ATIBHAV MITTAL
804598987

CS174A, HW#2, Part I

1.a) $P_{viewing} = A \times P_{torso}$

$P_{torso} = C \times P_{head}$

$P_{head} = D \times P_{ear}$

$P_{viewing} = (ACD)\, P_{ear}$

$M = A \times C \times D$ will transform a point in ear co-ordinate system to viewing co-ordinates.

b) For the tail camera, we would want to compute the co-ordinates w.r.t to the tail tip co-ordinates.

Assuming that world co-ordinates are the same as the torso co-ordinates,

we would use $B = KLM$

2. To calculate the position of a particle at time $(t+1)$ using Explicit Euler, given the force, velocity and position of the particle at time t, we do:

$$\vec{a}_t = \frac{\vec{F}_{t,\,total}}{m} \leftarrow \text{Using Newton's second law}$$

$$\vec{v}_{t+1} = \vec{V}_t + \vec{a}_t \Delta t = \vec{V}_t + \vec{a}_t(t+1-t)$$

$$= \vec{V}_t + \vec{a}_t$$

$$\vec{x}_{t+1} = \vec{x}_t + \vec{v}_{t+1}(t+1-t) = \vec{x}_t + \vec{v}_t + \vec{a}_t$$

3. The total force on a node in a spring-mass system can be calculated as:

$$\vec{F}_{total} = \vec{g} - \gamma\vec{v} + \vec{f}_{ext}$$

where $\vec{g}$ is the force on the node due to other springs connected to this node.
γ is the damping coefficient.
$\vec{f}_{ext}$ is the external force on the node.

```
zeroize_forces(num_nodes, nds);
spring_forces(num_springs, sprs);
damping_forces(num_nodes, nds);
external_forces(num_nodes, nds);
vscale(1/nds.mass, nds.force, accel);
vscale(dt, accel, delta_vel);
vplus(delta_vel, nds.velocity, nds.velocity);
vscale(dt, nds.velocity, delta_pos);
vinc(delta_pos, nds.position);
```

4. a) For a mass-spring model with non-zero length spring, we would need to implement an internal spring force

$$\vec{g}_i = \sum_{j \in N_i} \vec{g}_{ij} \quad (N_i \text{ is neighbor nodes of } i)$$

$$\vec{g}_{ij} = k_{ij} \, e_{ij} \, \frac{\vec{d}_{ij}}{\|\vec{d}_{ij}\|}$$

(k_{ij} is spring const b/w i and j
e_{ij} is spring deformation length
d_{ij} is distance between node i and j)

b) For a cloth-viscoelasticity model, we would have to account for the spring force associated with the rest length, i.e. rest length stiffness.

Rest length stiffness force $= C_{ij} \vec{r_i}$

$$C_{ij} = \frac{k_{ij} e_{ij} + \gamma_{ij} e'_{ij}}{\|\vec{r}_{ij}\|} \qquad \vec{r}_i = \text{position of } i^{th} \text{ particle}$$

c) $\theta \rightarrow$ temperature: We would have to account for a force due to heat-diffusion.

$$\theta^{t+\Delta t}_{u,v,w} = \theta^{t}_{u,v,w} + \frac{\Delta t \, c}{\mu \sigma} \left[\frac{\theta^t_{u+\Delta u,v,w} - 2\theta^t_{u,v,w} + \theta^t_{u-\Delta u,v,w}}{\Delta u^2} + \frac{\theta^t_{u,v+\Delta v,w} - 2\theta^t_{u,v,w} + \theta^t_{u,v+\Delta v,w}}{\Delta v^2} + \frac{\theta^t_{u,v,w+\Delta w} - 2\theta^t_{u,v,w} + \theta^t_{u,v,w-\Delta w}}{\Delta w^2} \right] + q$$

d) Internal forces of long range attraction and short term repulsion forces need to be modelled.

5. $m = 1$ $\quad \vec{x}(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\quad \vec{f}_{ext} = \begin{bmatrix} 2 \\ 14.7 \\ -5 \end{bmatrix}$ $\quad \vec{f}_{grav} = \begin{bmatrix} 0 \\ -9.8 \\ 0 \end{bmatrix}$

At $t=0$, both $\vec{f}_{ext}$ and $\vec{f}_{grav}$ act on particle.
Every time instant after, only $\vec{f}_{grav}$ acts on particle.

$$\vec{F}_{total,t} = \vec{f}_{ext,t} + \vec{f}_{grav,t}, \quad \vec{a}_t = \frac{F_{total,t}}{m} = F_{total,t}$$

$$\vec{V}_{t+1} = \vec{V}_t + \vec{a}_t\, dt = \vec{V}_t + \vec{a}_t \quad (dt = 1)$$

$$\vec{x}_{t+1} = \vec{x}_t + \vec{V}_{t+1}\, dt = \vec{x}_t + \vec{V}_t + \vec{a}_t \quad (dt = 1)$$

At $t=1$, $\quad \vec{a}_0 = \begin{bmatrix} 2 \\ 14.7 \\ -5 \end{bmatrix}$ $\quad \vec{x}_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\quad \vec{v}_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\vec{x}_1 = \begin{bmatrix} 2 \\ 14.7 \\ -5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 14.7 \\ -5 \end{bmatrix} \qquad \vec{V}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 14.7 \\ -5 \end{bmatrix} = \begin{bmatrix} 2 \\ 14.7 \\ -5 \end{bmatrix}$$

for all $t > 0$ $\quad a_t = \begin{bmatrix} 0 \\ -9.8 \\ 0 \end{bmatrix}$

$$\vec{x}_2 = \begin{bmatrix} 2 \\ 14.7 \\ -5 \end{bmatrix} + \begin{bmatrix} 2 \\ 14.7 \\ -5 \end{bmatrix} + \begin{bmatrix} 0 \\ -9.8 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 19.6 \\ -10 \end{bmatrix} \qquad \vec{V}_2 = \begin{bmatrix} 2 \\ 14.7 \\ -5 \end{bmatrix} + \begin{bmatrix} 0 \\ -9.8 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 4.9 \\ -5 \end{bmatrix}$$

$$\vec{x}_3 = \begin{bmatrix} 4 \\ 19.6 \\ -10 \end{bmatrix} + \begin{bmatrix} 2 \\ 4.9 \\ -5 \end{bmatrix} + \begin{bmatrix} 0 \\ -9.8 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 14.7 \\ -15 \end{bmatrix} \qquad \vec{V}_3 = \begin{bmatrix} 2 \\ 4.9 \\ -5 \end{bmatrix} + \begin{bmatrix} 0 \\ -9.8 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -4.9 \\ -5 \end{bmatrix}$$

$$\vec{x}_4 = \begin{bmatrix} 6 \\ 14.7 \\ -15 \end{bmatrix} + \begin{bmatrix} 2 \\ -4.9 \\ -5 \end{bmatrix} + \begin{bmatrix} 0 \\ -9.8 \\ 0 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \\ -20 \end{bmatrix}$$

Position of particle is $\begin{bmatrix} 8 \\ 0 \\ -20 \end{bmatrix}$

6. $\vec{P}_{eye} \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} \quad \vec{P}_{ref} = \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix} \quad \vec{V}_{up} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$

$$\vec{k} = \frac{\vec{P}_{eye} - \vec{P}_{ref}}{\|\vec{P}_{eye} - \vec{P}_{ref}\|} = \frac{\begin{bmatrix} 4 \\ 8 \\ 3 \end{bmatrix}}{\sqrt{4^2+8^2+3^2}} = \frac{1}{\sqrt{89}} \begin{bmatrix} 4 \\ 8 \\ 3 \end{bmatrix} = \begin{bmatrix} 0.424 \\ 0.848 \\ 0.318 \end{bmatrix}$$

$$\vec{i} = \frac{\vec{V}_{up} \times \vec{k}}{|\vec{V}_{up} \times \vec{k}|} = \frac{\left(\begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} \times \begin{bmatrix} 4 \\ 8 \\ 3 \end{bmatrix} \right)}{|\vec{V}_{up} \times \vec{k}|} = \frac{(-3, 3, -4)^T}{\sqrt{9+9+16}} = \frac{1}{\sqrt{34}} \begin{bmatrix} -3 \\ 3 \\ -4 \end{bmatrix}$$

$$j = \vec{k} \times \vec{i} = \frac{1}{\sqrt{89}} \cdot \frac{1}{\sqrt{34}} \left(\begin{bmatrix} 4 \\ 8 \\ 3 \end{bmatrix} \times \begin{bmatrix} -3 \\ 3 \\ -4 \end{bmatrix} \right) = \frac{1}{\sqrt{3026}} \begin{bmatrix} 23 \\ -25 \\ 36 \end{bmatrix}$$

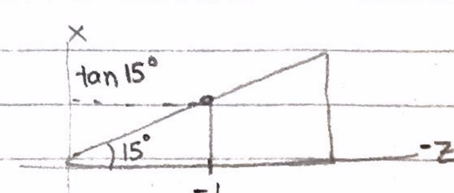
$$\vec{k} = \begin{bmatrix} 0.424 \\ 0.848 \\ 0.318 \end{bmatrix} \quad \vec{i} = \begin{bmatrix} -0.514 \\ 0.514 \\ -0.686 \end{bmatrix} \quad \vec{j} = \begin{bmatrix} 0.418 \\ -0.454 \\ 0.654 \end{bmatrix}$$

To compute the viewing transformation matrix, we need M_{cam}^{-1}

$$M_{cam}^{-1} = \begin{bmatrix} -0.514 & 0.514 & -0.686 & 0 \\ 0.418 & -0.454 & 0.654 & 0 \\ 0.424 & 0.848 & 0.318 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -10 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.514 & 0.514 & -0.686 & -2 \\ 0.418 & -0.454 & 0.654 & -10 \\ 0.424 & 0.848 & 0.318 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

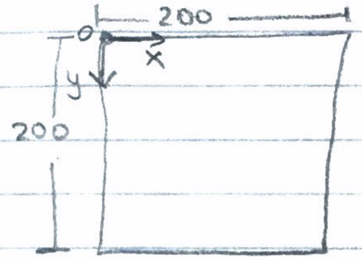
7. $$M_{proj} = \begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{l+r}{l-r} & 0 \\ 0 & \frac{2n}{t-b} & \frac{b+t}{b-t} & 0 \\ 0 & 0 & \frac{-f+n}{f-n} & \frac{2fn}{f-n} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



$r = \tan 15°$, $l = -\tan 15°$, $t = \tan 30°$, $b = -\tan 30°$, $f = 100$, $n = 1$
$= 0.268$ $\quad = -0.268$ $\quad = 0.577$ $\quad = -0.577$ $\quad f = 100$ $\quad n = 1$

$$M_{proj} = \begin{bmatrix} \frac{2(1)}{2(0.268)} & 0 & 0 & 0 \\ 0 & \frac{2(1)}{2(0.577)} & 0 & 0 \\ 0 & 0 & \frac{101}{99} & \frac{200}{99} \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 3.731 & 0 & 0 & 0 \\ 0 & 1.732 & 0 & 0 \\ 0 & 0 & 1.02 & 2.02 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

8. $range(x) = 0$ to 200
$range(y) = 0$ to 200
$n_x = 200$
$n_y = 200$



$$M_{vp} = \begin{bmatrix} \frac{n_x}{2} & 0 & 0 & \frac{n_x - 1}{2} \\ 0 & -\frac{n_y}{2} & 0 & \frac{n_y - 1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 100 & 0 & 0 & 99.5 \\ 0 & -100 & 0 & 99.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

9. Let the answer from problem 6 be a 4x4 matrix, M

- $(3,2,1,1)^T$
$$M \cdot \begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3.2 \\ -9 \\ 0.286 \\ 1 \end{bmatrix}$$

- $(0,0,-3,1)^T$
$$= M \begin{bmatrix} 0 \\ 0 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.058 \\ -11.96 \\ -3.95 \\ 1 \end{bmatrix}$$

- $(-2,-1,2,1)^T$
$$= M \begin{bmatrix} -2 \\ -1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -2.86 \\ -9.07 \\ -4.06 \\ 1 \end{bmatrix}$$

- $(1,5,-1,1)^T$
$$= M \begin{bmatrix} 1 \\ 5 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.742 \\ -12.506 \\ 1.346 \\ 1 \end{bmatrix}$$

These points give the vertices in the camera co-ordinate system.

10. Let P_1, P_2, P_3, P_4 be the points calculated from # 9 and M be the matrix calculated in # 7.

$$P_{1,ccs} = Mp_1 = \begin{bmatrix} -11.94 \\ -15.59 \\ 2.31 \\ 0.286 \end{bmatrix} \qquad P_{2,ccs} = Mp_2 = \begin{bmatrix} 0.216 \\ -20.71 \\ -2.009 \\ -3.95 \end{bmatrix}$$

$$P_{3,ccs} = Mp_3 = \begin{bmatrix} -7.46 \\ -1.73 \\ 4.06 \\ 2 \end{bmatrix} \qquad P_{4,ccs} = Mp_3 = \begin{bmatrix} 2.77 \\ -21.66 \\ 3.39 \\ 1.35 \end{bmatrix}$$

11. To convert to NDS, we divide by the 4^{th} co-ordinate.

$$P_{1,NDCS} = \frac{1}{0.286} \begin{bmatrix} -11.94 \\ -15.59 \\ 2.31 \\ 0.286 \end{bmatrix} = \begin{bmatrix} -41.75 \\ -54.51 \\ 8.077 \\ 1 \end{bmatrix}$$

$$P_{2,NDCS} = \frac{1}{-3.95} = \begin{bmatrix} -0.055 \\ 5.24 \\ 0.508 \\ 1 \end{bmatrix} \qquad P_{3,NDCS} = \frac{1}{2} \begin{bmatrix} -7.46 \\ -1.73 \\ 4.06 \\ 2 \end{bmatrix} = \begin{bmatrix} -3.73 \\ -0.865 \\ 2.03 \\ 1 \end{bmatrix}$$

$$P_{4,NDCS} = \frac{1}{1.35} \begin{bmatrix} 2.77 \\ -21.66 \\ 3.39 \\ 1.35 \end{bmatrix} = \begin{bmatrix} 2.05 \\ -16.04 \\ 2.51 \\ 1 \end{bmatrix}$$

12. Let's assume the answer from # 8 is a 4x4 matrix M, and the points p_1, p_2, p_3, p_4 be the answer from # 11.

$$p_{1,DCS} = M \cdot p_{1,NDCS} = \begin{bmatrix} 100 & 0 & 0 & 99.5 \\ 0 & -100 & 0 & 99.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -41.75 \\ -54.51 \\ 8.077 \\ 1 \end{bmatrix} = \begin{bmatrix} -4075.5 \\ 5550.5 \\ 8.077 \\ 1 \end{bmatrix}$$

$$p_{2,DCS} = M \cdot p_{2,NDCS} = \begin{bmatrix} 100 & 0 & 0 & 99.5 \\ 0 & -100 & 0 & 99.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -0.055 \\ 5.24 \\ 0.508 \\ 1 \end{bmatrix} = \begin{bmatrix} 94 \\ -424.5 \\ 0.508 \\ 1 \end{bmatrix}$$

$$p_{3,DCS} = M \cdot p_{3,NDCS} = \begin{bmatrix} 100 & 0 & 0 & 99.5 \\ 0 & -100 & 0 & 99.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3.73 \\ -0.865 \\ 2.03 \\ 1 \end{bmatrix} = \begin{bmatrix} -273.5 \\ 186 \\ 2.03 \\ 1 \end{bmatrix}$$

$$p_{4,DCS} = M \cdot p_{4,NDCS} = \begin{bmatrix} 100 & 0 & 0 & 99.5 \\ 0 & -100 & 0 & 99.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2.05 \\ -16.04 \\ 2.51 \\ 1 \end{bmatrix} = \begin{bmatrix} 304.5 \\ 1703.5 \\ 2.51 \\ 1 \end{bmatrix}$$