EE493 Protection of Power Systems I

Saeed Lotfifard

Washington State University

Chapter 8 of "Power System Analysis and Design", Fifth Edition, 2012, J. D. Glover, M. S. Saema, T. J. Overbye

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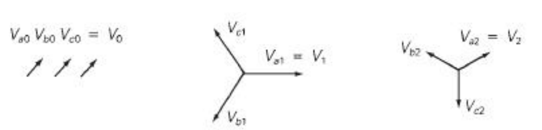
Symmetrical Components

Assume that a set of three-phase voltages designated Va, Vb, and Vc is given.
In accordance with Fortescue, these phase voltages are resolved into the following three sets of sequence components:

1. Zero-sequence components, consisting of three phasors with equal magnitudes and with zero phase displacement, as shown in Figure(a)
2. Positive-sequence components, consisting of three phasors with equal magnitudes, G120 phase displacement, and positive sequence, as in Figure (b)
3. Negative-sequence components, consisting of three phasors with equal magnitudes,G120 phase displacement, and negative sequence, as in Figure(c)



(a) Zero-sequence components (b) Positive-sequence components (c) Negative-sequence components

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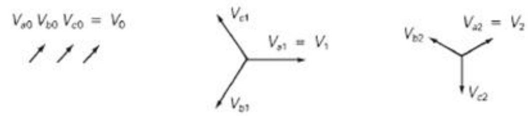
$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} V_{a0} \\ V_{b0} \\ V_{c0} \end{bmatrix} + \begin{bmatrix} V_{a1} \\ V_{b1} \\ V_{c1} \end{bmatrix} + \begin{bmatrix} V_{a2} \\ V_{b2} \\ V_{c2} \end{bmatrix}$$

According to the following



$V_{a0}=V_{b0}=V_{c0}$
$V_{c1}=a \times V_{a1}$ and $V_{c1}=a^2 \times V_{a1}$
$V_{c2}=a^2 \times V_{a2}$ and $V_{c2}=a \times V_{a2}$

Where

$$a = 1\angle 120^\circ$$

$$a^2 = (1\angle 120^\circ)\cdot(1\angle 120^\circ) = 1\angle 240^\circ$$

Therefore

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} V_{a0} \\ V_{b0} \\ V_{c0} \end{bmatrix} + \begin{bmatrix} V_{a1} \\ V_{b1} \\ V_{c1} \end{bmatrix} + \begin{bmatrix} V_{a2} \\ V_{b2} \\ V_{c2} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \times \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

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In this text we will work only with the zero-, positive-, and negative-sequence components of phase a, which are V_{a0}, V_{a1}, and V_{a2}, respectively. For simplicity, we drop the subscript a and denote these sequence components as V_0, V_1, and V_2. They are defined by the following transformation:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix}$$

where $\qquad \boldsymbol{V_p = AV_s}$

$$a = 1\angle 120^\circ = \frac{-1}{2} + j\frac{\sqrt{3}}{2}$$

$$a^2 = (1\angle 120^\circ)\cdot(1\angle 120^\circ) = 1\angle 240^\circ$$

$$V_a = V_0 + V_1 + V_2$$

$$V_b = V_0 + a^2 V_1 + aV_2$$

$$V_c = V_0 + aV_1 + a^2 V_2$$

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$$V_p = AV_s$$

$$V_p = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \qquad V_s = \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$V_s = A^{-1} V_p$$

$$A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

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EXAMPLE

Calculate the sequence components of the following balanced line-to-neutral voltages with *abc* sequence:

$$V_p = \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \begin{bmatrix} 277\underline{/0^\circ} \\ 277\underline{/-120^\circ} \\ 277\underline{/+120^\circ} \end{bmatrix} \quad \text{volts}$$

$$V_0 = \tfrac{1}{3}[277\underline{/0^\circ} + 277\underline{/-120^\circ} + 277\underline{/+120^\circ}] = 0$$

$$V_1 = \tfrac{1}{3}[277\underline{/0^\circ} + 277\underline{/(-120^\circ + 120^\circ)} + 277\underline{/(120^\circ + 240^\circ)}]$$

$$= 277\underline{/0^\circ} \quad \text{volts} = V_{an}$$

$$V_2 = \tfrac{1}{3}[277\underline{/0^\circ} + 277\underline{/(-120^\circ + 240^\circ)} + 277\underline{/(120^\circ + 120^\circ)}]$$

$$= \tfrac{1}{3}[277\underline{/0^\circ} + 277\underline{/120^\circ} + 277\underline{/240^\circ}] = 0$$

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EXAMPLE

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 10\underline{/0^\circ} \\ 0 \\ 10\underline{/120^\circ} \end{bmatrix}$$

$$I_0 = \tfrac{1}{3}[10\underline{/0^\circ} + 0 + 10\underline{/120^\circ}] = 3.333\underline{/60^\circ} \quad \text{A}$$

$$I_1 = \tfrac{1}{3}[10\underline{/0^\circ} + 0 + 10\underline{/(120^\circ + 240^\circ)}] = 6.667\underline{/0^\circ} \quad \text{A}$$

$$I_2 = \tfrac{1}{3}[10\underline{/0^\circ} + 0 + 10\underline{/(120^\circ + 120^\circ)}] = 3.333\underline{/-60^\circ} \quad \text{A}$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \times \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$a = 1\underline{/120^\circ} = \frac{-1}{2} + j\frac{\sqrt{3}}{2}$$

$$a^2 = (1\underline{/120^\circ}) \cdot (1\underline{/120^\circ}) = 1\underline{/240^\circ}$$