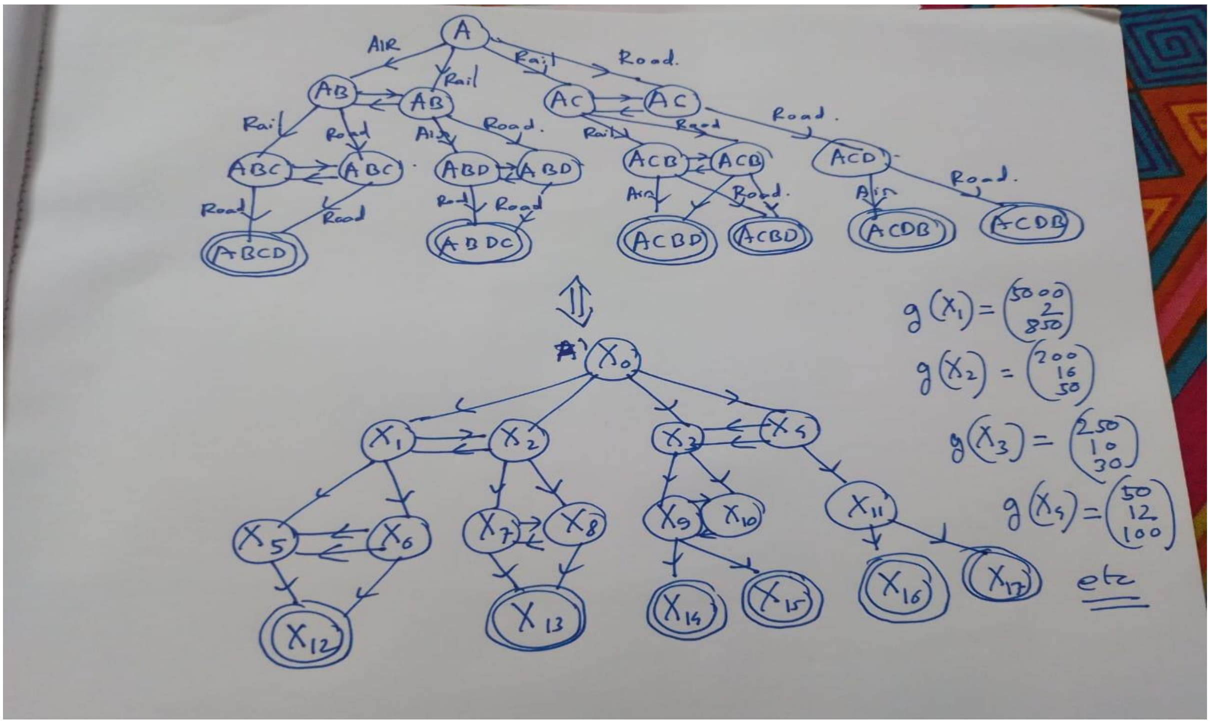
Q1. You are planning a multi-city tour. Between any two cities you have some transport options. Each option is associated with cost, time and carbon emission (note that the costs etc are symmetric between a pair of locations). You need to consider all these three criteria for planning your travel itinerary. Cast this in the state-space setting and construct the graph. Using multi-criteria A* algorithm, find the set of pareto-optimal paths that cover all 4 cities starting from A.

From	To	Mode	Cost	Time	C02 Emission
A	B	Air	5000	2	850
A	B	Rail	200	16	50
A	C	Rail	250	10	30
A	C	Road	50	12	100
B	C	Rail	100	8	35
B	C	Road	80	7	90
B	D	Air	2500	1	400
B	D	Road	600	10	70
C	D	Road	500	8	60

The state space graph nodes should indicate which nodes have been covered so far, and the edges should indicate movement from one node to another. For example, a node may indicate the state <AB>, from which there should be an edge to a node denoting <ABC>. The edge between them should denote the cost, which is in this case is a 3D vector. As we can have multiple paths between two locations, and the A* algorithm does not work with two parallel edges, we can have multiple nodes corresponding to same state, as shown below. These “duplicate nodes” can be connected by edges (both ways) with 0 weight.



After this the multi-objective A* can continue as usual. Any heuristic function can be used in each node, as long as it underestimates the total cost from that node to any goal node. Possible examples: i) consider the outgoing edges of each node and take the minimum value among them for each of the

3 objectives ii) consider the minimum value of each objective over all nodes (same heuristic value for all nodes) iii) just 0. While (ii) and (iii) are simpler, they are not good as they make you traverse the graph breadthwise, and it takes too long to reach a goal node.

[Marking policy: if the state-space graph is drawn reasonably, and a few steps of A* are shown, that will be sufficient for 10 marks. However, it was noticed that many students have only drawn the graph showing 4 nodes (A,B,C,D) and the paths connecting them. Furthermore, many students have solved the problem independently for each of the 3 objectives, and thus given separate optimal paths for cost minimization, time minimization etc. These are not correct and only partial marks will be given].

Q2. Your earnings for your first T years of service are [x(1), x(2), ….. x(T)]. Each year, you consume a part of these earnings (C(1), C(2), … C(T)), and save the rest (S(1), …. S(T)). The interest rate on your savings is 5% per annum. The utility function for your consumption is $U(t) = C(t)/t^2$ (where t is the number of years elapsed from the start).

i) How will you plan your consumptions/savings, such that your savings after 5 years (including interest) are at least 50% of the earnings?

ii) If you don't have that constraint on the savings but want to maximize it along with the consumption utility, how will you find an optimal solution?

[Please formulate the problems and indicate how they can be solved]

iii) Suppose you save a random fraction of your saving each year, i.e. S(t)=v(t)*x(t) where v(t) is chosen uniformly between 0.2 and 0.5. What is your expected utility value and expected savings (including interest) after 3 years?

Consider C(t)+S(t)=X(t) for each year.

So we have contraints: C(1)<=X(1), C(2)<=X(2), ….., C(5)<=X(5)

Interest in year 1: I(1)= r*S(1). Similarly, I(2) = r*(S(1)+S(2)), I(3) = r*(S(1)+S(2)+S(3)) etc

So total interest = 5r*S(1) + 4r*(S2) + 3r*S(3) + 2r*S(4) + r*S(5)

After 5 years: total savings with interest = Total income + total interest – total consumption

X(1) + X(2) + X(3) + X(4) + X(5) + 5r*(X(1)-C(1))+4r*(X(2)-C(2)) + … +r*(X(5)-C(5)) – C(1)– C(2)- C(3)-C(4)-C(5) = (1+5r)*X(1) + (1+4r)*X(2) … + (1+r)*X(5) – (1+5r)*C(1) –(1+4r)*C(2)-…… - (1+r)*C(5)

= B – b1*C(1) -b2*C(2)-b3*C(3)-b4*C(4)-b5*C(5) where B, b1, b2 etc are constants

i) Our objective: to maximize the utility: U(C) = C(1) + C(2)/4 + C(3)/9 + C(4)/16 + C(5)/25

Additional constraints:
B – b1*C(1) -b2*C(2)-b3*C(3)-b4*C(4)-b5*C(5) >= 0.5(X(1)+X(2)+X(3)+X(4)+X(5)),
i.e. D >= b1*C(1) + b2*C(2) + b3*C(3) + b4*C(4) + b5*C(5) where D is a constant

Essentially we have a problem of the type:

Maximize $U(C1,C2,C3,C4,C5) = a1*C1 + a2*C2 + a3*C3 + a4*C4 + a5*C5$

Subject to the constraints: $C1<=X1$, $C2<=X2$, $C3<=X3$, $C4<=X4$, $C5<=X5$, $b1*C1+b2*C2+b3*C3+b4*C4+b5*C5 <= D$

This can be solved by linear programming approach. We can add a slack variable for each constraint (total 6), consider the different basic solutions (at least 5 variables set to 0) and calculate the utility function for each of them. The basic feasible solution with highest value of utility is the best.

ii) We now have two objective functions: a) $f1(C)= a1*C1 + a2*C2 + a3*C3 + a4*C4 + a5*C5$

and b) $f2(C)=B – b1*C(1)– b2*C(2)- b3*C(3)-b4*C(4)-b5*C(5)$

We can use scalarization, where we attach weights W1 and W2, i.e. we now have the problem:

Maximize $W1*f1(C) + W2*f2(C)$ s.t. $C1<=X1$, $C2<=X2$, $C3<=X3$, $C4<=X4$, $C5<=X5$

We can now solve this using Linear Programming, as earlier.

iii) The utility function has already been expressed as a linear function of C(1), C(2) etc, and we know the relations between C(1), C(2) etc and S(1), S(2) etc.
Further, we know that $S(t)=v(t)*X(t)$, so $E(S(t)) = E(v(t))*X(t)$
As $v(t)\sim U(0.2,0.5)$, so $E(v(t))=0.35$, and hence $E(S(t)) = 0.35*X(t)$. This helps us calculate $E(C(t)) = X(t)-E(S(t)) = 0.65*X(t)$.
So, we can now calculate the expected utility $E(U) = a1*E(C1) + a2*E(C2) + … +a5*E(C5)$

[It was observed that there is some confusion regarding whether the interest remains in the saving or is available for consumption in the next year. This solution considers that the interest remains in the savings, but considering the other thing is also perfectly fine. The outline of the solution will remain unchanged, through the constraint inequalities may change. However, the total budget constraint equation in Prof. Dripto's slide is NOT valid here, since that assumes that there is no savings in the end and the entire earning including interest is consumed during the lifetime. Here we are considering only 5 years, not the entire lifetime, so there will be savings in the end. The periodic budget constraints are fine, though]