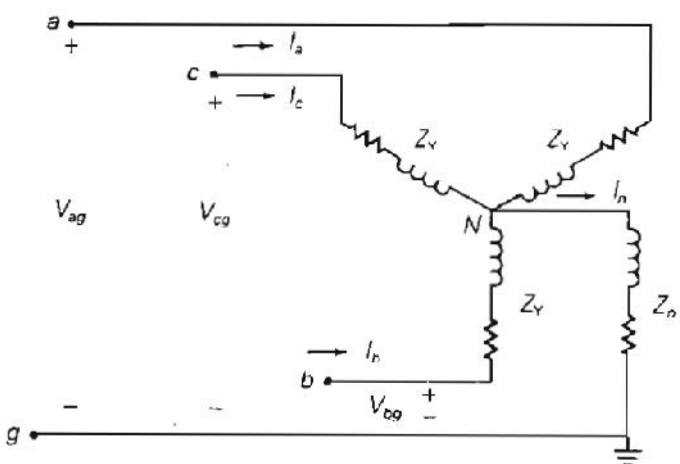
EE493 Protection of Power Systems I

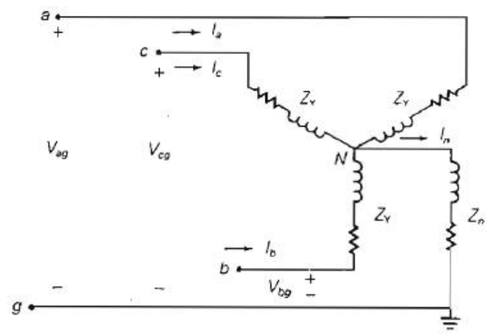
Saeed Lotfifard

Washington State University

Chapter 8 of “Power System Analysis and Design”, Fifth Edition, 2012, J. D. Glover, M. S. Saema, T. J. Overbye

Sequence Networks of Impedance Loads





$$V_{ag} = Z_Y I_a + Z_n I_n$$
$$= Z_Y I_a + Z_n(I_a + I_b + I_c)$$
$$= (Z_Y + Z_n) I_a + Z_n I_b + Z_n I_c$$

$$V_{bg} = Z_n I_a + (Z_Y + Z_n) I_b + Z_n I_c$$
$$V_{cg} = Z_n I_a + Z_n I_b + (Z_Y + Z_n) I_c$$

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \begin{bmatrix} (Z_Y + Z_n) & Z_n & Z_n \\ Z_n & (Z_Y + Z_n) & Z_n \\ Z_n & Z_n & (Z_Y + Z_n) \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$\boldsymbol{V_p = Z_p I_p}$$

$$\boldsymbol{V}_p = \boldsymbol{Z}_p \boldsymbol{I}_p$$

$$\boldsymbol{A}\boldsymbol{V}_s = \boldsymbol{Z}_p \boldsymbol{A}\boldsymbol{I}_s$$

$$\boldsymbol{V}_s = (\boldsymbol{A}^{-1}\boldsymbol{Z}_p\boldsymbol{A})\boldsymbol{I}_s$$

$$\boldsymbol{V}_s = \boldsymbol{Z}_s \boldsymbol{I}_s$$

$$\boldsymbol{Z}_s = \boldsymbol{A}^{-1}\boldsymbol{Z}_p\boldsymbol{A}$$

The impedance matrix $\boldsymbol{Z}_s$ is called the *sequence impedance matrix*

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$$\boldsymbol{Z}_s = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} (Z_Y + Z_n) & Z_n & Z_n \\ Z_n & (Z_Y + Z_n) & Z_n \\ Z_n & Z_n & (Z_Y + Z_n) \end{bmatrix}$$

$$\times \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$(1 + a + a^2) = 0$$

$$\boldsymbol{Z}_s = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} (Z_Y + 3Z_n) & Z_Y & Z_Y \\ (Z_Y + 3Z_n) & a^2 Z_Y & a Z_Y \\ (Z_Y + 3Z_n) & a Z_Y & a^2 Z_Y \end{bmatrix}$$

$$= \begin{bmatrix} (Z_Y + 3Z_n) & 0 & 0 \\ 0 & Z_Y & 0 \\ 0 & 0 & Z_Y \end{bmatrix}$$

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$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} (Z_Y + 3Z_n) & 0 & 0 \\ 0 & Z_Y & 0 \\ 0 & 0 & Z_Y \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

$$V_0 = (Z_Y + 3Z_n)I_0 = Z_0 I_0$$
$$V_1 = Z_Y I_1 = Z_1 I_1$$
$$V_2 = Z_Y I_2 = Z_2 I_2$$

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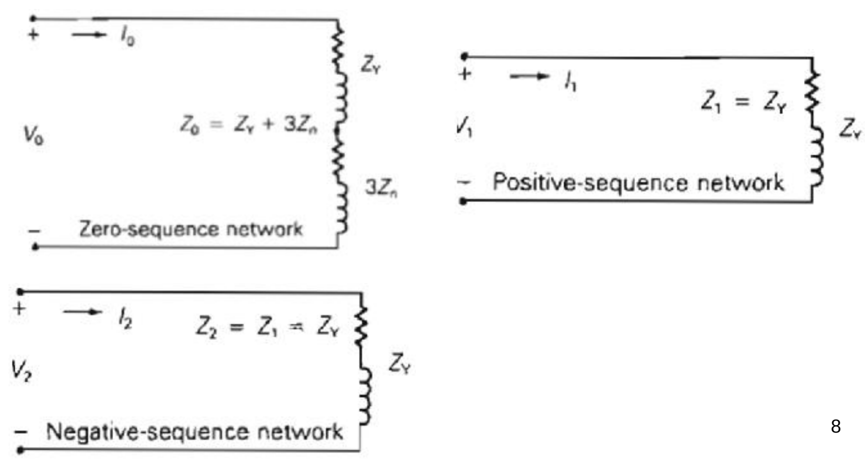
$$V_0 = (Z_Y + 3Z_n)I_0 = Z_0 I_0$$
$$V_1 = Z_Y I_1 = Z_1 I_1$$
$$V_2 = Z_Y I_2 = Z_2 I_2$$



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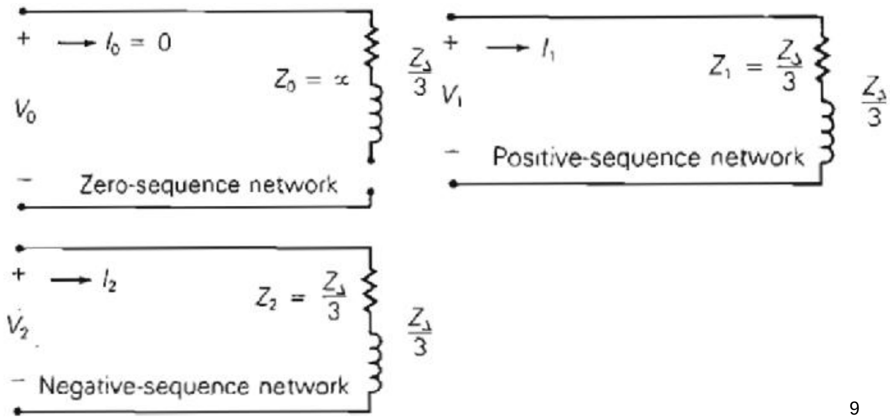
Δ load

Since the Δ load has no neutral connection, the equivalent Y load has an open neutral.



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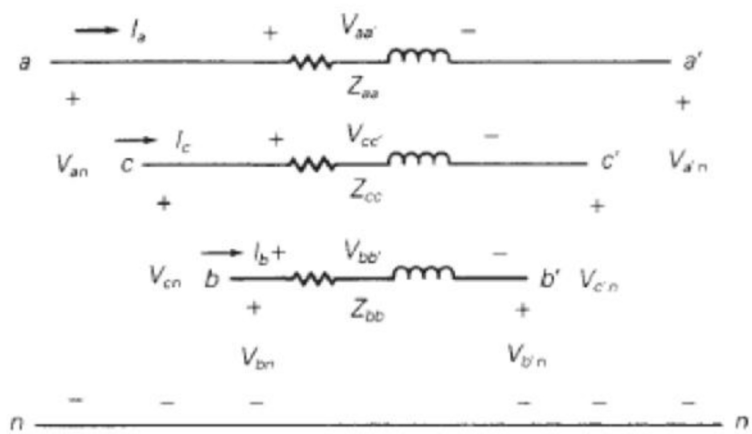
SEQUENCE NETWORKS OF SERIES IMPEDANCES



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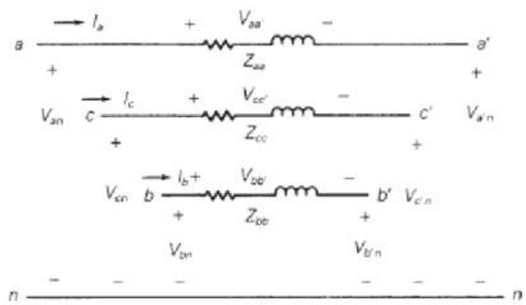

$$\begin{bmatrix} V_{an} - V_{a'n} \\ V_{bn} - V_{b'n} \\ V_{cn} - V_{c'n} \end{bmatrix} = \begin{bmatrix} V_{aa'} \\ V_{bb'} \\ V_{cc'} \end{bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ab} & Z_{bb} & Z_{bc} \\ Z_{ac} & Z_{cb} & Z_{cc} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$\boldsymbol{V_p} - \boldsymbol{V_{p'}} = \boldsymbol{Z_p I_p}$$

$$\boldsymbol{V_s} - \boldsymbol{V_{s'}} = \boldsymbol{Z_s I_s}$$

$$\boldsymbol{Z_s} = \boldsymbol{A^{-1} Z_p A}$$

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$$\boldsymbol{Z_s} = \begin{bmatrix} Z_0 & Z_{01} & Z_{02} \\ Z_{10} & Z_1 & Z_{12} \\ Z_{20} & Z_{21} & Z_2 \end{bmatrix}$$

$$\begin{bmatrix} Z_0 & Z_{01} & Z_{02} \\ Z_{10} & Z_1 & Z_{12} \\ Z_{20} & Z_{21} & Z_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ab} & Z_{bb} & Z_{bc} \\ Z_{ac} & Z_{bc} & Z_{cc} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$(1 + a + a^2) = 0,$$

$$Z_0 = \tfrac{1}{3}(Z_{aa} + Z_{bb} + Z_{cc} + 2Z_{ab} + 2Z_{ac} + 2Z_{bc})$$

$$Z_1 = Z_2 = \tfrac{1}{3}(Z_{aa} + Z_{bb} + Z_{cc} - Z_{ab} - Z_{ac} - Z_{bc})$$

$$Z_{01} = Z_{20} = \tfrac{1}{3}(Z_{aa} + a^2 Z_{bb} + a Z_{cc} - a Z_{ab} - a^2 Z_{ac} - Z_{bc})$$

$$Z_{02} = Z_{10} = \tfrac{1}{3}(Z_{aa} + a Z_{bb} + a^2 Z_{cc} - a^2 Z_{ab} - a Z_{ac} - Z_{bc})$$

$$Z_{12} = \tfrac{1}{3}(Z_{aa} + a^2 Z_{bb} + a Z_{cc} + 2a Z_{ab} + 2a^2 Z_{ac} + 2Z_{bc})$$

$$Z_{21} = \tfrac{1}{3}(Z_{aa} + a Z_{bb} + a^2 Z_{cc} + 2a^2 Z_{ab} + 2a Z_{ac} + 2Z_{bc})$$

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$$\left.\begin{array}{l} Z_{aa} = Z_{bb} = Z_{cc} \\ \\ Z_{ab} = Z_{ac} = Z_{bc} \end{array}\right\} \text{conditions for a symmetrical load}$$

and

then

$$Z_{01} = Z_{10} = Z_{02} = Z_{20} = Z_{12} = Z_{21} = 0$$

$$Z_0 = Z_{aa} + 2Z_{ab}$$

$$Z_1 = Z_2 = Z_{aa} - Z_{ab}$$

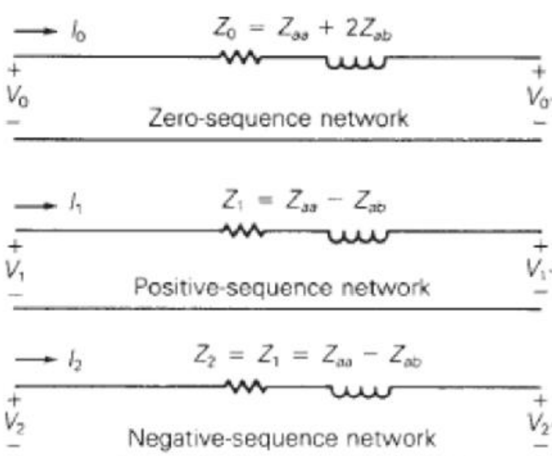
$$\boldsymbol{Z}_s = \begin{bmatrix} Z_0 & Z_{01} & Z_{02} \\ Z_{10} & Z_1 & Z_{12} \\ Z_{20} & Z_{21} & Z_2 \end{bmatrix} = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}$$

$\boldsymbol{V}_s - \boldsymbol{V}_{s'} = \boldsymbol{Z}_s \boldsymbol{I}_s$ becomes three uncoupled equations, written as follows:

$$V_0 - V_{0'} = Z_0 I_0$$

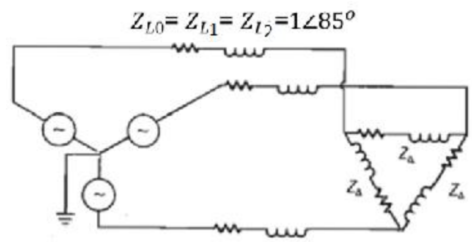
$$V_1 - V_{1'} = Z_1 I_1$$

$$V_2 - V_{2'} = Z_2 I_2$$



EXAMPLE

A Y-connected voltage source with the following unbalanced voltage is applied to the balanced line and load



$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \begin{bmatrix} 277\underline{/0^\circ} \\ 260\underline{/-120^\circ} \\ 295\underline{/+115^\circ} \end{bmatrix} \quad \text{volts}$$

The source neutral is solidly grounded. Using the method of symmetrical components, calculate the source currents I_a, I_b, and I_c.

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$V_0 = \tfrac{1}{3}(277\underline{/0^\circ} + 260\underline{/-120^\circ} + 295\underline{/115^\circ})$$

$$= 7.4425 + j14.065 = 15.912\underline{/62.11^\circ} \quad \text{volts}$$

$$V_1 = \tfrac{1}{3}(227\underline{/0^\circ} + 260\underline{/-120^\circ + 120^\circ} + 295\underline{/115^\circ + 240^\circ})$$

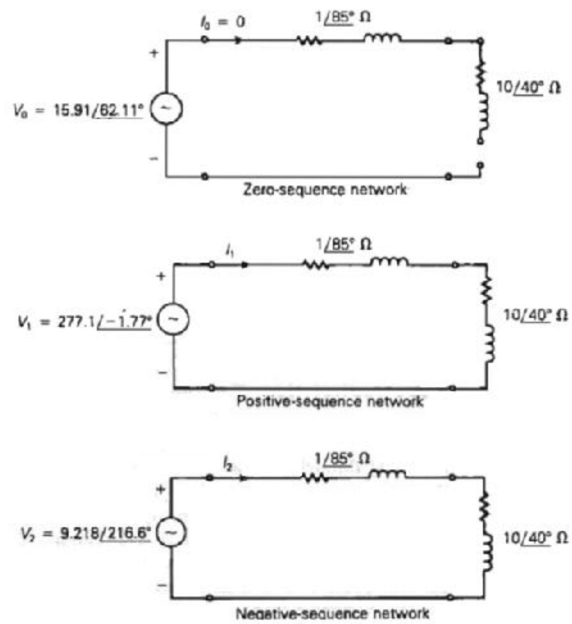
$$= \tfrac{1}{3}(277\underline{/0^\circ} + 260\underline{/0^\circ} + 295\underline{/-5^\circ})$$

$$= 276.96 - j8.5703 = 277.1\underline{/-1.772^\circ} \quad \text{volts}$$

$$V_2 = \tfrac{1}{3}(277\underline{/0^\circ} + 260\underline{/-120^\circ + 240^\circ} + 295\underline{/115^\circ + 120^\circ})$$

$$= \tfrac{1}{3}(277\underline{/0^\circ} + 260\underline{/120^\circ} + 295\underline{/235^\circ})$$

$$= -7.4017 - j5.4944 = 9.218\underline{/216.59^\circ} \quad \text{volts}$$



$$I_0 = 0$$

$$I_1 = \frac{V_1}{Z_{L1} + \frac{Z_\Delta}{3}} = \frac{277.1\underline{/-1.772^\circ}}{10.73\underline{/43.78^\circ}} = 25.82\underline{/-45.55^\circ} \quad \text{A}$$

$$I_2 = \frac{V_2}{Z_{L2} + \frac{Z_\Delta}{3}} = \frac{9.218\underline{/216.59^\circ}}{10.73\underline{/43.78^\circ}} = 0.8591\underline{/172.81^\circ} \quad \text{A}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

$$I_a = (0 + 25.82\underline{/-45.55^\circ} + 0.8591\underline{/172.81^\circ})$$

$$= 17.23 - j18.32 = 25.15\underline{/-46.76^\circ} \quad \text{A}$$

$$I_b = (0 + 25.82\underline{/-45.55^\circ + 240^\circ} + 0.8591\underline{/172.81^\circ + 120^\circ})$$

$$= (25.82\underline{/194.45^\circ} + 0.8591\underline{/292.81^\circ})$$

$$= -24.67 - j7.235 = 25.71\underline{/196.34^\circ} \quad \text{A}$$

$$I_c = (0 + 25.82\underline{/-45.55^\circ + 120^\circ} + 0.8591\underline{/172.81^\circ + 240^\circ})$$

$$= (25.82\underline{/74.45^\circ} + 0.8591\underline{/52.81^\circ})$$

$$= 7.441 + j25.56 = 26.62\underline{/73.77^\circ} \quad \text{A}$$