Recitation 5

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Problem 5.1

The Queen of Hearts is setting up court, putting people on trial for the crime of beating her at croquet. The queen's court consists of 12 subjects, not including the queen herself, and includes four roles: witnesses, prosecutors, defendants, and executioners (The Queen of Hearts, of course, is always the judge.). Every subject must serve in the court, and any subject may serve in any role.

How many ways are there for the 12 subjects to fill up the four roles in court under the following conditions?

a. Every subject of the Queen is unique. Not every role needs to be filled by subjects.

b. Subjects are indistinguishable - only the number of subjects serving in each role matters. Not every role needs to be filled.

c. Subjects are indistinguishable, and every role needs to be filled by at least one subject.

Problem 5.2

Alice runs across a blue caterpillar. The caterpillar takes off its hat and inside it are the numbers 1-10 written on small slips of paper. Alice will draw 5 numbers randomly with replacement. Determine the probability of the following

a. Each number drawn is greater than the previous number.

b. Each number drawn is greater than or equal to the previous number drawn.

c. The caterpillar got hungry and ate the two slips of paper with the largest numbers (so 1-8 remain). Alice still chooses 5 numbers with replacement. What is the probability that Alice can form two disjoint nonempty subsets of her 5 chosen numbers that sum to the same value.

Problem 5.3

The Knave of Hearts is accused of stealing the Queen's tarts. A trial is held where he will be judged by a jury of 12 animals. Each animal occupies one of 12 seats in the jury box. Alice is called up as a witness, but she accidentally knocks over the jury. The animals must return to the jury box, each animal picking a new seat equally likely at random (it could return to its original seat). The animals pick their seat independently of the other animals' choices so multiple animals could grab the same seat.

   a. What is the probability that each seat will be occupied by an animal?

   b. Suppose the seats in the jury box are arranged in a circle. If each animal picks its new seat either directly left or right of its original seat (equally likely at random), what is the probability that each seat will be occupied by an animal?

Problem 5.4

Alice is attempting to make her way through Wonderland, and needs to get through the Queen of Hearts's croquet grounds. Let A represent the event that Alice beats the Queen at croquet, let B be the event that Alice is put on trial in the Queen's court for cheating at croquet, and let C be the event that Alice escapes Wonderland safely.

   • $P(A) = 0.3$

   • $P(B|A) = 0.9$, and $P(B|\neg A) = 0.5$

   • $P(C|B) = 0.4$, and $P(C|\neg B) = 0.8$

Given the above information, what is the probability that:

a) Alice is put on trial for cheating at croquet?

b) Alice escapes Wonderland safely, if we know that she did not beat the queen at croquet?

c) Alice beats the Queen at croquet, if we know that she ultimately did escape Wonderland safely?