Indian Institute of Technology Kharagpur
Department of Mathematics
Optimization Techniques
Assignment

Spring 2023-2024 Submission Date: 15.09.2023

1. Four different metals namely, iron, copper, zinc and manganese are required to produce three commodities A, B and C. To produce one unit of A, 40 kg iron, 30 kg copper, 7 kg zinc and 4 kg manganese are needed. Similarly to produce one unit of B, 70 kg iron, 14 kg copper and 9 kg manganese are needed and for producing one unit of C, 50 kg iron, 18 kg copper and 8 kg zinc are required. The total available quantities of metals are 1 metric ton iron, 5 quintals of copper, 2 quintals of zinc and manganese each. The profits are Rs.300, Rs.200 and Rs.100 in selling per one unit of A, B and C respectively. Formulate the problem mathematically.

2. A coin to be minted contains 40% silver, 50% copper, 10% nickel. The mint has available alloys A, B, C and D having the following composition and costs:

	% Silver	% Copper	% Nickel	Costs
A	30	60	10	Rs. 11.00
B	35	35	30	Rs. 12.00
C	50	50	0	Rs. 16.00
D	40	45	15	Rs. 14.00

Present the problem of getting the alloys with specific composition at minimum cost in the form of an L.P.P.

3. An intermediate tableau of an L.P.P. by simplex method is given below in an incomplete form.

			c_j	-4	2	0	0	0	-M
c_B	B	x_B	b	a_1	a_2	a_3	a_4	a_5	a_6
-4			$\frac{24}{5}$			-$\frac{2}{5}$	0	$\frac{1}{5}$	
-M			$\frac{18}{5}$			$\frac{1}{5}$	-1	$\frac{2}{5}$	
-2			$\frac{63}{5}$						
$z_j - c_j$				0	0	-$\frac{1}{5}$M+$\frac{6}{5}$	M	-$\frac{2}{5}$M+$\frac{2}{5}$	0

(a) Complete the table.

(b) Find the entering and the departing vectors.

(c) Write down the next tableau and show that the next tableau gives the unique optimal solution $x_1 = 3, x_2 = 8$

4. Using simplex method, show that the inverse of the matrix $\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ is $\begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$

5. Solve the following degeneracy problem with the help of Charnes perturbation technique. Maximize, $z = 3x_1 + 4x_2$

subject to

$$x_1 + 2x_2 \leq 15$$
$$2x_1 + x_2 \leq 0$$
$$4x_1 + 7x_2 \leq 0$$
$$x_1, x_2 \geq 0.$$

6. Prove that $x_1 = 3$, $x_2 = 2$, $x_3 = 4$, $x_4 = 0$ is a feasible solution(F.S) of the set of equations but not a basic feasible solution (B.F.S.).
$$2x_1 + 5x_2 - 3x_3 + x_4 = 4$$
$$6x_1 + 16x_2 - 9x_3 + 5x_4 = 14$$
Reduce the F.S. to a B.F.S.

7. Solve the L.P.P (without using simplex method).
Maximize, $z = x_1 - x_2 + 2x_3 + 3x_4$
subject to
$$2x_1 + x_2 + 3x_3 + 2x_4 = 11$$
$$3x_1 - 3x_2 + 5x_3 + x_4 = 17$$
$$x_j \geq 0,\ \ j = 1, 2, 3, 4.$$

8. Find a basic feasible solution, if there be any, of the following set of linearly independent equations and if such solution exists, taking that basis as an admissible basis, calculate all y_j, $z_j - c_j$ $[j = 1, 2, ..., 4]$ and the value of the objective function corresponding to that B.F.S. (without using simplex method)
Maximize, $z = 2x_1 - 4x_2 - x_3 + 4x_4$
subject to
$$3x_1 - 5x_2 + x_3 - 2x_4 = 7$$
$$6x_1 - 10x_2 - x_3 + 5x_4 = 11$$
$$x_j \geq 0,\ \ j = 1, 2, 3, 4.$$

9. Solve the following L.P.P. by graphical method.

(a) Maximize, $z = 4x_1 + 7x_2$
subject to
$$2x_1 + 5x_2 \leq 40$$
$$x_1 + x_2 \leq 11$$
$$x_2 \geq 4$$
$$x_1, x_2 \geq 0.$$

(b) Maximize, $z = 4x_1 + x_2$
subject to
$$x_1 + 2x_2 \leq 3$$
$$4x_1 + 3x_2 = 6$$
$$3x_1 + x_2 \geq 3$$
$$x_1, x_2 \geq 0.$$

(c) Minimize, $z = -2x_1 + x_2$
subject to
$$x_1 + x_2 \geq 6$$
$$3x_1 + 2x_2 \geq 16$$
$$x_2 \leq 9$$
$$x_1, x_2 \geq 0.$$

(d) Maximize, $z = 2x_1 - 6x_2$
subject to
$$3x_1 + 2x_2 \leq 6$$
$$x_1 - x_2 \geq -1$$
$$-x_1 - 2x_2 \geq 1$$
$$x_1, x_2 \geq 0.$$

10. Solve the following L.P.P. with the help of the simplex algorithm.

(a) Maximize, $z = 4x_1 + 7x_2$
subject to

$$2x_1 + x_2 \leq 1000$$
$$x_1 + x_2 \leq 600$$
$$-x_1 - 2x_2 \geq -1000$$
$$x_1, x_2 \geq 0.$$

(b) Maximize, $z = 2x_1 + 2x_2$
subject to

$$x_1 - x_2 \geq -1$$
$$-0.5x_1 + x_2 \leq 2$$
$$x_1, x_2 \geq 0.$$

(c) Maximize, $z = 3x_1 + 6x_2 + 2x_3$
subject to

$$3x_1 + 4x_2 + x_3 \leq 2$$
$$x_1 + 2x_2 + 3x_3 \leq 1$$
$$x_1, x_2, x_3 \geq 0.$$

(d) Maximize, $z = 4x_1 + 10x_2$
subject to

$$2x_1 + x_2 \leq 50$$
$$2x_1 + 5x_2 \leq 100$$
$$2x_1 + 3x_2 \leq 90$$
$$x_1, x_2 \geq 0.$$

(e) Maximize, $z = 5x_1 + 4x_2 + x_3$
subject to

$$6x_1 + x_2 + 2x_3 \leq 12$$
$$8x_1 + 2x_2 + x_3 \leq 30$$
$$4x_1 + x_2 - 2x_3 \leq 16$$
$$x_1, x_2, x_3 \geq 0.$$

11. Maximize, $z = x_1 + x_2$
subject to

$$2x_1 + 3x_2 \leq 22$$
$$2x_1 + x_2 \leq 14$$
$$x_1 - x_2 \leq 4$$
$$3x_1 - 2x_2 \geq -6$$
$$x_1, x_2 \geq 0.$$

Verify the results by using graphical method.

12. Solve the following L.P.P problem by the Charnes method of penalties and prove that the problem has a finite optimal solution and finite value of the objective function.
Minimize, $z = 3x_1 + 5x_2$
subject to

$$x_1 + 2x_2 \geq 8$$
$$3x_1 + 2x_2 \geq 12$$
$$5x_1 + 6x_2 \leq 60$$
$$x_1, x_2 \geq 0.$$

13. Solve the following problem by Big M-method

(a) Maximize, $z = -2x_1 + 5x_2$
subject to

$$4x_1 - 5x_2 \geq -20$$
$$x_1 + x_2 \geq 10$$
$$x_2 \geq 2$$
$$x_1, x_2 \geq 0.$$

(b) Minimize, $z = 2x_1 + 3x_2$
subject to

$$2x_1 + 7x_2 \geq 22$$
$$x_1 + x_2 \geq 6$$
$$5x_1 + x_2 \geq 10$$
$$x_1, x_2 \geq 0.$$

(c) Minimize, $z = 4x_1 + 2x_2$
subject to

$$3x_1 + x_2 \geq 27$$
$$x_1 + x_2 \geq 21$$
$$x_1 + 2x_2 \geq 30$$
$$x_1, x_2 \geq 0.$$

(d) Prove that, Maximize, $z = 2x_1 + x_2 - x_3 + 4x_4$
subject to

$$3x_1 - 5x_2 + x_3 - 2x_4 = 7$$
$$6x_1 - 10x_2 - x_3 + 5x_4 = 11$$
$$x_j \geq 0, \; j = 1, 2, 3, 4$$

has an unbounded solution.

(e) Prove that, Minimize, $z = 3x_1 - x_2 + 10x_3$
subject to

$$x_1 + 2x_2 + 3x_3 \leq 6$$
$$4x_1 + x_2 + x_3 = 32$$
$$2x_1 + x_2 + 2x_3 \geq 72$$
$$x_j \geq 0, \; j = 1, 2, 3$$

has no feasible solution.

(f) Maximize, $z = 2x_1 + 5x_2$
subject to

$$2x_1 + x_2 \geq 12$$
$$x_1 + x_2 \leq 4$$

$x_1 \geq 0$ and x_2 is unrestricted in sign.

14. Solve the following L.P.P by two phase method.

(a) Maximize, $z = 5x_1 + x_2 - 2x_3 + x_4$
subject to

$$x_1 + 5x_2 - 8x_3 + 3x_4 = 6$$

$$3x_1 - x_2 + x_3 + x_4 = 2$$

$$x_j \geq 0,\ j = 1, 2, 3, 4.$$

(b) Maximize, $z = 4x_1 + x_2$
subject to

$$3x_1 - x_2 \leq 10$$

$$2x_1 + x_2 \geq 20$$

$$3x_1 - x_2 = -14$$

$$x_j \geq 0,\ j = 1, 2.$$

(c) Maximize, $z = 2x_1 + 3x_2$
subject to

$$2x_1 - 3x_2 \leq 10$$

$$-2x_1 + x_2 \leq -14$$

$$3x_1 + 4x_2 = 17$$

$$x_1, x_2 \geq 0.$$

15. Find the dual problem for the following problem.
Maximize, $z = x_1 - x_2 - x_4$
subject to

$$2x_1 + x_2 - x_3 = 10$$

$$x_2 + x_3 - x_4 \leq 0$$

$$x_1 + x_3 + 2x_4 \geq 6$$

$x_1, x_2, x_4 \geq 0$ and x_3 is unrestricted in sign.

16. Solve the following equations by the simplex method.

$$x_1 + x_2 = 1$$

$$2x_1 + x_2 = 3$$

Best wishes