

# Homework 7

*Due: Tuesday, 31 Mar 2015*

All homeworks are due at 11:00PM in the CS22 bin on the CIT second floor, next to the Fishbowl.

Include our cover sheet or equivalent, write your *full name* and CS login on each page of your homework, label all work with the problem number, and staple the entire handin before submitting it.

Be sure to fully explain your reasoning and show all work for full credit. Consult the style guide for more information.

## Problem 7.1

Consider a circuit with  $n$  inputs and one output. Let  $I_n$  be the set of each input to the circuit (i.e. each element in  $I_n$  represents a single input wire to the circuit). Consider some subset  $t \subseteq I_n$ ; we say this subset *satisfies* the circuit if the circuit evaluates to *true* when we set all input wires in  $t$  to *true*, and all wires not in  $t$  to *false*. Let  $T_n$  be the set of all  $t \subseteq I_n$  such  $t$  satisfies the circuit. Prove that for any  $n > 2$  there exists a circuit where:

- a.  $|T_n| = 1$
- b.  $|T_n| = 2^n - 1$
- c.  $|T_n| = 2^{n-1}$
- d.  $|T_n| = J_n$ , where  $J_n = 2^{n-1} - J_{n-1}$  and  $J_0 = 0$

## Problem 7.2

Prove that the  $\vee$  operator is associative if  $n$  expressions are combined only via the  $\vee$  operator. In other words, the placement of parentheses in  $A_1 \vee A_2 \vee \dots \vee A_n$  doesn't matter.

## Problem 7.3

The Queen of Hearts *still* can't decide if she wants to repaint the roses in her garden. However, she recently hired more servants, so she now has some

arbitrary number of gardeners  $n$  to help her decide.

- a. Again, the Queen decided to try a scheme where she changes her decision every time one of the gardeners switches his/her choice. To begin with, the Queen decides yes.

Prove that, for all integers  $n \geq 2$ , the Queen can construct such a circuit using only AND, OR, and NOT gates.

- b. The Queen wants to try a new scheme: she decides yes if and only if at least  $n - 1$  gardeners vote yes.

Prove that, for all integers  $n \geq 2$ , the Queen can construct such a circuit satisfying this criterion using only AND, OR, and NOT

#### Problem 7.4

Prove that there exists some integer in any set of 10 consecutive positive integers that is relatively prime to the product of the other 9.

#### Problem 7.5

The Mad Hatter and March Hare are setting up yet another tea party, and need to come up with new, exciting tea sets for their guests. They have 9 existing sets, each composed of a teapot, a cup, a spoon, and a saucer. The Hatter can create new tea sets by combining the individual components from existing sets.

- a. A new, stylish tea set can be put together by combining one teapot, one cup, one spoon, and one saucer from any existing set. How many distinct tea sets can the Hatter make, excluding the 9 existing tea sets?
- b. While putting together the new tea sets, the March Hare observes that tea sets #3 and #6 are incompatible - combining any items from tea sets #3 and #6 results in a garishly unfashionable set, regardless of what the other items in the set are. How many tea sets can the Hatter put together, provided that he is only willing to create fashionable tea sets? (Hint: Try splitting the problem into cases)
- c. Now suppose that only tea sets made with items from exactly 3 different existing tea sets are interesting for new guests. Recall that

sets #3 and #6 are unfashionable together. How many fashionable tea sets are also interesting for new guests?

- d. The Hatter, after much deliberation, creates 10 distinct new, fashionable, interesting tea sets. The March Hare brings in 5 new guests to entertain, and the two must decide how to distribute the new tea sets among their guests. No guest can leave without a tea set, and every tea set must be used exactly once - how many different ways are there to distribute the tea sets among the guests?

## Logisim Problem

In the following problem, you will make use of a program called Logisim to model and test your circuit. You can find a link on the CS22 website, and it is also available on the department machines (just enter `logisim` into a command line).

We will **only** be accepting a Logisim file (a `.circ` file) for this problem, and we require it to be named `eyes.circ`. Please do not name it anything else. Paper, email, or other handins will not be accepted. Please do not attempt to hand in the rest of your homework electronically.

To turn in this problem, log into your CS account and open a terminal. Run the command `cd /path/to/your/files` to navigate to where your `.circ` file is. The handin script will turn in everything in your current folder; if your folder contains other files, run the command `mkdir temp; cp eyes.circ temp; cd temp`. Turn in the contents of your current folder using `cs022handin circuits2`.

### Problem 7.6

It's Halloween and Cheshire Cat is putting up decorations. He has a creepy head in a jar and the eyes can be programmed to automatically open and close. He would like the eyes to first both be closed (00). Then just the right eye should open (01). Next the left eye should open so both eyes are open (11). Finally, the right eye should close (10). Using a clock, design a circuit with two outputs, one for each eye, that cycles between these four states in the given order.

Note: The eyes do not have to change state every time the clock changes.