1) Primal

Max

$z = 4x_1 + 3x_2$

s.t.

$x_1 \le 6$

$x_2 \le 8$

$x_1 + x_2 \le 7$

$3x_1 + x_2 \le 15$

$-x_2 \le 1$

$x_1, x_2 \ge 0$

Dual

Minimize.

$z' = 6w_1 + 8w_2 + 7w_3 + 15w_4 + w_5$

st

$w_1 + w_3 + 3w_4 \ge 4$

$w_2 + w_3 + w_4 - w_5 \ge 3$

$w_1, w_2 \ge 0$

Standard Form.

Max

$z = -(6w_1 + 8w_2 + 7w_3 + 15w_4 + w_5) - M(a_1 + a_2)$

st

$w_1 + w_3 + 3w_4 - S_1 + a_1 = 4$

$w_2 + w_3 + w_4 - w_5 - S_2 + a_2 = 3$

		-6	-8	-7	-15	-1	0	0	-M	-M		
v_B	C_B	w_1	w_2	w_3	w_4 ↓	w_5	S_1	S_2	a_1	a_2	B	Min Ratio
a_1	-M	1	0	1	$\boxed{3}$	0	-1	0	1	0	4	4/3 →
a_2	-M	0	1	1	1	-1	0	-1	0	1	3	3
$z_j - c_j$		-M+6	-M+8	-2M+7	-4M+15	M+1	M	M	0	0		

		-6	-8	-7	-15	-1	0	0	-M	-M		
v_B	C_B	w_1	w_2 ↓	w_3	w_4	w_5	S_1	S_2	a_1	a_2	B	Min Ratio
w_4	-15	1/3	0	1/3	1	0	-1/3	0	1/3	0	4/3	∞
a_2	-M	-1/3	$\boxed{1}$	2/3	0	-1	+1/3	-1	-1/3	1	5/3	5/3 →
$z_j - c_j$		$\frac{M}{3}$	-M	$-\frac{2}{3}M$	0	M	$\frac{1}{3}M$	M	$\frac{4}{3}M$	0		

		-6	-8	-7	-15	-1	0	0	-M	-M		
v_B	C_B	w_1	w_2	w_3 ↓	w_4	w_5	S_1	S_2	a_1	a_2	B	Min Ratio
w_4	-15	1/3	0	1/3	1	0	-1/3	0	$\frac{1}{3}$	0	4/3	4
w_2	-8	$-\frac{1}{3}$	1	$\boxed{2/3}$	0	-1	1/3	-1	$-\frac{1}{3}$	1	5/3	5/2 →
$z_j - c_j$		11/3	0	$-\frac{10}{3}$	0	9	7/3	8	M	M		

		-6	-8	-7	-15	-1	0	0	
v_B	C_B	w_1	w_2	w_3	w_4	w_5	S_1	S_2	B
w_4	-15	$\frac{1}{2}$	$-\frac{1}{2}$	0	1	+.75	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
w_3	-7	$-\frac{1}{2}$	3/2	1	0	-3/2	1/2	-3/2	5/2
$z_j - c_j$		2	5	0	0	.25	$\boxed{15}$	$\boxed{3}$	

$\forall j \; z_j - c_j \ge 0 \Rightarrow$ Optimality Reached.

$\Rightarrow z'^{*} = +\left(-15 \times \frac{1}{2} - 7 \times \frac{5}{2}\right) = \left(\frac{15+35}{2}\right) = 25$

at $w_1 = 1 \quad w_2 = 0 \quad w_3 = \frac{5}{2} \quad w_4 = \frac{1}{2} \quad w_5 = 0$

OR $x_1 = 4 \quad x_2 = 3$

b_2 $C_B B^{-1} A - C \geq$ & $x_B \geq 0$

$\downarrow$

$\cdot B^{-1} b \geq 0$

$$\begin{bmatrix} 1 & .25 & 2.75 \\ 0 & .25 & -.25 \\ 0 & .25 & .75 \end{bmatrix} \begin{bmatrix} 3 \\ b_2 \\ 2 \end{bmatrix} \cancel{b \geq 0} = \begin{bmatrix} \cancel{\geq} \\ 1 \\ 3 \end{bmatrix}$$

$\cancel{3 + \frac{1}{4} b_2 + 5.5 \geq 0 \Rightarrow b_2 \geq -34}$

$\cancel{\frac{1}{4} b_2 - \frac{1}{2} \geq 0 \Rightarrow b_2 \geq 2}$

$\cancel{\frac{1}{4} b_2 + 1.5 \geq 0 \Rightarrow b_2 \geq -6}$

$\cancel{\Rightarrow \boxed{b_2 \geq 2}}$

$3 + \frac{1}{4} b_2 + 5.5 \geq 0 \quad b_2 \geq -34$

$\frac{1}{4} b_2 - \frac{1}{2} = \frac{1}{4} \Rightarrow b_2 = 6$

$\frac{1}{4} b_2 + 1.5 = 3$

$b_2 = 6$

$\Rightarrow \boxed{b_2 = 6}$

9)

4) Max $Z = 2x_1 - 3x_2$

$x_1 - x_2 \leq 2$

$5x_1 + 4x_2 \leq 46$

$7x_1 + 2x_2 \geq 32$

$x_1, x_2 \geq 0$

$Z' - 2x_1 + 3x_2 - Ma_1 = 0$

$x_1 - x_2 + S_1 = 2$

$5x_1 + 4x_2 + S_2 = 46$

$7x_1 + 2x_2 - S_3 + a_1 = 32$

v_B	β_0	β_1	β_2	β_3	x_R	x_B	Ratio	x_1 ↓	x_2	S_3
Z	1	0	0	-M	-7M-2	0	—	-2	3	0
S_1	0	1	0	0	1	2	2 →	1	-1	0
S_2	0	0	1	0	5	46	9.2	5	4	0
a_1	0	0	0	1	7	32	4 4/7	7	2	-1
							Delta	-7M	-2M	M

$$x_R = \begin{bmatrix} 1 & 0 & 0 & -M \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 5 \\ 7 \end{bmatrix}$$

v_B	β_0	β_1	β_2	β_3	x_R	x_B	Ratio	S_1	x_2 ↓	S_3
Z	1	7M+2	0	-M	-9M+1	14M+4	—	0	3	0
x_1	0	1	0	0	-1	2	—	1	-1	0
S_2	0	-5	1	0	9	36	4	0	4	0
a_1	0	-7	0	1	9	18	2 →	0	2	-1
							Delta	7M	-7M	M

$$x_R = \begin{bmatrix} 1 & 7M+2 & 0 & -M \\ 0 & 1 & 0 & 0 \\ 0 & -5 & 1 & 0 \\ 0 & -7 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ 4 \\ 2 \end{bmatrix}$$

v_B	β_0	β_1	β_2	β_3	x_R	x_B	Ratio	S_1	a_1	S_3
Z	1	2.75	0	-0.5		32M+2		0	M	0
x_1	0	.2	0	.5		4		1	0	0
S_2	0	2	1	-1	1	18		0	0	0
x_2	0	-.5	0	.5		2		0	1	-1
							Delta	2.778	M	.5

Delta > 0

$\Rightarrow$ Optimality reached

$\Rightarrow$ Optimal sol^n = $x_1 = 4$, $x_2 = 2$
at $Z = 2$

5)

Max $z = -6x_1 - 11x_2$

s.t.

$-x_1 - x_2 \le -11$

$-2x_1 - 5x_2 \le -40$

Max $z = -6x_1 - 11x_2$

st

$-x_1 - x_2 + S_1 = -11$

$-2x_1 - 5x_2 + S_2 = -40$

		-6	-11	0	0		
C_B	ϑ_B	x_1	x_2	S_1	S_2	x_B	~~Ratio~~
0	S_1	-1	-1	1	0	-11	
0	S_2	-2	$\boxed{-5}$	0	1	-40	→
$z_i - c_j$		6	11	0	0		
$\frac{z_i - c_j}{x_R}$		-3	-2.2				

		-6	-11	0	0		
ϑ_B	C_B	x_1	x_2	S_1	S_2	x_B	
S_1	0	-.6	0	1	.2	-3	→
x_2	-11	.4	1	0	-.2	8	
$z_i - c_j$		1.6	0	0	2.2		
$\frac{z_i - c_j}{x_R}$		-2.6	—	—	—		

		-6	-11	0	0	
ϑ_B	C_B	x_1	x_2	S_1	S_2	x_B
x_1	-6	1	0	-1.6	-.3	5
x_2	-11	0	1	.64	.06	6
$z_i - c_j$		0	0	+ve	+ve	

∴ optimality reached.

$x_1 = 5$

$x_2 = 6$

$z = 96$

8)

Q ⇒ Max $Z = x_1 + 4x_2 - 2x_3 + 3x_4 + x_5$

st. $x_1 - 3x_2 + x_3 + 2x_4 + 6x_5 \le 3$

$2x_1 + x_2 + 3x_4 + 2x_5 \le 6$

$4x_1 + x_2 - x_4 + x_5 \le 2.$

Final Simplex Table.

		1	4	-2	3	1	0	0	0	
v_B	C_B	x_1	x_2	x_3	x_4	x_5	S_1	S_2	S_3	x_B
S_1	0	12.5	0	1	0	9.25	1	.25	2.75	10
x_4	3	-.5	0	0	1	.25	0	.25	-0.25	1
x_2	4	3.5	1	0	0	1.25	0	.25	.75	3

$\frac{1}{4} - \frac{2}{4} =$

i) c_1 x_1 is not part of basis

$\cancel{z_1 - c_1 \ge 0}$

$\cancel{12.5 - c_1 \ge 0}$

$12.5 - c_1 \ge 0$

$\cancel{c_1 \ge}$ $\boxed{c_1 \le 12.5}$

ii) c_3

x_3 is not part of basis

$z_3 - c_3 \ge 0$

$\Rightarrow -c_3 \ge 0$

$\boxed{c_3 \le 0}$

iii) c_4 x_4 is part of basis

$z_1 - c_1 \ge 0$ → $C_B \underbrace{B^{-1}A_1}_{\text{given}}$ $-\frac{1}{2}c_4 + \cancel{14} \overset{13}{} - \cancel{1} \ge 0$

$\boxed{26 \ge c_4}$

$z_3 - c_3 \ge 0$ → sat

$z_5 - c_5 \ge 0$ → $\frac{1}{4}c_4 + \frac{4}{5} - 1 \ge 0$

$z_7 - \cancel{c_7} \ge 0$ $\boxed{c_4 \ge -16}$

$z_8 - \cancel{c_8} \ge 0$ → $\frac{1}{4}c_4 + 1 \ge 0$ → $\boxed{c_4 \ge -4}$

→ $-\frac{1}{4}c_4 + 3 \ge 0$ $\boxed{c_4 \le 12}$

$\Rightarrow c_4 \in [-4, 12]$

ϑ_B	β_0	β_1	β_2	β_3	x_B	x_k	Ratio
z	1	1	0	2	22		
x_1	0	1	0	0	10		
S_2	0	-2	1	-1	-18		
x_3	0	0	0	1	6		

	a_1	a_2	a_3
	0	0	-3
	1	0	2
	0	0	0
	0	1	2
Δ			

ϑ_B	β_0	β_1	β_2	β_3	x_B	x_k	Ratio
z	1	0	$+\frac{1}{2}$	1.5	13	0	—
x_1	0	$1/3$	$1/3$	$-1/3$	4	$1/3$	$4/3$
x_3	0	$-2/3$	$1/3$	$2/3$	0	$-\frac{2}{3}$	—
x_2	0	$1/3$	$1/6$	$1/6$	3	$\boxed{\frac{1}{3}}$	1 →

	a_1	a_2	a_3
	0	0	0
	1	0	0
	0	0	1
	0	1	0
Δ	0	1.5	$\frac{1}{2}$

since $\Delta a_1 = 0$

Alternative Optima exists

$$x_k = \begin{bmatrix} 1 & 0 & \frac{1}{2} & 1.5 \\ 0 & \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \\ 0 & -\frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = [0, \tfrac{1}{3}, -\tfrac{2}{3}, \tfrac{1}{3}]$$

	β_0	β_1	β_2	β_3	x_B	x_k	Ratio
z	1	0	$\frac{1}{2}$	1.5	13		
x_1	0				1		
x_3	0				6		
[illegible]	0	$2/3$	$1/2$	$1/2$	9		

	a_1	a_2	a_3
	-3	0	0
	2	0	0
	0	0	1
	2	1	0
Δ	0	1.5	.5

$z^* = 13$

at $x_1 = 1,\ x_3 = 6,\ x_2 = 0$

OR

$x_1 = 4,\ x_2 = 3,\ x_3 = 0$

3) Max. (Standard)

$$z - x_1 - 3x_2 - 2x_3 = 0$$
$$x_1 + 2x_2 + S_1 = 10$$
$$2 \cdot x_1 + x_3 + S_2 = 8$$
$$2x_2 + x_3 + S_3 = 6$$

V_B	β_0	β_1	β_2	β_3	x_B	x_k	Ratio
z	1	0	0	0	0	-3	—
S_1	0	1	0	0	10	2	5
S_2	0	0	1	0	8	0	—
S_3	0	0	0	1	6	$\boxed{2}$	3 →

.	a_1	a_2 ↓	a_3
	-1	-3	-2
	1	2	0
	2	0	1
	0	2	1
Δ	-1	-3	-2

$$x_R = \begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{bmatrix} \times [-3, 2\ 0\ 2]^T = \begin{bmatrix} -3 \\ 2 \\ 0 \\ 2 \end{bmatrix}$$

β_1	β_2	β_3	x_B	x_R
0	0	0	0	-3
1	0	0	10	2
0	1	0	8	0
0	0	1	6	$\boxed{2}$

V_B	β_0	β_1	β_2	β_3	x_B	x_R	Ratio
z	1	0	0	$\frac{3}{2}$	9	-1	—
S_1	0	1	0	-1	4	$\boxed{1}$	4 →
S_2	0	0	1	0	8	2	4
x_2	0	0	0	$\frac{1}{2}$	3	0	—

	a_1 ↓	a_2	a_3
	-1	0	-2
	1	0	0
	2	0	1
	0	1	1
Δ	-1	$\frac{3}{2}$	$-\frac{1}{2}$

$$x_R = \begin{bmatrix} 1&0&0&\frac{3}{2} \\ 0&1&0&-1 \\ 0&0&1&0 \\ 0&0&0&\frac{1}{2} \end{bmatrix}_{4\times4} \times \cancel{[-1, 1, 2, 0]^T} \begin{bmatrix} -1 \\ 1 \\ 2 \\ 0 \end{bmatrix} = \cancel{\begin{bmatrix} -1 \\ 1 \\ 2 \\ 0 \end{bmatrix}}$$

$$= [-1, 1, 2, 0]^T$$

V_B	β_0	β_1	β_2	β_3	x_B	x_R	Ratio
z	1	1	0	.5	13	-1.5	—
x_1	0	1	0	-1	4	-1	—
S_2	0	-2	1	2	0	$\boxed{3}$	0 →
x_2	0	0	0	$\frac{1}{2}$	3	.5	6

	a_1	a_2	a_3 ↓
	0	0	-2
	1	0	0
	0	0	1
	0	1	1
Δ	1	$\frac{1}{2}$	-1.5

$$x_R = \begin{bmatrix} 1&1&0&\frac{1}{2} \\ 0&1&0&-1 \\ 0&-2&1&2 \\ 0&0&0&\frac{1}{2} \end{bmatrix} \times \begin{bmatrix} -2 \\ 0 \\ 1 \\ 1 \end{bmatrix} = [-1.5, -1, 3, \tfrac{1}{2}]$$

a) Primal.

Minimize $Z = x_1 + x_2$

st.

$-(x_1 + 2x_2) \leq -12$

$-(5x_1 + 6x_2) \leq -48$

$x_1, x_2 \geq 0$

Dual

~~Max~~ Min.

$Z' = -12w_1 - 48w_2$

st

$-w_1 - 5w_2 \geq -1$

$-2w_1 - 6w_2 \geq -1$

$w_1, w_2 \geq 0$

Standard form.

Max

$Z' = +12w_1 + 48w_2$

st

$w_1 + 5w_2 \leq +1$

$2w_1 + 6w_2 \leq +1$

$w_1, w_2 \geq 0$

Max $Z = +12w_1 + 48w_2$

$w_1 + 5w_2 + S_1 = +1$

$2w_2 + 6w_2 + S_2 = 1$

$w_1, w_2 \geq 0$

		+12	+48	0	0		
VB	CB	w_1	w_2 ↓	S_1	S_2	B	
S_1	0	1	5	1	0	1	1/5
S_2	0	2	$\boxed{6}$	0	1	1	$\frac{1}{6}$ →
$z_j - c_j$		-12	-48	0	0		

		12	48	0	0	
VB	CB	w_1	w_2	S_1	S_2	B
S_1	0	$-2/3$	0	1	$-5/6$	$\frac{1}{6}$
w_2	+48	$1/3$	1	0	$1/6$	$\frac{1}{6}$
$z_j - c_j$		4	0	0	8	

$\forall j \; z_j - c_j \geq 0 \Rightarrow$ Optimality reached.

$\Rightarrow Z' = 0 + \frac{48}{6}\cdot\frac{8}{16} = \cancel{16}\,8$

at

$w_1 = 0, w_2 = \frac{1}{6}$

OR

$x_1 = 0, x_2 = 8$

~~x_2~~

Gromory Constraint:

$$x_G = -\frac{1}{3} + \frac{1}{3}S_1 + \frac{1}{3}S_2$$

Additional Constraint $-\frac{1}{3}S_1 - \frac{1}{3}S_2 + x_G = -\frac{1}{3}$

v_B	C_B	x_1 (1)	x_2 (1)	S_1 (0)	S_2 (0)	x_G (0)	x_B
x_1	1	1	0	$\frac{1}{3}$	$-\frac{2}{3}$	0	$\frac{1}{3}$
x_2	1	0	1	0	1	0	2
x_G	0	0	0	$-\frac{1}{3}$	$-\frac{1}{3}$	1	$-\frac{1}{3}$ →
$z_i - c_i$		0	0	$\frac{1}{3}$	$\frac{1}{3}$	0	
				-1	-1		

v_B	C_B	x_1 (1)	x_2 (1)	S_1 (0)	S_2 (0)	x_G (0)	x_B	Ratio
x_1	1	1	0	0	-1	1	0	–
x_2	1	0	1	0	0	0	2	–
S_1	0	0	0	1	1	-3	1	1 →
$z_i - c_i$		0	0	0	-1	1		

v_B	C_B	x_1	x_2	S_1	S_2	x_G	x_B
x_1	1	1	0	1	0	-2	1
x_2	1	0	1	0	0	0	2
S_2	0	0	0	1	1	-3	1

v_B	C_B	x_1 (1)	x_2 (1)	S_1 (0)	S_2 (0)	x_G (0)	x_B
x_1	1	1	0	1	0	-2	1
x_2	1	0	1	-1	0	3	1
S_2	0	0	0	1	1	-3	1
$z_i - c_i$		0	0	0	0	1	

$\Rightarrow$ Optimality reached.

$\Rightarrow x_1 = 1,\ x_2 = 1 \quad z = 2.$

9) Since Problem is maximization. to convert to Minization $a_{ij} = -a_{ij}$

-62	-78	-50	-101	-82	
-71	-84	-61	-73	-59	
-87	-92	-111	-71	-81	
-48	-64	-87	-77	-80	
0	0	0	0	0	→ Artificial Row

i) Row Minima.

25	12	61	0	0
16	8	50	28	23
0	0	0	30	1
39	28	24	24	2
87	92	111	101	82

ii) Column Minima.

25	12	61	0	0
8	0	42	20	15
0	0	0	30	1
37	26	22	22	0
$\boxed{5}$	10	29	19	0

$n = 4$

we need '5'

iii)

25	17	61	$\boxed{0}$	5
3	$\boxed{0}$	37	25	15
0	5	$\boxed{0}$	30	6
32	26	17	17	$\boxed{0}$
$\boxed{0}$	10	29	14	0

$n = 5$

$\Rightarrow$ Optimal sol^n exists

Ans = 101 + 84 + 111 + 80

= 212 + 164 = 376

10) Minimize. ~~Maximize~~. Minize

-14	-6	-22	-11	-6
-18	-22	-14	-15	-9
-18	-12	-9	-12	-12
-10	-22	-16	-22	-8
-16	-16	-14	-10	-10

i) Row Minima

4	16	0	11	6
0	0	8	7	3
0	10	13	10	0
8	0	6	0	4
2	6	8	12	2

ii) Coulumn Minima.

4	16	$\boxed{0}$	11	6
0	$\boxed{0}$	8	7	3
0	10	13	10	$\boxed{0}$
8	0	6	$\boxed{0}$	4
$\boxed{0}$	4	6	10	0

$n=5$

$\Rightarrow$ Optimality reached.

$Cost = -22 -22 -12 -22 - 16$

$= -44 - 34 - 16$

$= -60 - 34 = -94.$

11)

				u_i
$8-\theta$ (9)	$4-\delta$ (8)	$8+\theta$ (5)	(7)	0
(4)	14 (6)	8 (8)	(7)	-2
$+\theta$ (5)	(8)	$13-\theta$ (9)	3 (5)	4
v_j: 9	8	5	1	

$n+m-1 = 6$

$BV = 5 \Rightarrow$ Degenerate Solution.

Put δ at Non-cyclic least cost Block

$d_{ij} = c_{ij} - z_{ij}$ $\quad z_{ij} = (u_i + v_j)$

$d_{14} = 7 - (0+1) = 6$

$d_{21} = 4 - (-2+9) = -3$

$d_{23} = 8 - (-2+5) = 5$

$d_{24} = 7 - (-2+1) = 8$

$d_{31} = 5 - (4+9) = -8 \rightarrow$

$d_{32} = 8 - (4+8) = -4$

$\Rightarrow \theta = 8$

				u_i
(9)	$4-\theta$ (8)	$8+\theta$ (5)	(7)	-4
(4)	14 (6)	(8)	(7)	-6
8 (5)	$+\theta$ (8)	$5-\theta$ (9)	3 (5)	0
v_j: 5	12	9	5	

$n+m-1 = 6$

$BV = 6 \Rightarrow$ Sol^n is non-degenerate

$d_{11} = 9 - (-4+5) = 8$

$d_{14} = 7 - (-4+5) = 6$

$d_{21} = 4 - (-6+9) = 5$

$d_{23} = 8 - (-6+9) = 5$

$d_{24} = 7 - (-6+5) = 8$

$d_{32} = 8 - (0+12) = -4 \rightarrow$

$\Rightarrow \theta = 4.$

				u_i
(9)	0 (8)	12 (5)	(7)	-4
(4)	14 (6)	(8)	(7)	-2
8 (5)	4 (8)	1 (9)	3 (5)	0
v_j: 5	8	9	5	

$d_{11} = 9 + 4 - 5 = 8$

$d_{12} = 8 + 4 - 8 = 4$

$d_{21} = 4 - 5 + 2 = 1$

$d_{23} = 8 + 2 - 9 = 1$

$d_{24} = 7 + 2 - 5 = 4$

$d_{14} = 7 + 4 - 5 = 6$

$\forall_{i,j}\ d_{ij} \geq 0 \Rightarrow$ optimality reached.

END

12

			v_j
0 [5]	10 [1]	0 [7]	-3
60 [6]	100 [4]	10 [6]	0
15 [3]	0 [2]	0 [5]	-3
0 [5]	0 [3]	40 [2]	-4
u_i: 6	4	6	

$d_{11} = 5-(-3+6) = 2$

$d_{13} = 7-(-3+6) = 4$

$d_{32} = 2-(-3+4) = 1$

$d_{34} = 5-(-3+6) = 2$

$d_{41} = 5-(-4+6) = 3$

$d_{42} = 3-(-4+4) = 3$

$\Rightarrow$ Optimality reached.

13) i) Row Minima.

R_1	R_2	R_3	R_4	R_5
		7		7
8		1		
	3		13	1

$\text{cost} = 7\times0 + 7\times6 + 8\times1 + 1\times-3 + 3\times-1 + 13\times0 + 1\times5$

$= 42 + \cancel{8-3-3}^{2} + 5$

$= 49$

ii) Column Minima.

C_1			8	6	
C_2	8	1			
C_3		2	0	7	8

$\text{cost} = 8\times0 + 6\times3 + 8\times1 + 1\times2 + 2\times-1 + 7\times0 + 8\times5$

$= 18 + 8 + \cancel{2-2} + 40$

$= 66$

iii) Matrix Minima

6				8
1		8		
1	3		13	

$\text{cost} = 6\times4 + 8\times6 + 1\times1 + 8\times-3 + 1\times3 + 3\times-1 + 13\times0$

$= \cancel{24} + 48 + 1 - \cancel{24} + 0$

$= 49$

iv) Northwest

8	3	3		
		5	4	
			9	8

$C = 8\times4 + 7\times3 + 0\times3$
$+ 5\times-3 + 4\times3$
$+ 9\times0 + 8\times5$
$= \cancel{24}^{21} + 21 + 0 \; \cancel{-15}^{-3} + 12 + 40$
$= 42 + 40 = 82$

v) VAM

6 $\boxed{4}$	0 $\boxed{7}$	0 $\boxed{0}$	0 $\boxed{3}$	8 $\boxed{6}$	2
1 $\boxed{1}$	0 $\boxed{2}$	8 $\boxed{-3}$	0 $\boxed{3}$	0 $\boxed{8}$	$\to$ 2
1 $\boxed{3}$	3 $\boxed{-1}$	0 $\boxed{4}$	13 $\boxed{0}$	0 $\boxed{5}$	2
				1	

$Cost = 6\times4 + 8\times6$
$+ 1\times1 + 8\times-3$
$+ 1\times3 + 3\times-1 + 13\times0$
$= \cancel{24} + 48 + 1 - \cancel{24} + 0$
$= 49$

14) Cutting Plane.

$Z = x_1 + x_2$

$3x_1 + 2x_2 \le 5$

$x_2 \le 2$

$x_1, x_2 \in$ integers.

		1	1	0	0		
y_B	C_B	x_1	x_2	S_1	S_2	x_B	
S_1	0	3	2	1	0	5	$5/3$
S_2	0	0	1	0	1	2	—
$z_i - c_i$		-1	-1	0	0		

		1	1	0	0		
y_B	C_B	x_1	x_2	S_1	S_2	x_B	Ratio
x_1	1	1	$2/3$	$1/3$	0	$5/3$	2.5
S_2	0	0	1	0	1	2	2
$z_i - c_i$		0	$-\frac{1}{3}$	$\frac{1}{3}$	0		

y_B	C_B	x_1	x_2	S_1	S_2	x_B
x_1	1	1	0	$\frac{1}{3}$	$-2/3$	$\frac{1}{3}$
x_2	1	0	1	0	1	2
$z_i - c_i$		0	0	$\frac{1}{3}$	$\frac{1}{3}$	

$\frac{5}{3} - \frac{4}{3}$

$\Rightarrow$ optimality reached but x_1 is not int.

15) Max $Z = -x_1 + 4x_2$

$$-10x_1 + 20x_2 \le 22$$
$$5x_1 + 10x_2 \le 49$$
$$x_1 \le 5$$
$$x_1, x_2 \in In^+$$

- $Z^* = 8.2$, $x_1 = 3.8$, $x_2 = 3$
  - $x_1 \ge 4$: $Z^* = 7.6$, $x_1 = 4$, $x_2 = 2.9$
    - $x_2 \ge 3$: —
    - $x_2 \le 2$: $Z^* = 4$, $x_1 = 4$, $x_2 = 2$
  - $x_1 \le 3$: $Z^* = 7.4$, $x_1 = 3$, $x_2 = 2.6$
    - $x_2 \ge 3$: —
    - $x_2 \le 2$: $Z^* = 6.2$, $x_1 = 1.8$, $x_2 = 2$
      - $x_1 \ge 2$: $Z^* = 6$, $x_1 = 2$, $x_2 = 2$ ✓
      - $x_1 \le 1$: $Z^* = 5.4$, $x_1 = 1$, $x_2 = 1.6$

Ans $Z^* = 6$
$x_1 = 2$
$x_2 = 2$.

16

	R_1	R_2	R_3	R_4	R_5
	∞	12	15	10	8
	8	∞	15	12	8
	9	11	∞	15	11
	7	12	19	∞	11
	9	12	16	10	∞

Row Operation

	A	B	C	D	E
A	∞	1	0	$\boxed{0}$	0
B	1	∞	0	2	0
C	2	$\boxed{0}$	∞	5	3
D	$\boxed{0}$	1	4	∞	3
E	2	1	1	$\boxed{0}$	∞

$n = 5$

No. of lines = 5

$\Rightarrow$ Sol^n exists

Coulum op → gives same as every colum has atleast '1' zero

sol^n

C → B
D → A
E → D

Case-I ✓
A → E
B → E
A → C → B → E → D → A

Case-II
A → E ✗
B → C

A → E → D → A
B → C → B

Ans Cost = 15 + 11 + 8 + 10 + 7
= 15 + 11 + 15 + 10 21 = 51.