UNCERTAINTY MANAGEMENT IN REASONING

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Introduction to Uncertainty in Economic Problems

→ incomplete information

probably ✓
approximately
accurate
eventually

possibility
↳ fuzzy

Deduction Using Propositional Logic: Example 1

Boolean variables a, b, c, d, … which can take values true or false.

Boolean formulae developed using well defined connectors ~, ∧, V, →, etc, whose meaning (semantics) is given by their truth tables.

Codification of sentences of the argument into Boolean Formulae.

Developing the Deduction Process as obtaining truth of a combined formula expressing the complete argument.

Determining the Truth or Validity of the formula and thereby proving or disproving the argument and Analyzing its truth under various interpretations.

If tax rate is increased then there will be more Govt. income. Tax rate is increased. So there will be more Govt. income

Coding: Variables

a: Tax rate is increased

b: There will be more Govt. Income

Coding the sentences:

F1: $a \rightarrow b$

F2: a

G: b

The final formula for deduction: $(F1 \wedge F2) \rightarrow G$, that is: $((a \rightarrow b) \wedge a) \rightarrow b$

a	b	$a \rightarrow b$	$(a \rightarrow b) \wedge a$	$((a \rightarrow b) \wedge a) \rightarrow b$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Deduction Using Propositional Logic: Example 2

Boolean variables a, b, c, d, … which can take values true or false.

Boolean formulae developed using well defined connectors ~, ∧, V, →, etc, whose meaning (semantics) is given by their truth tables.

Codification of sentences of the argument into Boolean Formulae.

Developing the Deduction Process as obtaining truth of a combined formula expressing the complete argument.

Determining the Truth or Validity of the formula and thereby proving or disproving the argument and Analyzing its truth under various interpretations.

If tax rate is increased then there will be more Govt. income. Tax rate is not increased. So there will not be more Govt. income

Coding: Variables

a: Tax rate is increased

b: There will be more Govt. Income

Coding the sentences:

F1: a → **b**

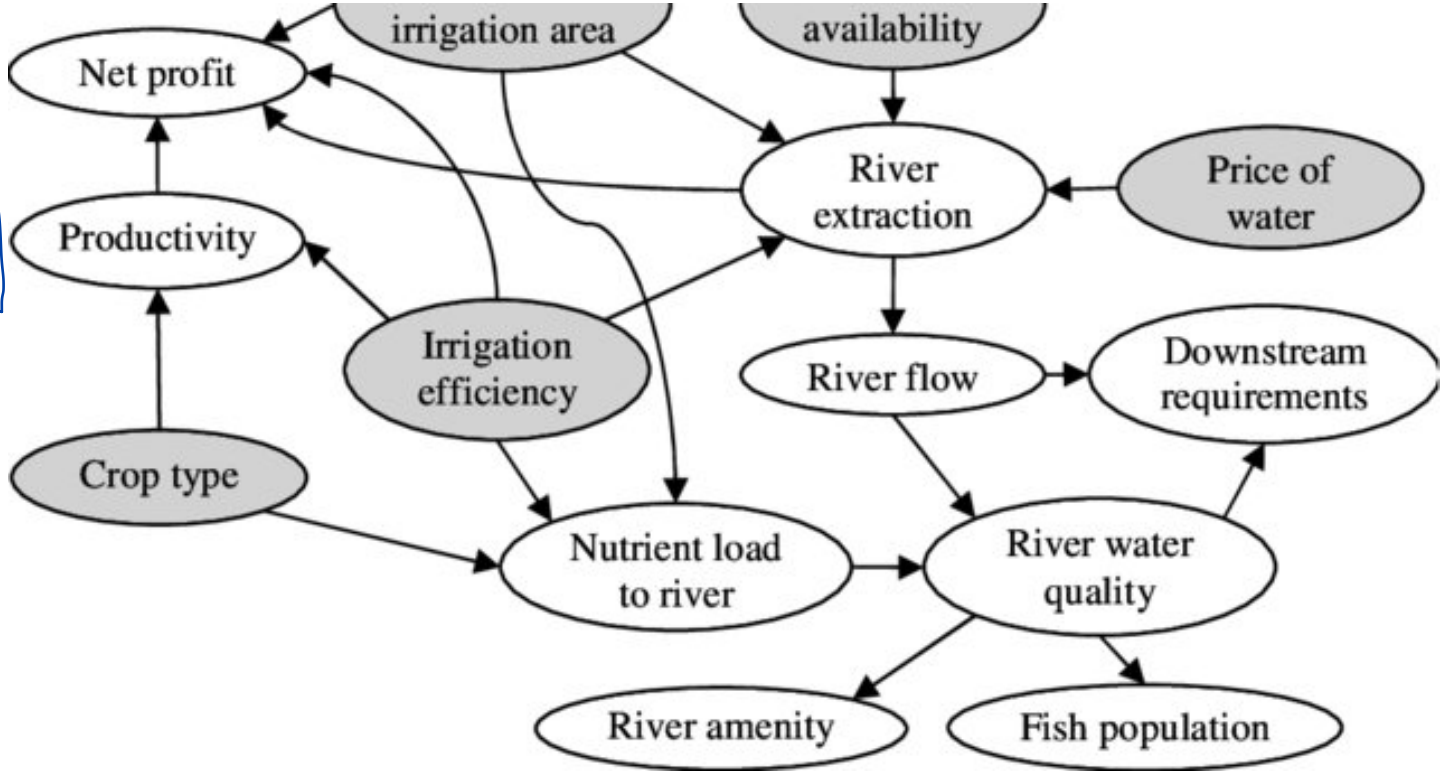
F2: ~a

G: ~b

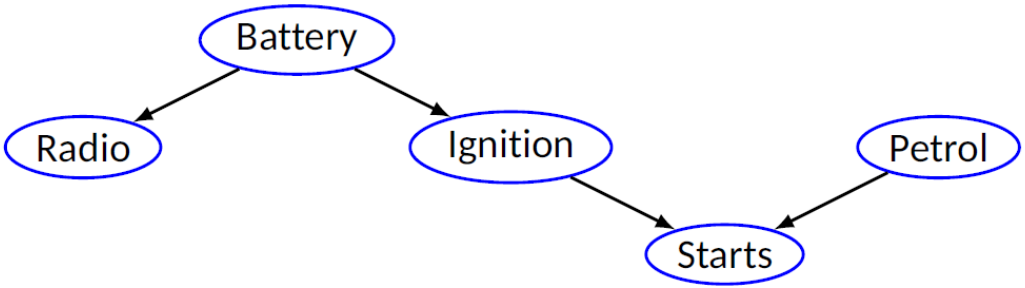
The final formula for deduction: (F1 ∧ F2) → G, that is: ((**a** → **b**) ∧ ~a) → ~b

a	b	a → b	(a → b) ∧ ~a	((a → b) ∧ ~a) → ~b
T	T	T	F	T
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Belief Networks: Causality & Probability

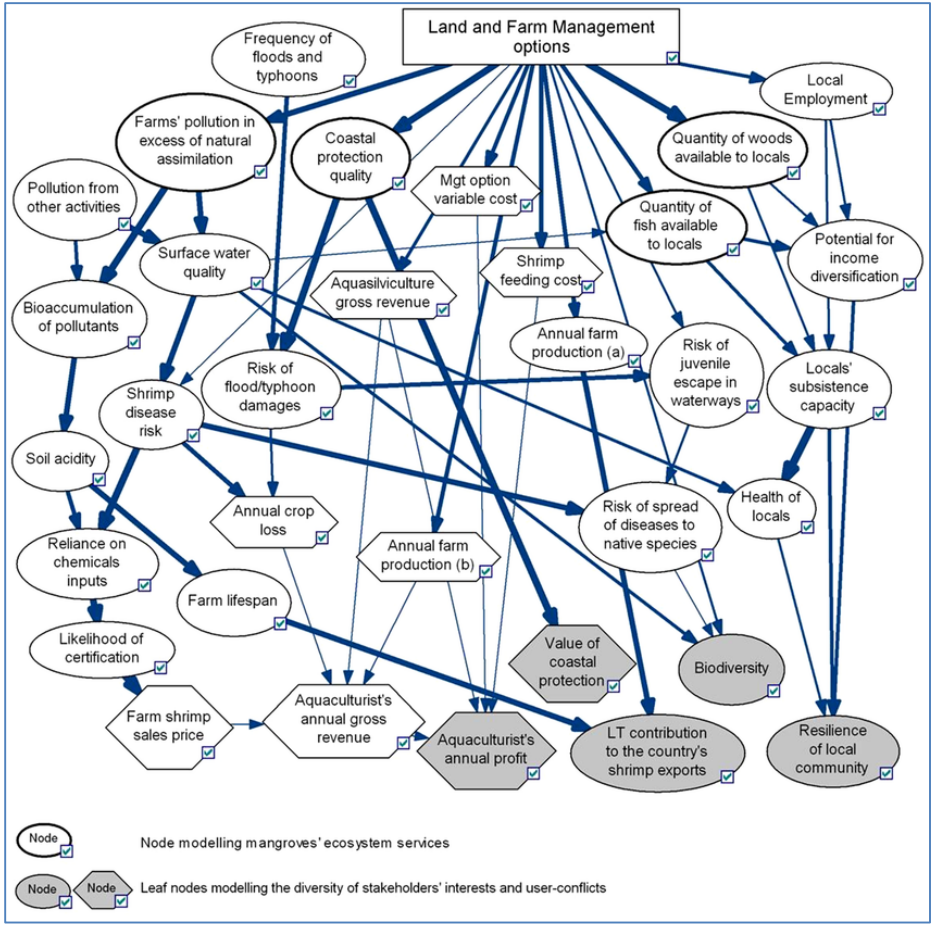


Belief Networks: Links and Meaning

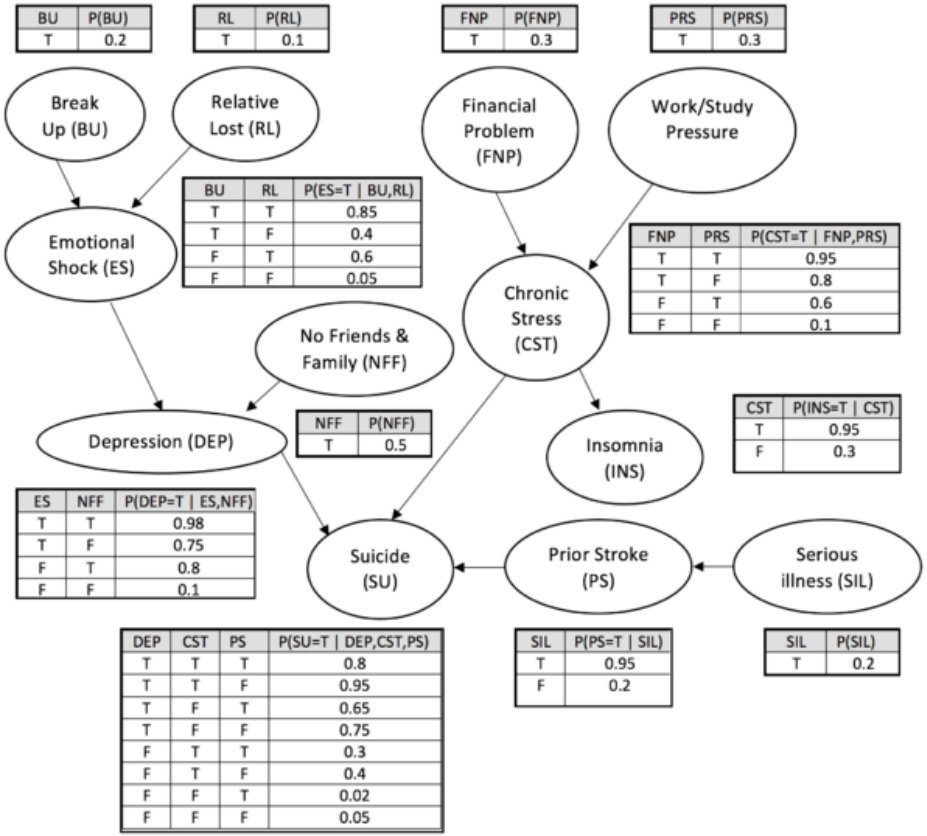


- Whether there is petrol and whether the radio plays are independent given evidence about whether the ignition takes place
- Petrol and Radio are independent if it is known whether the battery works
- Petrol and Radio are independent given no evidence at all.
- But they are dependent given evidence about whether the car starts.
- If the car does not start, then the radio playing is increased evidence that we are out of petrol.

Belief Networks: Economics Example



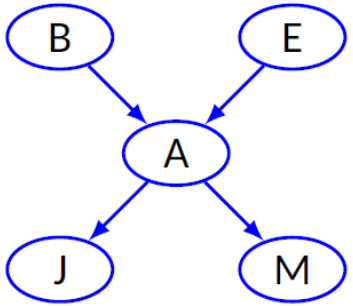
Belief Networks: Probability Assignments



Bayesian Networks: Example

$$
\begin{aligned}
&P(J \wedge M \wedge A \wedge \neg B \wedge \neg E) \\
&\quad = P(J|A) \times P(M|A) \times P(A|\neg B \wedge \neg E) \times P(\neg B) \times P(\neg E) \\
&\quad = 0.9 \times 0.7 \times 0.001 \times 0.999 \times 0.998 \\
&\quad = 0.00062
\end{aligned}
$$

$$
\begin{aligned}
&P(A) \\
&\quad = P(A\bar{B}\bar{E}) + P(A\bar{B}E) + P(AB\bar{E}) + P(ABE) \\
&\quad = P(A|\bar{B}\bar{E}) \times P(\bar{B}\bar{E}) + P(A|\bar{B}E) \times P(\bar{B}E) + P(A|B\bar{E}) \times P(B\bar{E}) + P(A|BE) \times P(BE) \\
&\quad = 0.001 \times 0.999 \times 0.998 + 0.29 \times 0.999 \times 0.002 + 0.95 \times 0.001 \times 0.998 \\
&\qquad\quad + 0.95 \times 0.001 \times 0.002 \\
&\quad = 0.0025
\end{aligned}
$$



$P(B)$
0.001

$P(E)$
0.002

B	E	$P(A)$
T	T	0.95
T	F	0.95
F	T	0.29
F	F	0.001

A	$P(J)$
T	0.90
F	0.05

A	$P(M)$
T	0.70
F	0.01

Bayesian Networks: Example

$P(J)$

$= P(JA) + P(J\bar{A})$

$= P(J|A) \times P(A) + P(J|\bar{A}) \times P(\bar{A})$

$= 0.9 \times 0.0025 + 0.05 \times (1 - 0.0025)$

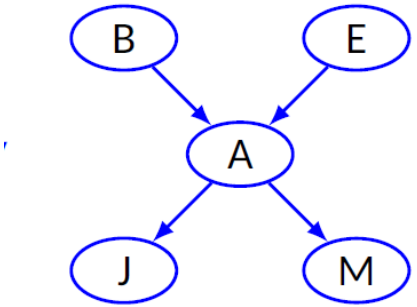
$= 0.052125$

$P(AB)$

$= P(ABE) + P(AB\bar{E})$

$= 0.95 \times 0.001 \times 0.002 + 0.95 \times 0.001 \times 0.998$

$= 0.00095$



$P(B)$
0.001

$P(E)$
0.002

B	E	$P(A)$
T	T	0.95
T	F	0.95
F	T	0.29
F	F	0.001

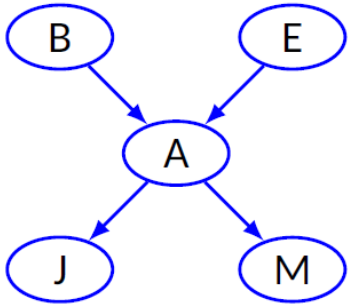
A	$P(J)$
T	0.90
F	0.05

A	$P(M)$
T	0.70
F	0.01

Bayesian Networks: Example

$$P(JB)$$
$$= P(JBA) + P(JB\bar{A})$$
$$= P(J|AB)\times P(AB) + P(J|\bar{A}B)\times P(\bar{A}B)$$
$$= P(J|A)\times P(AB) + P(J|\bar{A})\times P(\bar{A}B)$$
$$= 0.9\times 0.00095 + 0.05\times 0.00005$$
$$= 0.00086$$

$$P(J|B)$$
$$= \frac{P(JB)}{P(B)} = \frac{0.00086}{0.001} = 0.86$$



$P(B)$
0.001

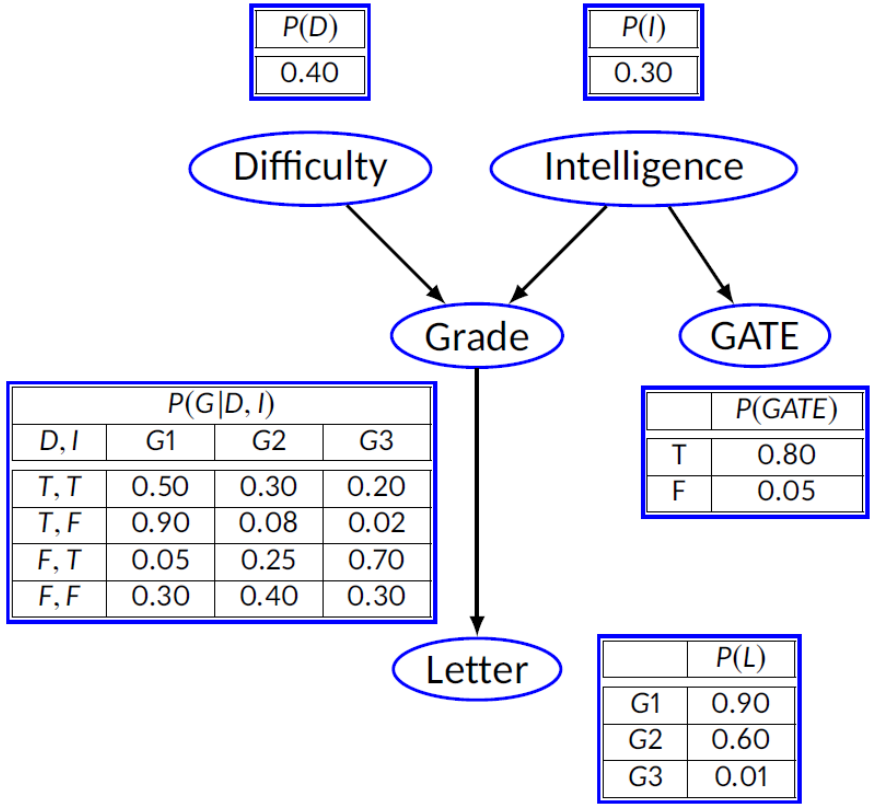
$P(E)$
0.002

B	E	$P(A)$
T	T	0.95
T	F	0.95
F	T	0.29
F	F	0.001

A	$P(J)$
T	0.90
F	0.05

A	$P(M)$
T	0.70
F	0.01

Belief Networks: Multiple Outcomes



Thank you

Any Questions?