Who am I: the Knowledge Change in *Fight Club*

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Outline

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Motivation

We plan to explore the knowledge change in the movie *Fight Club*. The film tells a story of a man, Jack, with schizophrenia who finally identifies his another personality, Tyler.

- The agents involved are the same person physically.
- The knowledge of Jack is always changing during the storyline.
- It is related to a interesting philosophical topic: identity.

Outline

A Brief Storyline

Consecutively through the movie there are cues hinted about the identity of Jack and Tyle:

- Step 1: Jack found Tyler has a totally same suitcase with his.
- Step 2: Tyler showed the method in making explosives.
- Step 3: Jack had a dream that he slept with Marla.
- Step 4: Jack found that Tyler had a same scar on his right hand.
- Step 5: Police suspected that it was Jack himself burnt Jack's condo.
- Step 6: Marla confirmed that Jack slept with Marla.
- Step 7: Tyler said it was Jack burnt Jack's condo.

Outline

First-order Epistemic-Doxastic Logic

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First-order Epistemic-Doxastic Logic

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The language of FOEDL:

Definition

Given a denumerable set of variables $\mathbf{X}$, and a denumerable set of unary predicate symbols, the language FOEDL is defined as:

$$\phi ::= (x \approx y) \mid Px \mid \neg\phi \mid (\phi \wedge \phi) \mid K_x\phi \mid B_x\phi$$

where $x \in \mathbf{X}$ and $P \in \mathbf{P}$.

Outline

Models and Semantics

We define the semantics of FOEDL over first-order Kripke models.

Definition

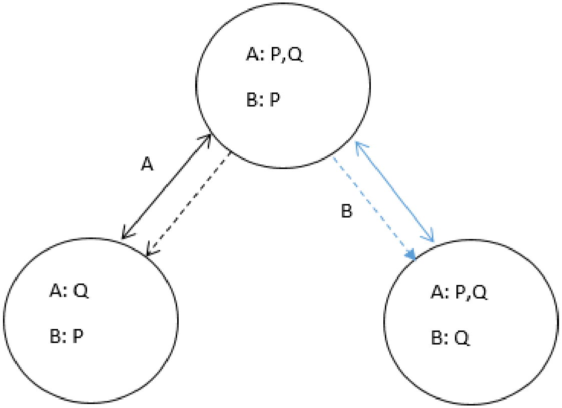
A first-order Kripke model M for FOEDL is a tuple $\langle W, A, R_K, R_B, \rho \rangle$ where:

- W is a non-empty set of possible worlds.
- A is a finite set of agents.
- $R_K : A \to 2^{W \times W}$ assign a binary relation $R_K(i)$ (also written R_{Ki}) between worlds, to each agent $i \in A$. R_{Ki} are equivalence relations.
- $R_B : A \to 2^{W \times W}$ assign a binary relation $R_B(i)$ (also written R_{Bi}) between worlds, to each agent $i \in A$. R_{Bi} are serial.
- $\rho : P \times W \to 2^A$ assigns an unary relation $\rho(P, w)$ between agents to each unary predicate P at each possible worlds w.
- $R_B \subseteq R_K$

Notice: for any formula ϕ, $\models K_x\phi \to B_x\phi$.

Models and Semantics

A sample for FOEDL-model:



where A and B are agents, P and Q are unary-predicates.

Models and Semantics

- $M, w, \sigma \models Px \;\Leftrightarrow\; \sigma_w(x) \in \rho(P, w)$
- $M, w, \sigma \neg\phi \;\Leftrightarrow\; M, w, \sigma \not\models \phi$
- $M, w, \sigma \models (\phi \wedge \psi) \;\Leftrightarrow\; M, w, \sigma \models \phi \, and \, M, w, \sigma \models \psi$
- $M, w, \sigma \models K_x\phi \Leftrightarrow\; M, w, \sigma \models \phi \, for \, all \, v \, s.t. \, wR_{\sigma_w(x)}v$
- $M, w, \sigma \models x \approx y \;\Leftrightarrow\; for \, any \, predicate \, P, \; [\sigma_w(x) \in \rho(P, w) \Leftrightarrow \sigma_w(y) \in \rho(P, w)]$

Remark

The semantics of $x \approx y$ is inspired by Leibniz Principle:

$$\forall x \forall y \forall P(x = y \leftrightarrow (Px \leftrightarrow Py))$$

Outline

The Meanings of Predicates

- P_1: can make explosives
- P_2: sleep with Marla
- P_3: burn Jack's condo
- P_4: has 'that' suitcase
- P_5: has a scar on right hand

Outline

Step 1: Suitcase

Jack and Tyler first meet on an airplane where it reveals they both got the same looking suitcase.

Knowledge change in (M,r)

$$\neg K_J P_4 T \longrightarrow K_J P_4 T$$
$$\neg K_T K_J P_4 T \longrightarrow K_T K_J P_4 T$$

Then

$(M,r) \models K_J P_4 T \wedge K_J P_4 J$
$(M,r) \models K_T K_J P_4 T$

Knowledge about the identity

$(M,r) \models \neg K_T \neg K_J (J \approx T)$

Step 4: Scar

Tyler burns a scar on Jack's right hand and reveals the same looking scar on his right hand.

Knowledge change in (M,r)

$$\neg K_J P_5 J \longrightarrow K_J P_5 J$$
$$\neg K_J P_5 T \longrightarrow K_J P_5 T$$

Then

$(M, r) \models K_J P_5 J \wedge K_J P_5 T$

Knowledge about the identity

$(M, r) \models \neg K_T \neg K_J (J \approx T)$
$(M, r) \models \neg K_J \neg (J \approx T)$

Step 2: Explosives

Tyler knows how to make explosives, Jack thinks it is possible he gained the knowledge from Tyler.

Knowledge change in (M,r)

$$\neg K_J P_1 T \longrightarrow K_J P_1 T$$
$$\neg K_J P_1 J \longrightarrow \neg K_J \neg P_1 J$$

Then

$(M, r) \models \neg K_J \neg (P_1 T \wedge P_1 J)$

Knowledge about the identity

$(M, r) \models \neg K_T \neg K_J (J \approx T)$
$(M, r) \models \neg K_J \neg (J \approx T)$

Step 3 + 6: To sleep with Marla

Tyler is sleeping with Marla while Jack is dreaming about that he is sleeping with her. Jack calls Marla for confirmation.

Knowledge change in (M,r)

$$\text{(dream)}\ B_J \neg P_2 J \longrightarrow \neg B_J \neg P_2 J$$
$$\text{(phone)}\ \neg B_J \neg P_2 J \longrightarrow B_J P_2 J$$

Then

$(M, r) \vDash B_J P_2 T \wedge \neg B_J \neg P_2 J$

after (phone)

$(M, r) \vDash B_J P_2 T \wedge B_J P_2 J$

Knowledge about the identity

$(M, r) \vDash \neg K_T \neg K_J (J \approx T)$

$(M, r) \vDash \neg K_J \neg (J \approx T)$

$(M, r) \vDash \neg K_J \neg K_M (J \approx T)$ (dream)

$(M, r) \vDash K_J K_M (J \approx T)$ (phone)

Step 5 + 7: Condo

The police suspects Jack with burning his own condo, Tyler suggests that that is the case. Later he admits that he burnt Jack's condo.

Knowledge change in (M,r)

(police) $B_J \neg P_3 J \longrightarrow \neg B_J \neg P_3 J$

(tyler) $\neg K_J P_3 T \longrightarrow K_J P_3 T$

Then

$(M,r) \models K_T P_3 J \wedge \neg B_J \neg P_3 J$

$(M,r) \models K_T P_3 T \wedge K_J P_3 T$

Knowledge about the identity

$(M,r) \models \neg K_T \neg K_J (J \approx T)$

$(M,r) \models \neg K_J \neg (J \approx T)$

$(M,r) \models K_J K_M (J \approx T)$

$(M,r) \models K_J K_T (J \approx T)$

Conclusion

Jack proves he is Tyler by killing Tyler with shooting himself on the head.

Knowledge about the identity

$(M, r) \vDash K_J K_M (J \approx T)$
$(M, r) \vDash K_J K_T (J \approx T)$
$(M, r) \vDash K_J (J \approx T)$

References

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Thanks for your attention!