EE493 Protection of Power Systems I

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slides 4-14 are from section 7.3 of "Power System Analysis and Design", Fifth Edition, 2012, J. D. Glover, M. S. Saema, T. J. Overbye

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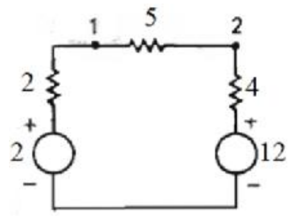
Calculating Z bus matrix



$$\frac{e_1-2}{2}+\frac{e_1-e_2}{5}=0$$

$$\frac{e_2-10}{4}+\frac{e_2-e_1}{5}=0$$

$$(\frac{1}{2}+\frac{1}{5})e_1+(\frac{-1}{5})e_2=1$$

$$(\frac{-1}{5})e_1+(\frac{1}{4}+\frac{1}{5})e_2=3$$

$$\begin{bmatrix} \frac{1}{2}+\frac{1}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{1}{4}+\frac{1}{5} \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}=\begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$Y_{bus}V_{bus}=I_{bus}$$

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The bus admittance matrix Y is formed by inspection as follows (this is similar to what we learned in circuit theory):

$$y_{ii} = \text{sum of admittances connected to node } i$$

$$y_{ij} = y_{ji} = -\text{sum of admittances connected from node } i \text{ to node } j$$

$$Y_{bus} V_{bus} = I_{bus}$$

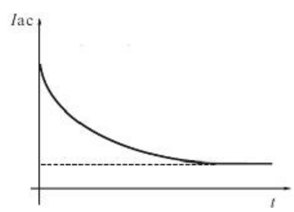
$$Y_{bus}^{-1} I_{bus} = V_{bus}$$

$$Z_{bus} = Y_{bus}^{-1}$$

$$Z_{bus} I_{bus} = V_{bus}$$

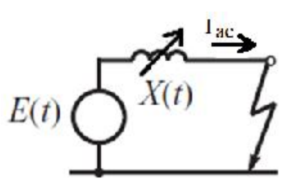
Modeling synchronous generators as a source behind a series impedance

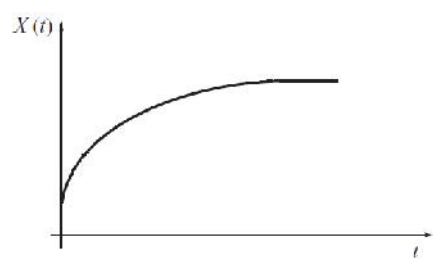
If a fault occurs at the terminal of a synchronous generator the amplitude of fundamental frequency short circuit current looks like the following



To generate such a short-circuit current using a constant voltage behind an impedance, the impedance should be variable impedance as follows

$$I_{ac} = \frac{E(t)}{X(t)} \rightarrow X(t) = E(t) \times \frac{1}{I_{ac}}$$





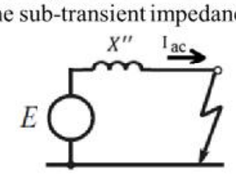
The short circuit current can be divided into several time intervals as shown in (a).

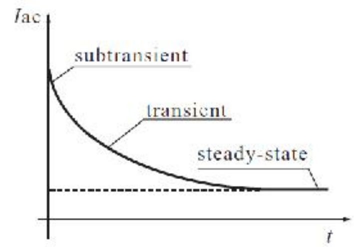
If we assume E(t) is a constant value equal to E, the impedance of the generator should look like (b) because $X(t) = \frac{E}{I_{ac}}$

impedance is also divided into several time intervals as shown in (b), $X'' < X' < X$.

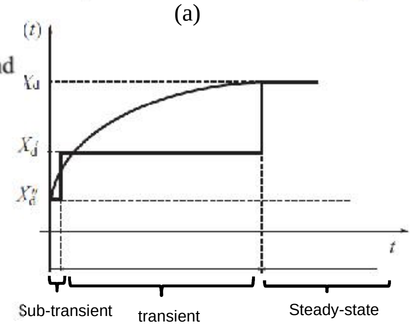
After a fault occurs, the sub-transient, transient, and steady-state, periods are characterized by the sub-transient reactance X'' , the transient reactance X', and the steady-state reactance X, respectively.

In power system protection analysis we are interested in very short period of time just after fault, because the protective relays will operate as fast as possible and disconnect the generators. Therefore, the synchronous generator is modeled as follows, where X'' is the sub-transient impedance of the generator





(a)



(b)

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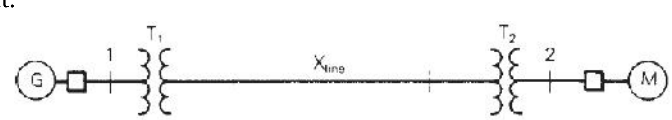
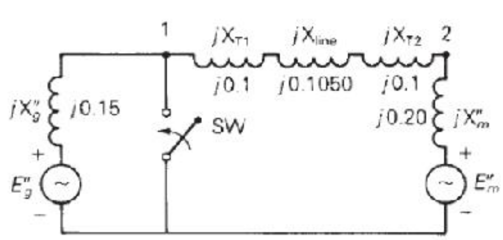
Symmetrical faults

lets start with an example:

The following figure shows a single-line diagram consisting of a synchronous generator feeding a synchronous motor through two transformers and a transmission line. A three-phase short circuit occurs at bus 1. The synchronous generator feeding a synchronous motor are modeled by constant voltages behind series impedances. The per-unit equivalent network is also shown where the provided values are in per-unit. The closing of switch SW represents the fault.





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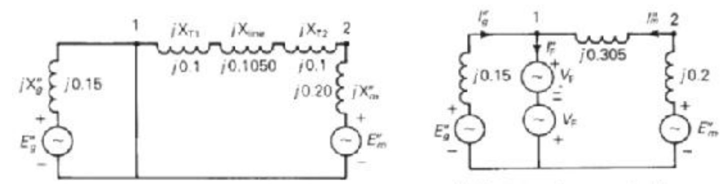
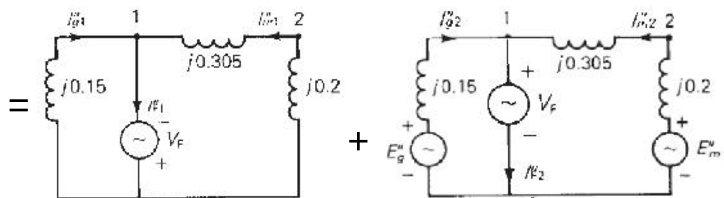
the fault is represented by two opposing voltage sources with equal phasor values VF in (b), which means (Vf)+(-vf)=0 to be equivalent to a short circuit fault. Vf is the voltage at the point of fault before fault occurrence. In Figure (b), Vf is equal to voltage of bus 1 when there was no fault



(a) Three-phase short circuit

(b) Short circuit represented by two opposing voltage sources

According superposition rule, (b) can be represented as following

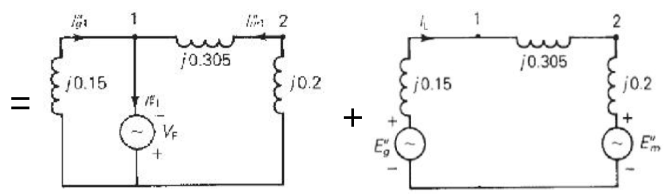


c

d

(c) Application of superposition

As VF equals the prefault voltage at the fault location, Figure (d) represents the system before the fault occurs. In (d), I_{F2}''=0 because we assume Vf is equal to voltage of point 1 therefore there is no voltage difference between point 1 and the source (Vf). Therefore, VF can be removed from the (d). As a results, Figures (c) and (d) can be represented as follows:



Circuit 1

Circuit 2

To study the faulted network (a), circuit s1 and 2 are studied separately. It means to calculate fault current/voltage at each location of the network (a), fist we should cancel all the sources of the network (i.e. make the voltage sources equal to zero) and only keep the Vf at fault point and solve the network (i.e. circuit 1). The value of the Vf is the voltage value before fault at point 1 which can be calculated by solving the pre-fault network. Next, we should solve circuit 2 which means we should keep all sources and ignore the fault (Circuit 2). Then to get the final result, we should add the results of circuit 1 and 2

Note: A systematic approach for solving Circuit 1

In a power system with N number of buses ZI=V,

$$\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} & \cdots & Z_{2N} \\ \vdots & & & & & \\ Z_{n1} & Z_{n2} & \cdots & Z_{nn} & \cdots & Z_{nN} \\ \vdots & & & & & \\ Z_{N1} & Z_{N2} & \cdots & Z_{Nn} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \\ \vdots \\ E_N \end{bmatrix}$$

where Z is Z bus matrix, I is current injections and each bus, and V is voltages at each bus of the network. In circuit 1, where all sources are short-circuited and only Vf is applied at fault point, all current injections are zero (because there is no source connected to the buses of the network to inject current) and only current injection at faulted point exists(because there is a voltage source to inject current). Therefore the following holds

$$\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} & \cdots & Z_{2N} \\ \vdots & & & & & \\ Z_{n1} & Z_{n2} & \cdots & Z_{nn} & \cdots & Z_{nN} \\ \vdots & & & & & \\ Z_{N1} & Z_{N2} & \cdots & Z_{Nn} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ -I''_{Fn} \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ -V_F \\ \vdots \\ E_N \end{bmatrix}$$

In above equation, according to nth row, the followings hold

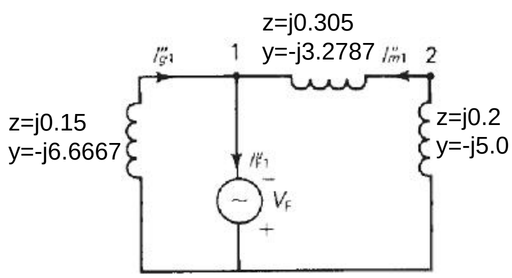
$$I''_{Fn} = \frac{V_F}{Z_{nn}}$$

In above equation, according to kth row, the followings hold

$$E_k = Z_{kn}(-I''_{Fn}) = \frac{-Z_{kn}}{Z_{nn}} V_F$$

Where n is the bus at which fault is applied, and k is any bus of the network.

Now, lets utilize the above systematic approach to solve Circuit 1. In Circuit 1, as shown bellow, n=1, and k=2



$$\boldsymbol{Y}_{bus} = -j\begin{bmatrix} 9.9454 & -3.2787 \\ -3.2787 & 8.2787 \end{bmatrix}$$

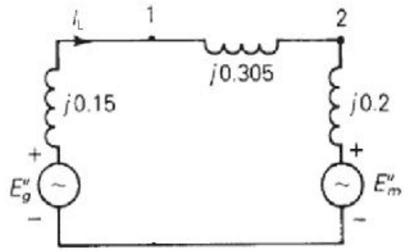
Inverting $\boldsymbol{Y}_{bus}$,

$$\boldsymbol{Z}_{bus} = \boldsymbol{Y}_{bus}^{-1} = +j\begin{bmatrix} 0.11565 & 0.04580 \\ 0.04580 & 0.13893 \end{bmatrix}$$

$Z_{nn}=Z_{11}=j0.11565$ and $Z_{kn}=Z_{21}=j0.04580$

$$E_k^{(1)} = \frac{-Z_{kn}}{Z_{nn}} V_F \rightarrow \quad E_2^{(1)} = \frac{-Z_{21}}{Z_{11}} V_F$$

The superscript (1) denotes the first circuit



The second circuit represents the prefault conditions. Neglecting prefault load current, all voltages throughout the second circuit are equal to the prefault voltage; that is, $E_k^{(2)} = V_F$ for each bus k.

Now, the total value of E_k is calculated by using superposition rule

$$E_k = E_k{}^{(1)} + E_k{}^{(2)} = \frac{-Z_{kn}}{Z_{nn}} V_F + V_F = \left(1 - \frac{Z_{kn}}{Z_{nn}}\right) V_F$$

Therefore, the total voltage of E_2(i.e. the summation of voltage of point 2 at circuit 2 and circuit 2) for k=2 and n=1

$$E_2 \;\; = \left(1 - \frac{Z_{21}}{Z_{11}}\right) V_F$$

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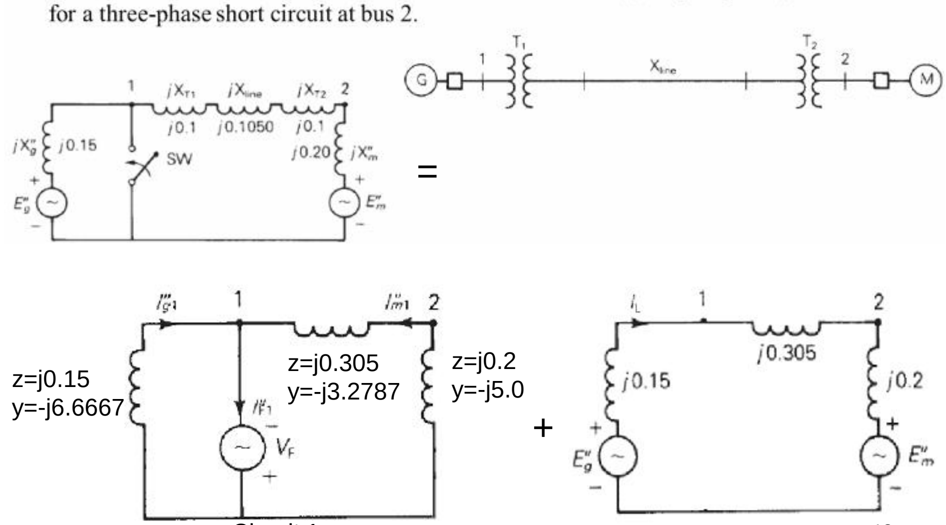
Now, assume prefault voltages are 1.05 per unit which means $E_g^{"} = E_m^{"} = V_f = 1.05$. (a) First determine the 2×2 bus impedance matrix. (b) For a three-phase short circuit at bus 1, use Zbus to calculate the subtransient fault current and the contribution to the fault current from the transmission line. (c) Repeat part (b) for a three-phase short circuit at bus 2.



Circuit 1 Circuit 2

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a.

Neglecting prefault load current, $E_g'' = E_m'' = V_F = 1.05\underline{/0^\circ}$ per unit.
the positive sequence bus admittance matrix is

$$Y_{bus} = -j\begin{bmatrix} 9.9454 & -3.2787 \\ -3.2787 & 8.2787 \end{bmatrix} \text{ per unit}$$

Inverting Y_{bus},

$$Z_{bus} = Y_{bus}^{-1} = +j\begin{bmatrix} 0.11565 & 0.04580 \\ 0.04580 & 0.13893 \end{bmatrix} \text{ per unit}$$

b. $$I_{F1}'' = \frac{V_F}{Z_{11}} = \frac{1.05\underline{/0^\circ}}{j0.11565} = -j9.079 \text{ per unit}$$

The voltages at buses 1 and 2 during the fault are,

$$E_1 = \left(1 - \frac{Z_{11}}{Z_{11}}\right) V_F = 0$$

$$E_2 = \left(1 - \frac{Z_{21}}{Z_{11}}\right) V_F = \left(1 - \frac{j0.04580}{j0.11565}\right) 1.05\underline{/0^\circ} = 0.6342\underline{/0^\circ}$$

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The current to the fault from the transmission line is obtained from the voltage drop from bus 2 to 1 divided by the impedance of the line and transformers T_1 and T_2:

$$I_{21} = \frac{E_2 - E_1}{j(X_{line} + X_{T1} + X_{T2})} = \frac{0.6342 - 0}{j0.3050} = -j2.079 \text{ per unit}$$

c. the subtransient fault current at bus 2 is

$$I_{F2}'' = \frac{V_F}{Z_{22}} = \frac{1.05\underline{/0^\circ}}{j0.13893} = -j7.558 \text{ per unit}$$

and,

$$E_1 = \left(1 - \frac{Z_{12}}{Z_{22}}\right) V_F = \left(1 - \frac{j0.04580}{j0.13893}\right) 1.05\underline{/0^\circ} = 0.7039\underline{/0^\circ}$$

$$E_2 = \left(1 - \frac{Z_{22}}{Z_{22}}\right) V_F = 0$$

The current to the fault from the transmission line is

$$I_{12} = \frac{E_1 - E_2}{j(X_{line} + X_{T1} + X_{T2})} = \frac{0.7039 - 0}{j0.3050} = -j2.308 \text{ per unit}$$

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