

2)

$$F = x_1 - x_2$$

$$L = (x_1 - x_2)$$

$$\begin{aligned} &+ \mu_1(1 - x_1 - x_2) + \\ &\mu_2(2x_1 + 3x_2 - 10) \\ &+ \mu_2(2 - 5x_1 - 2x_2) \\ &+ \mu_3(7x_2 - 2x_1 - 8) \end{aligned}$$

$$g_1: 2x_1 + 3x_2 - 10 \leq 0$$

$$g_2: 5x_1 + 2x_2 - 2 \geq 0$$

$$2 - 5x_1 - 2x_2 \leq 0$$

$$g_3: 7x_2 - 2x_1 - 8 \leq 0$$

	x_1	x_2
objective	+	-
g_1	+	+
g_2	-	-
g_3	-	+

$$\frac{\partial F}{\partial x_1} = 1$$

$$\frac{\partial F}{\partial x_2} = -1$$

Case 1)

INACTIVE ACTIVE ACTIVE
 g_1 g_2 g_3

RULE 1

RULE 2

RULE 3

Case 2)

g_2 g_1 g_3

✓

✓

✓

Case 3)

g_3 g_1 g_2

✓

✓

✓

Hence Case 3) works + Case 1) works + Case 2) works

Case 4)

	INACTIVE	INACTIVE	ACTIVE	RULE1	RULE2	RULE3
Case 4)	g_1	g_2	g_3	✓	✗	✓
Case 5)	g_1	g_3	g_2	✗	✓	✓
Case 6)	g_2	g_3	g_1	✗	✓	✓

Case 4) works.

I] Case 3

The ~~only~~ active condition that works is

$$\left. \begin{array}{l} g_1 \text{ or } g_2 \Rightarrow \text{active} \\ g_3 \Rightarrow \text{inactive} \end{array} \right\} \text{Case (3)}$$

$$\nabla L = \begin{bmatrix} 1 + 2\mu_1 - 5\mu_2 - 2\mu_3 \\ -1 + 3\mu_1 - 2\mu_2 + 7\mu_3 \end{bmatrix}$$

$$\begin{array}{l} g_3 < 0 \\ \mu_3 = 0 \end{array}$$

$$\begin{array}{l} \therefore 2\mu_1 - 5\mu_2 = -1 \times 3 \\ 3\mu_1 - 2\mu_2 = 1 \times 2 \end{array}$$

$$\begin{array}{r} 6\mu_1 - 15\mu_2 = -3 \\ -6\mu_1 + 4\mu_2 = 2 \end{array}$$

$$\begin{array}{l} -11\mu_2 = -5 \\ \mu_2 = \frac{5}{11} \end{array}$$

$$2\mu_1 = \frac{25}{11} - 1 = \frac{14}{11} \quad \mu_1 = \frac{7}{11}$$

$$\therefore \mu_2 = \frac{5}{11} \quad \mu_1 = \frac{7}{11}$$

$$\begin{array}{l} \therefore 2x_1 + 3x_2 = 10 \\ 5x_1 + 2x_2 = 2 \end{array}$$

\Rightarrow

$$\begin{array}{l} x_1 = 46/11 \\ x_2 = -14/11 \end{array}$$

does not satisfy Case 3

\rightarrow Hence case (3) doesn't work

II] Case 1) g_1 is inactive

$$\begin{aligned} 2 - 5x_1 - 2x_2 &= 0 \\ 7x_2 - 2x_1 - 8 &= 0 \end{aligned} \Rightarrow \begin{aligned} 5x_1 + 2x_2 &= 2 \times 2 \\ -2x_1 + 7x_2 &= 8 \times 5 \end{aligned}$$

$$\begin{aligned} 10x_1 + 4x_2 &= 4 \\ -10x_1 + 35x_2 &= 40 \\ \hline 39x_2 &= 44 \\ x_2 &= \frac{44}{39} \end{aligned}$$

$$5x_1 = 2 - \frac{88}{39} = -\frac{10}{39}$$

$$x_1 = -\frac{2}{39}$$

Checking for g_1

$$\Rightarrow g_1 \leq 0$$

Hence case ① works

also

$$\begin{aligned} 1 - 5\mu_2 - 2\mu_3 &= 0 \\ -1 - 2\mu_2 + 7\mu_3 &= 0 \end{aligned}$$

$$\begin{aligned} 5\mu_2 + 2\mu_3 &= 1 \times 2 \\ -2\mu_2 + 7\mu_3 &= 1 \times 5 \end{aligned}$$

$$10\mu_2 + 4\mu_3 = 2$$

$$-10\mu_2 + 35\mu_3 = 5$$

$$39\mu_3 = 7$$

$$\mu_3 = \frac{7}{39}$$

$$\mu_2 = \frac{5}{39}$$

III] Case 2) g_2 is inactive

$$2x_1 + 3x_2 = 10$$

$$7x_2 - 2x_1 = 8$$

$$10x_2 = 18$$

$$x_2 = \frac{18}{10}$$

$$2x_1 = 10 - \frac{54}{10} = \frac{46}{10}$$

$$x_1 = \frac{23}{10}$$

Checking g_2

$$2 - \frac{23}{2} - \frac{18}{5} < 0$$

Hence case ② works

$$\begin{aligned} 1 + 2\mu_1 - 2\mu_3 &= 0 \\ -1 + 3\mu_1 + 7\mu_3 &= 0 \end{aligned}$$

$$\Rightarrow \mu_1 = -1/4$$

$$\mu_3 = 1/4$$

IV] g_1 & g_2 are inactive
 g_3 is active.

$$7x_2 - 2x_1 = 8$$

Cannot determine

\therefore @ Case ①

$$x_1 = -\frac{2}{39}$$

$$x_2 = \frac{44}{39}$$

$$\mu_2 = \frac{5}{39}$$

$$\mu_3 = \frac{7}{39}$$

$$\begin{aligned} L &= -\frac{2}{39} + \frac{44}{39} + \frac{5}{39} (2 - 5x_1 - 2x_2) \\ &\quad + \frac{7}{39} (7x_2 - 2x_1 - 8) \\ &= \underline{-46/39} \end{aligned}$$

@ Case ②

$$x_1 = \frac{23}{10}$$

$$x_2 = \frac{18}{10}$$

$$\mu_1 = -1/4$$

$$\mu_3 = 1/4$$

$$L = 1/2$$

\Rightarrow ^{Global} Minimum occurs @

$$x_1 = -2/39 \text{ or } x_2 = 44/39$$

or value is $-46/39$