

Who am I: the Knowledge Change in *Fight Club*

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1 Introduction

- Motivation
- Storyline

2 Preliminaries

- First-order Epistemic-Doxastic Logic
- Models and Semantics

3 Change of the Knowledge in the Movie

- The Meanings of Predicates
- Steps and Conclusion

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We plan to explore the knowledge change in the movie *Fight Club*. The film tells a story of a man, Jack, with schizophrenia who finally identifies his another personality, Tyler.

- The agents involved are the same person physically.
- The knowledge of Jack is always changing during the storyline.
- It is related to an interesting philosophical topic: identity.

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A Brief Storyline

Consecutively through the movie there are cues hinted about the identity of Jack and Tyler:

- Step 1: Jack found Tyler has a totally same suitcase with his.
- Step 2: Tyler knows the method in making explosives.
- Step 3: Jack had a dream that he slept with Marla.
- Step 4: Jack found that Tyler had a same scar on his right hand.
- Step 5: Police suspected that it was Jack himself burnt Jack's condo.
- Step 6: Marla confirmed that Jack slept with Marla.
- Step 7: Tyler said it was Jack burnt Jack's condo.

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First-order Epistemic-Doxastic Logic

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First-order Epistemic-Doxastic Logic

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- 1 Epistemic logic is for describing knowledge. Doxastic logic is for expressing belief.
- 2 We need to talk about the properties of agents. So predicates should be introduced.

The language of FOEDL:

Definition

Given a denumerable set of variables \mathbf{X} , and a denumerable set of unary predicate symbols, the language FOEDL is defined as:

$$\phi ::= (x \approx y) \mid Px \mid \neg\phi \mid (\phi \wedge \phi) \mid K_x\phi \mid B_x\phi$$

where $x \in \mathbf{X}$ and $P \in \mathbf{P}$.

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Models and Semantics

We define the semantics of FOEDL over first-order Kripke models.

Definition

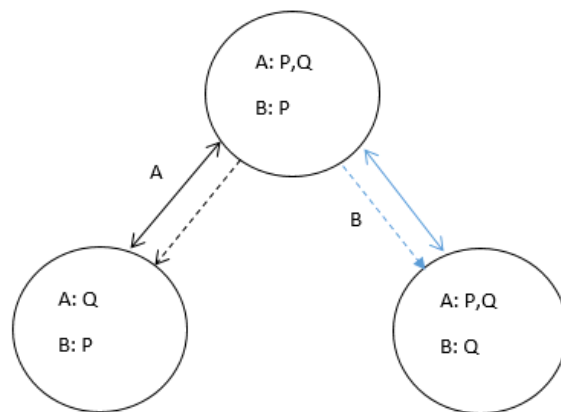
A first-order Kripke model M for FOEDL is a tuple $\langle W, A, R_K, R_B, \rho \rangle$ where:

- W is a non-empty set of possible worlds.
- A is a finite set of agents.
- $R_K : A \rightarrow 2^{W \times W}$ assign a binary relation $R_K(i)$ (also written R_{Ki}) between worlds, to each agent $i \in A$. R_{Ki} are equivalence relations.
- $R_B : A \rightarrow 2^{W \times W}$ assign a binary relation $R_B(i)$ (also written R_{Bi}) between worlds, to each agent $i \in A$. R_{Bi} are serial.
- $\rho : P \times W \rightarrow 2^A$ assigns an unary relation $\rho(P, w)$ between agents to each unary predicate P at each possible worlds w .
- $R_B \subseteq R_K$

Notice: for any formula ϕ , $\models K_x \phi \rightarrow B_x \phi$.

Models and Semantics

A sample for FOEDL-model:



where A and B are agents, P and Q are unary-predicates.

Models and Semantics

- $M, w, \sigma \models Px \Leftrightarrow \sigma_w(x) \in \rho(P, w)$
- $M, w, \sigma \models \neg\phi \Leftrightarrow M, w, \sigma \not\models \phi$
- $M, w, \sigma \models (\phi \wedge \psi) \Leftrightarrow M, w, \sigma \models \phi \text{ and } M, w, \sigma \models \psi$
- $M, w, \sigma \models K_x\phi \Leftrightarrow M, w, \sigma \models \phi \text{ for all } v \text{ s.t. } wR_{\sigma_w(x)}v$
- $M, w, \sigma \models x \approx y \Leftrightarrow \text{for any predicate } P, [\sigma_w(x) \in \rho(P, w) \Leftrightarrow \sigma_w(y) \in \rho(P, w)]$

Remark

The semantics of $x \approx y$ is inspired by Leibniz Principle:

$$\forall x \forall y \forall P (x = y \leftrightarrow (Px \leftrightarrow Py))$$

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The Meanings of Predicates

- P_1 : can make explosives
- P_2 : sleep with Marla
- P_3 : burn Jack's condo
- P_4 : has 'that' suitcase
- P_5 : has a scar on right hand

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Step 1: Suitcase

Jack and Tyler first meet on an airplane where it reveals they both got the same looking suitcase.

Knowledge change in (M, r)

$$\begin{aligned}\neg K_J P_4 T &\longrightarrow K_J P_4 T \\ \neg K_T K_J P_4 T &\longrightarrow K_T K_J P_4 T\end{aligned}$$

Then

$$\begin{aligned}(M, r) &\models K_J P_4 T \wedge K_J P_4 J \\ (M, r) &\models K_T K_J P_4 T\end{aligned}$$

Knowledge about the identity

$$(M, r) \models \neg K_T \neg K_J (J \approx T)$$

Step 4: Scar

Tyler burns a scar on Jack's right hand and reveals the same looking scar on his right hand.

Knowledge change in (M, r)

$$\neg K_J P_5 J \longrightarrow K_J P_5 J$$
$$\neg K_J P_5 T \longrightarrow K_J P_5 T$$

Then

$$(M, r) \models K_J P_5 J \wedge K_J P_5 T$$

Knowledge about the identity

$$(M, r) \models \neg K_T \neg K_J (J \approx T)$$

$$(M, r) \models \neg K_J \neg (J \approx T)$$

Step 2: Explosives

Tyler knows how to make explosives, Jack thinks it is possible he gained the knowledge from Tyler.

Knowledge change in (M, r)

$$\begin{aligned}\neg K_J P_1 T &\longrightarrow K_J P_1 T \\ \neg K_J P_1 J &\longrightarrow \neg K_J \neg P_1 J\end{aligned}$$

Then

$$(M, r) \models \neg K_J \neg (P_1 T \wedge P_1 J)$$

Knowledge about the identity

$$(M, r) \models \neg K_T \neg K_J (J \approx T)$$

$$(M, r) \models \neg K_J \neg (J \approx T)$$

Step 3 + 6: To sleep with Marla

Tyler is sleeping with Marla while Jack is dreaming about that he is sleeping with her. Jack calls Marla for confirmation.

Knowledge change in (M,r)

$$\text{(dream)} \quad B_J \neg P_2 J \longrightarrow \neg B_J \neg P_2 J$$

$$\text{(phone)} \quad \neg B_J \neg P_2 J \longrightarrow B_J P_2 J$$

Then

$$(M, r) \models B_J P_2 T \wedge \neg B_J \neg P_2 J$$

after (phone)

$$(M, r) \models B_J P_2 T \wedge B_J P_2 J$$

Knowledge about the identity

$$(M, r) \models \neg K_T \neg K_J (J \approx T)$$

$$(M, r) \models \neg K_J \neg (J \approx T)$$

$$(M, r) \models \neg K_J \neg K_M (J \approx T) \text{ (dream)}$$

$$(M, r) \models K_J K_M (J \approx T) \text{ (phone)}$$

Step 5 + 7: Condo

The police suspects Jack with burning his own condo, Tyler suggests that that is the case. Later he admits that he burnt Jack's condo.

Knowledge change in (M, r)

(police) $B_J \neg P_3 J \longrightarrow \neg B_J \neg P_3 J$

(tyler) $\neg K_J P_3 T \longrightarrow K_J P_3 T$

Then

$(M, r) \models K_T P_3 J \wedge \neg B_J \neg P_3 J$

$(M, r) \models K_T P_3 T \wedge K_J P_3 T$

Knowledge about the identity

$(M, r) \models \neg K_T \neg K_J (J \approx T)$

$(M, r) \models \neg K_J \neg (J \approx T)$

$(M, r) \models K_J K_M (J \approx T)$

$(M, r) \models K_J K_T (J \approx T)$

Jack proves he is Tyler by killing Tyler with shooting himself on the head.

Knowledge about the identity

$$(M, r) \models K_J K_M (J \approx T)$$

$$(M, r) \models K_J K_T (J \approx T)$$

$$(M, r) \models K_J (J \approx T)$$

References

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- ② Y. Wang and J. Seligman, “When names are not commonly known: Epistemic logic with assignments”, *ArXiv preprint arXiv:1805.03852*, 2018

Thanks for your attention!