

Big- M Method: LPP- Numerical Examples

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Numerical Example -b1: Condensed Tableau

$$\max : Z = x_1 + 3x_2 + x_3$$

Subject to

$$x_1 + x_2 + x_3 = 10$$

$$x_1 + 4x_2 + x_3 = 16$$

$$x_1, x_2, x_3 \geq 0$$

Numerical Example -b1:

$$\max : Z = x_1 + 3x_2 + x_3 - Ma_1 - Ma_2$$

Subject to

$$x_1 + x_2 + x_3 + a_1 = 10$$

$$x_1 + 4x_2 + x_3 + a_2 = 16$$

$$x_1, x_2, x_3 \geq 0$$

Artificial variables:

$$a_1, a_2 \geq 0$$

M is a large positive number.

Big-M Method:

Numerical Example -b1:

$$\max : Z = x_1 + 3x_2 + x_3 - M(a_1 + a_2)$$

Subject to

$$-x_1 - x_2 - x_3 + 10 = a_1$$

$$-x_1 - 4x_2 - x_3 + 16 = a_2$$

$$x_1, x_2, x_3 \geq 0$$

Artificial variables are Basic variables.

$$a_1, a_2 \geq 0$$

M is a large positive number.

Numerical Example -b1:

$$\begin{aligned}\max : Z &= x_1 + 3x_2 + x_3 - M(26 - 2x_1 - 5x_2 - 2x_3) \\ &= (1 + 2M)x_1 + (3 + 5M)x_2 + (1 + 2M)x_3 - 26M\end{aligned}$$

Subject to

$$\begin{aligned}-x_1 - x_2 - x_3 + 10 &= a_1 \\ -x_1 - 4x_2 - x_3 + 16 &= a_2 \\ x_1, x_2, x_3 &\geq 0 \\ a_1, a_2 &\geq 0\end{aligned}$$

Simplex Method: Condensed Tableau

Numerical Example (b1):

Table 0:

$-x_1$	$-x_2$	$-x_3$	1	XB
1	1	1	10	$= a_1$
1	4^*	1	16	$= a_2$
$-(2M+1)$	$-(5M+3)^*$	$-(2M+1)$	$-26M$	$= Z$

$$x_1 = 0, x_2 = 0, x_3 = 0, Z = -26M$$

Simplex Method: Condensed Tableau

Numerical Example (b1):

Table 1:

$-x_1$	$-a_2$	$-x_3$	1	XB
$3/4^*$	$-1/4$	$3/4$	6	$= a_1$
$1/4$	$1/4$	$1/4$	4	$= x_2$
$-(3M+1)/4$	$(5M+3)/4$	$-(3M+1)/4$	$-6M+12$	$=Z$

$$x_1 = 0, x_2 = 4, x_3 = 0, Z = -6M + 12$$

Big-M Method: Numerical Example-b1

Table 2 and 3 :

$-a_1$	$-a_2$	$-x_3$	1	XB
4/3	-1/3	1*	8	$= x_1$
-1/3	1/3	0	2	$= x_2$
M+1/3	M + 2/3	0*	14	$=Z$

Optimal Solution :

$$x_1^* = 8, x_2^* = 2, x_3^* = 0, Z^* = 14$$

$-a_1$	$-a_2$	$-x_1$	1	XB
4/3	-1/3	1	8	$= x_3$
-1/3	1/3	0	2	$= x_2$
M+1/3	M + 2/3	0	14	$=Z$

Alternate Optimal Solution :

$$x_1^* = 0, x_2^* = 2, x_3^* = 8, Z^* = 14$$

Trivial Example (b2):

$$\max : Z = x_1 + 3x_2$$

Subject to

$$x_1 + x_2 = 10$$

$$x_1 + 4x_2 = 16$$

$$x_1, x_2 \geq 0$$

Simplex Method: Condensed Tableau :New

Numerical Example (b2):

$$\max : Z = x_1 + 3x_2 - Ma_1 - Ma_2$$

Subject to

$$x_1 + x_2 + a_1 = 10$$

$$x_1 + 4x_2 + a_2 = 16$$

$$x_1, x_2 \geq 0$$

M : is a large positive number

Artificial variables (Basic variables) :

$$a_1, a_2 \geq 0$$

Use M to drive out artificial variables

Simplex Method: Condensed Tableau

Numerical Example (b2):

$$\begin{aligned}\max : Z &= x_1 + 3x_2 - Ma_1 - Ma_2 \\ &= x_1 + 3x_2 - M(a_1 + a_2) \\ &= x_1(2M + 1) + x_2(5M + 3) - 26M\end{aligned}$$

Subject to

$$-x_1 - x_2 + 10 = a_1$$

$$-x_1 - 4x_2 + 16 = a_2$$

$$x_1, x_2 \geq 0$$

Artifial variables (Basic variables) :

$$a_1, a_2 \geq 0$$

Simplex Method: Condensed Tableau

Numerical Example (b2):

Table 0:

$-x_1$	$-x_2$	1	XB
1	1	10	$= a_1$
1	* 4	16	$= a_2$
-2M-1	-5M-3	-26M	$= Z$

$$x_1 = 0, x_2 = 0, Z = -26M$$

Simplex Method: Condensed Tableau

Numerical Example (b2):

Table 1:

$-x_1$	$-a_2$	1	XB
* $3/4$	$-1/4$	6	$= a_1$
$1/4$	$1/4$	4	$= x_2$
$-(3M+1)/4$	$(5M+3)/4$	$-6M+12$	$=Z$

$$x_1 = 0, x_2 = 4, Z = -6M + 12$$

Simplex Method: Condensed Tableau

Numerical Example (b2):

Table 2:

$-a_1$	$-a_2$	1	XB
4/3	-1/3	8	$= x_1$
- 1/3	1/3	2	$= x_2$
M+1/3	M +2/3	14	$=Z$

Optimal Solution :

$$x_1^* = 8, x_2^* = 2, Z^* = 14$$

Numerical Example (b3):

$$\text{min : } Z = x_1 + 3x_2$$

Subject to

$$x_1 + x_2 \geq 10$$

$$x_1 + 4x_2 \geq 16$$

$$x_1, x_2 \geq 0$$

$$\text{max : } -Z = -x_1 - 3x_2$$

Simplex Method: Condensed Tableau

Numerical Example (b3):

$$\max : -Z = -x_1 - 3x_2 - Ma_1 - Ma_2$$

Subject to

$$x_1 + x_2 - x_3 + a_1 = 10$$

$$x_1 + 4x_2 - x_4 + a_2 = 16$$

$$x_1, x_2, x_3, x_4 \geq 0$$

M : is a large positive number

Surplus variables : $x_3, x_4 \geq 0$

Artificial variables (Basic variables) :

$$a_1, a_2 \geq 0$$

Use M to drive out artificial variables

Simplex Method: Condensed Tableau

Numerical Example (b3):

$$\max : -Z = -x_1 - 3x_2 - Ma_1 - Ma_2$$

$$= -x_1 - 3x_2 - M(a_1 + a_2)$$

$$= x_1(2M - 1) + x_2(5M - 3) - x_3M - x_4M - 26M$$

Subject to

$$-x_1 - x_2 + x_3 + 10 = a_1$$

$$-x_1 - 4x_2 + x_4 + 16 = a_2$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Artificial variables (Basic variables) :

$$a_1, a_2 \geq 0$$

Simplex Method: Condensed Tableau

Numerical Example (b3):

Table 0:

$-x_1$	$-x_2$	$-x_3$	$-x_4$	1	XB
1	1	-1	0	10	$= a_1$
1	* 4	0	-1	16	$= a_2$
$-2M+1$	$-5M+3$	M	M	$-26M$	$= -Z$

$$x_1 = 0, x_2 = 0, Z = 26M$$

Simplex Method: Condensed Tableau

Numerical Example (b3):

Table 1:

$-x_1$	$-a_2$	$-x_3$	$-x_4$	1	XB
* $3/4$	$-1/4$	-1	$1/4$	6	$= a_1$
$1/4$	$1/4$	0	$-1/4$	4	$= x_2$
$-(3M-1)/4$	$(5M-3)/4$	M	$-(M-3)/4$	$-6M-12$	$= -Z$

$$x_1 = 0, x_2 = 4, Z = 6M + 12$$

Simplex Method: Condensed Tableau

Numerical Example (b3):

Table 2:

$-a_1$	$-a_2$	$-x_3$	$-x_4$	1	XB
4/3	-1/3	-4/3	1/3	8	$= x_1$
- 1/3	1/3	1/3	-1/3	2	$= x_2$
M-1/3	M-2/3	1/3	2/3	-14	$= -Z$

Optimal Solution :

$$x_1^* = 8, x_2^* = 2, Z^* = 14$$

Numerical Example -4:

$$\min : Z = x_1 + 3x_2 + x_3$$

Subject to

$$x_1 + x_2 + x_3 = 10$$

$$x_1 + 4x_2 + x_3 = 16$$

$$x_1, x_2, x_3 \geq 0$$

Numerical Example -4:

$$\max : -Z = -x_1 - 3x_2 - x_3 - Ma_1 - Ma_2$$

Subject to

$$x_1 + x_2 + x_3 + a_1 = 10$$

$$x_1 + 4x_2 + x_3 + a_2 = 16$$

$$x_1, x_2, x_3 \geq 0$$

Artificial variables:

$$a_1, a_2 \geq 0$$

Big-M Method: Numerical Example-4

Table 0:

SIMP	CN	-1	-3	-1	b
CB	BV/NV	x_1	x_2	x_3	XB
-M	a_1	1	1	1	10
-M	a_2	1	* 4	1	16
*	-Z	-2M+1	- 5M+3	-2M+1	-26M

$$x_1 = 0, x_2 = 0, x_3 = 0, Z = 26M$$

Big-M Method: Numerical Example-4

Table 1:

SIMP	CN	-1	-M	-1	b
CB	BV/NV	x_1	a_2	x_3	XB
-M	a_1	$*3/4$	$-1/4$	$-3/4$	6
-3	x_2	$1/4$	$1/4$	$1/4$	4
*	-Z	$(-3M+1)/4$	$(5M-3)/4$	$(-3M+1)/4$	-6M-12

$$x_1 = 0, x_2 = 4, x_3 = 0, Z = 6M + 12$$

Big-M Method: Numerical Example-4

Table 2:

SIMP	CN	-M	-M	-1	b
CB	BV/NV	a_1	a_2	x_3	XB
-1	x_1	$4/3$	$-1/3$	1	8
-3	x_2	$-1/3$	$1/3$	0	2
*	-Z	$M-1/3$	$M-2/3$	0^*	-14

Optimal Solution :

$$x_1^* = 8, x_2^* = 2, x_3^* = 0, Z^* = 14$$

Alternate Optimal Solution :

$$x_1^* = 0, x_2^* = 2, x_3^* = 8, Z^* = 14$$

Numerical Example -5: Condensed Tableau

$$\max : Z = 4x_1 + 3x_2 + x_3$$

Subject to

$$x_1 + x_2 + x_3 = 10$$

$$x_1 + 4x_2 + x_3 = 16$$

$$x_1, x_2, x_3 \geq 0$$

Numerical Example -5:

$$\max : Z = 4x_1 + x_2 + x_3 - Ma_1 - Ma_2$$

Subject to

$$x_1 + x_2 + x_3 + a_1 = 10$$

$$x_1 + 4x_2 + x_3 + a_2 = 16$$

$$x_1, x_2, x_3 \geq 0$$

Artificial variables:

$$a_1, a_2 \geq 0$$

Big-M Method: Numerical Example-5

Table 0:

SIMP	CN	4	1	1	b
CB	BV/NV	x_1	x_2	x_3	XB
-M	a_1	1	1	1	10
-M	a_2	1	* 4	1	16
*	Z	-2M-4	- 5M-1	-2M-1	-26M

$$x_1 = 0, x_2 = 0, x_3 = 0, Z = -26M$$

Big-M Method: Numerical Example-5

Table 1:

SIMP	CN	4	-M	1	b
CB	BV/NV	x_1	a_2	x_3	XB
-M	a_1	$*3/4$	$-1/4$	$-3/4$	6
1	x_2	$1/4$	$1/4$	$1/4$	4
*	Z	$(-3M-15)/4$	$(5M+1)/4$	$(-3M-3)/4$	$-6M+4$

$$x_1 = 0, x_2 = 4, x_3 = 0, Z = -6M + 4$$

Big-M Method: Numerical Example-5

Table 2:

SIMP	CN	-M	-M	1	b
CB	BV/NV	a_1	a_2	x_3	XB
4	x_1	4/3	-1/3	1	8
1	x_2	-1/3	1/3	0	2
*	Z	M+5	M-1	3	34

Optimal Solution :

$$x_1^* = 8, x_2^* = 2, x_3^* = 0, Z^* = 34$$

Numerical Example -6: Condensed Tableau

$$\min : Z = 2x_1 + 3x_2 + 5x_3$$

Subject to

$$x_1 + x_2 + x_3 = 6$$

$$3x_1 + 2x_2 + x_3 = 16$$

$$x_1, x_2, x_3 \geq 0$$

Numerical Example -6:

$$\max : -Z = -2x_1 - 3x_2 - 5x_3 - Ma_1 - Ma_2$$

Subject to

$$x_1 + x_2 + x_3 + a_1 = 6$$

$$3x_1 + 2x_2 + x_3 + a_2 = 16$$

$$x_1, x_2, x_3 \geq 0$$

Artificial variables:

$$a_1, a_2 \geq 0$$

Big-M Method: Numerical Example-6

Table 0:

SIMP	CN	-2	-3	-5	b
CB	BV/NV	x_1	x_2	x_3	XB
-M	a_1	1	1	1	6
-M	a_2	*3	2	1	16
*	-Z	$-4M+2$	$-3M+3$	$-2M+5$	$-22M$

$$x_1 = 0, x_2 = 0, x_3 = 0, Z = 22M$$

Big-M Method: Numerical Example-6

Table 1:

SIMP	CN	-M	-3	-5	b
CB	BV/NV	a_2	x_2	x_3	XB
-M	a_1	$-1/3$	$1/3$	* $2/3$	$2/3$
-2	x_1	$1/3$	$2/3$	$1/3$	$16/3$
*	-Z	$(4M-2)/3$	$(5-M)/3$	$(13-2M)/3$	$-(32+2M)/3$

$$x_1 = 16/3, x_2 = 0, x_3 = 0, Z = (32 + 2M)/3$$

Big-M Method: Numerical Example-6

Table 2:

SIMP	CN	-M	-3	-M	b
CB	BV/NV	a_2	x_2	a_1	XB
-5	x_3	-1/2	1/2	3/2	1
-2	x_1	1/2	1/2	- 1/2	5
*	- Z	M+3/2	-1/2	M-13/2	-15

$$x_1 = 5, x_2 = 0, x_3 = 1, Z = 15$$

Big-M Method: Numerical Example-6

Table 3:

SIMP	CN	-M	-5	-M	b
CB	BV/NV	a_2	x_3	a_1	XB
-3	x_2	-1	2	3	2
-2	x_1	1	-1	-2	4
*	- Z	M+1	1	M-5	-14

Optimal Solution :

$$x_1^* = 4, x_2^* = 2, x_3^* = 0, -Z^* = -14, Z^* = 14$$

Numerical Example -7: Condensed Tableau

$$\max : Z = 2x_1 + 3x_2 + 5x_3$$

Subject to

$$x_1 + x_2 + x_3 = 6$$

$$3x_1 + 2x_2 + x_3 = 16$$

$$x_1, x_2, x_3 \geq 0$$

Numerical Example -7:

$$\max : Z = 2x_1 + 3x_2 + 5x_3 - Ma_1 - Ma_2$$

Subject to

$$x_1 + x_2 + x_3 + a_1 = 6$$

$$3x_1 + 2x_2 + x_3 + a_2 = 16$$

$$x_1, x_2, x_3 \geq 0$$

Artificial variables:

$$a_1, a_2 \geq 0$$

Big-M Method: Numerical Example-7

Table 0:

SIMP	CN	2	3	5	b
CB	BV/NV	x_1	x_2	x_3	XB
-M	a_1	1	1	1	6
-M	a_2	*3	2	1	16
*	Z	-4M-2	- 3M-3	-2M-5	-22M

$$x_1 = 0, x_2 = 0, x_3 = 0, Z = -22M$$

Big-M Method: Numerical Example-7

Table 1:

SIMP	CN	-M	3	5	b
CB	BV/NV	a_2	x_2	x_3	XB
-M	a_1	$-1/3$	$1/3$	$* 2/3$	$2/3$
2	x_1	$1/3$	$2/3$	$1/3$	$16/3$
*	Z	$(4M+2)/3$	$(-5-M)/3$	$(-13-2M)/3$	$(32-2M)/3$

$$x_1 = 16/3, x_2 = 0, x_3 = 0, Z = (32 - 2M)/3$$

Big-M Method: Numerical Example-7

Table 2:

SIMP	CN	-M	3	-M	b
CB	BV/NV	a_2	x_2	a_1	XB
5	x_3	-1/2	1/2	3/2	1
2	x_1	1/2	1/2	- 1/2	5
*	Z	M-3/2	1/2	M+13/2	15

Optimal Solution :

$$x_1^* = 5, x_2^* = 0, x_3^* = 1, Z^* = 15$$

Numerical Example -8: Condensed Tableau

$$\min : Z = x_1 + 3x_2 + x_3$$

Subject to

$$x_1 + x_2 + x_3 \geq 10$$

$$x_1 + 4x_2 + x_3 \geq 16$$

$$x_1, x_2, x_3 \geq 0$$

Numerical Example 8:

$$\max : -Z = -x_1 - 3x_2 - x_3 - Ma_1 - Ma_2$$

Subject to

$$x_1 + x_2 + x_3 - x_4 + a_1 = 10$$

$$x_1 + 4x_2 + x_3 - x_5 + a_2 = 16$$

$$x_1, x_2, x_3 \geq 0$$

Surplus variables:

$$x_4, x_5 \geq 0$$

Artificial variables:

$$a_1, a_2 \geq 0$$

Big-M Method: Numerical Example-8

Table 0:

SIMP	CN	-1	-3	-1	0	0	b
CB	BV/NV	x_1	x_2	x_3	x_4	x_5	XB
-M	a_1	1	1	1	-1	0	10
-M	a_2	1	* 4	1	0	-1	16
*	- Z	-2M+1	- 5M+3	-2M+1	M	M	-26M

$$x_1 = 0, x_2 = 0, x_3 = 0, -Z = -26M$$

Big-M Method: Numerical Example-8

Table 1:

SIMP	CN	-1	-M	-1	0	0	b
CB	BV/NV	x_1	a_2	x_3	x_4	x_5	XB
-M	a_1	$*3/4$	$-1/4$	$-3/4$	-1	$1/4$	6
-3	x_2	$1/4$	$1/4$	$1/4$	0	$-1/4$	4
*	-Z	$(1-3M)/4$	$(5M-3)/4$	$(1-3M)/4$	M	$(3-M)/4$	$-6M-12$

$$x_1 = 0, x_2 = 4, x_3 = 0, Z = 6M + 12$$

Big-M Method: Numerical Example-8

Table 2:

SIMP	CN	-M	-M	-1	0	0	b
CB	BV/NV	a_1	a_2	x_3	x_4	x_5	XB
-1	x_1	4/3	-1/3	1	-4/3	1/3	8
-3	x_2	-1/3	1/3	0	1/3	-1/3	2
*	- Z	M-1/3	M-2/3	0*	1/3	2/3	-14

Optimal Solution :

$$x_1^* = 8, x_2^* = 2, x_3^* = 0, Z^* = 14,$$

Alternate Optimal Solution :

$$x_1^* = 0, x_2^* = 2, x_3^* = 8, Z^* = 14$$

Numerical Example -1 : Practice Problem

$$\min : Z = 2x_1 + 3x_2 + x_3$$

Subject to

$$x_1 + x_2 + x_3 \geq 10$$

$$x_1 + 2x_2 + x_3 \geq 12$$

$$x_1 + 4x_2 + x_3 \leq 16$$

$$x_1, x_2, x_3 \geq 0$$

Find Optimal Solution :

$$x_1^* = *, x_2^* = *, x_3^* = *, Z^* = 12$$

Numerical Example -2: Practice Problem

$$\min : Z = x_1 + x_2 + x_3$$

Subject to

$$4x_1 + x_2 + x_3 \geq 20$$

$$x_1 + 3x_2 + x_3 \geq 12$$

$$x_1 + x_2 + 2x_3 \geq 10$$

$$x_1, x_2, x_3 \geq 0$$

Find Optimal Solution :

$$x_1^* = *, x_2^* = *, x_3^* = *, Z^* = 8$$

Numerical Example -3: Practice Problem

$$\min : Z = x_1 + 4x_2 + 4x_3$$

Subject to

$$x_1 + 2x_2 + x_3 \geq 16$$

$$x_1 + x_2 + 2x_3 \geq 14$$

$$4x_1 + x_2 + x_3 \leq 12$$

$$x_1, x_2, x_3 \geq 0$$

Find Optimal Solution :

$$x_1^* = *, x_2^* = *, x_3^* = *, Z^* = 39$$

Numerical Example -4: Practice Problem

$$\min : Z = x_1 + 6x_2 + 6x_3$$

Subject to

$$x_1 + 3x_2 + x_3 = 90$$

$$x_1 + x_2 + 3x_3 = 54$$

$$5x_1 + x_2 + x_3 \leq 45$$

$$x_1, x_2, x_3 \geq 0$$

Find Optimal Solution :

$$x_1^* = ***, x_2^* = ***, x_3^* = ***, Z^* = 212$$

Numerical Example -5: Practice Problem

$$\min : Z = x_1 + 4x_2 + 4x_3$$

Subject to

$$x_1 + 5x_2 + x_3 \geq 45$$

$$x_1 + x_2 + 5x_3 \geq 35$$

$$2x_1 + x_2 + x_3 \leq 25$$

$$x_1, x_2, x_3 \geq 0$$

Find Optimal Solution :

$$x_1^* = ***, x_2^* = ***, x_3^* = ***, Z^* = 51$$

Solve all the Practice Problems
Using Two-Phase Simplex Method.