

# Strong Induction

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## Requirements

1. Formally state the property that will be proved inductively.
2. Prove the property holds in the base case.
3. Formally state the inductive hypothesis.
4. Assume the inductive hypothesis, and prove the inductive step.
5. Conclude that the property holds in general.

## Example

Prove that every integer  $n \geq 2$  can be written as a product of one or more prime numbers.

## Proof

Let  $P(n)$  be the property “ $n$  can be written as a product of one or more prime numbers”.

Base case: 2 is prime, so is a product of exactly one prime number (itself), and  $P(2)$  is true.

Assume the inductive hypothesis, that for a particular  $k$ ,  $P(i)$  is true for all  $2 \leq i \leq k$ .

Inductive step: We must prove  $P(k+1)$ , that  $k+1$  is the product of one or more prime numbers.  $k+1$  is either prime or composite. If it is prime, then it is the product of exactly one prime number (itself), and  $P(k+1)$  is true. If it is composite, then by definition it is the product of two factors,  $k+1 = ab$ , where  $a$  and  $b$  are integers  $\geq 2$ . Since  $a$  and  $b$  are both greater than 1, they must also both be less than  $k+1$ . By the inductive hypothesis,  $a$  and  $b$  can each be written as a product of one or more primes. But since  $k+1 = ab$ , we can combine these two products to express  $k+1$  as a product of primes, so  $P(k+1)$  is true.

Since  $P(2)$  is true and  $P(2), \dots, P(k)$  together imply  $P(k+1)$ ,  $P(n)$  is true for all integers  $n \geq 2$ . □