CSCI 0220 Discrete Structures and Probability C. Klivans

Homework 6

Due: Tuesday, 17 Mar 2015

All homeworks are due at 11:00PM in the CS22 bin on the CIT second floor, next to the Fishbowl.

Include our cover sheet or equivalent, write your *full name* and CS login on each page of your homework, label all work with the problem number, and staple the entire handin before submitting it.

Be sure to fully explain your reasoning and show all work for full credit. Consult the style guide for more information.

Some relevant LaTeX commands for this homework: \Rightarrow, \vee, \wedge, \neg, \oplus, \heartsuit, \textsc{And}.

Problem 6.1

Let $S = \{p_1, p_2, \ldots, p_n\}$ be a set of propositions.

Consider propositions formed from the elements of S in the following way:

(1) You can use any p_i as well as the negation of any p_i.

(2) You can OR together any combination of p_i and $\neg p_i$. For example, the proposition $(p_1 \vee p_3 \vee \neg p_7)$ can be formed this way.

(3) You can AND together terms of the form from (2). For example,

$$(p_1 \vee p_3 \vee p_7) \wedge (\neg p_2 \vee p_1) \wedge (p_6 \vee p_1) \wedge (\neg p_3 \vee p_2)$$

can be formed this way.

The following is a short list of things that are *not* in the correct form: $p_1 \Rightarrow p_2$, $(p_1 \wedge p_3) \vee (p_1 \wedge \neg p_4)$, $\neg(p_1 \vee p_2)$.

Express the following propositions over the set $S = \{p, q, r, s\}$ in the given form, showing all steps.

a. $(p \Rightarrow q) \vee (r \oplus p)$

b. $p \Rightarrow ((q \Rightarrow r) \Rightarrow s)$

Problem 6.2

a. Can we express the OR ($\vee$) operation using only IMPLIES ($\Rightarrow$) operations? Justify your answer.

b. Can you express the IMPLIES ($\Rightarrow$) operation using only OR ($\vee$) operations? Justify your answer.

c. Can you use only AND ($\wedge$) and OR ($\vee$) operations to express all logical propositions? Justify your answer.

d. Let the $\heartsuit$ operator have the following definition

p	q	p $\heartsuit$ q
T	T	F
T	F	F
F	T	F
F	F	T

Prove that we can express all logical propositions using only the $\heartsuit$ operator.

Problem 6.3

Consider the following way to express a non-negative integer m:

$$m = a_k n^k + a_{k-1} n^{k-1} + \cdots + a_1 n + a_0$$

where $0 \leq a_i < n$ for all i, and $n \in \mathbb{Z}$.

For any fixed m and n, we will call the above (filled-in) equation the *expansion* of m in *base* n. So, the expansion of 8073 in base 10 would be $8 \cdot 10^3 + 7 \cdot 10 + 3$. This is a bit long-winded, so it is often shortened to the string of a_i's (*digits*), and the base is specified with a subscript, as in 8073_{10}. As a further example, $8073_9 = 8 \cdot 9^3 + 7 \cdot 9 + 3 = 5898_{10}$. Note that bases and single digits are always in base 10 (*decimal*).

a. Express each of the following decimal numbers in the given bases. Show both the expansion and the digit string within the given base. You do not need to show other work.

(i) $m = 1024$ for $n = 2$.

(ii) $m = 196$ for $n = 5$.

(iii) $m = 614$ for $n = 9$.

(iv) $m = 659\,918$ for $n = 16$. In the digit string, use the additional digits $A = 10, B = 11, \ldots, F = 15$.

b. Consider a binary number of the form $m = 1010\ldots0_2$, where the entire number is k digits long (i.e. there are $k - 3$ trailing zeroes). Express $6m$ as a binary string.

Not for points: Think about how computers might use properties of binary to make binary multiplication simpler.

c. Prove that every non-negative integer m may be expressed in base 3.

d. Let $n = 3$. Prove that m is even iff the sum of the a_is is even. (Hint: work $\mod 2$)

Logisim Problem

In the following problem, you will make use of a program called Logisim to model and test your circuit. You can find a link on the CS22 website, and it is also available on the department machines (just enter `logisim` into a command line).

We will **only** be accepting Logisim files (`.circ` files) for this problem, and we require them to be of the form `part_a.circ` and `part_b.circ`. Please do not combine them into one file or name them anything else. Paper, email, or other handins will not be accepted. Please do not attempt to hand in the rest of your homework electronically.

To turn in these problems, log into your CS account and open a terminal. Run the command `cd /path/to/your/files` to navigate to where your `.circ` files are. The handin script will turn in everything in your current folder; if your folder contains other files, run the command `mkdir temp; cp part_a.circ part_b.circ temp; cd temp`. Turn in the contents of your current folder using `cs022handin circuits`.

Problem 6.4

The Queen of Hearts can't decide if she wants to repaint the roses in her garden, so she makes her three gardeners help her choose.

a) Say she asks each of the gardeners to vote (either yes or no). The Queen will go with whichever option recieves the majority of the votes. Model this problem as a circuit.

b) The Queen decided this is too simple. Instead, the Queen will switch her choice every time one of the gardeners switches his/her choice. The initial state is that all three gardeners and the Queen say no.

   For example, say the Queen wants to paint her roses, and there is currently exactly one gardener who thinks she should. If one of the other two gardeners changes their mind, and decides the Queen *should* paint her roses, then the Queen will also change her mind – she will not paint her roses.

   Model this problem as a circuit (only using the AND, OR, and NOT gates).