Indian Institute of Technology Kharagpur
Department of Mathematics

Optimization Techniques
Assignment 2

Spring 2023-2024 Submission Date: 13^{th} November 2023

1. Solve the L.P.P.
   Maximize, $z_x = 4x_1 + 3x_2$
   subject to
   $$x_1 \leq 6$$
   $$x_2 \leq 8$$
   $$x_2 + x_2 \leq 7$$
   $$3x_1 + x_2 \leq 15$$
   $$-x_2 \leq 1$$
   $$x_1, x_2 \geq 0$$
   by using duality theory.

2. Solve the following L.P.P. by using duality theorem.
   Minimize, $z = x_1 + x_2$
   subject to
   $$x_1 + 2x_2 \geq 12$$
   $$5x_1 + 6x_2 \geq 48$$
   $$x_1, x_2 \geq 0.$$

3. Solve the following problem by revised simplex method and prove that alternative optimal solutions exist and the optimal solutions are degenerate
   Maximize, $z = x_1 + 3x_2 + 2x_3$
   subject to
   $$x_1 + 2x_2 \leq 10$$
   $$2x_1 + x_3 \leq 8$$
   $$2x_2 + x_3 \leq 6$$
   $$x_j \geq 0,\ j = 1, 2, 3.$$

4. Solve the L.P.P. by revised simplex method
   Maximize, $z = 2x_1 - 3x_2$
   subject to
   $$x_1 - x_2 \leq 2$$
   $$5x_1 + 4x_2 \leq 46$$
   $$7x_1 + 2x_2 \geq 32$$
   $$x_1, x_2 \geq 0.$$

5. Solve, if possible, by dual simplex method the following problem
   Minimize, $z = 6x_1 + 11x_2$
   subject to
   $$x_1 + x_2 \geq 11$$
   $$2x_1 + 5x_2 \geq 40$$
   $$x_1, x_2 \geq 0.$$

6. Using artificial constraint procedure, solve the following problems by dual simplex method and show that the problem has unbounded solution.
Maximize, $z = 2x_3$

subject to

$$-x_1 + 3x_2 - 7x_3 \geq 5$$
$$-x_1 + x_2 - x_3 \leq 1$$
$$3x_1 + x_2 - 10x_3 \leq 8$$
$$x_1, x_2, x_3 \geq 0.$$

7. (Sensitivity Analysis) Discuss the effect of discrete changes in the requirements for the following L.P.P.
Maximize, $z = x_1 - x_2 + 3x_3$

subject to

$$x_1 + x_2 + x_3 \leq 10$$
$$2x_1 - x_3 \leq 2$$
$$2x_1 - 2x_2 + 3x_3 \leq 0$$
$$x_1, x_2, x_3 \geq 0$$

so that the optimality of this problem remains undisturbed.

8. (Sensitivity Analysis) Given the following L.P.P.
Maximize, $z = x_1 + 4x_2 - 2x_3 + 3x_4 + x_5$

subject to

$$x_1 - 3x_2 + x_3 + 2x_4 + 6x_5 \leq 3$$
$$2x_1 + x_2 + 3x_4 + 2x_5 \leq 6$$
$$4x_1 + x_2 - x_4 + x_5 \leq 2$$
$$x_j \geq 0, j = 1, 2, ..., 5.$$

Find the ranges over which c_1, c_3, c_4 and b_3 can be changed so that the optimality of the current solution remains undisturbed [Assume that when c_1 changes, all other quantities remain unchanged etc.].

9. (Assignment Problem) The owner of a small machine shop has four machinists available to assign to jobs for the day. Five jobs are offered with expected profit for each machinist on each job as given in the adjacent tableau. Find the assignment of machinists to jobs that will result in a maximum profit.

	A	B	C	D	E
1	62	78	50	101	82
2	71	84	61	73	59
3	87	92	111	71	81
4	48	64	87	77	80

10. (Assignment Problem) Find the optimal assignment and the corresponding assignment cost from the following cost matrix.

	A	B	C	D	E
1	-14	-6	-22	-11	-6
2	-18	-22	-14	-15	-9
3	-18	-12	-9	-12	-12
4	-10	-22	-16	-22	-8
5	-16	-16	-14	-10	-10

11. (Transportation Problem) Find the I.B.F.S. to the following transportation problem using north-west corner rule and prove that the optimal solution is non-degenerate though the initial solution is degenerate.

	D_1	D_2	D_3	D_4	a_i
O_1	9	8	5	7	12
O_2	4	6	8	7	14
O_3	5	8	9	5	16
b_j	8	18	13	3	42

12. (Transportation Problem) Consider the following unbalanced transportation problem:

		TO			
		1	2	3	
FROM	I	5	1	7	10
	II	6	4	6	80
	III	3	2	5	15
		75	20	50	

Since there is not enough supply, some of the demands at these destinations may not be satisfied. Suppose there are penalty costs for every unsatisfied demand unit which are given by 5, 3, 2 for destinations 1, 2, 3 respectively. Find the optimal solution.

13. (Transportation Problem) Find the initial basic feasible solution of the following balanced T.P. by row minima method, column minima method, matrix minima method, north-west corner rule and VAM and compare their corresponding costs.

	D_1	D_2	D_3	D_4	D_5	a_i	
O_1	4	7	0	3	6	14	
O_2	1	2	-3	3	8	9	supply
O_3	3	-1	4	0	5	17	
b_j	8	3	8	13	8		
		Demand					

14. Solve the following Integer Programming Problem by cutting plane method:

$$\begin{aligned} &\text{Maximize } z = x_1 + x_2 \\ &\text{Subject to } \quad 3x_1 + 2x_2 \leq 5 \\ &\qquad\qquad\quad x_2 \leq 2 \\ &x_1, x_2 \geq 0 \text{ and are integers.} \end{aligned}$$

15. Solve using Branch and Bound Technique:

$$\begin{aligned} &\text{Max } Z = -x_1 + 4x_2 \\ &\text{Subject to } -10x_1 + 20x_2 \leq 22 \\ &\quad , \qquad\quad 5x_1 + 10x_2 \leq 49 \\ &\qquad\qquad\quad x_1 \leq 5 \\ &x_i \geq 0,\ x_i\text{'s are integers.} \end{aligned}$$

16. Solve the following travelling salesman problem.

	A	B	C	D	E
A	-	12	15	10	8
B	8	-	15	12	8
C	9	11	-	15	11
D	7	12	19	-	11
E	9	12	16	10	-