Linear Regression.

Cost Function

$$E(w) = \frac{1}{2}\sum_{i=0}^{n}\left(h_w(x^{(i)}) - y^{(i)}\right)^2$$

Weights updation.

$$W_j \leftarrow W_j - \alpha \frac{\partial E(w)}{\partial w_j} \quad \text{—— } (i)$$

Derivation.

For One Example.

$$\frac{\partial E(w)}{\partial w_j} = \frac{1}{2}\frac{\partial}{\partial w_j}\left(h_w(x) - y\right)^2$$

$$= \frac{1}{2}\cdot 2\left(h_w(x) - y\right)\frac{\partial}{\partial w_j}\left(\sum_{i=0}^{K} w_K x_K - y\right)$$

$$= \left(h_w(x) - y\right)\left(x_j \frac{\partial w_j}{\partial w_j} - 0\right)$$

$$\frac{\partial E(w)}{\partial w_j} = \left(h_w(x) - y\right)\cdot x_j \quad \text{—— } (ii)$$

Putting eq (ii) in eq (i).

$$W_j \leftarrow : W_j - \alpha \sum_{i=0}^{n}\left(h_w(x^{(i)}) - y^{(i)}\right)\cdot x_j^{(i)}$$

Logistic Regression.

Conventions

→ Hypothesis

$$h_w(x) = F(z) = a = \frac{1}{1+e^{-z}}$$

$$z = w^T x = \sum_{i=0}^{m} w_i x_i$$

Cost Function $J = J(w)$.

$$J(w) = \frac{1}{n} \sum_{i=0}^{n} \left[-y^i \log(h_w x^{(i)}) - (1-y^{(i)}) \log(1 - h_w(x^{(i)})) \right]$$

For One Example. assuming $a = h_w(x)$

$$J = -y \log a - (1-y) \log(1-a)$$

Finding Derivative of Cost Function w.r.t w_j

$\frac{\partial J}{\partial w_j}$ By using chain rule

$$\frac{\partial J}{\partial w_j} = \frac{\partial J}{\partial a} \times \frac{\partial a}{\partial z} \times \frac{\partial z}{\partial w_j} \quad \text{—— (i)}$$

Computing $\frac{\partial J}{\partial a}$.

$$\frac{\partial J}{\partial a} = -y \frac{\partial \log a}{\partial a} - (1-y) \frac{\partial}{\partial a} \log(1-a)$$

$$= \frac{-y}{a} - \frac{(1-y)}{(1-a)} (-1)$$

$$\frac{\partial J}{\partial a} = \frac{-y}{a} + \frac{(1-y)}{1-a}$$

Computing $\frac{\partial a}{\partial z}$

$$\frac{\partial a}{\partial z} = \frac{\partial}{\partial z}\left(\frac{1}{1+e^{-z}}\right)$$

$$= \frac{\partial}{\partial z}\left(1+e^{-z}\right)^{-1}$$

$$= -1\left(1+e^{-z}\right)^{-1-1}\left(0+e^{-z}(-1)\right)$$

$$= \frac{e^{z}}{\left(1+e^{-z}\right)^{2}}$$

$$= \frac{1}{1+e^{-z}}\left(\frac{e^{z}}{1+e^{-z}}\right)$$

$$= a\left(\frac{1+e^{-z}-1}{1+e^{-z}}\right)$$

$$= a\left(\frac{1+e^{-z}}{1+e^{-z}} - \frac{1}{1+e^{-z}}\right)$$

$$\frac{\partial a}{\partial z} = a(1-a)$$

Computing $\frac{\partial z}{\partial w_j}$

$$\frac{\partial z}{\partial w_j} = \frac{\partial}{\partial w_j}\sum_{i=0}^{m} w_i x_i$$

$$= x_j \frac{\partial}{\partial w_j} w_j$$

$$\frac{\partial z}{\partial w_j} = x_j$$

Putting $\frac{\partial J}{\partial a}$, $\frac{\partial a}{\partial z}$ and $\frac{\partial z}{\partial w_j}$ in eq (i)

$$\frac{\partial J}{\partial w_j} = \left(\frac{-y}{a} + \frac{(1-y)}{1-a}\right) \times a(1-a) \times x_j$$

$$\frac{\partial J}{\partial w_j} = \frac{-y(1-a)+a(1-y)}{a(1-a)} \times (a(1-a)) \times (x_j)$$

$$= (-y + \cancel{ay} + a - \cancel{ay}) \cdot x_j$$

$$\frac{\partial J}{\partial w_j} = (a-y) \cdot x_j$$

$$\frac{\partial J}{\partial w_j} = (h_w(x)-y) \cdot x_j$$

Weight updation equation.

$$W_j \leftarrow : W_j - \alpha \frac{\partial J(w)}{\partial w_j}$$

$$W_j \leftarrow : W_j - \alpha (h_w(x) - y) \cdot x_j$$

$$W_j \leftarrow : W_j + \alpha (y - h_w(x)) \cdot x_j$$