Recitation 1

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Problem 1.1

A partition of a set is a separation of the elements into non-overlapping distinct subsets, such that each element is a member of a subset. For example, the set $\{a, b, c\}$ has five possible partitions.

Given an arbitrary set S, define an equivalence relation over the set of partitions of S, $\mathbb{P}(S)$. Prove that the relation you define is an equivalence relation. Hint: it may help to write out $\mathbb{P}(\{a, b, c\})$, or some other example.

Problem 1.2

A relation R is a *partial order* over a set S if and only if it satisfies the following properties:

- Reflexivity: xRx for all $x \in S$
- Antisymmetry: If xRy and yRx, then $x = y$
- Transitivity: If xRy and yRz, then xRz

a. Define $R^{-1} = \{(x, y)|(y, x) \in R\}$. Prove that if R is a partial order, then so is R^{-1}.

b. Suppose R is a partial order over S. Prove that R^C is never a partial ordering on S (R^C denotes the complement of R, which is defined as $\{(a, b) \mid a, b \in S, (a, b) \notin R\}$).

Problem 1.3

Consider the nonempty set $S_n = \{1, 2, 3, \ldots, n\}$. Let E be the set of even cardinality subsets of S_n, and let O be the set of odd cardinality subsets of S_n. Prove that $|E| = |O|$ by defining a bijection from one set to the other. Be sure to prove that your function is in fact bijective.

Problem 1.4

The Mad Hatter is up to his old tricks again. He has gotten his hands on n cards, all indistinguishable Queens of Spades, and wants to arrange them into a set of piles of different numbers of cards placed along the edge of his table. How many ways are there for him to do this? Note that two arrangements are considered equal if they consist of the same number of piles, and counting from left to right, the sizes of each pile are equal.

Problem 1.5

Define the set of functions F as follows: $F = \{f : A \to B\}$ where $A, B \subseteq \mathbb{Z}$. Let a relation R on F be defined such that for any $g, h \in F$, gRh if and only if $g - h$ is a constant function; in other words, g is related to h if there exists a constant c such that $g(x) - h(x) = c$ for all $x \in A$.

Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 3, 5, 7\}$. Consider two functions $g, h \in F$ such that gRh. If $g(a) = 1$ and $h(b) = 1$ for some $a, b \in A$, prove that $g = h$.