EE493 Protection of Power Systems I

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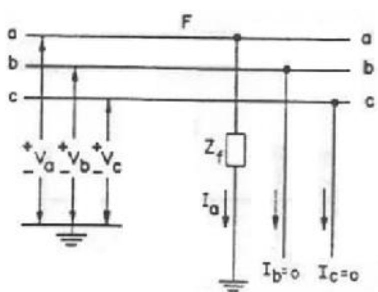
Chapter 9 of "Power System Analysis and Design", Fifth Edition, 2012, J. D. Glover, M. S. Saema, T. J. Overbye

Single Line-to-Ground (SLG) Fault

1. Circuit diagram



Single Line-to-Ground (SLG) Fault

2. Boundary conditions

$$I_b = I_c = 0$$

$$V_a = Z_f \cdot I_a$$

Single Line-to-Ground (SLG) Fault

3. Transformation

$$I_{0,1,2} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \cdot \begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix} = \frac{1}{3} I_a \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

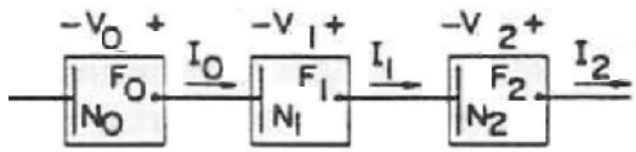
All sequence currents are equal, we have:

$$V_a = Z_f \cdot I_a = 3Z_f \cdot I_1$$

$$V_0 + V_1 + V_2 = 3Z_f \cdot I_1$$

Single Line-to-Ground (SLG) Fault

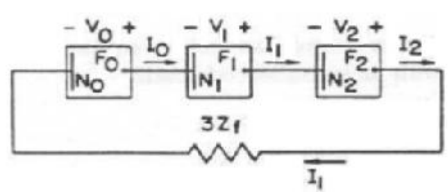
4. Sequence currents: the sequence currents are equal so the sequence networks must be connected in series:



Single Line-to-Ground (SLG) Fault

5. Sequence voltages

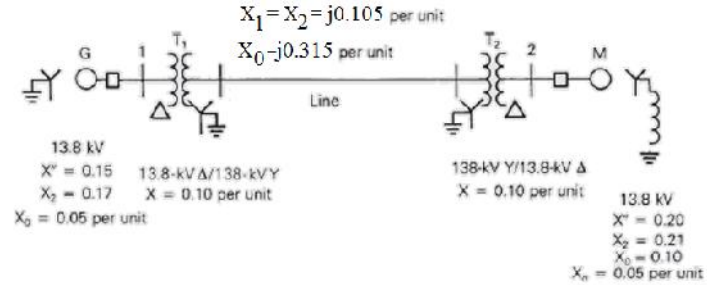
All voltages added up are equal to $3Z_f \cdot I_1$, hence the connection of sequence networks closes in a loop across $3Z_f$



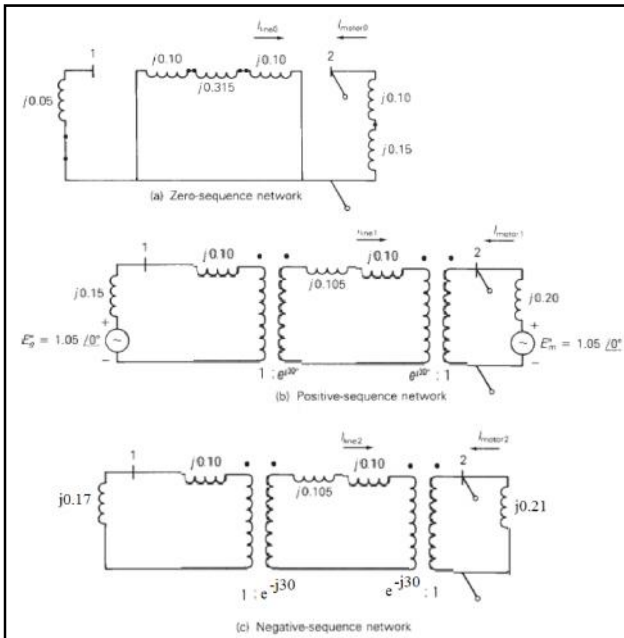
With the sequence network connection, we compute the sequence currents:

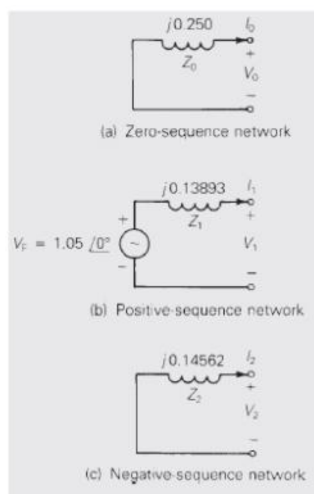
$$I_0 = I_1 = I_2 = \frac{V_F}{Z_0 + Z_1 + Z_2 + 3Z_f}$$

Example:

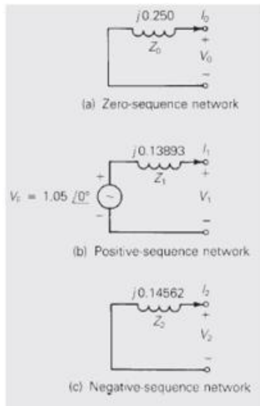


(a)Draw the per-unit zero-, positive-, and negative sequence networks (b) Reduce the sequence networks to their Thevenin equivalents, as viewed from bus 2. Prefault voltage is $V_F = 1.05\angle 0^\circ$ per unit. Pre-fault load current are neglected.





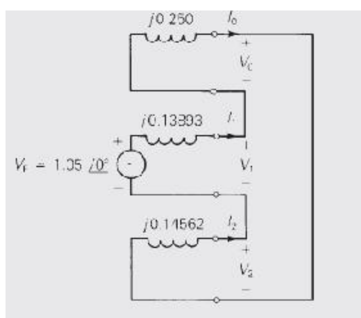
the positive-sequence Thévenin impedance at bus 2 is the motor impedance $j0.20$, as seen to the right of bus 2, in parallel with $j(0.15 + 0.10 + 0.105 + 0.10) = j0.455$, as seen to the left; the parallel combination is $j0.20 /\!/ j0.455 = j0.13893$ per unit. Similarly, the negative-sequence Thévenin impedance is $j0.21 /\!/ j(0.17 + 0.10 + 0.105 + 0.10) = j0.21 /\!/ j0.475 = j0.14562$ per unit. In the zero-sequence network of Figure 9.4, the Thévenin impedance at bus 2 consists only of $j(0.10 + 0.15) = j0.25$ per unit, as seen to the right of bus 2; due to the Δ connection of transformer T_2, the zero-sequence network looking to the left of bus 2 is open.



(a) Zero-sequence network

(b) Positive-sequence network

(c) Negative-sequence network



$$I_0 = I_1 = I_2 = \frac{1.05/0^\circ}{j(0.25 + 0.13893 + 0.14562)}$$
$$= \frac{1.05}{j0.53455} = -j1.96427 \quad \text{per unit}$$

$$I_a = I_0 + I_1 + I_2 = 3I_1$$

$I_a'' \quad 3(-j1.96427) \quad -j5.8928$ per unit

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ V_F \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1.05\underline{/0^\circ} \\ 0 \end{bmatrix} - \begin{bmatrix} j0.25 & 0 & 0 \\ 0 & j0.13893 & 0 \\ 0 & 0 & j0.14562 \end{bmatrix} \begin{bmatrix} -j1.96427 \\ -j1.96427 \\ -j1.96427 \end{bmatrix}$$

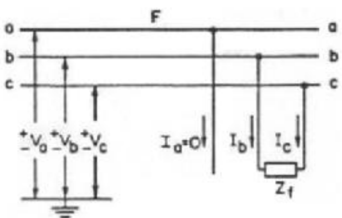
$$= \begin{bmatrix} -0.49107 \\ 0.77710 \\ -0.28604 \end{bmatrix} \quad \text{per unit}$$

Transforming to the phase domain, the line-to-ground voltages at faulted bus 2 are

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.49107 \\ 0.77710 \\ -0.28604 \end{bmatrix} = \begin{bmatrix} 0 \\ 1.179\underline{/231.3^\circ} \\ 1.179\underline{/128.7^\circ} \end{bmatrix} \quad \text{per unit}$$

Line-to-Line (LL) Fault

1. Circuit diagram



Line-to-Line (LL) Fault

2. Boundary conditions

$$I_a = 0$$
$$I_b = -I_c$$
$$V_b - V_c = Z_f \cdot I_b$$

Line-to-Line (LL) Fault

3. Transformation

$$I_{0,1,2} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \cdot \begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix} = \frac{j}{\sqrt{3}} I_b \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

The sequence currents:

$$I_0 = 0, I_1 = -I_2$$

$$V_b - V_c = Z_f \cdot I_b$$

$$(V_0 + a^2 V_1 + aV_2) - (V_0 + aV_1 + a^2 V_2) = Z_F(I_0 + a^2 I_1 + aI_2)$$

$$(a^2 - a)V_1 - (a^2 - a)V_2 = Z_F(a^2 - a)I_1$$

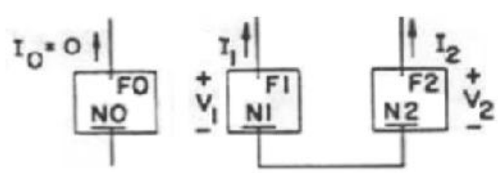
The sequence voltages:

$$Z_f \cdot I_1 = V_1 - V_2$$

Line-to-Line (LL) Fault

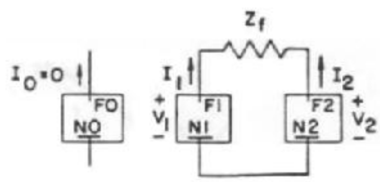
4. Sequence currents

$$I_0 = 0, I_1 = -I_2$$



Line-to-Line (LL) Fault

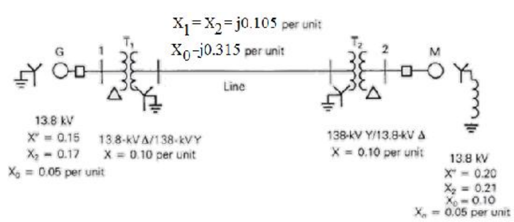
5. Sequence voltages

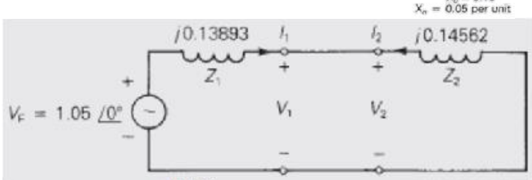


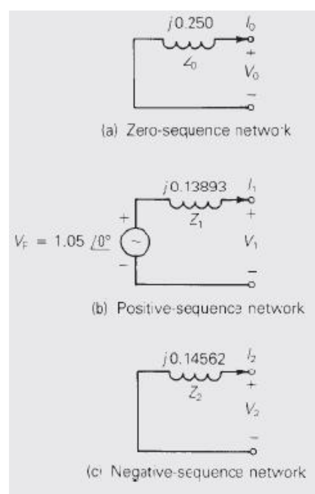
With the sequence network connection, we compute the sequence currents:

$$I_1 = \frac{V_F}{Z_1 + Z_2 + Z_f}$$

Calculate the subtransient fault current in per-unit and in kA for a bolted line-to-line fault from phase b to c at bus 2







$$I_1 = -I_2 = \frac{1.05\angle 0°}{j(0.13893 + 0.14562)} = 3.690\angle -90°$$

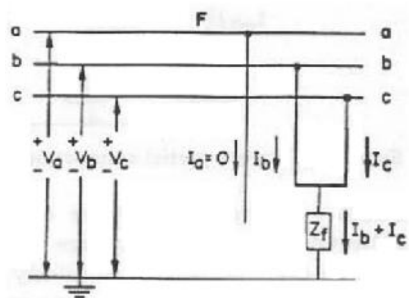
$$I_0 = 0$$

$$I_b = I_0 + a^2 I_1 + a I_2 = (a^2 - a) I_1 \quad = -j\sqrt{3} I_1$$

$$I_b'' = (-j\sqrt{3})(3.690\angle -90°) = -6.391 = 6.391\angle 180° \quad \text{per unit}$$

Double Line-to-Ground (2LG)Fault

1. Circuit diagram



Double Line-to-Ground (2LG)Fault

2. Boundary conditions

$$I_a = 0$$

$$V_b = V_c = (I_b + I_c)Z_f = 3I_0 Z_f$$

Double Line-to-Ground (2LG)Fault

3. Transformation

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \times \begin{bmatrix} V_a \\ 3I_{a0}Z_f \\ 3I_{a0}Z_f \end{bmatrix}$$

$$V_0 = \tfrac{1}{3}(V_a + 6I_0Z_f) = \tfrac{1}{3}(V_a + 6I_0Z_f) = \tfrac{1}{3}V_a + 2I_0Z_f$$

$$V_1 = \tfrac{1}{3}\left[(V_a + (a + a^2)(3I_0Z_f)\right]$$

$$V_2 = \tfrac{1}{3}\left[(V_a + (a^2 + a)(3I_0Z_f)\right]$$

$$V_1 = V_2 = \tfrac{1}{3}V_a - I_0Z_f$$

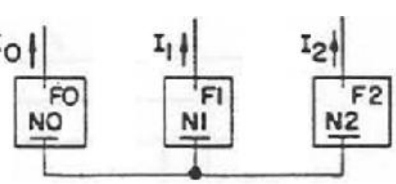
$$V_0 = V_1 + 3I_0Z_f$$

$$V_0 - V_1 = 3I_0Z_f$$

Double Line-to-Ground (2LG)Fault

4. Sequence currents

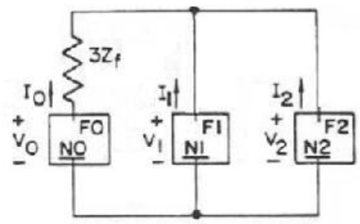
$$I_a = 0 = (I_0 + I_1 + I_2)$$



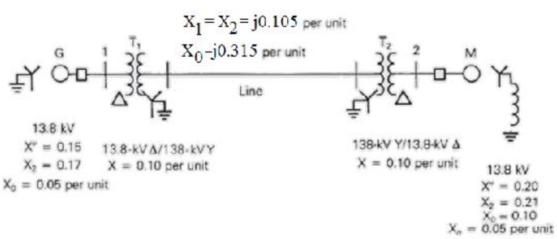
Double Line-to-Ground (2LG)Fault

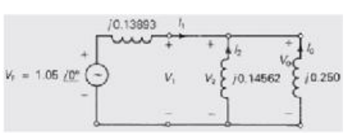
5. Sequence voltages

$$V_0 - V_1 = 3I_0 Z_f$$



Calculate (a) the subtransient fault current in each phase, (b) neutral fault current

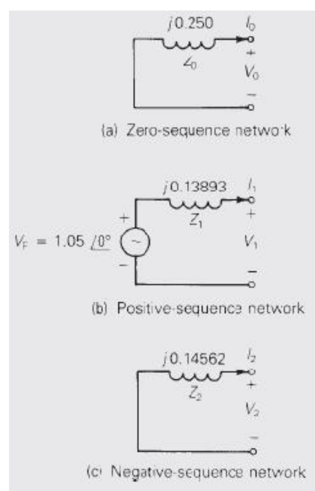




$$I_1 = \frac{1.05\underline{/0^\circ}}{j\left[0.13893 + \dfrac{(0.14562)(0.25)}{0.14562 + 0.25}\right]} = \frac{1.05\underline{/0^\circ}}{j0.23095} = -j4.5464 \quad \text{per unit}$$

$$I_2 = (+j4.5464)\left(\frac{0.25}{0.25 + 0.14562}\right) = j2.8730 \quad \text{per unit}$$

$$I_0 = (+j4.5464)\left(\frac{0.14562}{0.25 + 0.14562}\right) = j1.6734 \quad \text{per unit}$$



Transforming to the phase domain, the subtransient fault currents are:

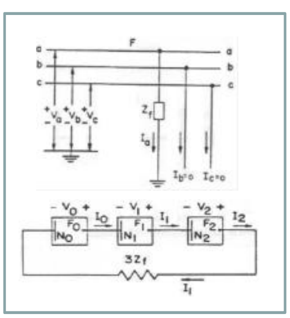
$$\begin{bmatrix} I_a'' \\ I_b'' \\ I_c'' \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} +j1.6734 \\ -j4.5464 \\ +j2.8730 \end{bmatrix} = \begin{bmatrix} 0 \\ 6.8983\underline{/158.66^\circ} \\ 6.8983\underline{/21.34^\circ} \end{bmatrix} \quad \text{per unit}$$

b. The neutral fault current is

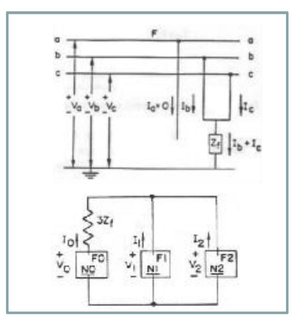
$$I_n = (I_b'' + I_c'') = 3I_0 = j5.0202 \quad \text{per unit}$$

Summary of unsymmetrical faults

L-G fault



L-L-G fault



L-L fault