GPS and Differential GPS: An Error Model for Sensor Simulation

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Abstract - This paper describes a software package that models various Global Positioning System (GPS) and Differential GPS receiver configurations. The model produces position errors in a locally-level coordinate system. The errors are statistically based on the errors associated with pseudorange measurements. An orbital model of the GPS satellites simulates the effects of satellite geometry on the position errors. The error model is used in the Real-Time Simulator at the NASA-Langley Research Center.

INTRODUCTION

The advent of the Global Positioning System (GPS) provided the aviation community with a new navigation sensor for enroute and terminal area operations. Research has proven the capability of GPS and Differential GPS (DGPS) for precision approaches and surface operations. Future avionics research will involve GPS in additional products.

To provide a platform for future research activities, the NASA-Langley Research Center required a GPS model for its real-time simulator. Position errors for various GPS and DGPS configurations are output from the sensor model. The position errors are referenced to locally-level X,Y,Z coordinates. The errors are used to "corrupt" aircraft position during simulation.

GPS position errors are determined by pseudorange errors and satellite geometry. Pseudorange errors are caused by the combined satellite errors, atmospheric errors, and receiver errors. Individual components of the pseudorange error are modeled as either a Gauss-Markov process (exponential auto-correlation) or as white noise. Pseudorange errors are dependent on the receiver used. The user may select from various stand-alone GPS or DGPS systems.

A simulation of the satellite constellation is used to determine satellite geometry. The effects from switching satellites and varying number of visible satellites are obtained from this simulation. The user can use an All in View satellite selection strategy based on a full constellation or a reduced set of satellites with a bad satellite geometry (high GDOP value).

This paper describes the pseudorange measurement models and how the pseudorange models are affected by receiver type. An explanation of the DOP calculations follow. The last section describes the software implementation of the GPS/DGPS model.

PSEUDORANGE MODELS

GPS pseudorange measurements, ρ, are noisy estimates of the range, r, from satellite to receiver. Pseudorange values are available from code and carrier phase measurements. The model for code pseudorange measurements is

$$\rho_{code}=r+\delta_{eph}+\delta_{iono}+\delta_{trop}+\delta_{SA}-cT+\delta_{mp}+v_{rcvr} \tag{1}$$

where δ_{eph} is the satellite ephemeris error, δ_{iono} is the ionosphere error, δ_{trop} is the troposphere error, δ_{SA} is the Selective Availability (SA) error, c is the speed of light, T is the receiver clock error, δ_{mp} is the multipath error, and v_{rcvr} is the receiver measurement noise.

The model for the L-band carrier phase measurement has similar error terms as the code pseudorange. The ionospheric delay is negative for phase and positive for code. The same atmospheric and satellite clock corrections are made for carrier phase. After multiplying by the carrier wavelength, λ, the carrier phase model (in meters) is

$$\lambda\phi=r+\delta_{eph}-\delta_{iono}+\delta_{trop}+\delta_{SA}-cT+\delta_{mp}+v_{rcvr}+\lambda N \tag{2}$$

The λN represents the integer ambiguity.

Gauss-Markov noise

The ephemeris, ionosphere, troposphere, Selective Availability, and multipath errors are modeled as Gauss-Markov processes. These processes have an exponential autocorrelation function with variance, σ^2, and time constant, $1/\beta$.

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$$R(\tau)=\sigma^2 e^{-\beta|\tau|} \quad (3)$$

The Gauss-Markov terms are modeled by

$$x_{k+1}=e^{-\beta*\Delta T}x_k+w_k \quad (4)$$

where x_k is the parameter being simulated, w_k is Gaussian white noise, and ΔT is the sample time. The standard deviation, σ, and time constant, $1/\beta$, are listed in Table 1 for the Gauss-Markov noise terms [1][8].

Error Parameter	Std. Dev. (meters)	Time (sec)
Ephemeris	3.0	1800
Ionosphere	5.0	1800
Troposphere	2.0	3600
Multipath, C/A standard	5.0	600
Multipath, C/A narrow	0.25	600
Multipath, P	1.0	600
Multipath, L1 carrier	0.048	600
Selective Availability	30.0	180

Table 1. Parameters for Gauss-Markov error sources

Multipath error is limited by the correlator width for code measurements. Carrier phase multipath is limited by the carrier wavelength.

Measurement noise

The receiver measurement noise term, v_{rcvr}, is the accuracy which the code or carrier can be tracked. The C/A and P code measurements can be tracked to within 1% of the correlator width. The carrier signal is tracked to within 1% of its wavelength. Measurement noise is modeled as Gaussian white noise. Measurement noise parameters are listed in Table 2.

Model	Std. Deviation (meters)
C/A code, standard correlator	3.0
C/A code, narrow correlator	0.1
P code	0.3
L1 Carrier	0.0019

Table 2. Measurement noise parameters

GPS RECEIVER MODELS

Each receiver model is susceptible to different pseudorange error sources. Assuming that all the error terms are independent of each other, the autocorrelation for the accumulated pseudorange error is the sum of the individual autocorrelation terms.

C/A code A C/A code receiver is affected by all the error terms in (1). Assuming a standard chip wide correlator, the total pseudorange noise (not including SA) has the autocorrelation shown in Figure 1. The total correlated noise can be modeled as a single Gauss-Markov process with a variance of 63 m^2 (1σ = 7.94 m) and a time constant of 1100 seconds. When SA is on, the total correlated noise is dominated by the SA noise as shown in Figure 2.

When a C/A code receiver uses a narrow code correlator, the multipath and measurement errors are reduced.

P code. Dual frequency P-code receivers can eliminate the ionosphere error. Multipath and measurement errors are also reduced with a standard P code correlator since the standard P code correlator is narrower than the C/A code correlator.

Carrier phase smoothing. Carrier phase smoothing receivers combine code and carrier phase measurements. Differencing the carrier phase is a good estimate of velocity without the integer ambiguity. The code pseudorange position is blended with the position derived by integrating velocity. A blending equation [8] is

$$r_k=W_k*\rho_k+(1-W_k)*(r_{k-1}+\phi_k-\phi_{k-1}) \quad (5)$$

The corrected pseudorange is r and the filter weighting value is W. W is initialized to 1 and then gradually reduced to a final value of 0.01.

The non-SA noise parameters for the stand-alone GPS models are given in Table 3. The different receiver types can significantly reduce the non-SA noise. The receiver errors are dominated though by SA noise when SA is on. The characteristics used for SA noise are found in Table 1.

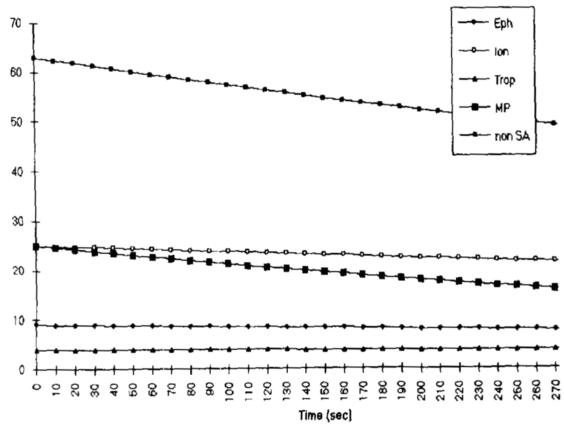

Figure 1. Autocorrelation of C/A code pseudorange errors (not including SA)

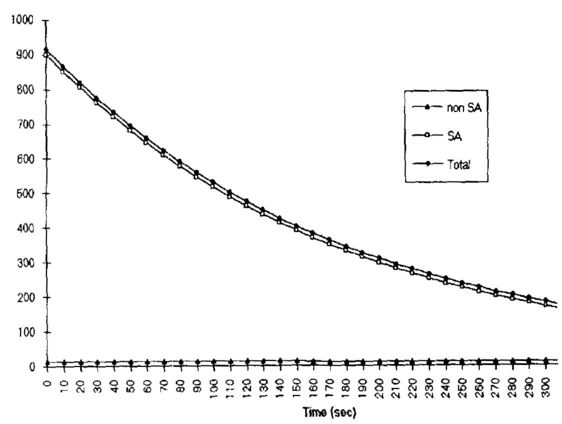

Figure 2. Autocorrelation of C/A code pseudorange errors with SA on.

Receiver Model	Gauss-Markov Noise		Measurement Noise (meters)
	Std. Dev. (meters)	Time (sec)	
C/A, Standard correlator	7.94	1100	3.00
C/A, Narrow correlator	6.16	1920	0.10
Two frequency P code	3.74	1845	0.30
Carrier phase smoothing	1.95	1100	0.0019

Table 3. Pseudorange error statistics for Stand-alone GPS systems with SA OFF

DIFFERENTIAL GPS MODELS

The DGPS models describe the positional error for a differential system. A base station sends corrections for code and/or carrier phase to the dynamic receiver. Orbital and atmospheric effects are negligible with baselines of less than 10-20 km are used.

For this simulation, the base receiver measurement and multipath errors are similar to those determined by Cannon and LaChapelle [8]. Multipath has a standard deviation of 0.25 meters and a 600 second time constant. Measurement error is 0.1 meters (1σ) .

The corrected code pseudorange is

$$\rho = r - c*T + (\delta_{mp} + v_{rcvr}) - (\delta_{base-mp} + v_{base-rcvr}) \tag{6}$$

The corrected carrier phase is

$$\lambda\phi = r - cT + (\lambda N + \delta_{mp} + v_{carrier})_{rcvr} - (\lambda N + \delta_{mp} + v_{carrier})_{base} \tag{7}$$

C/A and P code Differential receivers that use code measurements have pseudorange errors defined by (6). The errors are dominated by the multipath and measurement errors at the mobile receiver.

Carrier phase smoothing. One carrier-phase smoothing approach uses a complementary filter. Corrections for both code and carrier phase are transmitted to the mobile receiver. The corrected code and carrier phase are differenced. Filtering the difference, an estimate for integer ambiguity, $\lambda\Delta N_{filter}$, is obtained. Adding the filter output to the corrected carrier phase gives

$$\lambda\phi = r - cT + (\delta_{mp} + v)_{rcvr} - (\delta_{mp} + v)_{base} + \lambda(\Delta N - \Delta N_{filter}) \tag{8}$$

The measurement and multipath errors are twice the errors associated with carrier phase measurements.

Kinematic. Kinematic positioning double differences phase observations across two receivers and two satellites. The common system and atmospheric errors are removed. The double difference equation is

$$\nabla\Delta\phi = \Delta\phi^{i+1} - \Delta\phi^{i} = \nabla\Delta r + \lambda\nabla\Delta N + \nabla\Delta\delta_{mp} + \nabla\Delta v_{carrier} \tag{9}$$

The errors are uncorrelated multipath and measurement errors. The total multipath variance is four times the multipath error of a single carrier phase.

Table 4 describes the Gauss-Markov parameters and measurement noise for the DGPS receiver models. The white measurement noise is the combined variances of the two measurement noise parameters.

DOP PARAMETERS

Dilution of precision (DOP) parameters describe how geometry affects the position errors. Pseudorange error is translated to X-axis error according to

$$\sigma_x = XDOP * \sigma_\rho \tag{10}$$

where σ_x and σ_ρ are the standard deviation of the user's X-axis and the receiver's pseudorange errors, respectively. YDOP and VDOP values are used for error along the Y and Z axes.

DOP values are based on a linearized model of the pseudorange measurement used by the receivers. The coefficients h_x, h_y, and h_z form the direction cosines

$$\Delta\rho = h_x\Delta x + h_y\Delta y + h_z\Delta z + c\Delta T + v \tag{11}$$

Receiver Model	Gauss-Markov		Meas.
	Std. Dev. (meter)	Time (sec)	Std. Dev. (meters)
C/A, standard correlator	5.006	600	3.0000
C/A, narrow correlator	0.354	600	0.1414
Two frequency P code	1.030	600	0.3160
Carrier phase smoothing	0.202	600	0.0027
Kinematic	0.096	600	0.0038

Table 4. Pseudorange error statistics for DGPS systems

vector from a satellite to the receiver. Δx, Δy, Δz and $c\Delta T$ are the errors in the assumed 4-dimensional position. In a least squares approach to this multiple regression, the covariance matrix formed from the h_x, h_y, and h_z vectors from all satellites in view describes how accurately Δx, Δy, Δz. and $c\Delta T$ can be determined. The major diagonal of thecovariance matrix contains the error variance for Δx, Δy, Δz. and $c\Delta T$, respectively. The DOP value for any dimension is the square root of its error variance.

Other DOP parameters used in the simulation are HDOP and GDOP. HDOP describes the radial error in the X-Y plane and GDOP is the three dimensional radial error plus time error.

SOFTWARE IMPLEMENTATION

The GPS software routine provides independent error values for the X, Y, and Z axes. Each axis follows a similar algorithm to produce the errors. Three independent random number generators are used in each axis to produce normally distributed random numbers. The first random number generator is provides the Gauss-Markov noise related to non-SA errors. The second random number generator produces the Gauss-Markov noise for SA errors. The SA model is separate from the non-SA noise to simplify any future software modifications needed to change the SA characteristics. The third random number generator produces the white measurement noise. The block diagram showing the error algorithm for a single channel is shown in Figure 3.

The GPS error model is written in four C routines. The routines are gps_init_( ), gps_( ), calcdop_( ), and rnndhv_( ). gps_init_() is called once at the beginning of the simulation to initialize all GPS variables. gps_() provides the XYZ error values. gdop() determines the DOP values. rnndhv_() is a random number routine that returns a normally distributed number with zero mean and unity variance.

The software is written in C. FORTRAN calling conventions are used so the routines can be executed from either FORTRAN or C. Global variables are used to pass aircraft parameters and GPS data. The GPS model is written for minimal execution time while producing realistic error outputs.

gps_()

gps_() is the routine that provides the XYZ errors. The formal prototype is

void gps_(GPS_mode, SA, x_error, y_error, z_error)

GPS_mode denotes the receiver type. Model types are:

1- GPS, C/A code only, standard correlator
2- GPS, C/A code only, narrow correlator
3- GPS, C/A code with carrier phase smoothing
4- GPS, P code
5- DGPS, C/A code mobile receiver, wide correlator
6- DGPS, C/A code mobile receiver, narrow correlator
7- DGPS, P code mobile receiver
8- DGPS, C/A code narrow correlator with carrier phase smoothing
9- DGPS, Carrier phase tracking (kinematic)

SA enables or disables Selective Availability. On the real-time simulator, the gps_() routine executes in approximately 220 microseconds. The routine is executed at a 1 Hz update rate.

gps_init_()

gps_init_() initializes the data parameters used in the GPS simulation. It is called once at the beginning of the simulation. The formal procedure description is

void gps_init_(mode, SelAvail, UpdatePeriod)

The mode and SelAvail input parameters are used to set the variance of the Gauss-Markov noise for the selected GPS receiver type. The different receiver types are the same as those listed for gps_().

UpdatePeriod is set to the time between calls for the gps_() routine. The value of UpdatePeriod is

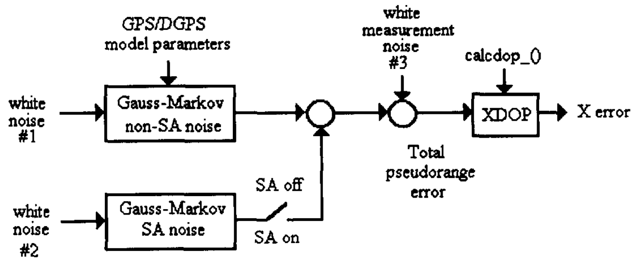

Figure 3. Algorithm for producing error in X axis. (Y and Z axes are similar.)

typically 1 second. gps_init_() uses UpdatePeriod to calculate the decay of the Gauss-Markov processes between updates.

calcdop_()

This routine determines the DOP parameters for the X, Y, and Z axes based on current receiver position and satellite geometry. The Primary 21 GPS constellation with 21 active and 3 spare satellites is simulated. The orbital parameters, argument of latitude and argument of longitude, were obtained from [2]. An orbital period of 43082 seconds (approximately 12 hours) and a Right Ascension period of 86164 seconds (approximately 24 hours) are used. A circular orbit of 26,560 km radius and a spherical earth of 6380 km radius are assumed.

The DOP values are determined using an all-in-view strategy with an elevation mask angle of 5 degrees. BadGDOP is an input that limits the number of satellites to the first 4.

The calcdop_() routine is a background task on the real-time simulator that executes in approximately 1.3 milliseconds. The receiver latitude, longitude, and altitude are inputs and the XDOP, YDOP, and VDOP values are outputs.

RESULTS

The GPS simulator outputs XYZ position errors that are dependent on the pseudorange errors and the DOP values. To examine the accuracy of the routine, the DOP values and the horizontal errors were tabulated.

DOP values for a receiver located at Denver were calculated every 10 seconds for one day. The results of the DOP analysis are shown in Table 5.

The horizontal accuracies for the different GPS and DGPS systems are listed in Table 6. The accuracy values are calculated assuming a 2σ or 95% accuracy based on the typical pseudorange errors for each receiver

	Ave	**Min**	**Max**
XDOP	0.7	0.5	0.9
YDOP	0.8	0.5	1.6
VDOP	1.1	0.7	1.8
HDOP	1.6	1.0	2.9
GDOP	2.1	1.4	4.1

Table 5. DOP values at Denver Stapleton airport for 1 day

type. To compare with other performance specifications, a Horizontal DOP value of 1.6 was used to convert pseudorange errors to horizontal errors.

SUMMARY

The GPS/DGPS sensor simulation models the error statistics for various receivers. The user can choose from seven different receiver configurations. The routine uses the code and carrier phase noise parameters to determine pseudorange errors. A model of the GPS satellite orbits is used to create the DOP values that translates pseudorange errors to XYZ errors.

GPS/DGPS RECEIVERS	SA off	SA on
GPS - C/A code wide correlator	27.2	99.8
GPS - C/A code narrow correlator	19.7	98.0
GPS - C/A code & carrier phase	6.3	32.5
GPS - P code	12.0	96.7
DGPS - C/A code wide correlator	-	18.7
DGPS - C/A code narrow correlator	-	1.2
DGPS - P code	-	3.4
DGPS - C/A code/carrier phase smoothing	-	0.6
DGPS - Carrier Phase tracking	-	0.3

Table 6. Horizontal accuracy in meters for GPS/DGPS receivers (95%) (HDOP=1.6)

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