# EE493 Protection of Power Systems I

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**Chapter 5 of “Power System Relaying”, 4th Edition, Wiley, 2014, by S. Horowitz and A. G. Phadke**

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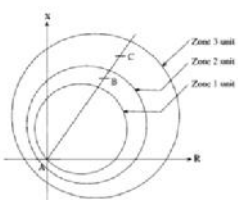


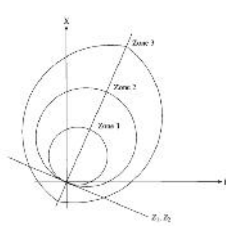

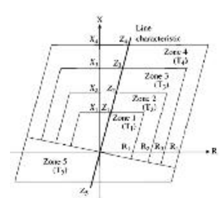

mho relays

quadrilateral relays

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Usually three zones are considered in distance protection of the line as follows:

Zone 1 = 80-85% of protected line impedance with delay of zero second (instantaneously)

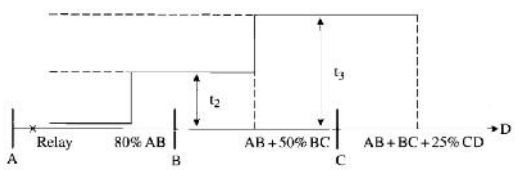
Biggest amount of the following values are considered for zone 2

Zone 2A= 120% of protected line
Zone 2B = Protected line + 50% of shortest second line
With the delay of typically 0.25 to 0.4 second.

Also for Zone 3, biggest amount of the following values are considered

Zone 3A = 1.2 (protected line + longest second line)
Zone 3B = protected line + longest second line +20-25% of shortest third line
With the delay typically between 0.6 to 1.5 second

In this course for the sake of simplicity just use 2B and 3B for zones 2 and 3 respectively.



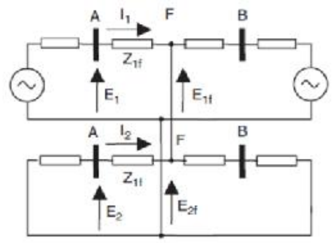



## Three-phase distance relays

On a three-phase power system, there are ten distinct types of possible faults: a three-phase fault, three phase-to-phase faults, three phase-to-ground faults and three double-phase-to-ground faults. The equations that govern the relationship between voltages and currents at the relay location are different for each of these faults. It is a fundamental principle of distance relaying that, regardless of the type of fault involved, the voltage and current used to energize the appropriate relay are such that the relay will measure the positive sequence impedance to the fault. Once this is achieved, the zone settings of all relays can be based upon the total positive sequence impedance of the line, regardless of the type of the fault. The following slides consider various types of fault, and determine the appropriate voltage and current inputs to be used for the distance relays responsible for each of these fault types.

*Phase-to-phase faults*

If you recall for line to line fault the sequence networks are connected as follows:




$$E_{1f} = E_{2f} = E_1 - Z_{1f} I_1 = E_2 - Z_{1f} I_2$$

where E1, E2, I1 and I2 are the symmetrical components of voltages and currents at the relay location. It is assumed that the positive and negative sequence impedances of the transmission line are equal. It follows from above equation that

$$\frac{E_1 - E_2}{I_1 - I_2} = Z_{1f}$$

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since the phase quantities at the relay location are given by

$$\begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} E_0 \\ E_1 \\ E_2 \end{bmatrix}$$

$$E_b = E_0 + \alpha^2 E_1 + \alpha E_2 \text{ and } E_c = E_0 + \alpha E_1 + \alpha^2 E_2$$

Therefore,

$$(E_b - E_c) = (\alpha^2 - \alpha)(E_1 - E_2) \text{ and } (I_b - I_c) = (\alpha^2 - \alpha)(I_1 - I_2)$$

Therefore,

$$\frac{E_b - E_c}{I_b - I_c} = \frac{E_1 - E_2}{I_1 - I_2} = Z_{1f}$$

Thus, a distance relay, to which the line-to-line voltage between phases b and c is connected, and which is supplied by the difference between the currents in the two phases, will measure the positive sequence impedance to the fault, when a fault between phases b and c occurs. Similar analysis will show that, for the other two types of phase-to-phase fault, when the corresponding voltage and current differences are used to energize the relays, the positive sequence impedance to the fault will be measured.

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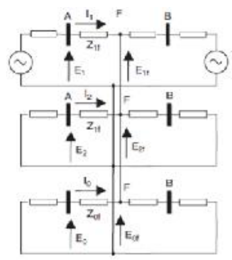
*Phase-to-phase-to-ground faults*

The symmetrical component diagram for a phase-b-to-c-to-ground fault is shown in the following figure. It should be clear from this figure that for this fault also, the performance equations for the positive and negative sequence parts of the equivalent circuit are exactly the same as those for the b-to-c fault.



According to above figure

$$E_{1f} = E_{2f} = E_1 - Z_{1f}I_1 = E_2 - Z_{1f}I_2$$

$$\frac{E_1 - E_2}{I_1 - I_2} = Z_{1f}$$

As we proved in previous slide

$$\frac{E_b - E_c}{I_b - I_c} = \frac{E_1 - E_2}{I_1 - I_2} = Z_{1f}$$

Therefore, in the case of b-c-g fault the same equation that was used for b-c fault is used to calculate positive sequence of the impedance between relay and fault location

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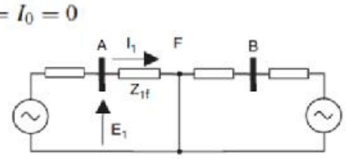
*Three-phase-faults*

For a three-phase fault at F, the symmetrical component diagram is as shown in the following figure. It should be noted that as three phase fault is a symmetrical fault and we assumed the system is balanced before fault, zero sequence and negative sequence current and voltages at relay location are zero.

$$E_2 = E_0 = 0$$
$$I_2 = I_0 = 0$$

According to the positive sequence network




$$E_1 = Z_{1f}I_1 \rightarrow \frac{E_1}{I_1} = Z_{1f}$$

As mentioned above in three phase faults $I_2$=$I_0$=0 and $E_{f0}$=$E_{f2}$=0. Therefore, according to the followings

$$\begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} E_0 \\ E_1 \\ E_2 \end{bmatrix}$$

for this case, $E_a = E_1$, $E_b = \alpha^2 E_1$ and $E_c = \alpha E_1$, and similar relations hold for the phase currents. Consequently, for a three-phase fault

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$$\frac{E_1}{I_1} = \frac{E_a}{I_a} = Z_{1f}$$

As $E_a=E_1$, and $E_b=a^2E_1$, $E_a-E_b=(1-a^2)E_1$ and the same relation holds for currents $I_a-I_b=(1-a^2)I_1$ Therefore

$$\frac{E_a - E_b}{I_a - I_b} = \frac{(1-a^2)E_a}{(1-a^2)I_a} = \frac{E_a}{I_a} = Z_{1f}$$

Therefore, for three phase faults the following holds

$$\frac{E_a - E_b}{I_a - I_b} = \frac{E_b - E_c}{I_b - I_c} = \frac{E_c - E_a}{I_c - I_a} = Z_{1f}$$

Which means the same equations that are used for phase-phase faults can be used for three-phase faults

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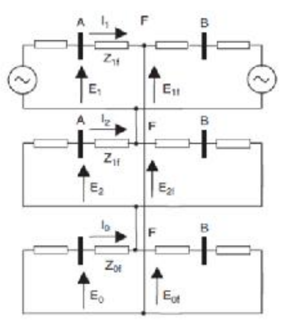
*phase-Ground faults*

For a fault between phase a and ground, the symmetrical component connection diagram is as shown in the following figure. The voltages and currents at the relay location for this case are given by




$$E_{1f} = E_1 - Z_{1f}I_1$$

$$E_{2f} = E_2 - Z_{1f}I_2$$

$$E_{0f} = E_0 - Z_{0f}I_0$$

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As the fault has happened at phase a, voltage of phase a at fault point is zero

$$E_{af} = 0$$

Using the following equation

$$\begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} E_0 \\ E_1 \\ E_2 \end{bmatrix}$$

We will have the following equation at fault point

$$\begin{aligned} E_{af} &= E_{0f} + E_{1f} + E_{2f} = 0 \\ &= (E_0 - Z_{0f} I_0) + (E_1 - Z_{1f} I_1) + (E_2 - Z_{2f} I_2) = 0 \\ &= (E_0 + E_1 + E_2) - Z_{1f}(I_1 + I_2) - Z_{0f} I_0 = 0 \\ &= (E_0 + E_1 + E_2) - Z_{1f}(I_0 - I_0 + I_1 + I_2) - Z_{0f} I_0 = 0 \\ &= (E_0 + E_1 + E_2) - Z_{1f}(I_0 + I_1 + I_2) - (Z_{0f} - Z_{1f}) I_0 = 0 \\ &= E_a - Z_{1f} I_a - (Z_{0f} - Z_{1f}) I_0 = 0 \end{aligned}$$

Therefore from last equation the following holds:

$$E_a = Z_{1f}\left(I_a + \frac{Z_{0f} - Z_{1f}}{Z_{1f}} I_0\right)$$

Now if we define a new current as follows

$$I'_a = I_a + \frac{Z_{0f} - Z_{1f}}{Z_{1f}} I_0$$

The following holds:

$$\frac{E_a}{I'_a} = Z_{1f}$$

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Thus, if the distance relay is energized with the phase a voltage, and the compensated phase a current (i.e $I'_a$ ), it also measures the positive sequence impedance to the fault.

It is notable that in compensated phase a current defined as follows

$$I'_a = I_a + \frac{Z_{0f} - Z_{1f}}{Z_{1f}} I_0$$

$Z_{0f}$ is the zero sequence impedance between relay location and fault location. If the total length of line is L miles, and distance between relay and fault location is K miles

$$Z_{0f} = \frac{K}{L} Z_0$$

Where $Z_0$ is the total zero sequence of the line. The same argument holds for $Z_{1f}$ therefore

$$I'_a = I_a + \frac{Z_{0f} - Z_{1f}}{Z_{1f}} I_0 = I_a + \frac{\frac{K}{L} Z_0 - \frac{K}{L} Z_1}{\frac{K}{L} Z_1} I_0 = I_a + \frac{\frac{K}{L}(Z_0 - Z_1)}{\frac{K}{L} Z_1} I_0 = I_a + \frac{Z_0 - Z_1}{Z_1} I_0$$

Therefore

$$I'_a = I_a + \frac{Z_0 - Z_1}{Z_1} I_0$$

In above equation Z0 and Z1 are total zero sequence and positive sequence impedance of the line and are known values.

m factor is defined as m=$\frac{Z_0 - Z_1}{Z_1}$. The factor *m* for most overhead transmission lines is a real number, and varies between 1.5 and 2.5. A good average value for m is 2.0, which corresponds to Z0 of a transmission line being equal to 3Z1.

$$I'_a = I_a + m I_0$$

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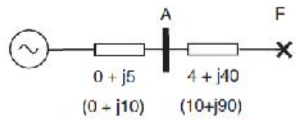
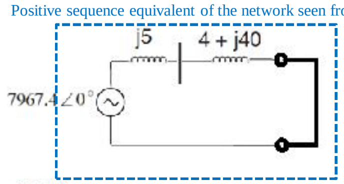
**Example:** Consider the simple system represented by the one-line diagram in the following figure. The system nominal voltage is 13.8 kV, and the positive and zero sequence impedances of the two elements are as shown in the figure. The zero sequence impedances are given in parentheses. Determine voltage and currents at relay location and the impedance calculated by the relay for three-phase, phase-to-phase and phase-to-ground faults




For **Three-phase fault** only the positive sequence current exists (i.e. $E_0=E_2=0$, and $I_0=I_2=0$)

Positive sequence equivalent of the network seen from fault point




$I_a=I_0+I_1+I_2$ $\quad I_a = I_1 = \dfrac{7967.4}{4+j45} = 176.36\angle 84.92^\circ$

In above figure $7967.4 = (13\,800/\sqrt{3})$ is the phase-to-neutral voltage. The phase a voltage at the relay location is given by

$$E_a = E_1 = 7967.4 - j5 \times 176.36\angle 84.92^\circ = 7089.49\angle -0.63^\circ$$

Thus, the fault impedance seen by the relay in this case is

$$Z_f = \frac{E_a - E_b}{I_a - I_b} = \frac{E_a}{I_a} = \frac{7089.49\angle -0.63^\circ}{176.36\angle -84.92^\circ} = 4 + j40\ \Omega$$



**Phase-to-phase fault**

For a b–c fault

$$I_1 = -I_2 = \frac{7967.4}{2 \times (4 + j45)} = 88.18\angle 84.92^\circ$$

Also, $I_b = -I_c = I_1(\alpha^2 - \alpha) = -j\sqrt{(3)}I_1 = 152.73\angle -174.92^\circ$. And, $(I_b - I_c) = 305.46\angle -174.92^\circ$. The positive and negative sequence voltages at the relay location are given by

$$E_1 = 7967.4 - j5 \times 88.18\angle -84.92^\circ = 7528.33\angle -0.3^\circ$$

$$E_2 = j5 \times 88.18\angle -84.92^\circ = 440.90\angle 5.08^\circ$$

and the phase b and c voltages at the relay location are

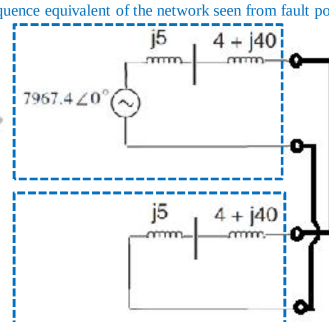
$$E_b = \alpha^2 E_1 + \alpha E_2 = 7528.33\angle -120.3^\circ + 440.90\angle 125.08^\circ$$
$$= -4051.3 - j6139.3$$

$$E_c = \alpha E_1 + \alpha^2 E_2 = 7528.33\angle 119.7^\circ + 440.90\angle -114.9^\circ$$
$$= -3916.09 + j6139.3$$

Thus, $E_b - E_c = 12279.37\angle -90.63^\circ$, and

$$\frac{E_b - E_c}{I_b - I_c} = \frac{12\,279.37\angle -90.63^\circ}{305.46\angle -174.92^\circ} = 4 + j40\ \Omega$$

Positive sequence equivalent of the network seen from fault point




Negative sequence equivalent of the network seen from fault point

**Phase-a-to-ground fault**

For this fault, the three symmetrical components of the current are equal:

$$I_1 = I_2 = I_0 = \frac{7967.4}{(0+\mathrm{j}10) + 2\times(0+\mathrm{j}5) + (10+\mathrm{j}90) + 2\times(4+\mathrm{j}40)}$$

$$= 41.75\angle -84.59^\circ$$

The symmetric components of the voltages at the relay location are

$$E_1 = 7967.4 - \mathrm{j}5\times 41.75\angle -84.59^\circ = 7759.58 - \mathrm{j}19.68$$

$$E_2 = -\mathrm{j}5\times 41.75\angle -84.59^\circ = -207.82 - \mathrm{j}19.68$$

$$E_0 = -(0+\mathrm{j}10)\times 41.75\angle -84.59^\circ = -415.64 - \mathrm{j}39.36$$

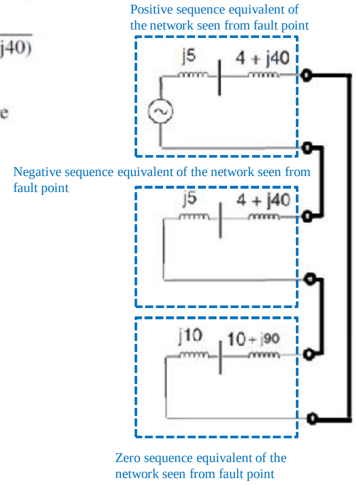
And the phase a voltage and current at the relay location are

$$E_a = E_1 + E_2 + E_0 = 7136.55\angle -0.63^\circ$$

$$I_a = I_1 + I_2 + I_0 = 125.25\angle -84.59^\circ$$

The zero sequence current compensation factor $m$ is given by

$$m = \frac{Z_0 - Z_1}{Z_1} = \frac{10+\mathrm{j}90 - 4 - \mathrm{j}40}{4+\mathrm{j}40} = 1.253\angle -1.13^\circ$$




and the compensated phase a current is $I'_a = I_a + mI_0 = 177.54\angle -84.92^\circ$; and, finally

$$\frac{E_a}{I'_a} = \frac{7136.55\angle -0.63^\circ}{177.54\angle -84.92^\circ} = 4 + \mathrm{j}40\ \Omega$$

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