AI60003 Midsem Solution Sketches and Marking Scheme

Question 1a

Correct answer – Full. Wrong Answer = 0

Answer:

i. Clearly $U(a,b) \leq 0$. And $U(a,b) = 0$ if $a = b = 10$. The price of the bundle (10,10) is = 20. When income is = 40 the utility maximizing bundle is affordable and hence it is the optimal.

ii. When income is = 10 the optimization problem is simply

$$\min_{a,b} \ (10-a)^2 + (10-b)^2 \ \text{ such that } a+b = 10$$

$$\Rightarrow a^* = b^* = 5$$

Question 1b

Correct answer – Full. Wrong Answer = 0

Answer:

Aditya wants 20 hours & Gaurav wants 30 hours. Anything more orless leads to some disutility.

II Aditya is given 90 hours of work & Gaurav is given 10 hours.Aditya's utility = $(-70)^2 = 4900$, Gaurav's utility = $(-20)^2 = 400$

Now if the researcher takes 20 hours from Aditya & allocates that toGaurav

Then Aditya is given 70 hours of work & Gaurav is given 30 hours.Aditya's utility = $(-50)^2 = 2500$

Gaurav's utility = $(0)^2 = 0$

Both are better off. Thus II is NOT pareto optimal.

I on the other hand is pareto optimal, because a reduction is beneficial for both. Unlike in II where one wanted a reduction in hours & the other wanted an increment.

Question 1c

Correct answer – Full. Wrong Answer = 0

Answer: It'll succeed when the first choice of every girl & every guy inunique.

Example

A -> 1,2,3,4,5	1 -> A,B,C,D,E
B -> 2,1,3,4,5	2-> B,A,C,D,E
C -> 3,1,2,4,5	3 -> C,A,B,D,E
D-> 4,1,2,3,5	4 -> D,A,B,C,E
E -> 5,1,2,3,4	5 -> E,A,B,C,D

Or Every one has the same preference

Question 1d

1 mark deducted in case of calculation errors.

Answer:

The PUT – CALL parity equation:

$S_t + P_E\ (S_t\ ,\mathrm{T}) = C_E\ (S_t\ ,\mathrm{T}) + \mathrm{E}.\ e^{-r(T-t)}$

E = 100

r = 0.1

$S_t = 120$

$C_{100}\ (120,\mathrm{T}) + 100.\ e^{-0.1(2/365)} = 120 + P_{100}\ (120,\mathrm{T})$

$P_{100}\ (120,\mathrm{T}) \geq 0$

Hence $C_{100}\ (120,\mathrm{T}) \geq 120 - 100.\ e^{-0.1(2/365)}$

Question 1e

There are two measures to solve this: either by classification accuracy or by information gain. If it is the second, then IG has to be calculated for the whole tree, i.e. entropy in the root (full dataset) minus weighted sum of entropies in all the leaf nodes. For calculating entropy at one level of split only, **1 mark has been deducted.** For wrong construction of tree**, 1 mark has been deducted**, and **0.5 marks** for mistakes in counting the number of correct and wrong classifications (in case of classification accuracy)

<u>Question 2a</u>.

Briefly discuss the idea of graph colouring & chromatic number of agraph and how it is applied in scheduling problems. **[5 marks]**

Marks awarded based on the clarity of exposition

<u>Question 2b</u>

Fundamental Theorem of LP **[2 marks]:** the most important point here is that the optimal solution is one of the basic feasible solutions, but you must also define what are the basic feasible solutions. They are the "corners" of the region formed by intersection of the objective and the constraints, formally defined as at least 'n' out of the 'n+m' ('n' basic and 'm' slack) variables being 0. **Failure to mention these correctly have cost 0.5 or 1 mark**.

<u>Question 2c</u>

Interior point vs exterior point [3 marks]

Main points: initial solution (inside or outside feasible region), how constraints are incorporated into objective, and how the weighing factor of the constraints is increased/decreased in each iteration.

Marks given as par important points mentioned.

The most common error has been mixing up the properties of one with the other (eg. barrier function is used for interior point), **in which case 1 mark has been deducted**.

<u>Question 2d</u>

Main points of difference: in case of linear regression, the target variable y is expressed as a weighted sum of the features (predictors), the weights are learnt from data by minimizing the squared error loss function, and these weights determine the importance of features. In case of decision trees, the importance of features is measured by the information gain obtained by splitting the dataset according to them. These should be explained using the formulae.
Marks have been given according to clarity of explanation.

Question 3

No part marks in part a. Calculation errors have been considered. Marks deducted for lack of clarity of exposition.

Answer:

Period 1:

Expenditure on education = ‘e’

Period 2:

Earning = w.h & h = e. δ

Education Loan payment = e (1 + R)Consumption

= c_2 , Savings = s_2

⇨ $c_2 + s_2 + e(1+R) = w.h$

Period 3:

Consumption $c_3 = s_2\,(1+R)$ (No savings in Period 3)

i.e $c_2 + \frac{c_3}{1+r} + e\,(1 + R) = w.e.\,\delta$ .... *(i)*

So the optimization problem is:

$$\max_{c_2,\,c_3} \boldsymbol{U(c_2) + \beta\, U(c_3)}$$

s.t: $c_2 + \frac{c_3}{1+R} + e\,(1 + R) = w.e.\,\delta$

The Lagrangian

$$\mathbf{L} = \boldsymbol{U(c_2)} + \beta\, \mathbf{U(c_3)} + \lambda\,(w.e.\,\delta - c_2 - \frac{c_3}{1+R} - e\,(1 + R))$$

The first order conditions:

$$\frac{\partial L}{\partial c_2} = 0 \;\Rightarrow\; U'(c_2) = \lambda \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

$$\frac{\partial L}{\partial c_2} = 0 \;\Rightarrow\; \beta U'(c_3) = \frac{\lambda}{1+R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(iii)}$$

$$\frac{\partial L}{\partial \lambda} = 0 \;\Rightarrow\; \text{w.e.}\,\delta - c_2 - \frac{c_3}{1+R} - \text{e}\,(1+\text{R}) = 0 \quad \ldots\ldots\ldots\ldots\ldots \text{(iv)}$$

$$\frac{\beta U'(c_3)}{U'(c_2)} = \frac{1}{1+R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(v)}$$

Using (iv) & (v) find the optimal consumptions c_2^* & c_3^*

So the optimal utility U (c_2^* , c_3^*) = g (e) (a function of 'e')

Now compute the optimal 'e'

<u>Question 4</u>

1 mark has been given for construction of the tree, along with all nodes and edges. But the state variables on nodes and costs on edges must not be confused. The state variables are <economy size, number of infections>.

In part b, it is clearly mentioned that economic loss is the edge weight. In part c, it is a multi-objective situation, so <economic loss, new infections> will be the edge weight. For example, the root variable state is <100,10>. If no lockdown is imposed, the new state is <99,40>. In part ii) the cost here is 100-99=1, but in part iii) the cost is <1,30> (40-10=30). **Confusing these have resulted in deduction of 1-2 marks.**

This is a small graph, with only 8 paths from root to leaves. Some of the leaves are not goals (eg. in part ii the leaves where there are more than 100 infections are not goals, this number is 300 in part iii). But it is not fine to manually calculate the total cost of each path and compare them. The government does not have the full graph in front of them to choose the optimal policy, it unfolds month-by-month. That is why it is necessary to use heuristic search like A*. **Just comparing the path costs without A* have caused deduction of 2 marks in each of parts ii and iii.**

Part ii is A* with scalar edge cost, part iii is Multi-objective A* with vector edge cost. Accordingly, the 'f', 'g', 'h' functions too should be scalar in part ii and vector in part iii. **Confusing these two have resulted in deduction of 1-2 marks**.

Choice of heuristic function is in your hands, but 0 is not a good choice. But it must be admissible, i.e. it should not overestimate the optimal cost to goal node. **Choosing inadmissible loss functions results in deduction of 1 mark**.

Question 5

Calculation errors have been considered, and marks given for approach.

The major errors here are as follows:

$E(revenue) = E(R1(x)) + E(R2(100-x)) = 2x + 100-x = 100+x$, similarly $E(jobs) = 3x/2 + 2(100-x) = 200-x/2$. Some students have written $E(revenue)=2x+x=3x$ or $E(jobs)=3x/2+2x$, **for which 1 mark has been deducted**.

The Bayesian Network should involve all random variables: E, F, X, R1, R2, J1, J2, U. The structure is given below. For most students, only some of the variables have been considered without the conditional distributions on the edges. **This has resulted in deduction of 1-2 marks.**

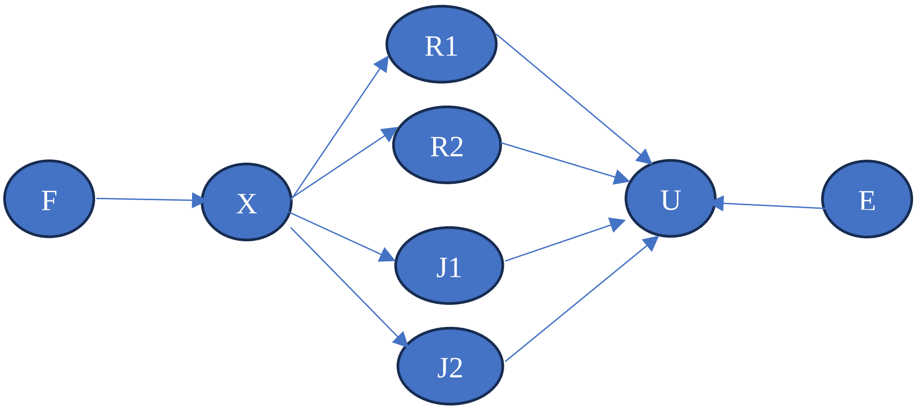

Table: $p(F=1)=0.3$, $p(F=0)=0.7$. $p(X=x|F=1) = (x-20)/30$, $p(X=x|F=0) = (x-50)/30$
$P(R1|X)= N(2X, 20)$, $P(J1|X) =U(X, 2X)$ etc
Also, $p(E=1)=0.6$, $p(E=0)=0.4$.
If $E=1$, $U= R1+R2+J1+J2$, if $E=0$ then $U=2*(J1+J2)$

In the last part, all 4 combinations of E and F must be considered (<E=1 F=0>, <E=0, F=1>, <E=F=1>, <E=F=0>) along with their respective probabilities. In each case, the expected revenue and jobs will be different, as specified in question. **Failure to consider these cases resulted in deduction of 1-2 marks.**