Probability

Requirements

1. Define all events and random variables explicitly.
2. Write out formulas before plugging in values.
3. Show all of your calculations.
4. Clearly state your answer.

Example

The Mathemagician prefers numbers over words, but his behavior is a little random. When someone sends him a letter consisting of numbers, he smiles with probability 0.6 and frowns with probability 0.4. When someone sends him a letter consisting of words, he smiles with probability 0.2 and frowns with probability 0.8. 95% of the letters he receives consist of numbers, and the rest consist of words. If you hand the Mathemagician a letter and observe him frown, what is the probability that the letter he received was written in words?

Answer.
Let W be the event that the Mathemagician receives a letter consisting of words.
Let N be the event that the Mathemagician receives a letter consisting of numbers.
Let F be the event that the Mathemagician frowns.

Using Bayes' formula,

$$\begin{aligned}
\Pr[W|F] &= \frac{\Pr[F|W] \times \Pr[W]}{\Pr[F|W] \times \Pr[W] + \Pr[F|N] \times \Pr[N]} \\
&= \frac{0.8 \times 0.05}{0.8 \times 0.05 + 0.4 \times 0.95} \\
&= \frac{0.04}{0.04 + 0.38} \\
&= \frac{0.04}{0.42} \\
&\approx 0.095
\end{aligned}$$

The probability that the letter was written in words is approximately 0.095.