Applied TSA - Assignment 6

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1. a. Our model is OLS $y[k] = x^T[k]\theta + \mathrm{e}[k]$. Lets start from sum of squared errors (SSE). here error is the unpredictable part; the white noise signal $\mathrm{e}[k]$.

$$\begin{aligned}|\mathrm{e}[k]|^2 &= |y - x^T[k]\theta|^2 \\ SSE &= \sum_{k=0}^{N-1} |y - x^T[k]\theta|^2 \\ &= \sum_{k=0}^{N-1} (y - x^T[k]\theta)^T (y - x^T[k]\theta)\end{aligned}$$

$\frac{SSE}{n-p}$ is the unbiased estimator for variance.

$$\frac{SSE}{n-p} = \sigma_\epsilon^2 = \frac{1}{n-p} \sum_{k=0}^{N-1} (y - x^T[k]\theta)^T (y - x^T[k]\theta)$$

Now taking expectation: Take non vector elements of equation and prove.

$$\begin{aligned}E(\sigma_\epsilon^2) &= E(\frac{1}{n-p} \sum_{k=0}^{N-1} (y - x^T[k]\theta)^2 \\ &= \frac{1}{n-p}(E\left(\sum_{i=0}^{N-1} (X_i - M)^2\right)) \\ &= \frac{1}{n-p}(\sum_{i=0}^{N-1} (\sigma^2 + \mu^2) - n\left(\frac{\sigma^2}{n} + \mu^2\right)) \\ &= \frac{1}{n-p}(n(\sigma^2 + \mu^2) - n\left(\frac{\sigma^2}{n} + \mu^2\right)) \\ &= \sigma^2\end{aligned}$$

b. An AR(p) model can be written as

$$\mathrm{v}[k] + a_1 \mathrm{v}[k-1] + \cdots + a_p \mathrm{v}[k-p] = \mathrm{e}[k]$$

Where $\mathrm{e}[k] \sim \mathcal{N}(0, \sigma_e^2)$.