Optimization Techniques
Assignment 1

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Q. 1→ cost of A = 300 let x_1 of A be sold
cost of B = 200 let x_2 of B be sold
cost of C = 100 let x_3 of C be sold

∴ max, (profit) $Z = 300x_1 + 200x_2 + 100x_3$
subject to

$$40x_1 + 70x_2 + 50x_3 \le 1000$$
$$30x_1 + 14x_2 + 18x_3 \le 500$$
$$7x_1 + 0x_2 + 8x_3 \le 200$$
$$4x_1 + 9x_2 + 0x_3 \le 200$$
$$x_1, x_2, x_3 \ge 0$$

Q. 2.

→ ~~percent~~ let A alloys be $= x_1$
let B alloys be $= x_2$
let C alloys be $= x_3$
let D alloys be $= x_4$

min $Z = 11x_1 + 12x_2 + 16x_3 + 14x_4$
Subject to

$$30x_1 + 35x_2 + 50x_3 + 40x_4 = 40$$
$$60x_1 + 35x_2 + 50x_3 + 45x_4 = 50$$
$$10x_1 + 30x_2 + 0x_3 + 15x_4 = 10$$
$$x_1, x_2, x_3, x_4 \ge 0$$
$$x_1, x_2, x_3, x_4 \le 1$$

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Q.3.

→ (a)

C_B	B	X_B	b	a_1	a_2	a_3	a_4	a_5	a_6
			c_j	-4	-2	0	0	0	$-M$
-4	a_1	$\cancel{x_1}$	$24/5$	1	0	$-2/5$	0	$1/5$	0
$-M$	a_6	x_6	$18/5$	0	0	$1/5$	-1	$2/5$	1
-2	a_2	x_2	$63/5$	0	1	$1/5$	0	$-3/5$	0
			$z_j - c_j$	0	0	$\frac{-M+6}{5}$	M	$\frac{-2M+2}{5}$	0

(b) x_5 is entering variable

$$\min \left\{ \left(\frac{24}{5}\right) \Big/ \left(\frac{1}{5}\right), \left(\frac{18}{5}\right) \Big/ \left(\frac{2}{5}\right) \right\}$$

$$\min \{ 24, 9 \}$$

So x_6 is leaving variable

(c)

C_B	$\cancel{X}_B$	X_B	b	a_1	a_2	a_3	a_4	a_5	a_6
			c_j	-4	-2	0	0	0	$-M$
-4	a_1	x_1	3	1	0	$-1/2$	$1/2$	0	$-1/2$
0	a_5	x_5	9	0	0	$1/2$	$-5/2$	1	$5/2$
-2	a_2	x_2	18	0	1	$1/2$	$-3/2$	0	$3/2$
			$z_j - c_j$	0	0	1	1	0	$M-1$

as $z_j - c_j \geq 0$

optimal solution is at $x_1 = 3, x_2 = 18$

Q.4.
→

Q.4.

→ Using simplex Method inverse of matrix :

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = A$$

let $b^T = [\,1 \quad 1 \quad 1\,]$

$$Z = x_1 + x_2 + x_3 - Mx_4 - Mx_5 - Mx_6$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$x_1 - x_2 + x_3 + x_4 = 1$$

$$\cancel{2x_1 - x_2 + x_5 = 1}$$

$$2x_1 - x_2 + 0x_3 + 0x_4 + x_5 = 1$$

$$x_1 + 0x_2 + 0x_3 + 0x_4 + 0x_5 + x_6 = 1$$

		c_j		1	1	1	-M	-M	-M
CB	B	XB	b	a_1	a_2	a_3	a_4	a_5	a_6
-M	a_4	x_4	1	1	-1	1	1	0	0
-M	a_5	x_5	1	2*	-1	0	0	1	0
-M	a_6	x_6	1	1	0	0	0	0	1
		$z_j - c_j$		$-(4M+1)$	$2M-1$	$-(M+1)$	0	0	0

		c_j		1	1	1	-M	-M	-M
CB	B	XB	b	a_1	a_2	a_3	a_4	a_5	a_6
-M	a_4	x_4	1/2	0	-1/2	1 *	1	-1/2	0
1	a_1	x_1	1/2	1	-1/2	0	0	1/2	0
-M	a_6	x_6	1/2	0	1/2	0	0	-1/2	1
		$z_j - c_j$		0	-3/2	$-(M+1)$	0	$2M+1/2$	0

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			c_j	1	1	1	$-M$	$-M$	$-M$
C_B	B	X_B	b	a_1	a_2	a_3	a_4	a_5	a_6
1	a_3	x_3	$1/2$	0	$-1/2$	1	1	$-1/2$	0
1	a_1	x_1	$1/2$	1	$-1/2$	0	0	$1/2$	0
$-M$	a_6	x_6	$1/2$	0	$1/2$*	0	0	$-1/2$	1
			$z_j - c_j$	0	$-\left(\frac{M}{2}+2\right)$	0	$1+M$	$\frac{3M}{2}$	0

			c_j	1	1	1	$-M$	$-M$	$-M$
C_B	B	X_B	b	a_1	a_2	a_3	a_4	a_5	a_6
1	a_3	x_3	1	0	0	1	1	-1	1
1	a_1	x_1	1	1	0	0	0	0	1
1	a_2	x_2	1	0	1	0	0	-1	2
			$z_j - c_j$						

			c_j	1	1	1	$-M$	$-M$	$-M$
C_B	B	X_B	b	a_1	a_2	a_3	a_4	a_5	a_6
1	a_1	x_1	1	1	0	0	0	0	1
1	a_2	x_2	1	0	1	0	0	-1	2
1	a_3	x_3	1	0	0	1	1	-1	1

$$\therefore \quad A^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$$

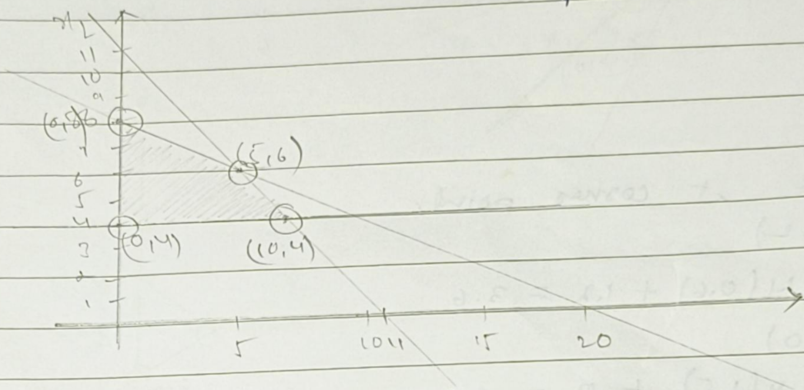
Ans-9 a) Max $z = 4x_1 + 7x_2$

s.t. $2x_1 + 5x_2 \le 40$

$x_1 + x_2 \le 11$

$x_2 \ge 4$

$x_1, x_2 \ge 0$



Value at corner points.

(i) (0,8)

$z = 4(0) + 7(8) = 56$

(ii) (0,4)

$z = 4(0) + 7(4) = 28$

(iii) (10,4)

$z = 4(10) + 7(4) = 68$ ✓

(iv) (5,6)

$z = 4(5) + 7(6) = 62$

$\boxed{z^* = 68}$ at $x_1 = 10,\ x_2 = 4$

Ans 9 b) Max $z = 4x_1 + x_2$

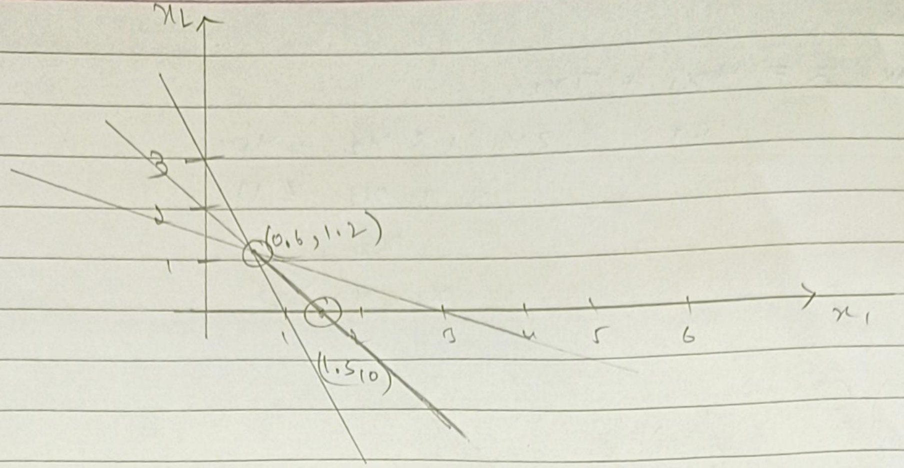
s.t.

$x_1 + 2x_2 \le 3$

$4x_1 + 3x_2 = 6$

$3x_1 + x_2 \ge 3$

$x_1, x_2 \ge 0$



value at corner point,

(i) (0.6, 1.2)

$z = 4(0.6) + 1.2 = 3.6$

(ii) (1.5, 0)

$z = 4(1.5) + 0 = 6$

$\boxed{z^* = 6}$ at $x_1 = 1.5,\ x_2 = 0$

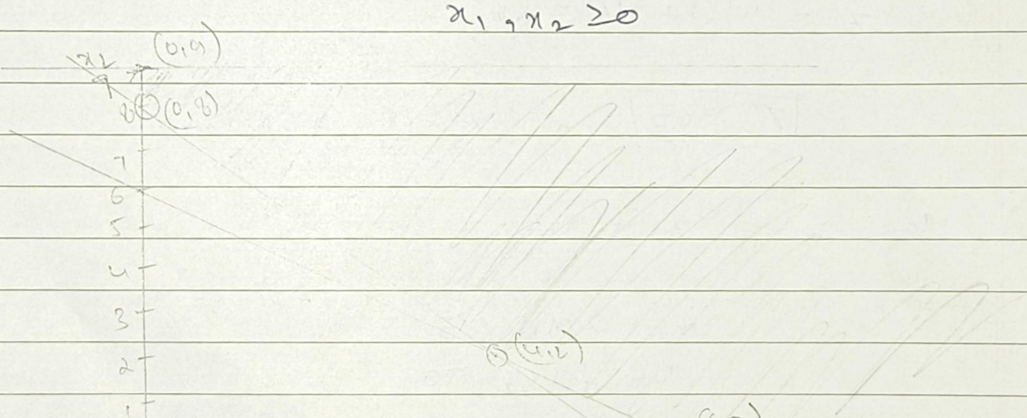
Ans-9 c) Minize, $z = -2x_1 + x_2$

s.t. $x_1 + x_2 \geq 6$

$3x_1 + 2x_2 \geq 16$

$x_2 \leq 9$

$x_1, x_2 \geq 0$



$\Rightarrow$ No global minimum value $\rightarrow$ $\boxed{\text{Unbounded solution}}$

The area of feasible region is unbounded.

minima at (0, 0)

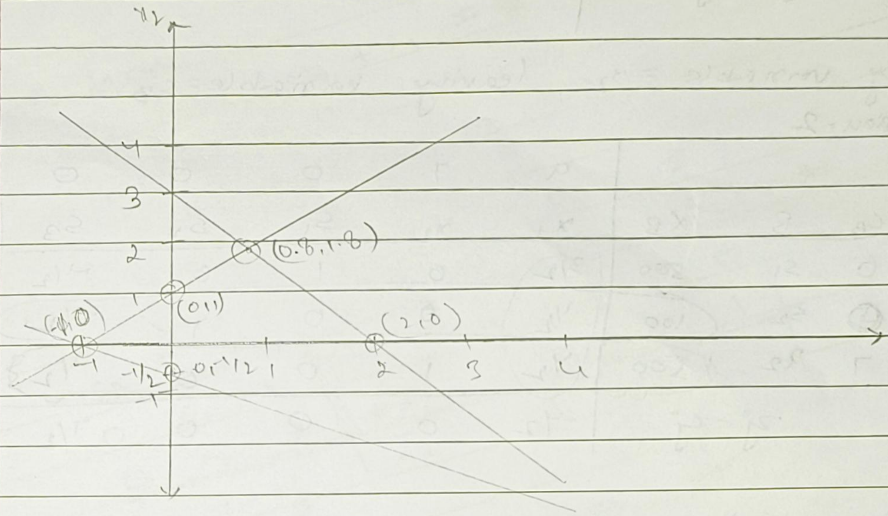
Ans-9 d)

Ans-10 a)

Ans 9 d) max $z = 2x_1 - 6x_2$

s.t.

$$3x_1 + 2x_2 \le 6$$
$$x_1 - x_2 \ge -1$$
$$-x_1 - 2x_2 \ge -1$$
$$x_1, x_2 \ge 0$$



No feasible region.

$\Rightarrow$ $\boxed{\text{No solution.}}$

Ans 10 a) max $z = 4x_1 + 7x_2$

s.t. $2x_1 + x_2 \le 1000$

$$x_1 + x_2 \le 600$$
$$-x_1 - 2x_2 \ge -1000$$
$$x_1, x_2 \ge 0$$

$\Rightarrow$

$$2x_1 + x_2 + S_1 = 1000$$
$$x_1 + x_2 + S_2 = 600$$
$$x_1 + 2x_2 + S_3 = 1000$$
$$x_1, x_2, S_1, S_2, S_3 \ge 0.$$

* Initial table

			4	7	0	0	0	
C_B	B	X_B	x_1	x_2	s_1	s_2	s_3	ratio
0	s_1	1000	2	1	1	0	0	1000
0	s_2	600	1	0	0	1	0	600
0	s_3	1000	1	2	0	0	1	500
	$z_j - c_j$		-4	-7	0	0	0	

entering variable $= x_2$, leaving variable $= s_3$.

* Iteration-2

			4	7	0	0	0	
C_B	B	X_B	x_1	x_2	s_1	s_2	s_3	ratio
0	s_1	500	3/2	0	1	0	-1/2	1000/3
0	s_2	100	1/2	0	0	1	-1/2	200
7	x_2	500	1/2	1	0	0	1/2	1000
	$z_j - c_j$		-1/2	0	0	0	7/2	

entering variable $= x_1$, leaving variable $= s_2$

* Iteration-3

			4	7	0	0	0	
C_B	B	X_B	x_1	x_2	s_1	s_2	s_3	ratio
0	s_1	200	0	0	1	-3	1	
4	x_1	200	1	0	0	2	-1	
7	x_2	400	0	1	0	-1	1	
	$z_j - c_j$		0	0	0	1	3	

No negative value in $z_j - c_j$

Hence solution is

$$z^* = 4(200) + 7(400) = 3600$$

$$\boxed{z^* = 3600}$$

$x_1 = 200$, $x_2 = 400$

Ans-10 b) max $z = 2x_1 + 2x_2$

s.t.

$x_1 - x_2 \geq -1$

$-0.5x_1 + x_2 \leq 2$

$x_1, x_2 \geq 0$

standard form

$-x_1 + x_2 + s_1 = 1$

$-0.5x_1 + x_2 + s_2 = 2$

* Initial Table

			2	2	0	0	
C_B	B	X_B	x_1	x_2	S_1	S_2	ratio
0	S_1	1	-1	1	1	0	-1 x
0	S_2	2	-0.5	1	0	1	-4 x
	$z_j - c_j$		-2	-2	0	0	

There is no global maxima.

The solution to this problem is unbounded.

Ans10 c) Max $z = 3x_1 + 6x_2 + 2x_3$

s.t. $3x_1 + 4x_2 + x_3 \le 2$

$x_1 + 2x_2 + 3x_3 \le 1$

$x_1, x_2, x_3 \ge 0$

Standard form.

$3x_1 + 4x_2 + x_3 + s_1 = 2$

$x_1 + 2x_2 + 3x_3 + s_2 = 1$

* Initial Table

			3	6	2	0	0		
C_B	B	X_B	x_1	x_2	x_3	s_1	s_2	ratio	
0	s_1	2	3	4	1	1	0	1/2	2
0	s_2	1	1	2	3	0	1	1/2	-
	$z_j - c_j$		-3	-6	-2	0	0		

leaving variable s_1, entering variable x_2.

*

			3	6	2	0	0	
C_B	B	X_B	x_1	x_2	x_3	s_1	s_2	ratio
6	x_2	1/2	3/4	1	1/4	1/4	0	2
0	s_2	0	-1/2	0	5/2	-1/2	1	0
	$z_j - c_j$		3/2	0	-1/2	3/2	0	

leaving variable = s_2, entering variable = x_3

*

			3	6	2	0	0
C_B	B	X_B	x_1	x_2	x_3	s_1	s_2
6	x_2	1/2	4/5	1	0	3/10	-1/10
2	x_3	0	-1/5	0	1	-1/5	2/5
	$z_j - c_j$		7/5	0	0	7/5	1/5

No negative value in $z_j - c_j$

$\Rightarrow z^* = 6(1/2) = 3$

$\boxed{z^* = 3}$ at $x_1 = 0$, $x_2 = 1/2$, $x_3 = 0$

Ans 10 d) Max $z = 4x_1 + 10x_2$

s.t. $2x_1 + x_2 \le 50$

$2x_1 + 5x_2 \le 100$

$2x_1 + 3x_2 \le 90$

$x_1, x_2 \ge 0$

Standard form

$2x_1 + x_2 + s_1 = 50$

$2x_1 + 5x_2 + s_2 = 100$

$2x_1 + 3x_2 + s_3 = 90$

* Initial table

			4	10	0	0	0	
C_B	B	X_B	x_1	x_2	s_1	s_2	s_3	Q
0	s_1	50	2	1	1	0	0	50
0	s_2	100	2	5	0	1	0	20
0	s_3	90	2	3	0	0	1	30
	$z_j - c_j$		-4	-10	0	0	0	

Leaving variable = s_2, entering variable = x_2

			4	10	0	0	0
C_B	B	X_B	x_1	x_2	s_1	s_2	s_3
0	s_1	30	8/5	0	1	-1/5	0
10	x_2	20	2/5	1	0	1/5	0
0	s_3	30	4/5	0	0	-3/5	1
	$z_j - c_j$		0	0	0	2	0

No negative value in $z_j - c_j$.

$\Rightarrow z^* = 10(20) = 200$

$\boxed{z^* = 200}$ at $x_1 = 0, x_2 = 20$, Ans 10 a) Max $z = 5x_1 + 4x_2 + x_3$

s.t. $6x_1 + x_1 + 2x_3 \le 12$

$8x_1 + 2x_2 + x_3 \le 30$

$4x_1 + x_2 - 2x_3 \le 16$

$x_1, x_2, x_3 \ge 0$

Standard form.

$6x_1 + x_2 + 2x_3 + s_1 = 12$

$8x_1 + 2x_2 + x_3 + s_2 = 30$

$4x_1 + x_2 - 2x_3 + s_3 = 16$

* Initial Table

			5	4	1	0	0	0	
CB	B	XB	x_1	x_2	x_3	s_1	s_2	s_3	Q
0	s_1	12	6	1	2	1	0	0	2
0	s_2	30	8	2	1	0	1	0	15/4
0	s_3	16	4	1	-2	0	0	1	4
	$z_j - c_j$		-5	-4	-1	0	0	0	

entering variable = x_1, leaving variable = s_1

			5	4	1	0	0	0	
CB	B	XB	x_1	x_2	x_3	s_1	s_2	s_3	Q
5	x_1	2	1	1/6	1/3	1/6	0	0	12
0	s_2	14	0	2/3	-5/3	-4/3	1	0	21
0	s_3	8	0	1/3	-10/3	-2/3	0	1	24
	$z_j - c_j$		0	-19/6	-4/3	5/6	0	0	

entering variable = x_2, leaving variable = x_1

			5	4	1	0	0	0
CB	B	XB	x_1	x_2	x_3	s_1	s_2	s_3
4	x_2	12	6	1	2	1	0	0
0	s_2	6	-4	0	-3	-2	1	0
0	s_3	4	-2	0	-4	-1	0	1
	$z_j - c_j$		19	0	5	4	0	0

as there is no negative value in $z_j - c_j$.

$\Rightarrow z^* = 4(12) = 48$

$\boxed{z^* = 48}$ at $x_1 = 0, x_2 = 12, x_3 = 0$

Ans-11 Max $z = x_1 + x_2$.

s.t. $2x_1 + 3x_2 \le 22$

$2x_1 + x_2 \le 14$

$x_1 - x_2 \le 4$

$3x_1 - 2x_2 \ge -6$

$x_1, x_2 \ge 0$

standard form.

$2x_1 + 3x_2 + s_1 = 22$

$2x_1 + x_2 + s_2 = 14$

$x_1 - x_2 + s_3 = 4$

$-3x_1 + 2x_2 + s_4 = 6$

$x_1, x_2, s_1, s_2, s_3, s_4 \ge 0$

* Initial Table

			1	1	0	0	0	0	
C_B	B	X_B	x_1	x_2	s_1	s_2	s_3	s_4	θ
0	s_1	22	2	3	1	0	0	0	11
0	s_2	14	2	1	0	1	0	0	7
0	s_3	4	1	-1	0	0	1	0	4
0	s_4	6	-3	2	0	0	0	1	-2×
	$z_j - c_j$		-1	-1	0	0	0	0	

entering variable $= x_1$, leaving variable $= s_3$

*

			1	1	0	0	0	0	
C_B	B	X_B	x_1	x_2	s_1	s_2	s_3	s_4	θ
0	s_1	14	0	5	1	0	-2	0	14/5
0	s_2	6	0	3	0	1	-2	0	2
1	x_1	4	1	-1	0	0	1	0	×
0	s_4	18	0	-1	0	0	3	1	×
	$z_j - c_j$		0	-2	0	0	1	0	

entering variable = x_2, leaving variable = s_2

*

		c_j	1	1	0	0	0	0	
C_B	B	X_B	x_1	x_2	s_1	s_2	s_3	s_4	θ
0	s_1	4	0	0	1	-4/3	4/3	0	3
1	x_2	2	0	1	0	1/3	-4/3	0	-3×
1	x_1	6	1	0	0	4/3	1/3	0	18
0	s_4	20	0	0	0	1/3	7/3	1	60/7
	$z_j - c_j$		0	0	0	2/3	-4/3	0	

entering variable = s_3, leaving variable = s_1

*

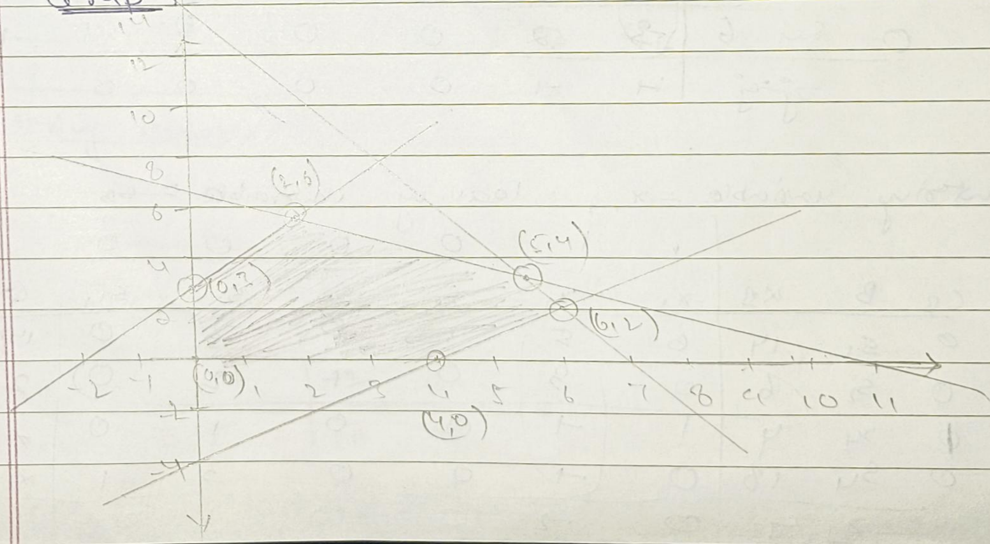
		c_j	1	1	0	0	0	0
C_B	B	X_B	x_1	x_2	s_1	s_2	s_3	s_4
0	s_3	3	0	0	3/4	-1/4	1	0
1	x_2	4	0	1	1/2	-1/2	0	0
1	x_1	5	1	0	-1/4	3/4	0	0
0	s_4	13	0	0	-7/4	13/4	0	1
	$z_j - c_j$		0	0	1/4	1/4	0	0

no negative value in $z_j - c_j$.

$\Rightarrow \quad z^* = 1(4) + 1(5) = 9$

$\boxed{z^* = 9}$ at $x_1 = 5$, $x_2 = 4$

* Graph



Value at corner points

(i) $(0,3)$

$\Rightarrow z = 3$

(ii) $(2,6)$

$\Rightarrow z = 8$

(iii) $(5,4)$

$\Rightarrow z = 9$ ✓

(iv) $(6,2)$

$\Rightarrow z = 8$

(v) $(4,0)$

$\Rightarrow z = 4$

Hence, $\boxed{z^* = 9}$ at $x_1 = 5$, $x_2 = 4$.

Hence verified.

<u>Ans 12</u> Max $z = 3x_1 + 5x_2$

s.t. $x_1 + 2x_2 \geq 8$

$3x_1 + 2x_2 \geq 12$

$5x_1 + 6x_2 \leq 60$

$x_1, x_2 \geq 0$

We need to solve this with Big-M method.

So, Max $z = 3x_1 + 5x_2 - MA_1 - MA_2$

s.t. $x_1 + 2x_2 - s_1 + A_1 = 8$

$3x_1 + 2x_2 - s_2 + A_2 = 12$

$5x_1 + 6x_2 + s_3 = 60$

$x_1, x_2, s_1, s_2, A_1, A_2, s_3 \geq 0$

→ Initial Table

			3	5	0	0	0	-M	-M	
C_B	B	X_B	x_1	x_2	s_1	s_2	s_3	A_1	A_2	Q
-M	A_1	8	1	2	-1	0	0	1	0	4
-M	A_2	12	3	2	0	-1	0	0	1	6
0	s_3	60	5	6	0	0	1	0	0	10
		$z_j - c_j$	-4M-3	-4M-5	M	M	0	0	0	

leaving variable = A_1, entering variable = x_2

			3	5	0	0	0	-M	-M	
C_B	B	X_B	x_1	x_2	s_1	s_2	s_3	A_1	A_2	Q
5	x_2	4	1/2	1	-1/2	0	0	1/2	0	8
-M	A_2	4	2	0	1	-1	0	-1	1	2
0	s_3	36	2	0	3	0	1	-3	0	18
			-2M-1/2	0	-M-5/2	M	0	2M+5/2	0	

entering variable = x_1, leaving variable = A_2

			3	5	0	0	0	-M	-M	
C_B	B	X_B	x_1	x_2	s_1	s_2	s_3	A_1	A_2	Q
5	x_2	3	0	1	-3/4	1/4	0	3/4	-1/4	×
3	x_1	2	1	0	1/2	-1/2	0	-1/2	1/2	4
0	s_3	32	0	0	2	1	1	-2	-1	16
			0	0	-9/4	-1/4	0	M+9/4	M+1/4	

entering variable = s_1, leaving variable = x_1

			3	5	0	0	0	-M	-M	
C_B	B	X_B	x_1	x_2	s_1	s_2	s_3	A_1	A_2	Q
5	x_2	6	3/2	1	0	-1/2	0	0	1/2	×
0	s_1	4	2	0	1	-1	0	-1	1	×
0	s_3	24	-4	0	0	3	1	0	-3	8
	z_j-c_j		9/2	0	0	-5/2	0	M	M+5/2	

entering variable = s_2, leaving variable = s_3

			3	5	0	0	0	-M	-M
C_B	B	X_B	x_1	x_2	s_1	s_2	s_3	A_1	A_2
5	x_2	10	5/6	1	0	0	1/6	0	0
0	s_1	12	2/3	0	1	0	1/3	-1	0
0	s_2	8	-4/3	0	0	1	1/3	0	-1
	z_j-c_j		7/6	0	0	0	5/6	M	M

Since there is no negative value in $z_j - c_j$

$z^* = 5(10)$, $\boxed{z^* = 50}$ at $x_1 = 0$, $x_2 = 10$

As we have found the optimal solution, it is proved that this problem has a finite optimal solution.

Q-14(a) Max $z = 5x_1 + x_2 - 2x_3 + x_4$

s.t. $x_1 + 5x_2 - 8x_3 + 3x_4 = 6$

$3x_1 - x_2 + x_3 + x_4 = 2$

$x_1, x_2, x_3, x_4 \geq 0$

standard form,

$x_1 + 5x_2 - 8x_3 + 3x_4 + A_1 = 6$

$3x_1 - x_2 + x_3 + x_4 + A_2 = 2$

Phase-I

auxilary function: $z' = -A_1 - A_2$

			0	0	0	0	-1	-1	
CB	B	XB	x_1	x_2	x_3	x_4	A_1	A_2	Q
-1	A_1	6	1	5	-8	3	1	0	6
-1	A_2	2	3	-1	1	1	0	1	2/3
	$z_j - c_j$		-4	-4	7	-4	0	0	

			0	0	0	0	-1	-1	
CB	B	XB	x_1	x_2	x_3	x_4	A_1	A_2	Q
-1	A_1	16/3	0	16/3	-25/3	8/3	1	-1/3	1
0	x_1	2/3	1	-1/3	1/3	1/3	0	1/3	×
			0	-16/3	25/3	-8/3	0	4/3	

			0	0	0	0	-1	-1	
CB	B	XB	x_1	x_2	x_3	x_4	A_1	A_2	Q
0	x_2	1	0	1	-25/16	8/16	3/16	-1/16	
0	x_1	1	1	0	-3/16	8/16	1/16	5/16	
			0	0	0	0	1	1	

as $z' = 0$

Phase-II

			5	1	-2	1	Q
C_B	B	X_B	x_1	x_2	x_3	x_4	~~A_1~~ A_2
~~1~~	x_2	1	0	1	$-25/16$	$8/16$	x
5	x_1	1	1	0	$-3/16$	$8/16$	x
			0	0	$-8/16$	2	

Hence, the problem has <u>unbounded solution</u>

<u>Ans-(4 b)</u> max $z = 4x_1 + x_2$

s.t. $3x_1 - x_2 \le 10$

$2x_1 + x_2 \ge 20$

$3x_1 - x_2 = -14$

Standard form

$3x_1 - x_2 + s_1 = 10$

$2x_1 + x_2 - S_2 + A_1 = 20$

$-3x_1 + x_2 + A_2 = 14$

<u>Phase-I</u>

auxilary function: $z' = -A_1 - A_2$

* <u>Initial table</u>

			0	0	0	0	-1	-1	
C_B	B	X_B	x_1	x_2	S_1	S_2	A_1	A_2	Q
0	S_1	10	3	-1	1	0	0	0	
-1	A_1	20	2	1	0	-1	1	0	20
-1	A_2	14	-3	1	0	0	0	1	14
	$z_j - c_j$		1	-2	0	1	0	0	

			0	0	0	0	-1	-1	
C_B	B	X_B	x_1	x_2	S_1	S_2	A_1	A_2	Q
0	S_1	24	0	0	1	0	0	1	x
-1	A_1	6	5	0	0	-1	1	-1	6/5
0	x_2	14	-3	1	0	0	0	1	x
	$z_j - c_j$		-5	0	0	1	0	2	

A

C_B	B	X_B	x_1	x_2	s_1	s_2	A_1	A_2
0	s_1	24	0	0	1	0	0	1
0	x_1	6/5	1	0	0	-1/5	1/5	-1/5
0	x_2	88/5	0	1	0	-3/5	3/5	2/5
	$z_j - c_j$		0	0	0	0	0	0

$z' = 0$.

$\Rightarrow$ Phase II

			4	1	0	0	
C_B	B	X_B	x_1	x_2	s_1	s_2	Q
0	s_1	24	0	0	1	0	x
4	x_1	6/5	1	0	0	-1/5	x
1	x_2	88/5	0	1	0	-3/5	x
		$z_j - c_j$	0	0	0	-7/5	

This problem has unbounded solution

Ans-15 Max $z = x_1 - x_2 - x_4$

s.t. $2x_1 + x_2 - x_3 = 10$ $\rightarrow \leq$ $\rightarrow \geq$

$x_2 + x_3 - x_4 \leq 0$

$x_1 + x_3 + 2x_4 \geq 6$

$x_1, x_2, x_4 \geq 0$, x_3 is unrestricted.

write $x_3 = x_3' - x_3''$ s.t. $x_3', x_3'' \geq 0$.

writing in canonical form.

* Max $z = x_1 - x_2 + 0x_3' + 0x_3'' - x_4$

s.t. $2x_1 + x_2 - x_3' + x_3'' + 0x_4 \leq 10$

$-2x_1 - x_2 + x_3' - x_3'' + 0x_4 \leq -10$

$0x_1 + x_2 + x_3' - x_3'' - x_4 \leq 0$

$-x_1 + 0x_2 - x_3' + x_3'' - 2x_4 \leq -6$

In the dual form,
let the variables be y_1, y_2, y_3, y_4

$\Rightarrow$ Min $z = 10y_1 - 10y_2 + 0y_3 - 6y_4$

s.t.

$$2y_1 - 2y_2 + 0y_3 - y_4 \geq 1$$
$$y_1 - y_2 + y_3 + 0y_4 \geq -1$$
$$-y_1 + y_2 + y_3 - y_4 \geq 0$$
$$y_1 - y_2 - y_3 + y_4 \geq 0$$
$$0y_1 + 0y_2 - y_3 - 2y_4 \geq -1$$
$$y_1, y_2, y_3, y_4 \geq 0$$

if we take $y' = y_1 - y_2$

$\Rightarrow$ min : $z = 10y' - 6y_4$

s.t

$$2y' - y_4 \geq 1$$
$$y' + y_3 \geq -1$$
$$-y' + y_3 - y_4 \geq 0$$
$$y' - y_3 + y_4 \geq 0$$
$$-y_3 - 2y_4 \geq -1$$

$\Rightarrow$ $y_3, y_4 \geq 0$ y' is unrestricted.

Ans-16

$$x_1 + x_2 = 1$$
$$2x_1 + x_2 = 3$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$A \; x = b$$
$$\Rightarrow x = A^{-1} b.$$

We will use product form of inverse of a matrix.

$B = B^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B_1 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $B_2 = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$

$\Rightarrow \quad c_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$e_1 = B^{-1} c_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\Rightarrow \eta_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\Rightarrow \quad E_1 = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

$\Rightarrow \quad B_1^{-1} = E_1 B^{-1} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

Now, $c_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\Rightarrow e_2 = B_1^{-1} c_2 = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\eta_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\Rightarrow E_2 = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$

$\Rightarrow B_2^{-1} = E_2 B_1^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} =$

$$\boxed{B_2^{-1} = \begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix}}$$

$\Rightarrow A^{-1} = \begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix}$

$\Rightarrow x = A^{-1} b$

$x = \begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$\Rightarrow \boxed{x_1 = 2} \quad \boxed{x_2 = -1}$