INDIAN INSTITUTE OF TECHNOLOGY MADRAS
Department of Chemical Engineering
CH 5350 Applied Time-Series Analysis

Assignment 3

Due: Saturday, September 27, 2014 11:00 PM

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1. [PACF and ARIMA process]

   (a) Write a function in R `mypacf.R` to compute the PACF of a given series using the Durbin-Levinson algorithm.

   (b) Compare the output from your function with that of the `pacf` in R for a (i) white-noise process and (ii) an ARMA(1,1) process: $H(q^{-1}) = \dfrac{1 + c_1 q^{-1}}{1 + d_1 q^{-1}}$ at lags $l = 0, 1, 2$. Choose $c_1 = 0.6$, $d_1 = -0.5$. Also compare it with the theoretical PACF obtained from `ARMAacf`.

   (c) For the series given in `a3_q1.Rdata`, fit an ARIMA model of appropriate orders.

2. [Processes with trend]

   Consider a series $x[k] = \beta_0 + \beta_1 k + v[k]$ where $\{v[k]\}$ is a stationary ARMA process. Consider building a model for $x[k]$ in two different ways

   (a) Fitting a linear model (in time) followed by an ARMA fit to the residuals

   (b) Using the differencing of the series approach (ARMA fit to the differenced series).

   Discuss the merits and demerits of these two methods. Evaluate these two methods on the series given in `a3_q2.Rdata`. In fitting a linear model to the series, use the `lm` and `residuals` routine in R. In both cases, the final model should be satisfactory in all important aspects.

3. [Discrete-Time Fourier Series]

   (a) Determine and sketch the magnitude and phase spectra of the following periodic signals: (i) $x[k] = 4\sin(\frac{\pi(k-2)}{3})$, (ii) $x[k] = \cos(\frac{2\pi}{3}k) + \sin(\frac{2\pi}{5}k)$ and (iii) $x[k] = \cos(\frac{2\pi}{3}k)\sin(\frac{2\pi}{5}k)$

   (b) Determine the periodic signal $x[k]$ with period $N_p = 8$ if its Fourier coefficients are given by $c_n = \cos(\dfrac{\pi n}{4}) + \sin(\dfrac{3\pi n}{4})$.

   (c) Consider the periodic signal $x[k] = 1, 0, 1, 2, 3, 2$ starting from $k = 0$. Verify Parseval's theorem for this case.

4. [Spectral density of a mixed process]

   Consider the series $y[k] = x[k] + e_1[k]$ where $x[k] = \phi_1 x[k-1] + e_2[k]$ where $e_1[k]$ and $e_2[k]$ are both white noise sequences with variances $\sigma^2_{e_1}$ and $\sigma^2_{e_2}$ respectively. It is given that $|\phi_1| < 1$ and $E(x[0]) = 0$. Show that the power spectrum of $y[k]$ is of the form

   $$\Phi_{yy}(f) = \frac{\sigma^2 |1 - \theta_1 e^{-j2\pi f}|^2}{|1 - \phi_1 e^{-j2\pi f}|^2}$$

   and provide expressions for θ_1 and σ^2.