Logical Equivalences

Given any statement variables p, q, and r, a tautology $\mathbf{t}$ and a contradiction $\mathbf{c}$, the following logical equivalences hold.

1. *Commutative Laws*:

$$p \wedge q \equiv q \wedge p \text{ and } p \vee q \equiv q \vee p$$

2. *Associative Laws*:

$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r) \text{ and } (p \vee q) \vee r \equiv p \vee (q \vee r)$$

3. *Distributive Laws*:

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r) \text{ and } p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

4. *Identity Laws*:

$$p \wedge \mathbf{t} \equiv p \text{ and } p \vee \mathbf{c} \equiv p$$

5. *Negation Laws*:

$$p \vee \neg p \equiv \mathbf{t} \text{ and } p \wedge \neg p \equiv \mathbf{c}$$

6. *Double Negative Law*:

$$\neg\neg p \equiv p$$

7. *Idempotent Laws*:

$$p \wedge p \equiv p \text{ and } p \vee p \equiv p$$

8. *Universal Bound Laws*:

$$p \vee \mathbf{t} \equiv \mathbf{t} \text{ and } p \wedge \mathbf{c} \equiv \mathbf{c}$$

9. *De Morgan's Laws*:

$$\neg(p \wedge q) \equiv \neg p \vee \neg q \text{ and } \neg(p \vee q) \equiv \neg p \wedge \neg q$$

10. *Absorption Laws*:

$$p \vee (p \wedge q) \equiv p \text{ and } p \wedge (p \vee q) \equiv p$$

11. *Negations of* $\mathbf{t}$ *and* $\mathbf{c}$:

$$\neg\mathbf{t} \equiv \mathbf{c} \text{ and } \neg\mathbf{c} \equiv \mathbf{t}$$

12. *Definition of Conditional*:

$$p \to q \equiv \neg p \vee q$$

13. *Definition of Biconditional*:

$$p \leftrightarrow q \equiv (\neg p \vee q) \wedge (\neg q \vee p)$$