INTERIOR POINT METHOD OF KARMARKAR

Scaling Algorithm for Linear Programming Problem

A mathematical model for a Linear Programming Problem can be stated as:

$$\max : Z = \sum_{j=1}^{n} c_j x_j \tag{1}$$

Subject to

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i, \quad i = 1, 2, ..., m \tag{2}$$

$$x_j \ \geq \ 0, \quad j = 1, 2, ..., n \tag{3}$$

It is assumed that all the b_i are positive and the constraints are all less than or equal to type ($\leq$). We introduce slack variables to transform the constraints to all equality types. Instead of less than or equal to type constraints,if some or all constraints are greater than or equal to type ($\geq$) or equality type ($=$), then it is also possible to convert the constraints all equality types by introducing artificial variables. The transformed problem can be stated as:

$$\max : Z = \sum_{j=1}^{N} c_j x_j \tag{4}$$

Subject to

$$\sum_{j=1}^{N} a_{ij} x_j = b_i, \quad i = 1, 2, ..., m \tag{5}$$

$$x_j \ \geq \ 0, \quad j = 1, 2, ..., N \tag{6}$$

Then find a trial feasible solution to the transformed problem, where all the components of the trial solution are strictly positive. After finding a trial solution, the main algorithm can be applied.

Algorithm:

Step 1.

Set $k = 0$. Let the trial solution be $X^{(k)} = (x_1^k, x_2^k, ..., x_N^k)^t$

Let $D = diag(x_1^k, x_2^k, ..., x_N^k)$ be a diagonal matrix.

Step 2.

Compute $\tilde{A}_{m\times N} = AD$ and $\tilde{C} = DC$

Step 3.

Compute $P = I - \tilde{A}^t(\tilde{A}\tilde{A}^t)^{-1}\tilde{A}$ and $C_p = P\tilde{C}$

Step 4.

Identify the negative component of C_p having largest absolute value, and set v equal to this absolute value.

Then compute $\tilde{X} = (1, 1, ..., 1)^t + \frac{\alpha}{v}C_p$

where $\alpha \in (0, 1)$, (say $\alpha = 0.5$)

Step 5.

Compute $X^{(k+1)} = D\tilde{X}$.

If $X^{(k)}$ is same as $X^{(k+1)}$, then the solution has converged to an optimal solution. Terminate the process. Otherwise, set k=k+1 and replace $X^{(k)}$ by $X^{(k+1)}$ and go to Step 1.

Numerical Example:

A numerical problem on a linear programming problem involving two decision variables is presented as:

$$\max : Z = x_1 + 2x_2 \tag{7}$$

Subject to

$$-x_1 + 3x_2 \leq 21 \tag{8}$$

$$x_1 + 3x_2 \leq 27 \tag{9}$$

$$4x_1 + 3x_2 \leq 45 \tag{10}$$

$$3x_1 + x_2 \leq 30 \tag{11}$$

$$x_1, x_2 \geq 0 \tag{12}$$