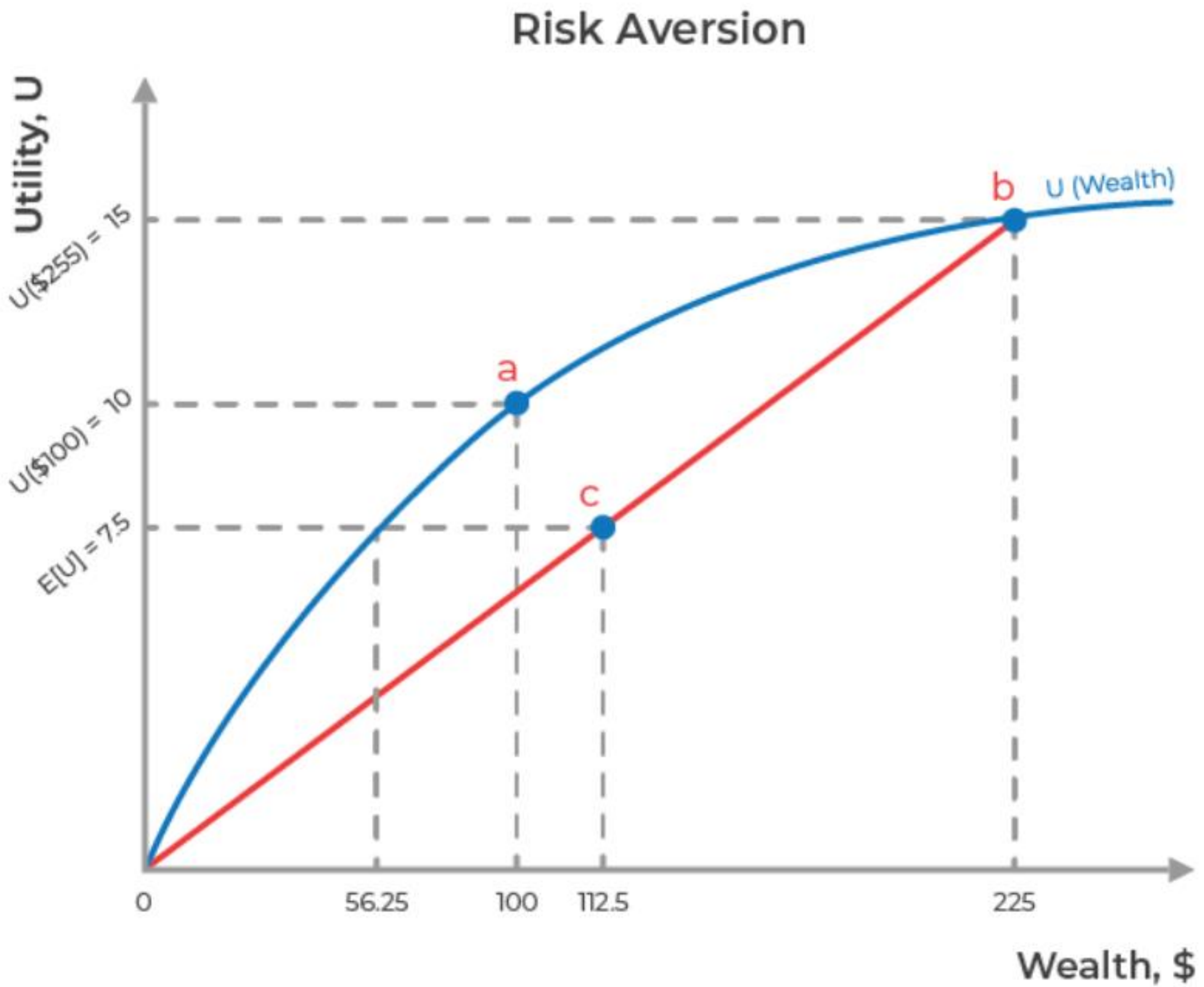
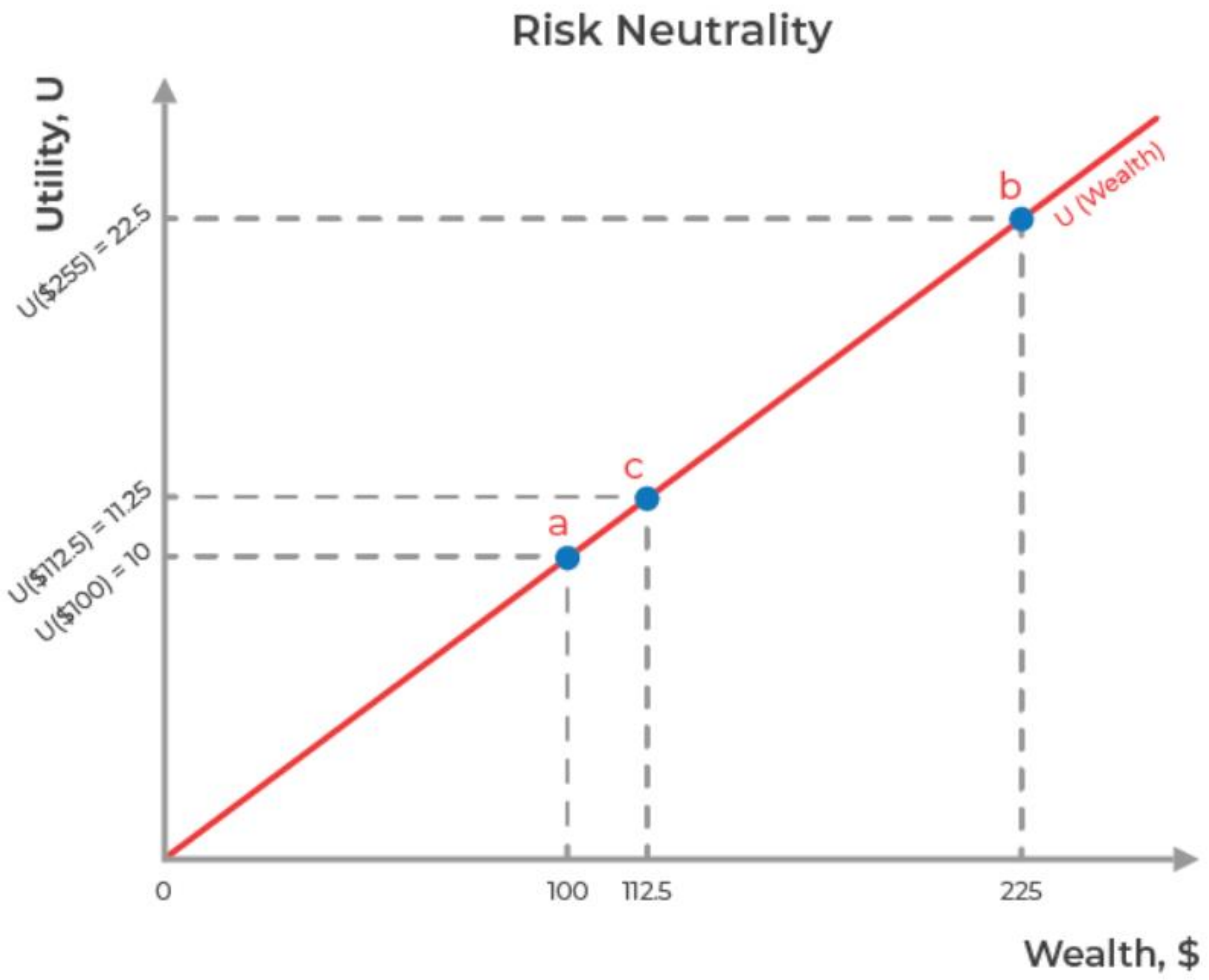
Uncertainty in Financial Markets:

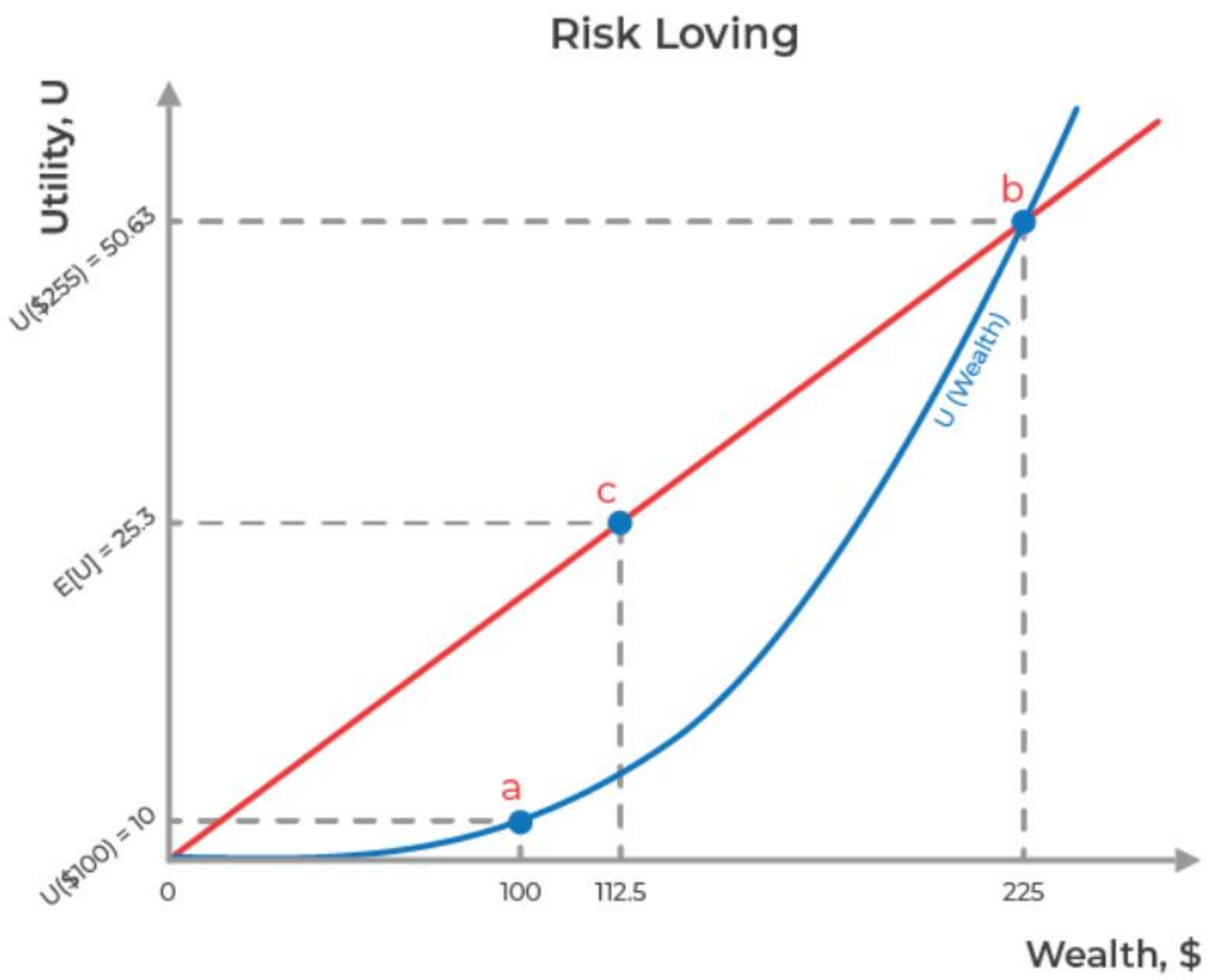
Idea of Hedging

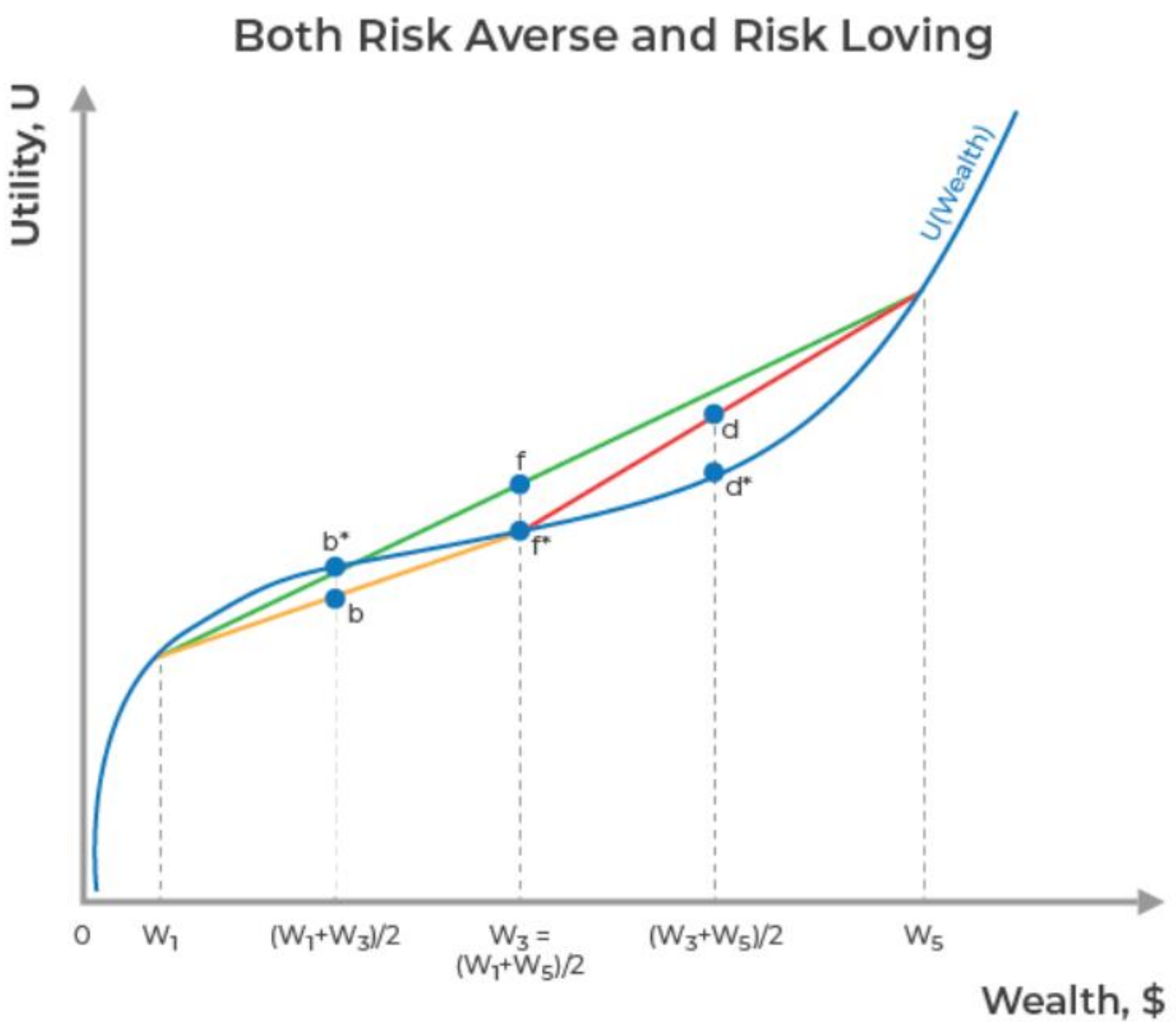
Expected Utility Theory



Risk Aversion
Utility, U
U($255) = 15
U($100) = 10
E[U] = 7.5
b
U (Wealth)
a
c
0
56.25
100
112.5
225
Wealth, $


Risk Neutrality
Utility, U
U($255) = 22.5
U($112.5) = 11.25
U($100) = 10
b
c
a
U (Wealth)
0
100
112.5
225
Wealth, $


Risk Loving
Utility, U
U($255) = 50.63
E[U] = 25.3
U($100) = 10
b
c
a
U (Wealth)
0
100
112.5
225
Wealth, $



Both Risk Averse and Risk Loving
Utility, U
U(Wealth)
f
d
d*
b*
f*
b
0
W1
(W1+W3)/2
W3 =
(W1+W5)/2
(W3+W5)/2
W5
Wealth, $

Cricket Betting

INDIA Vs AUSTRALLIA

- Virat bets on IND & offers a bet of 25:1

i.e If I bet Re. 1 with Virat that "AUS will win" & AUS indeed wins Virat will pay me Rs. 25 & if IND wins, I will pay Virat Re.1

- Steve bets on AUS & offers a bet of 6:5

i.e If I bet Re. 1 with Steve that "IND will win" & IND indeed wins Steve will pay me Rs. 6/5 & if AUS wins I will pay Steve Re. 1

- I have Rs. 100 in my wallet.

What Should I do?

- If I bet all my money on AUS against Virat:

Payoff (if AUS wins) = 25*100 = 2500
Payoff (if IND wins) = - 100

- If I bet all my money on IND against Steve:

Payoff (if IND wins) = 6/5.100 = 120
Payoff (if AUS wins) = - 100

- In either case the worst possible outcome is losing all money. (Win-all-Lose - all !!)

- Can I eliminate or reduce this risk?

Risk Free Strategy

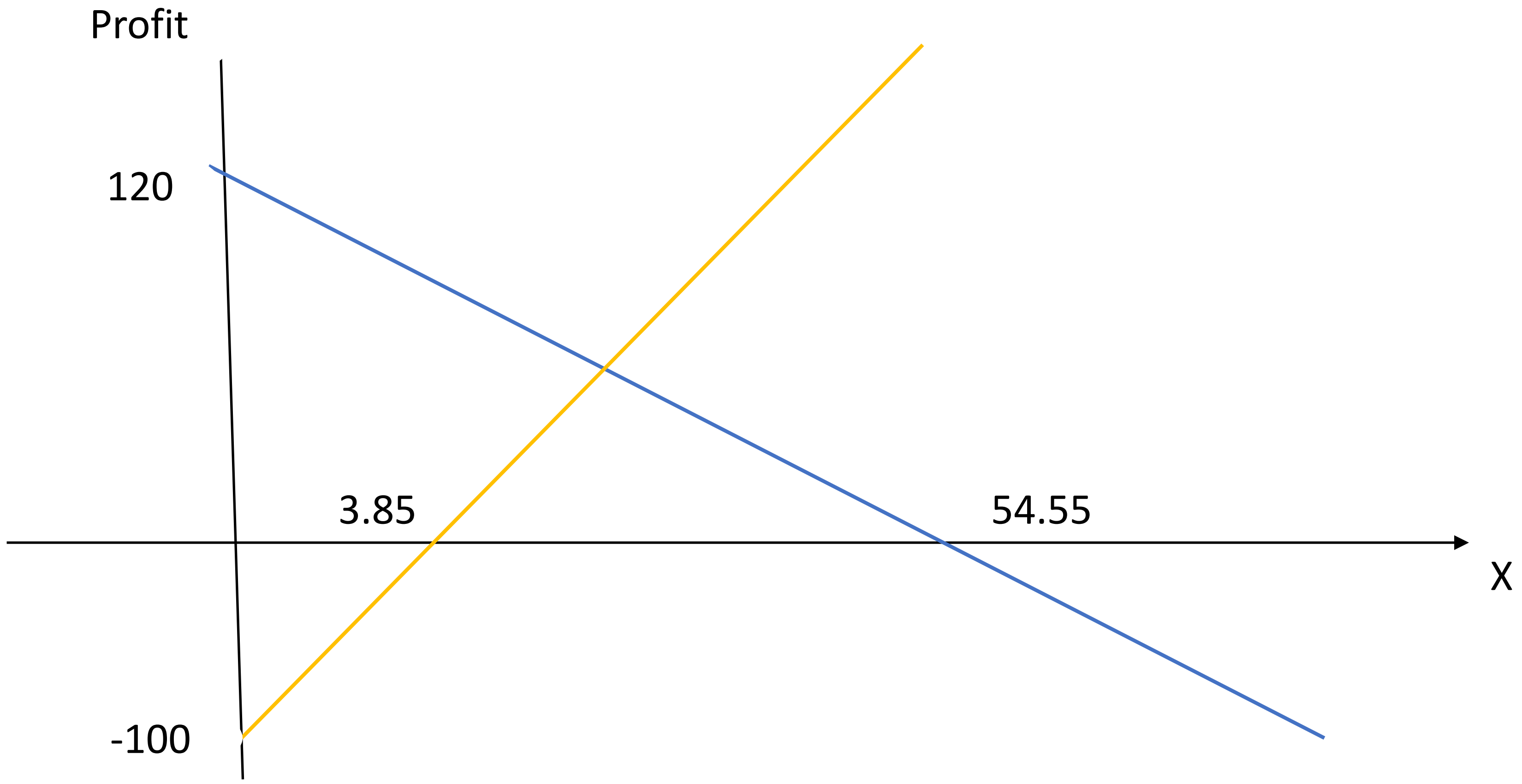
- Let's say I bet Rs. X with Virat & Rs. (100 - X) with Steve.

- If AUS wins: Profit = 26X – 100
- If IND wins: Profit = $\frac{6}{5}.(100 - X) + (100 - X) - 100 = 120 - \frac{11}{5}.X$

- Can I make positive profits, no matter who wins??

- 26X - 100 > 0 if X > 100/26 = 3.85
- $120 - \frac{11}{5}.X > 0$ if $X < 120/(\frac{11}{5}) = 54.55$

- Thus if X ∈ (3.85 , 54.55) Profit > 0, no matter which team wins.



Profit
120
3.85
54.55
X
-100

Maximum Risk – Free / Guaranteed Profit

- Clearly $\forall\ X \in (3.85, 54.55)$:

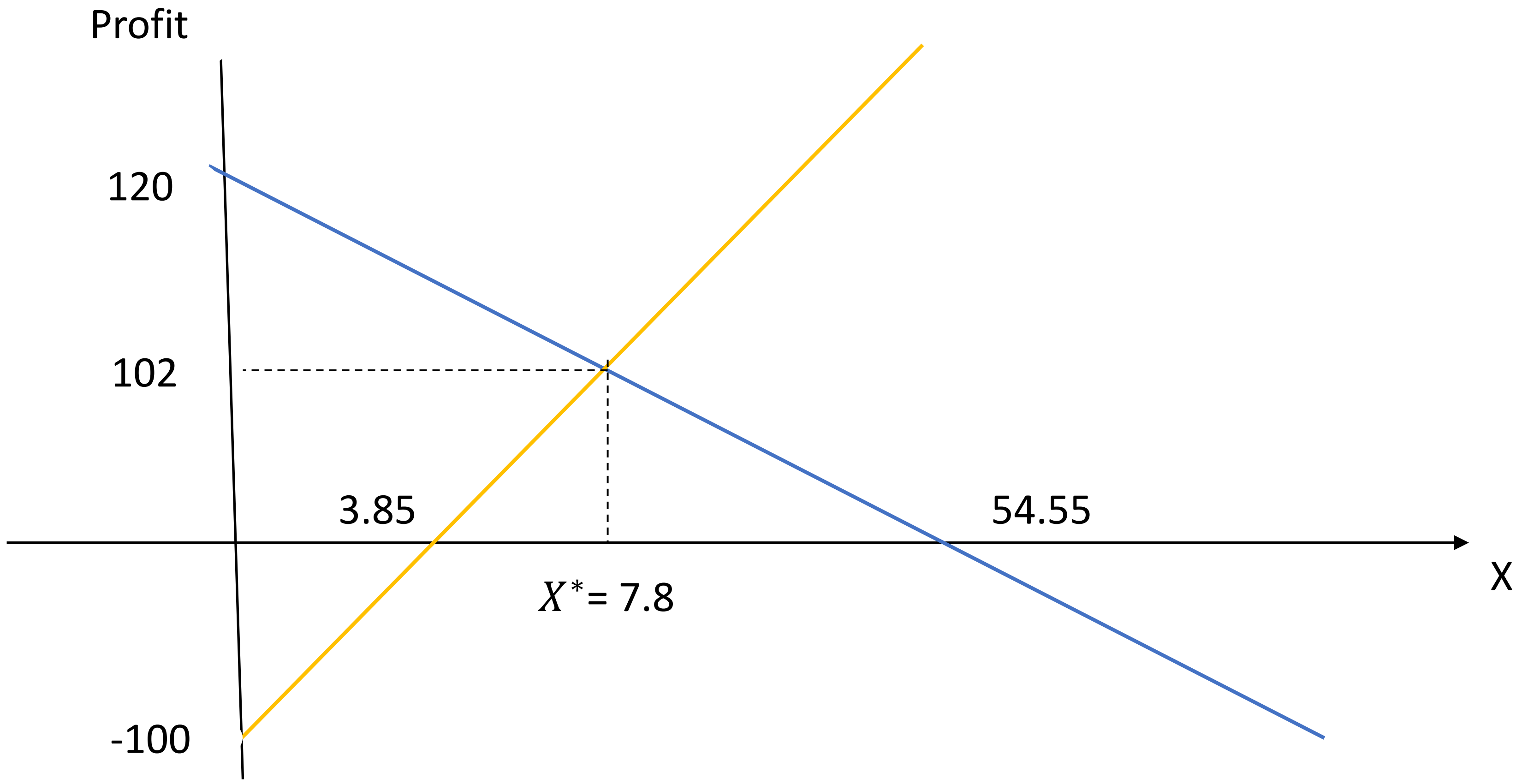
$$Min.\ \boldsymbol{guaranteed}\ Profit = min\ \{26X - 100\ , 120\ - \frac{11}{5}\text{X}\}$$

- So the gambler's optimization problem is:

$$\max_{X \in (3.85\ ,\ 54.55)} [\min\{26X - 100, 120 - \frac{11}{5}\ X\}]$$

- Risk free profit is maximized when 26X-100 = 120 - $\frac{11}{5}\ X$
- X* = 7.80
- The maximum risk – free profit = 102

(more than 100% profit… this too guaranteed… Wow!!!!!

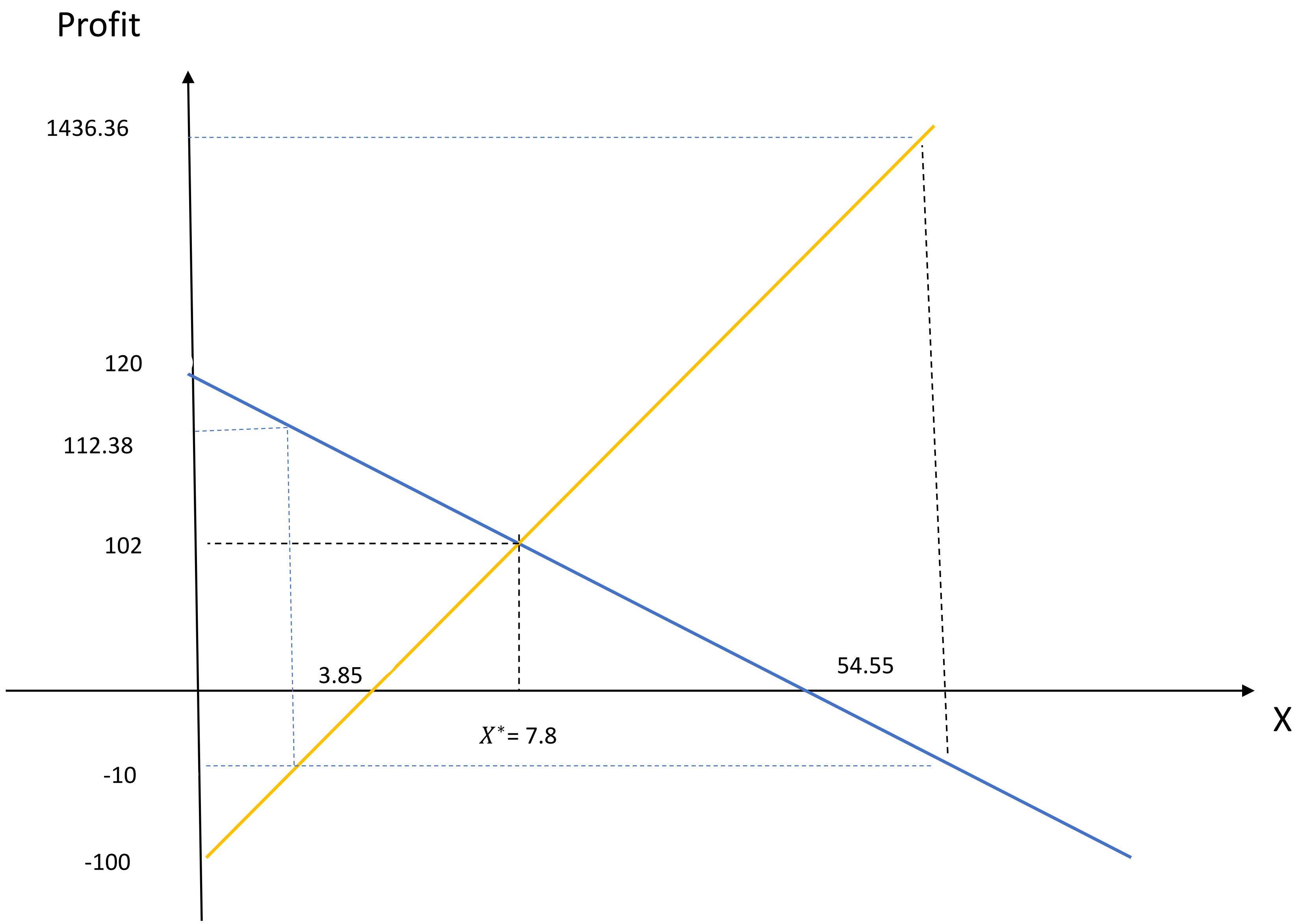

Profit
120
102
-100
3.85
54.55
X^*= 7.8
X

"Hedging" & "Arbitrage"

- The idea of mitigating or eliminating risk by betting on both sides is called "Hedging".
- A strategy generating guaranteed returns is called "Arbitrage".
- Examples of Arbitrage in financial markets (BSE – NSE arbitrage)

Hedging without arbitrage

- I'm greedy...I want to take some risk & earn more profit.
- If I bet all my money (i.e Rs.100) against Virat I can possibly earn a profit of Rs. 2500. But I can also lose all the money.
- I'm torn between "risk" & "return".
- Risk Management: Let's say I can at most afford a Rs. 10 loss.
- So the gambler wants to maximize profit, given that my maximum loss is Rs.10


Profit
1436.36
120
112.38
102
3.85
54.55
X
X^*= 7.8
-10
-100

Huge Profit & a Little Risk !!

- Max. risk - free return =102
- A risk of Rs.10
- Profit shoots up to 1436.36

"Fair Bet" & "Implied Probability"

- If a random variable X represents winnings from a bet then the bet is called "fair" if E(X) = 0

- Now let's assume both Virat & Steve offered "fair" bets according to them.

- Virat's expected payoff for Re.1 bet = $(+1).P_V\,(IND)$ + (-25). $P_V\,(AUS)$

Where $P_V\,(IND)$ is Virat's belief (implied Prob.) that IND will win.

- Hence if Virat believes he is offering a "fair bet" $P_V\,(IND)$ = 25/26 & P_V (AUS) = *1/26*

- Similarly we can compute Steve's implied probabilities if we know that he believes that he offered a "fair bet"

Financial Market Instruments

- SHORT- ing an asset
- Option Contracts (CALL & PUT)
- CALL -> Long Call
  -> Short Call
- PUT -> Long PUT
  -> Short PUT
- Portfolio Designing/Trading Strategies
- Straddle, Strangle & Butterfly
- PUT – CALL Parity

Example

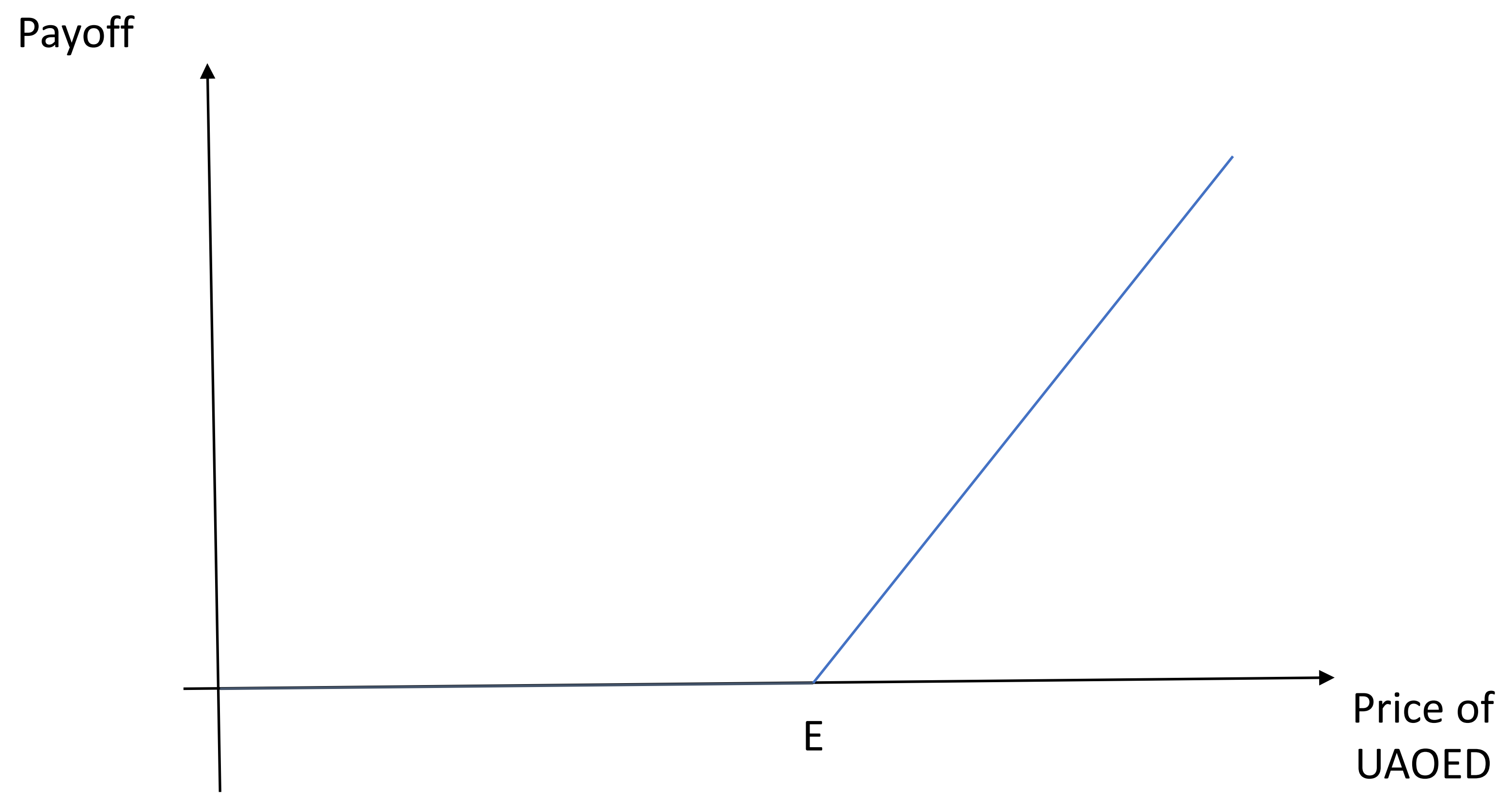
- TCS stock price today is 3416.
- I want to buy a TCS stock. But should I buy right now?
- I feel a market crash is round the corner on 10^{th} Sept. & the TCS price will go down to 3316 (i.e by 100)
- So it seems waiting till 10^{th} Sept. might be good.
- But what if the price goes up to 3500?
- Now I am confused. Should I take the risk of waiting till 10^{th} Sept?
- What if I have a right to buy TCS at 3416 on 10^{th} Sept. no matter what??
- If it goes down I will NOT exercise the right.
- So I have my downside protected.

- That right is a CALL option, where the Expiration date is 10th Sept. & Strike Price = 3416.
- Now why should TCS give you this right for free?
- Of course this "right" has a price. The price of the "CALL" option.
- So there is a Buyer & a Seller of the CALL option.
- The Buyer is said to go LONG on a CALL.
- The Seller is said to go SHORT on a CALL.

CALL Option

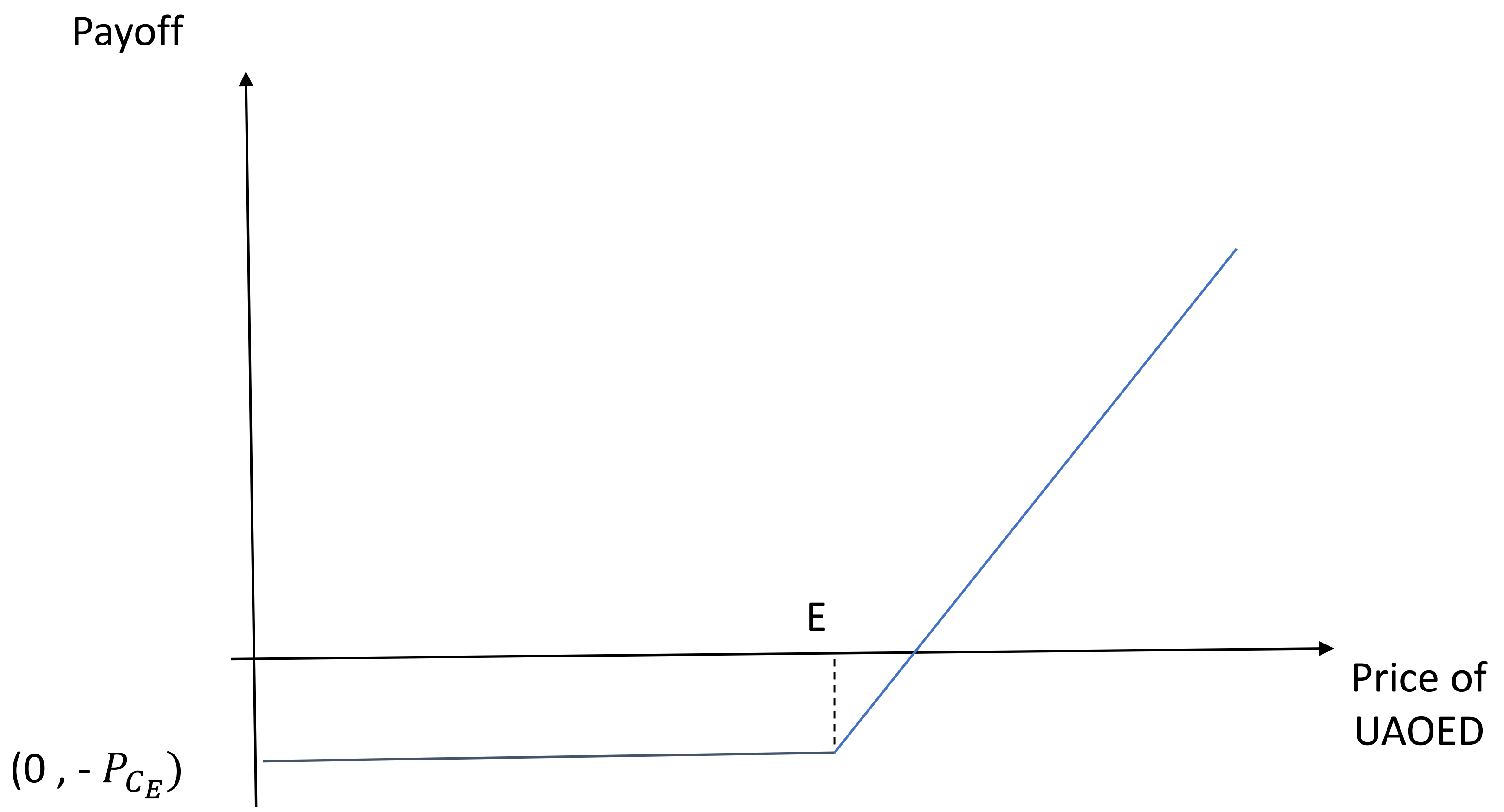
- A CALL Option is a contract which gives the owner the right to buy an asset at an agreed upon price at a specified date.
- If A sells B a CALL option $C_E(S,T)$ on the underlying set S, then B has the right to buy S from A at time t=T at a price E.
- S -> underlying Asset
- T -> Expiration time
- E -> Strike Price

LONG CALL Payoff



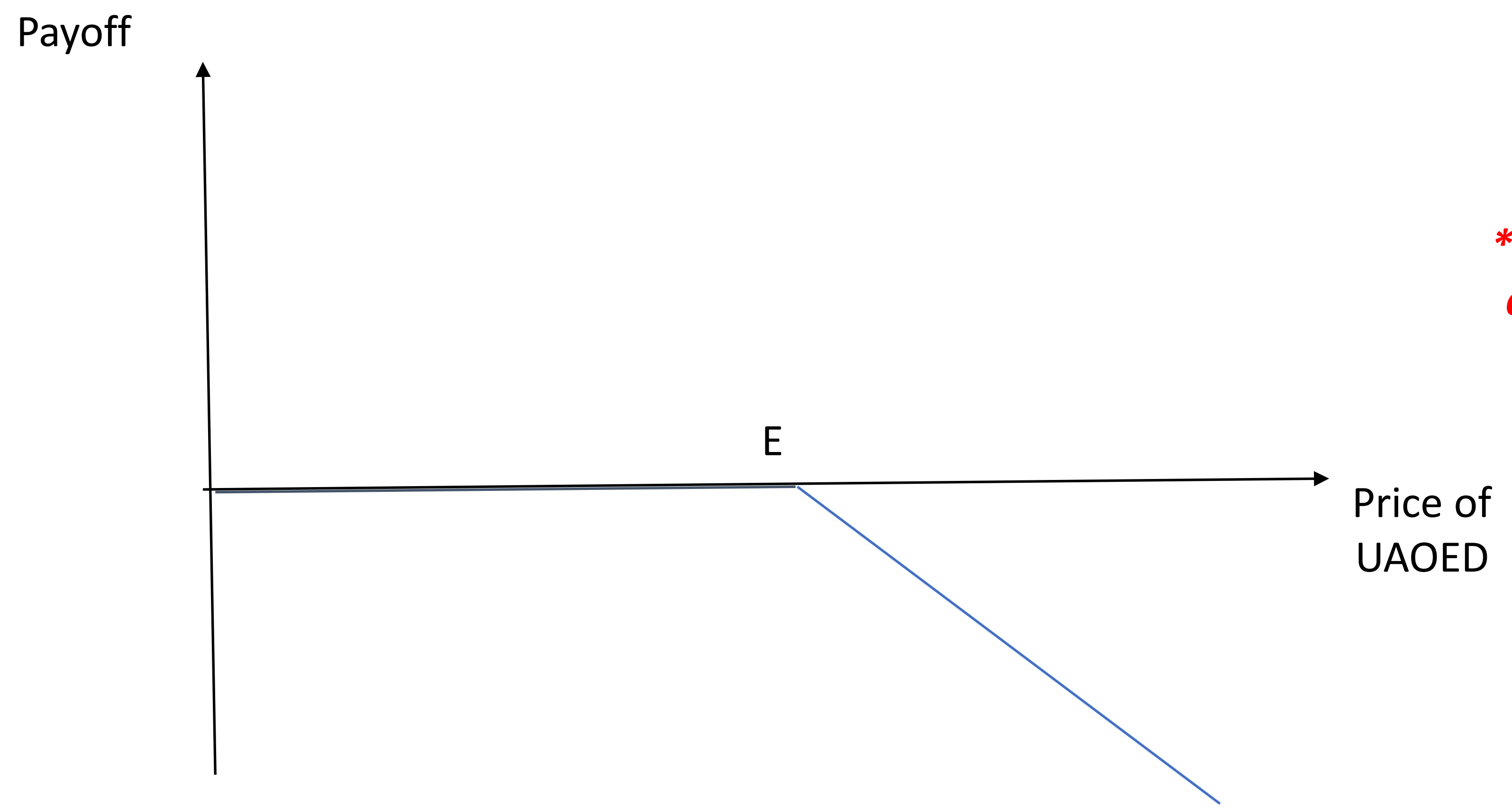
***UAOED: Underlying asset on Expiration date**

LONG CALL Payoff



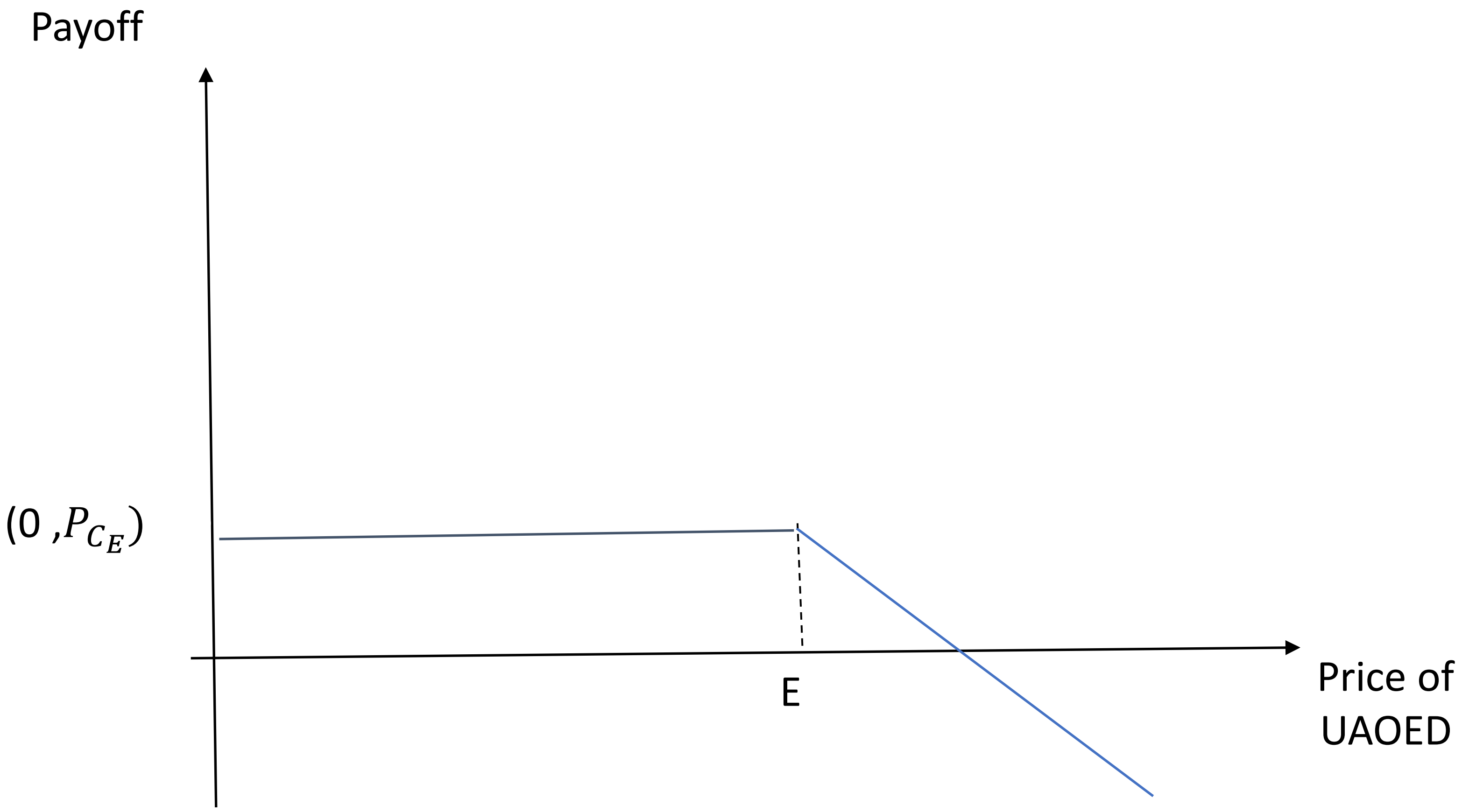
UAOED: Underlying asset on Expiration date

SHORT CALL Payoff



UAOED: Underlying asset on Expiration date

SHORT CALL Payoff



****UAOED: Underlying asset on Expiration date***

Example (Contd.)

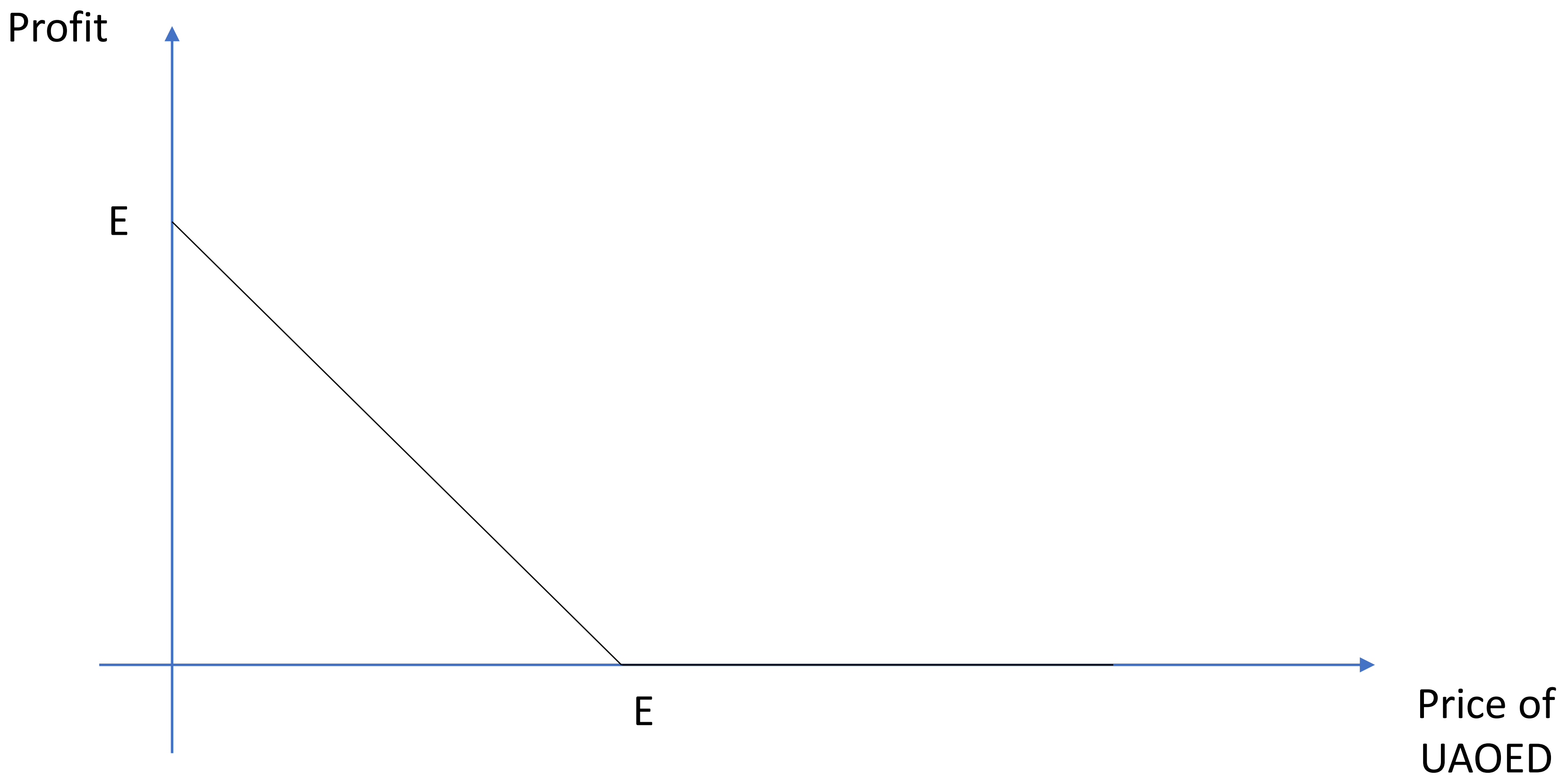
- Let’s say you bought a TCS stock today at 3416.
- Tomorrow the election results are to be declared.
- I feel that TCS price will go up to 3500 & I can make a profit by selling
- But if there is a hung assembly?
- Then the price can go down to 3300.
- So how to protect myself from the downward swing?
- What if I have a right to sell the stock at 3416 tomorrow?
- If the price goes up to 3500 I make a profit.
- If it goes down below 3416 even then I can sell it at 3416 (my buying price)

- That right is a PUT option, where the Expiration date is 1st Sept. & Strike Price = 3416.
- Now why should TCS give you this right for free?
- Of course this "right" has a price. The price of the “PUT" option.
- So there is a Buyer & a Seller of the PUT option.
- The Buyer is said to go LONG on a PUT.
- The Seller is said to go SHORT on a PUT.

PUT Option

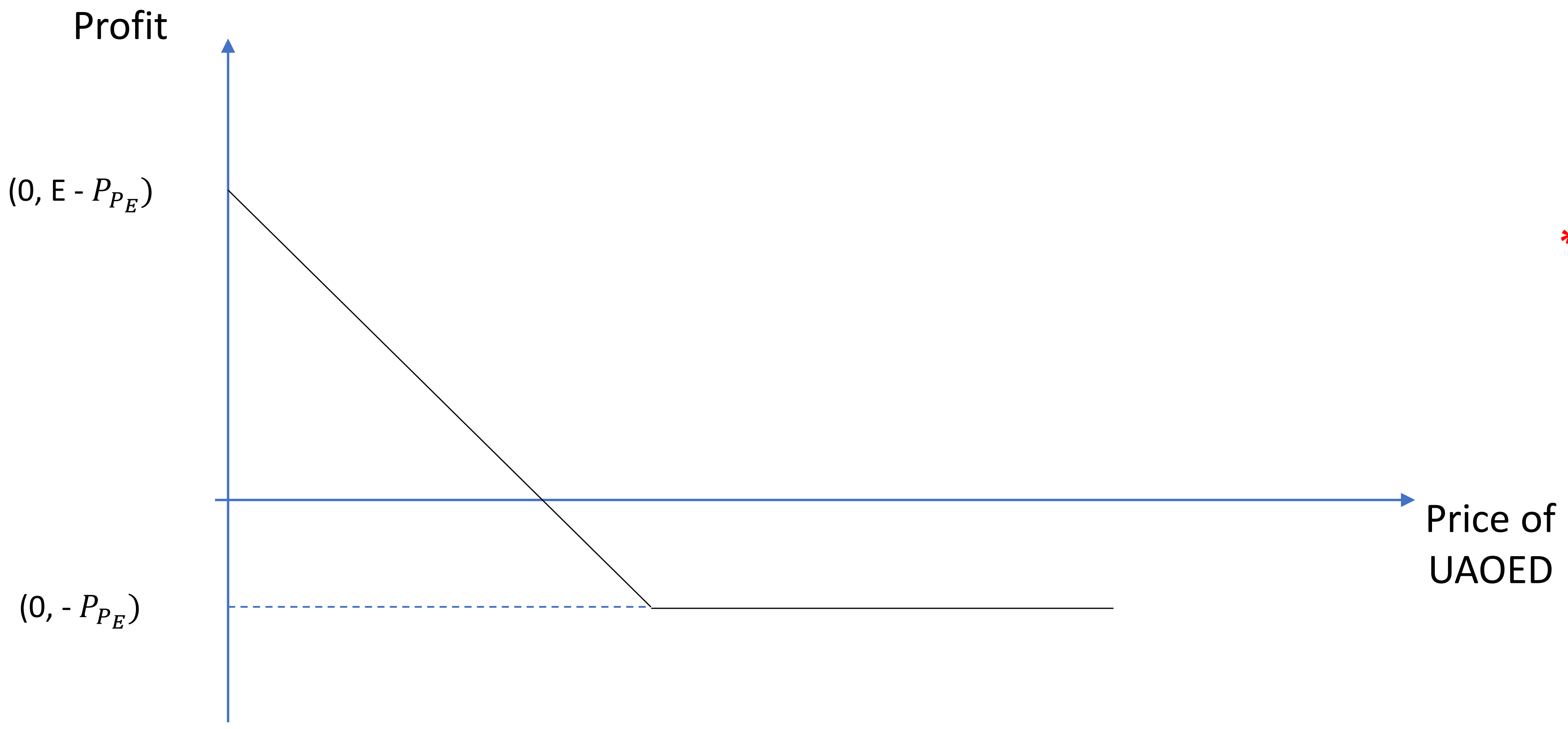
- A PUT Option is a contract which gives the owner the right to sell an asset at an agreed upon price at a specified date.
- If A sells B a PUT option P_E (S,T) on the underlying set S, then B has the right to sell S to A time t=T at a price E.
- S -> underlying Asset
- T -> Expiration time
- E -> Strike Price

LONG PUT



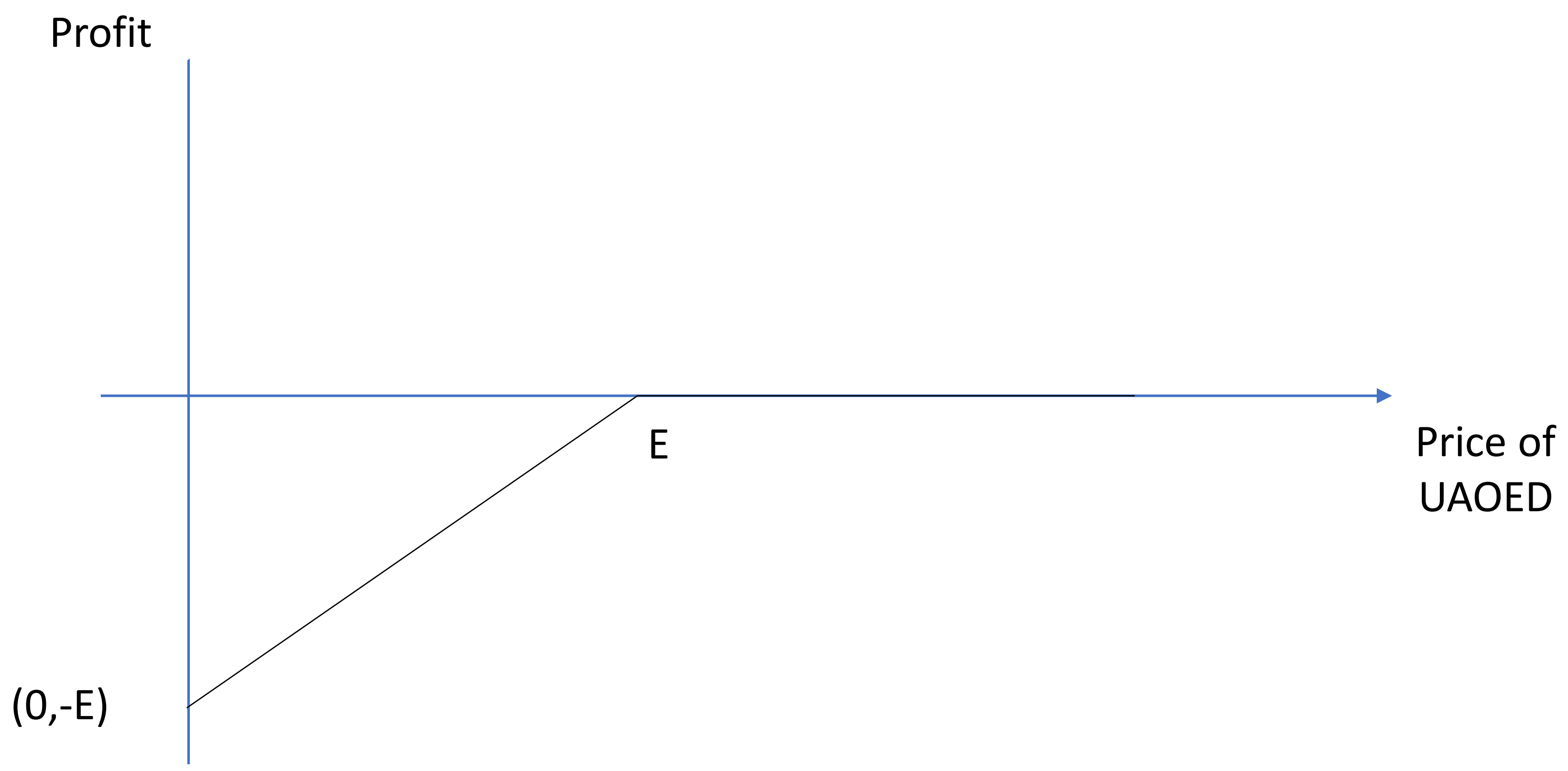
UAOED: Underlying asset on Expiration date

LONG PUT



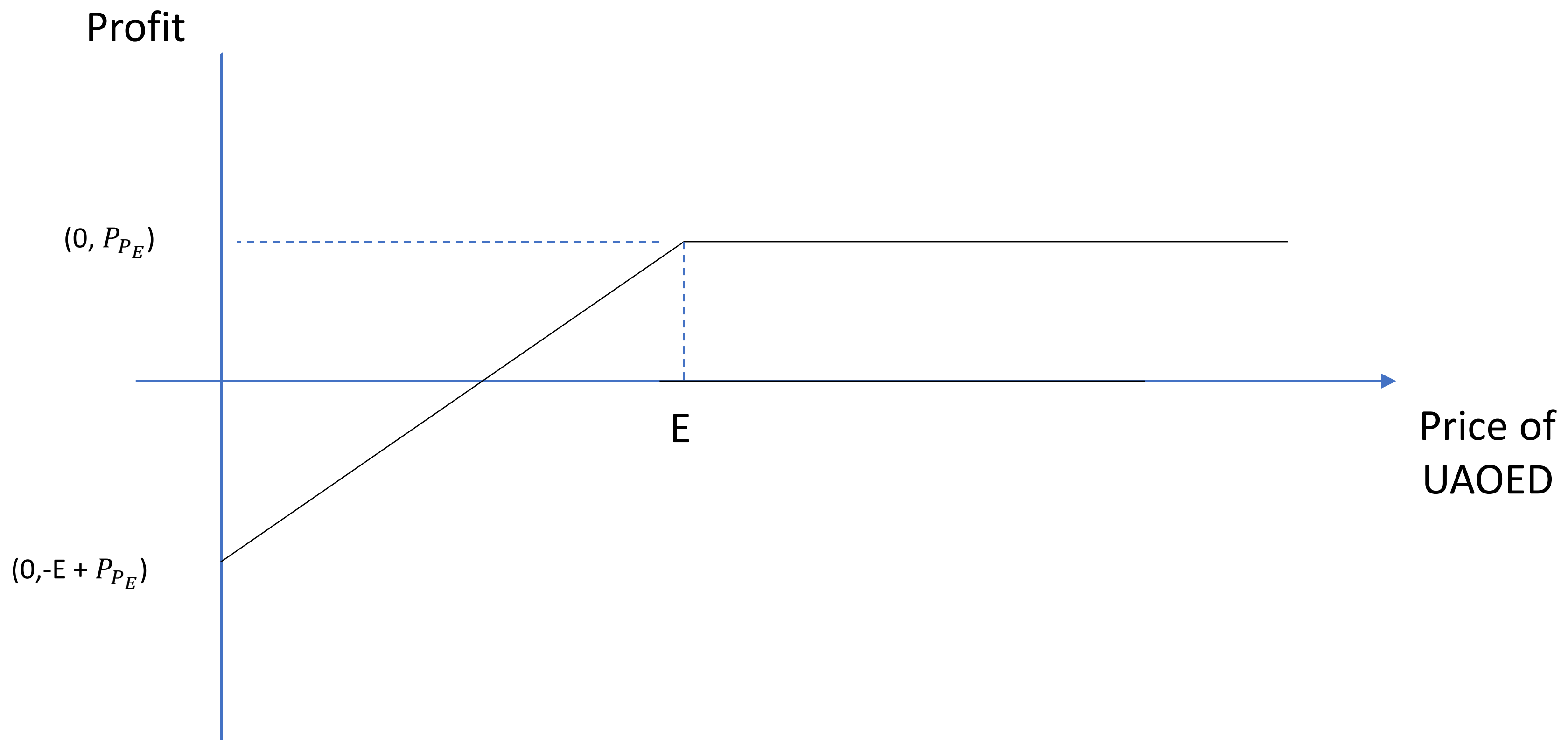
UAOED: Underlying asset on Expiration date

SHORT PUT



****UAOED: Underlying asset on Expiration date***

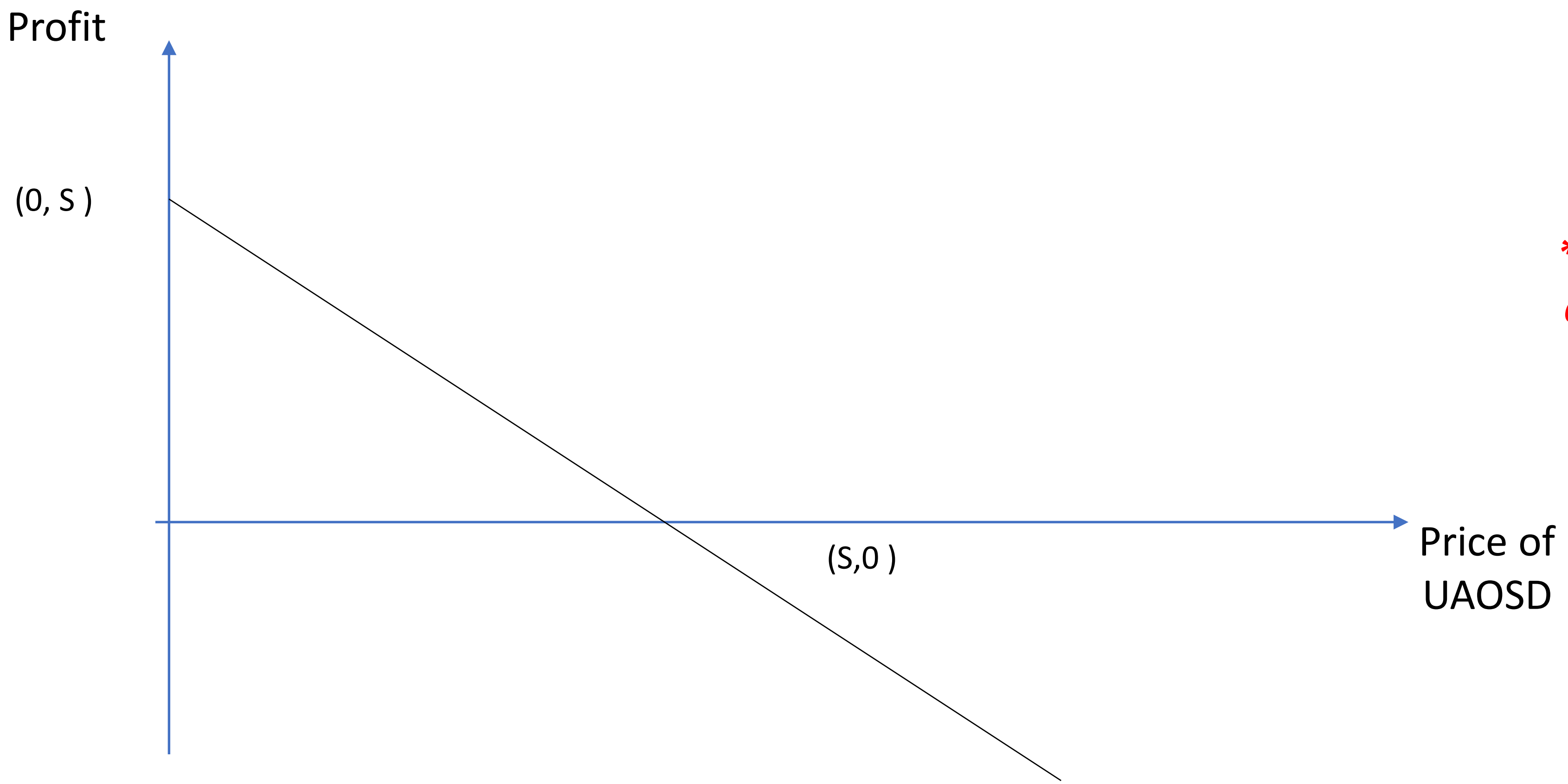
SHORT PUT



UAOED: Underlying asset on Expiration date

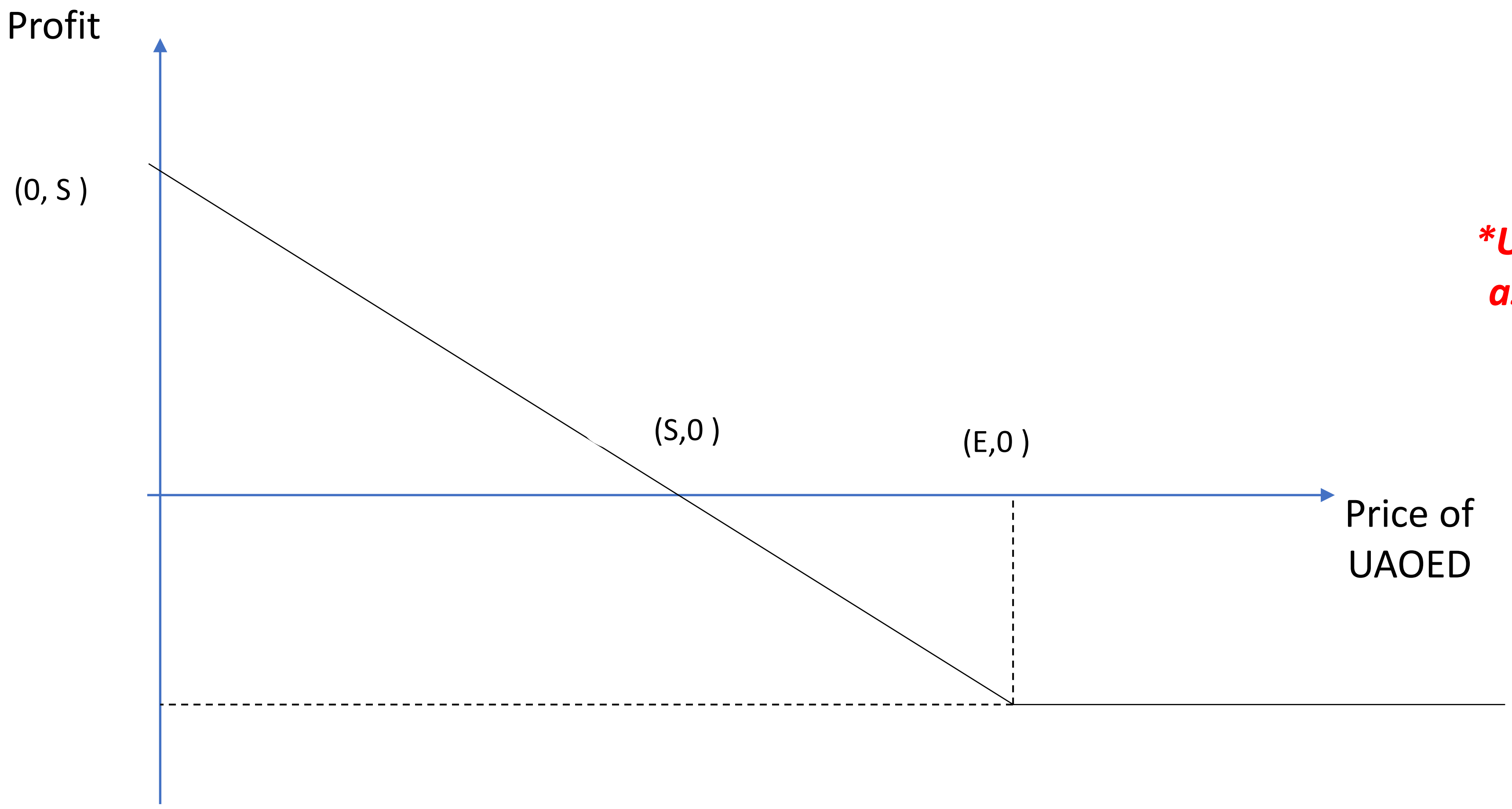
Shorting an Asset

Shorting a Stock



UAOSD: Underlying asset on Settlement date

Shorting with a Call



UAOED: Underlying asset on Expiration date


AMAZON VS FUTURE GROUP
WHAT IS INDIA'S RETAIL
MARKET FUTURE?

Trading Intuition

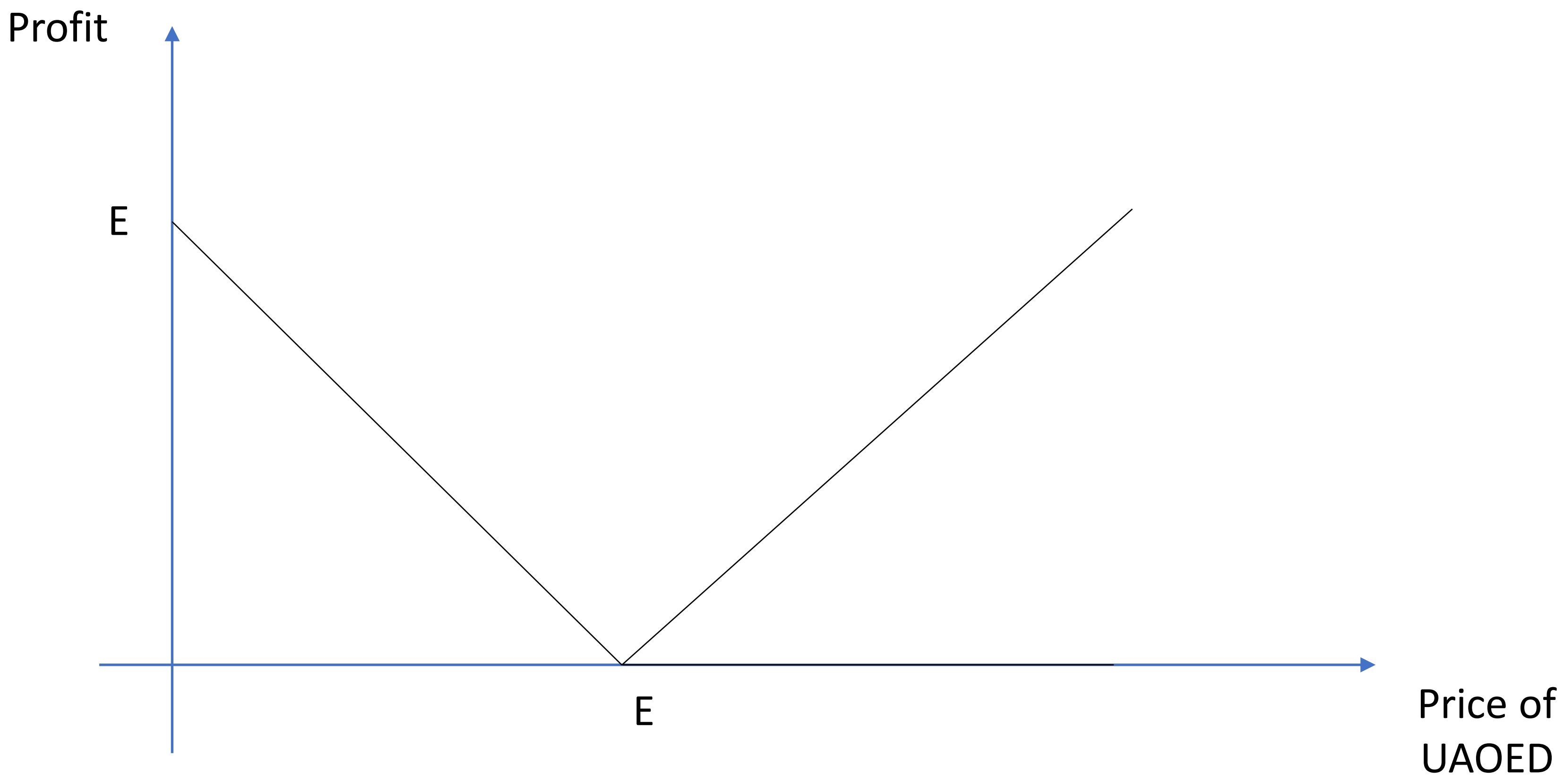
- If the REL – FUTURE deal happens: REL price will go up.
- If SC / SIAC terminates the deal, REL. price will go down.
- I don’t know what’s going to happen…
- But I see some action happening…whichever way the verdict goes..


What should I do now?
DreemBox™

STRADDLE

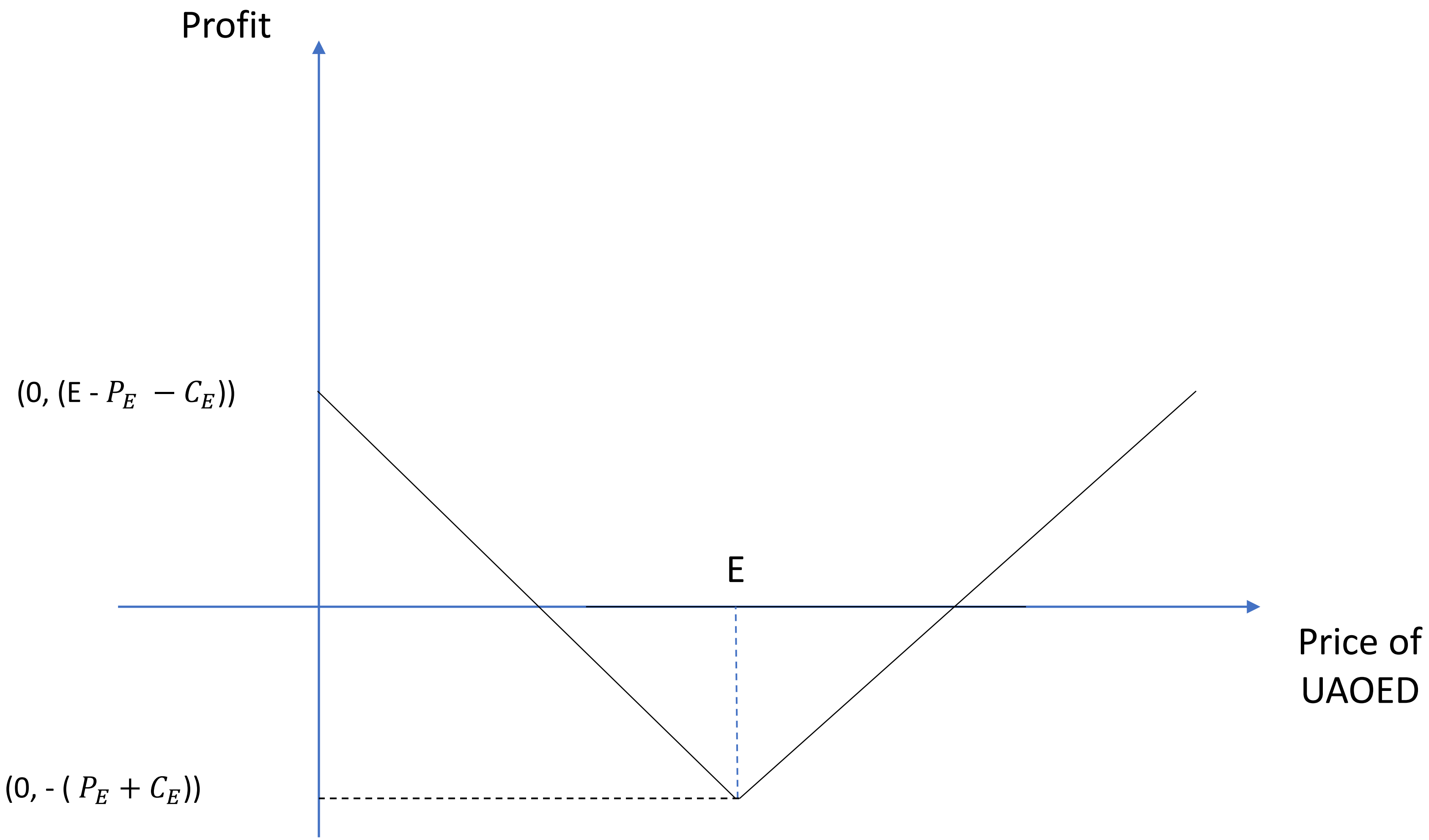
- I am betting on “movement” !!!
- I buy a CALL option C_E (S,T) & a PUT option P_E (S,T).
- Same Underlying asset & same expiration date.
- If $S_T > E$: I will exercise the CALL.
- If $S_T < E$: I will exercise the PUT.

STRADDLE - Payoff



UAOED: Underlying asset on Expiration date

STRADDLE - Payoff

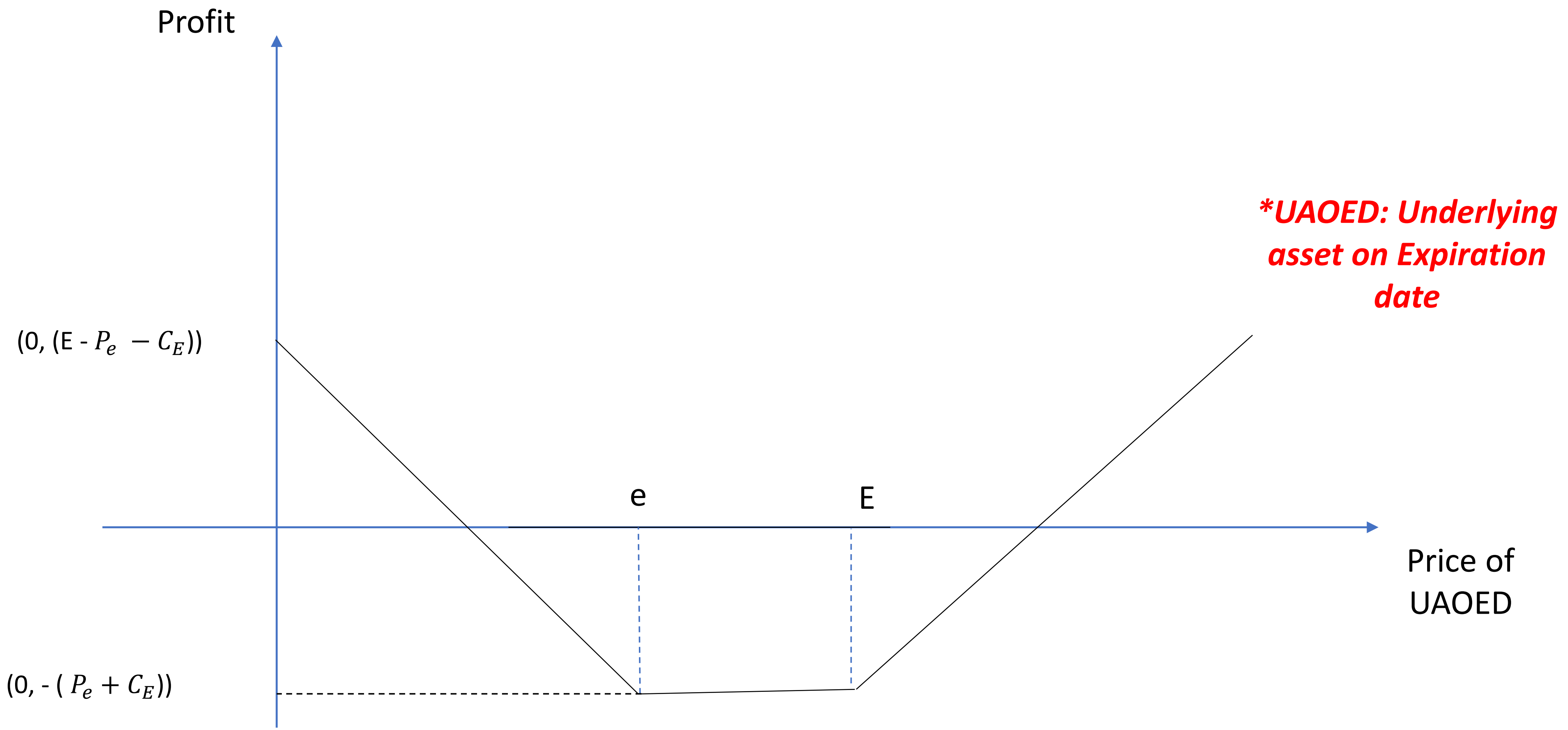


UAOED: Underlying asset on Expiration date

STRANGLE

- I am betting on “movement” !!!
- I also have a hunch about the direction…somewhat …
- I buy a CALL option C_E (S,T) & a PUT option P_e (S,T); e < E.
- Same Underlying asset & same expiration date.
- If $S_T > E$: I will exercise the CALL.
- If $S_T < e$: I will exercise the PUT.
- If $S_T \in (e, E)$ I exercise neither of the options.

STRANGLE - Payoff



Straddle vs Strangle

- If $e < E$, $P_e\ (S,T) < P_E\ (S,T)$
- So a “Strangle” Portfolio is cheaper than a “Straddle”.
- I am betting on the upward movement more…

Example

- Assume the stock is trading at Rs. 15 in April.
- Suppose a CALL option with Strike Rs. 15 & expiration date: June has a price of Rs. 2.
- Suppose a PUT option with Strike Rs. 15 & expiration date: June has a price of Re. 1.
- Buy a 100 – sized straddle (i.e 100 CALLs & 100 PUTs)
- Portfolio price = Rs. (1+2) x 100 = Rs. 300
- The straddle will increase in value if the stock moves higher (because of the long call option) or if the stock goes lower (because of the long put option).
- Profits will be realized as long as the price of the stock moves by more than Rs. 3 in either direction.

- A straddle has no directional bias.
- A strangle is used when the trader believes the stock has a better chance of moving in a certain direction, but would still like to be protected in the case of a negative move.
- For example, let's say you believe a company's results will be positive, meaning you require less downside protection. Instead of buying the put option with the strike price of $15 for $1, maybe you look at buying the $12.50 strike that has a price of $0.25.
- Now total cost of the Strangle portfolio: Rs. 100* (2.25) = Rs. 225 < Rs. 300
- So an upward movement in price of Rs. 2.25 is good enough to break even.

Portfolio Design

- Sell 2 - C_{50} (S_t ,T) calls.
- Buy 2 - C_{70} (S_t ,T) calls.
- Buy 3 - C_{90} (S_t ,T) calls.
- Sell 1 - C_{110} (S_t ,T) calls.
- Sell 2 - C_{120} (S_t ,T) calls.

$$C^* = 2.\, C_{50}\,(S_t\,,\mathrm{T}) + 1.\, C_{110}\,(S_t\,,\mathrm{T}) + 2.\, C_{120}\,(S_t\,,\mathrm{T})$$

$$-\, 2.\, C_{70}\,(S_t\,,\mathrm{T}) - 3.\, C_{90}\,(S_t\,,\mathrm{T})$$

Option Pricing

Present value of Money

- I have Rs. M_t in my wallet at time t.
- Interest rate = r
- Let's say Increase in money in the next Δt period = ΔM_t
- $\Delta M_t = M_t$ .r. Δt: $\frac{\Delta M_t}{\Delta t}$ = M_t .r
- If Δt is infinitesimally small: $\frac{dM}{dt} = M_t$ .r
- $M_t = M_0\, e^{r.t}$

PUT – CALL PARITY Eq.

- $S_t + P_E\ (S_t\ ,\mathrm{T})\ = C_E\ (S_t\ ,\mathrm{T})\ +\ \mathrm{E}.\ e^{-r(T-t)}$

S_t -> Price of the underlying asset at time = t
$P_E\ (S_t\ ,\ \mathrm{t})$: Price of a PUT option with strike price E & expiration date T
$C_E\ (S_t\ ,\mathrm{T})$: Price of a CALL option with strike price E & expiration date T

- The two options have the same Strike Price & Expiration date.

- The Equation is a simple reflection of No – Arbitrage Condition.

BSM – Option Pricing

Left to right: Robert Merton, Myron Scholes and Fisher Black