Two Phase Simplex Method: LPP- Examples

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Phase-I:

In this Phase, an artificial objective function f is used where we minimize the sum of artificial variables to zero. We try to drive out the all the artificial variables from the basis to make them zero. If they can not be removed i.e. all can not be made zero, we conclude that the problem is infeasible. If all the artificial variables are zero in Phase-I, we go to Phase-II.

Phase-II:

In this Phase-II, we replace the artificial objective function f by the original objective function Z using the last Tableau of Phase-I. Then we apply usual Simplex Method until an Optimal solution X^* is reached.

TPS: Point to Note

If an artificial variable is there in the basis at zero value at the end of Phase-I, we modify the departing variable rule. An artificial variable must not become positive from zero. So, we allow an artificial variable with negative a_{ij} value to depart. It is an important point to note.

Step 1. Establish the problem formulation in a form suitable for the implementation of the simplex algorithm (i.e., convert the objective function to a maximization form and convert all the constraints by adding proper, the slack, surplus, or artificial variables).

Step 2. The artificial objective function of Phase-I is constructed by changing all the coefficient of the variables in the original objective function as follows:

- **The coefficient of any artificial variables will be -1.**
- **The coefficient of all other variables in the objective will be zero.**

Step 3. Employ the simplex algorithm which is provided earlier on the problem constructed in Step 1 and 2. However, we may terminate the process (i.e., Phase-I) as soon as the value of f (the value of the artificial objective) is zero. If the simplex process ends with either $f = 0$ or all $z_j - c_j \geq 0$ and there are no artificial variables in the basis at a positive value, we go to Step 4 (i.e., Phase-II). Otherwise, the problem is (mathematically) infeasible and we stop.

Step 4. Assign the actual objective function coefficient (the original c_j's) to each variable except for the artificial variables. Any artificial variable in the basis at a zero level are given c_j value of 0 in Phase-II. Any artificial not in the basis may be dropped from the consideration by striking out their entire associated column in the tableau.

Step 5. **The first tableau of Phase-II is the final tableau of Phase-I except for the objective function coefficients and the indicator row values. We recompute the indicator row values (all $z_j - c_j$) and objective function Z value.**

Step 6. If no artificial variable were in the basis (at zero values) at the end of Phase-I, we simply use the simplex algorithm and proceed as usual manner. If, however, there are artificial variables in the basis, go to Step 7.

Step 7. **We must take sure that the artificial variables in the basis do not ever becomes positive form zero in Phase-II. This is accomplished by modifying the departing variable rule of the simplex algorithm as follows:**

- **Determine the entering variable and its associated column (j') is the usual manner.**
- **Examine the $a_{i,j'}$ values for each artificial variable. If any of these are negative, let an artificial with a negative $a_{i,j'}$ depart. Otherwise, employ the usual departing variable rule.**

Numerical Example -TPS1a:

$$\min : Z = x_1 + 3x_2 + x_3$$

Subject to

$$x_1 + x_2 + x_3 = 10$$

$$x_1 + 4x_2 + x_3 = 16$$

$$x_1, x_2, x_3 \geq 0$$

Numerical Example -TPS1a:
First Phase Objective Function:

$$\min : f = a_1 + a_2$$

$$\max : -f = -a_1 - a_2$$

Second Phase Objective Function :

$$\min : Z = x_1 + 3x_2 + x_3$$

$$\max : -Z = -x_1 - 3x_2 - x_3$$

Subject to

$$x_1 + x_2 + x_3 + a_1 = 10$$

$$x_1 + 4x_2 + x_3 + a_2 = 16$$

$$x_1, x_2, x_3 \geq 0$$

Artificial variables:

$$a_1, a_2 \geq 0$$

First Phase Objective Function:

$$\min : f = a_1 + a_2$$

$$\max : -f = -a_1 - a_2$$

Table 0:

SIMP	CN	0	0	0	b
CB	BV/NV	x_1	x_2	x_3	XB
-1	a_1	1	1	1	10
-1	a_2	1	* 4	1	16
*	-f	-2	- 5	-2	-26

First Phase Objective Function:

Table 1:

SIMP	CN	0	-1	0	b
CB	BV/NV	x_1	a_2	x_3	XB
-1	a_1	*3/4	-1/4	- 3/4	6
0	x_2	1/4	1/ 4	1/4	4
*	-f	-3/4	5/4	3/4	-6

First Phase Objective Function:
Table 2:

SIMP	CN	-1	-1	0	b
CB	BV/NV	a_1	a_2	x_3	XB
0	x_1	4/3	-1/3	1	8
0	x_2	-1/3	1/ 3	0	2
*	-f	1	1	0	0

$$\textit{Optimal} \quad \textit{Solution}:$$

$$a_1 = 0, a_2 = 0, x_1^* = 8, x_2^* = 2, x_3^* = 0, f^* = 0,$$

Final Table of Phase-I

First Table of Phase-II
Second Phase Objective Function:

$$\max : -Z = -x_1 - 3x_2 - x_3$$

Table 3:

SIMP	CN	0	0	0	b
CB	BV/NV	a_1	a_2	x_3	XB
0	x_1	0	0	1	8
0	x_2	0	0	0	2
*	*	0	0	0	0

Final Table of Phase-II
Second Phase Objective Function:

$$\max : -Z = -x_1 - 3x_2 - x_3$$

Table 4:

SIMP	CN	0	0	-1	b
CB	BV/NV	a_1	a_2	x_3	XB
-1	x_1	0	0	1	8
-3	x_2	0	0	0	2
*	-Z	0	0	0	-14

$$\textit{Optimal} \quad \textit{Solution}:$$

$$x_1^* = 8, x_2^* = 2, x_3^* = 0, -Z^* = -14, Z^* = 14,$$

Final Table of Phase-II
Second Phase Objective Function:

$$\max : -Z = -x_1 - 3x_2 - x_3$$

Table 4:

SIMP	CN	0	0	-1	b
CB	BV/NV	a_1	a_2	x_1	XB
-1	x_3	0	0	1	8
-3	x_2	0	0	0	2
*	-Z	0	0	0	-14

$$\textit{Alternate Optimal Solution}:$$

$$x_1^* = 0, x_2^* = 2, x_3^* = 8, -Z^* = -14, Z^* = 14$$

$$\textit{Min/Max has same solution}$$

Numerical Example -TPS1b: Condensed Tableau

$$\max : Z = x_1 + 3x_2 + x_3$$

Subject to

$$x_1 + x_2 + x_3 = 10$$

$$x_1 + 4x_2 + x_3 = 16$$

$$x_1, x_2, x_3 \geq 0$$

Numerical Example -TPS1b:
First Phase Objective Function:

$$\min : f = a_1 + a_2$$

$$\max : -f = -a_1 - a_2$$

Second Phase Objective Function :

$$\max : Z = x_1 + 3x_2 + x_3$$

Subject to

$$x_1 + x_2 + x_3 + a_1 = 10$$

$$x_1 + 4x_2 + x_3 + a_2 = 16$$

$$x_1, x_2, x_3 \geq 0$$

Artificial variables:

$$a_1, a_2 \geq 0$$

First Phase Objective Function:

$$\min : f = a_1 + a_2$$

$$\max : -f = -a_1 - a_2$$

Table 0:

SIMP	CN	0	0	0	b
CB	BV/NV	x_1	x_2	x_3	XB
-1	a_1	1	1	1	10
-1	a_2	1	* 4	1	16
*	-f	-2	- 5	-2	-26

First Phase Objective Function:

Table 1:

SIMP	CN	0	-1	0	b
CB	BV/NV	x_1	a_2	x_3	XB
-1	a_1	*3/4	-1/4	- 3/4	6
0	x_2	1/4	1/ 4	1/4	4
*	-f	-3/4	5/4	3/4	-6

First Phase Objective Function:
Table 2:

SIMP	CN	-1	-1	0	b
CB	BV/NV	a_1	a_2	x_3	XB
0	x_1	4/3	-1/3	1	8
0	x_2	-1/3	1/ 3	0	2
*	-f	1	1	0	0

$$\textit{Optimal} \quad \textit{Solution} :$$

$$a_1 = 0, a_2 = 0, x_1^* = 8, x_2^* = 2, x_3^* = 0, f^* = 0,$$

Final Table of Phase-I

First Table of Phase-II
Second Phase Objective Function:

$$\max : Z = x_1 + 3x_2 + x_3$$

Table 3:

SIMP	CN	0	0	0	b
CB	BV/NV	a_1	a_2	x_3	XB
0	x_1	0	0	1	8
0	x_2	0	0	0	2
*	*	0	0	0	0

Final Table of Phase-II
Second Phase Objective Function:

$$\max : Z = x_1 + 3x_2 + x_3$$

Table 4:

SIMP	CN	0	0	1	b
CB	BV/NV	a_1	a_2	x_3	XB
1	x_1	0	0	1	8
3	x_2	0	0	0	2
*	Z	0	0	0	14

$$Optimal \quad Solution :$$

$$x_1^* = 8, x_2^* = 2, x_3^* = 0, Z^* = 14,$$

Final Table of Phase-II
Second Phase Objective Function:

$$\max : Z = x_1 + 3x_2 + x_3$$

Table 5:

SIMP	CN	0	0	1	b
CB	BV/NV	a_1	a_2	x_1	XB
1	x_3	0	0	1	8
3	x_2	0	0	0	2
*	Z	0	0	0	14

$$\textit{Alternate} \quad \textit{Optimal} \quad \textit{Solution} :$$

$$x_1^* = 0, x_2^* = 2, x_3^* = 8, Z^* = 14,$$

Numerical Example -TPS-2

$$\max : Z = x_1 + 4x_2 + 4x_3$$

Subject to

$$x_1 + 2x_2 + x_3 = 16$$

$$x_1 + x_2 + 2x_3 = 14$$

$$4x_1 + x_2 + x_3 \leq 12$$

$$x_1, x_2, x_3 \geq 0$$

Numerical Example -TPS-2
Phase - I Method:

$$\min : f = a_1 + a_2$$

$$\max : -f = -a_1 - a_2$$

Subject to

$$x_1 + 2x_2 + x_3 + a_1 = 16$$

$$x_1 + x_2 + 2x_3 + a_2 = 14$$

$$4x_1 + x_2 + x_3 + s_1 = 12$$

$$x_1, x_2, x_3, a_1, a_2, s_1 \geq 0$$

Table 0:

SIMP	CN	0	0	0	b
CB	BV/NV	x_1	x_2	x_3	XB
-1	a_1	1	2	1	16
-1	a_2	1	1	2	14
0	s_1	4	1	1	12
*	-f	-2	-3	-3	-30

Table 1:

SIMP	CN	0	-1	0	b
CB	BV/NV	x_1	a_1	x_3	XB
0	x_2	1/2	1/2	1/2	8
-1	a_2	1/2	-1/2	3/2	6
0	s_3	7/2	-1/2	1/2	4
*	-f	-1/2	3/2	-3/2	-6

Table 2:

SIMP	CN	0	-1	-1	b
CB	BV/NV	x_1	a_1	a_2	XB
0	x_2	1/3	2/3	-1/3	6
0	x_3	1/3	-1/3	2/3	4
0	s_3	10/3	- 1/3	-1/3	2
*	-f	0	1	1	0

Optimal Solution **:**

$$a_1 = 0, a_2 = 0, x_1^* = 0, x_2^* = 6, x_3^* = 4, f = 0$$

Phase-II: First Tableau

Table 3:

SIMP	CN	0	0	0	b
CB	BV/NV	x_1	a_1	a_2	XB
0	x_2	1/3	0	0	6
0	x_3	1/3	0	0	4
0	s_3	10/3	0	0	2
*	*	0	0	0	0

$$Phase - II : Objective \ Function :$$

$$\max : Z = x_1 + 4x_2 + 4x_3$$

Phase-II: Final Tableau

Table 4:

SIMP	CN	1	0	0	b
CB	BV/NV	x_1	a_1	a_2	XB
4	x_2	1/3	0	0	6
4	x_3	1/3	0	0	4
0	s_3	10/3	0	0	2
*	Z	5/3	0	0	40

Phase-II: Final Tableau

Table 5:

SIMP	CN	1	0	0	b
CB	BV/NV	x_1	a_1	a_2	XB
4	x_2	1/3	0	0	6
4	x_3	1/3	0	0	4
0	s_3	10/3	0	0	2
*	Z	5/3	0	0	40

***Optimal Solution* :**

$$x_1^* = 0, x_2^* = 6, x_3^* = 4, Z = 40$$

Numerical Example -TPS-3

$$\max : Z = 4x_1 + 4x_2 + x_3$$

Subject to

$$x_1 + 6x_2 + x_3 = 40$$

$$6x_1 + x_2 + x_3 = 30$$

$$x_1 + x_2 + 3x_3 \leq 12$$

$$x_1, x_2, x_3 \geq 0$$

Numerical Example -TPS-3

$$\max : Z = 4x_1 + 4x_2 + x_3$$

Subject to

$$x_1 + 6x_2 + x_3 + a_1 = 40$$

$$6x_1 + x_2 + x_3 + a_2 = 30$$

$$x_1 + x_2 + 3x_3 + s_1 = 12$$

$$x_1, x_2, x_3 \geq 0$$

Phase - I Problem:

$$\min : f = a_1 + a_2$$

$$\max : -f = -a_1 - a_2$$

Table 0:

SIMP	CN	0	0	0	b
CB	BV/NV	x_1	x_2	x_3	XB
-1	a_1	1	6	1	40
-1	a_2	6	1	1	30
0	s_1	1	1	3	12
*	-f	-7	-7	-2	-70

Table 1:

SIMP	CN	-1	0	0	b
CB	BV/NV	a_2	x_2	x_3	XB
-1	a_1	-1/6	35/6	5/6	35
0	x_1	1/6	1/6	1/6	5
0	s_1	- 1/6	5/6	17/6	7
*	-f	7/6	-35/6	-5/6	-35

LPP: Phase-I : Final Tableau

Table 2:

SIMP	CN	-1	-1	0	b
CB	BV/NV	a_2	a_1	x_3	XB
0	x_2	-1/35	6/35	1/7	6
0	x_1	6/35	-1/35	1/7	4
0	s_1	-1/7	- 1/7	19/7	2
*	-f	1	1	0	0

$$Phase - I \quad : Solution$$

$$a_1 = 0, a_2 = 0, x_1^* = 4, x_2^* = 6, x_3^* = 0, f = 0$$

Table 3:

SIMP	CN	0	0	0	b
CB	BV/NV	a_2	a_1	x_3	XB
0	x_2	0	0	1/7	6
0	x_1	0	0	1/7	4
0	s_1	0	0	19/7	2
*	*	0	0	0	0

Phase- II : Objective Function:

$$\max : Z = 4x_1 + 4x_2 + x_3$$

Phase -II Final Tableau

Table 4:

SIMP	CN	0	0	1	b
CB	BV/NV	a_2	a_1	x_3	XB
4	x_2	0	0	1/7	6
4	x_1	0	0	1/7	4
0	s_1	0	0	19/7	2
*	Z	0	0	1/7	40

$$Optimal \quad Solution:$$

$$x_1^* = 4, x_2^* = 6, x_3^* = 0, Z^* = 40$$

Numerical Example -TPS-4

$$\max : Z = 8x_1 + 2x_2 + 8x_3$$

Subject to

$$4x_1 + x_2 + x_3 = 40$$

$$x_1 + 5x_2 + x_3 \leq 15$$

$$x_1 + x_2 + 4x_3 = 25$$

$$x_1, x_2, x_3 \geq 0$$

Numerical Example -TPS-4

$$\max : Z = 8x_1 + 2x_2 + 8x_3$$

Subject to

$$4x_1 + x_2 + x_3 + a_1 = 40$$

$$x_1 + 5x_2 + x_3 + s_1 = 15$$

$$x_1 + x_2 + 4x_3 + a_2 = 25$$

$$x_1, x_2, x_3, a_1, a_2, s_1 \geq 0$$

Phase - I Problem:

$$\min : f = a_1 + a_2$$

$$\max : -f = -a_1 - a_2$$

Table 0:

SIMP	CN	0	0	0	b
CB	BV/NV	x_1	x_2	x_3	XB
-1	a_1	4	1	1	40
0	s_1	1	5	1	15
-1	a_2	1	1	4	25
*	-f	-5	-2	-5	-65

Table 1:

SIMP	CN	-1	0	0	b
CB	BV/NV	a_1	x_2	x_3	XB
0	x_1	1/4	1/4	1/4	10
0	s_1	-1/4	19/4	3/4	5
-1	a_2	- 1/4	3/4	15/4	15
*	-f	5/4	-3/4	-15/4	-15

Phase I: Final Tableau

Table 2:

SIMP	CN	-1	0	-1	b
CB	BV/NV	a_1	x_2	a_2	XB
0	x_1	4/15	1/5	-1/5	9
0	s_1	-1/5	23/5	-1/5	2
0	x_3	-1/15	1/5	4/15	4
*	-f	1	0	1	0

$$Phase - I: \quad Solution:$$

$$a_1 = 0, a_2 = 0, x_1^* = 9, x_2^* = 0, x_3^* = 4, f = 0$$

Phase- II First Tableau

Table 3:

SIMP	CN	0	0	0	b
CB	BV/NV	a_1	x_2	a_2	XB
0	x_1	0	1/5	0	9
0	s_1	0	23/5	0	2
0	x_3	0	1/5	0	4
*	*	0	0	0	0

Phase- II : Objective Function:

$$\max : Z = 8x_1 + 2x_2 + 8x_3$$

Phase - II Final Tableau

Table 4:

SIMP	CN	0	2	0	b
CB	BV/NV	a_1	x_2	a_2	XB
8	x_1	0	1/5	0	9
0	s_1	0	23/5	0	2
8	x_3	0	1/5	0	4
*	Z	0	6/5	0	104

Optimal Solution :

$$x_1^* = 9, x_2^* = 0, x_3^* = 4, Z^* = 104$$

Numerical Example -TPS-5

$$\max : Z = x_1 + 4x_2 + 4x_3$$

Subject to

$$x_1 + 2x_2 + x_3 = 16$$

$$x_1 + x_2 + 2x_3 = 14$$

$$4x_1 + x_2 + x_3 \leq 12$$

$$x_1, x_2, x_3 \geq 0$$

Numerical Example -TPS-5
Phase - I Method:

$$\min : f = a_1 + a_2$$

$$\max : -f = -a_1 - a_2$$

Subject to

$$x_1 + 2x_2 + x_3 + a_1 = 16$$

$$x_1 + x_2 + 2x_3 + a_2 = 14$$

$$4x_1 + x_2 + x_3 + s_1 = 12$$

$$x_1, x_2, x_3, a_1, a_2, s_1 \geq 0$$

Phase - I: Method

Table 0:

SIMP	CN	0	0	0	b
CB	BV/NV	x_1	x_2	x_3	XB
-1	a_1	1	2	1	16
-1	a_2	1	1	2	14
0	s_1	4	1	1	12
*	-f	-2	-3	-3	-30

Table 1:

SIMP	CN	0	-1	0	b
CB	BV/NV	x_1	a_1	x_3	XB
0	x_2	1/2	1/2	1/2	8
-1	a_2	1/2	-1/2	3/2	6
0	s_3	7/2	-1/2	1/2	4
*	-f	-1/2	3/2	-3/2	-6

Table 2:

SIMP	CN	0	-1	-1	b
CB	BV/NV	x_1	a_1	a_2	XB
0	x_2	1/3	2/3	-1/3	6
0	x_3	1/3	-1/3	2/3	4
0	s_3	10/3	- 1/3	-1/3	2
*	-f	0	1	1	0

Optimal Solution **:**

$$a_1 = 0, a_2 = 0, x_1^* = 0, x_2^* = 6, x_3^* = 4, f = 0$$

Phase-II: First Tableau

Table 3:

SIMP	CN	0	0	0	b
CB	BV/NV	x_1	a_1	a_2	XB
0	x_2	1/3	0	0	6
0	x_3	1/3	0	0	4
0	s_3	10/3	0	0	2
*	*	0	0	0	0

$$Phase - II : Objective\ Function :$$

$$\max : Z = x_1 + 4x_2 + 4x_3$$

Phase-II: Final Tableau

Table 4:

SIMP	CN	1	0	0	b
CB	BV/NV	x_1	a_1	a_2	XB
4	x_2	1/3	0	0	6
4	x_3	1/3	0	0	4
0	s_1	10/3	0	0	2
*	Z	5/3	0	0	40

Optimal Solution :

$x_1^* = 0, x_2^* = 6, x_3^* = 4, Z = 40$

Numerical Example -TPS-6

$$\min : Z = x_1 + 4x_2 + 4x_3$$

Subject to

$$x_1 + 2x_2 + x_3 = 16$$

$$x_1 + x_2 + 2x_3 = 14$$

$$4x_1 + x_2 + x_3 \leq 12$$

$$x_1, x_2, x_3 \geq 0$$

Numerical Example -TPS-6
Phase - I Method:

$$\min : f = a_1 + a_2$$

$$\max : -f = -a_1 - a_2$$

Subject to

$$x_1 + 2x_2 + x_3 + a_1 = 16$$

$$x_1 + x_2 + 2x_3 + a_2 = 14$$

$$4x_1 + x_2 + x_3 + s_1 = 12$$

$$x_1, x_2, x_3, a_1, a_2, s_1 \geq 0$$

Table 0:

SIMP	CN	0	0	0	b
CB	BV/NV	x_1	x_2	x_3	XB
-1	a_1	1	2	1	16
-1	a_2	1	1	2	14
0	s_1	4	1	1	12
*	-f	-2	-3	-3	-30

Phase I- Method

Table 1:

SIMP	CN	0	-1	0	b
CB	BV/NV	x_1	a_1	x_3	XB
0	x_2	1/2	1/2	1/2	8
-1	a_2	1/2	-1/2	3/2	6
0	s_1	7/2	-1/2	1/2	4
*	-f	-1/2	3/2	-3/2	-6

Table 2:

SIMP	CN	0	-1	-1	b
CB	BV/NV	x_1	a_1	a_2	XB
0	x_2	1/3	2/3	-1/3	6
0	x_3	1/3	-1/3	2/3	4
0	s_1	10/3	- 1/3	-1/3	2
*	-f	0	1	1	0

***Optimal Solution* :**

$$a_1 = 0, a_2 = 0, x_1^* = 0, x_2^* = 6, x_3^* = 4, f = 0$$

Phase-II: First Tableau

Table 3:

SIMP	CN	0	0	0	b
CB	BV/NV	x_1	a_1	a_2	XB
0	x_2	1/3	0	0	6
0	x_3	1/3	0	0	4
0	s_1	10/3	0	0	2
*	*	0	0	0	0

$$\textit{Phase} - \textit{II} : \textit{Objective Function} :$$

$$\min : -Z = -x_1 - 4x_2 - 4x_3$$

Phase-II: Second Tableau

Table 4:

SIMP	CN	- 1	0	0	b
CB	BV/NV	x_1	a_1	a_2	XB
-4	x_2	1/3	0	0	6
-4	x_3	1/3	0	0	4
0	s_3	10/3	0	0	2
*	-Z	-5/3	0	0	-40

Phase-II: Final Tableau

Table 5:

SIMP	CN	0	0	0	b
CB	BV/NV	s_1	a_1	a_2	XB
-4	x_2	-1/10	0	0	58/10
-4	x_3	-1/10	0	0	38/10
-1	x_1	3/10	0	0	6/10
*	-Z	1/2	0	0	-39

***Optimal Solution* :**

$$\mathbf{x}_1^* = 6/10, x_2^* = 58/10, x_3^* = 38/10, Z = 39$$

Numerical Example -1 : Practice Problem

$$\min : Z = 2x_1 + 3x_2 + x_3$$

Subject to

$$x_1 + x_2 + x_3 \geq 10$$

$$x_1 + 2x_2 + x_3 \geq 12$$

$$x_1 + 4x_2 + x_3 \leq 16$$

$$x_1, x_2, x_3 \geq 0$$

$$\textit{Find}\quad \textit{Optimal}\quad \textit{Solution}:$$

$$x_1^* = ***, x_2^* = ***, x_3^* = ***, Z^* = 12$$

Numerical Example -2: Practice Problem

$$\min : Z = x_1 + x_2 + x_3$$

Subject to

$$4x_1 + x_2 + x_3 \geq 20$$

$$x_1 + 3x_2 + x_3 \geq 12$$

$$x_1 + x_2 + 2x_3 \geq 10$$

$$x_1, x_2, x_3 \geq 0$$

$$\textit{Find} \quad \textit{Optimal} \quad \textit{Solution} :$$

$$x_1^* = ***, x_2^* = ***, x_3^* = ***, Z^* = 8$$

Two-Phase Simplex Method:

Numerical Example -3: Practice Problem

$$\min : Z = x_1 + 4x_2 + 4x_3$$

Subject to

$$x_1 + 2x_2 + x_3 \geq 16$$

$$x_1 + x_2 + 2x_3 \geq 14$$

$$4x_1 + x_2 + x_3 \leq 12$$

$$x_1, x_2, x_3 \geq 0$$

$$\textit{Find} \quad \textit{Optimal} \quad \textit{Solution} :$$

$$x_1^* = ***, x_2^* = ***, x_3^* = ***, Z^* = 39$$

Numerical Example -4: Practice Problem

$$\min : Z = x_1 + 6x_2 + 6x_3$$

Subject to

$$x_1 + 3x_2 + x_3 = 90$$

$$x_1 + x_2 + 3x_3 = 54$$

$$5x_1 + x_2 + x_3 \leq 45$$

$$x_1, x_2, x_3 \geq 0$$

$$\textit{Find} \quad \textit{Optimal} \quad \textit{Solution} :$$

$$x_1^* = ***, x_2^* = ***, x_3^* = ***, Z^* = 212$$

Numerical Example -5: Practice Problem

$$\min : Z = x_1 + 4x_2 + 4x_3$$

Subject to

$$x_1 + 5x_2 + x_3 \geq 45$$

$$x_1 + x_2 + 5x_3 \geq 35$$

$$2x_1 + x_2 + x_3 \leq 25$$

$$x_1, x_2, x_3 \geq 0$$

$$\textit{Find} \quad \textit{Optimal} \quad \textit{Solution} :$$

$$x_1^* = ***, x_2^* = ***, x_3^* = ***, Z^* = 51$$

Solve all the Practice Problems
Using Two-Phase Simplex Method.