Induction

Requirements

1. Formally state the property that will be proved inductively.
2. Prove the property holds in the base case.
3. Formally state the inductive hypothesis.
4. Assume the inductive hypothesis, and prove the inductive step.
5. Conclude that the property holds in general.

Example

Prove that for all integers $n \geq 1$,

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

Proof:

Let $P(n)$ be the property "$1 + 2 + \cdots + n = n(n+1)/2$".

Base case: $P(1)$ states that $1 = 1(2)/2$. This is true.

Assume the inductive hypothesis, that for a particular k, $P(k)$ is true.

Inductive step: We must prove $P(k+1)$, that $1 + 2 + \cdots + (k+1) = (k+1)((k+1)+1)/2$. We know $1 + 2 + \cdots + k + (k+1) = k(k+1)/2 + k + 1$ by the inductive hypothesis, and

$$\begin{aligned} \frac{k(k+1)}{2} + k + 1 &= \frac{k(k+1) + 2(k+1)}{2} \\ &= \frac{(k+2)(k+1)}{2} \\ &= \frac{(k+1)((k+1)+1)}{2} \end{aligned}$$

which was what we wished to show.

Because $P(1)$ is true and for all k, $P(k)$ implies $P(k+1)$, $P(n)$ is true for all integers $n \geq 1$ by induction. $\square$