

Recitation 6

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Problem 6.1

The Queen of Hearts is choosing between two fertilizers to buy for her rose garden, X and Y . The number of roses that will grow per year using X is given by a random variable T_X with mean 20 and variance 4. The number of roses that will grow per year using Y is given by an random variable T_Y with mean 18 and variance 9. The Queen will make a yearly profit in \$ of $Q = (T - 20)^2$ per year, where T is the number of roses grown.

- If the Queen buys fertilizer Y , find an upper bound for the probability that she grows at least 30 roses.
- If the Queen buys fertilizer X , find a lower bound for the probability that she grows between 10 and 30 tomatoes.
- Calculate the expected profit using each fertilizer. Which should the Queen pick to maximize her expected profit?
- Suppose that both T_X and T_Y follow binomial distributions. For each fertilizer, find the probability that the Queen earns at least \$100.

Problem 6.2

Prove that every connected graph contains a spanning tree.

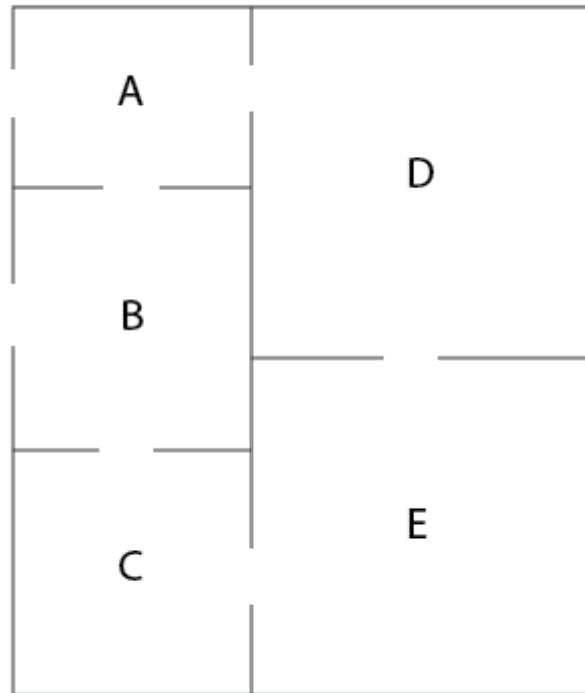
Problem 6.3

The degrees of the vertices of a graph form a *multiset* (a set that can have repeated terms). Show that there can be no *simple graph* that has a multiset of degrees equal to:

- $[3, 2, 2, 1, 3]$
- $[0, 3, 3, 4, 4, 5, 6, 7]$
- $[1, 2, 3, 3, 5, 6, 6]$

Problem 6.4

The Carpenter is constructing a house for the Walrus. The potential floorplan is below, with openings representing doorways.



Upon reviewing construction plans, the Walrus decides that he would like a way to travel through the house, starting and ending outside, by going through each doorway exactly once. What is the minimum number of doorways the Carpenter needs to add, and where (between which two rooms) must each be placed?