Recitation - Midterm 1 Review

Date: Feb 23/25, 2015

Problem - Midterm 1 Review.1

Prove the following equality using the element method: $X \cap Y = (X \cup Y) \setminus (X^c \cup Y^c)$

Problem - Midterm 1 Review.2

The Count is counting money and thinks that it is possible to make any positive multiple of 5 cents except for 5 and 15 using only quarters and dimes. Prove that he is correct using strong induction.

Problem - Midterm 1 Review.3

Let S be a finite set. Define $\mathcal{R}(S)$ to be the set of relations on S. Define a relation R on $\mathcal{R}(S)$ as follows:

$$R = \{(P, Q) \mid P, Q \in \mathcal{R}(S), \forall a, b \in S, \text{ if } aPb, \text{ then } aQb\}$$

a) Is R an equivalence relation, partial ordering, or neither?

b) Suppose we restrict $\mathcal{R}$ to relate only *complete equivalence relations* on S. A relation $R \in R(S)$ is complete if for all $a, b \in S$, $(a, b) \in R$ or $(b, a) \in R$ or both. Do any of your answers above change? Which, and why?

c) Prove that the equivalence classes of an equivalence relation that is also a partial ordering have size 1.

Problem - Midterm 1 Review.4

For $n \in \mathbb{Z}^+$, define $[n] = \{1, 2, ..., n\}$.
Let $Q = \{(x, y) \mid x, y \in [9] \text{ and } x < y\}$.
Let $S = \{(a, b, c) \mid a, b, c \in [9] \text{ and } a + b + c = 10\}$.

Prove by constructing a bijection that $|Q| = |S|$.