CSCI 0220 **Discrete Structures and Probability** C. Klivans

Recitation 2

Date: Feb 15, 2015

Problem 2.1

Suppose f is a bijection between two sets A and B. Prove that equivalence relations are pushed through the bijection. That is, if R is an equivalence relation on A, then R' such that $f(x)R'f(y)$ iff xRy is also an equivalence relation.

Problem 2.2

Let $C(n)$ be the number of 0/1 strings of length n that do not contain consecutive 1s. For example, $C(4) = 8$ because there are 8 0/1 strings of length 4 without consecutive 1s: 0000, 0001, 0010, 0100, 1000, 0101, 1010, and 1001.

Prove that $\forall n \in \mathbb{Z}^+$, $C(n) = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^{n+2} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+2}\right)$.

Hint: show that for any $n \geq 3$, $C(n) = C(n-1) + C(n-2)$. Also, you may use the following without proof:

- $1 + \frac{1+\sqrt{5}}{2} = (\frac{1+\sqrt{5}}{2})^2$
- $1 + \frac{1-\sqrt{5}}{2} = (\frac{1-\sqrt{5}}{2})^2$.

Problem 2.3

A function on the positive integers is said to be multiplicative if $f(ab) = f(a)f(b)$ whenever $gcd(a, b) = 1$. Prove by induction that a multiplicative function is completely determined by its value on prime powers.

Note: A prime power is simply any number of the form p^k, where p is prime and k is a positive integer.