EE 523 Homework - 2

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1 (a) Given $P_{L_i} = P_{L_i}^D + P_{L_i}^S$; 100 MW

$Q_{L_i} = Q_{L_i}^D + Q_{L_i}^S$; 50 MVAR

$$V_i \longrightarrow \boxed{0.5 \ \frac{1}{1+5s}} \longrightarrow P_{L_i}^D$$

$$P_{L_i}^S = 0.5\, V_i^{0.7}$$

$$Q_{L_i}^S = 0.8\, V_i^{0.6}$$

$$V_i \longrightarrow \boxed{0.2 \ \frac{1}{1+s \cdot 2}} \longrightarrow Q_{L_i}^D$$

for $t < 10$ sec

$V = 1$ pu ; $P_{L_i}^S = 0.5(1)^{0.7} = 0.5$ pu ; $P_{L_i}^D = (1)(0.5) = 0.5$ pu

$Q_{L_i}^S = 0.8(1)^{0.6} = 0.8$ pu ; $Q_{L_i}^D = (1)(0.2) = 0.2$ pu

at $t = 10^+$ sec

$V = 0.9$ p.u. $P_{L_i}^S = 0.5(0.9)^{0.7} = 0.464$ pu

$Q_{L_i}^S = 0.8(0.9)^{0.6} = 0.751$ pu

Steady state values of $P_{L_i}^D$ & $Q_{L_i}^D$

$P_{L_i}^D = (0.9)(0.5) = 0.45$ pu

$Q_{L_i}^D = (0.9)(0.2) = 0.18$ pu

at $t = 20^+$ sec

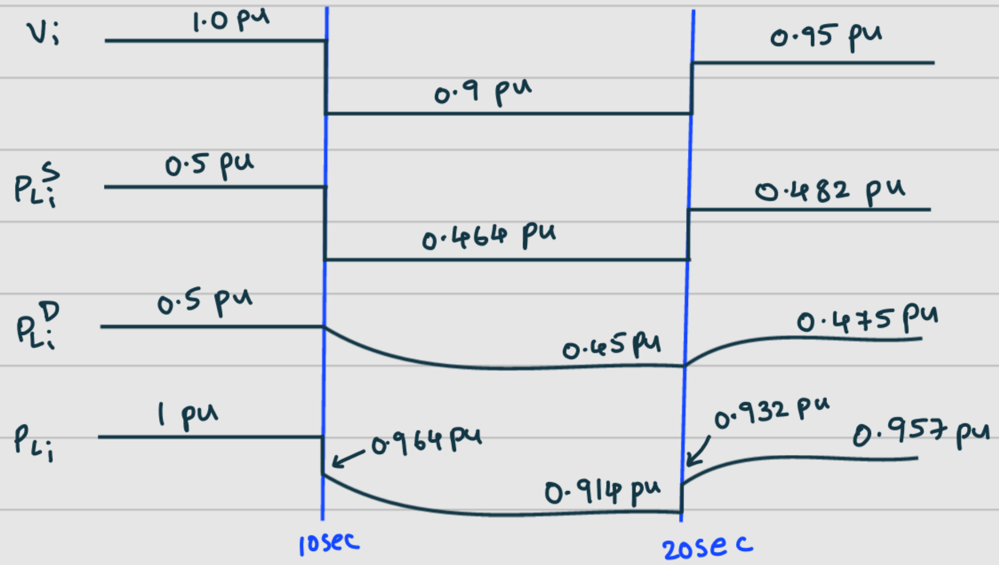
$V = 0.95$ p.u. $P_{L_i}^S = 0.5(0.95)^{0.7} = 0.482$ pu

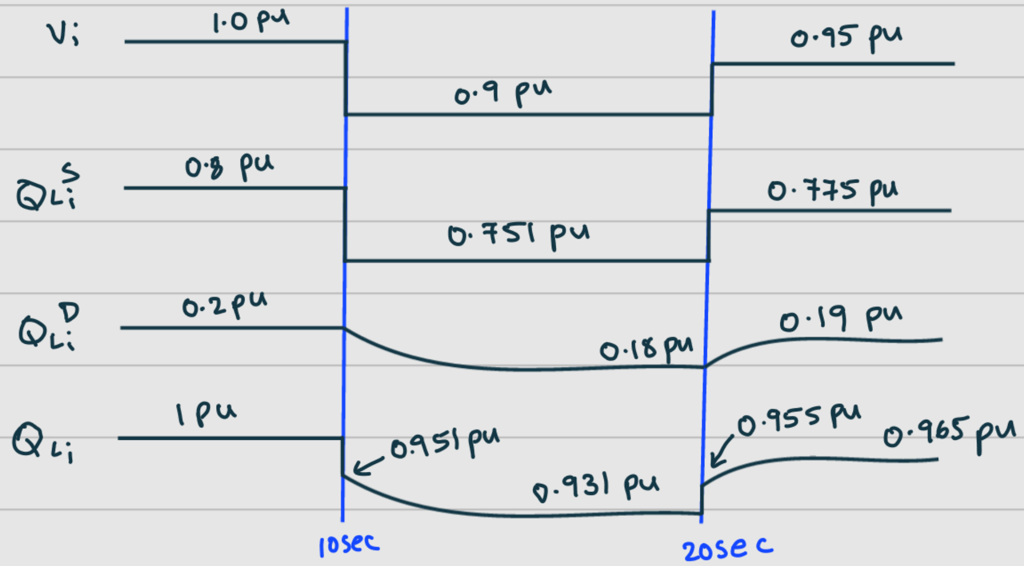
$Q_{L_i}^S = 0.8(0.95)^{0.6} = 0.775$ pu

Steady state values of $P_{L_i}^D$ & $Q_{L_i}^D$

$P_{L_i}^D = (0.95)(0.5) = 0.475$ pu

$Q_{L_i}^D = (0.95)(0.2) = 0.19$ pu





1 b Given

$$V_i \longrightarrow \boxed{0.5 \dfrac{1}{1+0.002s+0.025s^2}} \longrightarrow P_{Li}^{D}$$

$$V_i \longrightarrow \boxed{0.2 \dfrac{1}{1+0.002s+0.05s^2}} \longrightarrow Q_{Li}^{D}$$

Consider CE for P_{Li}^{D} $\quad 1+0.002s+0.025s^2=0$

$\div 0.025 \Rightarrow 40+0.08s+s^2=0$

Compare with $\quad \omega_n^2+2\xi\omega_n s+s^2=0$

$\omega_n^2=40$

$\underline{\underline{\omega_n=\sqrt{40}}}$

$2\xi\omega_n=0.08$

$\xi=\dfrac{0.08}{2\omega_n}=\dfrac{0.08}{2\sqrt{40}}=0.0063$

$\xi<1 \Rightarrow$ underdamped system

$M_p=e^{-\pi\xi/\sqrt{1-\xi^2}}=e^{-\pi 0.0063/\sqrt{1-0.0063^2}}$

$M_p=98.04\%$

$T_s=\dfrac{4}{\xi\omega_n}=\dfrac{4}{0.0063\sqrt{40}}=\underline{\underline{100\text{ sec}}}$

$T_p=\dfrac{4}{\omega_n\sqrt{1-\xi^2}}=\dfrac{4}{\sqrt{40}\sqrt{1-0.0063^2}}=\underline{\underline{0.632\text{ sec}}}$

Consider CE for $Q_{L_i}^D$ $\quad 1+0.002s+0.05s^2=0$

$\div 0.05 \Rightarrow 20+0.04s+s^2=0$

Compare with $\quad \omega_n^2+2\xi\omega_n s+s^2=0$

$\omega_n^2=20 \quad \Big| \quad 2\xi\omega_n=0.04$

$\underline{\underline{\omega_n=\sqrt{20}}} \quad \Big| \quad \xi=\dfrac{0.04}{2\omega_n}=\dfrac{0.04}{2\sqrt{20}}=0.0044$

$\xi<1 \Rightarrow$ underdamped system

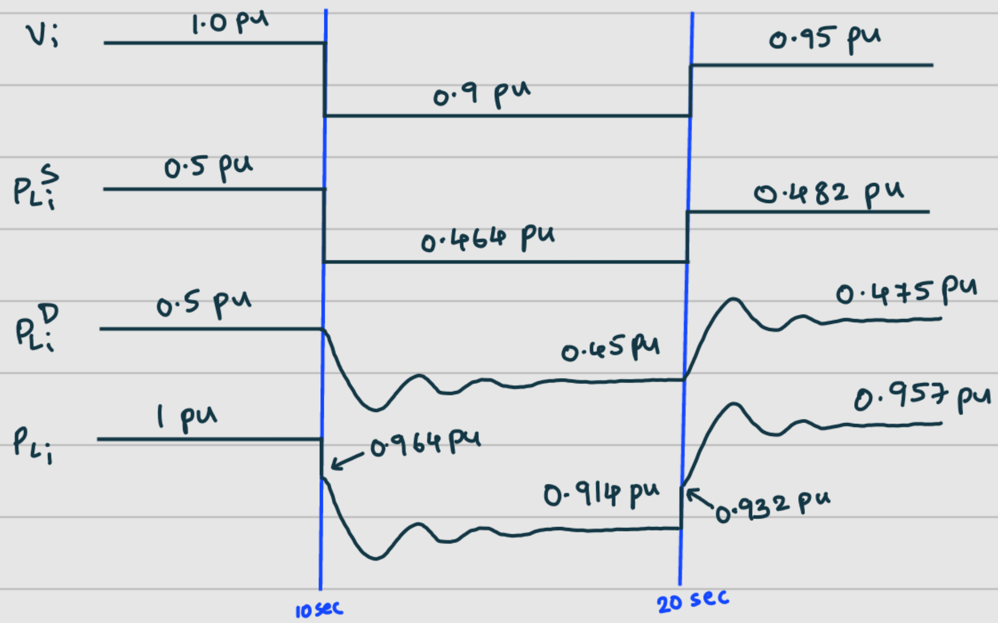
$M_p=e^{-\pi\xi/\sqrt{1-\xi^2}}=e^{-\pi 0.0044/\sqrt{1-0.0044^2}}$

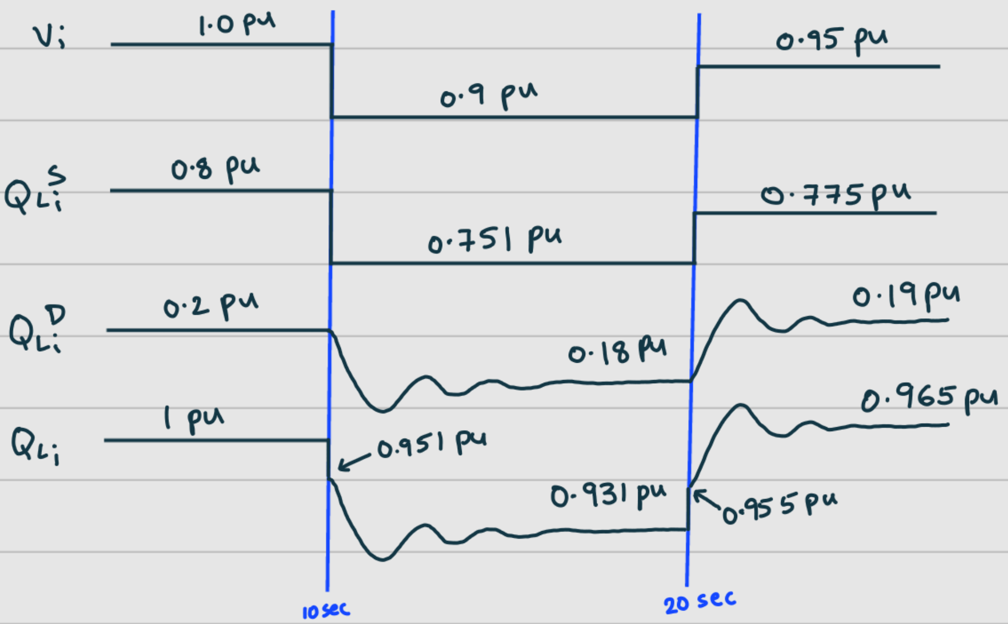
$\underline{\underline{M_p=98.62\ \%}}$

$T_s=\dfrac{4}{\xi\omega_n}=\dfrac{4}{0.0044\sqrt{20}}=\underline{\underline{203\ \text{sec}}}$

$T_p=\dfrac{4}{\omega_n\sqrt{1-\xi^2}}=\dfrac{4}{\sqrt{20}\sqrt{1-0.0044^2}}=\underline{\underline{0.894\text{sec}}}$

Assuming the response settles down before next disturbance





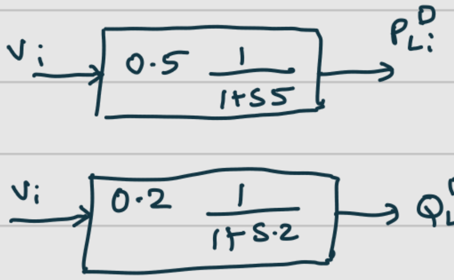
1 c a

Given $P_{L_i} = P_{L_i}^{D} + P_{L_i}^{S}$; 100 MW

$Q_{L_i} = Q_{L_i}^{D} + Q_{L_i}^{S}$; 50 MVAR

$P_{L_i}^{S} = 0.5 V_i^{0.7}$

$Q_{L_i}^{S} = 0.8 V_i^{0.6}$



for $t < 10$ sec

$V = 1$ pu ; $P_{L_i}^{S} = 0.5(1)^{0.7} = 0.5$ pu ; $P_{L_i}^{D} = (1)(0.5) = 0.5$ pu

$Q_{L_i}^{S} = 0.8(1)^{0.6} = 0.8$ pu ; $Q_{L_i}^{D} = (1)(0.2) = 0.2$ pu

at $t = 10^{+}$ sec

$V = 1.05$ p.u. $P_{L_i}^{S} = 0.5(1.05)^{0.7} = 0.517$ pu

$Q_{L_i}^{S} = 0.8(1.05)^{0.6} = 0.824$ pu

Steady state values of $P_{L_i}^{D}$ & $Q_{L_i}^{D}$

$P_{L_i}^{D} = (1.05)(0.5) = 0.525$ pu

$Q_{L_i}^{D} = (1.05)(0.2) = 0.21$ pu

at $t = 20^{+}$ sec

$V = 0.95$ p.u. $P_{L_i}^{S} = 0.5(0.95)^{0.7} = 0.482$ pu

$Q_{L_i}^{S} = 0.8(0.95)^{0.6} = 0.775$ pu

Steady state values of $P_{L_i}^{D}$ & $Q_{L_i}^{D}$

$P_{L_i}^{D} = (0.95)(0.5) = 0.475$ pu

$Q_{L_i}^{D} = (0.95)(0.2) = 0.19$ pu

at $t = 30^{+}$ sec

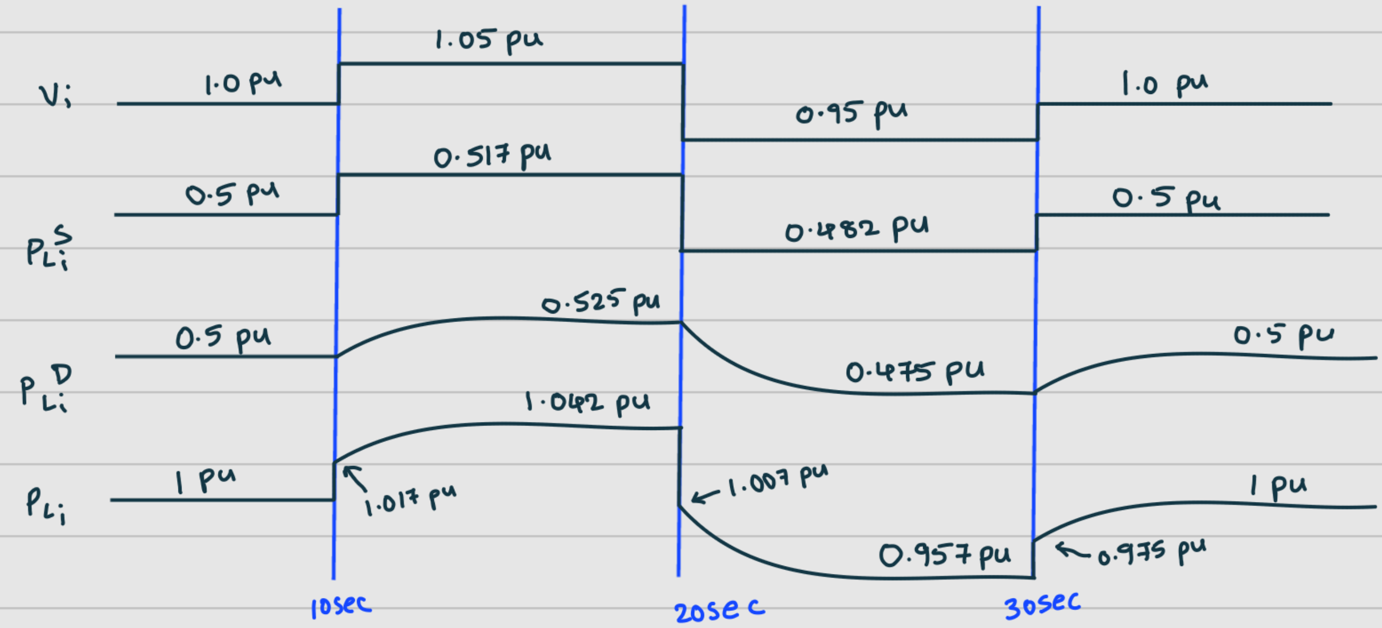
$V = 1$ pu ; $P_{L_i}^{S} = 0.5(1)^{0.7} = 0.5$ pu

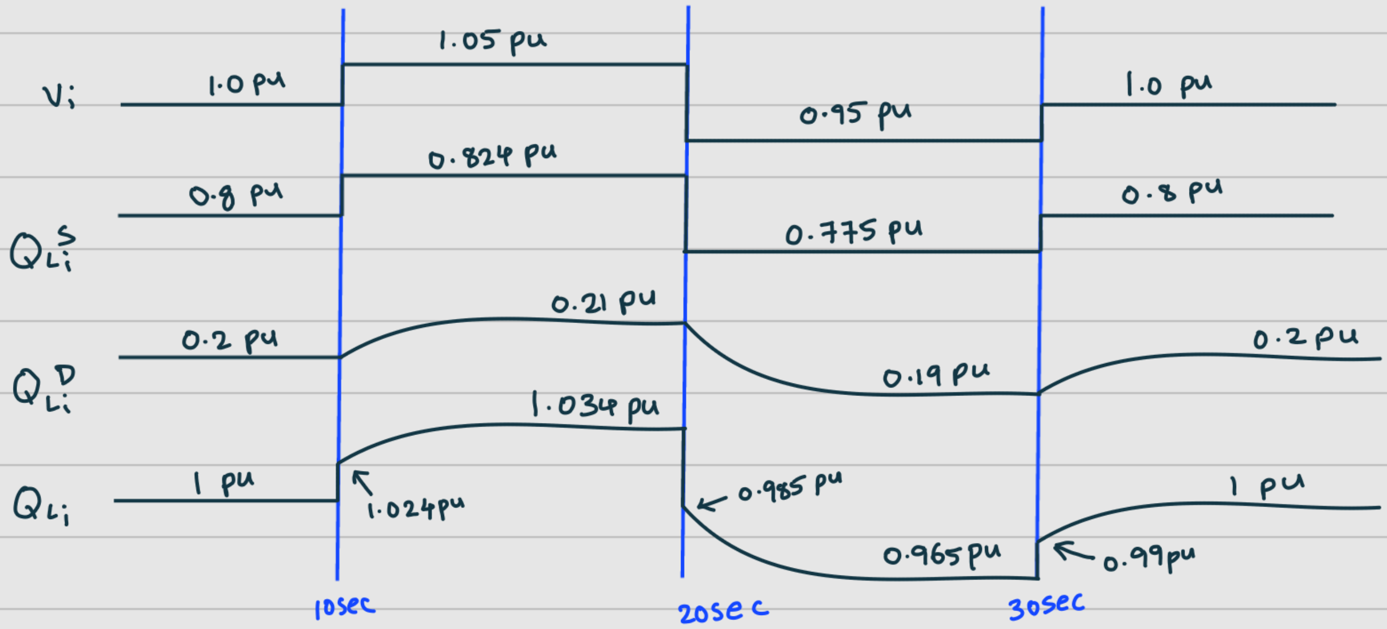
$Q_{L_i}^{S} = 0.8(1)^{0.6} = 0.8$ pu

Steady state values of $P_{L_i}^{D}$ & $Q_{L_i}^{D}$

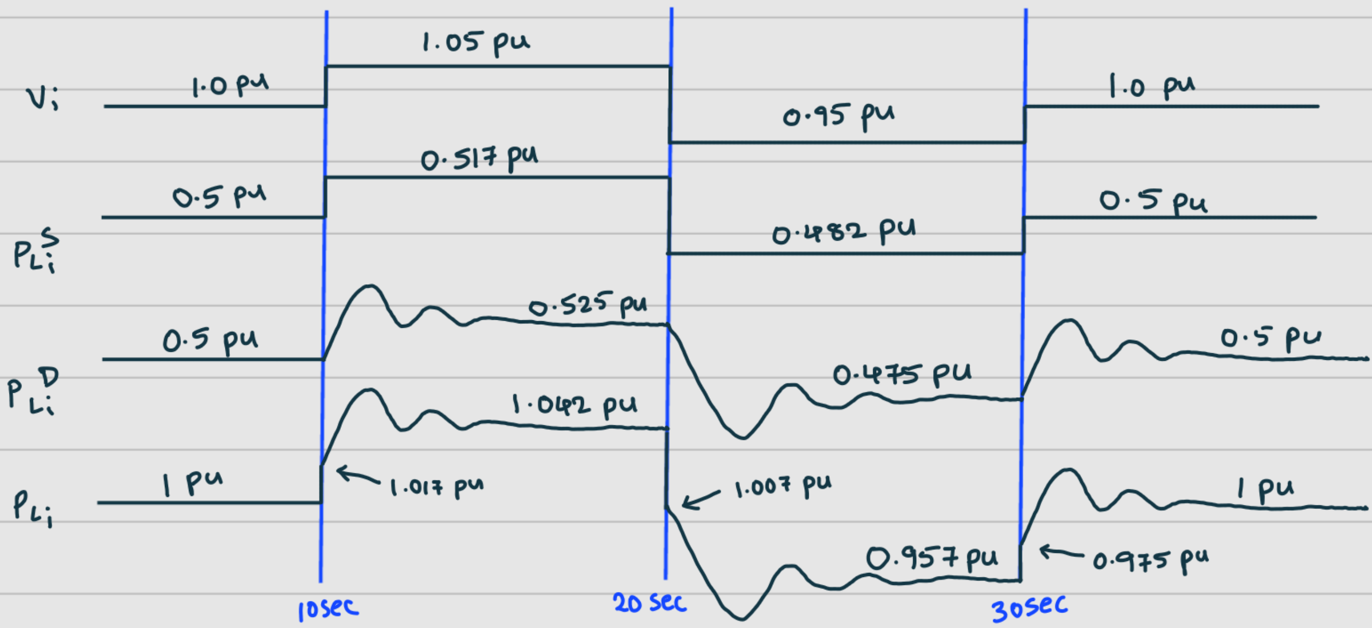
$P_{L_i}^{D} = (1)(0.5) = 0.5$ pu

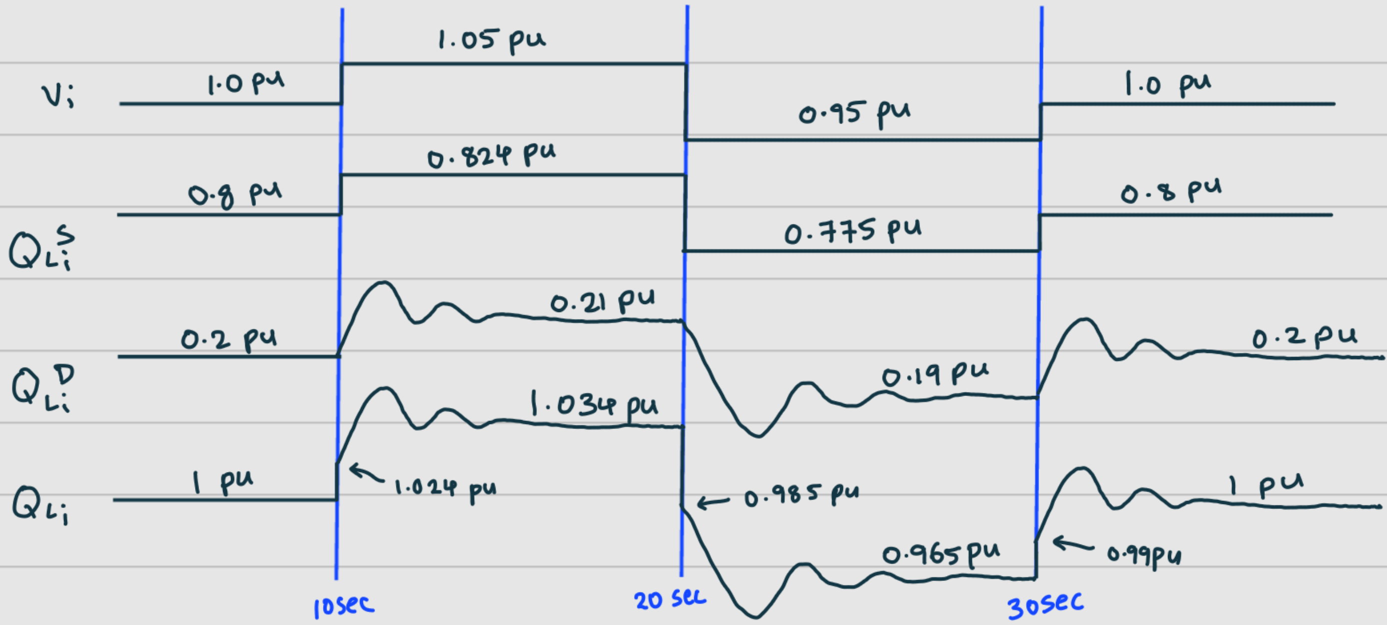
$Q_{L_i}^{D} = (1)(0.2) = 0.2$ pu





1 c b the response follows same values as 1 c a however a second order response is observed.





1 d a Given $P_L^S = 0.2 + 0.2V + 0.1V^2$ $\qquad V_i \rightarrow \boxed{0.5 \dfrac{1}{1+s5}} \rightarrow P_L^D \qquad V_i \rightarrow \boxed{0.2 \dfrac{1}{1+s2}} \rightarrow Q_L^D$

$Q_L^S = 0.1V + 0.1V^2$

for $t < 10$ sec

$V = 1$ pu ; $P_{L_i}^S = 0.2 + 0.2(1) + 0.1(1)^2 = 0.5$ pu

$Q_{L_i}^S = 0.1(1) + 0.1(1)^2 = 0.2$ pu

$P_{L_i}^D = (1)(0.5) = 0.5$ pu

$Q_{L_i}^D = (1)(0.2) = 0.2$ pu

at $t = 10^+$ sec

$V = 0.9$ p.u. $\quad P_{L_i}^S = 0.2 + 0.2(0.9) + 0.1(0.9)^2 = 0.461$ pu

$Q_{L_i}^S = 0.1(0.9) + 0.1(0.9)^2 = 0.171$ pu

Steady state values of $P_{L_i}^D$ & $Q_{L_i}^D$

$P_{L_i}^D = (0.9)(0.5) = 0.45$ pu

$Q_{L_i}^D = (0.9)(0.2) = 0.18$ pu

at $t = 20^+$ sec

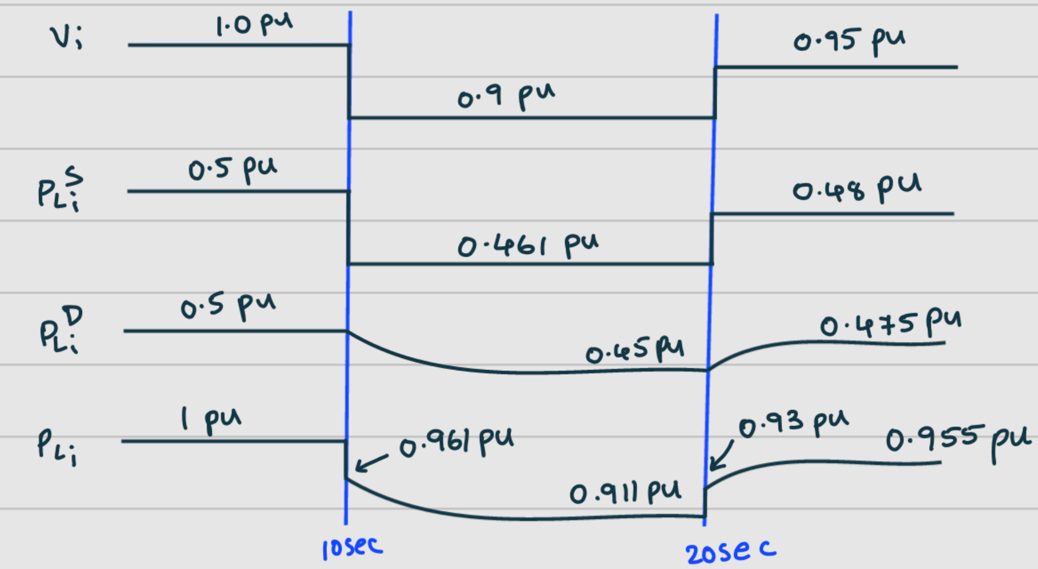
$V = 0.95$ p.u. $\quad P_{L_i}^S = 0.2 + 0.2(0.95) + 0.1(0.95)^2 = 0.48$ pu

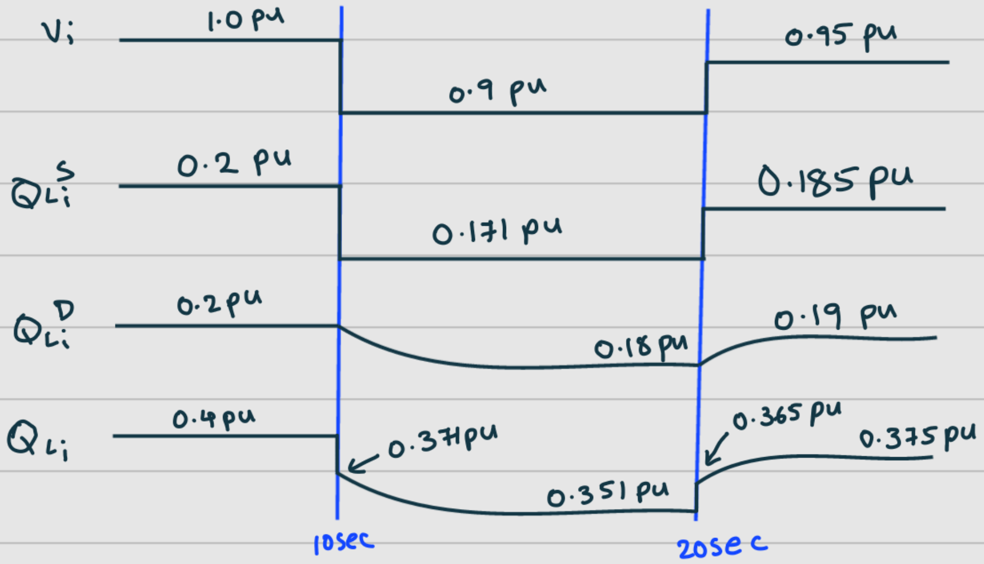
$Q_{L_i}^S = 0.1(0.95) + 0.1(0.9)^2 = 0.185$ pu

Steady state values of $P_{L_i}^D$ & $Q_{L_i}^D$

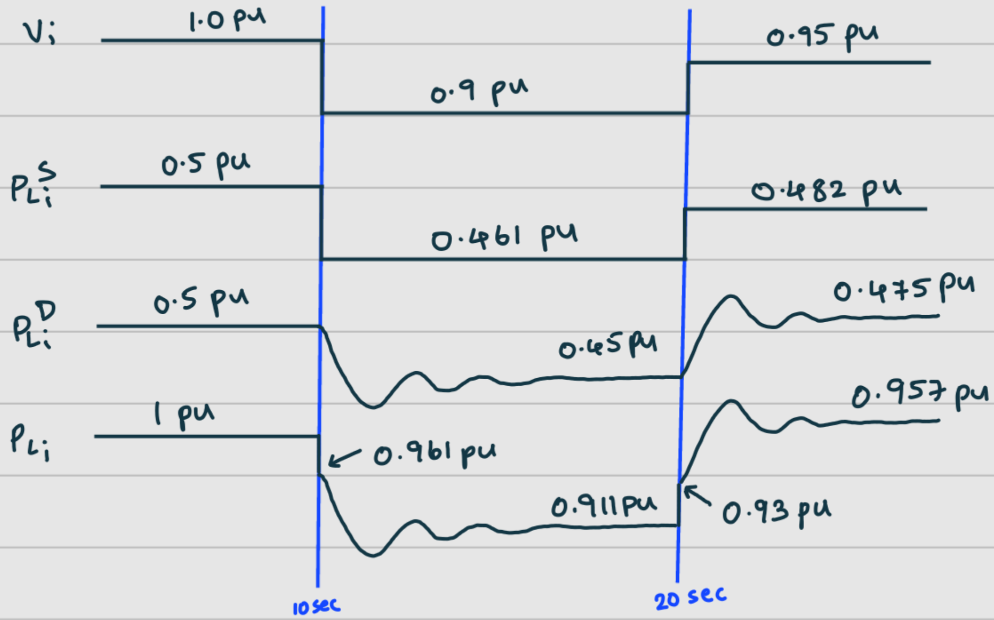
$P_{L_i}^D = (0.95)(0.5) = 0.475$ pu

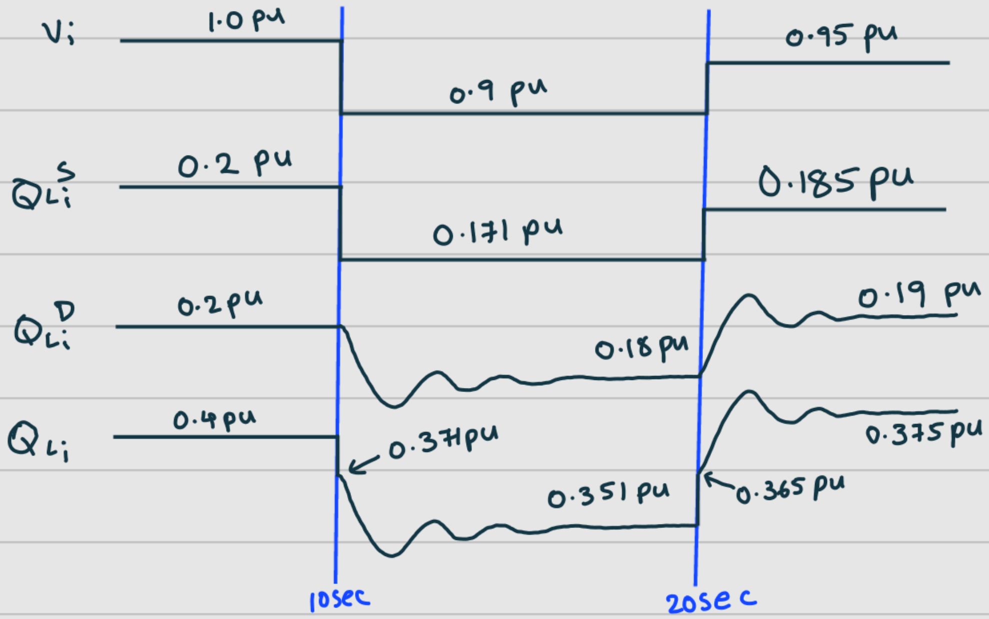
$Q_{L_i}^D = (0.95)(0.2) = 0.19$ pu





1 d b the response follows same values as 1 d a however, a second order response is observed.





1 d c a Given $P_L^S = 0.2 + 0.2V + 0.1V^2$

$Q_L^S = 0.1V + 0.1V^2$

$V_i \rightarrow \boxed{0.5 \cdot \frac{1}{1+s5}} \rightarrow P_L^D$

$V_i \rightarrow \boxed{0.2 \cdot \frac{1}{1+s2}} \rightarrow Q_L^D$

for $t < 10$ sec

$V = 1\,pu\,;\; P_{L_i}^S = 0.2 + 0.2(1) + 0.1(1)^2 = 0.5\,pu$

$Q_{L_i}^S = 0.1(1) + 0.1(1)^2 = 0.2\,pu$

$P_{L_i}^D = (1)(0.5) = 0.5\,pu$

$Q_{L_i}^D = (1)(0.2) = 0.2\,pu$

at $t = 10^+$ sec

$V = 1.05\,p.u.\quad P_{L_i}^S = 0.2 + 0.2(1.05) + 0.1(1.05)^2 = 0.52\,pu$

$Q_{L_i}^S = 0.1(1.05) + 0.1(1.05)^2 = 0.215\,pu$

Steady state values of $P_{L_i}^D$ & $Q_{L_i}^D$

$P_{L_i}^D = (1.05)(0.5) = 0.525\,pu$

$Q_{L_i}^D = (1.05)(0.2) = 0.21\,pu$

at $t = 20^+$ sec

$V = 0.95\,p.u.\quad P_{L_i}^S = 0.2 + 0.2(0.95) + 0.1(0.95)^2 = 0.48\,pu$

$Q_{L_i}^S = 0.1(0.95) + 0.1(0.9)^2 = 0.185\,pu$

Steady state values of $P_{L_i}^D$ & $Q_{L_i}^D$

$$P_{L_i}^D = (0.95)(0.5) = 0.475\,pu$$

$$Q_{L_i}^D = (0.95)(0.2) = 0.19\,pu$$

at $t = 30^+$ sec

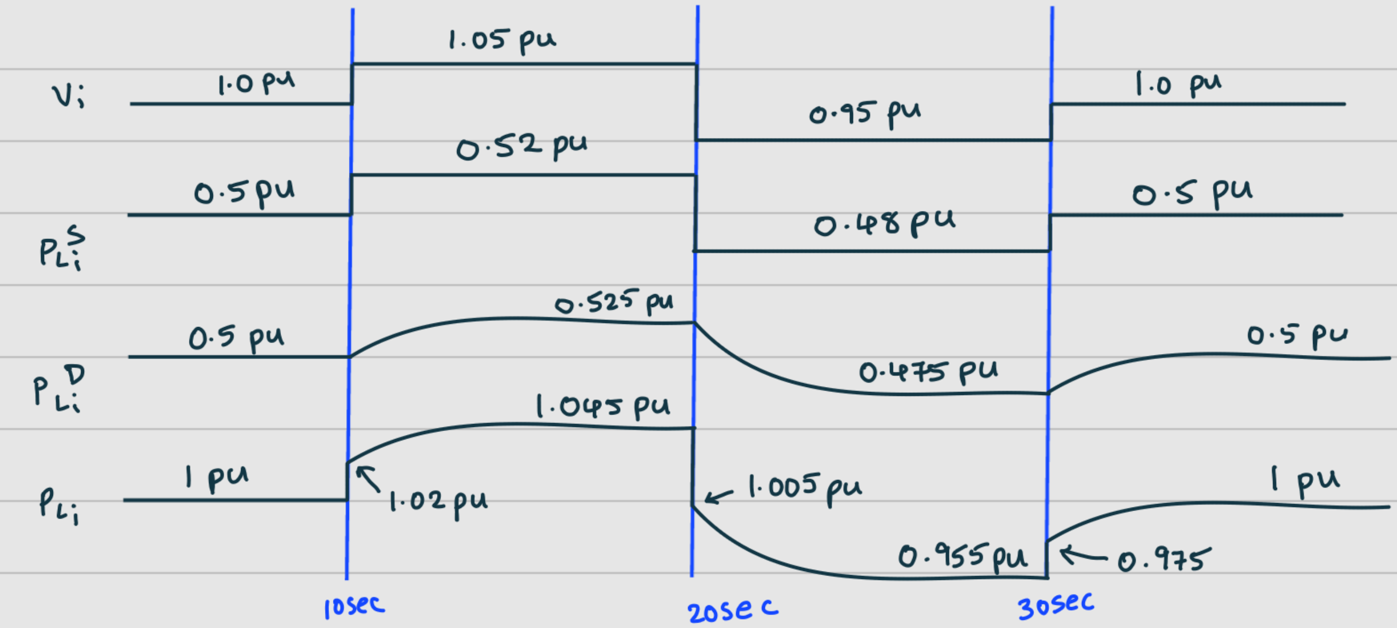
$V = 1\,pu$; $P_{L_i}^S = 0.2 + 0.2(1) + 0.1(1)^2 = 0.5\,pu$

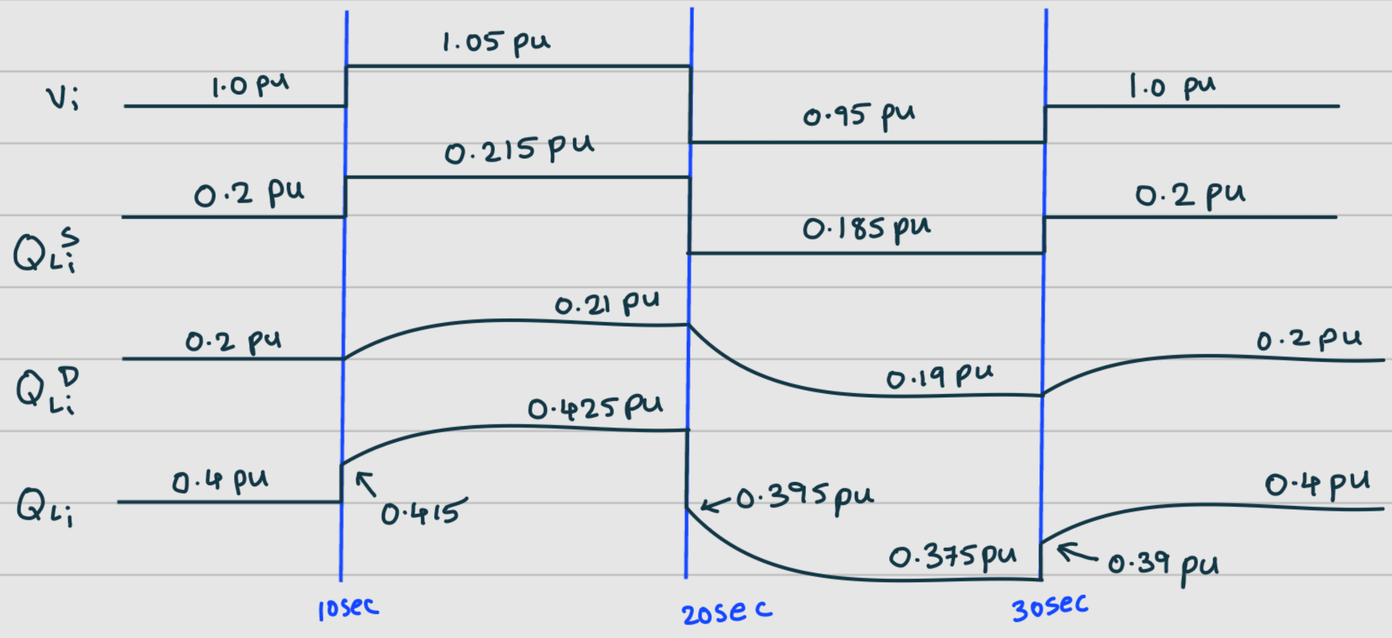
$$Q_{L_i}^S = 0.1(1) + 0.1(1)^2 = 0.2\,pu$$

Steady state values of $P_{L_i}^D$ & $Q_{L_i}^D$

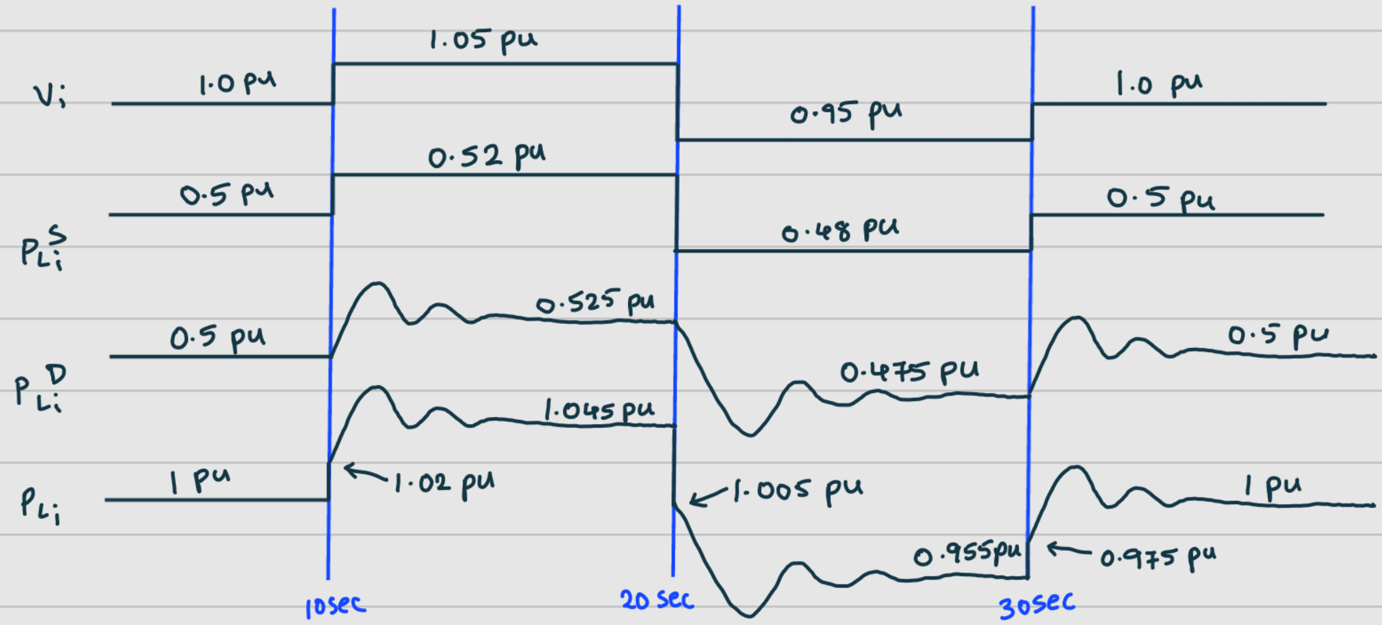
$$P_{L_i}^D = (1)(0.5) = 0.5\,pu$$

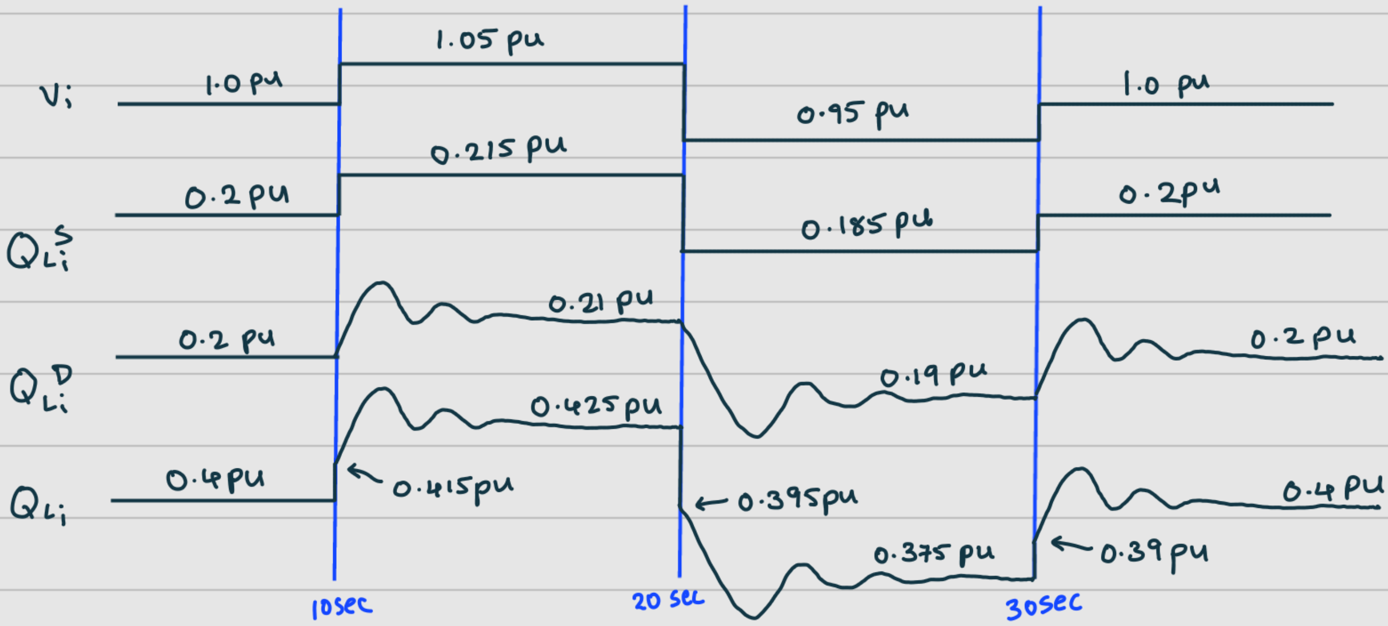
$$Q_{L_i}^D = (1)(0.2) = 0.2\,pu$$





1 d c b the response follows same values as 1 d c a however, a second order response is observed.





2 Assume first order ZIP model

$$P_L^S = P_{L0} + M_p V + G_p V^2 ; \quad P_L^D = \frac{K_p^D}{1+sb_p} V$$

At $t = 5^-$ sec

$V = 1$ pu ; $P_L = 0.9$ pu

$$P_{L0} + M_p + G_p + K_p^D = 0.9 \quad \text{— (1a)}$$

At $t = 5^+$ sec

$V = 0.9$ pu ; $P_L = 0.842$ pu

$$P_{L0} + 0.9M_p + 0.81G_p + K_p^D = 0.842 \quad \text{(2a)}$$

At $t = 20^-$ sec

$V = 0.9$ pu ; $P_L = 0.792$

$$P_{L0} + 0.9M_p + 0.81G_p + 0.9K_p^D = 0.792 \quad \text{— (3a)}$$

(2a) − (3a) $\Rightarrow$ $0.1 K_p^D = 0.05$

$$\underline{\underline{K_p^D = 0.5}}$$

At $t = 20^+$ sec

$V = 1.1$ pu ; $P_L = 0.912$ pu

$$P_{L0} + 1.1M_p + 1.21G_p + 0.9K_p^D = 0.912 \quad \text{— (4a)}$$

(1a) − (2a) $\Rightarrow$ $0.1M_p + 0.19G_p = 0.058$ — (5a)

(4a) − (3a) $\Rightarrow$ $0.2M_p + 0.4G_p = 0.12$ — (6a)

$2 \times$ (5a) $\Rightarrow$ $0.2M_p + 0.38G_p = 0.116$ — (7a)

(6a) − (7a) $\Rightarrow$ $0.02G_p = 0.004$

$$\underline{\underline{G_p = 0.2}}$$

$G_p = 0.2$ in (5a)

$$0.1M_p + 0.19(0.2) = 0.058$$

; $$Q_L^S = Q_{L0} + M_Q V + G_Q V^2 ; \quad Q_L^D = \frac{K_Q}{1+sb_Q} V$$

At $t = 5^-$ sec

$V = 1$ pu ; $Q_L = 0.3$ pu

$$Q_{L0} + M_Q + G_Q + K_Q^D = 0.3 \quad \text{— (1b)}$$

At $t = 5^+$ sec

$V = 0.9$ pu ; $Q_L = 0.29$ pu

$$Q_{L0} + 0.9M_Q + 0.81G_Q + K_Q^D = 0.29 \text{ pu} \quad \text{— (2b)}$$

At $t = 20^-$ sec

$V = 0.9$ pu ; $Q_L = 0.27$

$$Q_{L0} + 0.9M_Q + 0.81G_Q + 0.9K_Q^D = 0.27 \quad \text{— (3b)}$$

(2b) − (3b) $\Rightarrow$ $0.1 K_Q^D = 0.02$

$$\underline{\underline{K_Q^D = 0.2}}$$

At $t = 20^+$ sec

$V = 1.1$ pu $\quad Q_L = 0.29$ pu

$$Q_{L0} + 1.1M_Q + 1.21G_Q^2 + 0.9K_Q^D = 0.29 \quad \text{— (4b)}$$

(1b) − (2b) $\Rightarrow$ $0.1M_Q + 0.19G_Q = 0.01$ — (5b)

(4b) − (3b) $\Rightarrow$ $0.2M_Q + 0.4G_Q = 0.02$ — (6b)

$2 \times$ (5b) $\Rightarrow$ $0.2M_Q + 0.38G_Q = 0.02$ — (7b)

(6b) − (7b) $\Rightarrow$ $0.02G_Q = 0$

$$\underline{\underline{G_Q = 0}}$$

$G_Q = 0$ in (5b)

$$0.1M_Q = 0.01$$

$$0.1\, M_P = 0.02$$
$$M_P = 0.2$$

$M_P = 0.2$, $G_P = 0.2$, $K_P^D = 0.5$ in (1a)

$$P_{L0} + 0.2 + 0.2 + 0.5 = 0.9$$
$$P_{L0} = 0$$

$$P_L^S = 0.2V + 0.2V^2$$

From graph, $b_P = 24 - 20 = 4$ sec

$$P_L^D = \frac{0.5}{1+S4} V$$

$$M_Q = 0.1$$

$M_Q = 0.1$, $G_Q = 0$, $K_Q^D = 0.2$ in (1b)

$$Q_{L0} + 0.1 + 0 + 0.2 = 0.3$$
$$Q_{L0} = 0$$

$$Q_L^S = 0.1V$$

From graph, $b_Q = 22 - 2 = 2$ sec

$$Q_L^D = \frac{0.2}{1+S.2} V$$