Power System Protection II

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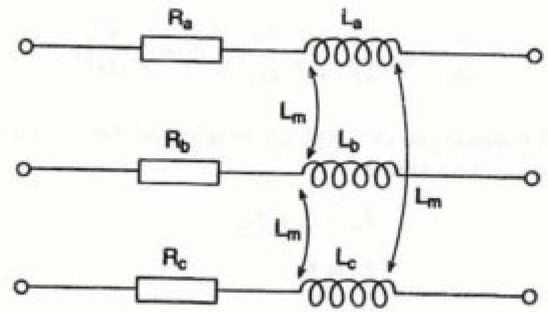
EE 511

7.2 Representation of transmission lines



Elemental length of distributed transmission line represented by series resistance and self and mutual inductances

If it is assumed that R_k, L_k are the resistance and inductance per unit length of the kth phase, L_{kl} is the mutual

$$dv_a = (R_a dx) i_a + (L_a dx)\frac{di_a}{dt} + (L_{ab} dx)\frac{di_b}{dt} + (L_{ac} dx)\frac{di_c}{dt}$$

$$\frac{dv_a}{dx} = \left(R_a + L_a\frac{d}{dt}\right) i_a + L_{ab}\frac{di_b}{dt} + L_{ac}\frac{di_c}{dt} \qquad (7.1)$$

Similarly, the voltage-current relations for phases b and c would be

$$\frac{dv_b}{dx} = L_{ba}\frac{di_a}{dt} + \left(R_a + L_b\frac{d}{dt}\right) i_b + L_{bc}\frac{di_c}{dt} \qquad (7.2)$$

$$\frac{dv_c}{dx} = L_{ca}\frac{di_a}{dt} + L_{cb}\frac{di_b}{dt} + \left(R_c + L_c\frac{d}{dt}\right) i_c \qquad (7.3)$$

If the line is assumed to be ideally transposed, we have

$$
\begin{aligned}
&R_a = R_b = R_c = R_s \\
&L_a = L_b = L_c = L_s \\
&L_{ab} = L_{ac} = L_{ba} = L_{bc} = L_{ca} = L_{cb} = L_m
\end{aligned}
\tag{7.4}
$$

$$
\begin{aligned}
\frac{dv_a}{dx} &= \left(R_s + L_s \frac{d}{dt}\right) i_a + L_m \frac{di_b}{dt} + L_m \frac{di_c}{dt} \\
\frac{dv_b}{dx} &= L_m \frac{di_a}{dt} + \left(R_s + L_s \frac{d}{dt}\right) i_b + L_m \frac{di_c}{dt} \\
\frac{dv_c}{dx} &= L_m \frac{di_a}{dt} + L_m \frac{di_b}{dt} + \left(R_s + L_s \frac{d}{dt}\right) i_c
\end{aligned}
\tag{7.5}
$$

Single-phase to ground fault

Assume a solid single-phase to ground fault occurs on phase 'a' at a distance x from the relay location. The instantaneous value of the voltage v_a, which is the voltage of phase 'a' at the relaying point, can be calculated using eqns. 7.1 for untransposed lines and eqns. 7.5 or 7.7 for assumed ideally transposed lines.

Using the instantaneous values of the voltages, currents and the rate of change of the currents, the voltage v_a can be obtained using eqn. 7.1 such that

$$v_a = xR_d i_a + xL_a \frac{d}{dt}\left(i_a + \frac{L_{ab}}{L_a} i_b + \frac{L_{ac}}{L_a} i_c\right)$$

or

$$v_a = xR_d i_x + xL_a \frac{di_y}{dt} \tag{7.8}$$

$$i_x = i_a$$

$$i_y = i_a + (L_{ab}/L_a)i_b + (L_{ac}/L_a)i_c$$

Equations relating to transposed lines can be expressed in the same way, and in this case eqns. 7.5 are used to obtain the relationship given in eqn. 7.9

$$v_a = xR_s i_x + xL_s \frac{di_y}{dt} \tag{7.9}$$

where

$$i_x = i_a$$

and

$$i_y = i_a + (L_m/L_s)i_b + (L_m/L_s)i_c$$

7.2.2 *Phase-to-phase and three-phase faults*

When the fault involves two or three phases, the voltage between the faulted phases, say 'a' and 'b', can be found as follows:

$$v_a - v_b = x\left(R_a + L_a \frac{d}{dt}\right) i_a + xL_{ab}\frac{di_b}{dt} - x\left[L_{ba}\frac{di_a}{dt} + \left(R_b + L_b \frac{d}{dt}\right) i_b\right]$$

This equation can be reduced to the more succinct form

$$v_a - v_b = xR_a i_x + x(L_a - L_{ab})\frac{di_y}{dt} \tag{7.11}$$

where

$$i_x = i_a - (R_b/R_a) i_b$$

and

$$i_y = i_a - \frac{(L_b - L_{ab}) i_b}{L_a - L_{ab}}$$

The above analysis shows that the behaviour of the transmission line under fault conditions is governed by a differential equation having the general form of eqn. 7.13.

$$v = Ri_x + L\frac{di_y}{dt} \qquad (7.13)$$

Now, to estimate L and R the following approaches are used

7.3 Differential equation protection with selected limits

If eqn. 7.13 is integrated once over the time interval t_1 to t_2 and again over the period from t_3 to t_4, the following equations are obtained:

$$R\int_{t_1}^{t_2} i_x dt + L(i_{y2} - i_{y1}) = \int_{t_1}^{t_2} v\, dt \tag{7.14}$$

$$R\int_{t_3}^{t_4} i_x dt + L(i_{y4} - i_{y3}) = \int_{t_3}^{t_4} v\, dt \tag{7.15}$$

7.3.3 Graphical interpretation of digital filtering by integration over selected limits

The filtering of harmonics from waveforms by using integration over selected limits can be usefully explained graphically. Consider a waveform $i(t)$ and assume that it consists of only fundamental and third harmonic components, which can in turn be described by

$$\begin{aligned} i(t) &= i_1(t) + i_3(t) \\ &= I_1 \cos(\omega_0 t + \theta_1) + I_3 \cos(3\omega_0 t + \theta_3) \end{aligned}$$

When $i(t)$ is integrated from $t_1 = 0$ to $t_2 = \alpha/\omega_0$ and from $t_3 = (\pi/3)/\omega_0$ to $t_4 = (\pi/3 + \alpha)/\omega_0$ the corresponding integrations of the third harmonic components are equal to the areas shown shaded in Figure 7.2. These areas are equal to each other but have opposite signs and, when added together, they are cancelled and therefore effectively filtered out from the original waveform $i(t)$.

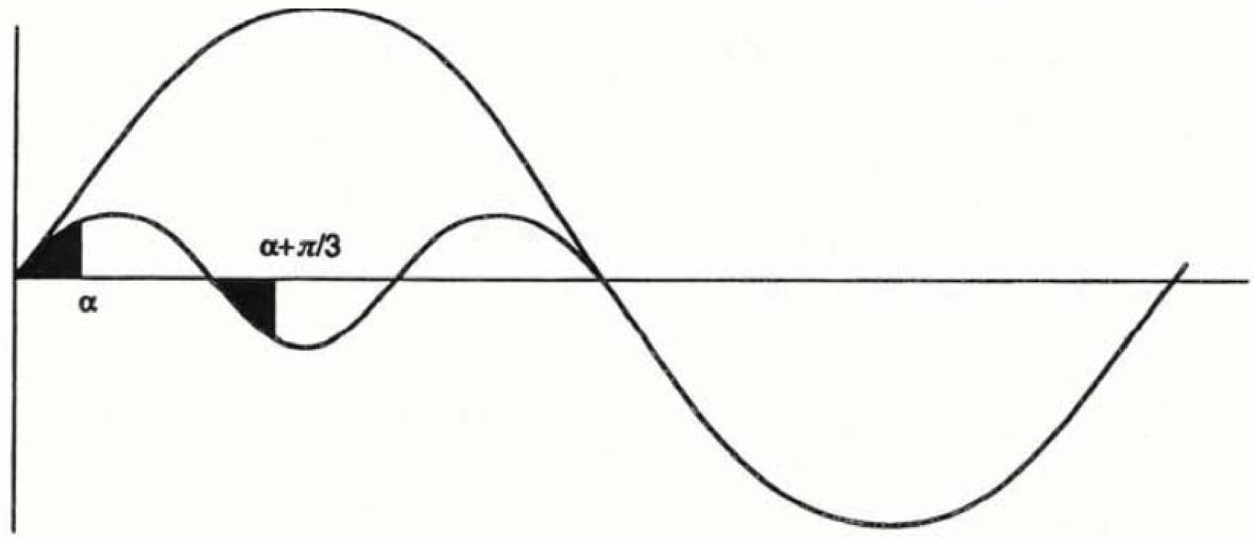


Figure 7.2 Physical interpretation of digital filtering by integration over selected limits

7.3.4 Filtering of multiple harmonic components

We have seen how it is possible to eliminate, or filter out, nth harmonic components by simply adding two integrations together. This is because the nth harmonic contained in the first integration period is cancelled out if the second integration is performed over limits that are related to nth harmonic order. The same basic approach can be used to eliminate any other harmonic m. This is achieved by simply adding a third integration over limits related to mth order harmonic. Therefore, the limits under this condition would be from $t_5 = (\pi/m)/\omega_0$ to $t_6 = (\pi/m + \alpha)/\omega_0$.

The equation used to eliminate nth and mth harmonics simultaneously from a waveform $i(t)$ would then be as follows:

$$\int_0^{a/\omega_0} i(t)dt + \int_{\pi/n\omega_0}^{(a+\pi/n)/\omega_0} i(t)dt + \int_{\pi/m\omega_0}^{(a+\pi/m)/\omega_0} i(t)dt \tag{7.20}$$

However, the number of integrations can be reduced to only two if the value of α is chosen to be equal to the angle corresponding to a full cycle of the mth harmonic order (i.e. $\alpha = 2\pi/m$). In this way we can ensure the elimination of the effect of the mth harmonic by the first integration, as the integration over a full cycle of a sinusoid is always equal to zero. It is then only required to eliminate the effect of the nth harmonic, which can be done by addition of a second integration. Thus, for removing two harmonics of order n and m together with any multiples thereof, the following equation would be used:

$$\int_0^{2\pi/m\omega_0} i(t)\,dt + \int_{\pi/n\omega_0}^{(\pi/n+2\pi/m)/\omega_0} i(t)\,dt \tag{7.21}$$

By applying the above described principles to eqn. 7.13, it is possible to calculate R and L so that any number of harmonics are eliminated. For example, in order to remove the third and fifth harmonics, eqn. 7.13 can be integrated over

$$\left(0, \frac{2\pi/5}{\omega_0}\right)$$

and again over the interval

$$\left[\frac{\pi/3}{\omega_0}, \left(\frac{\pi/3 + 2\pi/5}{\omega_0}\right)\right].$$

The resulting equation, when added, gives

$$L\left[\int_{0}^{2\pi/5\omega_0} di_y + \int_{\pi/3\omega_0}^{(\pi/3+2\pi/5)/\omega_0} di_y\right] + R\left[\int_{0}^{2\pi/5\omega_0} i_x dt + \int_{\pi/3\omega_0}^{(\pi/3+2\pi/5)/\omega_0} i_x dt\right]$$
$$= \left[\int_{0}^{2\pi/5\omega_0} v dt + \int_{\pi/3\omega_0}^{(\pi/3+2\pi/5)/\omega_0} v dt\right] \tag{7.22}$$

In general, this principle can be extended to any number of harmonics by making a sufficient number of integrations.

7.4 Simultaneous differential equation techniques

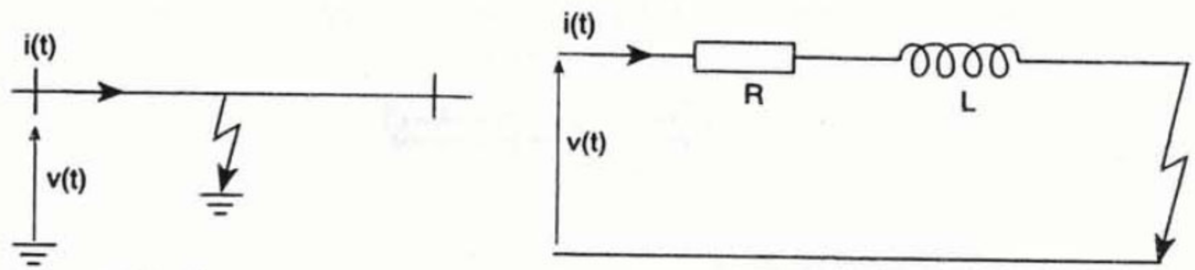


Figure 7.3 Representation of transmission line using lumped-series circuit parameters

$$v = Ri + L\frac{di}{dt} \tag{7.23}$$

$$\frac{di}{dt} = \frac{i_{k+1} - i_{k-1}}{2\Delta t} \tag{7.24}$$

$$v_k \simeq Ri_k + L\frac{(i_{k+1} - i_{k-1})}{2\Delta t} \tag{7.25}$$

Similarly, by using the following sets of samples at t_{k+1}, eqn. 7.23 becomes

$$v_{k+1} \simeq Ri_{k+1} + L\frac{i_{k+2} - i_k}{2\Delta t} \tag{7.26}$$

In matrix form, eqns. 7.25 and 7.26 can be combined to give

$$\left[\begin{array}{c:c} i_k & \dfrac{i_{k+1} - i_{k-1}}{2\Delta t} \\ \hdashline i_{k+1} & \dfrac{i_{k+2} - i_k}{2\Delta t} \end{array}\right] \left[\begin{array}{c} R \\ \hdashline L \end{array}\right] = \left[\begin{array}{c} v_k \\ \hdashline v_{k+1} \end{array}\right] \tag{7.27}$$

$$R \simeq \frac{v_k(i_{k+2} - i_k) - v_{k+1}(i_{k+1} - i_{k-1})}{i_k(i_{k+2} - i_k) - i_{k+1}(i_{k+1} - i_{k-1})} \tag{7.29}$$

$$L \simeq 2\Delta t\,\frac{(i_k v_{k+1} - i_{k+1} v_k)}{i_k(i_{k+2} - i_k) - i_{k+1}(i_{k+1} - i_{k-1})} \tag{7.30}$$

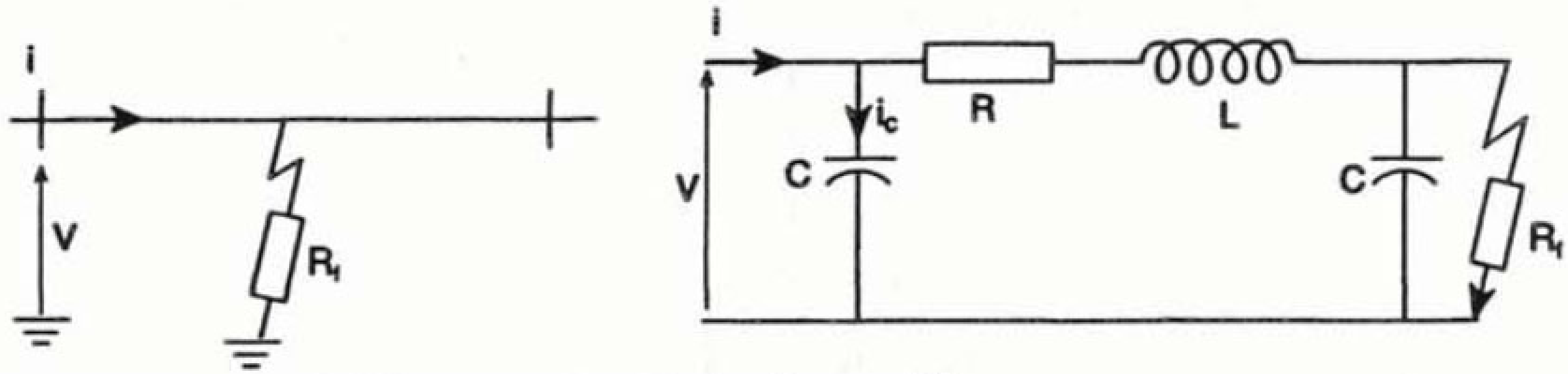


Figure 7.4 Single PI circuit transmission-line model

It is assumed Rf=0 then

$$R(i-i_c)+L\frac{d(i-i_c)}{dt}=v \qquad (7.31)$$

$i_c = Cdv/dt$

$$Ri+L\frac{di}{dt}-RC\frac{dv}{dt}-LC\frac{d^2v}{dt^2}=v \qquad (7.32)$$

$$\left.\begin{aligned}\left.\frac{di}{dt}\right|_{t=t_k} &= \frac{i_{k+1}-i_{k-1}}{2\Delta t}\\ \left.\frac{dv}{dt}\right|_{t=t_k} &= \frac{v_{k+1}-v_{k-1}}{2\Delta t}\end{aligned}\right\} \tag{7.33}$$

$$\left.\frac{d^2v}{dt^2}\right|_{t=t_k} = \frac{v_{k+1}-2v_k+v_{k-1}}{(\Delta t)^2} \tag{7.34}$$

Substituting current and voltage samples at $t=t_k$ and their corresponding derivatives described by eqns. 7.33 and 7.34, we obtain

$$Ri_k + L\frac{(i_{k+1}-i_{k-1})}{2\Delta t} - RC\frac{(v_{k+1}-v_{k-1})}{2\Delta t} - LC\frac{(v_{k+1}-2v_k+v_{k-1})}{(\Delta t)^2} = v_k$$

Similarly, at instants of time t_{k+1}, t_{k+2} and t_{k+3}, we obtain

$$Ri_{k+1}+L\frac{(i_{k+2}-i_k)}{2\Delta t}-RC\frac{(v_{k+2}-v_k)}{2\Delta t}-LC\frac{(v_{k+2}-2v_{k+1}+v_k)}{(\Delta t)^2}=v_{k+1}$$

$$Ri_{k+2}+L\frac{(i_{k+3}-i_{k+1})}{2\Delta t}-RC\frac{(v_{k+3}-v_{k+1})}{2\Delta t}-LC\frac{(v_{k+3}-2v_{k+2}+v_{k+1})}{(\Delta t)^2}=v_{k+2}$$

$$Ri_{k+3}+L\frac{(i_{k+4}-i_{k+2})}{2\Delta t}-RC\frac{(v_{k+4}-v_{k+2})}{2\Delta t}-LC\frac{(v_{k+4}-2v_{k+3}+v_{k+2})}{(\Delta t)^2}=v_{k+3}$$

The above equations can be written in the matrix form

$$\begin{bmatrix} i_k & \dfrac{i_{k+1}-i_{k-1}}{2\Delta t} & -\dfrac{v_{k+1}-v_{k-1}}{2\Delta t} & -\dfrac{v_{k+1}+2v_k+v_{k-1}}{(\Delta t)^2} \\[2ex] i_{k+1} & \dfrac{i_{k+2}-i_k}{2\Delta t} & -\dfrac{v_{k+2}-v_k}{2\Delta t} & -\dfrac{v_{k+2}-2v_{k+1}+v_k}{(\Delta t)^2} \\[2ex] i_{k+2} & \dfrac{i_{k+3}-i_{k+1}}{2\Delta t} & -\dfrac{v_{k+3}-v_{k+1}}{2\Delta t} & -\dfrac{v_{k+3}-2v_{k+2}+v_{k+1}}{(\Delta t)^2} \\[2ex] i_{k+3} & \dfrac{i_{k+4}-i_{k+2}}{2\Delta t} & -\dfrac{v_{k+4}-v_{k+2}}{2\Delta t} & -\dfrac{v_{k+4}-2v_{k+3}+v_{k+2}}{(\Delta t)^2} \end{bmatrix} \begin{bmatrix} R \\ L \\ CR \\ CL \end{bmatrix} = \begin{bmatrix} v_k \\ v_{k+1} \\ v_{k+2} \\ v_{k+3} \end{bmatrix} \tag{7.35}$$