# AI60003 AI for Economics

# Practice Problems

**Q1 Linear programming**

I have a total budget of 100. There are two projects (P1 P2), in which we can make investments $X_1$, $X_2$. The revenue obtained from these projects are $(3X_1+5)$, $(2X_2+4)$. Further, there is a rule that I have to make a fair allocation of budget to the projects – the investment to no project can be more than twice of the other. What is the optimal allocation to the projects?

**Solution sketch**:

Utility function $U(X_1, X_2) = 3X_1 + 2X_2 + 9$

Constraints: $X_1 + X_2 <=100$, $X_1<2X_2$, $X_2<2X_1$

Modified constraints: $X_1 + X_2 + a – 100 = 0$, $X_1 - 2X_2 +b =0$, $X_2-2X_1+c=0$, s.t. $X_1$, $X_2$, a, b, c >=0

Basic solutions: at least 2 of these 5 should be 0.

$X_1=0$, $X_2=0$ => $U(X_1, X_2) = 9$ (feasible solution with low utility)

$X_1=0$, a=0 => $X_2=100$ (constraint 3 violated, infeasible), $X_1=0$, b=0 => $X_1 - 2X_2 = 0$, i.e. $X_2=0$! (already considered), same with $X_1=0$, c=0

b=0, c=0 => X1 = X2 = 0 ! (already considered)

a=0, b=0 => X1 = 200/3, X2 = 100/3, $U(X_1, X_2) = 200 + 200/3 + 9 =276$

a=0, c=0 => X1 = 100/3, X2 = 200/3, $U(X_1, X_2) = 100 + 400/3 + 9 =242$

By fundamental theorem of LP, the best solution is one of the basic solutions. Hence, optimal allocation: ($X_1 = 200/3$, $X_2 = 100/3$)

**Q2. Nonlinear programming**.

I have a total budget of 100. There are two projects (P1 P2), in which we can make investments $X_1$, $X_2$. The revenue obtained from these projects are $X_1^2 + 2X_2^2$. Further, there is a rule that the first project should receive an investment of at least 25. What is the optimal allocation to the projects?

**Solution Sketch**:

$U(X_1, X_2) = -X_1^2 - 2X_2^2$ (we want to maximize the revenue, so minimize its negative)

Constraints: $X_1 + X_2 <= 100$, $X_1>= 25$, i.e. $g_1(X_1, X_2) = X_1 + X_2 – 100$, $g_2(X_1, X_2) = 25 - X_1$

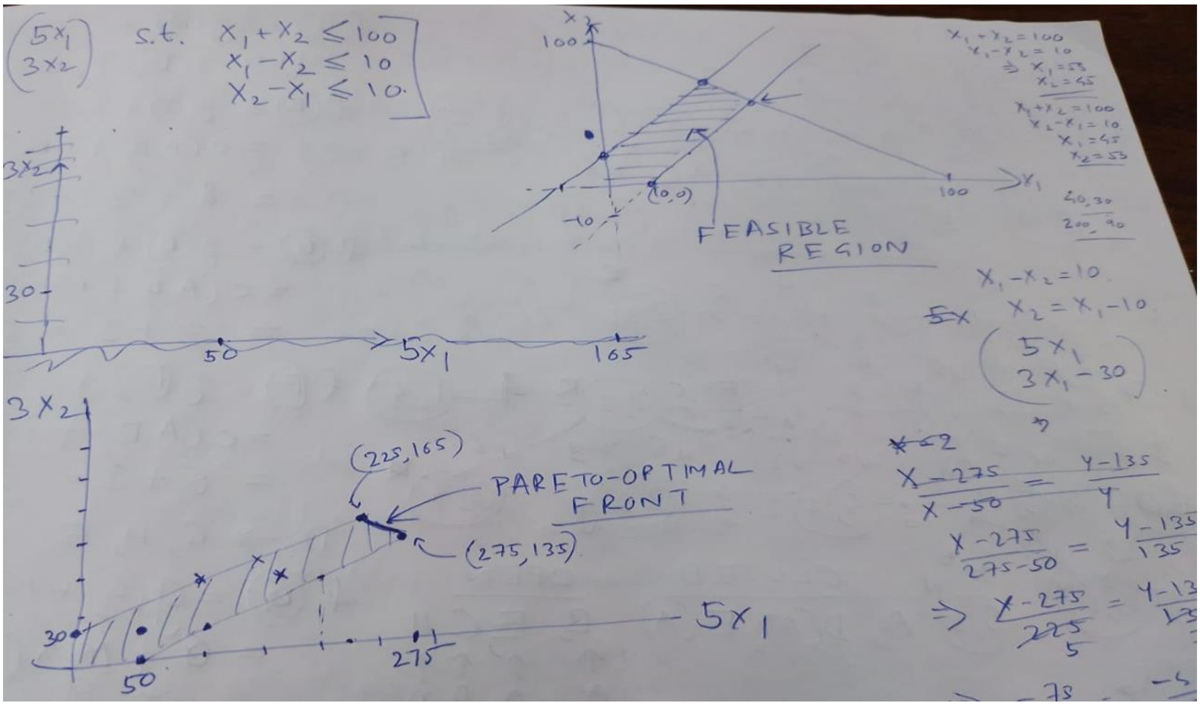
Interior point approach: define barrier function $B(X_1, X_2) = -\log(-g_1(X_1, X_2)) -\log(-g_2(X_1, X_2))$

Now define augmented objective function $U(X_1, X_2) + \mu B(X_1, X_2)$, Initial point: ($X_1 =0$, $X_2 =0$) [inside feasible region]

Proceed according to steps shown in the notes on Nonlinear Optimization (Pages 6-8)

**Q3. Multicriteria Optimization**

The government has two priority sectors: public health and job creation. When x1 and x2 amounts of funds are allocated to these sectors, the utility functions for them are 5x1 and 3x2 respectively. However, the total fund allocation cannot exceed 100. Also, the ministers handling these departments have a pact that the difference between the spendings in these departments will not exceed 10. Characterize the pareto-optimal solution to this problem.




Pareto-optimal front: the line connecting (225,165) and (275,135). Note that these two solutions are pareto-optimal, as there is no point in the feasible region which dominates them.

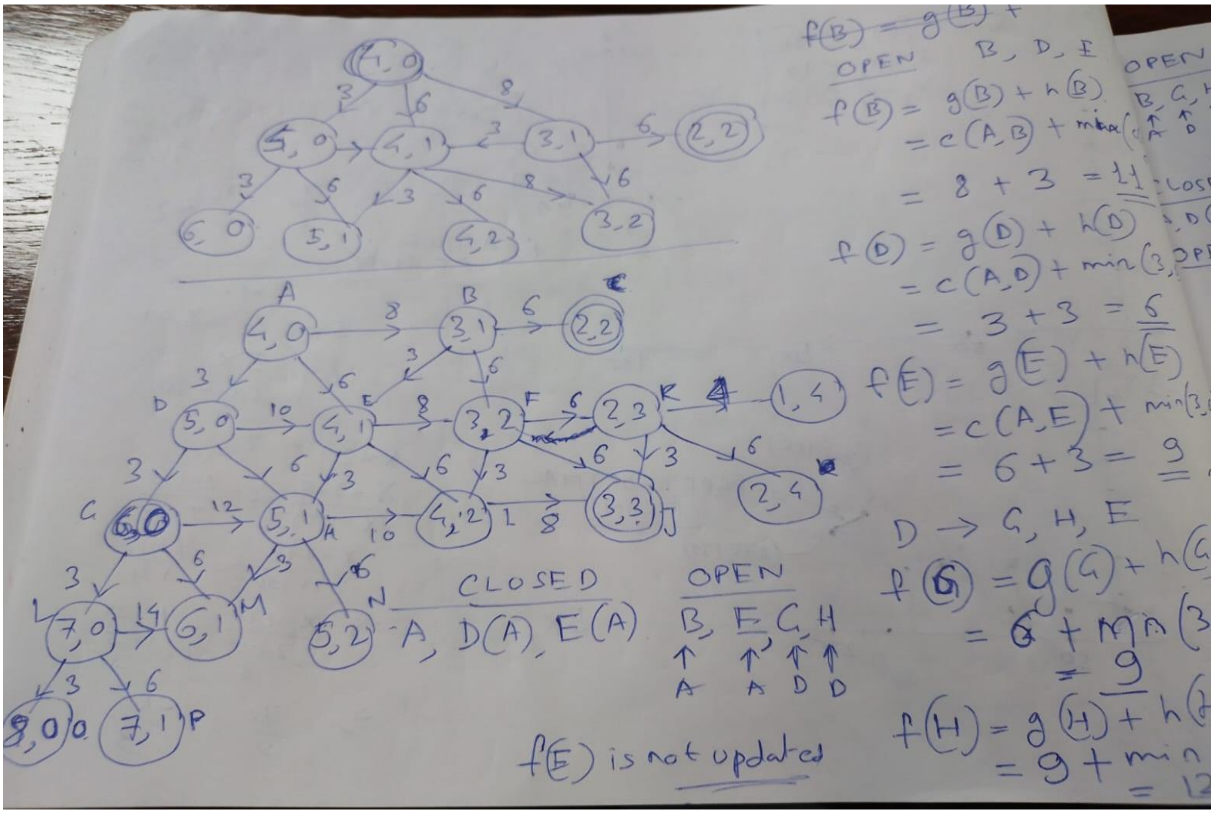
**Q4. A-star algorithm**

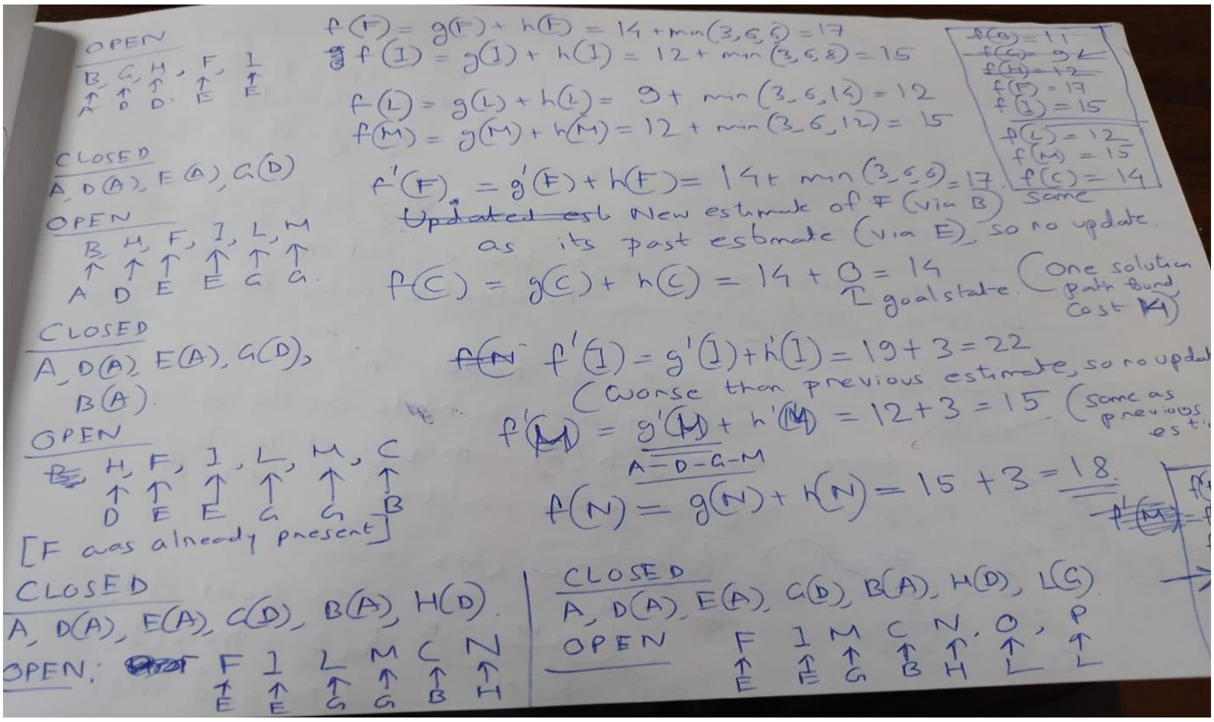
There are two departments – one of them currently has budget 4, and the other has budget 0. The government wants to raise or redistribute funds such that both departments have equal funds. Since Dep 1 is already well-established, raising funds for it is easier than doing so for Dep 2, which is novice. Further, any attempt to redistribute will face resistance from Dep 1 as it is strong. This can be done step-by-step. In each step, the government can do any of the following:

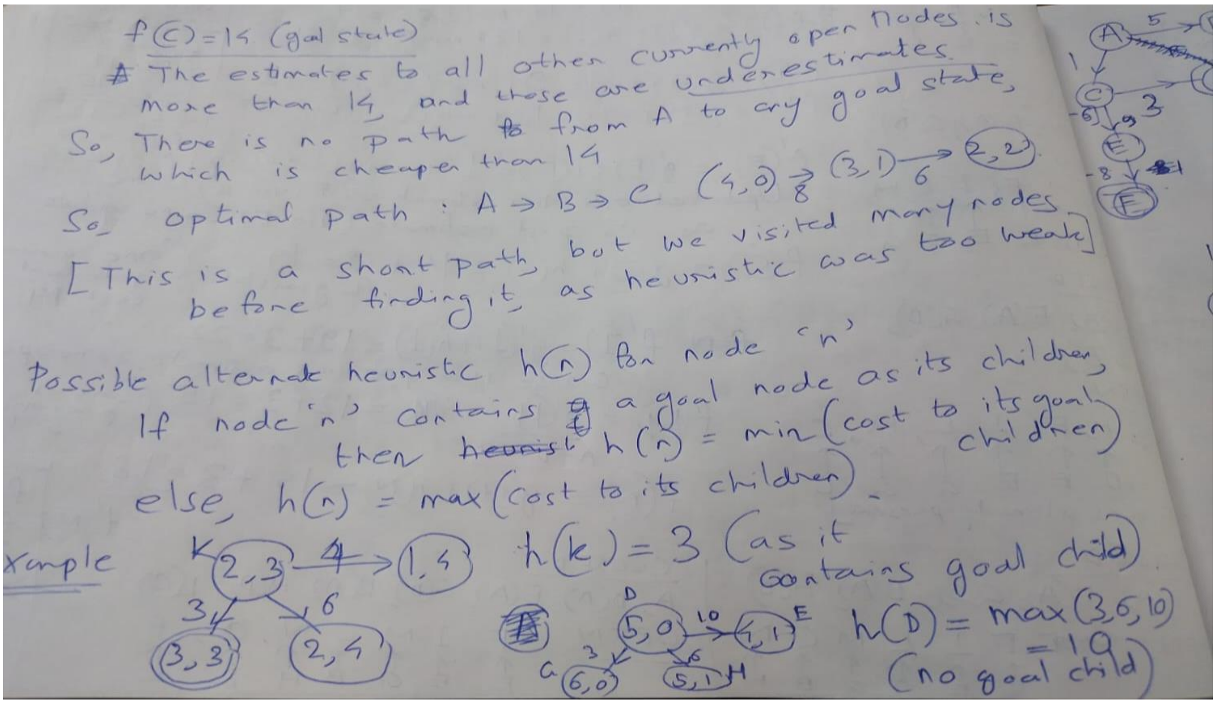
i) raise 1 unit of funding and allocate to first department, which has a cost of 'k'
ii) raise 1 unit of funding and allocate to second department, which has a cost of '2k'
iii) transfer 1 unit of funding from first to second department, which has a cost proportional to the current budget of the first department (consider 'm' as the constant)

For m=2 and k=3, find the optimal sequence of moves by the government by which they can achieve their objective. At any node, use the minimum of the costs to its children nodes as the heuristic measure. On seeing the performance, can you suggest a better heuristic?

**Solution sketch**: Define each node in state space as (x, y) where x = current budget of first dept, y = current budget of second dept.





f(C) = 14 (goal state)

* The estimates to all other currently open nodes is more than 14 and these are underestimates.

So, There is no path ~~to~~ from A to any goal state, which is cheaper than 14

So optimal path: A → B → C (4,0) $\xrightarrow{8}$ (3,1) $\xrightarrow{6}$ (2,2)

[This is a short path, but we visited many nodes before finding it, as heuristic was too weak]

Possible alternate heuristic h(n) for node 'n'

If node 'n' contains a goal node as its children, then ~~heurist~~ h(n) = min(cost to its goal children)

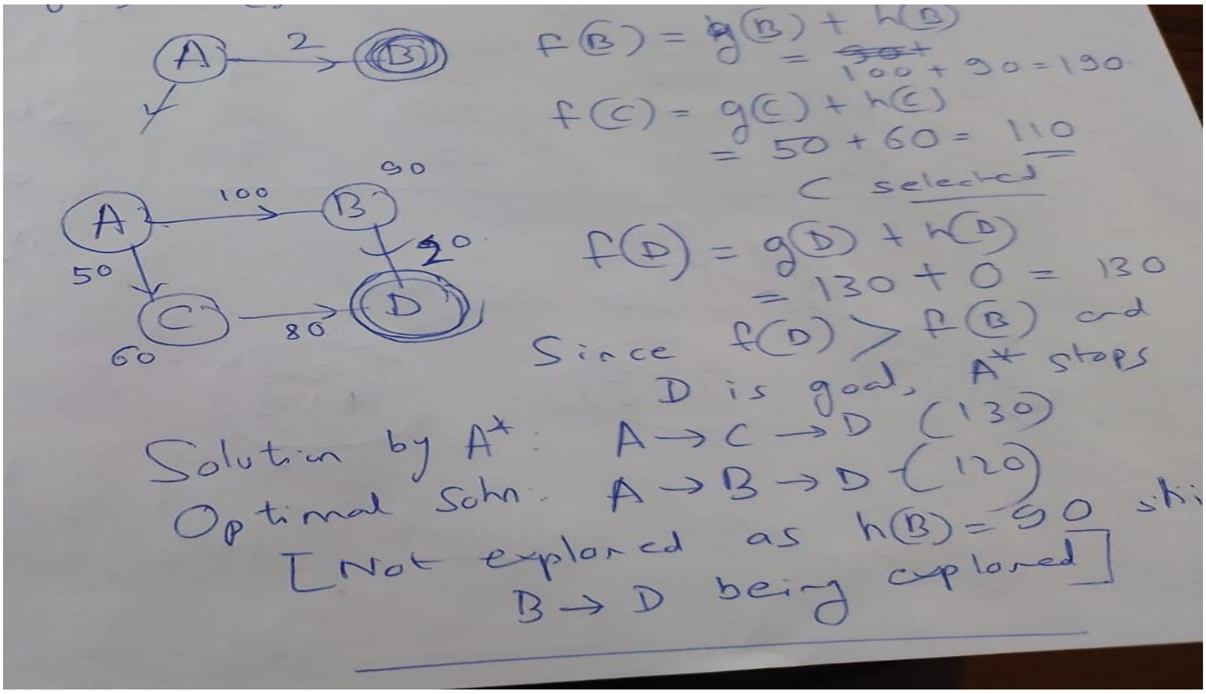
else, h(n) = max(cost to its children).

Example: K (2,3) $\xrightarrow{4}$ (1,4) h(K) = 3 (as it contains goal child)

D (5,0) $\xrightarrow{10}$ E (4,1) h(D) = max(3,6,10) = 10 (no goal child)

## Q5. A-star algorithm

Discuss with examples, why the A* algorithm won't work if the heuristic involved overestimates the actual cost to the goal node?



f(B) = g(B) + h(B) = 100 + 90 = 190

f(C) = g(C) + h(C) = 50 + 60 = 110

C selected

f(D) = g(D) + h(D) = 130 + 0 = 130

Since f(D) > f(B) and D is goal, A* stops

Solution by A*: A → C → D (130)

Optimal Soln. A → B → D (120)

[Not explored as h(B) = 90 B → D being explored]

## Q6. Multi-objective A* (maximization)

A set of n projects (P) are to be completed by the Government. There are a set of r agencies (A) who apply for taking up the projects. We can assume r >= n. Each agency submits bids for some projects, giving the cost requirement for each project that they bid for. For each project they also have to guarantee a minimum amount of revenue generation and number of jobs created. Each agency can be assigned at most one project. Given a total budget B available with the Government for completion of ALL projects, we are to assign projects to various agencies (one agency can get at most one project) so as to maximize the two objectives, namely total revenue generation and total job creation within the total budget B. The set of non-dominated valid solutions are to be reported. If all projects cannot be completed within the budget B through any combination, then a failure is reported. Develop a multi-objective state space heuristic search model for this problem with the following:

a) Definition of State and its validity condition

b) State Transformation Rules

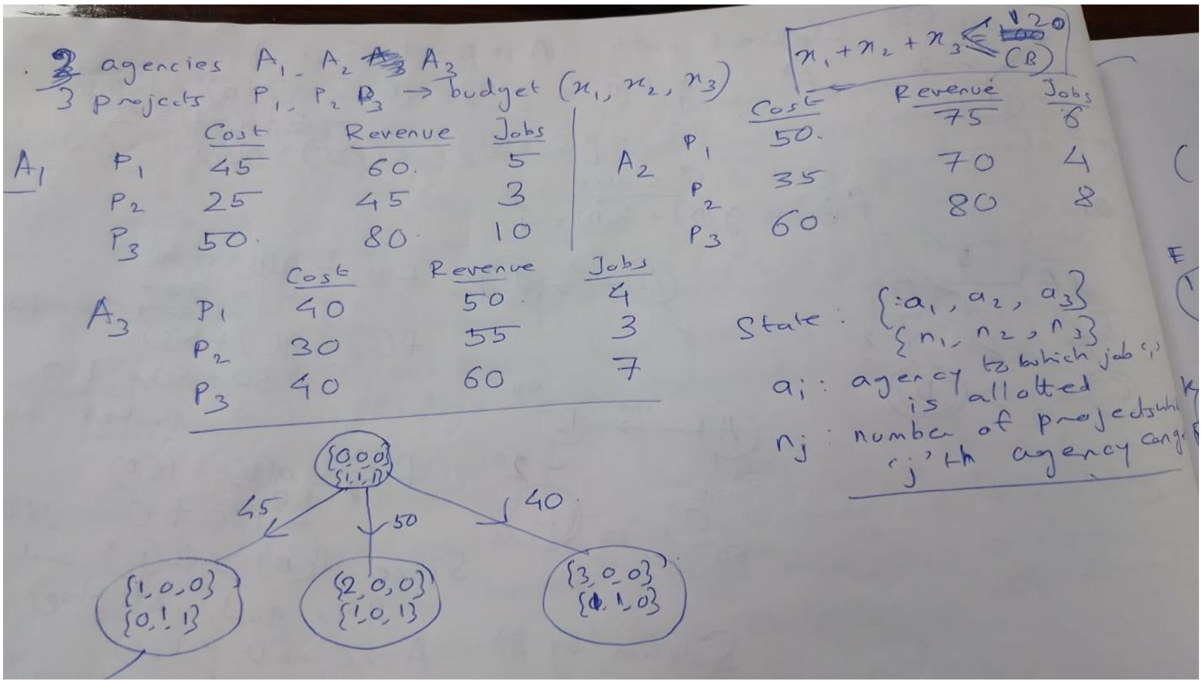
c) Start State

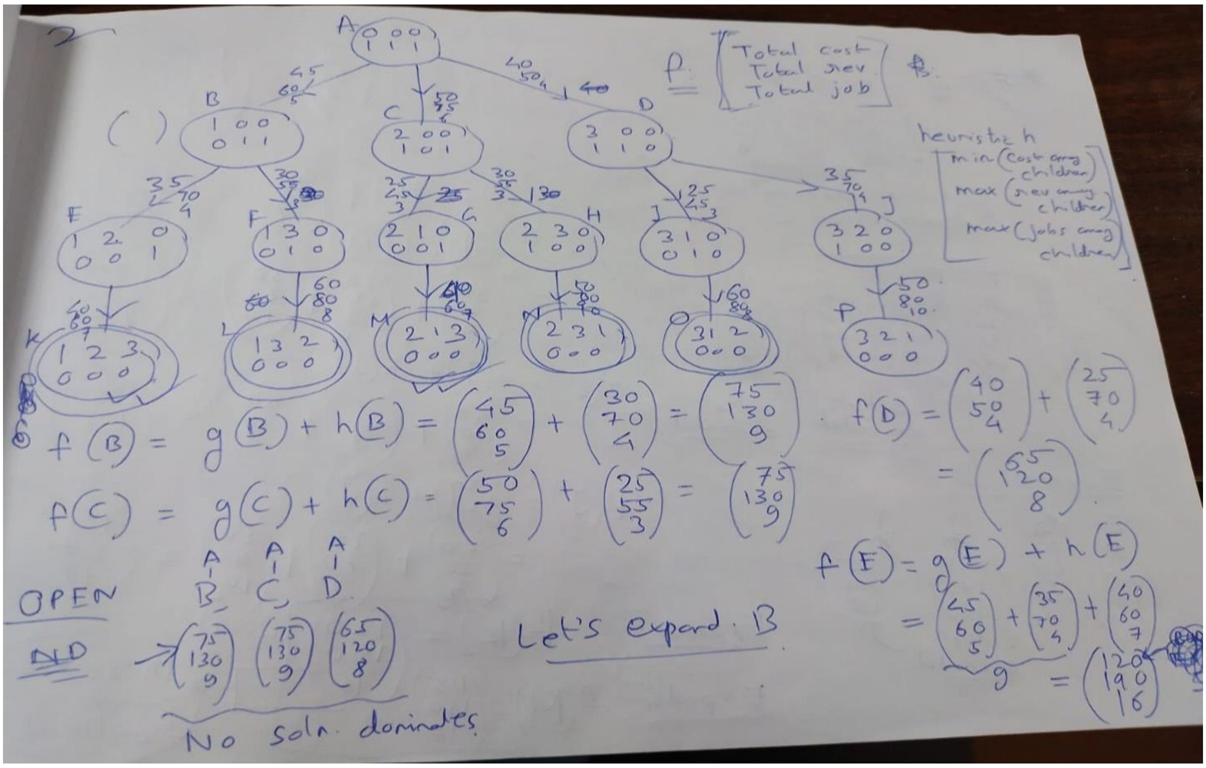
d) Goal Condition

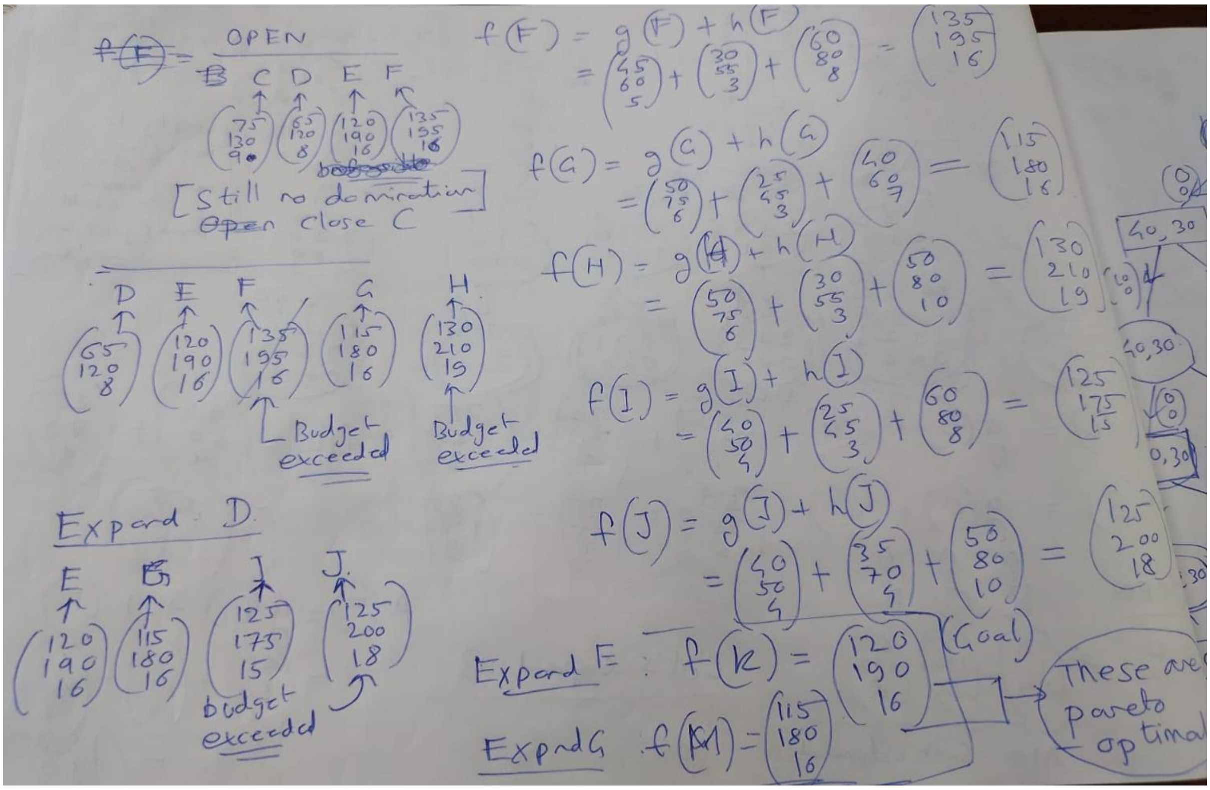
e) The multi-objective cost g(n) of any state

f) A good multi-objective Heuristic estimate h(n), which must **overestimate** the two objectives (remember, this is a maximization problem).

Create a non-trivial example problem of 3 projects and 3 agencies where each agency bids for every project and there are at least 3 solutions. Show the working of a Multi- objective A* algorithm



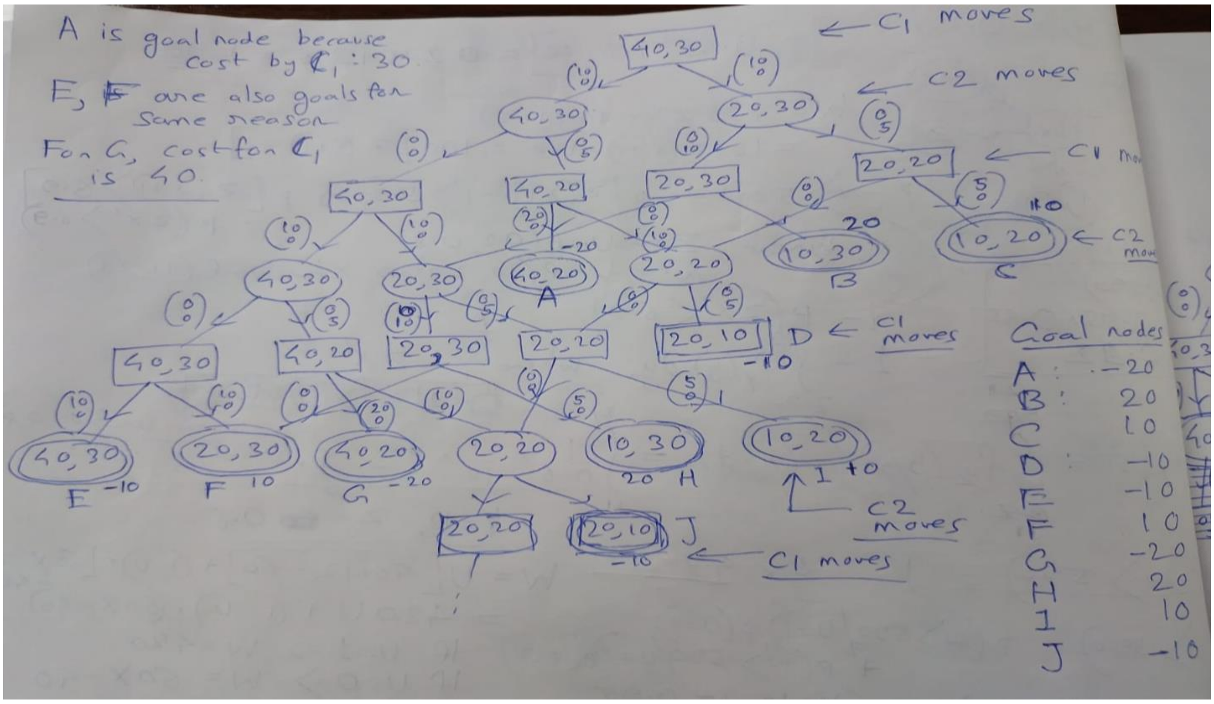


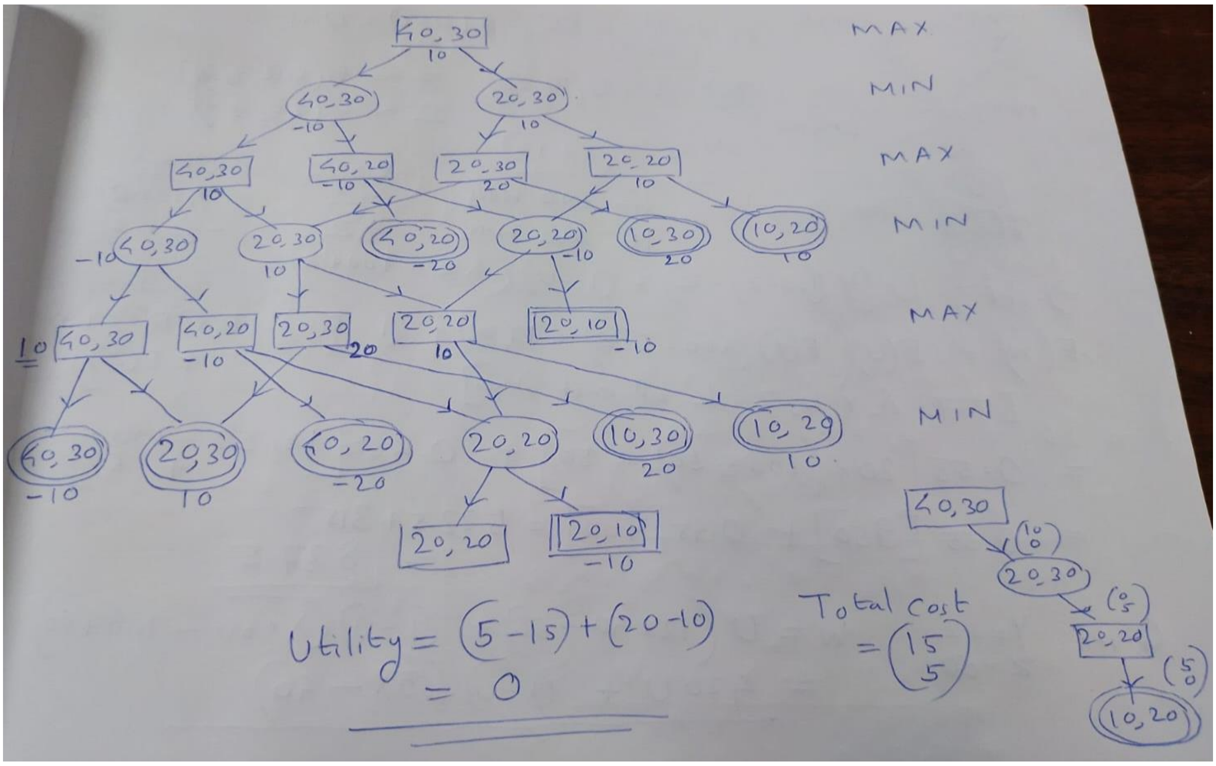




### Q7. Game Tree

Two companies C1 and C2 are selling an article for prices 40 and 30 respectively. They take turns to revise their prices. During its turn, C1 can either maintain its current price, or slash it by 50%. If it maintains the price, the cost of that step is max(0, excess price of C1 wrt C2). If it slashes price, the cost of that step is half of the reduction amount. Similarly, during its turn, C2 can either maintain its price, or slash its price by 10 units. The costs for these steps is similar to those C1. The price race stops either when any one company reaches the minimum possible price, which is 10, or the total cost it incurs is 25. The utility of each goal state is (price offered by C2-price offered by C1). Calculate the game-tree, along with the valuations of each intermediate node. Identify the optimal moves at each step.

**Solution sketch**: define state space (x,y) where x is price offered by C1, y is price offered by C2. Each edge is associated with vector (a b) where a is loss suffered by C1, b is loss suffered by C2 in current step



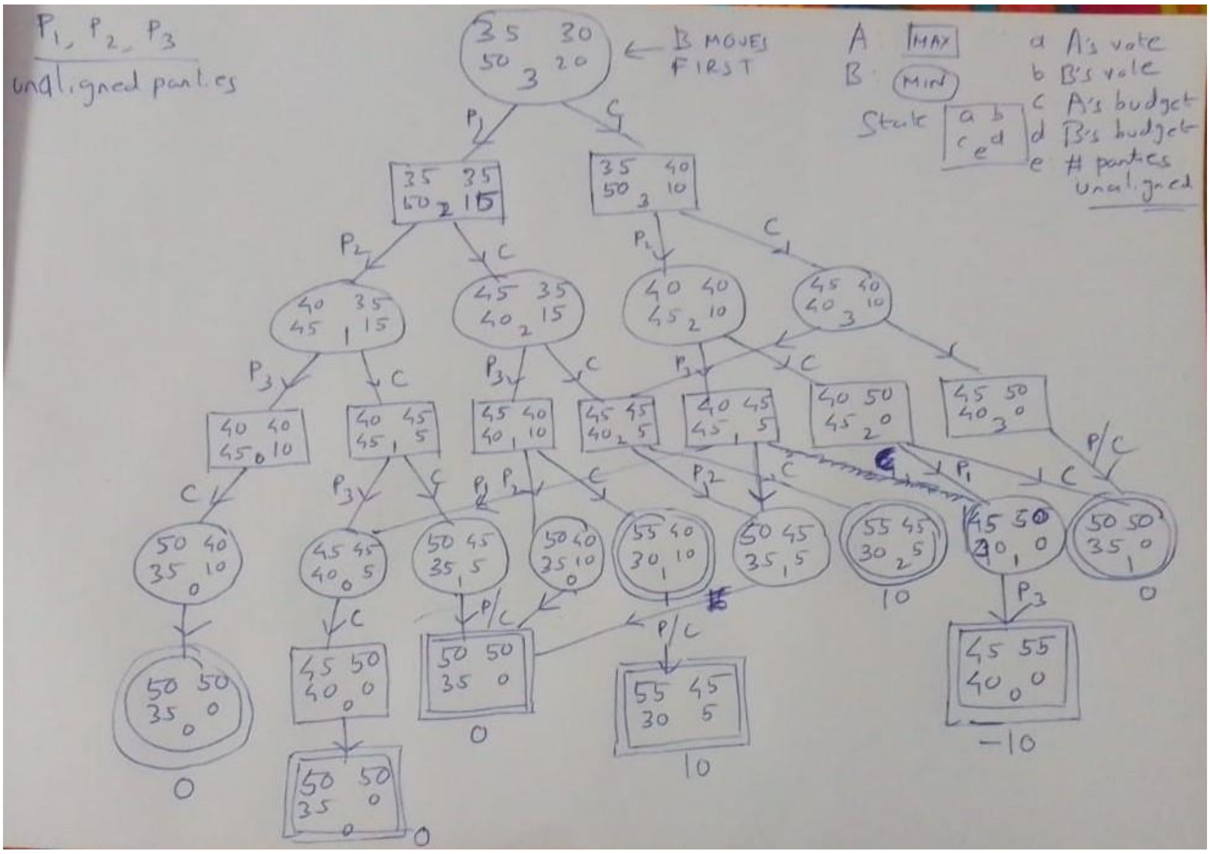
**Q8. Game Tree**

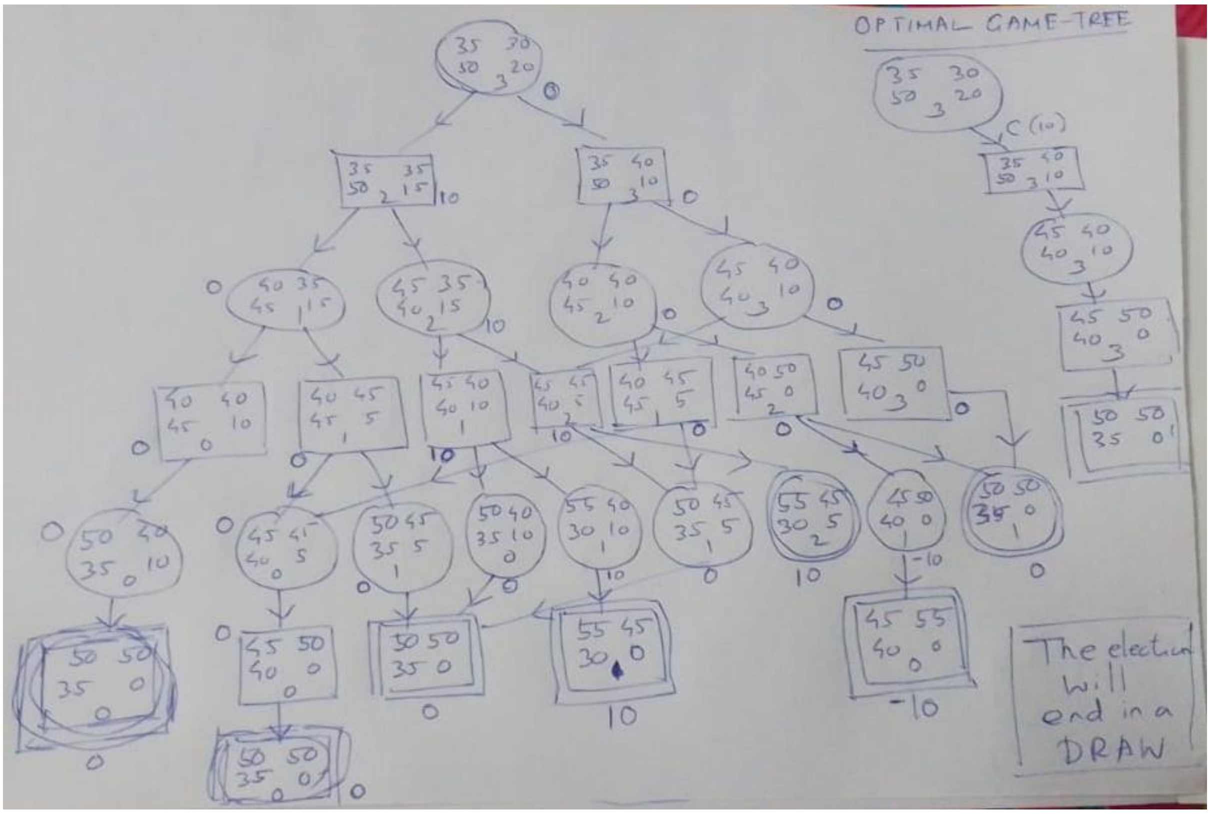
Two major political alliances A and B are gearing up for the general elections in a country of 100 million people. Alliance A currently has the total support of 35 million people, while B has a total support of 30 million people. There are 3 smaller parties that are unaligned, each of which have support of 5 million people. The remaining 20 million people are not aligned with any party, and can be influenced by campaigning. The alliances A and B have budgets of 50 million and 20 million respectively, which they can use for campaigning.

The two alliances take turns to make their next political move. In each step, either they add one of the remaining unaligned parties to their alliance (the votes of the new party will be fully transferred to the alliance), or convince the unaligned people (including the supporters of the remaining unaligned parties) to vote for them, at the rate of 1 unit per person (i.e. win x million votes by spending x millions). However, there is a limit of 10 million for each move. Further, if an alliance which is currently not in lead wants to add an unaligned party, it must spend 5 millions to do so, while the leading alliance can add a party for free. If an alliance is not in able to do either of these (i.e. 0 budget and trailing), they forfeit their move. The process continues till all the people have been aligned to an alliance.

Represent this situation as a two-player game between the alliances. Define an appropriate state space representation, and draw the min-max graph. Alliance B moves first. The utility of each goal state is defined as (number of people favouring A - number of people favouring B). Is there a way in which alliance B can win? What is the outcome if both alliances always take optimal decisions?

**Solution sketch**: Define state space as (a,b,c,d,e) where a=A's total vote, b=B's total vote, c=A's remaining budget, d=B's remaining budget, e=number of unaligned parties left.

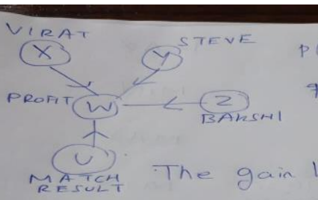





It is possible for alliance B to win, provided A makes a suboptimal move. Similarly, A can also win if B makes a suboptimal move. But if both alliances move optimally, the election will end in a Draw with both sides securing equal votes.

Q9. Uncertainty and Bayesian Network

In the India-Australia match, Virat bets X:1 on India, where X~Unif(10,20) [X is an integer only]. Steve bets Y:1 Australia, where Y follows a Gaussian distribution with mean 20 and variance 10. Prof. Bakshi chooses Z uniformly between 20 and 80. He bets Z against Virat on Australia, and (100-Z) against Steve on India. The odds of India's victory is 55:45. Describe the situation using a Bayesian Network. What is Prof. Bakshi's expected gain W? If it is known that Y=12 and Z=60, what is the probability that Prof. Bakshi had more gains than expected?




$P(U=1) = 0.55$ (INDIA), $P(U=0) = 0.45$ (AUS)

$X \sim U(10, 20)$. ~~$Z \sim U(0.2, 0.8)$~~ $Z \sim U(20, 80)$.

$Y \sim N(20, 10)$

| The gain W: | Virat | Steve |
|---|---|---|
| If IND WINS: | $-Z$ | $Y(100-Z)$ |
| If AUS WINS: | $XZ$ | $-(100-Z)$ |

$\Rightarrow W = U(Y(100-Z) - Z) + (1-U)(XZ - 100 + Z)$

$E(W) = E(U)(E(Y)(100 - E(Z)) - E(Z)) + (1 - E(U))(E(X)E(Z) - 100 + E(Z))$

[$X, Y, U, Z \perp$ of each other]

$= 0.55[20 \times (100 - 50) - 50] + 0.45[15 \times 50 - 100 + 50]$

$= 0.55[950] + 0.45[700] = 522.5 + 315$

$= 837.5$

Given: $Y = 12$, $Z = 60$

$W = U(12 \times 40 - 60) + (1-U)(X \times 60 - 100 + 60)$

$= 420U + (1-U)(60X - 40)$

$$P(W > 837.5) = P(W > 837.5 / U=1)\, \underbrace{P(U=1)}_{0.55} + P(W > 837.5 / U=0)\, \underbrace{P(U=0)}_{0.45}$$

If $U=1$, $W = 420$. So $P(W > 837.5 / U=1) = 0$!

If $U=0$, $W = 60X - 40$. So $P(W > 837.5 / U=0) = P(60X - 40 > 837.5)$

$$= P(X > 14.625)$$

$$= \frac{20 - 14.625}{20 - 10} = 0.5375$$

Hence $P\left(W > 837.5 \,/\, \begin{smallmatrix} Y=12 \\ Z=60 \end{smallmatrix}\right)$

$$= 0.55 \times 0 + 0.45 \times 0.5375$$

$$\approx 0.24$$

Q10. Linear Regression

We are trying to predict the annual income (Y) of a person. Consider the following dataset which records their Masters' Degree marks (M), inherited income (I) and age (A). You are required to solve this as a LASSO regression problem, for which you are given three possible coefficient vectors w1=[0.04 0.4 0.1], w2=[0.06 0 0.04] and w3=[0 0.6 0.1]. Which model will you prefer for different values of λ? What is your interpretation of the model? LASSO loss = $(y - w.x)^2 + \lambda||w||_1$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| M | 0 | 0 | 65 | 78 | 55 | 0 | 85 | 50 | 70 |
| I | 1 | 2 | 0 | 5 | 2 | 0 | 10 | 3 | 5 |
| A | 40 | 42 | 27 | 31 | 40 | 55 | 25 | 50 | 28 |
| Y | 2 | 3 | 3 | 5 | 6 | 8 | 8 | 10 | 15 |

**Hint**: For each data-point, find the LASSO loss using all three values of w and add them up. Find which 'w' has the least loss. Repeat using different values of λ.

Q11. Decision Tree

We are trying to predict whether a startup company will survive or not. We have considered several attributes in the dataset provided below. Construct a decision stump (i.e. decision tree of depth 1) by choosing the most discriminative feature.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Seed funding (lakhs) | 45 | 80 | 12 | 56 | 37 | 105 | 85 | 19 | 21 |
| Age of main founder (years) | 31 | 36 | 43 | 29 | 23 | 27 | 31 | 26 | 22 |
| Number of founders | 3 | 4 | 3 | 2 | 5 | 8 | 7 | 1 | 1 |
| Survived? | Y | Y | Y | Y | N | N | N | N | N |

**Hint**: Choose each feature, and choose 2 or 3 “splits”, eg. for seed funding you may choose 0-30, 30-80, 80+ as the three splits, similarly for age of main founder you may choose <30, 30-45, >45 as the splits. For each feature, find the relative frequencies of ‘Y’ and ‘N’ among the data-points in each split, and calculate their entropy. Find which split gives you the maximum reduction of entropy with respect to the full dataset.