EE493 Protection of Power Systems I

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Chapter 8 of "Power System Analysis and Design", Fifth Edition, 2012, J. D. Glover, M. S. Saema, T. J. Overbye

Reminder: Ideal transformer

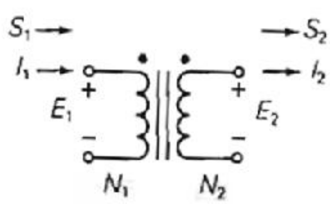
$$N_1 I_1 = N_2 I_2$$

$$\frac{E_1}{N_1} = \frac{E_2}{N_2}$$

$$a_t = \frac{N_1}{N_2}$$

$$E_1 = \left(\frac{N_1}{N_2}\right) E_2 = a_t E_2$$

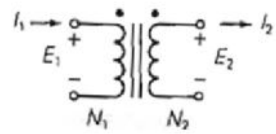
$$I_1 = \left(\frac{N_2}{N_1}\right) I_2 = \frac{I_2}{a_t}$$



$$a_t = \frac{N_1}{N_2}$$

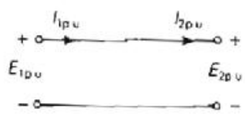
Reminder: Ideal transformer in per unit

$$E_{1\text{p.u.}} = \frac{E_1}{V_{\text{base}1}} = \frac{\left(\frac{N_1}{N_2}\right) E_2}{\left(\frac{N_1}{N_2}\right) V_{\text{base}2}} = \frac{E_2}{V_{\text{base}2}} = E_{2\text{p.u.}}$$



$$I_{1\text{p.u.}} = \frac{I_1}{I_{\text{base}1}} = \frac{\left(\frac{N_2}{N_1}\right) I_2}{\left(\frac{N_2}{N_1}\right) I_{\text{base}2}} = \frac{I_2}{I_{\text{base}2}} = I_{2\text{p.u.}}$$

According to above equations, in per unit systems, primary voltage in per-unit is equal to secondary voltage in per-unit. Therefore, the following (which represents a wire) shows the model of transformer in per-unit

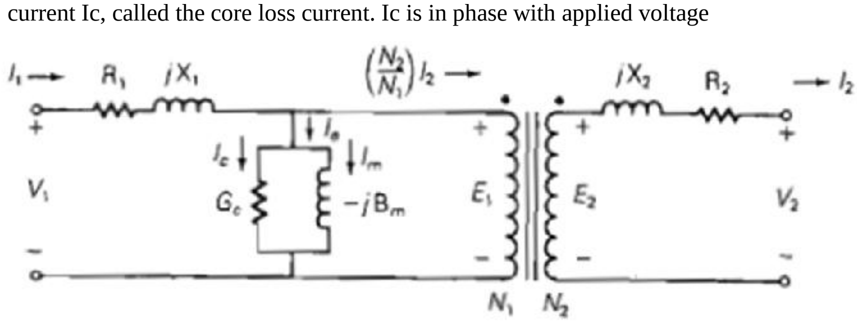


Ideal transformer

Reminder: **EQUIVALENT CIRCUITS FOR PRACTICAL TRANSFORMERS**

In practical transformers the leakage impedances of windings are modeled by series impedances Z=R+jX at both sides of transformer. The impact of exciting (magnetizing) current is model by a shunt inductance and core loss which is modeled by a resistor in parallel with a magnetizing inductance

The magnetizing current is the current that flows in the primary winding when the primary voltage is applied with the secondary unloaded(i.e. open circuit). Magnetizing current lags the applied voltage by 90^0 and can be represented by a shunt inductor with susceptance B_m mhos.
The shunt branch that is represented by a resistor with conductance Gc mhos carries a current Ic, called the core loss current. Ic is in phase with applied voltage

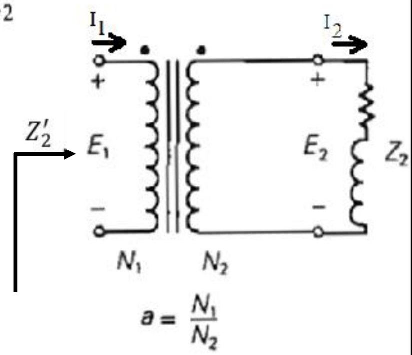


Reminder: Equivalent of impedance seen from primary side of transformer

$$Z_2 = \frac{E_2}{I_2}$$

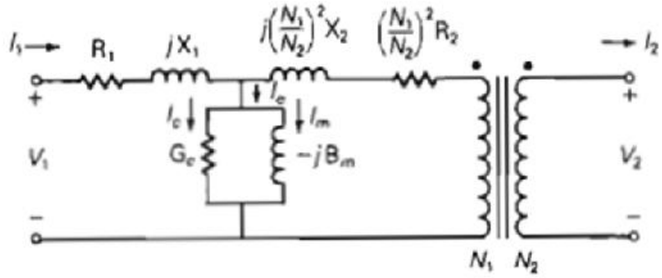
Z_2' which is the impedance seen from primary side of the transformer is

$$Z_2' = \frac{E_1}{I_1} = \frac{a_t E_2}{I_2/a_t} = a_t^2 Z_2 = \left(\frac{N_1}{N_2}\right)^2 Z_2$$



Reminder: **EQUIVALENT CIRCUITS FOR PRACTICAL TRANSFORMERS**

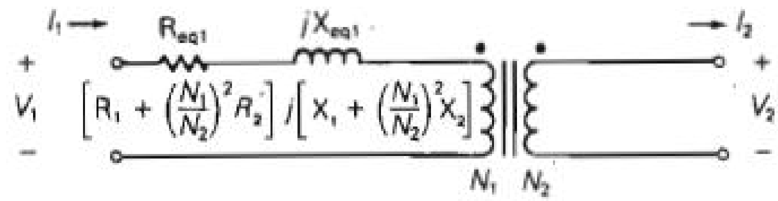
Therefore, the impedance of the secondary side of transformer can be moved to the primary side using the equation that is mentioned in previous slide



(a) R_2 and X_2 are referred to winding 1

Reminder: **EQUIVALENT CIRCUITS FOR PRACTICAL TRANSFORMERS**

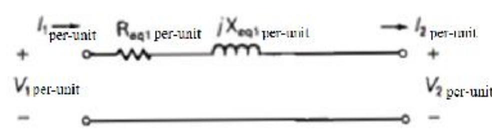
Since the exciting current is usually less than 5% of rated current, neglecting it in power system studies is often valid.



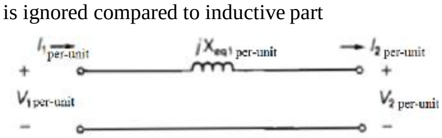
(b) Neglecting exciting current

Reminder: **EQUIVALENT CIRCUITS FOR PRACTICAL TRANSFORMERS**

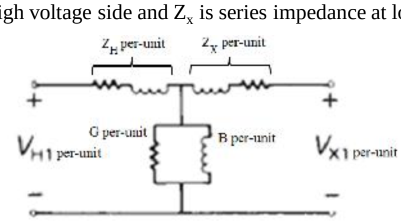
Now if the impedances are represented in per-unit values, the equivalent of the practical transformer in per-unit system becomes the following. It should be noted as we discussed in few slides before, the equivalent of ideal transformer is just a wire. That is why the ideal transformer part of the previous slide is replaced by a wire.



Usually resistive part is ignored compared to inductive part

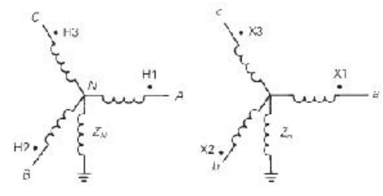


If the shunt elements are not ignored, the equivalent of practical transform is as follows. Z_H is series impedance at high voltage side and Z_x is series impedance at low voltage side

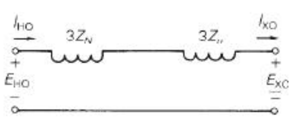


PER-UNIT SEQUENCE MODELS OF THREE-PHASE RANSFORMERS

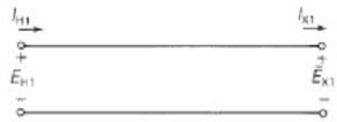
Ideal Yg–Yg transformer:



(a) Schematic representation



(b) Per-unit zero-sequence network



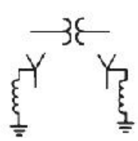
(c) Per-unit positive-sequence network

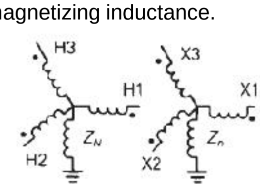
(d) Per-unit negative-sequence network

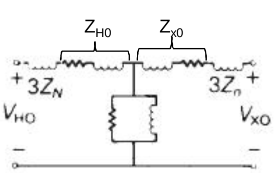
Practical Yg–Yg transformer:

The leakage impedances of windings are modeled by series impedances . We assume the phase a, b, and c windings have equal leakage impedances Z=R+jX, therefore $Z_0=Z_1=Z_2$at both sides of transformer

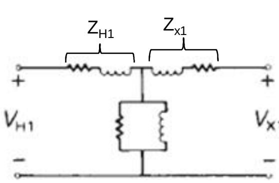
The impact of exciting (magnetizing) current is model by a shunt inductance and core loss is modeled by a resistor in parallel with a magnetizing inductance.



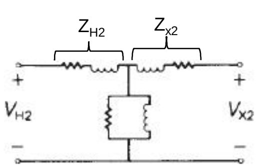




Per-unit
zero-sequence
network



Per-unit
positive-sequence
network

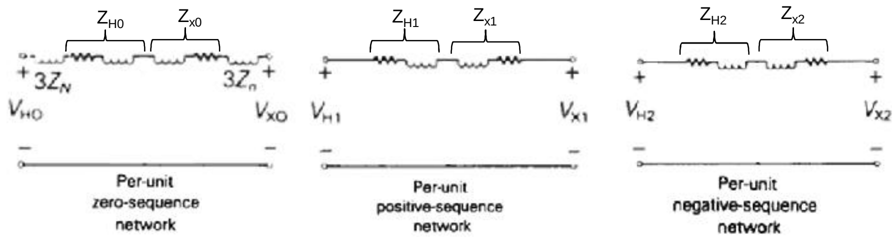


Per-unit
negative-sequence
network

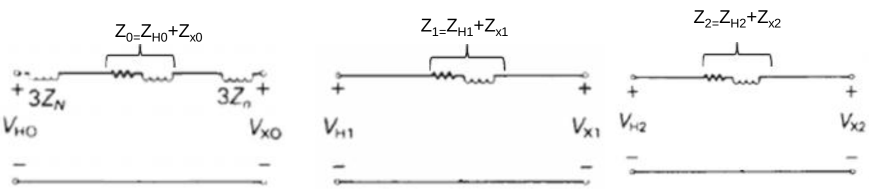
Z_H is impedance at High voltage side in per-unit and Zx is impedance of low voltage side in per-unit.

Practical Yg–Yg transformer:

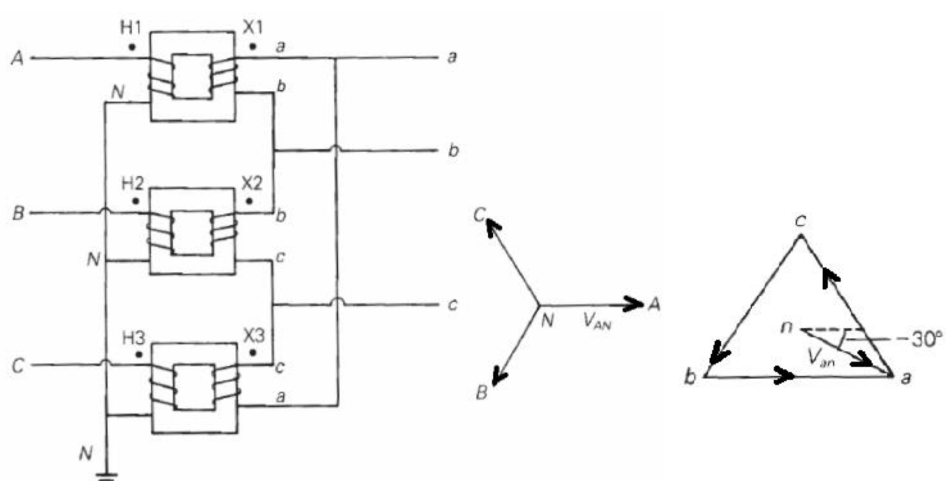
Usually shunt element is ignored



Instead of having impedance of each side separately, we can add them up and have the total equivalent impedance. So, when some one provides the impedance of the transformer, he/she means the total impedance (summation of primary and secondary impedances) otherwise he/she explicitly mentions the value of each side. Therefore, we will use the followings as the equivalent of Yg-Yg transformers



Positive and negative sequence of Yg-Δ connected three-phase transformer



In the above figure, V_{ab} is in phase with V_{AN} , and V_{bc} is in phase with V_{BN} and V_{ca} is in phase with $V_{CN.}$ This is because V_{AN} and V_{ab} are the primary and secondary sides of the first transformer, and V_{BN} and V_{bc} are the primary and secondary sides of the second transformer, V_{CN} and V_{ca} are the primary and secondary sides of the third transformer.

The labeling of the windings and the schematic representation are in accordance with the American standard, which is as follows:

In either a Y–Δ or Δ–Y transformer, positive-sequence quantities on the high-voltage side shall lead their corresponding quantities on the low-voltage side by 30.

As shown in the figure of previous slide, V_{AN} leads V_{an} by 30

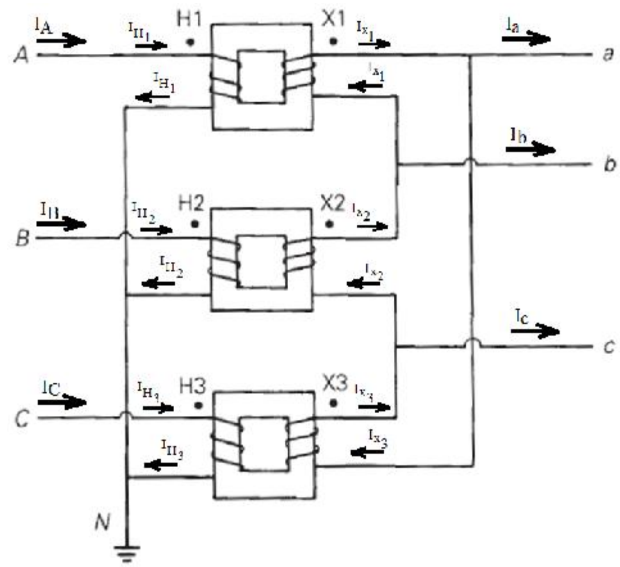
According to above statement
V_{AN} leads V_{an} by 30
and
I_A leads I_a by 30

For the negative sequence (in which the direction of phasor rotation is opposite of the direction of rotation of positive sequence) the following statement holds

In either a Y–Δ or Δ–Y transformer, negative-sequence quantities on the high-voltage side shall lead their corresponding quantities on the low-voltage side by -30.

According to above statement
V_{AN} leads V_{an} by -30
and
I_A leads I_a by -30

Zero sequence of Yg-Δ connected three-phase transformer



In above figure $I_{X_1} = \frac{N_H}{N_X} I_{H_1}$, therefore they are in phase. The same argument holds for I_{X_2} *and* I_{H_2}, as well as I_{X_3} *and* I_{H_3}.
According to above figure,
$I_a = I_{X_1} - I_{X_3} = \frac{N_H}{N_X}(I_{H_1} - I_{H_3})$
$I_b = I_{X_2} - I_{X_1} = \frac{N_H}{N_X}(I_{H_2} - I_{H_1})$
$I_c = I_{X_3} - I_{X_2} = \frac{N_H}{N_X}(I_{H_3} - I_{H_2})$

Zero sequence of Yg-Δ connected three-phase transformer

Now, if zero sequence currents are applied at the H side, $I_{H_1}=I_{H_2}=I_{H_3}$ because by definition zero sequence components are three equal vectors (their amplitude and phase angles are the same)

Therefore,

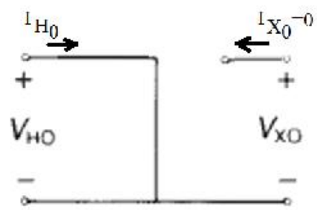
$$I_a = I_{X_1} - I_{X_3} = \frac{N_H}{N_X}(I_{H_1} - I_{H_3}) = \frac{N_H}{N_X}(I_{H_1} - I_{H_1}) = 0$$

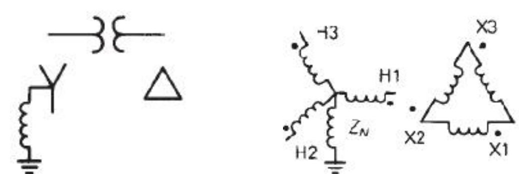
$$I_b = I_{X_2} - I_{X_1} = \frac{N_H}{N_X}(I_{H_2} - I_{H_1}) = \frac{N_H}{N_X}(I_{H_2} - I_{H_2}) = 0$$

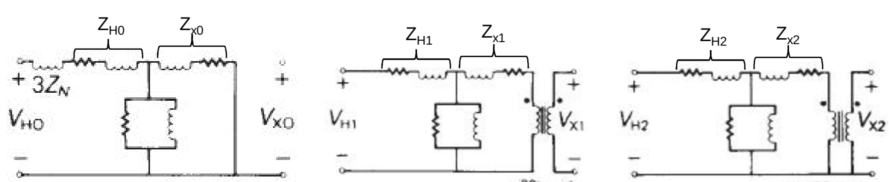
$$I_c = I_{X_3} - I_{X_2} = \frac{N_H}{N_X}(I_{H_3} - I_{H_2}) = \frac{N_H}{N_X}(I_{H_3} - I_{H_3}) = 0$$

$$I_0 = \frac{1}{3}(I_a + I_b + I_c) = 0$$

Which means the Δ side of transformer appears as an open circuit in zero sequence domain. The following shows this schematically



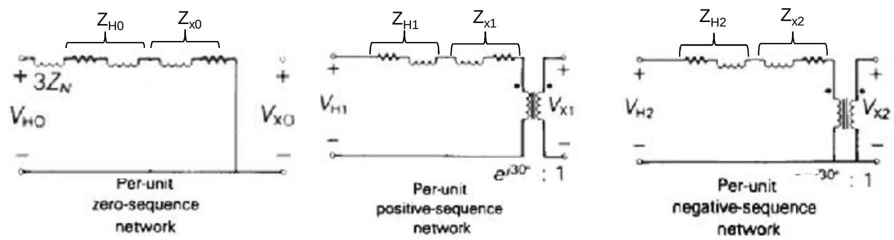




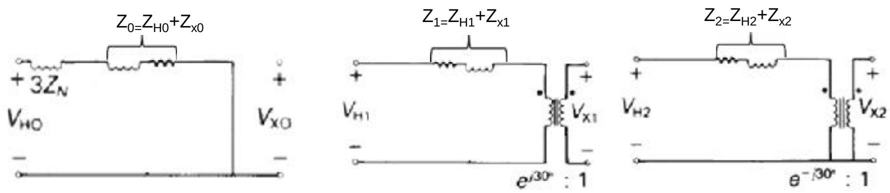
Z_H is impedance at High voltage side in per-unit and Zx is impedance of low voltage side in per-unit.

The positive-sequence voltages and currents on the high-voltage side of the Y–Δ transformer lead the corresponding quantities on the low-voltage side by 30.
For negative sequence, the high-voltage quantities lag by 30.

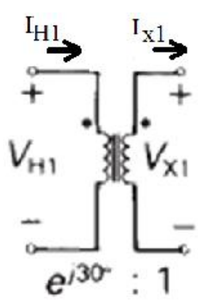
Usually shunt element is ignored



Instead of having impedance of each side separately, we can add them up and have the total equivalent impedance. So, when some one provides the impedance of the transformer, he/she means the total impedance (summation of primary and secondary impedances) otherwise he/she explicitly mentions the value of each side. Therefore, we will use the followings as the equivalent of Yg-Δ transformers



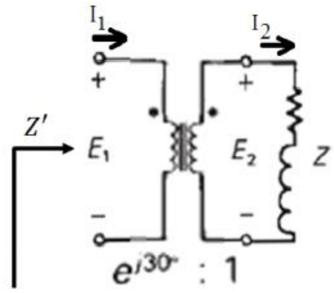
It is important to mention that the following symbol (phase shifter) means you have to add 30 degrees to voltage and current phasors when you go from X side to H side (it does not change the amplitudes as the ratio is 1)



$$I_{H1} = I_{X1}\angle + 30$$

$$V_{H1} = V_{X1}\angle + 30$$

Example:
In the following network determine Z'

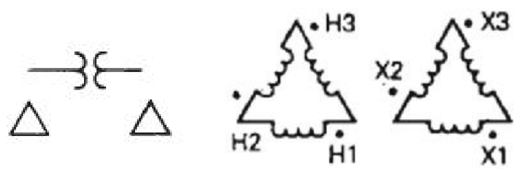


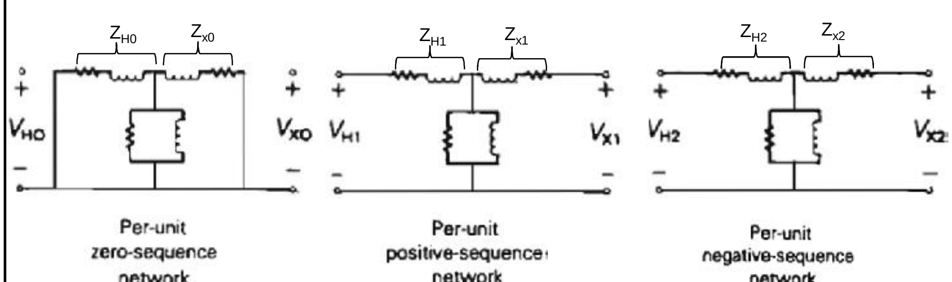
$$Z = \frac{E_2}{I_2}$$

Z_2' which is the impedance seen from left hand side is

$$Z' = \frac{E_1}{I_1} = \frac{E_2\angle+30}{I_2\angle+30} = \frac{E_2}{I_2} = Z$$

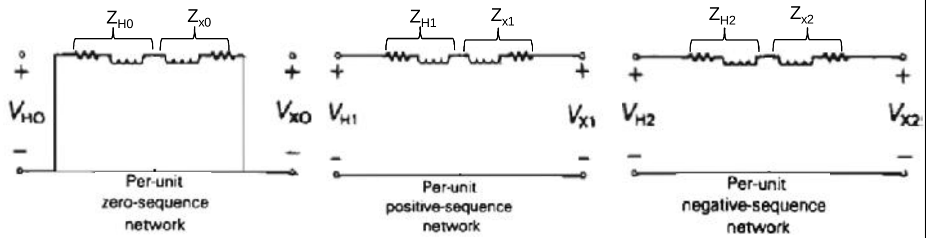
which means the above symbol does not change impedance values. It is because it change phase angle of voltage by 30 degrees and phase angle of current by 30 degrees. Z=$\frac{V}{I}$, *therefore the phase angle change of the voltage is canceled out by phase angle change of the current*



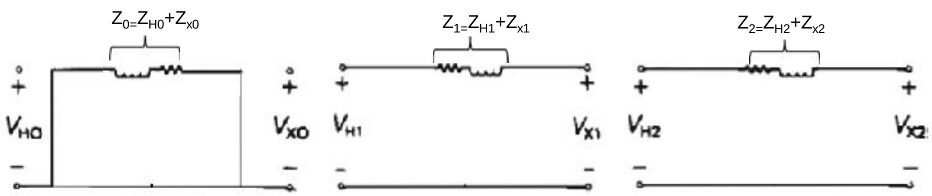


Z_H is impedance at High voltage side in per-unit and Zx is impedance of low voltage side in per-unit.

Usually shunt element is ignored

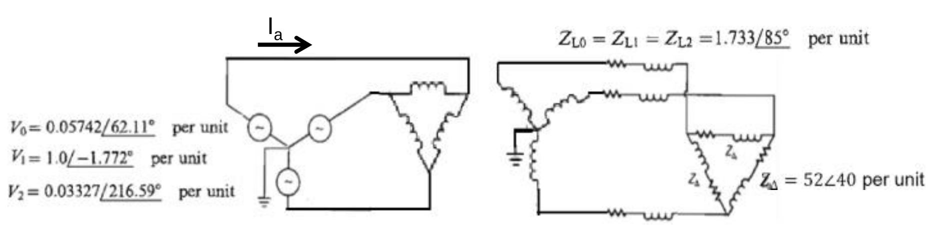


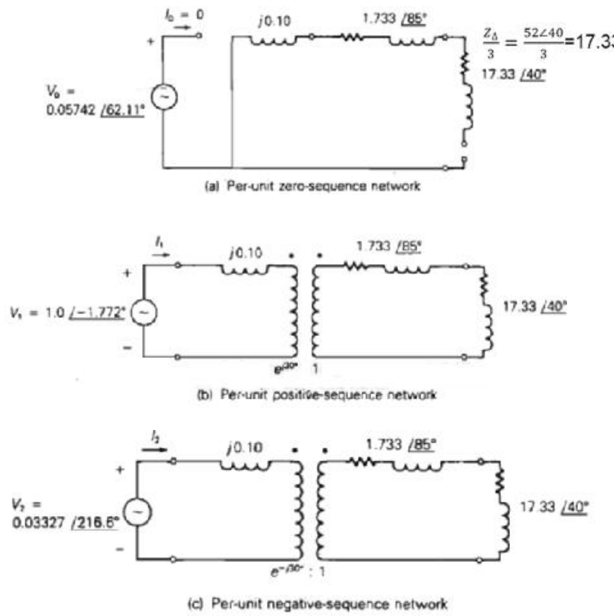
Instead of having impedance of each side separately, we can add them up and have the total equivalent impedance. So, when some one provides the impedance of the transformer, he/she means the total impedance (summation of primary and secondary impedances) otherwise he/she explicitly mentions the value of each side. Therefore, we will use the followings as the equivalent of Δ-Δ transormers



EXAMPLE

In the following network, the transformer leakage reactance is X_{eq}=0.10 per unit; winding resistances and exciting current are neglected. Calculate the phase a source current I_a in per unit





The positive-sequence voltages and currents on the high-voltage side of the Y–Δ transformer lead the corresponding quantities on the low-voltage side by 30.
For negative sequence, the high-voltage quantities lag by 30.

The per-unit sequence networks are shown the sequence components of the source currents are

$$I_0 = 0$$

$$I_1 = \frac{V_1}{jX_{eq} + Z_{L1} + Z_{load1}} = \frac{1.0\angle -1.772°}{j0.10 + 1.733\angle 85° + 17.33\angle 40°}$$

$$= \frac{1.0\angle -1.772°}{13.43 + j12.97} = \frac{1.0\angle -1.772°}{18.67\angle 44.0°} = 0.05356\angle -45.77° \quad \text{per unit}$$

$$I_2 = \frac{V_2}{jX_{eq} + Z_{L2} + Z_{load2}} = \frac{0.03327\angle 216.59°}{18.67\angle 44.0°}$$

$$= 0.001782\angle 172.59° \quad \text{per unit}$$

The phase a source current is then

$$I_a = I_0 + I_1 + I_2$$

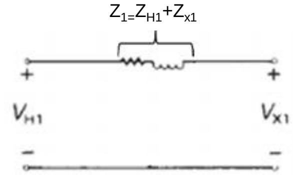
$$= 0 + 0.05356\angle -45.77° + 0.001782\angle 172.59°$$

$$= 0.03511 - j0.03764 = 0.05216\angle -46.19° \quad \text{per unit}$$

Y-Y and Yg-Y connected three-phase transformer

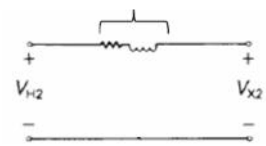
Positive sequence and Negative sequence network equivalent of Yg-Y, and Y-Y connected three phase transformers are exactly similar to Yg-Yg transformers. Because the way the neutral point is grounded does not have any impacts on positive sequence and negative sequence networks. It only affected zero sequence network. Therefore, the positive sequence and negative sequence equivalent network of Yg-Y, and Y-Y connected transformers are

$Z_{1=}Z_{H1}+Z_{x1}$



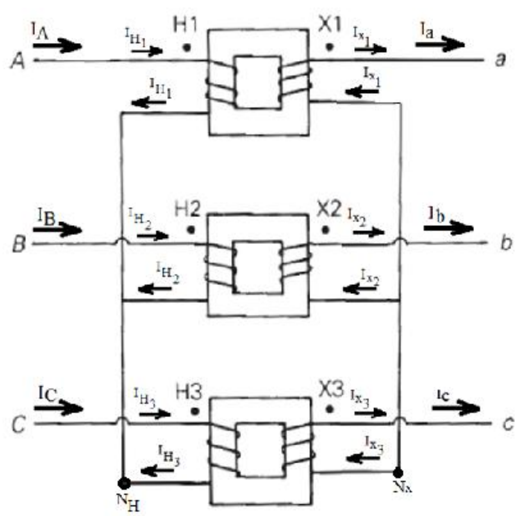
Positive sequence

$Z_{2=}Z_{H2}+Z_{x2}$



Negative sequence

Zero sequence of Y-Y connected three-phase transformer



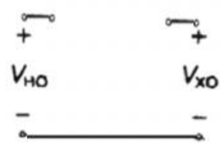
If we write KCL at $N_{H,}$ the following holds
$I_{H1}+I_{H2}+I_{H3}=0$

KCL at $N_{x,}$ gives the following equation
$I_{x1}+I_{x2}+I_{x3}=0$

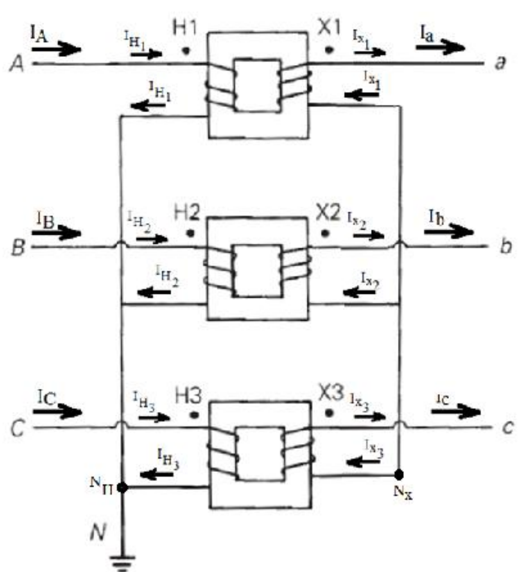
Now assume $I_{H1}=I_{H2}=I_{H3}$.It means zero sequence phasors are applied to high voltage side (Recall that by definition zero sequence components are 3 phasors that have same amplitide and phase angle.)
$I_{H1}+I_{H2}+I_{H3}=0 \rightarrow I_{H1}+I_{H1}+I_{H1}=0 \rightarrow 3I_{H1}=0 \rightarrow I_{H1}=0 \rightarrow$

$I_{H3}=I_{H2}=I_{H1}=0$ (1)
Moreover, as $I_{x1} = \frac{N_H}{N_x}(I_{H1}),\ I_{x2} = \frac{N_H}{N_x}(I_{H2}),\ I_{x3} = \frac{N_H}{N_x}(I_{H3}),$
$I_{x1} = I_{x2} = I_{x3}=0$ (2)

(1) and (2) mean no zero current can flow in primary and secondary sides of Y-Y transformers. Therefore, in zero sequence domain the Y-Y connected transformer appears as open circuit in primary and secondary sides



Zero sequence of Yg-Y connected three-phase transformer



If we write KCL at N_x, the following equation holds
$I_{x1}+I_{x2}+I_{x3}=0$

as $I_{x1} = \frac{N_H}{N_x}(I_{H1})$, $I_{x2} = \frac{N_H}{N_x}(I_{H2})$, $I_{x3} = \frac{N_H}{N_x}(I_{H3})$, we can rewrite the above equation as

$$\frac{N_H}{N_x}I_{H1} + \frac{N_H}{N_x}I_{H2} + \frac{N_H}{N_x}I_{H2} = 0$$

Now assume $I_{H1}=I_{H2}=I_{H3}$. It means zero sequence phasors are applied to high voltage side (Recall that by definition zero sequence components are 3 phasors that have same amplitide and phase angle.) The above equation becomes

$$\frac{N_H}{N_x}I_{H1} + \frac{N_H}{N_x}I_{H1} + \frac{N_H}{N_x}I_{H1} = 0 \rightarrow 3\frac{N_H}{N_x}I_{H1} = 0 \rightarrow I_{H1} = 0 \rightarrow$$

$$I_{H3} = I_{H2} = I_{H1} = 0 \quad (1)$$

And as $I_{x1} = \frac{N_H}{N_x}(I_{H1})$, $I_{x2} = \frac{N_H}{N_x}(I_{H2})$, $I_{x3} = \frac{N_H}{N_x}(I_{H3})$,
$I_{x1} = I_{x2} = I_{x3}=0$ (2)

(1) and (2) mean no zero current can flow in primary and secondary sides of Yg-Y transformers.
Therefore, in zero sequence domain the Yg-Y connected transformer appears as open circuit in primary and secondary sides