Constrained Uni /Multi-Objective Decision Making in Economics

Topics

- A Consumer’s choice Problem (Utility Maximization)
- Life’s Optimization Problem
- Air Travel choice Problem
- Efficiency vs Equity

Consumer Behaviour

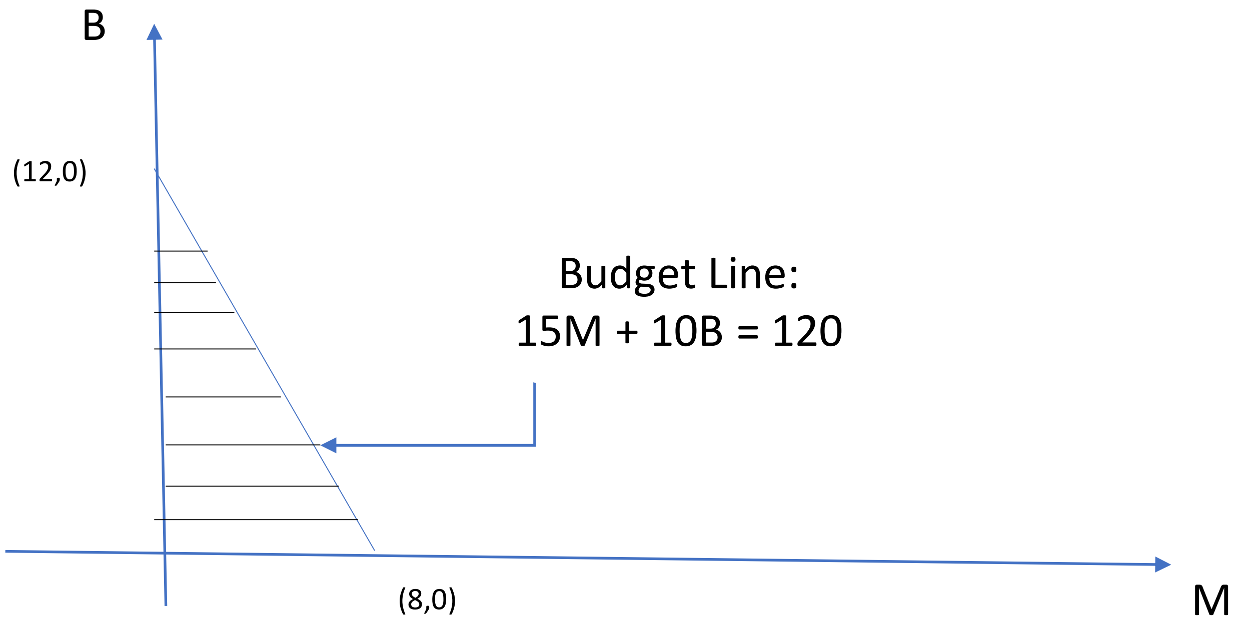
A Consumer's choice Problem

- Two consumable items: Banana & Mango
- Price of 1 Banana = Rs. 10
- Price of 1 Mango = Rs. 15
- Consumer has Rs. 120
- How should he/she allocate money between Mangoes & Bananas?
- Depends on his taste…what he loves…& How much…Right??
- The consumers taste is represented by a Utility Function U: $R^2 \rightarrow R$
- $\max_{M,B} U\ (M, B)$: subject to the constraint 15M + 10B $\leq$ 120

Consumer's Optimization Problem

- Decision variables: M $\in R_+$ & B $\in R_+$
- Objective function: U(M,B)
- Constraint: 15M + 10B ≤ 120

Budget Set



All (M,B) ∈  are affordable bundles.

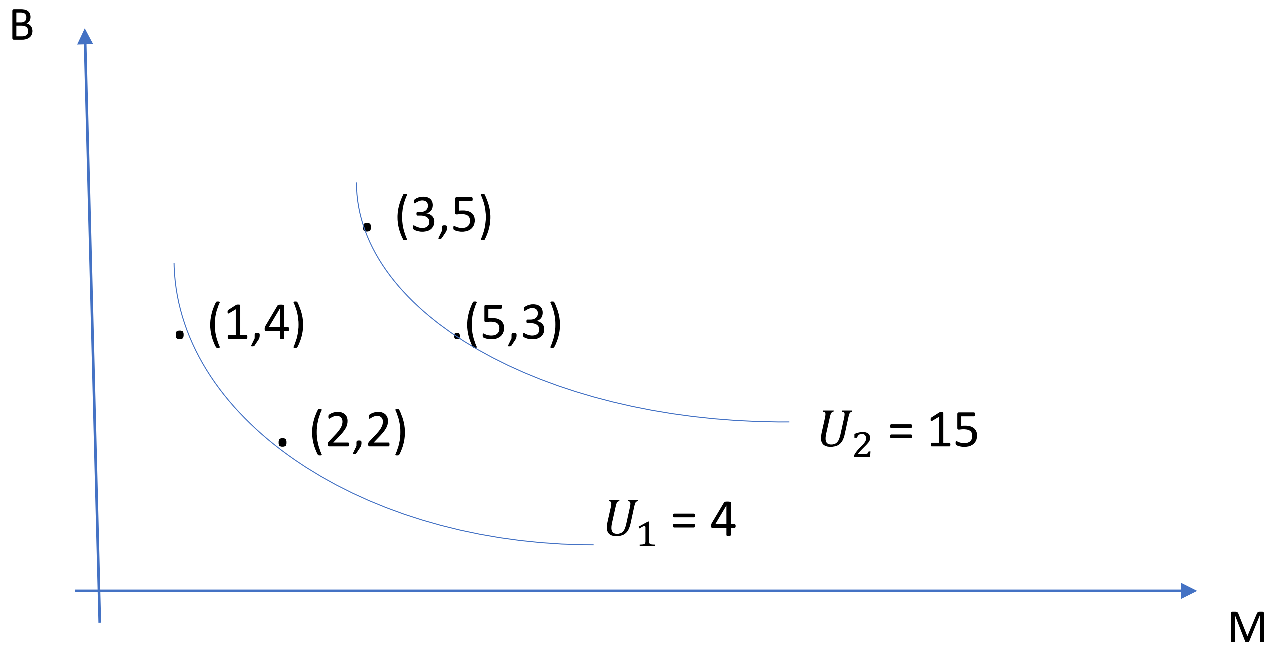
Example

- U(M,B) = M.B

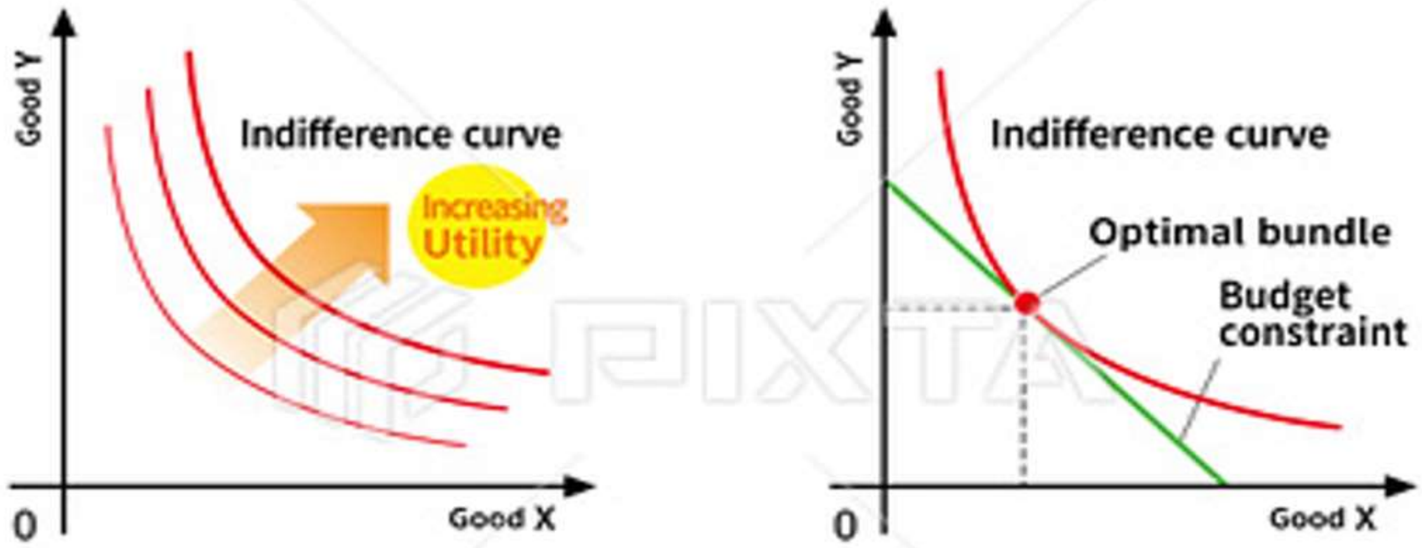
- $\max_{M,B} U\,(M, B) = M.B$: subject to the constraint 15M + 10B ≤ 120

- i.e $\max_{M,B} U\,(M, B) = M.B$ such that (M,B) ∈

Indifference curves: U(M,B) = M.B



Optimal Choice / Optimal Bundle



Indifference curve and Budget constraint

Optimization Problem - Revised

- $\max\limits_{M,B} U\ (M, B) = M.B$: subject to the constraint 15M + 10B $= 120$
- $\equiv \max\limits_{M,B} U\ (M, B) = M.B$: subject to the constraint B = (24 – 3M)/2
- $\equiv \max\limits_{M}$ M(24 – 3M)
- M^* = 4 & therefore B^* = 6
- (M^*, B^*) = (4,6) is the optimal bundle.

Practice Problems

- U(M,B) = $M^2 + B^2$
- U(M,B) = min {M,B}. (e.g left shoe & right shoe)
- U(M,B) =


Plan a Great Life

The Problem

- Suppose you will be alive for T+1 years (starting from your Grad. Day)
- Let's say you know that in year 'n' you will make income = y_n
- Each year you can save = s_n & consume = c_n
- Interest rate = r (remains constant throughout life)
- Consumption in youth "feels better" than in Old Age.
- World with 100 % inheritance tax ☺... No Dad's money !!!!
- $\max_{c_0, c_1, \ldots\ldots c_T} \sum_{n=0}^{T} [f(n) \,.\, (log_e\, c_n)]$; $f(n)$ is the depreciation

function. $f(n)$ is decreasing in 'n'

The Periodic Budget Constraints

- $c_0 = y_0 - s_0$
- $c_1 = y_1 + s_0(1 + r) - s_1$
- $c_2 = y_2 + s_1(1 + r) - s_2$
- $c_3 = y_3 + s_2(1 + r) - s_3$

…….

- $c_{T-1} = y_{T-1} + s_{T-2}(1 + r) - s_{T-1}$
- $c_T = y_T + s_{T-1}(1 + r)$

The Periodic Budget Constraints

- $c_0 = y_0 - s_0$: $c_0 = y_0 - s_0$
- $c_1 = y_1 + s_0(1+r) - s_1$: $\frac{c_1}{1+r} = \frac{y_1}{1+r} + s_0 - \frac{s_1}{1+r}$
- $c_2 = y_2 + s_1(1+r) - s_2$: $\frac{c_2}{(1+r)^2} = \frac{y_2}{(1+r)^2} + \frac{s_1}{1+r} - \frac{s_2}{(1+r)^2}$
- $c_3 = y_3 + s_2(1+r) - s_3$: $\frac{c_3}{(1+r)^3} = \frac{y_3}{(1+r)^3} + \frac{s_2}{(1+r)^2} - \frac{s_3}{(1+r)^3}$

……. ……….

- $c_{T-1} = y_{T-1} + s_{T-2}(1+r) - s_{T-1}$: $\frac{c_{T-1}}{(1+r)^{T-1}} = \frac{y_{T-1}}{(1+r)^{T-1}} + \frac{s_{T-2}}{(1+r)^{T-2}} - \frac{s_{T-1}}{(1+r)^{T-1}}$
- $c_T = y_T + s_{T-1}(1+r)$: $\frac{c_T}{(1+r)^T} = \frac{y_T}{(1+r)^T} + \frac{s_{T-1}}{(1+r)^{T-1}}$

The Consolidated Budget Constraint

$$c_0 + \frac{c_1}{1+r} + \frac{c_2}{(1+r)^2} + \frac{c_3}{(1+r)^3} \ldots\ldots + \frac{c_{T-1}}{(1+r)^{T-1}} + \frac{c_T}{(1+r)^T}$$

$$= y_0 + \frac{y_1}{1+r} + \frac{y_2}{(1+r)^2} + \frac{y_3}{(1+r)^3} \ldots\ldots + \frac{y_{T-1}}{(1+r)^{T-1}} + \frac{y_T}{(1+r)^T}$$

Life's Optimization Problem

$$\max_{c_0, c_1, \ldots\ldots c_T} \sum_{n=0}^{T} [f(n) \,.\, (log_e \, c_n)]$$

Subject to the constraints:

$$\sum_{n=0}^{T} \frac{c_n}{(1+r)^n} = \sum_{n=0}^{T} \frac{y_n}{(1+r)^n}$$

$c_n \geq 0 \ \forall$ n = 0,1,2,……T

Let’s get real !!

- You don’t know what my lifetime income stream $\{y_0, y_1, y_2 \ldots\ldots y_T\}$
- What is the depreciation function $f(n)$ for you??
- Will the interest rate remain same all the while?
- Won’t you invest in other asset classes?

Depreciation function

- Typically $f(n) = \beta^n \ \forall\ n = 0,1\ldots T \quad \& \ \beta \in (0,1)$
- Lower the β lesser value you assign to future: MYOPIC
- Higher the β higher value you assign to the future: FAR - SIGHTED
- Of course you can have other functional forms too for $f(n)$
- Depends on your personality type !!

Income Estimation

- Let's say life starts at 35 ☺

- From your grad day to the time you turn 35 let's say your annual incomes are given by $\{y_0, y_1, y_2 \ldots\ldots y_{14}\}$

- Fit the model on the data: $\{y_0, y_1, y_2 \ldots\ldots y_{14}\}$ & income streams of other individuals

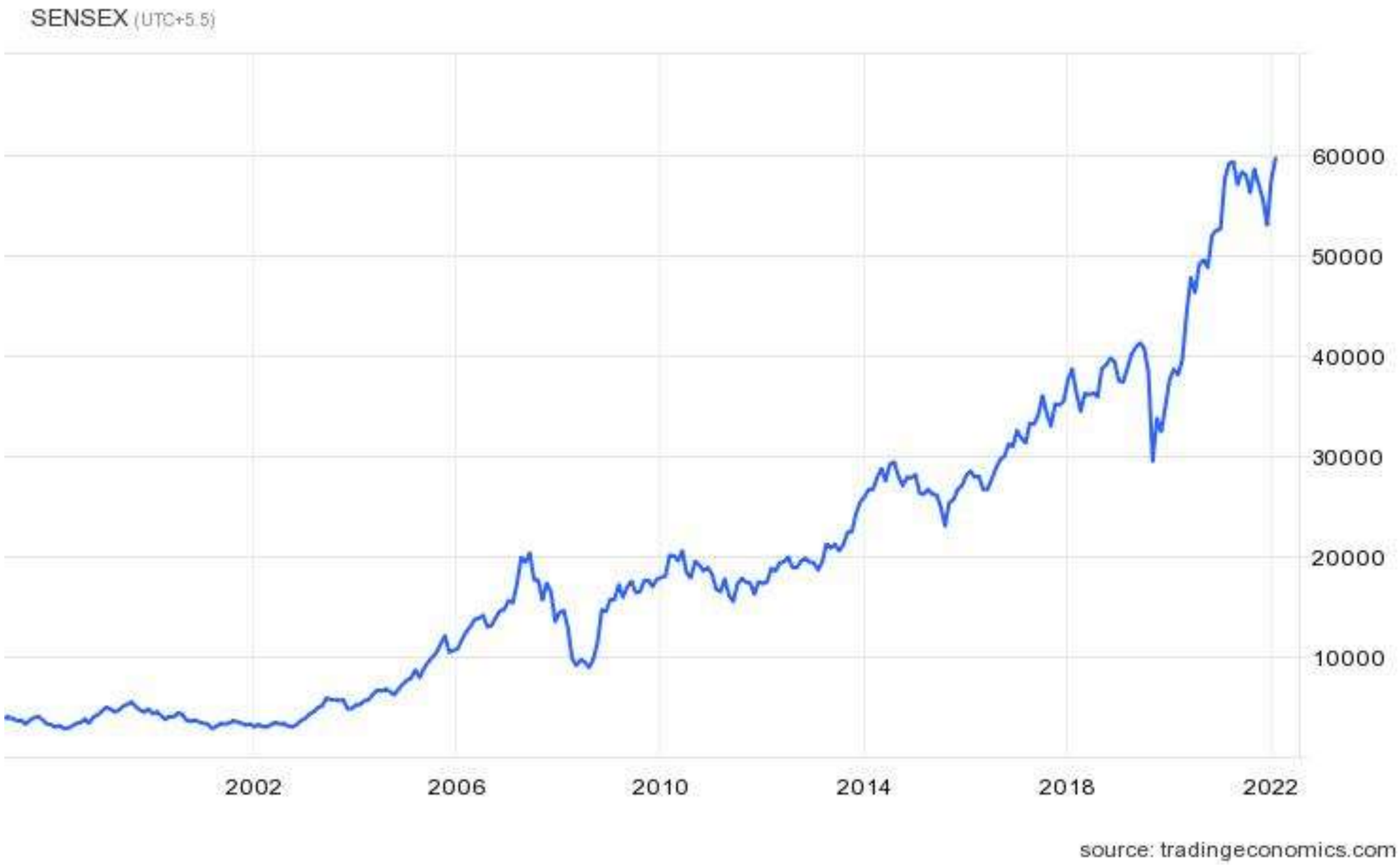
- Get estimates: $\widehat{y_{15}}$, $\widehat{y_{16}} \ldots\ldots . \widehat{y_T}$

- Define $\widetilde{y_n}$ = $\widehat{y_{n+15}}$ (* $\widetilde{y_0}$ = $\widehat{y_{15}}$ + $\bar{\bar{S}}$; $\bar{\bar{S}}$ = savings till age 35)

- So your annual income flow from age 35 is given by:

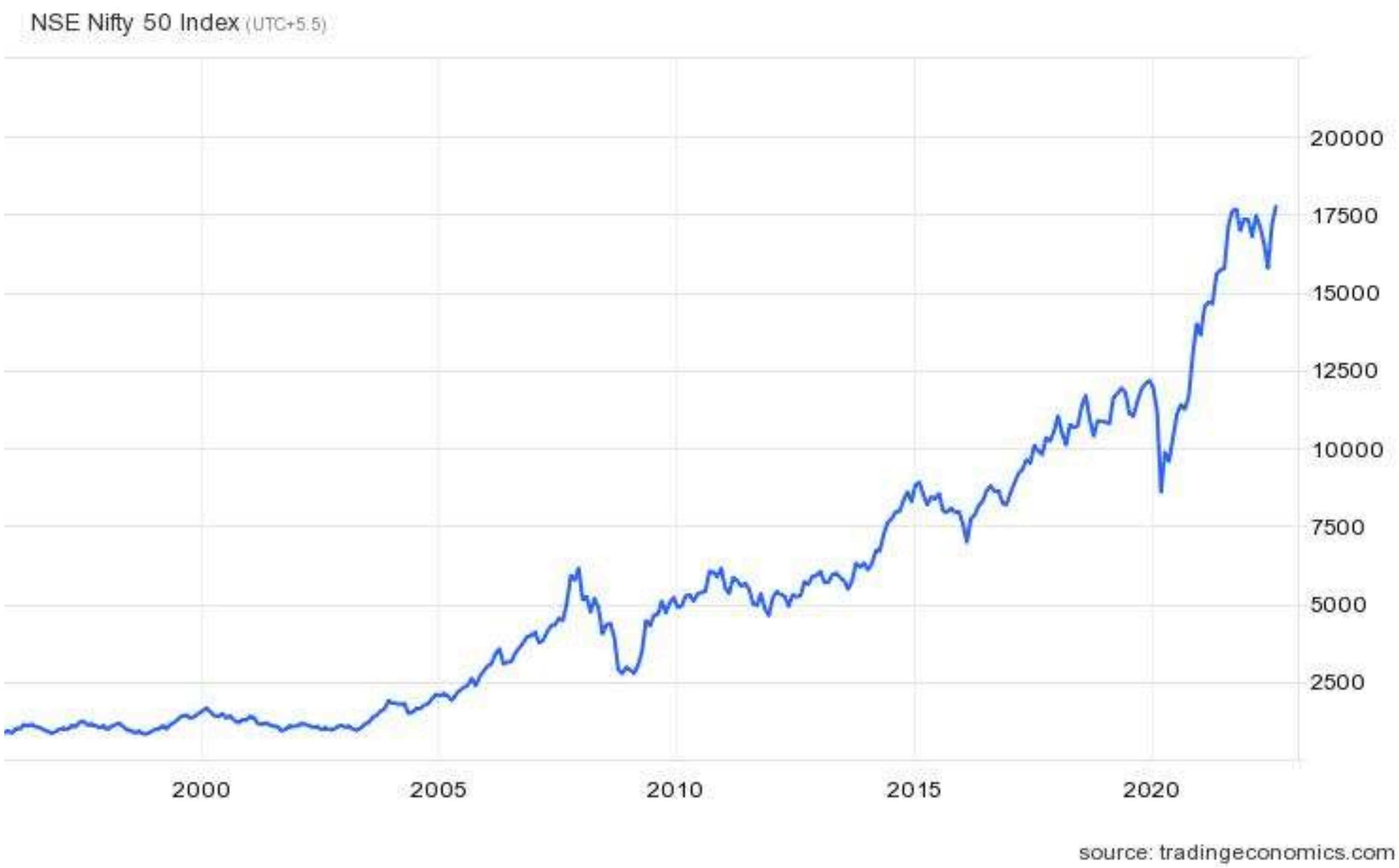
$\{\widetilde{y_0}$, $\widetilde{y_1}, \ldots\ldots \widetilde{y_{T'}}\}$; T' = T - 15

Sensex


SENSEX (UTC+5.5)
60000
50000
40000
30000
20000
10000
2002
2006
2010
2014
2018
2022
source: tradingeconomics.com

Nominal Int. rate (savings)

NIFTY 50



NSE Nifty 50 Index (UTC+5.5)
20000
17500
15000
12500
10000
7500
5000
2500
2000
2005
2010
2015
2020
source: tradingeconomics.com

Investment Returns

- CAGR of Sensex (1979 – 2019) -> 16.1%
- CAGR of NIFTY 50 (2001 – 2021) -> 14%
- Avg. interest rate (going forward) -> 6%
- Constant Portfolio: γ_1 % in Sensex fund
  γ_2 % in NIFTY 50
  (1- $\gamma_1 - \gamma_2$) % in FD / savings A/c
- r = 16.1 γ_1 + 14 γ_2 + 6 (1- $\gamma_1 - \gamma_2$)

Life's Optimization Problem (at 35 !)

$$\max_{c_0, c_1, \ldots.. c_{T'}} \sum_{n=0}^{T'} [f(n) \,.\, (log_e \; c_n)]$$

Subject to the constraints:

$$\sum_{n=0}^{T'} \frac{c_n}{(1+r)^n} = \sum_{n=0}^{T'} \frac{\widetilde{y_n}}{(1+r)^n}$$

$c_n \geq 0 \; \forall$ n = 0,1,2,…… T'

- Solving the optimization problem yields optimal periodic consumptions $({c_0}^*, {c_1}^*, {c_2}^* \ldots\ldots\ {c_{T'}}^*)$

- $\sum_{n=0}^{T'}[f(n) \, . \, (log_e \, {c_n}^*)$ = V^* (The optimal utility)

- $V^*(\gamma_1 \, , \gamma_2 \, , T')$

- Given that you know T' , maximize $V^*(\gamma_1 \, , \gamma_2 \, , T')$ w.r.t γ_1 & γ_2

Let’s Fly !! (Multi – Objective Choice)

Kolkata –> Delhi

AIRLINE	TIME (Hrs)	COST (Rs. '000)
INDIGO	2	8
SPICE JET	3	6
Go AIR	3	8
AIR INDIA	4	5
VISTARA	4	7
AIR ASIA	5	5
JET AIRWAYS	5	6
AKASA	5	7
KINGFISHER	6	7

Optimization Problem

- Decision Variable: **Airline** $\in$ {Indigo , SpiceJet, GoAir, Air India, Vistara, Air Asia, Jet Airways, Akasa, Kingfisher}

- Here the consumer / buyer has two objectives.

i. Minimize cost

ii. Minimize Time (of travel)

- Let's assume for the time being that your bank a/c balance > Rs. 8000

i.e no budget constraint.

Let’s NOT be DUMB !

Indigo vs GoAir

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Go AIR	3	8
AIR INDIA	4	5
VISTARA	4	7
AIR ASIA	5	4
JET AIRWAYS	5	6
AKASA	5	7
KINGFISHER	6	7

Air India vs Vistara

AIRLINE	TIME (Hrs)	COST (Rs. '000)
INDIGO	2	8
SPICE JET	3	6
Go AIR	3	8
AIR INDIA	4	5
VISTARA	4	7
AIR ASIA	5	4
JET AIRWAYS	5	6
AKASA	5	7
KINGFISHER	6	7

Air Asia vs {Jet Airways, Akasa, Kingfisher}

AIRLINE	TIME (Hrs)	COST (Rs. ‘000)
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SPICE JET	3	6
Go AIR	3	8
AIR INDIA	4	5
VISTARA	4	7
AIR ASIA	5	4
JET AIRWAYS	5	6
AKASA	5	7
KINGFISHER	6	7

Re-Inspecting the "Smart" Choices

AIRLINE	TIME (Hrs)	COST (Rs. '000)
INDIGO	2	8
SPICE JET	3	6
AIR INDIA	4	5
AIR ASIA	5	4

Pareto Optimal Set

- {INDIGO , SPICEJET , AIR INDIA , AIR ASIA} form a pareto optimal set.

- In this set moving from one Airline to another will NOT lead to an improvement in one objective function without incurring a loss in the other objective function.

- Pareto Optimal Set is also called Dominant Set of Rank -1.

Re-Inspecting the “DUMB” choices

AIRLINE	TIME (Hrs)	COST (Rs. ‘000)
Go AIR	3	8
VISTARA	4	7
JET AIRWAYS	5	6
AKASA	5	7
KINGFISHER	6	7

Vistara vs {Akasa, Kingfisher}

AIRLINE	TIME (Hrs)	COST (Rs. '000)
Go AIR	3	8
VISTARA	4	7
JET AIRWAYS	5	6
AKASA	5	7
KINGFISHER	6	7

Dominant Set of Rank-2

- {Go AIR, VISTARA, JET AIRWAYS} form a dominant set of Rank -2
- With in this set moving from one Airline to another will NOT lead to an improvement in one objective function without incurring a loss in the other objective function.

{Akasa, Kingfisher}

- Akasa dominates Kingfisher.
- Dominant set of Rank -3 = {Akasa}
- Dominant set of Rank -4 = {Kingfisher}

The Hierarchy of Choice

- DSR-1 -> “SMART” (pareto optimal)
- DSR-2 -> “DUMB”
- DSR - 3 -> “DUMBER”
- DSR – 4 -> “DUMBEST”

Constraints

- The consumer might face two kinds of constraints:

i. Budget constraint
ii. Time constraint

- A consumer might have either one of them or both.

- the constraints determine the set of feasible alternatives.

Constraint Example 1: Budget = Rs. 6500

- The feasible set of alternatives are:

AIRLINE	TIME (Hrs)	COST (Rs. '000)
SPICE JET	3	6
AIR INDIA	4	5
AIR ASIA	5	4
JET AIRWAYS	5	6

- The Pareto Optimal (DSR -1) set: {SPICE JET, AIR INDIA, AIR ASIA}

- DSR -2: { JET AIRWAYS}

Movie Time !!

Two Competing Theatres

- Two Movie Theatres (of same quality) in a neighbourhood.
- Total market size (per annum) = Rs. 1 Cr.
- Marketing budget of theatre 1 = Rs. 1 L & that of Theatre – 2 is Rs 2 L.
- Then Theatre 1 grabs 1/3 of the market size & Theatre 2 grabs 2/3.
- Market share is proportional to the marketing budget.

- If the marketing budget of firm j is = X_j where j ∈ $\{1,2\}$

- Then the market share of firm j is given by

$$
\begin{aligned}
\pi_j &= \frac{X_j}{X_1+X_2} \text{ if } X_j > 0 \;\&\; X_i > 0 \\
&= 1 \quad \text{if } X_j > 0 \;\&\; X_i = 0 \\
&= 0 \quad \text{if } X_j = 0 \;\&\; X_i > 0
\end{aligned}
$$

$$\pi_i = \pi_j = 0 \quad \text{if } X_j = 0 \;\&\; X_i = 0$$

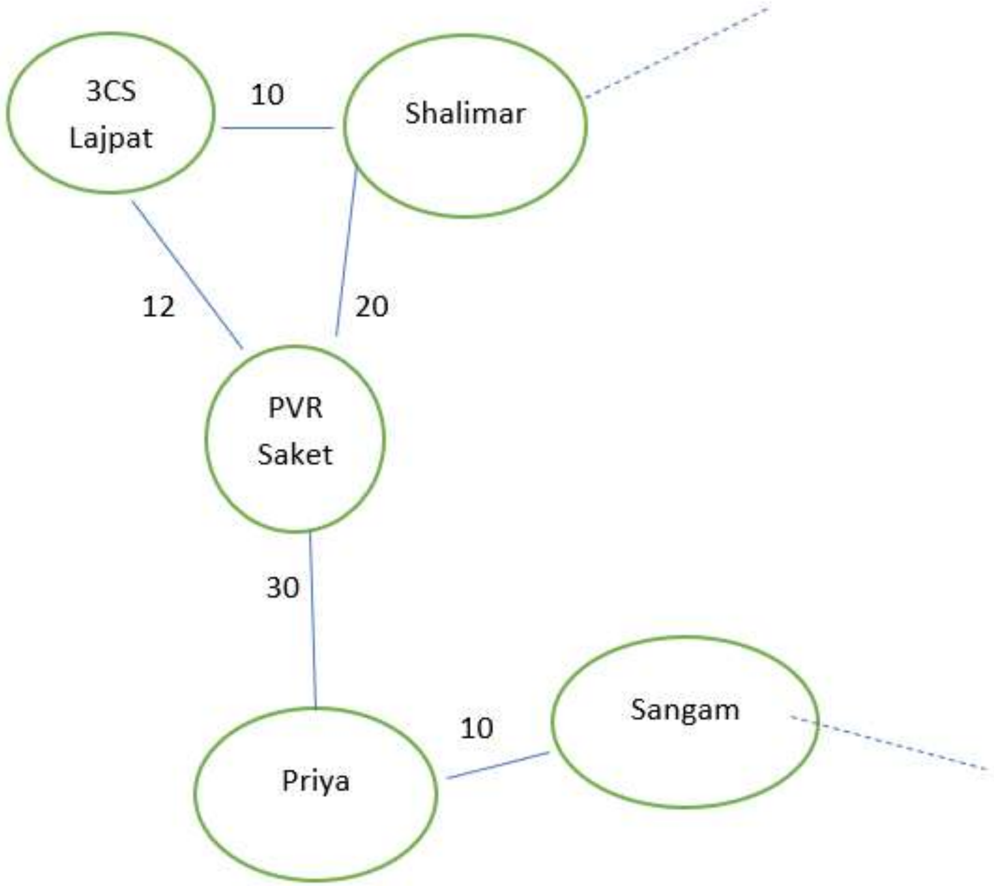
Herfindahl Hirschman Index (HHI) – Inequality Measure

- Given that the total market size is = 1, if firm j ∈ $\{1,2, \ldots . N\}$ grabs a market share = π_j then HHI = $\sum_{j=1}^{N} {\pi_j}^2$.
- Clearly HHI is minimum if π_j = 1/N (all firms grab equal share)
- HHI is maximum (=1) if π_j = 1 for some j (monopoly)
- In the example: HHI = $(1/3)^2 + (2/3)^2$ = 5/9

A Network of Contests


DELHI
CINEMA HALLS
To Chandigarh
To Sahranpur
To Rohtak
To Moradabad
To Jaipur
To Agra
N
PVR
Prashant Vihar
Rithala
Pitampura
ROHINI
Dr.Mukherjee Nagar
NH 1
Mustafabad
WAZIRABAD RD.
Rasulpur
Mangolpuri
Shakurpur Colony
GTB Nagar
Batra
Yamuna Vihar
Ashok Nagar
Nand Nagri
Dilshad Garden
Rajendra Nagar
Old Secretariat
Shahdara
NH 10
Mundka
Pira Garhi
ROHTAK RD.
Tri Nagar
Shastri Nagar
Alpana
Delhi University
NH 24
Kamaruddin Nagar
Punjabi Bagh
Moti Nagar
Milan
Patel Nagar
Liberty
Seelampur
Vivek Vihar
Sahibabad
Bakarwala
Chandan Garden
Satyam
Patel Nagar
Sadar Bazar
Filmistan
Chandni Chowk
Krishna Nagar
Anand Vihar
GHAZIABAD
Tagore Garden
Vishal Cinema
Vivek
Jhandewalan
Gagan Vihar
Vaishali
Vikaspur
Tilak Nagar
Shiela
Golcha
Kaushambi
PVR Vikaspuri
Rajouri Garden
Pusa Institute
Rachna
Connaught Place
Delite
Lakshmi Nagar
Mohan Garden
VIKAS MARG
Indraprastha Extens
Satyam
PVR Naraina
Naraina
Rivoli
Regal
Makanpur Colony
Janakpuri
NH-24 BYPASS
Kalyanpuri
Uttam Nagar
Sansad Bhawan
Pragati Maidan
India Gate
Games Village
Mayur Vihar
Gharoli Dairy Farm
Sagarpur
Delhi Cantt
Sunder Nagar
Manglapuri
Lodi Garden
DWARKA
South Campus (Delhi University)
Chanakya Puri
NOIDA
Palam
Jhankar
Shalimar
Alka
Goela Dairy
Domestic Airport
R.K. Puram
Safdarjung Airport
New Friends Colony
8
Sangam
3CS
Lajpat Nagar
MATHURA RD.
Golf Course
Chhawala
Vasant Vihar
Indira Gandhi International Airport
PVR Priya
Green Park
Satyam Nehru Place
Nehru Place
Munirka
Hauz Khas
Ymuna River
Vasant Kunj
Kalkaji
NH 2
Isapur Khera
Bijwasan
Malviya Nagar
Greater Kailash
Sarita Vihar
NOIDA LINK ROAD
Shankar Vihar
Rangpuri Pahari
Qutab Institutional Area
Saket
PVR Saket
Alaknanda
NH 8
New Palam Vihar
Rajokri
Mehrauli
Okhla
Palam Vihar
Dundahera
Chhatarpur Mandir
Khanpur
Badarpur
OLD GURGAON ROAD
Nathupur
Sangam Vihar
Agra Canal
DLF Cyber City
MEHRAULI GURGAON ROAD
Sultanpur
GURGAON
LEGEND
Existing Metro
Outer Ring Road
Ring Road
Major Roads
Other Roads
Railway Line
Water Bodies
Greenery
Cinema Halls
Suraj Kund
IFFCO Chowk
DLF-I
Fountain Chowk
Map not to Scale
Copyright © 2010 www.mapsofindia.com
(Updated on 11th August 2010)
Arangpur
Green Fields
Tilpat
FARIDABAD

Graphical Representation



Objective of every Theatre (Agent)

- Maximize their own market share – the “prize”
- They allocate – the marketing budget.
- Distribution mechanism – Tullock Contest (1980)

Model

- S: set of agents (firms), who are engaged in a network of conflicts (for market share).

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- W$(e_{ij}) = w_{ij}$ i.e the weight on the edge connecting agent i & j signifies the common valuation of the prize which the two agents are fighting over. w_{ij} is the market size firms i & j are contesting over.
- Of course W$(e_{ij}) = 0$ if $e_{ij} \notin E$.

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- Of course W(e_{ij}) $= 0$ if $e_{ij} \notin E$.
- Also each agent i $\in S$ has “arms” endowment $\bar{X}_i$ (marketing budget of firm i).

Theatre j's Optimization Problem

- Theatre 'j of course wants to maximize his total market share.

- Let X_{jk} is the marketing allocation by Theatre 'j' against theatre 'k'

- $\max_{\{X_{jk}\}_{k\in N(j)}} \sum_{k\in N(j)} \frac{X_{jk}}{X_{jk}+X_{kj}} . W_{jk}$

s.t $\sum_{k\in N(j)} X_{jk} = \bar{X}_j$

- The above optimization problem is solved simultaneously by all j ∈ V

- Solving the optimization problem will yield the optimal marketing allocations in all markets $\{X_{jk}^{*}\}_{k\in N(j)}$ ∀ j ∈ V

Market Shares

- The market share of theatre ‘j’ , $\pi_j{}^* = \sum_{k \in N(j)} [\frac{X_{jk}^*}{X_{jk}^* + X_{kj}^*}].\, W_{jk}$

- The resulting $HHI^* = \sum_{j \in V} [\pi_j{}^*]^2$

- Clearly HHI^* is a function of the marketing budget vector $(\bar{X}_i)_{i \in V}$

The Policy Maker's Problem

- How to allocate (or re-allocate with taxes / subsidies) the marketing budgets.
- Objective: Minimize HHI