LPP: All Methods:- Numerical Examples

Prof. M.P. Biswal

Department of Mathematics

IIT- Kharagpur
E-Mail: mpbiswal@maths.iitkgp.ac.in

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Numerical Example (1):

$$\max : Z = x_1 + 3x_2$$

Subject to

$$x_1 + x_2 \leq 10$$

$$x_1 + 4x_2 \leq 16$$

$$x_1, x_2 \geq 0$$

Numerical Example (1):
Introduce Slack Variables to transform the inequations to equations.

$$\max : Z = x_1 + 3x_2 + 0x_3 + 0x_4$$

Subject to

$$x_1 + x_2 + x_3 = 10$$

$$x_1 + 4x_2 + x_4 = 16$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Numerical Example (1):
There are four non-negative variables ($n = 4$)
but only two equations $m = 2$.
Check it: $r(A) = r(A|b) = 2$
By equating any two ($=n - m$) variables to zero,
we solve the system $\binom{4}{2} = 6$ times.
That is all the non-basic variables are zero.

$$x_1 + x_2 + x_3 = 10$$

$$x_1 + 4x_2 + x_4 = 16$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Basic Feasible Solution : Table

SL.	x_1	x_2	x_3	x_4	Z
1.*	0*	0*	10	16	0
2.	0	10	0	-24	NO
3.*	0*	4*	6	0	12
4.*	10*	0*	0	6	10
5.	16	0	-6	0	NO
6.*	8*	2*	0	0	14

BFS=Extreme Points= (0,0), (0,4),(10,0),(8,2)

$$\textit{Optimal} \quad \textit{Solution}:$$

$$x_1^* = 8, x_2^* = 2, \max : Z^* = 14$$

Numerical Example (2):

$$\min : Z = x_1 + 3x_2$$

Subject to

$$x_1 + x_2 \leq 10$$

$$x_1 + 4x_2 \leq 16$$

$$x_1, x_2 \geq 0$$

Numerical Example (2):
Introduce Slack Variables to transform the inequations to equations.

$$\min : Z = x_1 + 3x_2 + 0x_3 + 0x_4$$

Subject to

$$x_1 + x_2 + x_3 = 10$$

$$x_1 + 4x_2 + x_4 = 16$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Numerical Example (2):
There are four non-negative variables ($n = 4$) but only two equations $m = 2$.
Check it: $r(A) = r(A|b) = 2$
By equating any two ($= n - m$) variables to zero, we solve the system $\binom{4}{2} = 6$ times.
That is all the non-basic variables are zero.

$$x_1 + x_2 + x_3 = 10$$

$$x_1 + 4x_2 + x_4 = 16$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Basic Feasible Solution : Table

SL.	x_1	x_2	x_3	x_4	Z
1.*	0*	0*	10	16	0
2.	0	10	0	-24	NO
3.*	0*	4*	6	0	12
4.*	10*	0*	0	6	10
5.	16	0	-6	0	NO
6.*	8*	2*	0	0	14

BFS=Extreme Points= (0,0), (0,4),(10,0),(8,2)

***Optimal Solution* :**

$$x_1^* = 0, x_2^* = 0, \min : Z^* = 0$$

Numerical Example (3):

$$\min : Z = x_1 + 3x_2$$

Subject to

$$x_1 + x_2 \geq 10$$

$$x_1 + 4x_2 \geq 16$$

$$x_1, x_2 \geq 0$$

Numerical Example (3):
Introduce surplus Variables to transform the inequations to equations.

$$\min : Z = x_1 + 3x_2 + 0x_3 + 0x_4$$

Subject to

$$x_1 + x_2 - x_3 = 10$$

$$x_1 + 4x_2 - x_4 = 16$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Numerical Example (3):
There are four non-negative variables ($n = 4$)
but only two equations $m = 2$.
Check it: $r(A) = r(A|b) = 2$
By equating any two ($= n - m$) variables to zero,
we solve the system $\binom{4}{2} = 6$ times.
That is all the non-basic variables are zero.

$$x_1 + x_2 - x_3 = 10$$

$$x_1 + 4x_2 - x_4 = 16$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Basic Feasible Solution : Table

SL.	x_1	x_2	x_3	x_4	Z
1.	0	0	-10	- 16	NO
2.*	0*	10*	0	24	30
3.	0	4	-6	0	NO
4.	10	0	0	-6	NO
5.*	16*	0	6	0	16
6.*	8*	2*	0	0	14

BFS=Extreme Points= (16,0), (0,10),(8,2)

Optimal Solution **:**

$$x_1^* = 8, x_2^* = 2, \min : Z^* = 14$$

Numerical Example (4):

$$\max : Z = -x_1 - 3x_2$$

Subject to

$$x_1 + x_2 \geq 10$$

$$x_1 + 4x_2 \geq 16$$

$$x_1, x_2 \geq 0$$

Numerical Example (4):
Introduce surplus Variables to transform the inequations to equations.

$$\min : Z = x_1 + 3x_2 + 0x_3 + 0x_4$$

Subject to

$$x_1 + x_2 - x_3 = 10$$

$$x_1 + 4x_2 - x_4 = 16$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Numerical Example (4):
There are four non-negative variables ($n = 4$) but only two equations $m = 2$.
Check it: $r(A) = r(A|b) = 2$
By equating any two ($=n - m$) variables to zero, we solve the system $\binom{4}{2} = 6$ times.
That is all the non-basic variables are zero.

$$x_1 + x_2 - x_3 = 10$$

$$x_1 + 4x_2 - x_4 = 16$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Basic Feasible Solution : Table

SL.	x_1	x_2	x_3	x_4	Z
1.	0	0	-10	- 16	NO
2.*	0*	10*	0	24	-30
3.	0	4	-6	0	NO
4.	10	0	0	-6	NO
5.*	16*	0	6	0	-16
6.*	8*	2*	0	0	-14

BFS=Extreme Points= (16,0), (0,10),(8,2)

Optimal Solution :

$$x_1^* = 8, x_2^* = 2, \max : Z^* = -14$$

Numerical Example (5):

$$\max : Z = x_1 + 3x_2$$

Subject to

$$x_1 + x_2 \geq 10$$

$$x_1 + 4x_2 \geq 16$$

$$x_1, x_2 \geq 0$$

Numerical Example (5):
Introduce surplus Variables to transform the inequations to equations.

$$\max : Z = x_1 + 3x_2 + 0x_3 + 0x_4$$

Subject to

$$x_1 + x_2 - x_3 = 10$$

$$x_1 + 4x_2 - x_4 = 16$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Numerical Example (5):
There are four non-negative variables ($n = 4$)
but only two equations $m = 2$.
Check it: $r(A) = r(A|b) = 2$
By equating any two ($=n - m$) variables to zero,
we solve the system $\binom{4}{2} = 6$ times.
That is all the non-basic variables are zero.

$$x_1 + x_2 - x_3 = 10$$

$$x_1 + 4x_2 - x_4 = 16$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Basic Feasible Solution : Table

SL.	x_1	x_2	x_3	x_4	Z
1.	0	0	-10	- 16	NO
2.*	0*	10*	0	24	30
3.	0	4	-6	0	NO
4.	10	0	0	-6	NO
5.*	16*	0	6	0	16
6.*	8*	2*	0	0	14

BFS=Extreme Points= (16,0), (0,10),(8,2)
This LPP is feasible but unbounded.
No Optimal Solution.

Numerical Example (6):

$$\max : Z = x_1 + 3x_2$$

Subject to

$$x_1 + x_2 \leq 10$$

$$x_1 + 4x_2 \geq 16$$

$$x_1, x_2 \geq 0$$

Numerical Example (6):
Introduce slack/surplus Variables to transform the inequations to equations.

$$\max : Z = x_1 + 3x_2 + 0x_3 + 0x_4$$

Subject to

$$x_1 + x_2 + x_3 = 10$$

$$x_1 + 4x_2 - x_4 = 16$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Numerical Example (6):
There are four non-negative variables ($n = 4$)
but only two equations $m = 2$.
Check it: $r(A) = r(A|b) = 2$
By equating any two ($=n - m$) variables to zero,
we solve the system $\binom{4}{2} = 6$ times.
That is all the non-basic variables are zero.

$$x_1 + x_2 + x_3 = 10$$

$$x_1 + 4x_2 - x_4 = 16$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Basic Feasible Solution : Table

SL.	x_1	x_2	x_3	x_4	Z
1.	0	0	10	- 16	NO
2.*	0*	10*	0	24	30
3.*	0*	4*	6	0	12
4.	10	0	0	-6	NO
5.	16	0	-6	0	NO
6.*	8*	2*	0	0	14

BFS=Extreme Points= (0,10),(0,4),(8,2)

Optimal Solution :

$$x_1^* = 0, x_2^* = 10, \max : Z^* = 30$$

Numerical Example (7):

$$\min : Z = x_1 + 3x_2$$

Subject to

$$x_1 + x_2 \leq 10$$

$$x_1 + 4x_2 \geq 16$$

$$x_1, x_2 \geq 0$$

Numerical Example (7):
Introduce slack/surplus Variables to transform the inequations to equations.

$$\min : Z = x_1 + 3x_2 + 0x_3 + 0x_4$$

Subject to

$$x_1 + x_2 + x_3 = 10$$

$$x_1 + 4x_2 - x_4 = 16$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Numerical Example (7):
There are four non-negative variables ($n = 4$)
but only two equations $m = 2$.
Check it: $r(A) = r(A|b) = 2$
By equating any two ($= n - m$) variables to zero,
we solve the system $\binom{4}{2} = 6$ times.
That is all the non-basic variables are zero.

$$x_1 + x_2 + x_3 = 10$$

$$x_1 + 4x_2 - x_4 = 16$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Basic Feasible Solution : Table

SL.	x_1	x_2	x_3	x_4	Z
1.	0	0	10	- 16	NO
2.*	0*	10*	0	24	30
3.*	0*	4*	6	0	12
4.	10	0	0	-6	NO
5.	16	0	-6	0	NO
6.*	8*	2*	0	0	14

BFS=Extreme Points= (0,10),(0,4),(8,2)

Optimal Solution :

$$x_1^* = 0, x_2^* = 4, \min : Z^* = 12$$

Numerical Example (8):

$$\max : Z = x_1 + 3x_2$$

Subject to

$$x_1 + x_2 \geq 10$$

$$x_1 + 4x_2 \leq 16$$

$$x_1, x_2 \geq 0$$

Numerical Example (8):
Introduce slack/surplus Variables to transform the inequations to equations.

$$\max : Z = x_1 + 3x_2 + 0x_3 + 0x_4$$

Subject to

$$x_1 + x_2 - x_3 = 10$$

$$x_1 + 4x_2 + x_4 = 16$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Numerical Example (8):
There are four non-negative variables ($n = 4$)
but only two equations $m = 2$.
Check it: $r(A) = r(A|b) = 2$
By equating any two ($=n - m$) variables to zero,
we solve the system $\binom{4}{2} = 6$ times.
That is all the non-basic variables are zero.

$$x_1 + x_2 - x_3 = 10$$

$$x_1 + 4x_2 + x_4 = 16$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Basic Feasible Solution : Table

SL.	x_1	x_2	x_3	x_4	Z
1.	0	0	-10	16	NO
2.	0	10	0	-24	NO
3.	0	4	-6	0	NO
4.*	10*	0*	0	6	10
5.*	16*	0*	6	0	16
6.*	8*	2*	0	0	14

BFS=Extreme Points= (10,0),(16,0),(8,2)

Optimal Solution :

$$x_1^* = 16, x_2^* = 0, \max : Z^* = 16$$

Numerical Example (9):

$$\min : Z = x_1 + 3x_2$$

Subject to

$$x_1 + x_2 \geq 10$$

$$x_1 + 4x_2 \leq 16$$

$$x_1, x_2 \geq 0$$

Numerical Example (9):
Introduce slack/surplus Variables to transform the inequations to equations.

$$\min : Z = x_1 + 3x_2 + 0x_3 + 0x_4$$

Subject to

$$x_1 + x_2 - x_3 = 10$$

$$x_1 + 4x_2 + x_4 = 16$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Numerical Example (9):
There are four non-negative variables ($n = 4$)
but only two equations $m = 2$.
Check it: $r(A) = r(A|b) = 2$
By equating any two ($=n - m$) variables to zero,
we solve the system $\binom{4}{2} = 6$ times.
That is all the non-basic variables are zero.

$$x_1 + x_2 - x_3 = 10$$

$$x_1 + 4x_2 + x_4 = 16$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Basic Feasible Solution : Table

SL.	x_1	x_2	x_3	x_4	Z
1.	0	0	-10	16	NO
2.	0	10	0	-24	NO
3.	0	4	-6	0	NO
4.*	10*	0*	0	6	10
5.*	16*	0*	6	0	16
6.*	8*	2*	0	0	14

BFS=Extreme Points= (10,0),(16,0),(8,2)

$$\textit{Optimal} \quad \textit{Solution}:$$

$$x_1^* = 10, x_2^* = 0, \min : Z^* = 10$$

Numerical Example (10):

$$\min : Z = x_1 + 3x_2$$

Subject to

$$x_1 + x_2 \leq 10$$

$$x_1 + 4x_2 \geq 44$$

$$x_1, x_2 \geq 0$$

Numerical Example (10):
Introduce slack/surplus Variables to transform the inequations to equations.

$$\min : Z = x_1 + 3x_2 + 0x_3 + 0x_4$$

Subject to

$$x_1 + x_2 + x_3 = 10$$

$$x_1 + 4x_2 - x_4 = 44$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Numerical Example (10):
There are four non-negative variables ($n = 4$)
but only two equations $m = 2$.
Check it: $r(A) = r(A|b) = 2$
By equating any two ($=n - m$) variables to zero,
we solve the system $\binom{4}{2} = 6$ times.
That is all the non-basic variables are zero.

$$x_1 + x_2 + x_3 = 10$$

$$x_1 + 4x_2 - x_4 = 44$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Basic Feasible Solution : Table

SL.	x_1	x_2	x_3	x_4	Z
1.	0	0	10	-44	NO
2.	0	10	0	-4	NO
3.	0	11	-1	0	NO
4.	10	0	0	-34	NO
5.	44	0	-34	0	NO
6.	-4/3	34/3	0	0	NO

BFS=Extreme Points= Nil
There is no Basic Feasible Solution.
Some x_j are negative. The LPP is infeasible.

Numerical Example (11):

$$\max : Z = x_1 + 3x_2$$

Subject to

$$x_1 + x_2 \leq 10$$

$$x_1 + 2x_2 \leq 11$$

$$x_1 + 4x_2 \leq 16$$

$$x_1, x_2 \geq 0$$

Numerical Example (11):
Introduce Slack Variables to transform the inequations to equations.

$$\max : Z = x_1 + 3x_2 + 0x_3 + 0x_4 + 0x_5$$

Subject to

$$x_1 + x_2 + x_3 = 10$$

$$x_1 + 2x_2 + x_4 = 11$$

$$x_1 + 4x_2 + x_5 = 16$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Numerical Example (11):
There are five non-negative variables ($n = 5$)
but only three equations $m = 3$.
Check it: $r(A) = r(A|b) = 3$
By equating any two ($=n - m$) variables to zero,
we solve the system $\binom{5}{2} = 10$ times.
That is all the non-basic variables are zero.

$$x_1 + x_2 + x_3 = 10$$

$$x_1 + 2x_2 + x_4 = 11$$

$$x_1 + 4x_2 + x_5 = 16$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

SL.	x_1	x_2	x_3	x_4	x_5	Z
1.*	0*	0*	10	11	16	0
2.	0	10	0	-9	-24	No
3.	0	11/2	9/2	0	-6	No
4.*	0*	4*	6	3	0	12
5.*	10*	0*	0	1	6	10
6.	11	0	-1	0	5	No
7.	16	0	-6	-5	0	No
8.*	9*	1*	0	0	3	12
9.	8	2	0	-1	0	No
10.*	6*	5/2*	3/2	0	0	27/2*

BFS=Extreme Points= (0,0), (0,4),(10,0),(9,1),(6,5/2)

***Optimal Solution* :**

$$x_1^* = 6, x_2^* = 5/2, \max : Z^* = 27/2$$

Numerical Example (12):

$$\max : Z = x_1 + 4x_2$$

Subject to

$$x_1 + x_2 \leq 10$$

$$x_1 + 2x_2 \leq 11$$

$$x_1 + 4x_2 \leq 16$$

$$x_1, x_2 \geq 0$$

Numerical Example (12):
Introduce Slack Variables to transform the inequations to equations.

$$\max : Z = x_1 + 4x_2 + 0x_3 + 0x_4 + 0x_5$$

Subject to

$$x_1 + x_2 + x_3 = 10$$

$$x_1 + 2x_2 + x_4 = 11$$

$$x_1 + 4x_2 + x_5 = 16$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Numerical Example (12):
There are five non-negative variables ($n = 5$)
but only three equations $m = 3$.
Check it: $r(A) = r(A|b) = 3$
By equating any two ($=n - m$) variables to zero,
we solve the system $\binom{5}{2} = 10$ times.
That is all the non-basic variables are zero.

$$x_1 + x_2 + x_3 = 10$$

$$x_1 + 2x_2 + x_4 = 11$$

$$x_1 + 4x_2 + x_5 = 16$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Basic Feasible Solution : Table

SL.	x_1	x_2	x_3	x_4	x_5	Z
1.*	0*	0*	10	11	16	0
2.	0	10	0	-9	-24	No
3.	0	11/2	9/2	0	-6	No
4.*	0*	4*	6	3	0	16*
5.*	10*	0*	0	1	6	10
6.	11	0	-1	0	5	No
7.	16	0	-6	-5	0	No
8.*	9*	1*	0	0	3	13
9.	8	2	0	-1	0	No
10.*	6*	5/2*	3/2	0	0	16*

BFS=Extreme Points= (0,0), (0,4),(10,0),(9,1),(6,5/2)

***Optimal Solution* :**

$$x_1^* = 6, x_2^* = 5/2, \max : Z^* = 16$$
$$x_1^* = 0, x_2^* = 4, \max : Z^* = 16$$

Basic Feasible Solution Method: LPP

Numerical Example (13):

$$\min : Z = x_1 + 3x_2$$

Subject to

$$x_1 + x_2 \geq 10$$

$$x_1 + 2x_2 \geq 11$$

$$x_1 + 4x_2 \geq 16$$

$$x_1, x_2 \geq 0$$

Numerical Example (13):
Introduce surplus Variables to transform the inequations to equations.

$$\min : Z = x_1 + 3x_2 + 0x_3 + 0x_4 + 0x_5$$

Subject to

$$x_1 + x_2 - x_3 = 10$$

$$x_1 + 2x_2 - x_4 = 11$$

$$x_1 + 4x_2 - x_5 = 16$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Numerical Example (13):
There are five non-negative variables ($n = 5$)
but only three equations $m = 3$.
Check it: $r(A) = r(A|b) = 3$
By equating any two ($= n - m$) variables to zero,
we solve the system $\binom{5}{2} = 10$ times.
That is all the non-basic variables are zero.

$$x_1 + x_2 - x_3 = 10$$

$$x_1 + 2x_2 - x_4 = 11$$

$$x_1 + 4x_2 - x_5 = 16$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Basic Feasible Solution : Table

SL.	x_1	x_2	x_3	x_4	x_5	Z
1.	0	0	-10	- 11	-16	No
2.*	0*	10*	0	9	24	30
3.	0	11/2	-9/2	0	6	No
4.	0	4	-6	-3	0	No
5.	10	0	0	-1	-6	No
6.	11	0	1	0	-5	No
7.*	16*	0*	6	5	0	16
8.	9	1	0	0	-3	No
9.*	8*	2*	0	1	0	14*
10.	6	5/2	-3/2	0	0	No

BFS=Extreme Points= (0,10), (16,0),(8,2)

***Optimal Solution* :**

$$x_1^* = 8, x_2^* = 2, \min : Z^* = 14$$

Numerical Example (14):

$$\min : Z = x_1 + 4x_2$$

Subject to

$$x_1 + x_2 \geq 10$$

$$x_1 + 2x_2 \geq 11$$

$$x_1 + 4x_2 \geq 16$$

$$x_1, x_2 \geq 0$$

Numerical Example (14):
Introduce surplus Variables to transform the inequations to equations.

$$\min : Z = x_1 + 4x_2 + 0x_3 + 0x_4 + 0x_5$$

Subject to

$$x_1 + x_2 - x_3 = 10$$

$$x_1 + 2x_2 - x_4 = 11$$

$$x_1 + 4x_2 - x_5 = 16$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Numerical Example (14):
There are five non-negative variables ($n = 5$)
but only three equations $m = 3$.
Check it: $r(A) = r(A|b) = 3$
By equating any two ($=n - m$) variables to zero,
we solve the system $\binom{5}{2} = 10$ times.
That is all the non-basic variables are zero.

$$x_1 + x_2 - x_3 = 10$$

$$x_1 + 2x_2 - x_4 = 11$$

$$x_1 + 4x_2 - x_5 = 16$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Basic Feasible Solution : Table

SL.	x_1	x_2	x_3	x_4	x_5	Z
1.	0	0	-10	- 11	-16	No
2.*	0*	10*	0	9	24	40
3.	0	11/2	-9/2	0	6	No
4.	0	4	-6	-3	0	No
5.	10	0	0	-1	-6	No
6.	11	0	1	0	-5	No
7.*	16*	0*	6	5	0	16*
8.	9	1	0	0	-3	No
9.*	8*	2*	0	1	0	16*
10.	6	5/2	-3/2	0	0	No

BFS=Extreme Points= (0,10), (16,0),(8,2)

$$\textit{Optimal} \quad \textit{Solution}:$$

$$x_1^* = 8, x_2^* = 2, \min : Z^* = 16$$
$$x_1^* = 16, x_2^* = 0, \min : Z^* = 16$$

Numerical Example (15):

$$\min : Z = x_1 + 3x_2$$

Subject to

$$x_1 + x_2 \leq 10$$

$$x_1 + 2x_2 = 11$$

$$x_1 + 4x_2 \geq 16$$

$$x_1, x_2 \geq 0$$

Numerical Example (15):
Introduce slack/surplus/artificial Variables to transform the inequations to equations.

$$\min : Z = x_1 + 3x_2 + 0x_3 + 0x_4 + 0x_5$$

Subject to

$$x_1 + x_2 + x_3 = 10$$

$$x_1 + 2x_2 + x_4 = 11$$

$$x_1 + 4x_2 - x_5 = 16$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

where x_3 is a slack variable,
x_4 is an artificial variable,
x_5 is a surplus variable.

Numerical Example (15):
There are five non-negative variables ($n = 5$)
but only three equations $m = 3$.
Check it: $r(A) = r(A|b) = 3$
By equating any two ($=n - m$) variables to zero,
we solve the system $\binom{5}{2} = 10$ times.
That is all the non-basic variables are zero.

$$x_1 + x_2 + x_3 = 10$$

$$x_1 + 2x_2 + x_4 = 11$$

$$x_1 + 4x_2 - x_5 = 16$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

SL.	x_1	x_2	x_3	x_4	x_5	Z
1.	0	0	10	11	-16	NO
2.	0	10	0	-9	24	NO
3.	0	11/2	9/2	0	6	33/2
4.	0	4	6	3*	0	$x_4 > 0$
5.	10	0	0	1	-6	NO
6.	11	0	-1	0	-5	NO
7.	16	0	-6	-5	0	NO
8.	9	1	0	0	-3	NO
9.	8	2	0	-1	0	NO
10.	6	5/2	3/2	0	0	27/2

BFS=Extreme Points= (0,11/2), (6,5/2)

***Optimal Solution* :**

$$x_1^* = 6, x_2^* = 5/2, \min : Z^* = 27/2$$

Numerical Example (16):

$$\max : Z = x_1 + 3x_2$$

Subject to

$$x_1 + x_2 \leq 10$$

$$x_1 + 2x_2 = 11$$

$$x_1 + 4x_2 \geq 16$$

$$x_1, x_2 \geq 0$$

Numerical Example (16):
Introduce slack/surplus/artificial Variables to transform the inequations to equations.

$$\max : Z = x_1 + 3x_2 + 0x_3 + 0x_4 + 0x_5$$

Subject to

$$x_1 + x_2 + x_3 = 10$$

$$x_1 + 2x_2 + x_4 = 11$$

$$x_1 + 4x_2 - x_5 = 16$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

where x_3 is a slack variable,
x_4 is an artificial variable,
x_5 is a surplus variable.

Numerical Example (16):
There are five non-negative variables ($n = 5$)
but only three equations $m = 3$.
Check it: $r(A) = r(A|b) = 3$
By equating any two ($=n - m$) variables to zero,
we solve the system $\binom{5}{2} = 10$ times.
That is all the non-basic variables are zero.

$$x_1 + x_2 + x_3 = 10$$

$$x_1 + 2x_2 + x_4 = 11$$

$$x_1 + 4x_2 - x_5 = 16$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Basic Feasible Solution : Table

SL.	x_1	x_2	x_3	x_4	x_5	Z
1.	0	0	10	11	-16	NO
2.	0	10	0	-9	24	NO
3.	0	11/2	9/2	0	6	33/2
4.	0	4	6	3*	0	$x_4 > 0$
5.	10	0	0	1	-6	NO
6.	11	0	-1	0	-5	NO
7.	16	0	-6	-5	0	NO
8.	9	1	0	0	-3	NO
9.	8	2	0	-1	0	NO
10.	6	5/2	3/2	0	0	27/2

BFS=Extreme Points= (0,11/2), (6,5/2)

$$\textit{Optimal} \quad \textit{Solution}:$$

$$x_1^* = 0, x_2^* = 11/2, \max : Z^* = 33/2$$

Numerical Example (17):

$$\max : Z = x_1 + 3x_2$$

Subject to

$$x_1 + x_2 \leq 10$$

$$x_1 + 4x_2 \leq 16$$

$$3x_1 + 4x_2 \geq 60$$

$$x_1, x_2 \geq 0$$

Numerical Example (17):
Introduce slack/surplus Variables
to transform the inequations to equations.

$$\max : Z = x_1 + 3x_2 + 0x_3 + 0x_4 + 0x_5$$

Subject to

$$x_1 + x_2 + x_3 = 10$$

$$x_1 + 4x_2 + x_4 = 16$$

$$3x_1 + 4x_2 - x_5 = 60$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

where x_3 is a slack variable,
x_4 is a slack variable,
x_5 is a surplus variable.

Numerical Example (17):
There are five non-negative variables ($n = 5$)
but only three equations $m = 3$.
Check it: $r(A) = r(A|b) = 3$
By equating any two ($= n - m$) variables to zero,
we solve the system $\binom{5}{2} = 10$ times.
That is all the non-basic variables are zero.

$$x_1 + x_2 + x_3 = 10$$

$$x_1 + 4x_2 + x_4 = 16$$

$$3x_1 + 4x_2 - x_5 = 60$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Basic Feasible Solution : Table

SL.	x_1	x_2	x_3	x_4	x_5	Z
1.	0	0	10	16	-60	NO
2.	0	10	0	-24	-20	NO
3.	0	4	6	0	-44	NO
4.	0	15	-5	-44	0	NO
5.	10	0	0	6	-30	NO
6.	16	0	-6	0	-12	NO
7.	20	0	-10	-4	0	NO
8.	8	2	0	0	-28	NO
9.	-20	30	0	-84	0	NO
10.	22	-3/2	-21/2	0	0	NO

BFS=Extreme Points= Nil
There is no Basic Feasible Solution.
Some x_j are negative. The LPP is infeasible.

Numerical Example (18):

$$\min : Z = x_1 + 3x_2$$

Subject to

$$x_1 + x_2 \leq 10$$

$$x_1 + 4x_2 \leq 16$$

$$3x_1 + 4x_2 \geq 60$$

$$x_1, x_2 \geq 0$$

Numerical Example (18):
Introduce slack/surplus Variables
to transform the inequations to equations.

$$\min : Z = x_1 + 3x_2 + 0x_3 + 0x_4 + 0x_5$$

Subject to

$$x_1 + x_2 + x_3 = 10$$

$$x_1 + 4x_2 + x_4 = 16$$

$$3x_1 + 4x_2 - x_5 = 60$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

where x_3 is a slack variable,
x_4 is a slack variable,
x_5 is a surplus variable.

Numerical Example (18):
There are five non-negative variables ($n = 5$)
but only three equations $m = 3$.
Check it: $r(A) = r(A|b) = 3$
By equating any two ($=n - m$) variables to zero,
we solve the system $\binom{5}{2} = 10$ times.
That is all the non-basic variables are zero.

$$x_1 + x_2 + x_3 = 10$$

$$x_1 + 4x_2 + x_4 = 16$$

$$3x_1 + 4x_2 - x_5 = 60$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

SL.	x_1	x_2	x_3	x_4	x_5	Z
1.	0	0	10	16	-60	NO
2.	0	10	0	-24	-20	NO
3.	0	4	6	0	-44	NO
4.	0	15	-5	-44	0	NO
5.	10	0	0	6	-30	NO
6.	16	0	-6	0	-12	NO
7.	20	0	-10	-4	0	NO
8.	8	2	0	0	-28	NO
9.	-20	30	0	-84	0	NO
10.	22	-3/2	-21/2	0	0	NO

BFS=Extreme Points= Nil

There is no Basic Feasible Solution.

Some x_j are negative. The LPP is infeasible.

Numerical Example (19):

$$\max : Z = -x_1 - 3x_2$$

Subject to

$$x_1 + x_2 \geq 10$$

$$x_1 + 2x_2 \geq 11$$

$$x_1 + 4x_2 \geq 16$$

$$x_1, x_2 \geq 0$$

Feasible region is not bounded.
This feasible region can not be enclosed in a circle of a finite radius.

Numerical Example (19):
Introduce surplus Variables to transform the inequations to equations.

$$\max : Z = -x_1 - 3x_2 + 0x_3 + 0x_4 + 0x_5$$

Subject to

$$x_1 + x_2 - x_3 = 10$$

$$x_1 + 2x_2 - x_4 = 11$$

$$x_1 + 4x_2 - x_5 = 16$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Numerical Example (19):
There are five non-negative variables ($n = 5$)
but only three equations $m = 3$.
Check it: $r(A) = r(A|b) = 3$
By equating any two ($= n - m$) variables to zero,
we solve the system $\binom{5}{2} = 10$ times.
That is all the non-basic variables are zero.

$$x_1 + x_2 - x_3 = 10$$

$$x_1 + 2x_2 - x_4 = 11$$

$$x_1 + 4x_2 - x_5 = 16$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Basic Feasible Solution : Table

SL.	x_1	x_2	x_3	x_4	x_5	Z
1.	0	0	-10	- 11	-16	No
2.*	0*	10*	0	9	24	-30
3.	0	11/2	-9/2	0	6	No
4.	0	4	-6	-3	0	No
5.	10	0	0	-1	-6	No
6.	11	0	1	0	-5	No
7.*	16*	0*	6	5	0	-16
8.	9	1	0	0	-3	No
9.*	8*	2*	0	1	0	-14*
10.	6	5/2	-3/2	0	0	No

BFS=Extreme Points= (0,10), (16,0),(8,2)

***Optimal Solution* :**

$$x_1^* = 8, x_2^* = 2, \max : Z^* = -14$$

Numerical Example (20):

$$\max : Z = x_1 + 3x_2$$

Subject to

$$x_1 + x_2 \geq 10$$

$$x_1 + 2x_2 \geq 11$$

$$x_1 + 4x_2 \geq 16$$

$$x_1, x_2 \geq 0$$

Feasible region is not bounded.
This feasible region can not be enclosed in a circle of a finite radius.

Numerical Example (20):
Introduce surplus Variables to transform the inequations to equations.

$$\max : Z = x_1 + 3x_2 + 0x_3 + 0x_4 + 0x_5$$

Subject to

$$x_1 + x_2 - x_3 = 10$$

$$x_1 + 2x_2 - x_4 = 11$$

$$x_1 + 4x_2 - x_5 = 16$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Numerical Example (20):
There are five non-negative variables ($n = 5$) but only three equations $m = 3$.
Check it: $r(A) = r(A|b) = 3$
By equating any two ($=n - m$) variables to zero, we solve the system $\binom{5}{2} = 10$ times.
That is all the non-basic variables are zero.

$$x_1 + x_2 - x_3 = 10$$

$$x_1 + 2x_2 - x_4 = 11$$

$$x_1 + 4x_2 - x_5 = 16$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Basic Feasible Solution : Table

SL.	x_1	x_2	x_3	x_4	x_5	Z
1.	0	0	-10	- 11	-16	No
2.*	0*	10*	0	9	24	30*
3.	0	11/2	-9/2	0	6	No
4.	0	4	-6	-3	0	No
5.	10	0	0	-1	-6	No
6.	11	0	1	0	-5	No
7.*	16*	0*	6	5	0	16
8.	9	1	0	0	-3	No
9.*	8*	2*	0	1	0	14
10.	6	5/2	-3/2	0	0	No

BFS=Extreme Points= (0,10), (16,0),(8,2)

No Optimal Solution.

This LPP is feasible but unbounded.