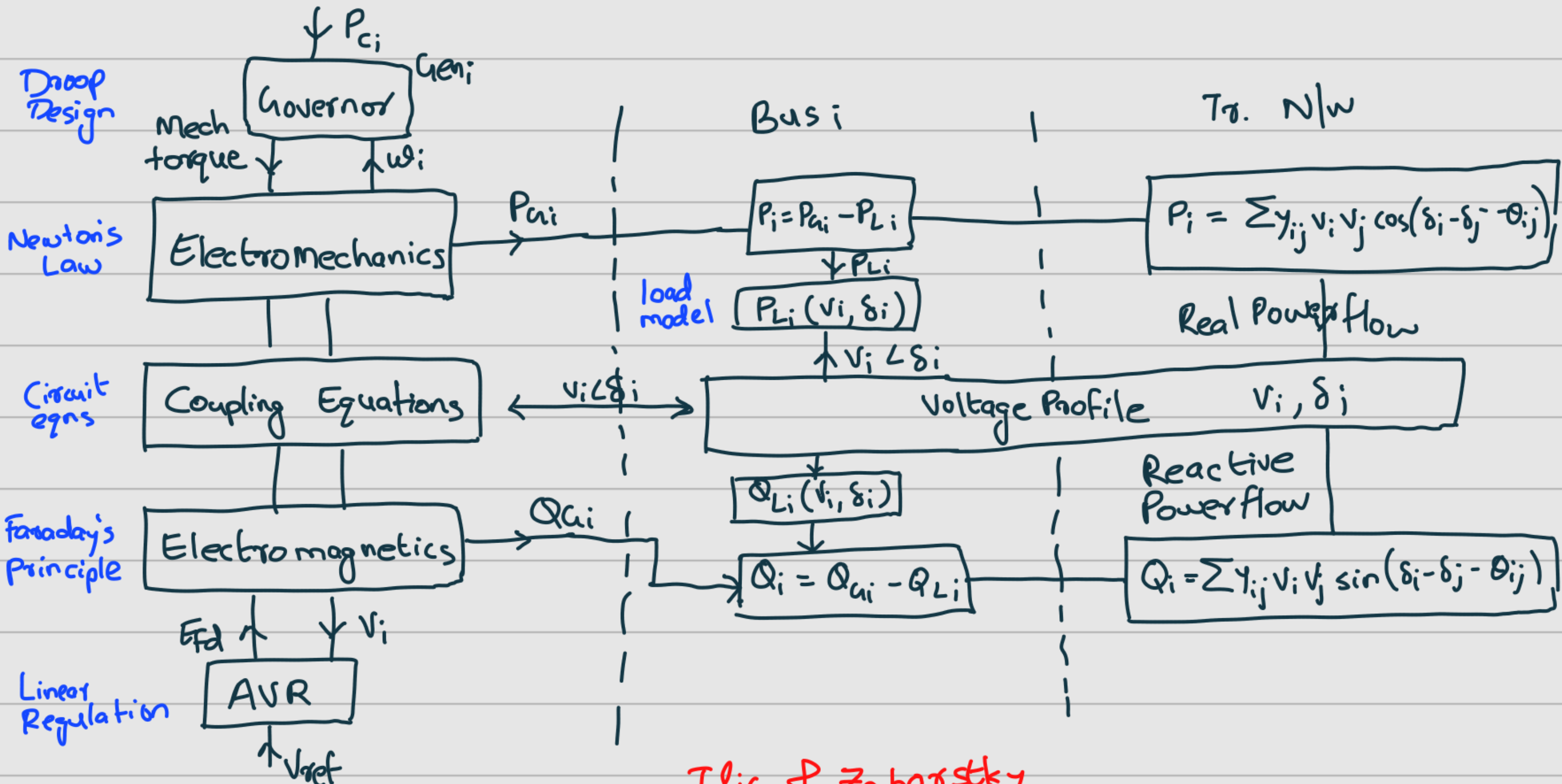
EMTP, PSCAD

TSA, PSLF, PSSE, PowerWorld

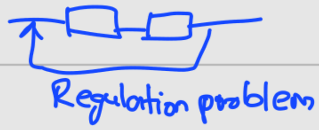
$\dot{x} = f(x, y)$ — Gen, control

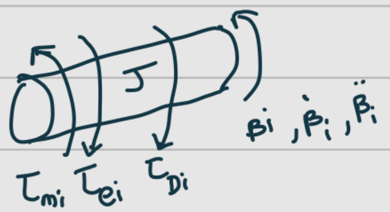
$0 = g(x, y)$ — Instantaneous eqn (TL)





Ilic & Zaborsky





$$J_i \ddot{\beta}_i = \tau_{mi} - \tau_{ei} - \tau_{Di}$$

$\dot{x} = f(x,y)$ — Elec Mech.
— Elec. Mag.
— Governor
— AVR

$0 = g(x,y)$ Real Power Flow
Reactive Power Flow

Homework 1: Powerflow Analysis of the 11 bus Kundur System ⇒ 14 on page

100 MVA base

Load Models

- Residential — Homes: Lighting, heating (quadratic), cooling, appliances — motors, PE (const power) } Aggregate model
- Commercial — WSU: Lighting, motors, electronics
- Industrial — SEL: Large motors, lighting, heating, cooling

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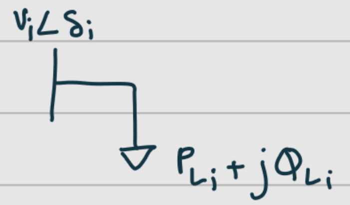
Load Models



Aggregate Loads (Functional Models)

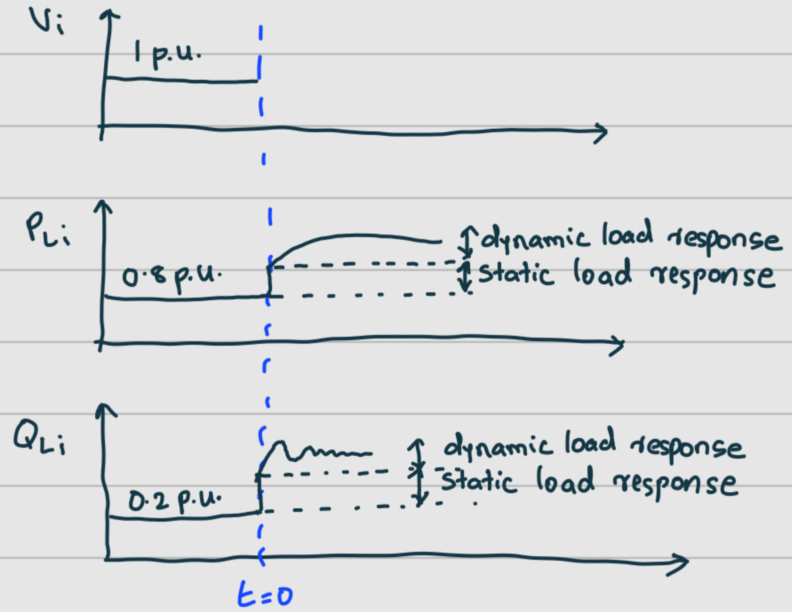
- Residential
- Commercial
- Industrial

P_{L_i} a function of V_i and δ_i

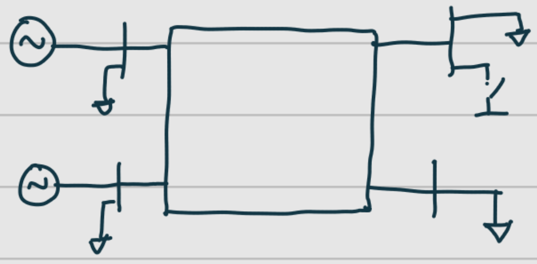
Q_{L_i} a function of V_i and δ_i (ω_i)

$\dot{\delta}_i = \frac{d\delta_i}{dt} \sim \omega_i$ (bus freq)

dependence of freq is ignored



t=0 ⇒ switch in 50 MVAR shunt cap.



$$P_{L_i} = P_{L_i}^{static} + P_{L_i}^{dynamic} = P_{L_i}^{S} + P_{L_i}^{D}$$

$$Q_{L_i} = Q_{L_i}^{S} + Q_{L_i}^{D}$$

Residential : $P_{L_i}^{D} = 0 = Q_{L_i}^{D}$ — specifications

Industrial : $P_{L_i}^{D} = 50\% \, P_{L_i}$ (assumption) — estimations

$P_{L_i}^{Static}$ = Instantaneous load change

$P_{L_i}^{Dynamic}$ = Load change with inertia or time constant

Static VAR Compensator (SVC)

⇒ Very fast voltage control

Static Exciter : Very fast

} PE components

$P_{L_i}^{S}$ = ZIP load model
- Const Impedance (Z) heating, LED
- Const Current (I) Flourescent
- Const Power (P) A/C, LED

(EE Engg)

$$P_{L_i}^{S} = P_{L_{oi}}^{S} + M_i V_i + G_i V_i^2$$

50% 25% 25%

$$P_{L_i}^{S} = 0.5 + 0.25 V_i + 0.25 V_i^2$$

Provided same Voltage base

$$Q_{L_i}^{S} = Q_{L_{io}} + H_i V_i + \overset{\text{susceptance}}{B_i} V_i^2$$

const. power | Const current | Const Imp

① $P_{L_i} = 1$ p.u. $\quad Q_{L_i} = 0.3$ pu $\quad V_i = 1$ p.u.

50% static $\quad$ 66.67% static

25% P, 25% I, 50% Z $\quad$ 50% P, 25% I, 25% Z

$P_{L_i}^S = 0.5$ pu (25% P, 25% I, 50% Z) $\qquad Q_{L_i}^S = 0.2$ pu (50% P, 25% I, 25% Z)

$P_{L_i}^S = 0.125 + 0.125\,V_i + 0.25\,V_i^2 \qquad Q_{L_i}^S = 0.1 + 0.05\,V_i + 0.05\,V_i^2$

② $P_{L_i} = 0.5 \Rightarrow$ 40% P, 30% I, 30% Z $\qquad V_i = 1.05$ pu

$P_{L_{i0}} = 0.5 \times 0.4 = 0.2$ pu

30% I $= M_i V_i = 0.3 \times 0.5 = 0.15$

$$M_i = 0.15/1.05 = 0.143$$

30% Z $= (0.5)(0.3) = 0.15 = G_i V_i^2$

$$G_i = \frac{0.15}{1.05^2} = 0.136\,V_i^2$$

$P_{L_i}^S = 0.2 + 0.143\,V_i + 0.136\,V_i^2 \qquad$ check $V_i = 1.05 \Rightarrow P_{L_i} = 0.2 + 0.15 + 0.15$

$P_{L_i}^S$ = Exponent Model (Mech Engg)

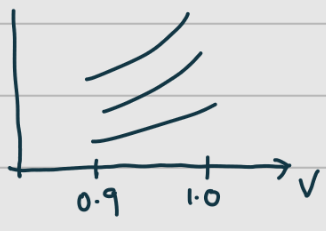
$P_{L_i}^S = K_{P_i}^S V_i^{\alpha_{P_i}} \qquad \alpha_{P_i} \in \mathbb{R},\ \alpha_{P_i} \geq 0$

$\alpha_{P_i} = 0 \Rightarrow$ const P

$\alpha_{P_i} = 1 \Rightarrow$ const I

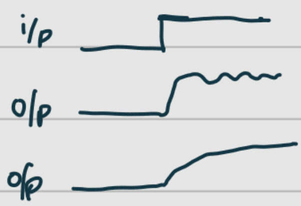
$\alpha_{P_i} = 2 \Rightarrow$ const Z

$\alpha_{P_i} = 0.5$



$Q_{L_i}^S = K_{Q_i}^S V_i^{\alpha_{Q_i}}$

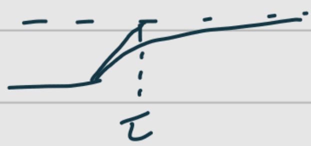
Dynamic Component



2nd (damping & freq)

1st (time const)

$1 - e^{-1} = 0.63 = 63\%$



$$V_i \longrightarrow \boxed{\frac{1+a_{1P_i}s}{1+sb_{1P_i}+s^2b_{2P_i}}} \longrightarrow P_{Li}^D$$

if only first order resp, $a_{1P_i}=0,\ b_{2P_i}=0,\ b_{1P_i}=\tau$

$$V_i \longrightarrow \boxed{K_{P_i}D.\ \frac{1}{1+sb_{1P_i}+s^2b_{2P_i}}} \longrightarrow P_{Li}^D$$

T_r = Rise time
T_s = Settling time
M_p = Peak overshoot
freq, damp. ratio

(?) P_{Li} = 50% static + 50% dynamic

$P_{Li} = 0.25 + 0.125V_i + 0.125V_i^2 + P_{Li}^D$

$P_{Li}^D = 0.5\,\frac{1}{1+s4}\,V_i$

$P_{Li} = ?$

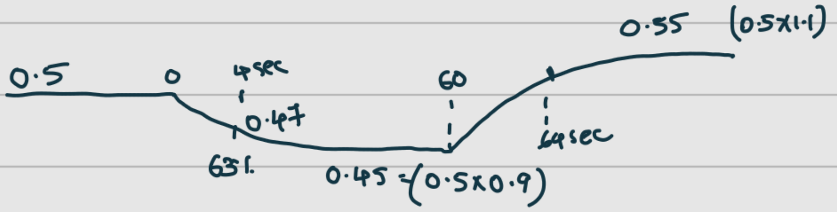
(A) $P_{Li}^{static} = 0.25 + 0.125V_i + 0.125V_i^2$

Controls dynamic Power

$t<0 \Rightarrow P_{Li}^S = 0.5$ pu

$0<t<60 \Rightarrow V_i = 0.9\text{pu} \Rightarrow P_{Li}^S = 0.25 + 0.125(0.9) + 0.125(0.9)^2 = 0.464$ pu

$t>60 \Rightarrow V_i = 1.1\text{pu} \Rightarrow P_{Li}^S = 0.25 + 0.125(1.1) + 0.125(1.1)^2 = 0.539$ pu



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$$V_i \longrightarrow \boxed{\frac{1+a_{1P_i}s}{1+sb_{1P_i}+s^2b_{2P_i}}} \longrightarrow P_{Li}^D$$

$a_{1P_i}=0 \Rightarrow$ std second order s/m

$s^2 + 2\xi\omega_n s + \omega_n^2$

ξ - damp. ratio

ω_n - natural freq.

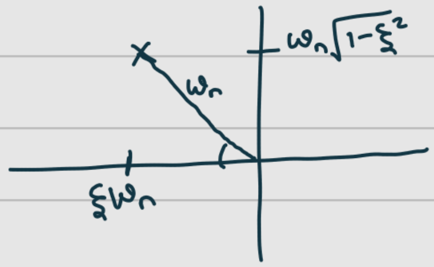
$\xi > 1 \Rightarrow$ Overdamped

$\hookrightarrow$ represent as first order s/m with time delay

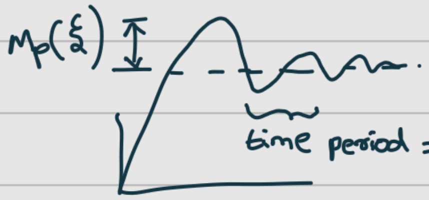
$\xi < 1 \Rightarrow$ underdamped

$s^2 + 2\xi\omega_n + \omega_n^2$

Roots are $-\xi\omega_n \pm j\sqrt{\omega_n^2(1-\xi^2)}$, $\omega_d = \omega_n\sqrt{1-\xi}$ – damped osc. freq



$$\xi = \frac{-\text{Real part}}{\sqrt{Re^2 + Im^2}}$$



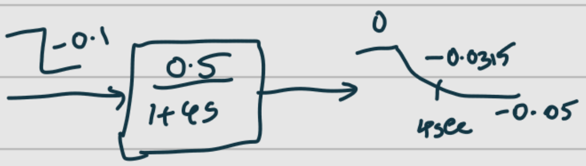
time period $= \dfrac{2\pi}{\omega_d}$

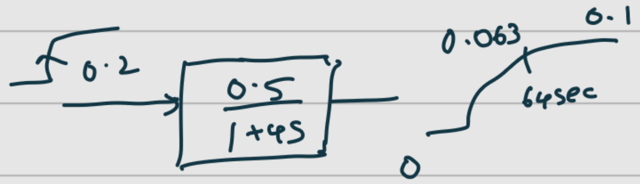
T_r = rise time = Time from 10% to 90%

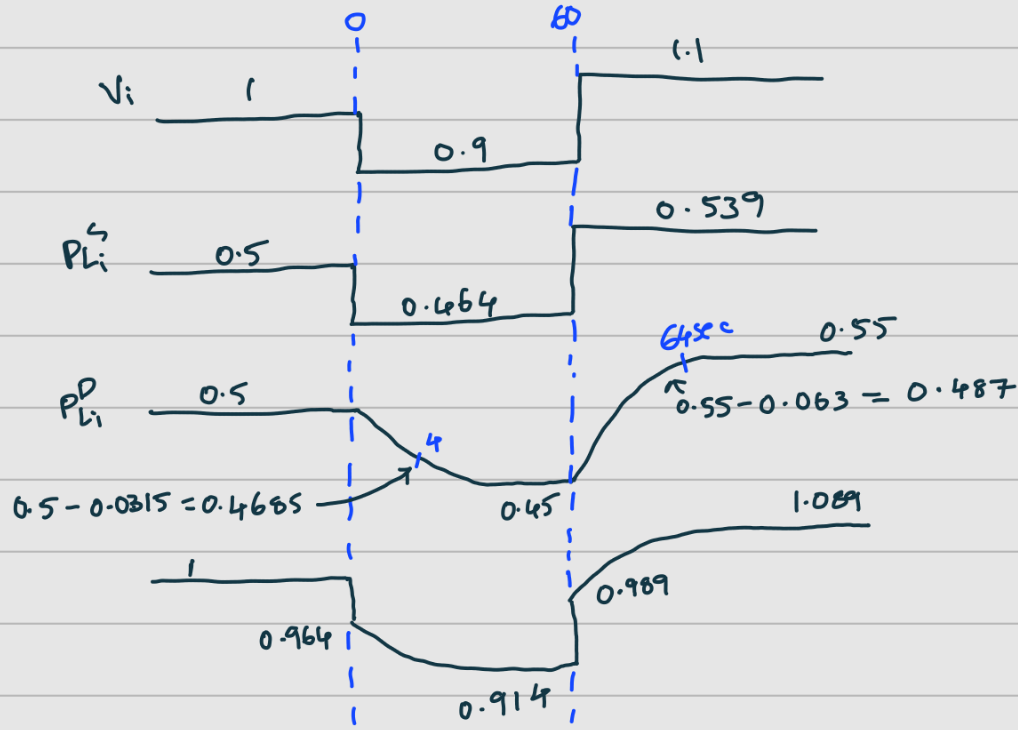
$T_r =$

T_s = settling time = Time taken to stay within either 5% of final value

$T_s =$

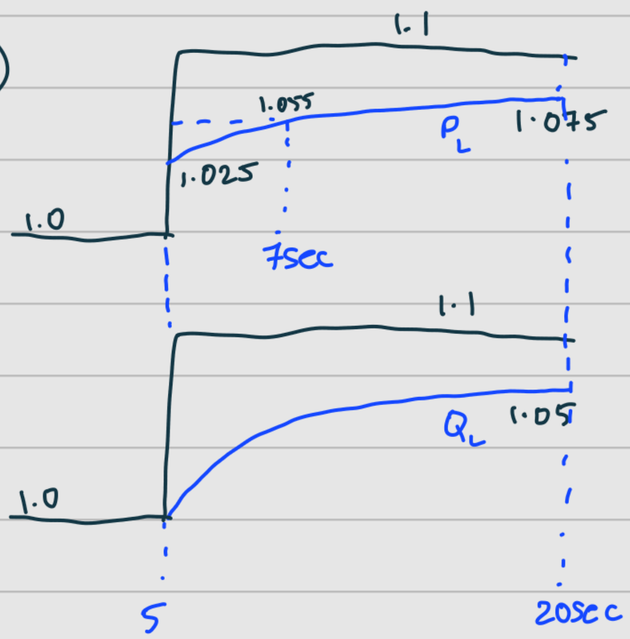






$f_d << 60Hz$

?



if heating load, $M=0$

Assume $P_L = P_{L0} + MV + \cancel{G}V^2 + P_L^D$

(Since no 2nd order)

$$P_L^D = \frac{K_p^D}{1+s\tau_p} V$$

$P_L = K_p^S V^{\alpha_p^S}$

$P_{L0} = ?$ $M = ?$ $K_p^D = ?$ $b_{1p} = ?$

$t = 0^-$; $V = 1$; $P_L = 1$

$P_{L0} + M + K_p^D = 1$ — (1)

$t = 0^+$; $V = 1.1$; $P_L = 1.025$

$P_{L0} + (1.1 \times M) + K_p^D = 1.025$ — (2)

(2) − (1) ⇒ $0.1M = 0.025$

$\underline{\underline{M = 0.25}}$

$t = 20$; $V = 1.1$; $P_L = 1.075$

$P_{L0} + M(1.1) + K_p^D(1.1) = 1.075$ — (3)

(3) − (2) ⇒ $0.1 K_p^D = 0.05$

$\underline{\underline{K_p^D = 0.5}}$

From graph, $\tau = 7 - 5 = \underline{\underline{2sec}}$

$\underline{\underline{b_{1p} = 2}}$

in (1) $P_{L0} + 0.25 + 0.5 = 1$

$\underline{\underline{P_{L0} = 0.25}}$

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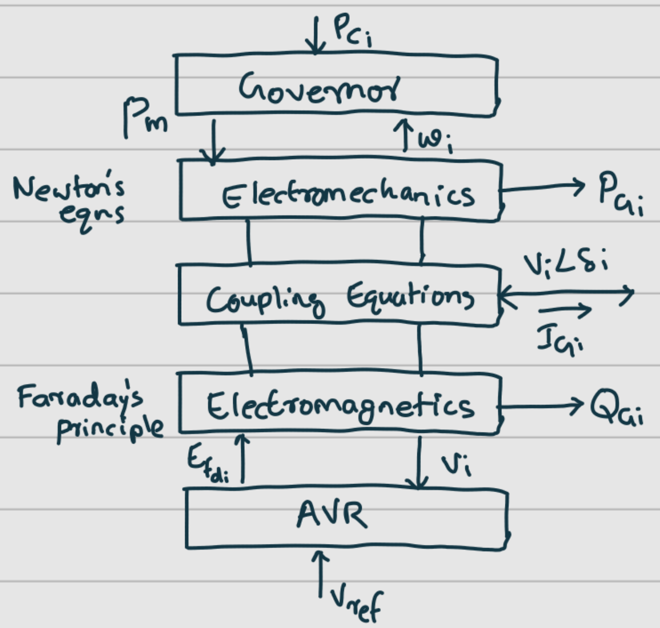
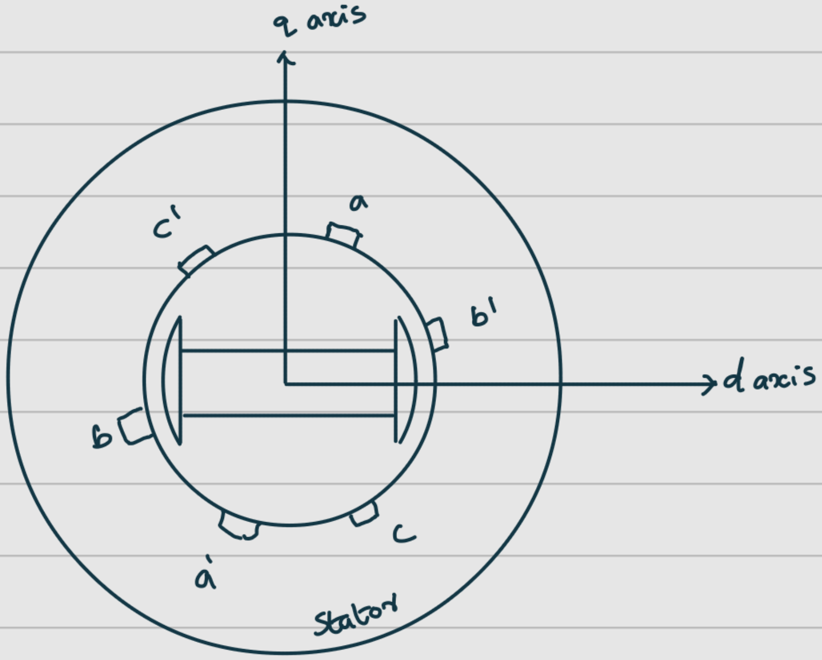
HW 1 : $\varepsilon = 0.01$ MW tolerance for Power-Flow Convergence

All power mismatches should be less than ε $|\Delta P_i| < \varepsilon$ $|\Delta Q_i| < \varepsilon$ $\forall i$



ElectroMechanics



$$P = 2 \Rightarrow N = 3600 \qquad f_e = 60Hz \qquad \text{Turbo, steam}$$
$$f_m = 60Hz$$

hydro $N = 600$ rpm $\quad P = 12 \quad$ 36 slots salient pole rotor
$f = 60Hz$

Design is different

β - d axis displacement w.r.t. Axis a (Ang. displacement of rotor)

$\dot{\beta}$ - Rotor speed (mech)

$P = 2 \Rightarrow$ elec qts = mech qty $\Rightarrow \frac{P}{2} = 1$

$\dot{\beta} = \omega$

At S.S. $\omega = 2\pi f = 120\pi$ rad/sec

$= 377$ rad/sec

$\ddot{\beta} = \dot{\omega}$ - Angular acceleration

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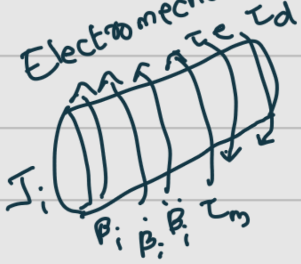
Electromechanics



β_i - ang. displ. of the d-axis w.r.to axis a

ω_i - ang. speed of the rotor

τ_m - Mechanical Torque

τ_e - Torque that gets converted to elec. engy

τ_d - Frictional torque

$$J_i \ddot\beta_i = \tau_m - \tau_e - \tau_D \quad \text{Newton's Equations}$$

Objective: All the sync. m/cs must stay in synchronism

Loss of synchronism

— under stress – fault
  – loss of excitation
  – loss of mech power

β_i can deviate a lot from the other units

θ_i - Angular displacement of the q-axis w.r. to 60Hz rotating frame

β_i – absolute angular displacement (of d axis)

θ_i – relative angular displacement (of q axis) w.r.t. a common 60Hz rotating frame

$$\theta_i = \beta_i + \frac{\pi}{2} - \omega_s t \quad , \quad \omega_s = 2\pi 60$$

$$\boxed{\dot\theta_i = \dot\beta_i - \omega_s = \omega_i - \omega_s}$$

$$\ddot\theta_i = \ddot\beta_i$$

$$J_i \ddot\beta_i = T_m - T_e - T_d$$

$$\boxed{J_i \ddot\theta_i = T_m - T_e - T_d}$$

$$(\text{Torque})\left(\begin{smallmatrix}\text{Ang.}\\ \text{speed}\end{smallmatrix}\right) = \text{Elec. power} = (\text{Voltage})(\text{current})$$

$$J_i \omega_i \ddot\theta_i = T_{m_i}\omega_i - T_{e_i}\omega_i - T_{d_i}\omega_i$$

$$= P_{m_i} - P_{e_i} - P_{d_i}$$

P_{m_i} - mech energy i/p into the rotor

P_{e_i} - elec. energy o/p from the rotor

P_{d_i} - elec. damping power

$\omega_i \approx \omega_s$ (assumption)

$$J_i \omega_s \ddot{\theta}_i = P_{m_i} - P_{e_i} - P_{d_i}$$

$\div S_{rating_i}$

$$= P_{m_i}(p.u.) - P_{e_i}(p.u.) - P_{d_i}(p.u.)$$

$$\underbrace{\frac{J_i \omega_s}{S_{rating_i}}} \ddot{\theta}_i = P_{m_i} - P_{e_i} - P_{d_i}$$

H_i = Inertia time constant = $\frac{\text{stored KE in rotor shaft}}{S_{rating_i}}$

Mech systems			Elec Systems
$P_{m_i} \Rightarrow \sum P_{a_i}$	=		$\sum P_{L_i}$ + losses

steam/water

(True all the time)

Governor Valves Seconds to minutes $\Rightarrow$ Stored energy in rotor shaft $\Rightarrow$ very fast Instantaneous

Buffer

$$\sum \frac{1}{2} J_i \omega_i^2$$

$$KE_i = \frac{1}{2} J_i \omega_i^2$$

$$H_i = \frac{1/2\, J_i \omega_s^2}{S_{rating_i}} = \text{Inertia time constant}$$

$H_i = 4$ seconds $\Rightarrow$ M/c can supply its rated power for 4 seconds with no mech. energy i/p

$H_1 = 4$ sec, $H_2 = 10$ sec $\rightarrow$ not comparable rating should be given

$$\frac{J_i \omega_s}{S_{rating}} \ddot{\theta}_i = P_{m_i} - P_{e_i} - P_{d_i}$$

$$H_i = \frac{1/2\, J_i \omega_s^2}{S_{rating_i}}$$

$$\frac{2H_i}{\omega_s} = \frac{J_i \omega_s}{S_{rating_i}}$$

$$\boxed{\frac{2H_i}{\omega_s}\ddot{\theta}_i = P_{m_i} - P_{e_i} - P_{d_i}} \quad (2)$$

$$\dot{\theta}_i = \omega_i - \omega_s \quad (1)$$

$$\omega_i(pu) = \frac{\omega_i}{\omega_s}$$

$$\dot{\theta}_i = \left(\frac{\omega_i}{\omega_s} - 1\right)\omega_s$$

$$\dot{\theta}_i = (\omega_i(pu) - 1)\,\omega_s$$

$$\boxed{\dot{\theta}_i = (\omega_i - 1)\,\omega_s} \quad (1)$$

- Swing eqns

$\theta_i =$

$$\ddot{\theta}_i = \dot{\omega}_i\,\omega_s$$

$$\frac{2H_i}{\omega_s}\omega_s\dot{\omega}_i = P_{m_i} - P_{e_i} - P_{d_i}$$

$$\boxed{2H_i\dot{\omega}_i = P_{m_i} - P_{e_i} - P_{d_i}} \quad (2)$$

θ_i = Relative ang. displ. w.r.t. 60 Hz of the q axis $= \beta_i + \frac{\pi}{2} - \omega_s t$

ω_i = Rotor speed in p.u. $= \dot{\beta}_i/\omega_s$

P_{m_i} = Mech Torque i/p

P_{e_i} = Elec. Power Converted $= P_{a_i}$ + Core losses

P_{d_i} = Damping power $= \underbrace{K_{D_i}}_{\text{damping constant}}(\omega_i - 1)$

Frictional torque — is very small

Damper winding / Amortisseur Winding

02/01/24

Synchronous Machine Models

Electro Mechanics : Governor

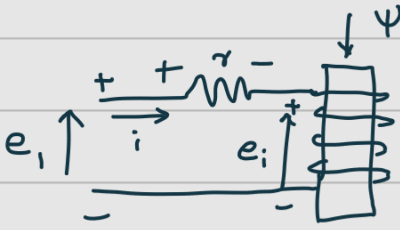
Electro Magnetics : AVR

$$\dot{\theta}_i = (\omega_i - 1)\,\omega_s$$

$$2H_i\dot{\omega}_i = P_{m_i} - P_{e_i} - P_{d_i}$$

$$P_{e_i} = P_{a_i} + R_{s_i}|I_{a_i}|^2$$

$$P_{d_i} = K_{D_i}(\omega_i - 1)$$



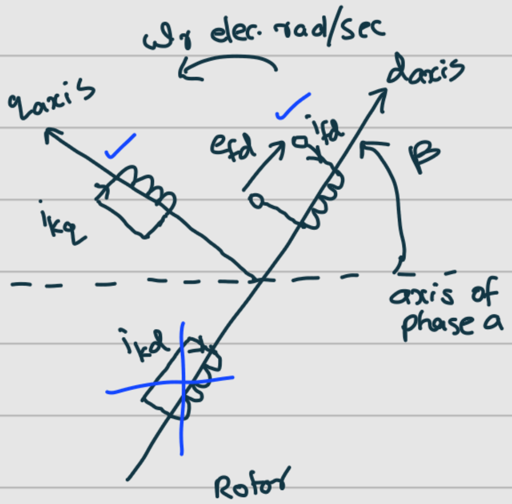
$e_i = \dfrac{d\psi}{dt}$ Faraday's Principle

$e_1 = e_i + r_i$

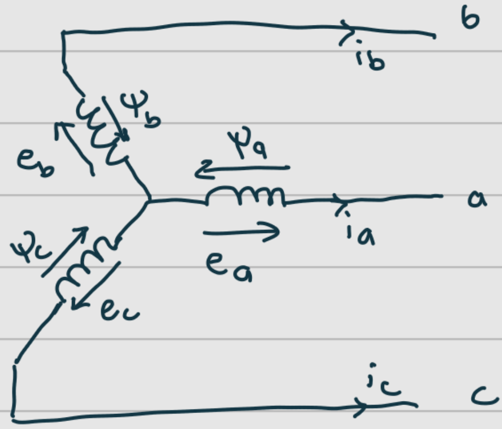
$e_1 = \dfrac{d\psi}{dt} + r_i$

Rotor Field : Flowing in

Stator Axis a : Flowing out
b
c



Rotor



Stator

$\cancel{i_{fd}}, i_{1q}, \cancel{i_{2q}}$

$e_{fd} = \dfrac{d\psi_{fd}}{dt} + R_{fd}\, i_{fd}$

$k = 1, 2, 3 \}$ damped

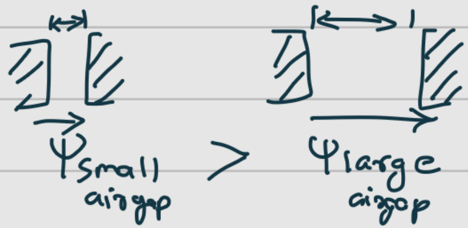
$0 = \dfrac{d\psi_{1q}}{dt} + R_{1q}\, i_{1q}$

$e_a = \dfrac{d\psi_a}{dt} - R_a i_a \;;\quad e_b = \dfrac{d\psi_b}{dt} - R_b i_b \;;\quad e_c = \dfrac{d\psi_c}{dt} - R_c i_c$

$\psi = Li$

$\psi_a \Rightarrow i_a, i_b, i_c, i_{fd}, i_{1q}$ (ignoring saturation)

$$\begin{bmatrix} \psi_a \\ \psi_b \\ \psi_c \\ \psi_{fd} \\ \psi_{1q} \end{bmatrix} = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & L_{afd} & L_{a1q} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_{fd} \\ i_{1q} \end{bmatrix}$$ — mag. ckt. eqn.



$L_{\substack{small \\ airgap}} > L_{\substack{large \\ airgap}}$

$$\psi_a = -i_a\left[L_{aa0} + L_{aa2}\cos 2\beta\right] + i_b\left[L_{ab0} + L_{aa2}\cos\left(2\beta + \frac{\pi}{3}\right)\right]$$
$$+ i_c\left[L_{ac0} + L_{aa2}\cos\left(2\beta - \frac{\pi}{3}\right)\right] + i_{fd} L_{afd}\cos\theta + i_{kd} L_{akd}\cos\theta$$
$$- i_{kq} L_{akq}\sin\theta$$

- Highly non-linear
- Functions of rotor position (β)

} Difficult to solve

Park Transformation

$$\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + 2\frac{\pi}{3}\right) \\ -\sin\theta & -\sin\left(\theta - 2\frac{\pi}{3}\right) & -\sin\left(\theta + 2\frac{\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

$$[\psi] = [L]\,[i]$$

↳ constant matrix after Park Tr.

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 1 \\ \cos\left(\theta - \frac{2\pi}{3}\right) & -\sin\left(\theta - \frac{2\pi}{3}\right) & 1 \\ \cos\left(\theta + \frac{2\pi}{3}\right) & -\sin\left(\theta + \frac{2\pi}{3}\right) & 1 \end{bmatrix}\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix}$$

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} \sqrt{2}\,V\cos(\omega_s t + \delta) \\ \sqrt{2}\,V\cos(\omega_s t + \delta - 2\pi/3) \\ \sqrt{2}\,V\cos(\omega_s t + \delta + 2\pi/3) \end{bmatrix}$$

$$\theta = \beta + \frac{\pi}{2} - \omega_s t$$

$$\begin{bmatrix} v_d \\ v_q \\ v_0 \end{bmatrix} = P(\beta)\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} V\sin(\theta - \delta) \\ V\cos(\theta - \delta) \\ 0 \end{bmatrix}$$

$V\angle\delta \Rightarrow v_d = V\sin(\theta - \delta)$

$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \xrightarrow{\text{Park}}$ $\theta_i = \beta_i + \frac{\pi}{2} - \omega_s t$ $\quad \underbrace{v_q = V\cos(\theta - \delta)}_{\text{Rotor freq}}$

(60Hz)

$$V_d + jV_q = V\angle\delta \,\angle\left(\tfrac{\pi}{2}-\theta\right)$$

$$V\angle\delta \,\angle\left(\tfrac{\pi}{2}-\theta\right) = V\angle\left(\tfrac{\pi}{2}+\delta-\theta\right)$$

$$V_d = V\cos\left(\tfrac{\pi}{2}+\delta-\theta\right) = -V\sin(\delta-\theta)$$
$$= V\sin(\theta-\delta)$$

$$V_q = V\cos\left(\tfrac{\pi}{2}+\delta-\theta\right) = V\cos(\delta-\theta)$$
$$= V\cos(\theta-\delta)$$

02/06/24

Course Project: IEEE Tras. after 2000 (Rec: IEEE Trans. on Power Systems)

IEEE Trans. on Smart Grid, Electric Delivery

$$i_a = I\cos(\omega_s t+\phi) \Rightarrow \bar{I}_a = I\angle\phi$$
$$i_b = I\cos(\omega_s t+\phi-2\pi/3)$$
$$i_c = I\cos(\omega_s+\phi+2\pi/3)$$

Apply Park Transformation

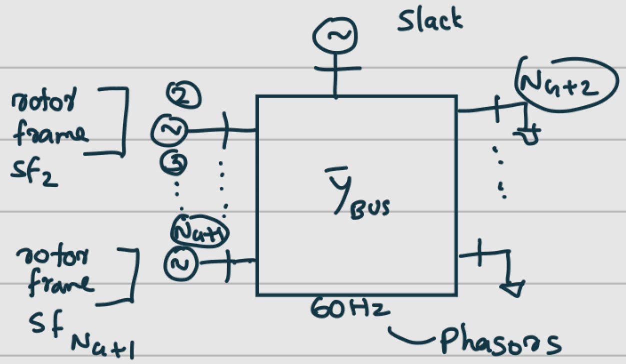
$$i_d = I\sin(\theta-\phi)$$
$$i_q = I\cos(\theta-\phi)$$

$$\theta = \beta-\omega_s t+\pi/2$$

$$\underbrace{I_d + jI_q}_{\text{Rotor frame}} = \bar{I}\angle\theta-\pi/2$$

rotor speed (ω) ⇐ Rotor frame

→ Phasor w. r. to. 60Hz rot. frame (ω_s)



$V_d + jV_q = V\angle\delta \angle \pi/2 - \theta \xleftarrow{\text{Park}}$

Park Trans. $V_i \angle\delta_i \angle \pi/2 - \theta_i = V_{d_i} + jV_{q_i}$

Inv. Park Trans. $(V_{d_i} + jV_{q_i}) \angle \theta_i - \pi/2 = V_i \angle\delta_i$

Stator $\begin{cases} e_d = p\psi_d - \psi_q \omega_r - R_a i_d \\ e_q = p\psi_q + \psi_d \omega_r - R_a i_q \end{cases}$ assume balanced $e_0 = 0$ $\qquad p = \frac{d}{dt}$

rotor $\begin{cases} e_{fd} = p\psi_{fd} + R_{fd} i_{fd} \\ 0 = p\psi_{1q} + R_{1q} i_{1q} \end{cases}$ assume only 1 damper wdg

$$\psi_d = -(L_{ad} + L_l) i_d + L_{ad} i_{fd} \qquad 3.127$$
$$\psi_q = -(L_{aq} + L_l) i_q + L_{aq} i_{1q}$$

$$\psi_{fd} = L_{ffd} i_d - L_{ad} i_d$$
$$\psi_{1q} = L_{11q} i_{1q} - L$$

Parameters that can be estimated by std tests
↳ OC test, SC test, Step test

$E_q' = \frac{L_{ad}}{L_{ffd}} \psi_{fd} \rightarrow$ Main induced voltage from field wdg

$E_d' = \frac{L_{aq}}{L_{11q}} \psi_{1q} \rightarrow$ Main induced voltage along the q axis

$X_d = \qquad X_d' = \qquad X_q = \qquad X_q'$

8 eqns $\xrightarrow[\omega]{\text{reduce}}$ 4 eqns : Two Axis Model

$\psi_{fd} \Rightarrow \quad T_{d0}' \dot{E}_q' = -E_q' - (X_d - X_d') I_d + E_{fd}$

$\psi_{1q} \Rightarrow \quad T_{q0}' \dot{E}_d' = -E_d' - (X_q - X_q') I_q$

$$E_q' = V_q + R_a I_q + x_d' I_d$$
$$E_d' = V_d + R_a I_d + X_q' I_q$$

$i_d \longleftrightarrow I_d$

$i_q \longleftrightarrow I_q$

$v_d = e_d \longleftrightarrow V_d$

$v_q = e_q \longleftrightarrow V_q$

$$V_d + jV_q = V\angle\delta \angle \pi/2 - \theta \qquad I_d + jI_q = \bar{I}_a \angle \tfrac{\pi}{2} - \theta$$

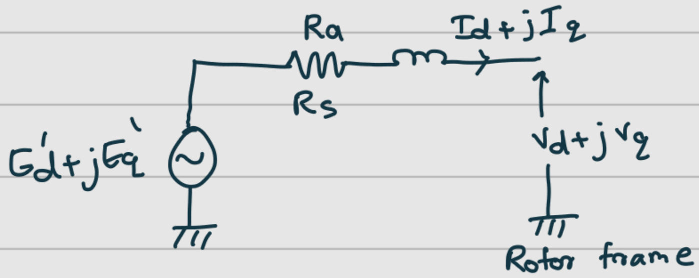
$$V\angle\delta = V_d + jV_q \angle \theta - \pi/2 \qquad I\angle\delta = I_d + jI_q \angle \theta - \pi/2$$

$$P_a + jQ_a = V\angle\delta\ I_a^*$$
$$= \left[(V_d + jV_q)\angle\theta - \pi/2\right]\left[I_d + jI_q \angle \theta - \pi/2\right]^*$$

$$= (V_d + jV_q)(I_d + jI_q)^*$$
$$= (V_d + jV_q)(I_d - jI_q)$$

$$\left.\begin{aligned} P_a &= V_d I_d + V_q I_q \\ Q_a &= V_q I_d - V_d I_q \end{aligned}\right\}$$ Park frame
Rotor frame

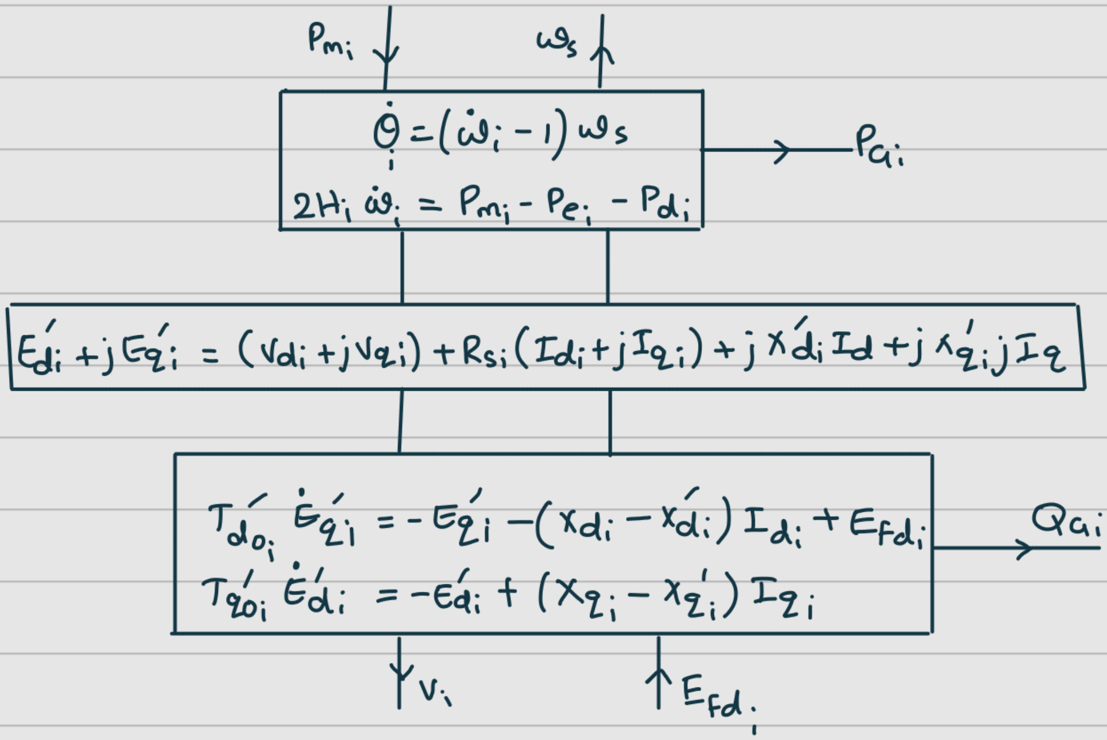
$$E_q' = V_q + R_a I_q + X_d' I_d$$
$$E_d' = V_d + R_a I_d - X_q' I_q$$



$X_d' \neq X_q'$ (salency effect)

$$E_d' + jE_q' = V_d + jV_q + R_a(I_d + jI_q) + jX_d' I_d + jX_q' jI_q$$

$$E_d' = V_d + R_a I_d - X_q' I_q$$
$$E_q' = V_q + R_a I_q - X_d' I_d$$



02/08/24

Swing
$$\begin{cases} \dot{\theta}_i = (\omega_i - 1)\,\omega_s \\ 2H_i \dot{\omega}_i = P_{m_i} - P_{e_i} - P_{d_i} \end{cases}$$

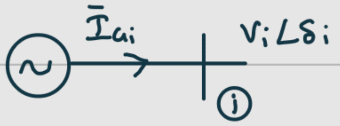
$$P_{d_i} = K_{D_i}(\cos \quad)$$
$$P_{e_i} = P_{a_i} + R_{s_i} |I_{a_i}|^2$$

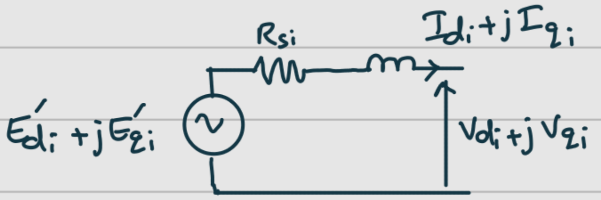
flux decay
$$\begin{cases} T'_{do_i} E'_{q_i} = -E'_{q_i} - (x_{d_i} - x'_{d_i}) i_{d_i} + E_{fd_i} \\ T'_{qo_i} E'_{d_i} = -E'_{d_i} - (x_{q_i} - x'_{q_i}) i_{q_i} \end{cases}$$

stator coupling
$$\left\{ E'_{d_i} + jE'_{q_i} = (V_{d_i} + jV_{q_i}) + R_{s_i}(I_{d_i} + jI_{q_i}) + jx'_{d_i} I_{d_i} + jx'_{q_i}(jI_{q_i}) \right.$$

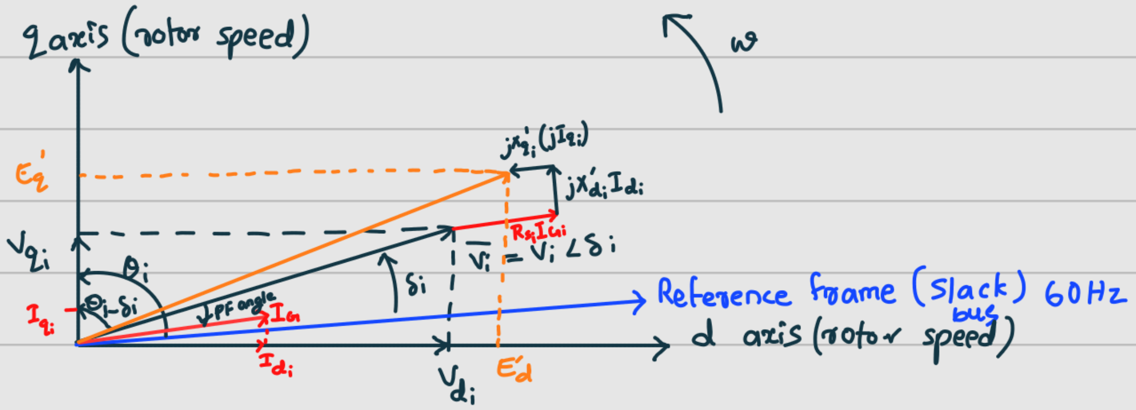
Park
$$V_{d_i} + jV_{q_i} = v_i \angle \delta_i \angle \tfrac{\pi}{2} - \theta_i \quad , \quad I_{d_i} + jI_{q_i} = \bar{I}_{a_i} \angle \tfrac{\pi}{2} - \theta_i$$
$$P_{a_i} = V_{d_i} I_{d_i} + V_{q_i} I_{q_i} \quad , \quad Q_{a_i} = V_{q_i} I_{d_i} - V_{d_i} I_{q_i}$$

$\theta_i, \omega_i, E'_{q_i}, E'_{d_i}$





$$E_{d_i} + jE_{q_i} = V_{d_i} + jV_{q_i} + R_{s_i}(I_{d_i} + jI_{q_i}) + jX'_{d_i} I_{d_i} + jX'_{q_i}(jI_{q_i})$$



θ_i – relative angle

$$V_{q_i} = V_i \cos(\theta_i - \delta_i)$$
$$V_{d_i} = V_i \sin(\theta_i - \delta_i)$$

Steady State

$$T'_{d_i} \dot{E}'_{q_i} = -E'_{q_i} - (x_{d_i} - x'_{d_i}) I_{d_i} + E_{fd_i} = 0 \quad (1)$$
$$T'_{q_i} \dot{E}'_{d_i} = -E'_{d_i} + (x_{q_i} - x'_{q_i}) I_{q_i} = 0 \quad (2)$$

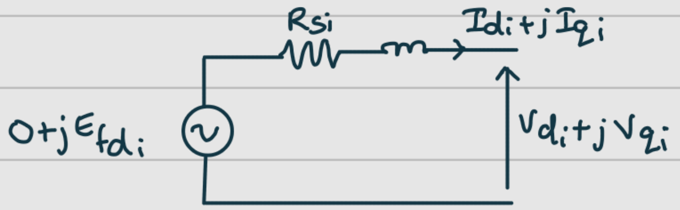
$$E'_{d_i} + jE'_{q_i} = V_{d_i} + jV_{q_i} + R_s(I_{d_i} + jI_{q_i}) + jx'_{d_i} I_{d_i} + jx'_{q_i}(jI_{q_i})$$

Transient
$$\begin{cases} E'_{d_i} = V_{d_i} + R_{s_i} I_{d_i} - X'_{q_i} I_{q_i} \\ E'_{q_i} = V_{q_i} + R_{s_i} I_{q_i} + X'_{d_i} I_{d_i} \quad (3) \end{cases}$$

$$(1) \Rightarrow E'_{q_i} = -(x_{d_i} - x'_{d_i}) I_{d_i} + E_{fd_i} \quad (4)$$

$$(4) = (3) \Rightarrow V_{q_i} + R_{s_i} I_{q_i} + x'_{d_i} I_{d_i} = -(x_{d_i} - x'_{d_i}) I_{d_i} + E_{fd_i}$$

Steady state
$$\begin{cases} E_{fd_i} = V_{q_i} + R_{s_i} I_{q_i} + x_{d_i} I_{d_i} \\ 0 = V_{d_i} + R_{s_i} I_{d_i} - x_{q_i} I_{q_i} \end{cases}$$

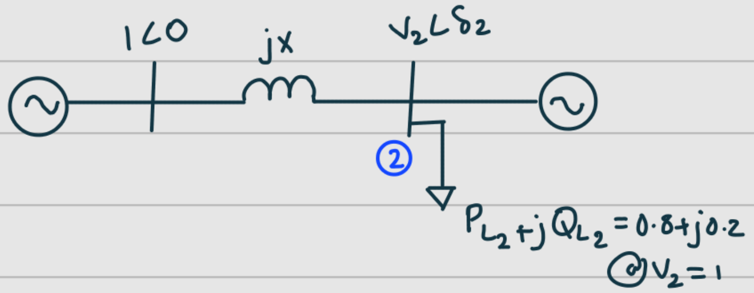


$$0 + jE_{fd_i} = V_{d_i} + jV_{q_i} + R_{s_i}(I_{d_i} + jI_{q_i}) + jx_{d_i} I_{d_i} + jx_{q_i}(jI_{q_i})$$

X_{d_i} – Steady state reactance along d axis
X_{q_i} – Steady state reactance along q axis

X_d' in transient $\Rightarrow$ X_{d_i} in steady state

Example



$P_L = 50\%P,\ 25\%I,\ 25\%Z$
$Q = 60\%P, 40\%Z$

$P_{L_2} = 0.8(0.5 + 0.25V_2 + 0.25V_2^2)$
$Q_{L_2} = 0.2(0.6 + 0.4V_2^2)$

$$\begin{bmatrix} -j/x & j/x \\ j/x & -j/x \end{bmatrix} \Rightarrow P_{G_2} - P_{L_2} = \frac{V_2 \cdot 1}{x}\sin(\delta_2)$$

$$Q_{G_2} - Q_{L_2} = \frac{V_2^2 - V_2\cos(\delta_2)}{x}$$

$$P_{ij} = \frac{V_i V_j}{x_{ij}}\sin(\delta_i - \delta_j)$$

$Q_{ij} =$

Gen dynamics $\dot{x} = f(x, y)$
$0 = g(x, y)$
DAG
Differential Algebraic Eqns

N/w constraints
Power flow equations

$$x = \begin{bmatrix} \theta_2 \\ \omega_2 \\ E_{q_2}' \\ E_{d_2}' \end{bmatrix} \qquad y = \begin{bmatrix} \delta_2 \\ V_2 \end{bmatrix}$$

$$\dot{\theta}_2 = (\omega_2 - 1)\omega_s$$

$$2H_2\dot{\omega}_2 = P_{m_2} - P_{e_2} - K_{d_2}(\omega_2 - 1)$$

$$T'_{do2}\dot{E}'_{q_2} = -E'_{q_2} - (x_{d_{i_2}} - x'_{d_{i_2}})I_{d_2} + E_{fd_2}$$

$$T'_{q02}\dot{E}'_{d_2} = -E'_{d_2} + (x_{q_{i2}} - x'_{q_{i2}})I_{q_2}$$

I_a - 60Hz phasor domain

I_d, I_q - Park frame
Rotor frame

$$P_{e_2} = P_{a_2} + R_{s_2}(|I_{a_2}|^2) = P_{a_2} + R_{s_2}(I_{d_2}^2 + I_{q_2}^2)$$

$$P_{a_2} = V_{d_2}I_{d_2} + V_{q_2}I_{q_2}$$

$$Q_{a_2} = V_{q_2}I_{d_2} - V_{d_2}I_{q_2}$$

$$V_{d_2} = V_2\sin(\theta_2 - \delta_2)$$

$$V_{q_2} = V_2\cos(\theta_2 - \delta_2)$$

Six independent variable: $\underbrace{\theta, \omega, E'_d, E'_q}_{\text{Dynamic}}, \underbrace{\delta_2, V_2}_{\text{Power Flow}}$

02/13/24

$$P_{e_2} = P_{a_2} + R_{s_2}(|I_{a_2}|^2)$$

$$= P_{a_2} + R_{s_2}(I_{d_2}^2 + I_{q_2}^2)$$

$$I_{d_2} + jI_{q_2} = \bar{I}_{a_2}\underline{/\pi/2 - \theta_2}$$

$$|I_{a_2}|^2 = I_{d_2}^2 + I_{q_2}^2$$

$$E'_{d_2} + jE'_{q_2} = V_{d_2} + jV_{q_2} + R_{s_2}(I_{d_2} + jI_{q_2}) + jx'_{d_2}I_{d_2} + jx'_{q_2}\,jI_{q_2}$$

$$E'_{d_2} = V_{d_2} + R_{s_2}I_{d_2} - x'_{q_2}$$

$$E'_{q_2} =$$

$$V_{d_2} = V_2\sin(\theta_2 - \delta_2), \quad V_{q_2} = V_2\cos(\theta_2 - \delta_2)$$

$$\begin{bmatrix} E'_{d_2} - V_{d_2} \\ E'_{q_2} - V_{q_2} \end{bmatrix} = \begin{bmatrix} R_{s2} & -X'_{q_2} \\ X'_{d_2} & R_{s_2} \end{bmatrix} \begin{bmatrix} I_{d_2} \\ I_{q_2} \end{bmatrix}$$

$$\begin{bmatrix} I_{d_2} \\ I_{q_2} \end{bmatrix} = \begin{bmatrix} R_{s_2} & -X'_{q_2} \\ X'_{d_2} & R_{s_2} \end{bmatrix}^{-1} \begin{bmatrix} E'_{d_2} - V_{d_2} \\ E'_{q_2} - V_{q_2} \end{bmatrix}$$

$$= I_{d_2}(\theta_2, E'_{q_2}, E'_{d_2}, \delta_2, V_2)$$
$$I_{q_2}(\theta_2, E'_{q_2}, E'_{d_2}, \delta_2, V_2)$$

Assume $R_{s_2} = 0$ (stator resistance is zero)

$$I_{d_2} = \frac{E'_{q_2} - V_{q_2}}{X'_{d_2}} = \frac{E'_{q_2} - V_2 \cos(\theta_2 - \delta_2)}{X'_{d_2}}$$

$$I_{q_2} = \frac{E'_{d_2} - V_{d_2}}{-X'_{q_2}} = \frac{E'_{d_2} - V_2 \sin(\theta_2 - \delta_2)}{-X'_{q_2}}$$

$$P_{G_2} = V_{d_2} I_{d_2} + V_{q_2} I_{q_2} = P_{G_2}(\theta_2, E'_{q_2}, E'_{d_2}, \delta_2, V_2)$$
$$Q_{G_2} = V_{q_2} I_{d_2} - V_{d_2} I_{q_2} = Q_{G_2}(\theta_2, E'_{q_2}, E'_{d_2}, \delta_2, V_2)$$

Swing eqn: $2H_2 \dot{\omega}_2 = P_{m_2} - P_{e_2} - K_{d_2}(\omega_2 - 1)$

$R_s = 0 \Rightarrow P_{e_2} = P_{G_2} = P_{G_2}(\theta_2, E'_{q_2}, E'_{d_2}, \delta_2, V_2)$

$$T'_{d_{02}} \dot{E}'_{q_2} = -E'_{q_2} - (X_{d_2} - X'_{d_2}) I_{d_2} + E_{fd_2}$$
$$T'_{q_{02}} \dot{E}'_{d_2} = -E'_{d_2} + (X_{q_2} - X'_{q_2}) I_{q_2}$$

$$\dot{x} = f(x, y)$$
$$0 = g(x, y)$$

$$P_{G_2} - P_{L_2} = \frac{V_2 \cdot 1}{X} \sin\delta_2 = 0$$

$$P_{G_2} = P_{G_2}(\theta_2, E'_{q_2}, E'_{d_2}, \delta_2, V_2)$$

$$P_{L_2} = 0.8(0.5 + 0.25V_2 + 0.25V_2^2)$$
$$P_{L_2} = P_{L_2}(V_2)$$

$$Q_{G_2} - Q_{L_2} - \frac{V_2^2 - V_2\cos\delta_2}{X} = 0$$

$$Q_{G_2} = Q_{G_2}(\theta_2, E_{q_2}', E_{d_2}', \delta_2, V_2)$$

$$Q_{L_2} = 0.2($$
$$Q_{L_2} = Q_{L_2}(V)$$

$$\begin{bmatrix} \dot{\theta} \\ \dot{\omega} \\ \dot{E}_{q_2}' \\ \dot{E}_{d_2}' \end{bmatrix} = \begin{bmatrix} (\omega_2 - 1)\omega_s \\ \frac{1}{2H_2}[\quad] \\ \frac{1}{T_{d0_2}'}[\quad] \\ \frac{1}{T_{q0_2}'}[\quad] \end{bmatrix} \qquad f(x,y)$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} P_{G_2} - P_{L_2} - \frac{V_2}{X}\sin\delta_2 \\ Q_{G_2} - Q_{L_2} - \frac{V_2^2 - V_2\cos\delta_2}{X} \end{bmatrix} \qquad g(x,y)$$

<u>Type I model</u>

$\#(x)\ 4N_G$ $\qquad$ $\#(y)\ 2(N-1)$

Swing eqns (Electromechanics)
Two axis flux decay model (Electromagnetics)
ZIP Load Model (no dynamic load)

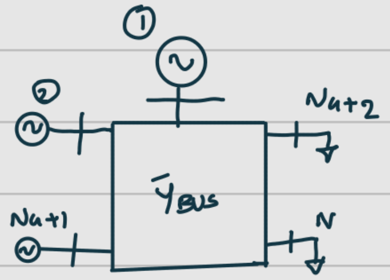
$\dot{y} =$

$\dot{x} =$

Saliency effect $x'_d \neq x'_i$

Assume

02/14/24



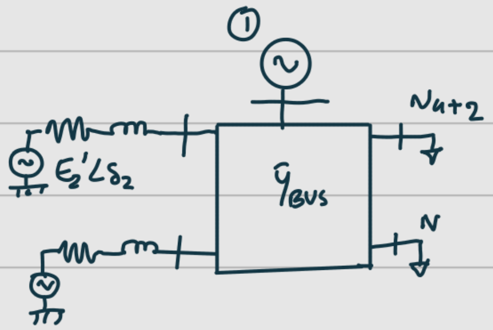
$$\dot{x} = f(x, y)$$
$$0 = g(x, y)$$

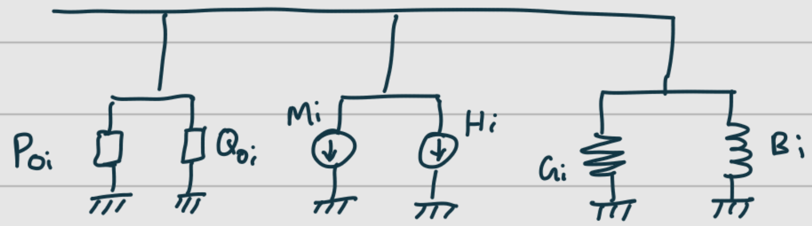
Type 1

Network Reduction

Assumptions

$$0 = g(x,y) \rightarrow y = h(x) \implies \dot{x} = f(x, h(x)), \quad \dot{x} = \tilde{f}(x) \text{ — ODE}$$



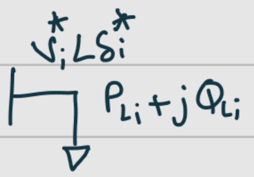


$$\left.\begin{aligned} P_{Li}^{s} &= P_{0i} + M_i V_i + G_i V_i^2 \\ Q_{Li}^{s} &= Q_{0i} + H_i V_i + B_i V_i^2 \end{aligned}\right\} \text{ZIP Model}$$

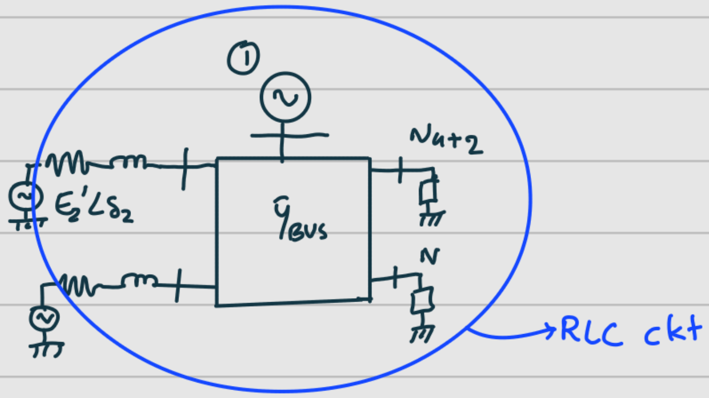
To apply Thevenin / Norton — we need linear n/w

$\implies$ Assume const power is not present

const current source makes calc. difficult $\rightarrow$ Assume not present

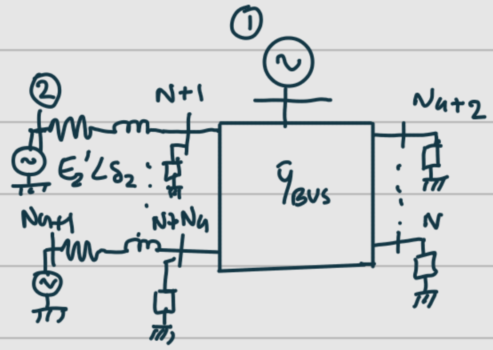


$$\bar{Y}_{Li} = \frac{P_{Li} - jQ_{Li}}{|V_i^*|^2}$$



Adding additional nodes in the system

$$\bar{V}_{Gen} = \begin{bmatrix} 1\angle 0 \\ E_2' \angle \delta_2 \\ \vdots \\ E_{N_G+1}' \angle \delta_{N_G+1} \end{bmatrix} \qquad \bar{V}_{rest} = \begin{bmatrix} V_{N_G+2} \\ \vdots \\ V_{N+N_G} \end{bmatrix}$$

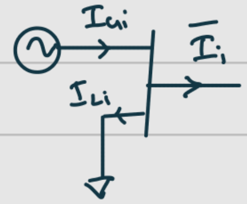


$$\bar{Y}_{Net\,(N+N_G \times N+N_G)}$$

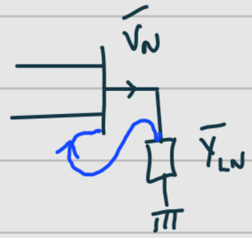
$$\bar{V}_{Gen} = \begin{bmatrix} \overset{1\angle 0}{E_1' \angle \delta_1} \\ E_2' \angle \delta_2 \\ \vdots \\ E_{N_G+1} \angle \delta_{N_G+1} \end{bmatrix} \qquad \bar{V}_{rest} = \begin{bmatrix} \bar{V}_{N_G+2} \\ \vdots \\ \bar{V}_N \\ \bar{V}_{N+1} \\ \vdots \\ \bar{V}_{N+N_G} \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{Load terminals} \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{Gen terminals} \end{matrix}$$

$$I_{Gen} = \begin{bmatrix} \bar{I}_{G_1} \\ \vdots \\ \bar{I}_{G_{N+1}} \end{bmatrix} \qquad \bar{I}_{rest} = \begin{bmatrix} \bar{I}_{G_{N+2}} \\ \vdots \\ \bar{I}_{G_{N+N_G}} \end{bmatrix}$$

$$\begin{bmatrix} I_{Gen} \\ I_{rest} \end{bmatrix} = \begin{bmatrix} \bar{Y}_{Net} \end{bmatrix} \begin{bmatrix} \bar{V}_{Gen} \\ \bar{V}_{rest} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} \bar{V}_{Gen} \\ \bar{V}_{rest} \end{bmatrix}$$



$$\begin{bmatrix} \bar{I}_1 \\ \vdots \\ \bar{I}_n \end{bmatrix} = \begin{bmatrix} \bar{Y}_{BUS} \end{bmatrix} \begin{bmatrix} \bar{V}_1 \\ \vdots \\ \bar{V}_n \end{bmatrix}$$



$$\bar{I}_N = \cancel{I_{G_N}}^{0} - \cancel{I_{L_N}}^{0} = 0$$

Zero Injection bus

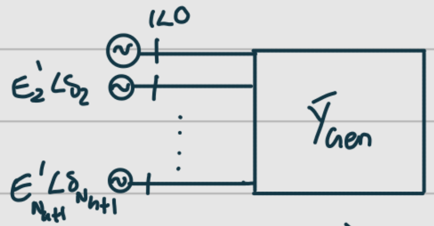
$$\Rightarrow I_{rest} = 0$$

$$\begin{bmatrix} I_{Gen} \\ I_{rest} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_{Gen} \\ V_{rest} \end{bmatrix}$$

$$\bar{Y}_{21} \bar{V}_{Gen} + \bar{Y}_{22} \bar{V}_{rest} = 0$$

$$\boxed{\bar{V}_{rest} = -\bar{Y}_{22}^{-1} \bar{Y}_{21} \bar{V}_{Gen}}$$

Thevenin equivalency



Reduced Network

N/W Reduction Principle

Assumptions
1. Gen $\rightarrow$ linear ckt
   $X_d' = X_q'$ (no saliency)
2. Load $\rightarrow$ linear ckt
   pure const Z

$$\begin{bmatrix} I_{G_1} \\ I_{G_2} \\ \vdots \\ I_{G_{N_{G+1}}} \end{bmatrix} = \bar{Y}_{Gen} \begin{bmatrix} \bar{V}_{G_1} \\ \bar{V}_{G_2} \\ \vdots \\ V_{G_{N_{G+1}}} \end{bmatrix}$$

$$\bar{I}_G = \sum_{j=1}^{N_{G+1}} \bar{Y}_{G_{ij}} \cdot \bar{V}_{Gj}$$

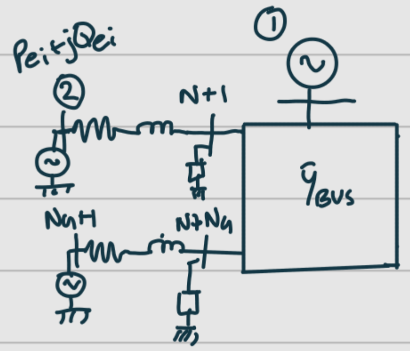
$$= \sum_{j=1}^{N_{G+1}} Y_{G_{ij}} \angle \theta_{G_{ij}} \, E_j' \angle \delta_j$$

$$I_{d_i} + j I_{q_i} = \bar{I}_G \angle \tfrac{\pi}{2} - \theta_i$$

$$= \sum_{j=1}^{N_{G+1}} Y_{G_{ij}} E_j \angle \delta_j + \theta_{G_{ij}} + \tfrac{\pi}{2} - \theta_i$$

$$I_{d_i} = \sum_{j=1}^{N_{G+1}} Y_{G_{ij}} E_j' \cos\left(\delta_j + \theta_{G_{ij}} + \tfrac{\pi}{2} - \theta_i\right)$$

$$\boxed{\begin{aligned} I_{d_i} &= \sum_{j=1}^{N_a+1} Y_{a_{ij}} E_j' \sin(\theta_i - \delta_j - \theta_{a_{ij}}) \\ I_{q_i} &= \sum_{j=1}^{N_a+1} Y_{a_{ij}} E_j' \cos(\theta_i - \delta_j - \theta_{a_{ij}}) \end{aligned}}$$



$$P_{e_i} = \sum_{j=1}^{N_a+1} Y_{a_{ij}} E_i' E_j' \cos(\gamma_i - \gamma_j - \theta_{a_{ij}})$$

Type 2 Model

$$\dot{\theta}_2 = (\omega_i - 1)\omega_s$$

$$2H_i\dot{\omega}_i = P_{M_i} - P_{e_i} - K_{D_i}(\omega_i - 1)$$

$$P_{e_i} = \sum_{j=1}^{N_a+1} Y_{a_{ij}} E_i' E_j' \cos(\gamma_i - \gamma_j - \theta_{a_{ij}}) \qquad P_{a_i}\,(R_s=0)$$

$$T_{do_i}\dot{E}_{q_i}' = -E_{q_i}' - (x_d - x_{d_i}')I_{d_i} + E_{fd_i}$$

$$T_{qo_i}\dot{E}_{d_i}' = -E_{d_i}' + (x_{q_i} - x_{q_i}')I_{q_i}$$

changes in type 1 (we need to solve n/w for these parameters)

$$I_{d_i} = \sum_{j=1}^{N_a+1} Y_{a_{ij}} E_j' \sin(\theta_i - \gamma_j - \theta_{ij})$$

$$I_{q_i} = \sum_{j=1}^{N_a+1} Y_{a_{ij}} E_j' \cos(\theta_i - \gamma_j - \theta_{ij})$$

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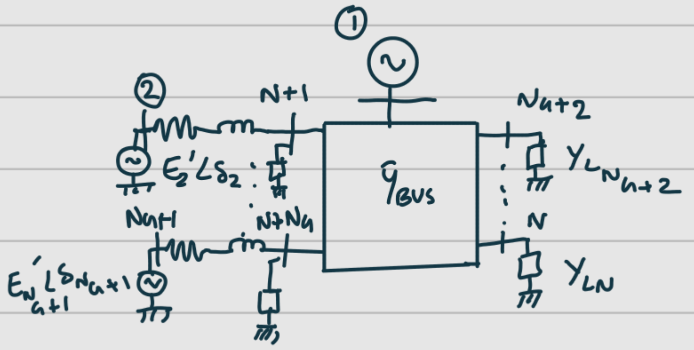
Network Reduction Principle

1. Machine saliency can be ignored
2. Loads are of constant impedance type

$$\dot{x} = f(x)$$



Y_{Net}

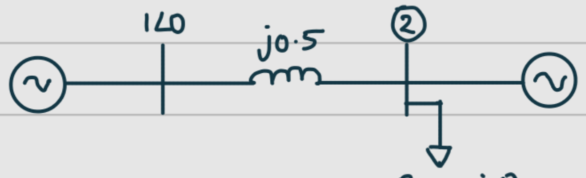
$$\begin{bmatrix}\bar{I}_{gen}\\ \bar{I}_{rest}\end{bmatrix}=\begin{bmatrix}Y_{11} & Y_{12}\\ Y_{21} & Y_{22}\end{bmatrix}\begin{bmatrix}\bar{V}_{gen}\\ \bar{V}_{rest}\end{bmatrix}$$

$$I_{gen}=\left(\bar{Y}_{11}-\bar{Y}_{12}\,\bar{Y}_{22}^{-1}\,\bar{Y}_{21}\right)\bar{V}_{gen}$$

$$Y_{gen}=\left(\bar{Y}_{11}-\bar{Y}_{12}\,\bar{Y}_{22}^{-1}\,\bar{Y}_{21}\right)$$

Type 2 Model: eqns

Example



$P_{m_2}=1.0$ $\quad x'_{d_2}=0.3$

$R_{s_2}=0$ $\quad x'_{q_2}=0.2$

$V_2=1.0$

$P_{L_2}+jQ_{L_2}$

$0.5+j0.2$ $\xrightarrow[\text{conv. to admittance}]{}$ $\dfrac{(P_{L_2}+jQ_{L_2})^*}{|V_i|^2}=\underline{\underline{0.5-j0.2}}$

To verify,

$I_{L_2}=V_2\angle\delta_2\,\bar{Y}_{L_2}$

$=1\angle\delta_2\,(0.5-j0.2)$

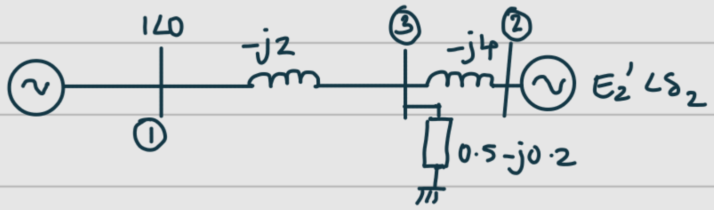
$P_{L_2}+jQ_{L_2}=V_2\angle\delta_2\,\bar{I}_{L_2}^*$

$=1\angle\delta_2\,[1\angle\delta_2(0.5-j0.2)]$

$=0.5+j0.2$

$x_2'=\dfrac{0.3+0.2}{2}=\underline{\underline{j0.25}}\xrightarrow{adm}\underline{\underline{-j4}}$

Admittance Diagram



$$\left[\begin{array}{cc|c}-j2 & 0 & j2\\ 0 & -j4 & j4\\ \hline j2 & j4 & 0.5-j6.2\end{array}\right]=\bar{Y}_{net}$$

$$\bar{Y}_{gen}=\left(\bar{Y}_{11}-\bar{Y}_{12}\,\bar{Y}_{22}^{-1}\,\bar{Y}_{21}\right)=\begin{bmatrix}-j2 & 0\\ 0 & -j4\end{bmatrix}-\begin{bmatrix}j2\\ j4\end{bmatrix}\begin{bmatrix}0.5-j6.2\end{bmatrix}^{-1}\begin{bmatrix}j2 & j4\end{bmatrix}$$

$$=\begin{bmatrix}0.05-j1.35 & 0.1+j1.28\\ 0.1+j1.28 & 0.2-j1.43\end{bmatrix}$$

$$\bar{Y}_{Gen} \approx \begin{bmatrix} 1.35\angle -90 & 1.28\angle 90 \\ 1.28\angle 90 & 1.43\angle -90 \end{bmatrix}$$

$$P_{e_2} = P_{a_2} = \sum_{j=1}^{2} Y_{a_{ij}} E_i' E_j' \cos(r_i - r_j - \theta_{a_{ij}})$$

$$= 1.28\, E_2' E_1' \cos(r_2 - r_1 - 90^\circ) + 1.43\, E_2 E_2' \cos(r_2 - r_2 + 90)$$

$$I_{d_i} = \sum_{j=1}^{N+1} Y_{a_{ij}} E_j' \sin(\theta_i - r_j - \theta_{a_{ij}})$$

$$I_{d_2} = 1.28\, \cancelto{1}{E_1'} \sin(\theta_2 - \cancelto{0}{r_1} - \theta_{a_{21}}) + 1.43 E_2' \sin(\theta_2 - r_2 - \theta_{a_{22}})$$

$$I_{q_2} = 1.28\, \cancelto{1}{E_1'} \cos(\theta_2 - \cancelto{0}{r_1} - \theta_{a_{21}}) + 1.43 E_2' \cos(\theta_2 - r_2 - \theta_{a_{22}})$$

Model :-

$$\dot{\theta}_2 = (\omega_2 - 1)\,\omega_s$$
$$2H_2 \dot{\omega}_2 = P_{m_2} - P_{e_2} - K_{D_2}(\omega_2 - 1)$$
$$T_{do_2}' \dot{E}_{q_2}' = -E_{q_2}' - (x_{d_2} - x_{d_2}') I_{d_2} + E_{fd_2}$$
$$T_{qo_2}' \dot{E}_{d_2}' = -E_{d_2}' + (x_{q_2} - x_{q_2}') I_{q_2}$$
$$E_2' \angle r_2 = (E_{d_2}' + j E_{q_2}') \angle \theta_2 - \pi/2$$

Classical Model (Angle Stability Model)

Goal: Do the machines stay in synchronism?

θ_i s small? $\longrightarrow$ Angle stable

large? $\longrightarrow$ Angle instability

$$\frac{1}{\omega_s}\dot{\theta}_i = \omega_i - 1$$
$$\sqrt{\frac{2H_i}{\omega_s}} = \sqrt{\frac{10}{400}} \approx 0.15 \text{ sec}$$

$$\dot{\theta}_i = (\omega_i - 1)\,\omega_s \qquad \omega_s = 2\pi 60 = 377$$
$$2H_i \dot{\omega}_i = P_{m_i} - P_{e_i} - K_{D_i}(\omega_i - 1) \qquad H_i \approx 5\text{–}10$$
$$T_{do_i}' \dot{E}_{q_i}' = \qquad T_{do_i}' \approx 10$$
$$T_{qo_i}' \dot{E}_{d_i}' = \qquad T_{qo_i}' \approx 5$$

Angle $\sim$ 0.15 sec

Voltage E_d' E_q' $\sim$ 5 to 10 sec

Assumption: E_d' & E_q' are constant

$$E_i' \angle \gamma_i = \underbrace{E_{d_i}' + j E_{q_i}'}\ \angle \theta_i - \pi/2$$

E_i' is const.

$$\gamma_i = \angle E_{d_i}' + j E_{q_i}' + \theta_i - \pi/2$$

ψ_{1q} ψ_{fd}

$$E_{d_i}' \ll E_{q_i}' \Rightarrow \angle E_{d_i}' + j E_{q_i}' = \pi/2$$

$$\gamma_i = \theta_i$$

Classical Model

coupled pendulum (Physics)

Swing Equations

Energy Function Theory

$$\dot{\theta}_i = (\omega_i - 1)\,\omega_s$$

$$2H_i \dot{\omega}_i = P_{m_i} - P_{e_i} - K_{D_i}(\omega_i - 1)$$

$$P_{e_i} = \sum_{j=1}^{N_a+1} Y_{a_{ij}} E_i' E_j' \cos(\cancel{\gamma_i}^{\theta_i} - \cancel{\gamma_j}^{\theta_j} - \theta_{a_{ij}})$$

$$P_{e_i} = \sum_{j=1}^{N_a+1} Y_{a_{ij}} E_i' E_j' \cos(\theta_i - \theta_j - \theta_{a_{ij}})$$

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Euler's Method

Runge-Kutta Method

Initial Conditions

$\theta_i \Leftarrow \delta_i$

$\omega_i \Leftarrow 1\,pu$

$E_{q_i}' \Leftarrow 1.2\,pu$

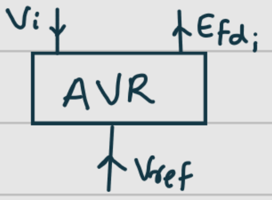
$E_{d_i}' \Leftarrow 0.2\,pu$

HW 3: Specify X_d X_d', H, ... in common base 100 MVA

$H = 4\,sec$ for 900 MVA

$H = 9 \times 4 = 36\,sec$ for 100 MVA

Exciter Models



Automatic Voltage Regulator

- Adjust E_{fd_i} to keep V_i close to V_{ref_i}

- DC Machine
- AC Machine + Rectifier
- Static Exciters $\Rightarrow$ Thyristor based (modern) very fast

speed of operation

IEEE Standard Models

- Kundur book - Many models
- Static, AC, DC

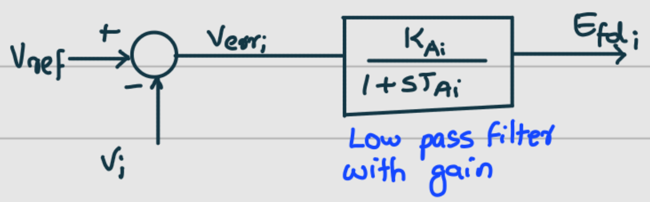
Static	AC	DC
1a	1a	1a
1b	1b	1b
⋮	⋮	⋮

Static Exciter Models: (First Order Model)

$$V_i < V_{ref_i} \Rightarrow E_{fd_i}\uparrow \Rightarrow E'_{q_i}\uparrow \Rightarrow V_i\uparrow$$

$$T'_{do_i}\dot{E}'_{q_i} = -E'_{q_i} - (x_{d_i} - x'_{d_i}) I_{d_i} + E_{fd_i}$$

$$V_i > V_{ref_i} \Rightarrow E_{fd_i}\downarrow \Rightarrow E'_{q_i}\downarrow \Rightarrow V_i\downarrow$$



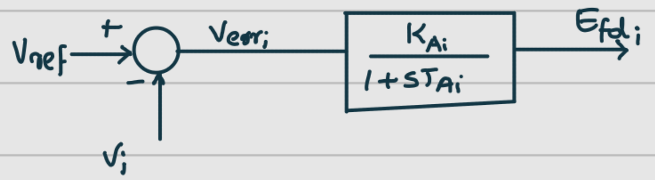
Low pass filter with gain

V_i - Bus Voltage
V_{ref_i} - Reference Voltage
E_{fd_i} - Exciter Voltage
K_{A_i} - Amplifier Gain (large) High gain 100-500
T_{A_i} - Amplifier Time constant (small)

$$E_{fd_i} = \frac{K_{A_i}}{1+sT_{A_i}}(V_{ref_i} - V_i)$$

$$(1+sT_{A_i})E_{fd_i} = K_{A_i}(V_{ref_i} - V_i) \xrightarrow[\text{Domain}]{\text{Time}} E_{fd_i} + T_{A_i}\dot{E}_{fd_i} = K_{A_i}(V_{ref_i} - V_i)$$

$$\boxed{T_{A_i}\dot{E}_{fd_i} = -E_{fd_i} + K_{A_i}(V_{ref} - V_i)}$$



$K_{A_i} \Rightarrow$ large 100 − 500

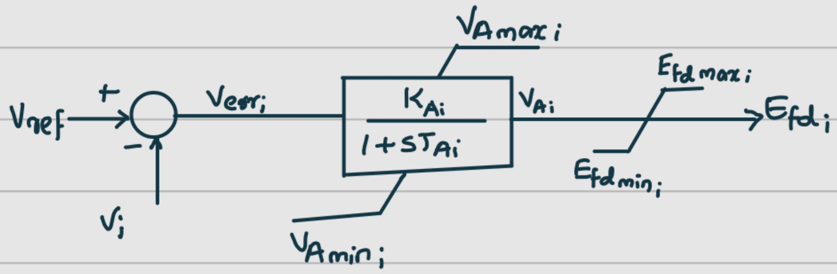
At SS : $s = 0$

$K_{A_i}(V_{ref_i} - V_i) = E_{fd_i}$

$V_{ref} - V_i$ will be small $= \frac{E_{fd_i}}{K_{A_i}}$

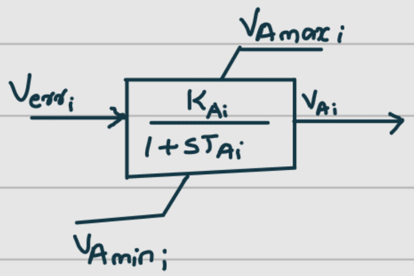
During fault, $(V_{ref_i} - V_i)$ large E_{fd_i} shoots up → burn out / accident

- Provide limiters



Non wind up limiter: $V_{A min_i}$ − Minimum limit for amplifier voltage
$V_{A max_i}$ − Maximum " " " "

wind up limiter: $E_{fd min_i}$ − Minimum excitation needed for machine to operate
$E_{fd max_i}$ − Maximum " " " "



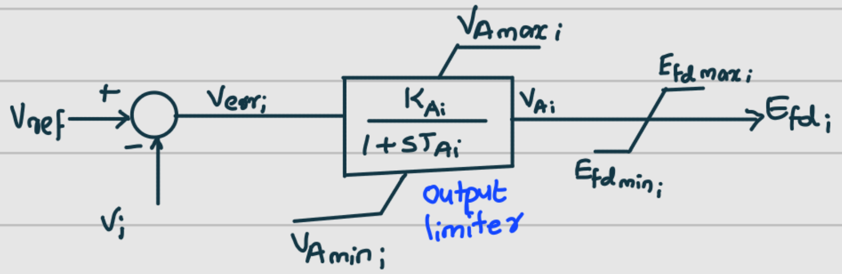
V_{A_i} cannot change below $V_{A min_i}$
increase above $V_{A max_i}$

$$\dot{V}_{A_i} = \begin{cases} 0 & \text{if } V_{A_i} = V_{A min_i} \ \& \ \dot{V}_{A_i} < 0 \\ 0 & \text{if } V_{A_i} = V_{A max_i} \ \& \ \dot{V}_{A_i} > 0 \\ \dot{V}_{A_i} & \text{otherwise} \end{cases}$$

$$T_{A_i} \dot{V}_{A_i} = -V_{A_i} + K_{A_i} V_{err_i}$$

intended control $V_{A_i}dot = [-V_{A_i} + K_{A_i} V_{err_i}] / T_{A_i}$

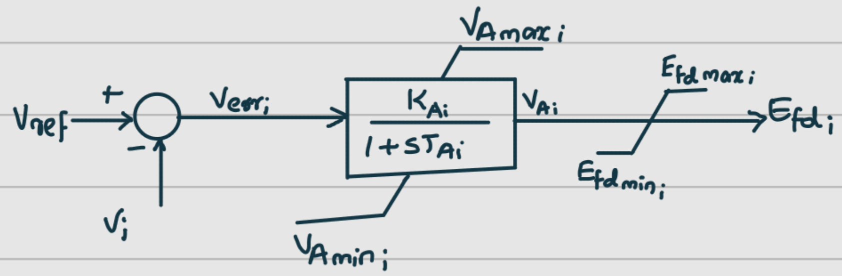
actual $$\dot{V}_{A_i} = \begin{cases} 0 & \text{if } V_{A_i} = V_{A min_i} \ \& \ V_{A_i}dot < 0 \\ 0 & \text{if } V_{A_i} = V_{A max_i} \ \& \ V_{A_i}dot > 0 \\ V_{A_i}dot & \text{otherwise} \end{cases}$$



Wind up limiter

$$E_{fd_i} = \begin{cases} E_{fd\max_i} & \text{if } V_{A_i} > E_{fd\max_i} \\ E_{fd\min_i} & \text{if } V_{A_i} < E_{fd\min_i} \\ V_{A_i} & \text{otherwise} \end{cases}$$

02/21/24



static exciter

simplest model

$$T_{A_i}\dot{V}_{A_i} = -V_{A_i} + K_{A_i}(V_{ref_i} - V_i)$$

$$V_{A_i}dot = \frac{1}{T_{A_i}}\left[-V_{A_i} + k_{A_i}(V_{ref_i} - V_i)\right] \longrightarrow$$ Internal voltage derivative from the control logic

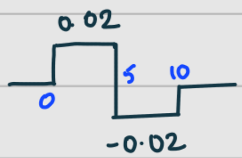
Non wind up limit (Inherent limit)

$$\dot{V}_{A_i} = \begin{cases} 0 & \text{if } V_{A_i} = V_{A\min_i} \ \& \ V_{A_i}dot < 0 \\ 0 & \text{if } V_{A_i} = V_{A\max_i} \ \& \ V_{A_i}dot > 0 \\ V_{A_i}dot & \text{otherwise} \end{cases}$$ Actual Control

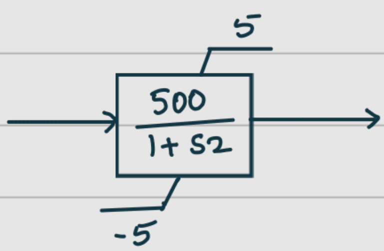
Wind up limiter (Imposed by us)

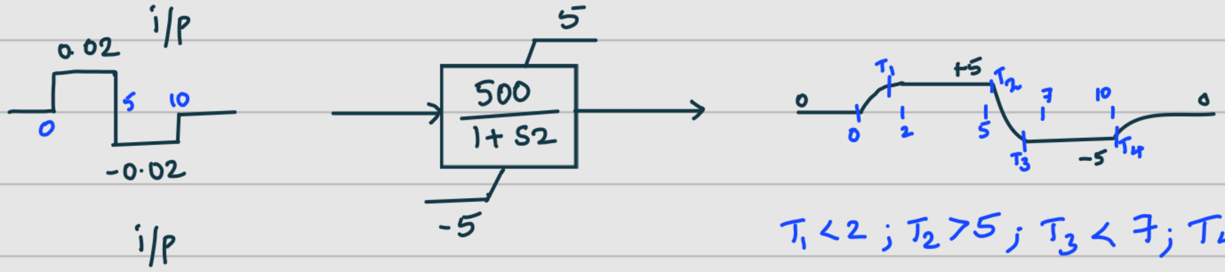
$$E_{fd_i} = \begin{cases} E_{fd\max_i} & \text{if } V_{A_i} > E_{fd\max_i} \\ E_{fd\min_i} & \text{if } V_{A_i} < E_{fd\min_i} \\ V_{A_i} & \text{otherwise} \end{cases}$$

Example



i/p





$T_1 < 2$; $T_2 > 5$; $T_3 < 7$; $T_4 > 10$

$2V_A\,dot = -V_A + 500V_{err}$

$V_r = 0.02$

$V_A = 5 = V_{Amax}$

$V_A\,dot = 5 + 500(0.2) = 5 > 0$

@ $t = 5$ sec $\Rightarrow V_{err} = -0.02$
5+

$V_A = 5$

$2V_Adot = -5 + 500(0.2) = -15 < 0$

@ $t = T_2$ sec $\Rightarrow V_A = 0.5$, $V_{err} \Rightarrow -0.02$

$2V_Adot =$

@ $t = 10$ sec $\Rightarrow V_A = -5$, $V_{err} = 0$

$2V_Adot = -(-5) + 500(0) = +5 > 0$

No delay between when the signal changes and when the output changes

Example: Waveform Given

$t = 0$, $V = 1$, $V_{ref} = 1.03$, $V_{err} = 0.03$, $E_{fd} = 3$

$E_{fd} = K_A\,(V_{ref} - V) + E_{fd0}$

$3 = K_A\,(0.03) + E_{fd0}$ —— 1

@ $t = 25^-$: $V_{err} = 0$

$E_{fd} = K_A V_{err} + E_{fd_0}$

$0 = K_A(0) + E_{fd_0}$

$\underline{\underline{E_{fd_0} = 0}}$

$E_{fd_0} = 0$ in ①

$3 = K_A(0.03) + 0$

$\underline{\underline{K_A = 100}}$

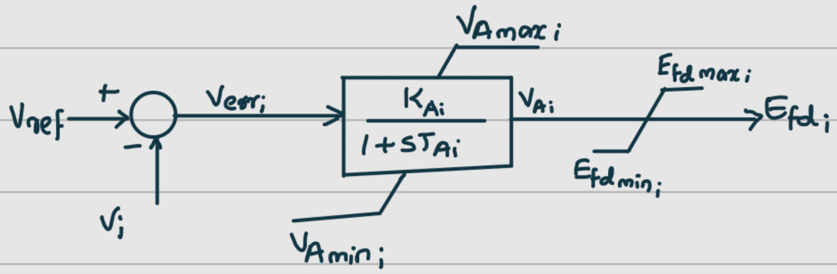
From graph, $T_A = 5-4 = \underline{\underline{1\,sec}}$,

@ $t = 14^-$: $V_{err} = -0.07$ either $V_{Amin} = -5$ or $\cancel{E_{fdmin}} = -5$

confirmed at $t = 20^+$

@ $t = 20^-$: $V_{err} = 1.03 - 0.95 = 0.08$ $V_{Amax} = 4$ (o/p change exactly when i/p change) (no windup)

02/22/24



Static Exciter

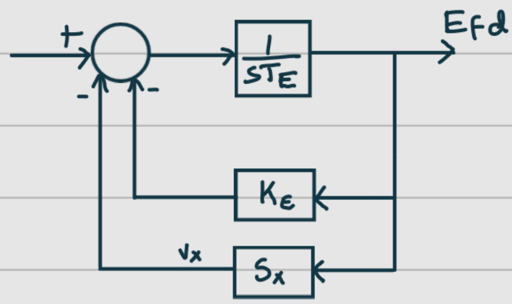
First order delay type

IEEE DC1A Model

- DC field voltage generated by DC Machine

self excited

Seperately excited



T_E = Ramp rate of exciter

$K_E < 0 \Rightarrow$ self excited

$K_E > 0 \Rightarrow$ Sep. excited

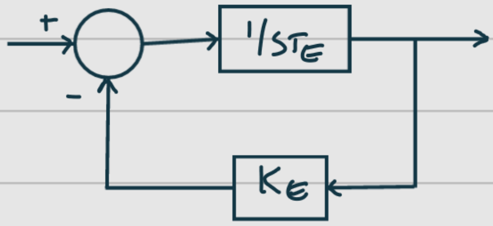
self excited

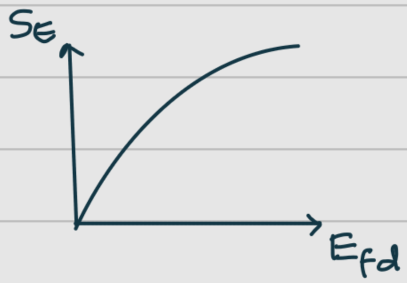
unstable $\Rightarrow$ saturation helps

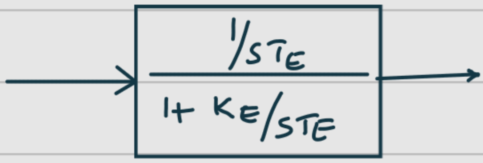
sep excited

$$S_x = \left(A_{ex}\, e^{B_{ex} E_{fd}}\right) / E_{fd}$$

A_{ex}, B_{ex} – very small 0.001–0.0004



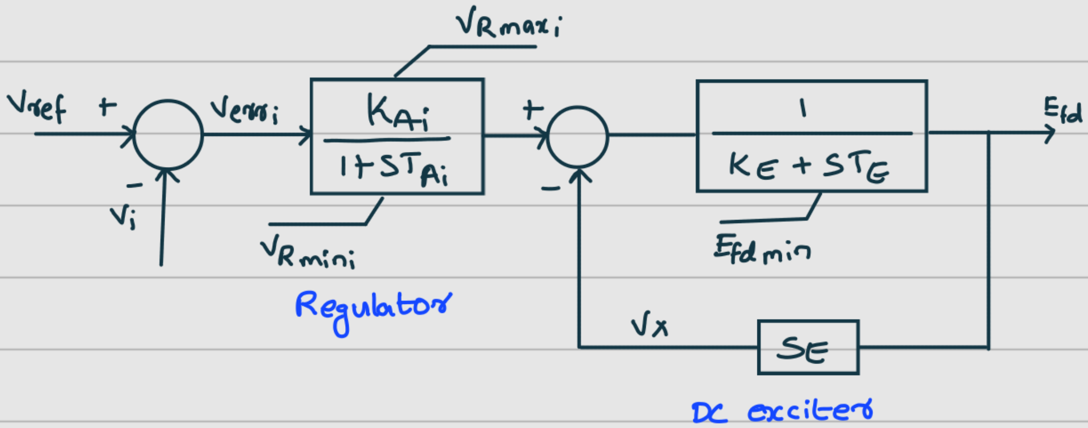




Hard limit { Non windup – state limit
Windup – Output limit

Saturation – soft limit

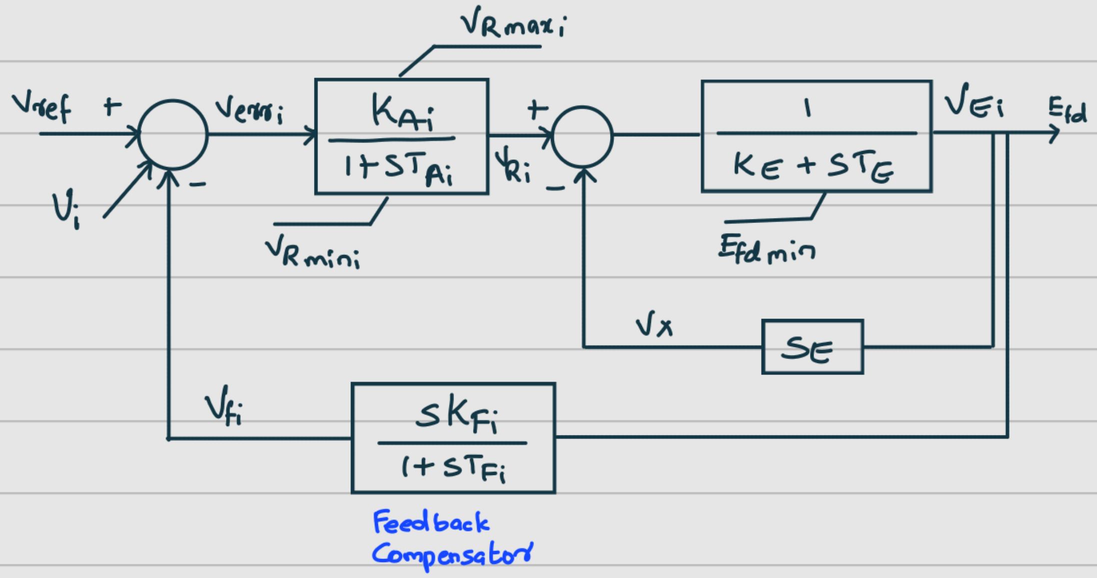
no upper limit – sat.



– Open loop unstable

– Design a compensator

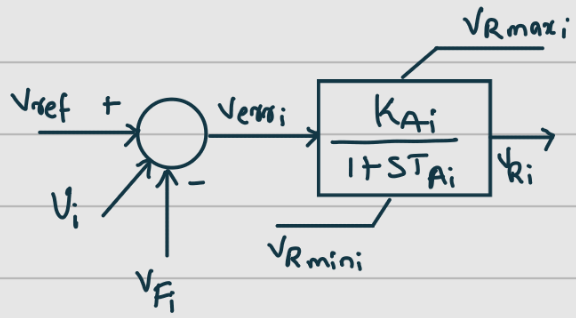
$K_E < 0$ (self)



Under steady state : $V_{fi} = 0$ washout filter

washout filter does not disrupt voltage regulation in steady state

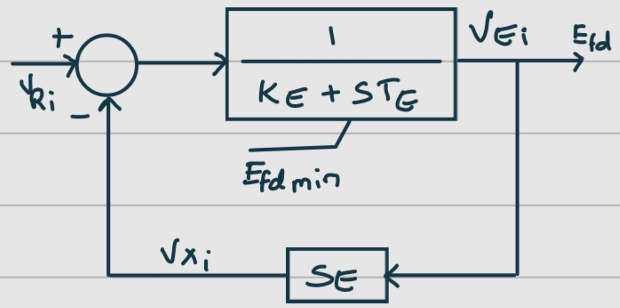
Regulator



$$V_{Ri}\,dot = \frac{1}{T_{Ai}}\left[-V_{Ri} + K_{Ai}\left(V_{ref_i} - V_i - V_{Fi}\right)\right]$$

$$\dot{V}_{Ri} = \begin{cases} 0 & V_{Ri} = V_{Rmaxi} \text{ \& } V_{Ri}\,dot > 0 \\ 0 & V_{Ri} = V_{Rmini} \text{ \& } V_{Ri}\,dot < 0 \\ V_{Ri}\,dot & \text{otherwise} \end{cases}$$

Exciter

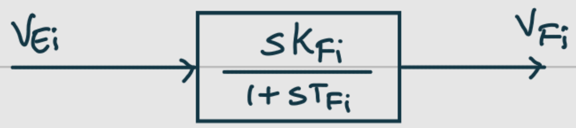


$$V_{E_i} = \frac{1}{K_{E_i} + sT_{E_i}} (V_{R_i} - V_{x_i})$$

$$T_{E_i} V_{E_i dot} = -K_{E_i} V_{E_i} + 1 (V_{R_i} - V_{x_i})$$

$$\dot{V}_{E_i} = \begin{cases} 0 & V_{E_i} = E_{fd min} \ \& \ V_{E_i dot} < 0 \\ V_{E_i dot} & \text{otherwise} \end{cases}$$

Feedback Compensator



$$V_{F_i} = \frac{sK_{F_i}}{1+sT_{F_i}} V_{E_i}$$

$$V_{F_i} + sT_{F_i} V_{F_i} = sK_{F_i} V_{E_i}$$

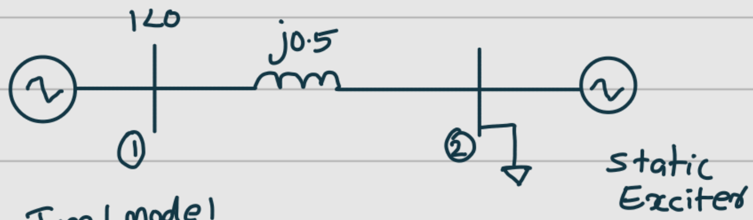
$$T_{F_i} \dot{V}_{F_i} = -V_{F_i} + K_{F_i} \dot{V}_{E_i}$$

substitute $\dot{V}_{E_i}$ from the exciter equation

Static First Order Model : Dynamic State : V_{A_i}

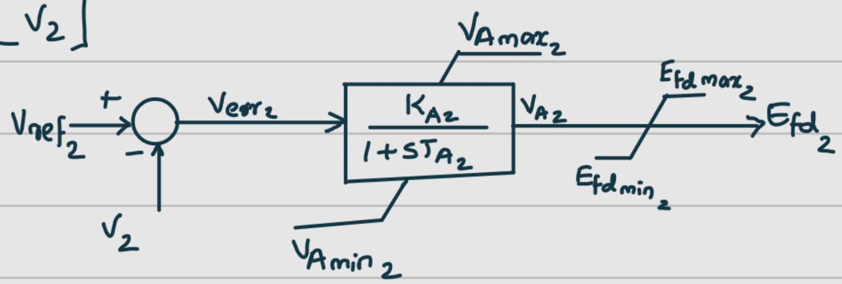
DC1A Exciter : Three Dynamic State : V_{R_i}, V_{E_i} and V_{F_i}

Example



Type1 model

$$x = \begin{bmatrix} \theta_2 \\ \omega_2 \\ E'_{q_2} \\ E'_{d_2} \\ V_{A_2} \end{bmatrix} \qquad y = \begin{bmatrix} \delta_2 \\ V_2 \end{bmatrix}$$



DC1A Exciter:

$$X = \begin{bmatrix} \theta_2 \\ \omega_2 \\ E'_{q2} \\ E'_{d2} \\ V_{R2} \\ V_{E2} \\ V_{F2} \end{bmatrix} \qquad y = \begin{bmatrix} \delta_2 \\ V_2 \end{bmatrix}$$

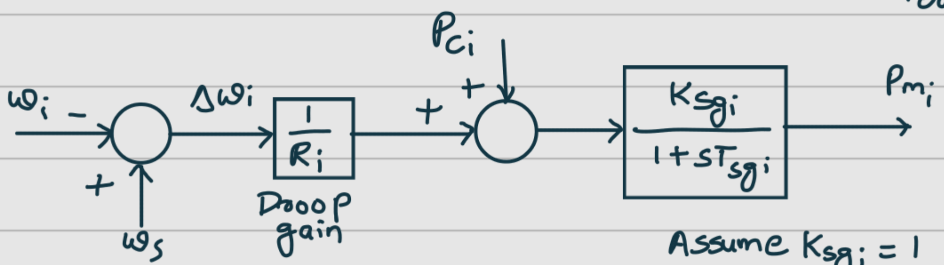
02/27/24

Governor

Objective :- ① Keeps rotor freq close to 60 Hz (speed governor)

② Keeps generator power o/p close to P_{c_i} when freq is at 60 Hz

└ Power schedule for i



Assume $K_{sg_i} = 1$

T_{sg_i} = Tens of sec to minutes

$\omega_i < \omega_s \Rightarrow P_{m_i}\uparrow \Rightarrow$ More torque $\Rightarrow \omega_i \uparrow$

$\omega_i > \omega_s \Rightarrow P_{m_i}\downarrow \Rightarrow$ Less torque $\Rightarrow \omega_i \downarrow$

$\omega_i = \omega_s \Rightarrow P_{m_i} = P_{c_i}$ (schedule i)

$$R_i = \frac{\text{pu change in freq}}{\text{pu change in power o/p}} = \frac{\Delta\omega_i}{\Delta P_i}$$

Steady state : $s = 0$

$$P_{m_i} = P_{c_i} + \frac{1}{R_i}(\omega_s - \omega_i)$$

$$R_i = \frac{\omega_s - \omega_i}{P_{c_i} - P_{m_i}} = \frac{\text{pu change in freq}}{\text{pu change in } P_{G_i}}$$

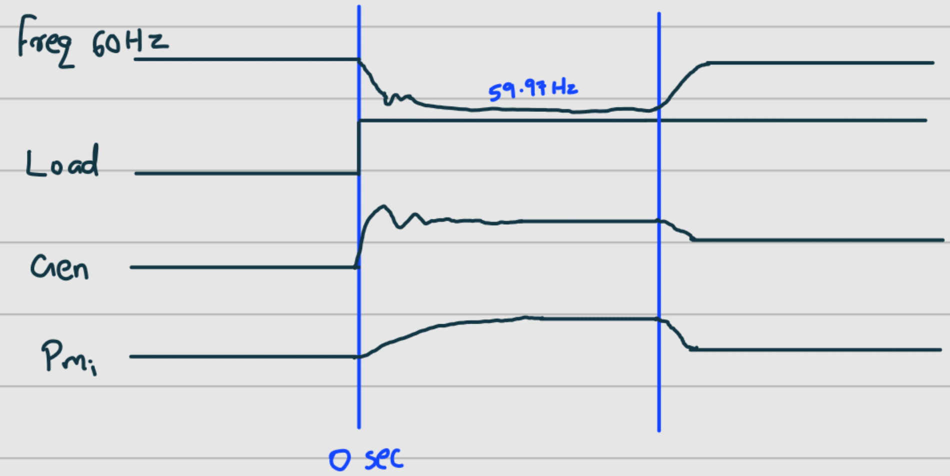
$R_i = 5\%$ means 5% change in freq will lead to 100% change in power

$R_i = 5\%$ required droop setting for Western Interconnection

$R_1 = 2\%$ $\qquad R_2 = 50\%$ $\qquad$ if same base

$\frac{1}{R_1} = 50$ $\qquad \frac{1}{R_2} = 20$ $\qquad$ more power from Gen 1

② Suppose we have a total capacity of 100 GW in WECC [Western Electricity Coordination Council]. Suddenly we lose a generator 1000 MW.



$$\underbrace{\sum P_{G_i}}_{\text{Go up by releasing inertial KE into electrical energy}} = \underbrace{\sum P_{L_i}}_{\text{suddenly goes up by 100MW}} + \text{Losses}$$

$$\Delta P_{pu} = \frac{-1000\,MW}{100\,GW} = \frac{-1000}{100 \times 10^3} = -0.01\,pu$$

All machines have droop of 5%. $R = 5\% = 0.05$

$$R = \frac{\Delta \omega}{\Delta P} \Rightarrow \Delta \omega = R(\Delta P)$$

$$\Delta \omega = (0.05)(-0.01)$$

$$\Delta \omega = -0.0005\,pu$$

$$\Delta f = (-0.0005)60 = \underline{\underline{-0.03\,Hz}} \qquad \underline{\underline{f_{new} = 59.97\,Hz}}$$

$R = 5\%$ $\quad 0.05 = \frac{\Delta \omega}{\Delta P}$ $\quad \Delta \omega = -0.0005\,pu$

Change in Chief Joseph (2GW) P_{m_i}?

$$\Delta P = -\frac{\Delta \omega}{\Delta P} = -\frac{(0.0005)}{0.05} = +0.01\,pu$$

$$\Delta P = (0.01)(2000) = 20\,MW$$

$$P_{new} = 1000 + 20 = \underline{\underline{1020\,MW}}$$

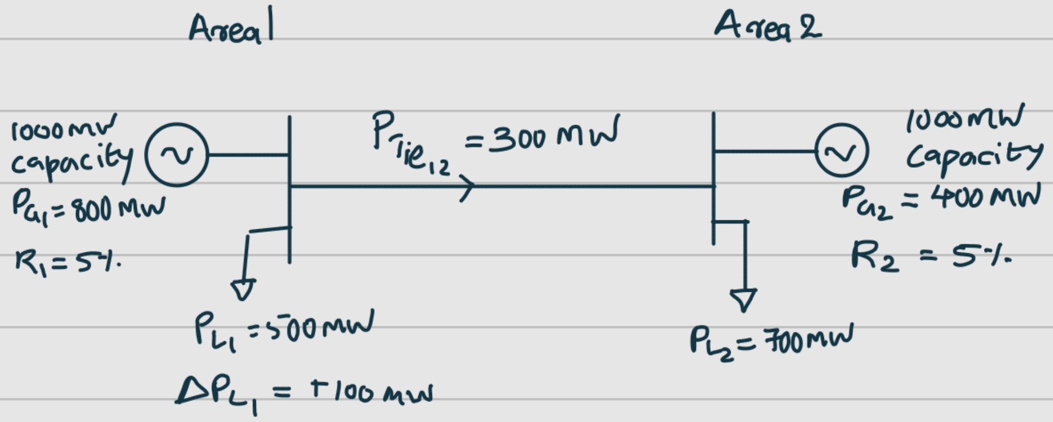
BPA - WECC

more resp ← DOE US : Bonneville Power Administration 5%

$P_{Tie.} \uparrow$ (problem)

CA : Pvt. companies

$R \sim 10\%$ to 20%



$R = 5\%$ Total Capacity = 2000 MW

$\Delta P_{L_i} = 100$ MW

$$\Delta P = \frac{-100}{2000} = \underline{\underline{-0.05\,pu}}$$

$$R = \frac{\Delta\omega}{\Delta P} \qquad \Delta\omega = R\Delta P = (0.05)(-0.05) = -0.0025\,pu$$

$$f_{new} = 60 + (-0.0025 \times 60)$$
$$= 60 - 0.15$$
$$= \underline{\underline{59.85\,Hz}}$$

$$\Delta P_{G_1} = \frac{\Delta\omega}{R} = \frac{+0.0025}{0.05} = +0.05\,pu$$

$$\Delta P_{G_1} = (0.05)(1000) = \underline{\underline{50\text{ MW}}} \qquad P_{G_1} = 850\text{ MW}$$

$$\Delta P_{G_2} = \underline{\underline{+50\text{ MW}}} \qquad P_{G_2} = 450\text{ MW}$$

$P_{Tie_{12}} = 250$ MW — change in contract $f\downarrow$

AGC will make (Automatic Generation Control)(LFC - Load Freq. Control)

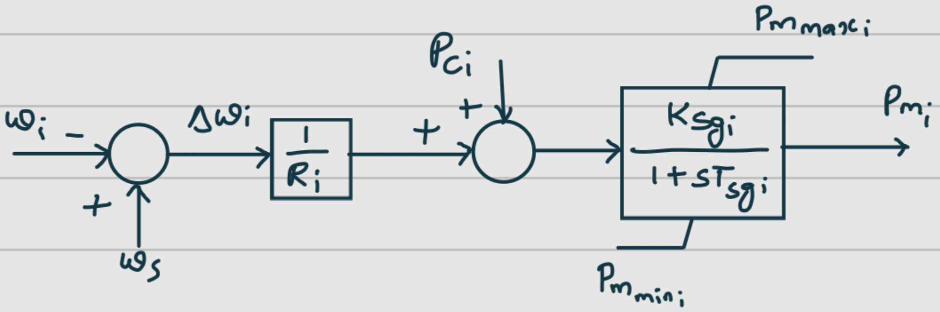
$P_{G_1} = 900$ MW

$P_{G_2} = 400$ MW

$P_{Tie} = 300$ MW

$f = 60$ Hz

02/29/24



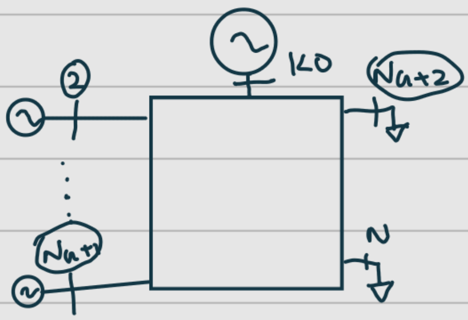
$$\boxed{P_{m_i} = \frac{1}{1+sT_{sg_i}}\left[P_{c_i} + \frac{1}{R_i}(\omega_s - \omega_i)\right]}$$

$$T_{sg_i} \, P_{m_i}\text{dot} = -P_{m_i} + P_{c_i} + \frac{1}{R_i}(1-\omega_i)$$

$$\dot{P}_{m_i} = \begin{cases} 0 & P_{m_i} = P_{m_{i\,max}} \text{ and } P_{m_i}\text{dot} > 0 \\ 0 & P_{m_i} = P_{m_{i\,min}} \text{ and } P_{m_i}\text{dot} < 0 \\ P_{m_i}\text{dot} & \text{otherwise} \end{cases}$$

P_{c_i} = Generator Schedule for Gen i (from control schedule) − ED, OPF

Slack Generators - Varied to keep up with the load demand



Two axis model
+
Static first order exciter
+
Governor

Gen$_i$:
$\left.\begin{matrix}\theta_i \\ \omega_i\end{matrix}\right\}$ Elec. Mech

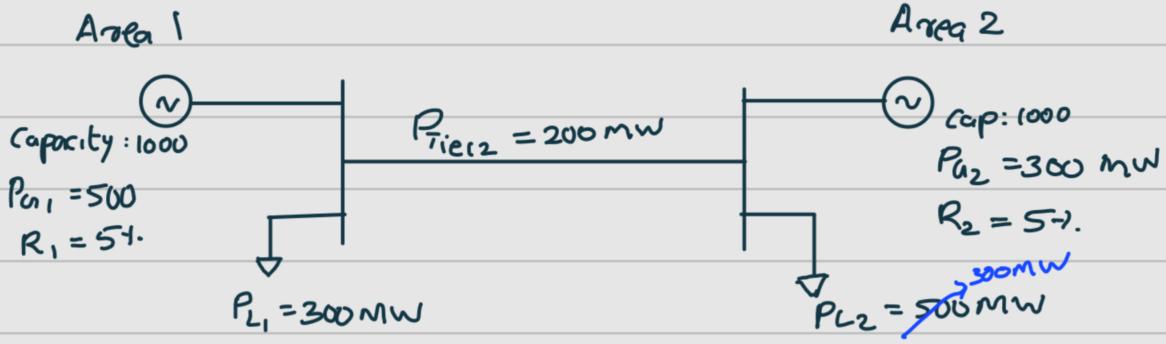
$\left.\begin{matrix}E'_{q_i} \\ E'_{d_i}\end{matrix}\right\}$ Elec. mag

V_{A_i} } Exciter

P_{m_i} } Governor

$\#(x) = 6\,N_G$

$\#(Y) = 2(N-1)$



$\Delta P_{L_2} = -200\,MW = \Delta P_L$

$= \dfrac{-200}{2000} = \underline{\underline{-0.1\,pu}} \Rightarrow f\uparrow$

(sync.)

$\Delta f_1 = \Delta f_2 = \Delta f = R\Delta P = (0.05)(+0.1)$

$= -0.005\,pu$

$= (-0.005)(60)$

$= \underline{\underline{0.3\,Hz}}$

$f_{new} = \underline{\underline{60.3\,Hz}}$

$\Delta f_1 = 0.005\,pu, \; R_1 = 0.05$

$\Delta P_1 = \dfrac{\Delta\omega}{R} = \dfrac{-0.005}{0.05} = -0.1\,pu. = \underline{\underline{-100\,MW}}$

$\Delta f_2 = 0.005\,pu, \; R_2 = 0.05$

$\Delta P_2 = -0.1\,pu = \underline{\underline{-100\,MW}}$

$P_{G_1} = 500 - 100 = 400\,MW$ $\qquad P_{Tie} = 100\,MW \neq 200\,MW$

$P_{G_2} = 300 - 100 = 200\,MW$ $\qquad$ AGC

<u>Area Control Error</u> (ACE_i)

$ACE_i = \Delta P_{Net_i} + B_i \Delta f_i$

$P_{Net_i} = \sum_i^{j} P_{Tie_{ij}}$ (Net Area <u>Export</u>)

B_i = Bias factor for area i

$= 1/R_i$

$\Delta P_{Net_i} = P_{Net_i}^{actual} - P_{Net_i}^{contract}$

$\Delta f_i = f_i^{actual} - f_i^{contract} = f_i^{actual} - 1$

Area 1

$$P_{Net_1}^{contract} = P_{Tie_{12}} = 200\,MW = \frac{200}{1000} = 0.2\,pu$$

$$P_{Net_1}^{actual} = P_{Tie_{12}} = 100\,MW = 0.1\,pu$$

$$\Delta P_{Net} = 0.1 - 0.2 = -0.1\,pu$$

$$f_1^{actual} = 60.3 = 1.005\,pu$$
$$f_1^{contract} = 1\,pu$$

$$ACE_1 = \Delta P_{Net_1} + B_1 (f_1^{act} - 1)$$ → correction to the slack buses in area 1 (ACE_1)

$$= -0.1 + \frac{1}{0.05}(1.005 - 1)$$

$$= -0.1 + 20(+0.005)$$
$$= -0.1 + 0.1$$
$$= 0$$

Area 2

$$P_{Net_2}^{contract} = -200\,MW = -0.2\,pu$$

$$P_{Net_2}^{actual} = -100\,MW = -0.1\,pu$$

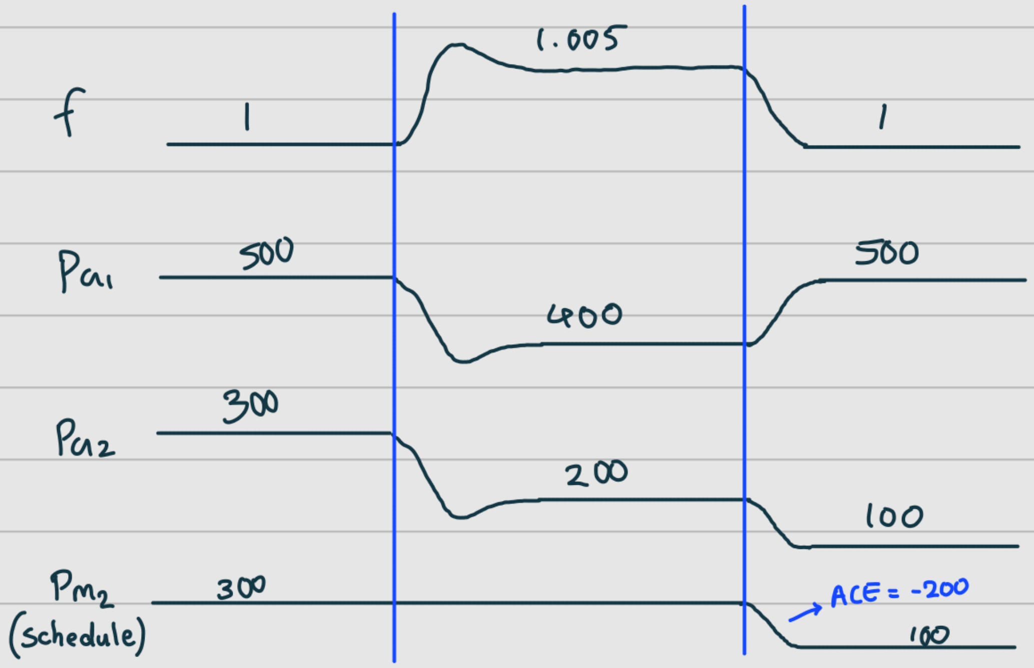
$$\Delta P_{Net_2} = -0.1 - (-0.2) = +0.1\,pu$$

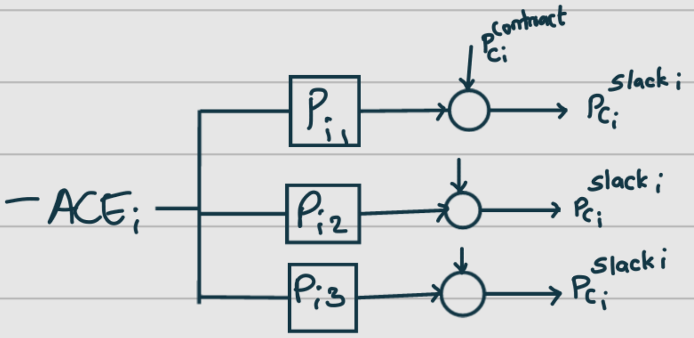
$$ACE_2 = \Delta P_{Net_2} + B_2 (f_2^{act} - 1)$$
$$= 0.1 + 20(0.005)$$
$$= +0.2\,pu = 200\,MW$$ (+ve → over production)

$-ACE_i$ = slack bus correction for area i

$ACE_2 = 200\,MW \Rightarrow \Delta P_{G_2} = -200\,MW$

(AGC action)





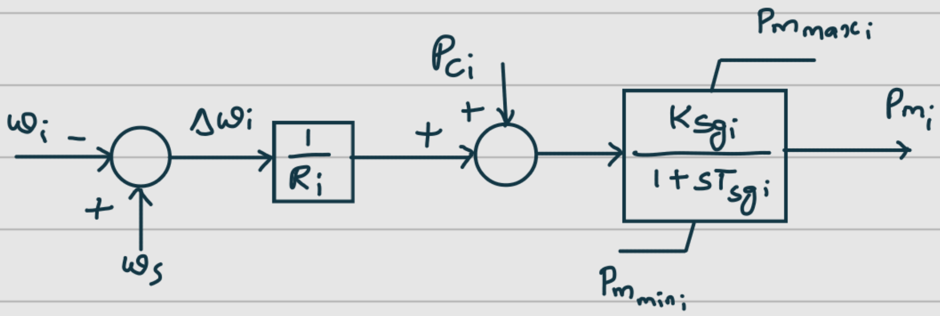
Hydro-pref. for slack

Nuclear - not pref.

NW

- Grand Coulee
- Chief Joseph
- John Dey

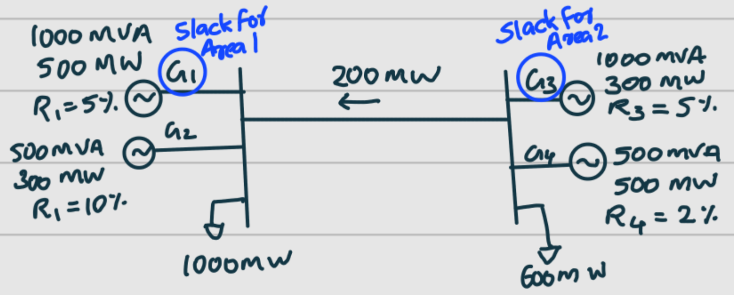
03/05/24



AGC

Area Control Error, $ACE_i = \Delta P_{Net_i} + B_i \Delta f_i$

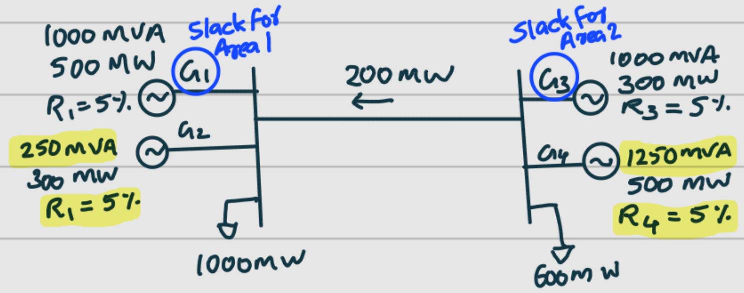
$$\Delta P_{Net_i} = P_{Net_i}^{actual} - P_{Net\,i}^{contract}$$



Area 1 load 1000 MW ⟶ 800 MW

Find Governor Response
Freq. Response
AGC Response

$$R_i = \frac{\Delta \omega_i}{\Delta P_i} \qquad \Delta\omega_i = R_i \Delta P_i = R_i \frac{\Delta P_i}{P_{rated_i}} \qquad R_i \uparrow \quad P_{rated_i} \uparrow$$



$$\Delta P_L = \frac{-200}{1000+250+1000+1250} = \frac{-200}{3500} = \underline{\underline{-0.057}}\ pu$$

$$\Delta f = +R\Delta P_L = 0.05 \times 0.057 = \underline{\underline{0.00286}}\ pu = (0.00286)(60) = \underline{\underline{0.17 Hz}}$$

$$f_{new} = \underline{\underline{60.17 Hz}}$$

$$\Delta P_1 = \frac{-\Delta f}{R_1} = \frac{-0.00286}{0.05} = \underline{\underline{-0.057 pu}} = \underline{\underline{-57 MVA}}$$

$$\Delta P_2 = (-0.057)250 = -14.25\ MW$$

$$\Delta P_3 = (-0.057)1000 = -57\ MW$$

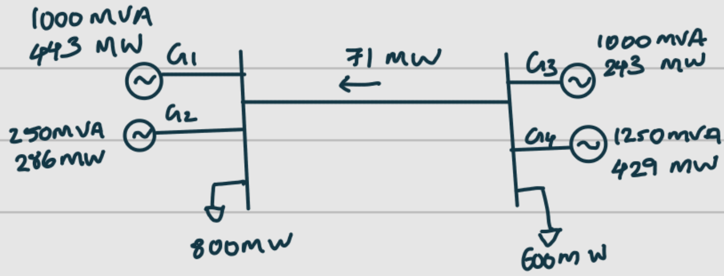
$$\Delta P_4 = (-0.057)1250 = -71.4\ MW$$

$$\Delta P_1 + \Delta P_2 + \Delta P_3 + \Delta P_4 = -200\ MW$$

$P_{1gen} = 442.86$ MW $\qquad P_{3gen} = 242.86$ MW

$P_{2gen} = 285.75$ MW $\qquad P_{4gen} = 428.57$ MW

$$P_{Tie_{21}} = 71.43$$



$P_{Net_1}^{contract} = -200\ MW$ $\qquad ACE_1 = 200 \quad ACE_2 = 0$

$P_{Net_1}^{actual} = -71\ MW$

$\Delta P_{Net_1} = -71 - (-200) = 129\ MW = \frac{129}{1250} = 0.10\ pu$

$\Delta f_1 = \Delta f = 0.00286 \qquad B_1 = \frac{1}{R_1} = \frac{1}{0.05} = 20$

$ACE_1 = 0.10 + 20(0.00286) = \underline{\underline{0.16\ pu}} = 0.16 \times 1250 = \underline{\underline{200\ MW}}$

$ACE_2 =$

$P_{Net_2}^{actual} = 71\ MW$

$\Delta P_{Net\,2} = -129\ MW = \frac{-129}{2250} = \underline{\underline{-0.057\ pu}}$

$B_2 = \frac{1}{R_2} = \underline{\underline{20}}$

$\underline{\underline{ACE_2 = -0.057 + 20(0.00286) = 0}}$

AGC Action:-

$ACE_1 = 200\ MW \Rightarrow G_1$ is the slack bus for Area 1

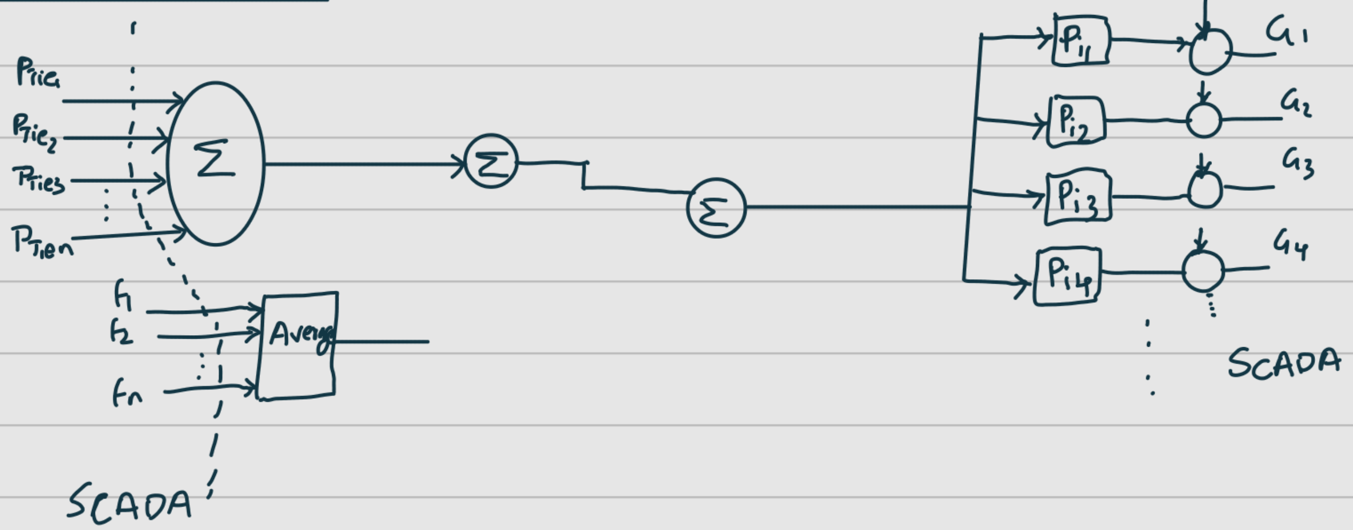
$P_{C_1}^{AGC} = 500 - 200 = 300\ MW$ (slack bus)

$P_{C_2}^{AGC} = 300\ MW$ (non slack bus) $\qquad \underline{\underline{P_{Tie} = -200\ MW}}$

$P_{C_3}^{AGC} = 300\ MW$

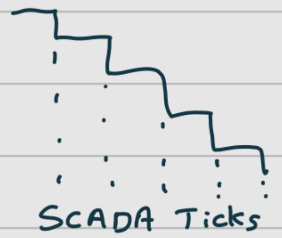
$P_{C_4}^{AGC} = 500\ MW$

AGC Architecture



$ACE_i \Rightarrow$ Incremental Control in closed loop

$ACE_i = 20 \Rightarrow$



03/06/24

- Check Dr. Mani's Notes

03/07/24

$$\dot{x} = f(x,y)$$
$$0 = g(x,y)$$
$$\Downarrow$$
$$0 = f(x,y)$$
$$0 = g(x,y)$$
$$\Downarrow$$
$$(x_e, y_e)$$

$$\dot{x} = f(x)$$
$$\Downarrow$$
$$0 = f(x)$$
$$\Downarrow$$
$$x_e$$
$$\Delta\dot{x} = J\Delta x$$
$$J = \left.\frac{\partial f}{\partial x}\right|_{x_e}$$

$$\Delta\dot{x} = J\Delta x$$
$$J = A - BD^{-1}C$$
$$A = \left.\frac{\partial f}{\partial x}\right|_{(x_e,y_e)}, \quad B = \left.\frac{\partial f}{\partial y}\right|_{(x_e,y_e)}$$
$$C = \left.\frac{\partial g}{\partial x}\right|_{(x_e,y_e)}, \quad D = \left.\frac{\partial g}{\partial y}\right|_{(x_e,y_e)}$$

if all eigen values of J have -ve real parts. Then X_e is a small signal stable eqbm point

Example

$$\dot{x} = -x + 2xy$$
$$0 = y^2 - x - 1$$

$\Rightarrow$ eqbm points

$$0 = -x + 2xy = x(-1+2y) \quad \text{—①}$$
$$0 = y^2 - x - 1 \quad \text{—②}$$

$x = 0$ or $y = 1/2$

$x = 0 \Rightarrow y^2 - 1 = 0$
$y = \pm 1$

$y = 1/2 \Rightarrow \frac{1}{4} - x - 1 = 0$
$x = \frac{-3}{4}$

$$(0,1), (0,-1), (-3/4, 1/2)$$

$$J = A - BD^{-1}C$$
$$A = -1 + 2y\big|_{(x_e,y_e)}, \quad B = 2x\big|_{(x_e,y_e)}$$

$C = -1, \quad D = 2y \Big|_{(x_e, y_e)}$

$x_{e_1} = (0, 1) \Rightarrow A = 1, \ B = 0, \ C = -1, \ D = 2$

$J = A - BD^{-1}C = 1$

$(0,1)$ is ss unstable

$x_{e_2} = (0, -1) \Rightarrow A = -3, \ B = 0, \ C = -1, \ D = -2$

$J = -3 - 0 = -3$

$(0,-1)$ is ss stable

$x_{e_3} = (-3/4, 1/2) \Rightarrow A = 0, \ B = -3/2, \ C = -1, \ D = 1$

$J = 0 - (-3/2)(1)^{-1}(-1) = -3/2$

$(-3/4, 1/2)$ is ss stable

$\dot{\underline{x}} = \underline{J}\,\underline{x}, \quad \underline{x} \in \mathbb{R}^n$

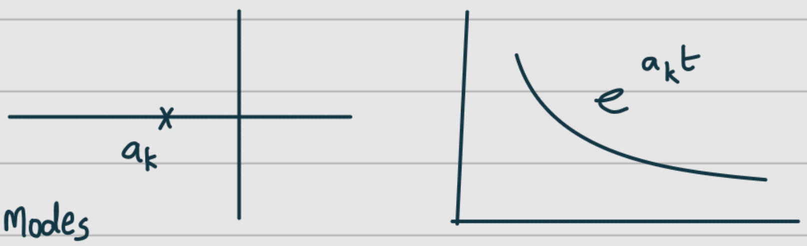
$x(t) = ? \Rightarrow$ eigen values of J

n eigen values $\Rightarrow \lambda_k, \ k = 1, 2 \ldots n$

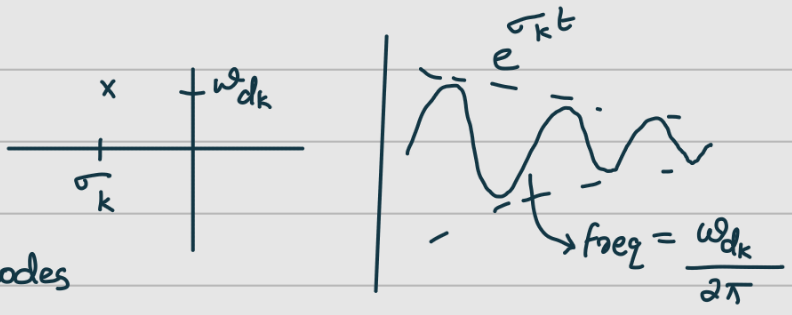
$\Downarrow$

modes of the s/m

Modal Responses $\Rightarrow e^{\lambda_k t}$



Real Modes



Complex Modes

⇒ Assume that all eigen values of $\underline{J}$ have -ve real parts

⇒ Multiple eigen values ⇒ $\begin{bmatrix} \sigma & 1 \\ 0 & \sigma \end{bmatrix}$ $\begin{bmatrix} \sigma & 1 & 0 \\ 0 & \sigma & 1 \\ 0 & 0 & \sigma \end{bmatrix}$ → Jordan Canonical forms

⇒ All eigen values are distinct

Eigen Vector

λ_k is a eigen value of $\underline{J}$ if

$\exists\ \underline{v}_k \neq 0 \ni \underline{J}\,\underline{v}_k = \lambda_k \underline{v}_k$

$\underline{v}_k$ is a right eigen vector for J

$(\underline{J} - \lambda \underline{I})\underline{v}_k = 0 \Rightarrow \det(\lambda \underline{I} - \underline{J}) = 0$ for any eigen value

$\underline{P}^{-1} = [\underline{v}_1, \underline{v}_2, \dots \underline{v}_n]$

$\underline{z} = \underline{P}\underline{x} \Rightarrow \dot{\underline{z}} = \underline{P}\dot{x} = \underline{P}\,\underline{J}\,\underline{x} = \underline{P}\,\underline{J}\,\underline{P}^{-1}\underline{z}$

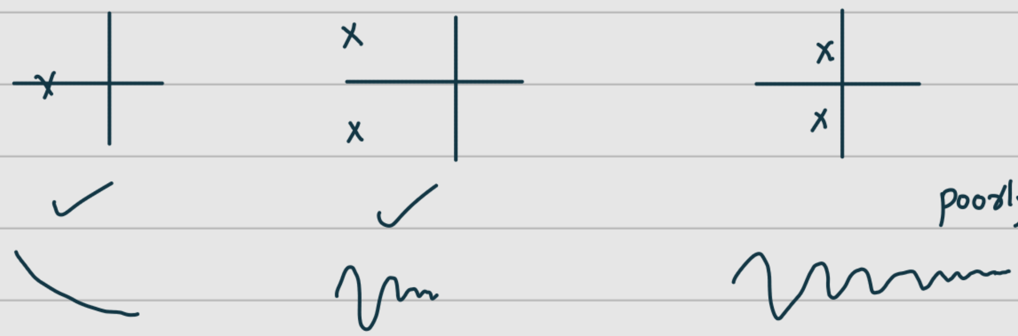
$\dot{\underline{z}} = \begin{bmatrix} \lambda_1 & & \underline{0} \\ & \ddots & \\ \underline{0} & & \lambda_n \end{bmatrix} \underline{z}$ ⇐ Jordan State Span

$\dot{z}_k = \lambda_k z_k$ Jordan form

Preposition: Assume that all eigenvalues of $\underline{J}$ are distinct

$$x_i(t) = \sum_{k=1}^{n} \sum_{j=1}^{n} v_{k_i} \omega_{k_j} e^{\lambda_k t} x_{j0}$$

$e^{\lambda_k t} \Rightarrow k^{th}$ modal response



poorly damped responses

$$e^{\lambda_k t} \longrightarrow \text{Observability} \Rightarrow \text{what states respond to this mode?}$$

$$\longrightarrow \text{Controllability} \Rightarrow \text{What states have an impact on the mode}$$

$$\dot{\underline{x}} = \underline{J}\,\underline{x} \quad \Longrightarrow \quad x_i(t)$$

$$\underline{x}_0 \qquad\qquad \Uparrow\ \underline{x} = \underline{P}^{-1}\underline{z}$$

$$\Downarrow\ \underline{z} = P\,x$$

Jordan $\dot{z}_k = \lambda_k z_k \quad \Longrightarrow \quad z_k(t) = e^{\lambda_k t} z_{k0}$

$$P^{-1} = [v_1 \cdots v_n]$$

$$\dot{\underline{x}} = \underline{J}\,\underline{x} \Rightarrow \underline{J}\,\underline{v}_k = \lambda_k v_k$$

$$\Downarrow$$

$$J\,[\underline{v}_1, \dots v_n] = (\underline{v}_1 \cdots \underline{v}_n)\begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{bmatrix}$$

$$[\underline{v}_1 \cdots \underline{v}_n]^{-1}\,\underline{J}\,[\underline{v}_1 \cdots \underline{v}_n] = \begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{bmatrix}$$

$$\underline{z} = \underline{P}\,x, \quad \underline{x} = \underline{P}^{-1}\underline{z}$$

$$\dot{\underline{z}} = \underline{P}\,\dot{\underline{x}} = \underline{P}\,\underline{J}\,\underline{x}$$

$$= \underline{P}\,\underline{J}\,\underline{P}^{-1}\underline{z} = \begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{bmatrix}\begin{bmatrix} z_1 \\ \vdots \\ z_n \end{bmatrix}$$

$$P^{-1} = [\underline{v}_1 \cdots \underline{v}_n]$$

$\dot{z}_k = \lambda_k z_k$ Jordan form

$$[\underline{v}_1 \cdots \underline{v}_n]^{-1} = \begin{bmatrix} \underline{w}_1^T \\ \vdots \\ \underline{w}_n^T \end{bmatrix}$$ — eigen value property

$$\underline{z} = \underline{P}\,\underline{x} = [\underline{v}_1 \cdots \underline{v}_n]^{-1} x$$

$$= \begin{bmatrix} w_1^T \\ \vdots \\ w_n^T \end{bmatrix}\underline{x}$$

$$\underline{w}_k^T \underline{J} = \lambda_k \underline{w}_k^T$$

$$\dot{\underline{x}} = \underline{J}\,\underline{x}, \quad \underline{x}_0$$

$$\Downarrow \; \underline{z} = \underline{P}\,\underline{x} \qquad\qquad \Uparrow \; x = P^{-1} z$$

$$\dot{z}_k = \lambda_k z_k \implies z_k(t) = e^{\lambda_k t} z_{k0}$$

$$z_0$$

$$x_0 \implies z_0 = P x_0$$

$$\begin{bmatrix} z_{10} \\ \vdots \\ z_{n0} \end{bmatrix} = \begin{bmatrix} \underline{w}_1^T \\ \vdots \\ \underline{w}_n^T \end{bmatrix} \begin{bmatrix} x_{10} \\ \vdots \\ x_{n0} \end{bmatrix}$$

$$\begin{bmatrix} z_{10} \\ \vdots \\ z_{k0} \\ \vdots \\ z_{n0} \end{bmatrix} = \begin{bmatrix} w_{11} & \cdots & w_{1n} \\ \vdots & & \vdots \\ w_{k1} & \cdots & w_{kn} \\ \vdots & & \vdots \\ w_{n1} & \cdots & w_{nn} \end{bmatrix} \begin{bmatrix} x_{10} \\ \vdots \\ x_{n0} \end{bmatrix}$$

$$z_{k0} = \sum_{j=1}^{n} w_{kj} x_{j0}$$

$$z_k(t) = e^{\lambda_k t} z_{k0}$$

$$= \sum_{j=1}^{n} w_{kj} e^{\lambda_k t} x_{j0}$$

$$x_i(t) = ?$$

$$x(t) = P^{-1} z(t)$$

$$\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} \underline{v}_1 & \cdots & \underline{v}_n \end{bmatrix} \begin{bmatrix} z_1 \\ \vdots \\ z_n \end{bmatrix}$$

$$
\begin{bmatrix} x_1 \\ \vdots \\ x_k \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} v_{11} & \cdots & v_{n1} \\ \vdots & & \vdots \\ v_{1k} & & v_{nk} \\ \vdots & & \vdots \\ v_{1n} & \cdots & v_{n} \end{bmatrix} \begin{bmatrix} z_1 \\ \vdots \\ z_n \end{bmatrix}
$$

$$
x_i(t) = \sum_{k=1}^{n} v_{ki} \, z_k(t)
$$

$$
x_i(t) = \sum_{k=1}^{n} \sum_{j=1}^{n} \underbrace{v_{ki}}_{\text{Obs.}} \underbrace{w_{ji}}_{\text{Con.}} e^{\lambda_k t} x_{j0}
$$

$$PJP^{-1} = \begin{bmatrix} \lambda_1 & & \underline{0} \\ & \ddots & \\ \underline{0} & & \lambda_n \end{bmatrix} \to \text{diagonal}$$

$$\dot{\underline{z}} = \begin{bmatrix} \dot{z}_1 \\ \vdots \\ \dot{z}_n \end{bmatrix} = \begin{bmatrix} \lambda_1 & & \underline{0} \\ & \ddots & \\ \underline{0} & & \lambda_n \end{bmatrix} z$$

$$\boxed{\dot{z}_k = \lambda_k z_k}, \quad z_{k0}$$

$$\boxed{z_k(t) = e^{\lambda_k t} z_{k0}}$$

$$\underline{x_0} \Rightarrow \underline{z_0} = \underline{P}^{-1} \underline{x}$$

$$\begin{bmatrix} z_{10} \\ \vdots \\ z_{n0} \end{bmatrix} = \begin{bmatrix} \underline{v_1} & \cdots & \underline{v_n} \end{bmatrix} \begin{bmatrix} x_{10} \\ \vdots \\ x_{n0} \end{bmatrix}$$

$$\begin{bmatrix} z_{10} \\ \vdots \\ z_{k0} \\ \vdots \\ z_{n0} \end{bmatrix} = \begin{bmatrix} v_{11} & \cdots & v_{n1} \\ \vdots & & \vdots \\ v_{1k} & \cdots & v_{nk} \\ \vdots & & \vdots \\ v_{1n} & \cdots & v_{nn} \end{bmatrix} \begin{bmatrix} x_{10} \\ \vdots \\ x_{n0} \end{bmatrix}$$

numbering is opp. as normal matrix

$$z_{k0} = \sum_j v_{jk} x_{j0}$$

$$x_i(t) = \sum_{k=1}^{n} \sum_{j=1}^{n} v_{ki} w_{kj} e^{\lambda_k t} x_{j0}$$

03/19/24

$\dot{X} = JX$

Modal response

Eigen values with large real part

- Not of interest 
- Ignored - Not a problem

Eigen values with small real part

 poorly damped mode

- What mode is causing this response?
- How can we improve the damping of this mode?

Proposition:

$$x_i(t) = \sum_{j=1}^{n} \sum_{k=1}^{n} \underbrace{v_{ki}}_{\text{Right E.V}} \underbrace{w_{kj}}_{\text{Left eig. vect}} \underbrace{x_{j0}}_{} \underbrace{e^{\lambda_k t}}_{k^{th} \text{ modal response}}$$

Effect of mode λ_k on state x_i (observability)

Perturbation in state x_j

Effect of state x_j on mode λ_k (controllability)

$v_{k100} = 0$ means mode λ_k will not be seen in $x_{100}(t)$

$x_{k500} = 0$ means x_{500} will have no effect in λ_k

v_{k50} is large

$\Rightarrow x_{50}$ will show a big response to mode λ_k

Poorly damped mode λ_{100}

$|v_{100}|$ $\quad w_{100} = [1 \;\; 0 \;\; 0 \;\; 0.7 \;\; 0 \;\; 0]$

$\Downarrow$

$$\begin{array}{c} ① \\ \\ \\ \\ ㊳ \\ \\ \end{array} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.7 \\ 1 \\ 0.8 \end{bmatrix}$$

x_1 will show no effect from λ_{100}, λ_{100} will not seen in x_1

x_{38} will show poorly damped λ_{100}

$w_{100} = [1 \quad 0 \quad 0 \quad 0.7 \quad 0 \quad 0]$

x_{1_0} will have large impact on λ_{100}

Any disturbance in x_2 will not excite λ_{100}

state x_{38} excites λ_{100} and responds k mode λ_{100}

$v_{k_{38}}$ & $w_{k_{38}}$ are large for $k=100$

Mode x_{38} participates strongly in the mode λ_{100}

Selective Modal Analysis (Verghese)

$\lambda_k \Rightarrow v_k \quad , \quad w_k = [w_{k_1} \; w_{k_2} \; \cdots \; w_{kn}]$

$$\begin{bmatrix} v_{k1} \\ v_{k2} \\ \vdots \\ v_{kn} \end{bmatrix}$$

$\Rightarrow [v_{kj} \, w_{kj}] \quad i=1:n$

$\quad j=1:n$

$$\begin{bmatrix} v_{k_1}w_{k_1} & & & \\ & v_{k_2}w_{k_2} & & \\ & & \ddots & \\ & & & v_{kn}w_{kn} \end{bmatrix}$$

$$p_i^k = \begin{bmatrix} |v_{k_1}||w_{k_1}| \\ \vdots \\ |v_{kn}||w_{kn}| \end{bmatrix}$$

- Divide by the largest entry
- maximum entry

$38^{th} =$

$$p_i^k = \frac{|v_{ki}||w_{ki}|}{\max_i(|v_{ki}||w_{ki}|)}$$

Modes

- Local Modes
- Intra area mode
- Intra area mode

03/20/24

April 9^{th} 9 AM – 12 PM

- Comprehensive
- Closed book, closed notes
- Four single sided 8.5" x 11" formula sheets
- Matrix calculator. Max 3x3 matrices

$\dot{x} = Jx \; , \; x_0$

$$x_i(t) = \sum_{j=1}^{n} \sum_{k=1}^{n} v_{ki} \, w_{kj} \, x_{j0} \, e^{\lambda_k t}$$

$$p_i^k = \frac{|v_{ki}| \, |w_{ki}|}{\max_i \left(|v_{ki}| \, |w_{ki}| \right)}$$

$p_i^k < \varepsilon \Rightarrow$ Set to zero

states with $p_i^k > \varepsilon$ are said to participate

- → Local
- → Intra area $\Rightarrow$ multiple
- → Inter area $\Rightarrow$ across multiple control areas

Load Serving Authority $\Rightarrow$ AGC

$\Rightarrow$ Balancing of generation Vs load

$$\underline{\dot{x}} = \underline{f}(\underline{x}, \underline{y})$$
$$\underline{0} = \underline{g}(\underline{x}, \underline{y})$$

Type 1

$\Downarrow$

$$\underline{0} = \underline{f}(\underline{x_e}, \underline{y_e})$$
$$\underline{0} = \underline{g}(\underline{x_e}, \underline{y_e})$$

$$\underline{\dot{x}} = \underline{f}(\underline{x})$$

Type 2 or 3

$$\underline{f}(\underline{x_e}) = 0$$

$$\underline{A} = \left.\frac{\partial \underline{f}}{\partial \underline{x}}\right|_{(\underline{x_e},\, \underline{y_e})} \qquad \underline{B} = \left.\frac{\partial \underline{f}}{\partial \underline{y}}\right|_{(\underline{x_e},\, \underline{y_e})} \qquad J = \left.\frac{\partial \underline{f}}{\partial \underline{x}}\right|_{(x_e)}$$

$$\underline{C} = \left.\frac{\partial \underline{g}}{\partial \underline{x}}\right|_{(\underline{x_e},\, \underline{y_e})} \qquad \underline{D} = \left.\frac{\partial \underline{g}}{\partial \underline{y}}\right|_{(\underline{x_e},\, \underline{y_e})}$$

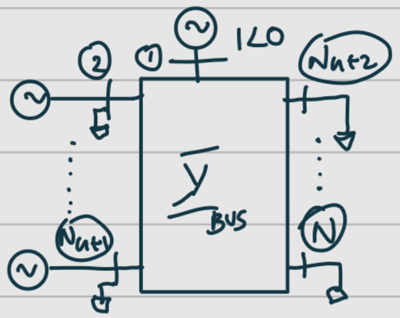
$$J = A - BD^{-1}C$$

λ_k

v_k

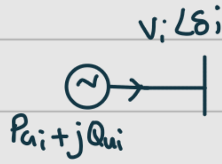
w_k

Modal analysis



Power flow solution (HW1) $\Rightarrow$ known

θ_i
ω_i
E'_{qi}
E'_{di}



$P_{Gi} = \checkmark$
$V_i = \checkmark$ } Specified variables

$\Downarrow$

$\delta_i = \checkmark$
$Q_{Gi} = \checkmark$ } Power flow solution

Internal states?
(at steady state)

ED or OPF $\rightarrow$ P_{Gi}^{sch} (SCADA) AGC

Power plant: Hourly Schedule $\rightarrow$ P_{Gi}^{sch}, V_i^{sch}

$\Downarrow$
Seasonal

Ferranti Effect
(light load)

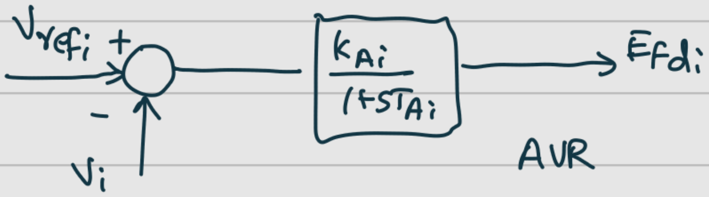
$P_{ai} = P_{ai}^{sch}\,(P_{ci})$

$V_i = V_i^{sch}$

(plant operators)

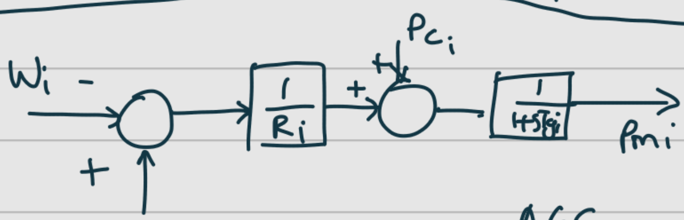
V_i^{sch}

(not from SCADA)



AVR

$V_i \sim V_{ref_i}$



AGC

Power plant voltage control:-

Plant operator will adjust V_{ref_i} to keep $V_i = V_i^{sch}$

MW side: $P_{ai} = P_{ai}^{sch.}$
(terminal)

$P_{m_i} = P_{a_i} + \text{Core losses}$

$R_{s_i} |I_{a_i}|^2$

Assume
$R_{s_i} = 0$
for EE523

Steady State

$$\left.\begin{aligned} V_i &= V_i^{sch} \\ P_{a_i} &= P_{a_i}^{sch} \end{aligned}\right\} \text{Power flow}$$

Solve for V_{ref_i} and P_{c_i} so that $V_i = V_i^{sch}$ & $P_{a_i} = P_{a_i}^{sch}$ as specified

Equilibrium points $\Rightarrow$ Decoupled fashion for each generator

θ_i

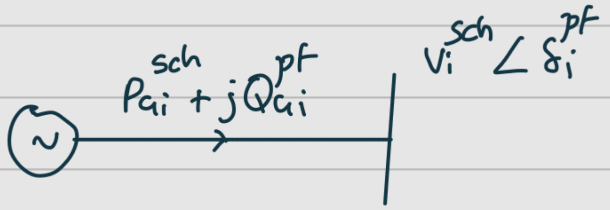
ω_i

E'_{q_i}

E'_{d_i}

E_{fd_i}

P_{m_i}

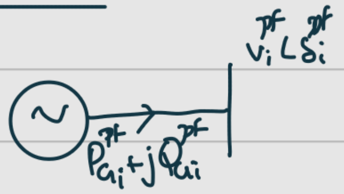


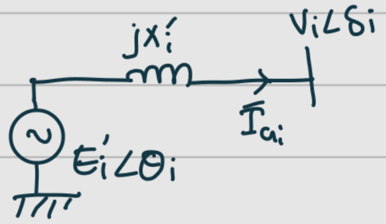
Reactive OPF

Obj: min(losses)

V_i as the control variable

Type 3 Model





$\theta_i = ?$

$\omega_1 = ?$

$E'_i = ?$

$P_{a_i}^{pf} + jQ_{a_i}^{pf}, \quad V_i^{pf} \angle \delta_i^{pf}$

$$\bar{I}_{a_i}^{pf} = \frac{P_{a_i}^{pf} - jQ_{a_i}^{pf}}{V_i^{pf} \angle -\delta_i^{pf}} \checkmark$$

$$E'_i \angle \theta_i = V_i^{pf} \angle \delta_i^{pf} + jX'_1 \bar{I}_{a_i}$$

E'_i ✓

θ_i ✓

In ss ω_i should be 1

$\dot{\theta}_i = (\omega - 1)\omega_s = 0 \Rightarrow \omega_i = 1$ ✓

Equilibrium solution for Type 3

$$\dot{x} = f(\underline{x})$$

$$\Downarrow$$

$$J = \frac{\partial f}{\partial \underline{x}} \Big|_{\underline{x}_e} \Rightarrow \lambda_k \checkmark$$

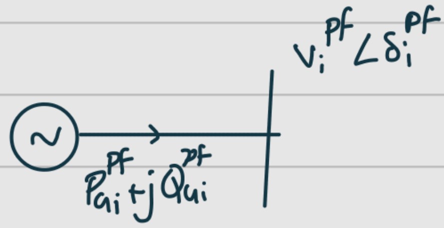
Type 1 Model

$EMe \begin{cases} \theta_i \\ \omega_i \end{cases}$

$EMa \begin{cases} E'_{qi} \\ E'_{di} \end{cases}$

$AVR \{ E_{fdi}$

$Gov \{ P_{mi}$



P_{ci} and $V_{ref_i} \Rightarrow V_i^{sch}$

$\Downarrow_{sch}$

P_{ai}

$$\dot{\theta}_i = (\omega_i - 1) = 0$$

$$2H_i \dot{\omega}_i = P_{mi} - P_{ai} - (\omega_i - 1) K_{Di} = 0$$

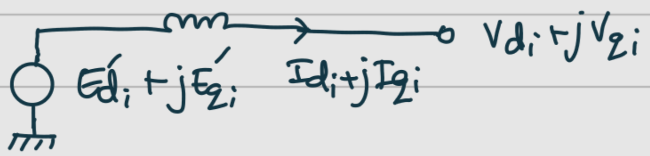
$$T'_{doi} \dot{E}'_{qi} = -E_{qi} - (x_{di} - x'_{di}) I_{di} + E_{fdi} = 0$$

$$T'_{qoi} \dot{E}'_{di} = -E_{di} + (x_{qi} - x'_{qi}) I_{qi} = 0$$

$$T_A V_{Ai} = -V_{Ai} + K_{Ai}(V_{ref} - V_i) = 0$$

$$E_{fd} = V_A \quad \text{(ignoring limits)}$$

$$T_{sgi} \dot{P}_{mi} = -P_{mi} + \frac{1}{R_i}(1 - \omega_i) + P_{ci} = 0$$



$$P_{ai} = V_{di} I_{di} + V_{qi} I_{qi} = P_{ai}^{pf}$$

$$Q_{ai} = V_{qi} I_{di} - V_{di} I_{qi} = Q_{ai}^{pf}$$

eight eqns in eight unknown

$$V_{di} = V_i \sin(\theta_i - \delta_i)$$

$$V_{qi} = V_i \cos(\theta_i - \delta_i)$$

$$\Rightarrow 0 = F_i(\underline{x}_{gi}) = 0$$

$\Rightarrow$ 8 unknowns

NR $\Rightarrow$ fsolve

NR good with good initial condition

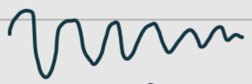
$$\left.\begin{array}{l} \theta_i \longrightarrow \delta_i^{pf} + 10^\circ \text{ or } 20^\circ \\ \omega_i \longrightarrow 1 \\ E'_{qi} \longrightarrow 1.5 \\ E'_{di} \longrightarrow 0.3 \\ E_{fd} \longrightarrow 2 \\ P_{mi} \rightarrow P_{ai}^{sch} \\ P_{ci} \rightarrow P_{ai}^{sch} \end{array}\right\} \text{fsolve}$$

$A = ?$ $\quad B = ?$

$C = ?$ $\quad D = ?$

$$\underline{J} = \underline{A} - \underline{B}\,\underline{D}^{-1}\underline{C}$$

03/21/24



$\text{freq} = \frac{1}{\text{period}}$

$\Rightarrow \lambda_k\,? \Rightarrow$ Participation Factors

$\Leftarrow x_i\,?$

Design controls to improve the damping of λ_k

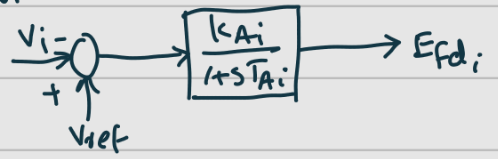
Generators associated with x_i

Electromechanical Modes — local (1–2 Hz)

0.1 to 2Hz — Intra area (0.5 – 1 Hz)

Inter area (0.1 – 0.5Hz)

AVR:



Steady state: $V_i \approx V_{ref_i} \Rightarrow K_{Ai} \uparrow$

Root locus

$E_{fd_i} = K_{A_i}(V_{ref_i} - V_i)$

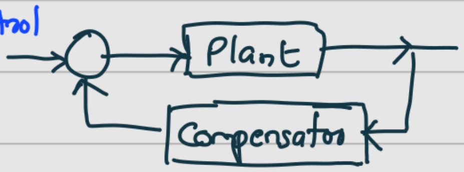
(High Gain Control)

⇒ Can make the closed loop damping to become low

⇒ poorly damped modes

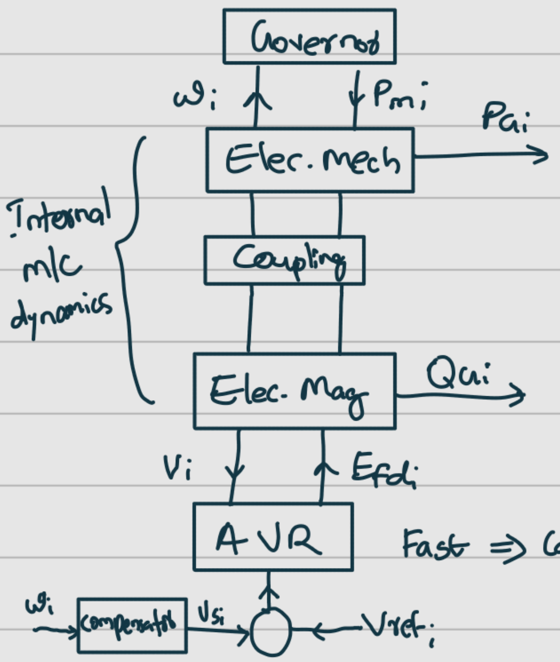
Objective: To keep all modes with at least 5% damping ratio

Feedback Control design



Compensator Design

Design Techniques
- S-domain (Laplace) — Root Locus
- Freq domain (Fourier) — Bode plot ✓ power
- Time domain — State space aerospace



Governor should be used

↳ slow sec to mins

∴ Use AVR (larger loop) fast

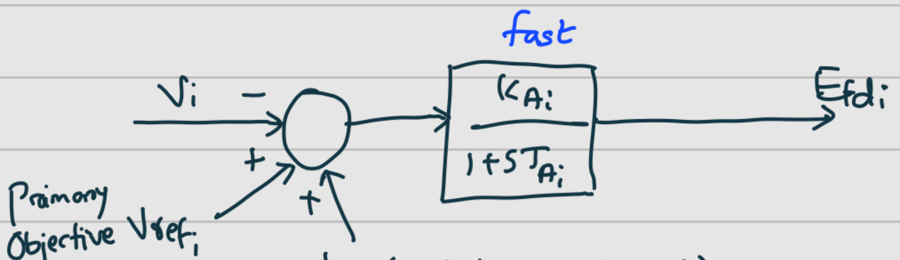
Fast ⇒ Can control elec. mech. modes

$\Downarrow$

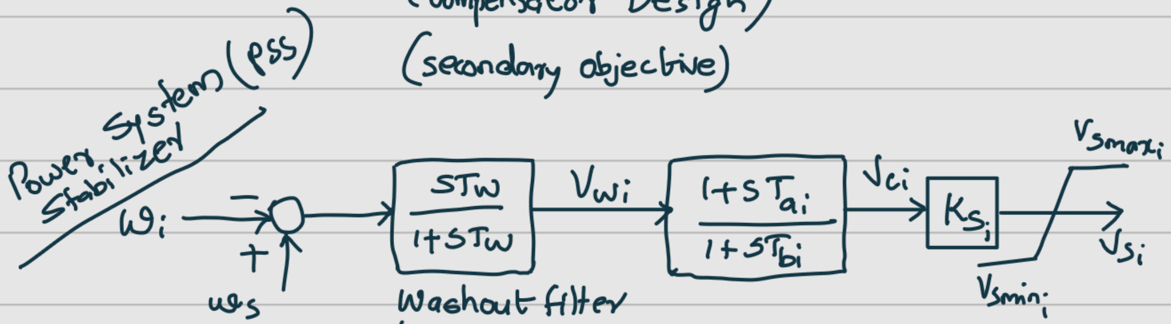
E_{fd_i}

$\Downarrow$

$\omega_i \Leftarrow P_{a_i} \Leftarrow E'_{q_i}, E'_{d_i}$



V_{si} (stabilizer Signal)
(compensator Design)
(secondary objective)



$V_{smax_i} = \pm 2\% \; \pm 0.02$

$V_{smin_i} = \pm 1\% \; \pm 0.01$

Washout filter
highpass filter

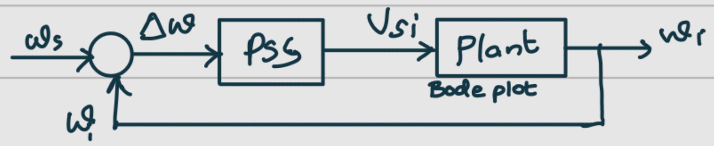
Make o/p zero
when i/p reach ss.

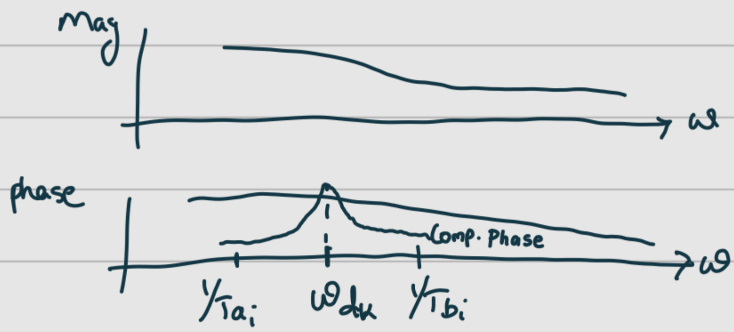
$T_{a_i}, T_{b_i}, K_{s_i}$

$\Rightarrow$ Compensator Parameters

$\Rightarrow$ Read any book on linear Control System Design

$\Rightarrow$ Bode plot designs





$\lambda_k \Rightarrow -\sigma_k \pm j\omega_{d_k}$

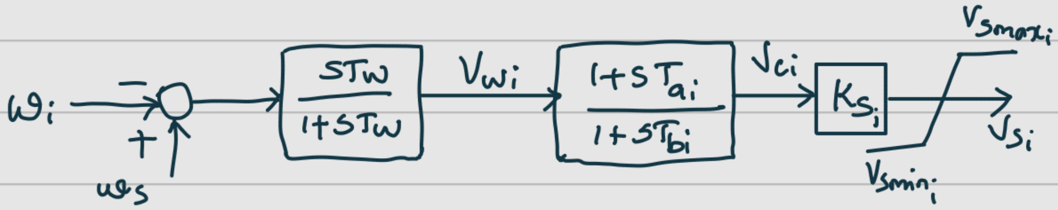
Design the compensator to provide adequate phase lead at ω_{d_k}

① $\omega_{d_k} = \sqrt{\frac{1}{T_{a_i}} \frac{1}{T_{b_i}}}$

② Place T_{a_i} & T_{b_i} so that the phase lead is adequate for the target damping

③ Design K_{s_i} to keep the system stable

- Implement PSS in the model
- Recalculate J and eigen values
- Check if the damping has improved



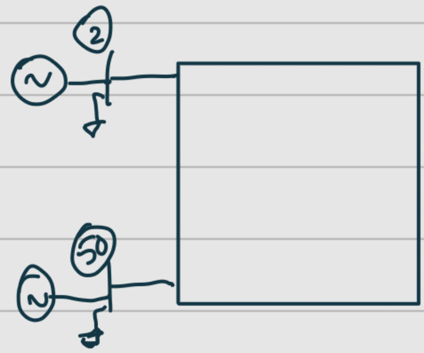
$$V_{w_i} = \frac{sT_{w_i}}{1+sT_{w_i}}(\omega_s - \omega_i)$$

① $T_{w_i}\dot{V}_{w_i} = -V_{w_i} + T_{w_i}(-\dot{\omega}_i)$

$$2H\dot{\omega}_i = P_{m_i} - P_{e_i} - K_{D_i}(\omega_i - 1)$$

② $T_{b_i}\dot{V}_{c_i} = -V_{c_i} + V_{w_i} + T_{a_i}\dot{V}_{w_i}$

θ_2
ω_2
E'_{q_2}
E'_{d_2}
V_{A_2}
P_{m_2}
V_{W_2}
V_{C_2}



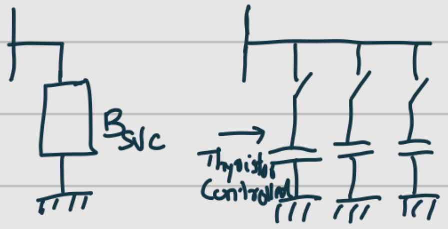
We can't have PSS in type 3 model (no control action occurs)

DC1A Exciter + PSS + Governor

$\underbrace{\theta_{50}, \omega_{50}}_{E_{mech}}, \underbrace{E'_{q_{50}}, E'_{d_{50}}}_{E_{mag}}, \underbrace{P_{m_{50}}}_{Gov}$

$\underbrace{V_{R50}, V_{E50}, V_{F50}}_{DC1A}, \underbrace{V_{W50}, V_{C50}}_{PSS}$

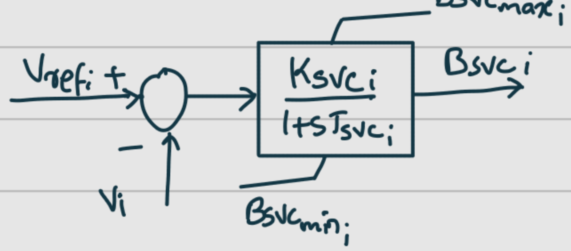
SVC



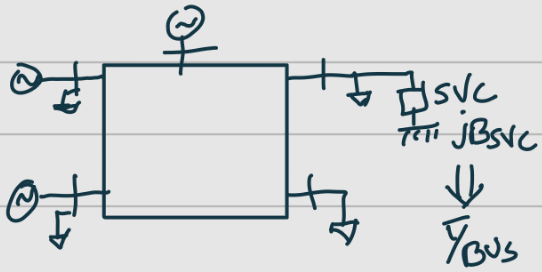
min to Max

Vary B_{svc} fast to

shunt cap banks
Reactor banks } Fast Voltage Control



SVC will keep V_i close to V_{ref_i} ((critical loads)



$\Downarrow$

Y_{BUS} includes a variable

B_{svc_i} dot

$$K_{svc_i} \dot{B}_{svc_i} = -B_{svc_i} + K_{svc_i}(V_{ref_i} - V_i)$$

$$\Downarrow$$

$$\dot{B}_{svc_i} = \begin{cases} 0 & B_{svc_i} = B_{svc_i max} \text{ \& } B_{scv_i} \text{ dot} > 0 \\ \vdots & \end{cases}$$

03/26/24

$$I = \det(\lambda \underline{I} - J) = 0$$

$\lambda_i \Rightarrow A v_i = \lambda_i v_i \Rightarrow$ Right e.v. (unit vectors)

$w_i A = \lambda_i w_i^T \Rightarrow$ Left e.v.

$$\begin{bmatrix} \underline{v}_1 & \underline{v}_2 & \cdots & \underline{v}_n \end{bmatrix}^{-1} = \begin{bmatrix} w_1^T \\ w_2^T \\ \vdots \\ w_n^T \end{bmatrix}$$

$$J = \begin{bmatrix} -2 & 3 \\ 0 & -4 \end{bmatrix} \Rightarrow \lambda_1 = -2, \ \lambda_2 = -4$$

$$\begin{bmatrix} -2 & 3 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} = -2 \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix}$$

$-2v_{11} + 3v_{12} = -2v_{11} \Rightarrow v_{12} = 0$

$-4v_{12} = -2v_{12} \Rightarrow v_{12} = 0$

$v_{11} = 1 \Rightarrow$ unity vector

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 3 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} = -4 \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix}$$

$-2v_{21} + 3v_{22} = -4v_{21}$

$3v_{22} = -2v_{21}$

$v_{21} = -\frac{3}{2} v_{22} \Rightarrow [$

$-4v_{22} = -4v_{22} \Rightarrow$ Ignore

$$\begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} = \begin{bmatrix} -3/2 \\ 1 \end{bmatrix} \Big/ \sqrt{1 + \left(\frac{3}{2}\right)^2}$$

$$= \begin{bmatrix} -3/\sqrt{\frac{10}{4}} \\ 2/\sqrt{10} \end{bmatrix}$$

$$\begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} = \begin{bmatrix} -0.83 \\ 0.63 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -0.83 \\ 0 & 0.63 \end{bmatrix}^{-1} = \begin{bmatrix} w_1^T \\ w_2^T \end{bmatrix} = \Big[1$$

$$\lambda_1 = v_1 w_1^T = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 1.317 \end{bmatrix} = \begin{bmatrix} 1 & 1.317 \\ 0 & 0 \end{bmatrix}$$

$p_1^1 = 1, \quad p_2^1 = 0 \Rightarrow$ state x_1

x_1 participates in λ_1

$$\lambda_2 \Rightarrow \begin{bmatrix} -0.83 \\ 0.63 \end{bmatrix} \begin{bmatrix} 0 & 1.587 \end{bmatrix}$$

$$p_1^2 = 0 \quad p_2^2 = \underbrace{1.587 \quad 1.03}_{max}$$

x_2 participates in λ_2

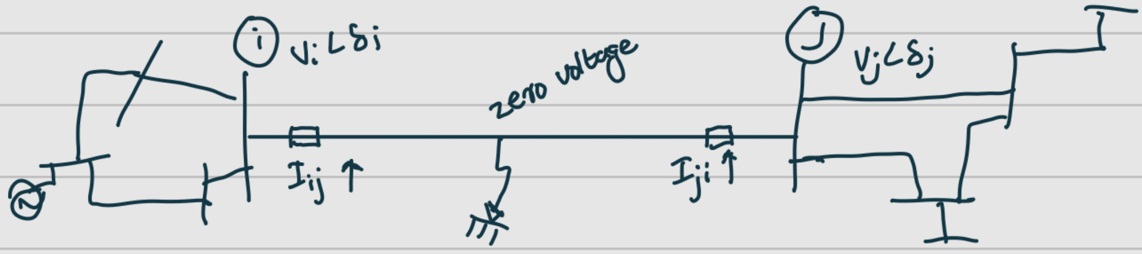
Transient Stability

Large Disturbance $\begin{cases} \text{Fault} \\ \text{Generator outages} \end{cases}$

Fault $-$ $\left.\begin{array}{l} LG \\ LL \\ 2LG \end{array}\right\}$ unbalanced $-$ Symmetrical Components

$\left.\begin{array}{l} 3LG \end{array}\right\}$ balanced

Assume balanced solid faults (zero fault impedance)



$t<0 \Rightarrow$ System is operating nominally 60 Hz & voltages and currents are acceptable

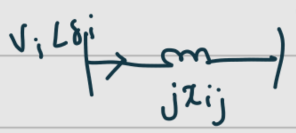
$t=0^+ \Rightarrow$ Fault occurs on the line from bus i to bus j

$0<t<t_c \Rightarrow$ Fault-on period

$t=t_c^+ \Rightarrow$ Protection will trip the line i to j (fault cleared)

$t>t_c \Rightarrow$ Post-fault period $\Rightarrow$ Voltages recovered $\Rightarrow P_{ei}$ recovered

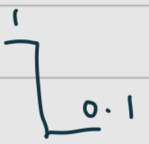
$\Rightarrow$ Sync. Torque will pull back S.M. back to 60 Hz



$$P_{ij} = \frac{V_i V_j}{x_{ij}} \sin(\delta_i - \delta_j)$$

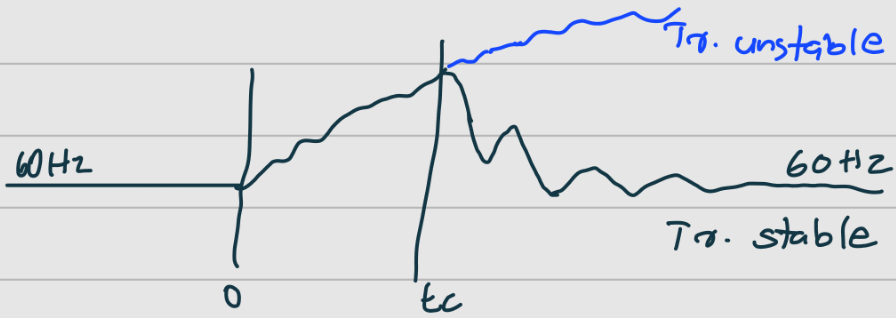
$P_{ij} \downarrow$

P_{ei}



$$2H\dot{\omega}_i = P_{mi} - P_{ei}$$

$\omega_i \uparrow \Rightarrow$ M/Cs accelerates near fault



t_c small $\Rightarrow$ Acceleration manageable

$\Rightarrow$ M/Cs recover

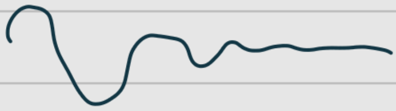
$\Rightarrow$ Tr. stability

t_c large $\Rightarrow$ Too much acceleration

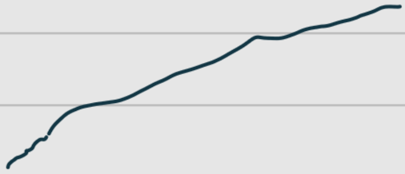
$\Rightarrow$ Sync. Torque is not able to keep M/Cs together

$\Rightarrow$ Tr. Instability

slow initial speed



Large initial speed



Prefault

$t<0$

Line i-j present

$\bar{Y}_{BUS}^{pre}$

$\Downarrow$

① $\dot{\underline{x}} = \underline{f}^{pre}(\underline{x}, \underline{y})$

$\underline{0} = \underline{g}^{pre}(\underline{x}, \underline{y})$

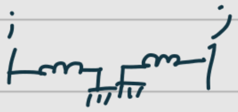
$\Downarrow$

(x_s^{pre}, y_s^{pre})

$\boxed{t<0}$

$\xrightarrow{t=0}$

Fault-on



$\bar{Y}_{BUS}^{fault}$

$\Downarrow$

② $\dot{\underline{x}} = \underline{f}(\underline{x}, \underline{y})$

$0 = \underline{g}^{fault}(\underline{x}, \underline{y})$

$\boxed{0 < t < t_c}$

Integrate ② from $(\underline{x}_s^{pre}, \underline{y}_s^{pre})$ for $0 < t < t_c$

(x_c, y_c)

clearing state

$\Rightarrow$ (x_c, y_c) $t=t_c$

Post fault

$t = t_c^+$ or $t > t_c$

$\bar{Y}_{BUS}^{post} \Rightarrow$ (no line from i to j)

$\dot{\underline{x}} = \underline{f}(\underline{x}, \underline{y})$

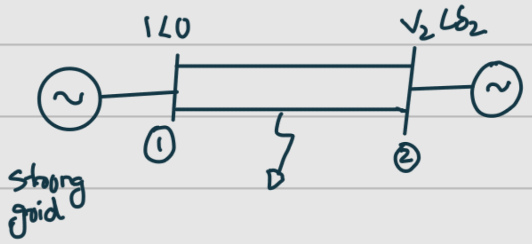
$\underline{0} = g^{post}(\underline{x}, \underline{y})$

Integrate ③ from (x_c, y_c) at $t = t_c$ for $t > t_c$

Prev class notes (03/28/24)

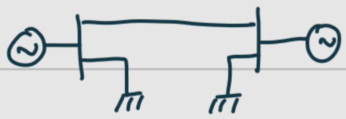
04/02/24

Equal Area Criterion: Analytical Method



Prefault

Fault on



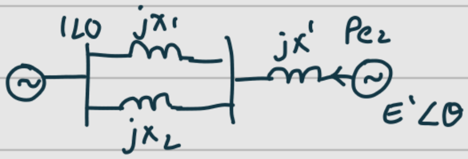
Post fault

Assumption: Use classical model, lines are lossless

$$\dot{\theta}_2 = (\omega_2 - 1)\,\omega_s$$

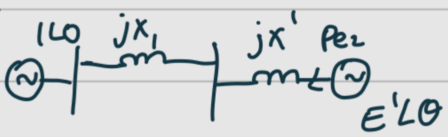
$$2H_2\dot{\omega}_2 = P_{m_2} - P_{e_2} - K_{D_2}(\omega_2 - 1)$$

Prefault:

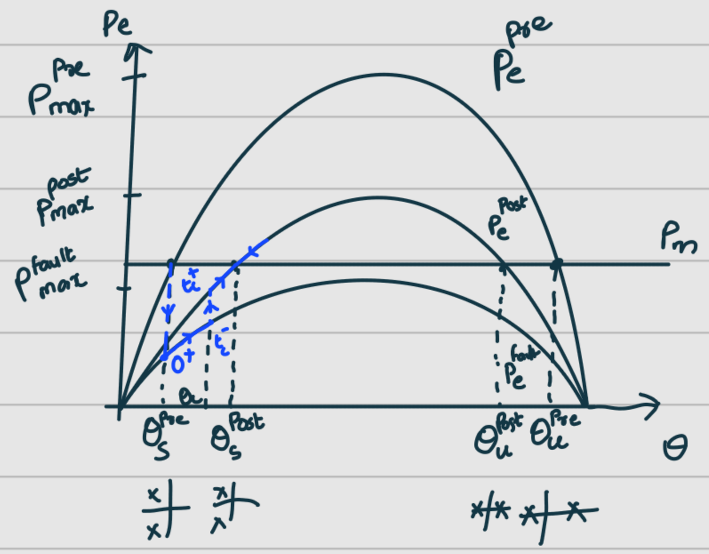


$$P_e^{pre} = \frac{E' \cdot 1}{x' + (x_1 \| x_2)} \sin\theta = P_{max}^{pre} \sin\theta$$

Post fault



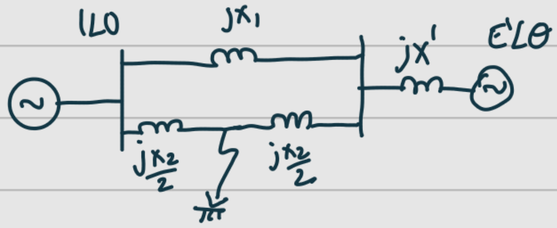
$$P_e^{post} = \frac{E' \cdot 1}{x' + x_1} \sin\theta = P_{max}^{post} \sin\theta$$

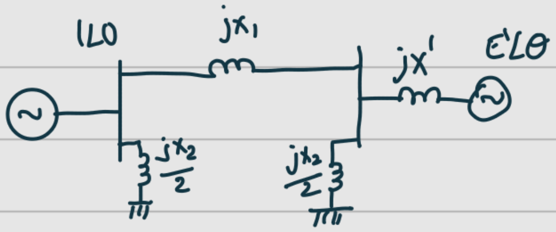


$$x' + x_1 \| x_2 < x' + x_1$$

$$\frac{E'}{x' + (x_1 \| x_2)} > \frac{E'}{x' + x_1}$$

Fault on



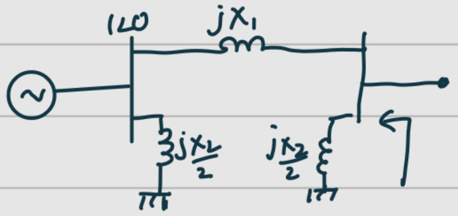


$$Y_{net} = \begin{array}{c} \\ \end{array}\begin{bmatrix} \bar{Y}_{11} & \bar{Y}_{12} \\ \bar{Y}_{21} & Y_{22} \end{bmatrix}\begin{array}{ll} 1,2 & \text{Gen} \\ 3 & \text{rest} \end{array}$$

$$\bar{Y}_{Gen} = \bar{Y}_{11} - \bar{Y}_{12}\bar{Y}_{22}^{-1}\bar{Y}_{21}$$

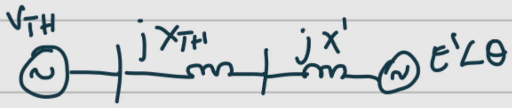
$$P_{e_2} = \sum_{j=1}^{2} Y_{G_{ij}} E_i' E_j' \cos(\theta_i - \theta_j - \theta_{G_{ij}})$$

Second approach : Thevenin

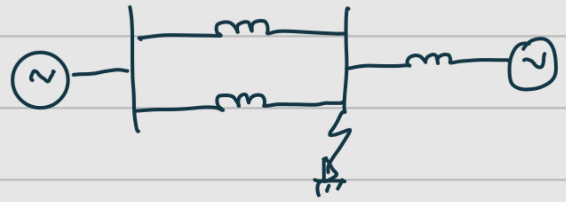


$$V_{TH} = \frac{jx_2/2}{jx_1 + jx_2/2} \cdot 1\angle 0 = \frac{x_2}{2x_1 + x_2}\angle 0$$

$$X_{TH} = x_1 \| x_2/2$$



$$P_e^{fault} = \frac{E' V_{TH}}{x' + X_{TH}} \sin\theta = P_{max}^{fault} \sin\theta$$



$$P_e^{fault} = \frac{E'.0}{x}\sin\theta = 0$$

$$\dot{\theta} = (\omega - 1)\,\omega_s$$

$$2H\dot{\omega} = P_m - P_e - \cancelto{0}{K_D(\omega-1)}$$

Prefault: s/m is in steady state

$$\dot{\theta} = (\omega - 1)\,\omega_s$$

$$2H\dot{\omega} = P_m - P_e^{pre} = P_m - P_{max}^{pre}\sin\theta$$

$$\dot{\theta} = (\omega-1)\,\omega_s = 0 \Rightarrow \omega = 1$$

$$2H\dot{\omega} = 0 \Rightarrow P_m - P_{max}^{pre}\sin\theta = 0$$

$$P_m = P_{max}^{pre}\sin\theta$$

<u>Pre fault</u> : s/m in steady state

$$\theta = \theta_s^{pre}, \quad \omega = 1, \quad \forall\, t < 0$$

<u>Fault appears at $t=0$:</u> Pe changes to P_e^{fault}

$$P_m > P_e^{fault\text{-}on} \Rightarrow 2H\dot{\omega} = P_m - P_e^{fault} > 0$$

$$\dot{\omega} > 0 \Rightarrow \omega\uparrow$$

Machine accelerates

$$0^+ : \omega > 1 \Rightarrow \dot{\theta} = (\omega-1)\,\omega_s > 0$$

$$\Rightarrow \theta\uparrow$$

<u>Post fault</u>

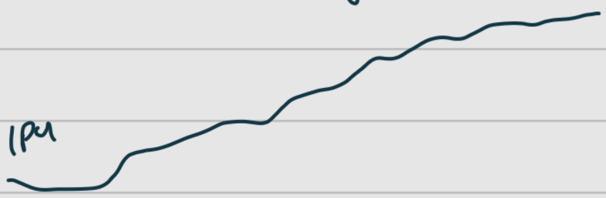
$$2H\dot{\omega} = P_m - P_e^{post}$$

$$\theta < \theta_s^{post} \Rightarrow P_m > P_e^{post} \Rightarrow \dot{\omega} > 0 \Rightarrow \omega\uparrow$$

Machine loses all the momentum before θ_u^{post}

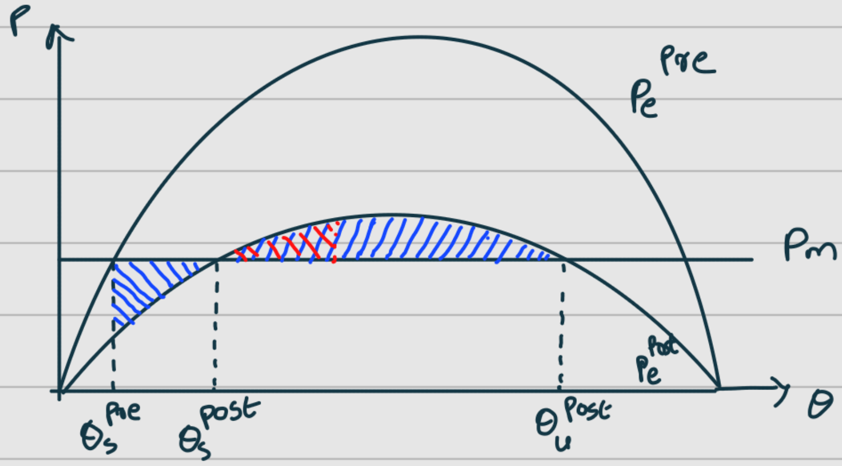
Stable

Momentum takes the angle past θ_u^{post}



unstable

Assumption: Fault clearing is instantaneous

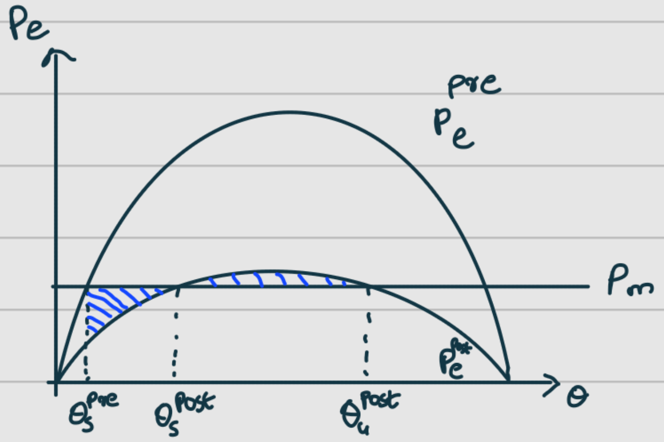


$$A_{acc} = \int_{\theta_s^{pre}}^{\theta_s^{post}} (P_m - P_e^{post})\, d\theta$$

$A_{acc} < A_{dec}^{max} \Rightarrow$ Tr. stable

$$A_{dec} = \int_{\theta_s^{post}}^{\theta_u^{post}} (P_e^{post} - P_m)\, d\theta$$

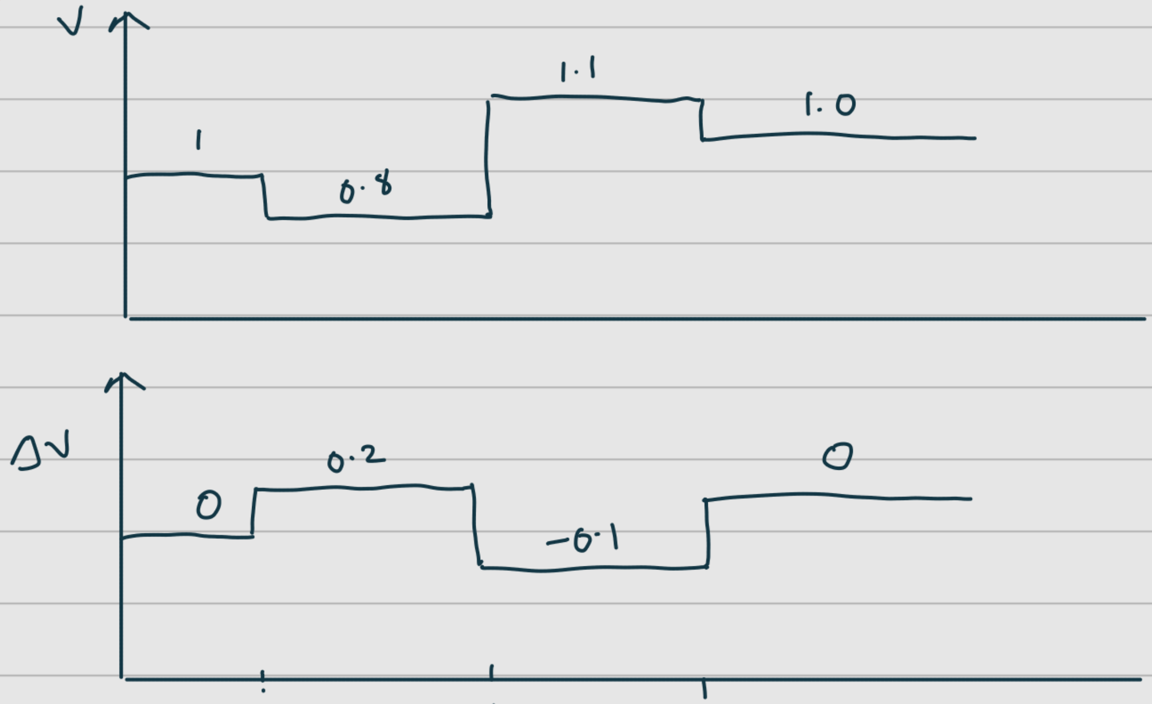
$A_{acc} > A_{dec}^{max} \Rightarrow$ Tr. unstable



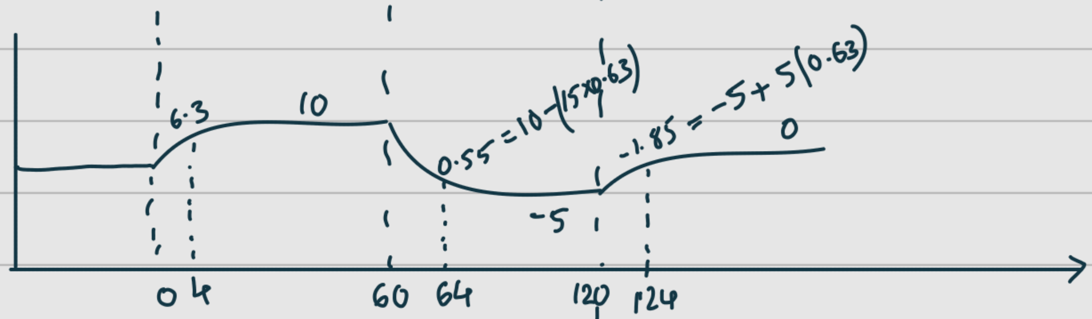
$A_{acc} > A_{dec}^{max}$

Tr. unstable

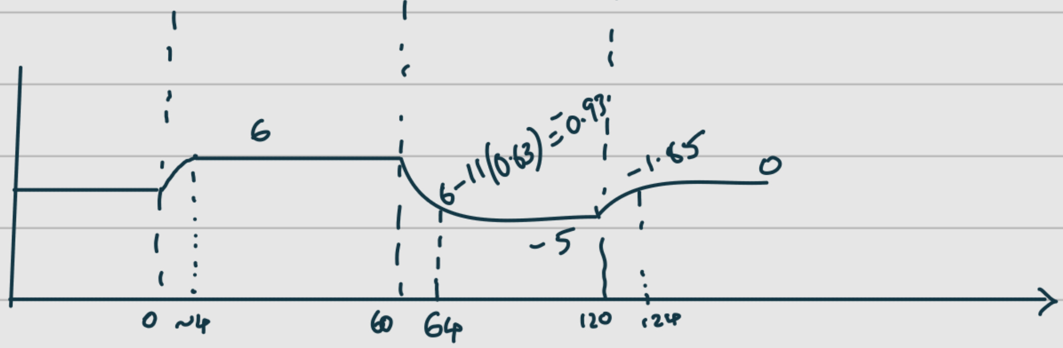
04/04/24



(a)



(b)



$$4 V_A \text{dot} = -V_A + 50 \Delta V \qquad V_A = \frac{50}{1+4s} \Delta V$$

$4V_A dot = -V_A + 50\Delta V$

$t = 4^-, V_A = 6, \Delta V = 0.2 \Rightarrow 4V_A dot = -6 + 50(0.2)$

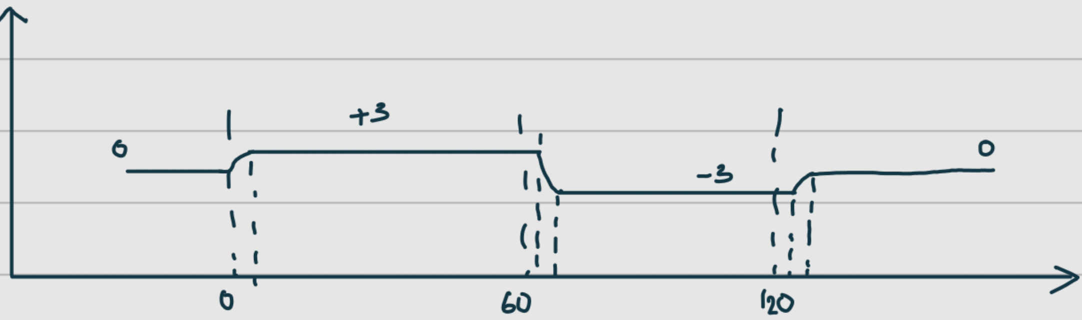
$= 4 > 0$

$t = 60^-, V_A = 6, \Delta V = 0.2 \Rightarrow V_A dot > 0$

$t = 60^+, V_A = 6, \Delta V = -0.1 \Rightarrow 4V_A dot = -6 + 50(-0.1)$

$= -11 < 0$

(c)



04/09/24 **Review Class**

(1)

$$\dot{\theta}_2 = (\omega_2 - 1)\omega_s$$

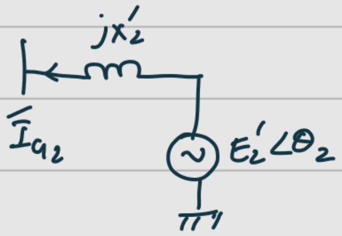
$$2H_2\dot{\omega}_2 = P_{m_2} - P_{e_2} - K_{D_2}(\omega_2 - 1)$$

Initialization

$\bar{Y}_{Net} \Rightarrow \bar{Y}_{Gen} \Rightarrow$ $\quad E_2' \angle \theta_2 = V_2 \angle \delta_2 + j x_2' \bar{I}_{G_2}$

$V_2^{pf} \angle \delta_2^{pf}$ $\quad \Rightarrow E_2'$ and θ_2 at s.s.

$P_{m_2} = P_{G_2}^{pf}$



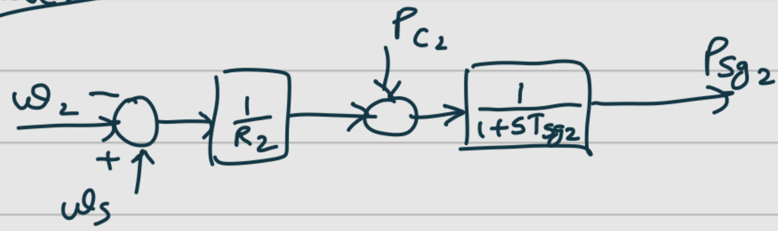
$$\underline{J} = \left.\begin{bmatrix} 0 & \omega_s \\ -\frac{\partial P_{G_2}}{\partial \theta_2}\frac{1}{2H_2} & -\frac{K_{D_2}}{2H_2} \end{bmatrix}\right|_{\substack{\theta_2 = \theta_2^+ \\ \omega_2 = 1}}$$

Assume Classical model (Type 3)

$E' = \text{const.} \Rightarrow$ ignore E_{q_2}', E_d', E_{fd_2} — No AVR

if type 3 mentioned $\Rightarrow$ No AVR, No gov.

Governor



$P_{m_2} = P_{sg_2}$

$P_{m_2} = P_{sg_2}$

(2)

Fault-on s/m model

$z_g = 0$ (assume)

new

Y_{BUS}^{fault}

$\Downarrow$

Y_{Net}^{fault}

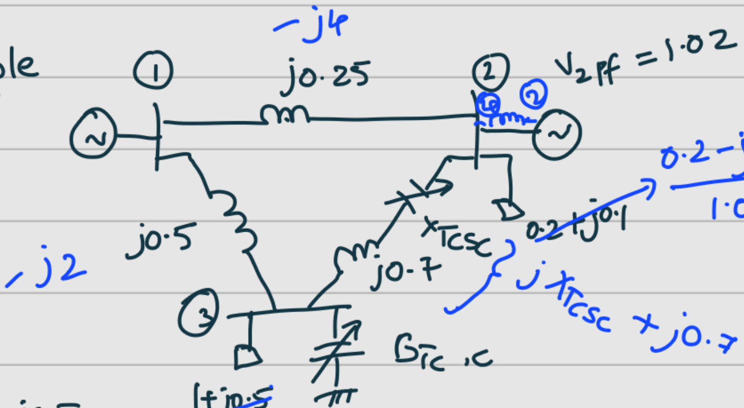
$P_{a_2}^{fault} = ? \Leftarrow \bar{Y}_{Gen}^{fault} \Leftarrow Y_{Net}^{fault}$

$\dot{\theta}_2 = (\omega_2 - 1)\omega_s$

$2H_2 \dot{\omega}_2 = P_{m_2} - P_{a_2}^{fault} - K_{D_2}(\omega_2 - 1)$

Assume E_2' & P_{m_2} remain const. [for prefault, fault on, post fault]

(2) from sample



$\frac{0.2 - j0.1}{1.02^2} =$

$jx_{TCSC} + j0.7$

adm $= \frac{1}{jx_{TCSC} + j0.7} = \frac{-j}{x_{TCSC} + 0.7}$

$\frac{1 - j0.5}{1^2}$

$$Y_{BUS} = \begin{bmatrix} -j6 & j4 & j2 \\ j4 & -\frac{j}{x_{TCSC}+0.7} - j4 + \frac{0.2-j0.1}{(1.02)^2} & \frac{j}{x_{TCSC}+0.7} \\ j2 & \frac{j}{x_{TCSC}+0.7} & -\frac{j}{x_{TCSC}+0.7} - j2 + 1 - j0.5 + jB_{SVC} \end{bmatrix}$$

$\boxed{\text{SVC keeps } V_3 \text{ @ } 1}$

Not given in qn
should be added to qr

$$\left.\begin{aligned} x_d' &= 0.4 \\ x_q' &= 0.3 \end{aligned}\right\} 0.35$$

$$Y_{red} = \begin{array}{c|c} \left[\begin{array}{c|c} & \\ \hline & \end{array}\right] & \begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \end{array} \end{array} \quad \begin{array}{l} \bar{V}_{gen} \\ \bar{V}_{rest} \end{array}$$

$$\bar{Y}_{gen} = \bar{Y}_{11} - \bar{Y}_{12}\bar{Y}_{22}^{-1}\bar{Y}_{21}$$

Type 2

$$\dot{\theta}_2 = (\omega_2 - 1)\omega_s$$

$$2H_2\dot{\omega}_2 = P_{m_2} - P_{e_2} - K_{D_2}(\omega_2 - 1)$$

$$T_{do_2}'\dot{E}_{q_2}' = -E_{q_2}' - (x_{d_2} - x_{d_2}')I_{d_2} + E_{fd_2}$$

$$T_{qo_2}'\dot{E}_{d_2}' = -E_{d_2}' + (x_{q_2} - x_{q_2}')I_{q_2}$$

$$I_{d_2} = \sum Y_{a_{ij}} E_j' \cos(\theta_i - r_j - \theta_{a_{ij}})$$

$$I_{q_2} = \sum Y_{a_{ij}} E_j' \sin(\theta_i - r_j - \theta_{a_{ij}})$$

$$P_{e_2} = \sum Y_{a_{ij}} E_i' E_j' \cos(r_i - r_j - \theta_{a_{ij}})$$

from block diagram

$$P_{LP} = \frac{k_{LP}}{1+sT_{LP}}(P_{23}^{*} - P_{ref})$$

$$T_{LP}\dot{P}_{LP} = -P_{LP} + k_{LP}(P_{23}^{*} - P_{23})$$

$$P_{23} = (Y_{BUS})_{2,3} V_2 V_3 \cos(\delta_2 - \delta_3 - \angle \bar{Y}_{23})$$ from Y_{BUS}

$$\begin{bmatrix} \bar{V}_{gen} \\ \bar{I}_{rest} \; 0 \end{bmatrix} = \begin{bmatrix} \bar{Y}_{11} & \bar{Y}_{12} \\ \bar{Y}_{21} & \bar{Y}_{22} \end{bmatrix} \begin{bmatrix} \bar{V}_{gen} \\ \bar{V}_{rest} \end{bmatrix}$$

$$\bar{Y}_{21}\bar{V}_{gen} + \bar{Y}_{22}\bar{V}_{rest} = 0$$

$$\begin{bmatrix} \bar{V}_3 \\ \bar{V}_4 \end{bmatrix} = \bar{V}_{rest} = -\bar{Y}_{22}^{-1}\bar{Y}_{21}\bar{V}_{gen}$$

$\bar{V}_3 = \bar{V}_3$ (original)

$\bar{V}_4 = \bar{V}_2$ (original)

$$\bar{V}_{gen} = \begin{bmatrix} 1\angle 0 \\ E_2' \angle \gamma_2 \end{bmatrix}$$ state variables

$\Downarrow$

$P_{23} = ?$

V_3 will be fn (or in matrix form)

$P_{23} = ?$

$V_3 = ?$

Type 2:

$$\underline{x} = \begin{bmatrix} \theta_2 \\ \omega_2 \\ E'_{q_2} \\ E'_{d_2} \\ P_{CP_2} \\ x_{TCSC} \\ B_{SVC} \end{bmatrix}$$

$P_{23} = ?$

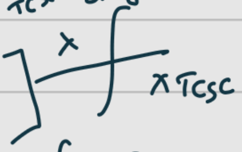
$V_3 = ?$

Type 1:

$$\underline{x} = \begin{bmatrix} \\ \\ \\ \end{bmatrix} \qquad \underline{y} = \begin{bmatrix} \delta_2 \\ \delta_3 \\ V_2 \\ V_3 \end{bmatrix}$$

directly given as State Variables

TCSC block diagram



from TCSC

$$T_2 \dot{x} = -x + k\left(P_{LP} + T_1 \dot{P}_{LP}\right)$$

$$V \longrightarrow \boxed{\frac{1}{s^2 + a_1 s + a_2}} \longrightarrow P_L^D$$

$$P_L^D = \frac{1}{s^2 + a_1 s + a_2} V$$

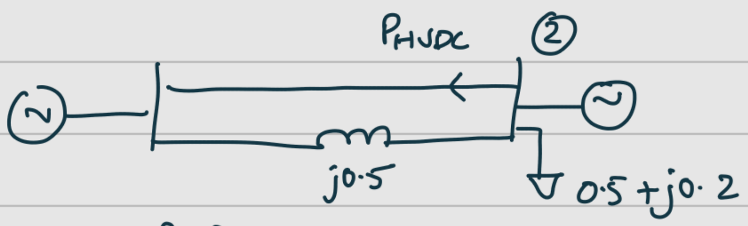
$$s^2 P_L^D + a_1 s P_L^D + a_2 P_L^D = V$$

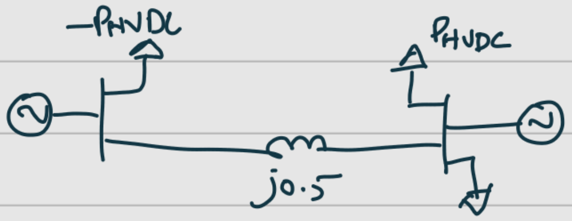
$$\ddot{P}_L^D + a_1 \dot{P}_L^D + a_2 P_L^D = V$$

$$\dot{P}_L^D = \tilde{P}_L^D$$

$$\dot{\tilde{P}}_L^D = -a_1 \tilde{P}_L^D - a_2 P_L^D + V$$

$$\begin{bmatrix} P_L^D \\ \tilde{P}_L^D \end{bmatrix}$$





if type 2 : Const. P_{HVDC} $\rightarrow$ convert to resistor

04/17/24

Review Class

HW Only type 2 & 3

- Use Euler's Integration

$$\dot{x} = f(x)$$

AVR & Gov. included. Parameters from Kundur book.

1^{st} Order AVR

Consider limits

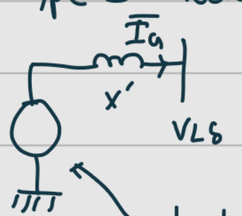
1^{st} Step: Initialization (soln will be posted)

↳ f eqns will be zero

Use numerical differencing

$$\left.\frac{\partial f}{\partial x}\right|_{x^*} = \frac{f(x^* + \Delta x) - f(x^*)}{\Delta x}$$

for type 3 model

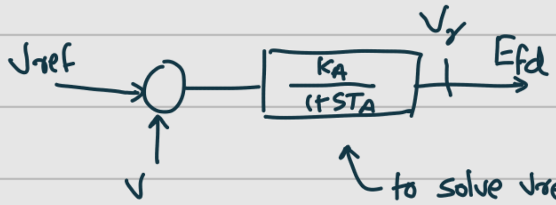


calculate $E'\angle\theta$

→ easy

but for type 2 → solve eqns

Assume V_{ref} - unknown



↳ to solve V_{ref}, use eqn from this block

E_{fd} → from E_q', E_d'

$E_{fd} = 1.5$ $\quad$ $E_q' = 1.3$ $\quad$ θ = slightly above δ

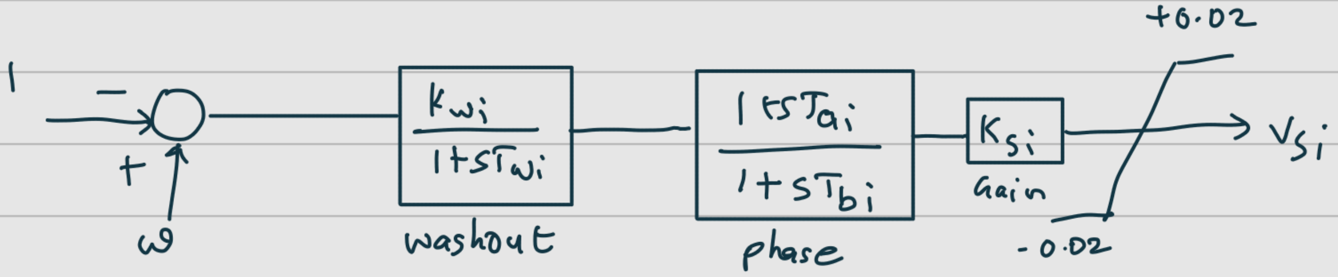
$V_{ref} = V$ $\quad$ E_d' 0.2, 0.3

Set up either NR or fsolve.
check with soln.

In $Q_3 \longrightarrow$ Type 2 (not type 1 & 3)

PSS – simple phase lead (Trial & error) Gain & lead value

04/24/24



Kundur book

Bode Plot compensator design (control system) phase lead

$T_{wi} = 20$ sec (assume)

Tune T_{ai}, T_{bi}, K_{si} to improve damping (Trial & error)