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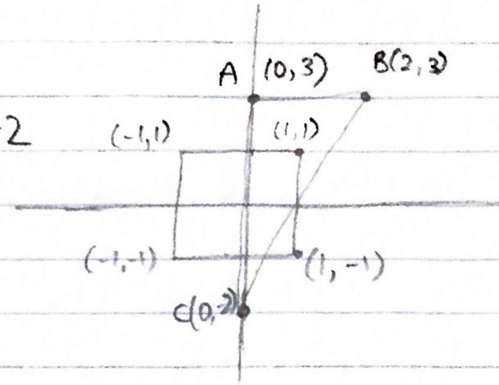
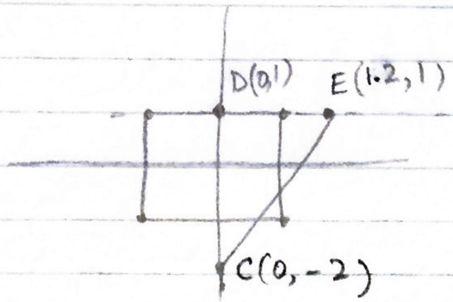
CS174A (HW3)

1. Equation of AB : $y = 3$
Equation of AC : $x = 0$
Equation of BC : $y = 2.5x - 2$

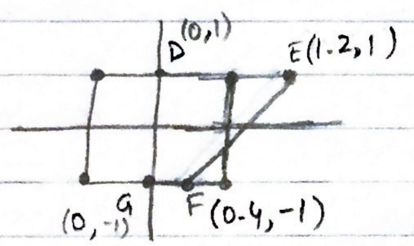


a)



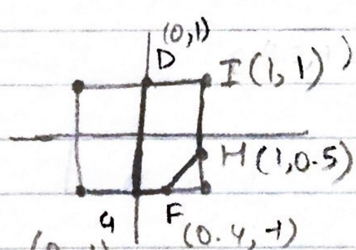
To calculate co-ords of C:
Intersection of $y = 1$, $y = 2.5x - 2$
$2.5x = 3$
$x = 1.2$

b)



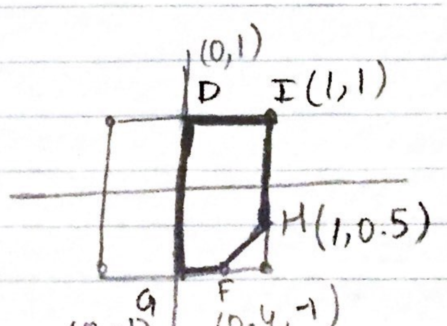
Co-ords of F: Intersection of $y = 2.5x - 2$, $y = -1$
$x = \frac{1}{2.5} = 0.4$
F : (0.4, −1)
Co-ords of G: Intersection of $x = 0$, $y = -1$
G = (0, −1)

c)



Co-ords of I: (1,1) (Intersection of $x = 1$, $y = 1$)
Co-ords of H: Intersection of $y = 2.5x - 2$, $x = 1$
$\Rightarrow y = 2.5 - 2 = y = 0.5$
H: (1, 0.5)

d)



The Final polygon is given by DIHFG.

2\. a) Implicit equation:

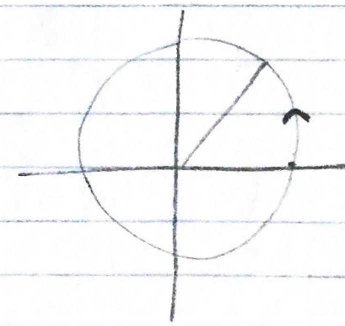
$$(x-x_0)^2 + (y-y_0)^2 - r^2 = 0$$

b) For a naive algorithm, we need the explicit equation

$$y = y_0 \pm \sqrt{r^2-(x-x_0)^2}$$

```
DrawCircle(x0, y0, r):
    for x from (x0-r) to (x0+r):
        y = y0 + sqrt(r^2 - (x - x0)^2)
        SetPixel(x, Round(y))
        y = y0 - sqrt(r^2 - (x - x0)^2)
        SetPixel(x, Round(y))
    end
end
```

c) For first octant, move either north or north-west.
decide between $(x-1, y+1)$ and $(x, y+1)$



```
DrawCircle(x0, y0, r):
    int x, y, d, dW, dNW, end_x;
    end_x = Round(x0 + r cos(45°));    y = y0
    for (x = x0 + r; x >= end_x;        ){
        Set Pixel(x, y);
        d = (x - x0)^2 + (y - y0)^2 - r^2
        if (d < 0)
            y = y + 1;
        else {
            x = x - 1;  y = y + 1;
        }
}
```

For second octant,
we similarly decide between the pixels $(x-1, y+1)$ and $(x-1, y)$
if $d < 0$

$x = x - 1;\ y = y + 1$

else

$y = x - 1$

For third octant, decide b/w pixels $(x-1, y)$, $(x-1, y-1)$

if $d < 0$

$x = x - 1$

else: $x = x - 1,\ y = y - 1$

For 4^{th} octant, decide b/w pixels $(x, y-1)$ $(x-1, y-1)$

if $d < 0$

$x = x - 1,\ y = y - 1$

else

$y = y - 1$

For 5^{th} octant, decide b/w pixels $(x, y-1)$ and $(x+1, y-1)$

if $d < 0$

$y = y - 1$

else

$x = x + 1,\ y = y - 1$

For 6^{th} octant decide b/w pixels $(x+1, y-1)$ and $(x+1, y)$

if $d < 0$

$y = y - 1,\ x = x + 1$

else

$x = x + 1$

For 7^{th} octant, decide b/w pixels $(x+1, y)$ and $(x+1, y+1)$

if $d < 0$:

$x = x + 1$

else : $x = x + 1,\ y = y + 1$

For 8^{th} octant, decide b/w pixels $(x, y+1)$ and $(x+1, y+1)$

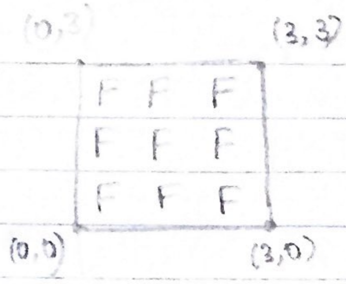
if $d < 0$:

$x = x + 1,\ y = y + 1$

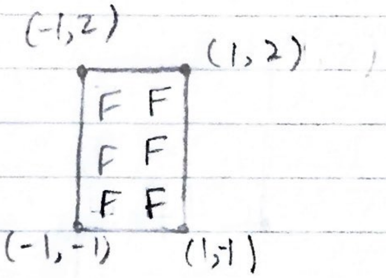
else

$y = y + 1$

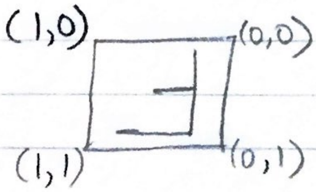
3. a)



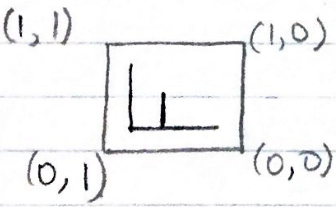
b)



c)



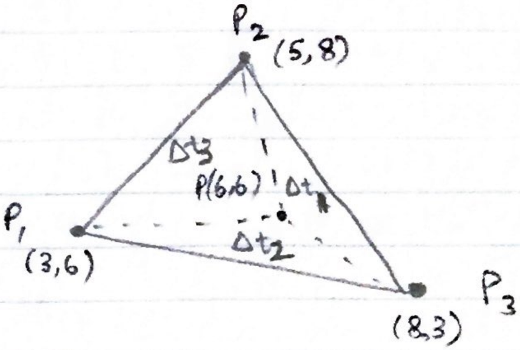
d)



4. $\alpha = \Delta t_2$

$\beta = \Delta t_3$

$\gamma = \Delta t_1$



$$\Delta t_1 = \frac{\overset{((P_2-P)\times(P_3-P))}{|(-1,2,0)\times(2,-3,0)|}}{|(3,-5,0)\times(-2,-2,0)|} = \frac{|(0,0,-1)|}{|(0,0,-16)|} = \frac{1}{16}$$

$$\Delta t_2 = \frac{|(P_1-P)\times(P_3-P)|}{|(0,0,-16)|} = \frac{|(-3,0,0)\times(2,-3,0)|}{|(0,0,-16)|}$$

$$= \frac{|(0,0,9)|}{|(0,0,-16)|} = \frac{9}{16}$$

$$\Delta t_3 = 1 - \Delta t_1 - \Delta t_2 = \frac{6}{16}$$

Barycentric Co-ordinates of P : $\left(\frac{1}{16}, \frac{9}{16}, \frac{6}{16}\right)$

$$r,g,b \text{ at point} = \frac{1}{16}(0.3,0.2,0.9) + \frac{9}{16}(0.5,0.5,0.9) + \frac{6}{16}(0.7,0.2,0.7)$$

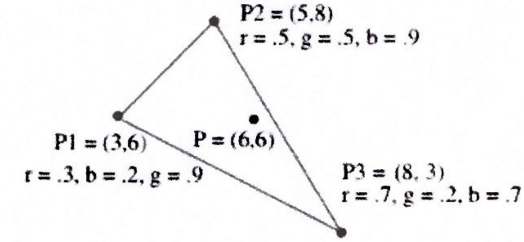
$$= \left(\frac{0.3+4.5+4.2}{16}, \frac{0.2+4.5+1.2}{16}, \frac{0.9+8.1+4.2}{16}\right)$$

$$= \left(\frac{9}{16}, \frac{5.9}{16}, \frac{13.2}{16}\right) = (0.56, 0.37, 0.83)$$

5. b) Local illumination takes place between viewing and projection transformations. This is because we still want the light reflections/ interactions on all objects before our scene is modified by projection and clipping.

0.4

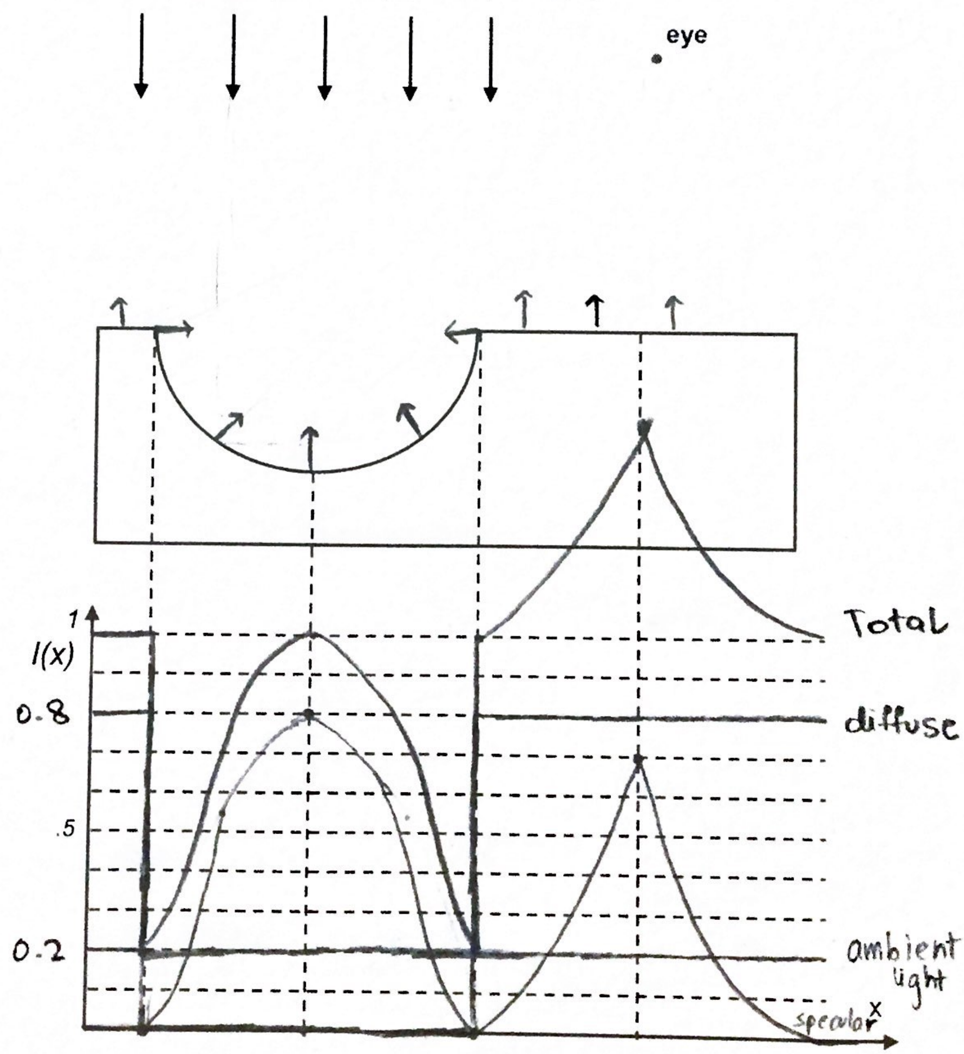
4. **Interpolation** (4 pts)
Find the barycentric coordinates for P, and use them to interpolate the (r, g, b) color component at that point. Show your work.



5. **Local Illumination** (18 pts)
a) (16 pts) Sketch the illumination that would be computed for the above scene using the Phong illumination model. The scene is lit from above using a directional light source that is coming directly from above. Use 4 sketches: one for ambient, one for diffuse, one for specular and one for the total illumination. The Phong illumination model is given by:

$$I = I_d k_d (\mathbf{n} \cdot \mathbf{l}) + I_s k_s (\mathbf{r} \cdot \mathbf{v})^n + I_a k_a$$

where $I_d = I_a = I_s = 1.0$, $k_a = 0.2$, $k_d = 0.8$, $k_s = 0.7$, $n = 100$.



6. $I_a = (0.1, 0.2, 0.1)$

$I_L = (1.0, 1.0, 0.9)$

$k_d = (0.3, 0.8, 0.9)$

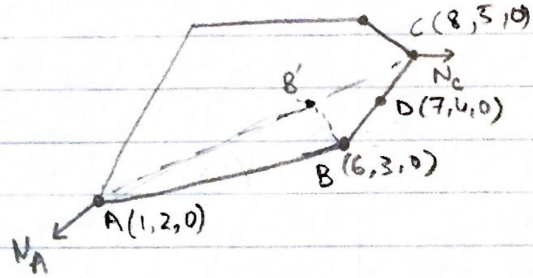
$k_a = (0.1, 0.1, 0.1)$

$k_s = (1,1,1)$

$n = 20$

a) $\vec{N_A} = (-1,-1,0)$ $\quad$ $\vec{N_c} : (1,0,0)$

$\hat{N_A} = \left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0\right)$ $\quad$ $\hat{N_c} = (1,0,0)$



$\vec{AC} = (7,3,0)$

$\vec{AB} = (5,1,0)$

$$\vec{AB'} = \frac{\vec{AC}}{|\vec{AC}|}\left(\frac{\vec{AB}\cdot\vec{AC}}{|\vec{AC}|}\right) = \vec{AC}\left(\frac{\vec{AB}\cdot\vec{AC}}{|\vec{AC}|^2}\right) = \frac{38}{58}\vec{AC}$$

$$= \frac{19}{29}(7,3,0) = \left(\frac{133}{29}, \frac{57}{29}, 0\right)$$

Point $B' = (1,2,0) + \left(\frac{133}{29}, \frac{57}{29}, 0\right) = \left(\frac{162}{29}, \frac{115}{29}, 0\right)$

$$|\vec{AB'}| = \sqrt{\left(\frac{133}{29}\right)^2 + \left(\frac{57}{29}\right)^2} = \frac{144.7}{29} \sim 5$$

$$|\vec{B'C}| = \sqrt{\left(8 - \frac{162}{29}\right)^2 + \left(5 - \frac{115}{29}\right)^2} = 2.6$$

$$\vec{N_B} = \frac{2.6}{7.6}\vec{N_A} + \frac{5}{7.6}\vec{N_C} = 0.34\left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0\right) + 0.65(1,0,0)$$

$$= (0.411, -0.2438, 0)$$

$\hat{N_B} : (0.86, -0.51, 0)$

b) Blinn - Phong Lighting model

$$I_{total} = k_a I_a + I_L k_d (n \cdot l) + I_L k_s (H \cdot N)^n$$

For point C: $\vec{H_c} = (0,-1,0)$

Ambient $= (0.01, 0.02, 0.01)$

Diffuse $: (0.3, 0.8, 0.81) [(1,0,0) \cdot (0,1,0)] = (0,0,0)$

Specular $= (1,1,0.9) [(0,-1,0) \cdot (1,0,0)]^{20} = (0,0,0)$

Total $= (0.01, 0.02, 0.01)$

For point D:

Since its flat shaded, it has the same lighting as C.

Ambient $= (0.01, 0.02, 0.01)$

Specular $: (0,0,0)$

Diffuse $= (0,0,0)$

Total $= (0.01, 0.02, 0.01)$

For Point B:

$$\vec{H_B} = \frac{(2,-2,0)+(2,-4,0)}{|(4,-6,0)|} = \left(\frac{4}{\sqrt{52}}, \frac{-6}{\sqrt{52}}, 0\right) = (0.55, -0.83, 0)$$

$$\vec{N_B} = (0.86, -0.51, 0) \qquad \vec{l} = \frac{(2,-2,0)}{\sqrt{8}} = \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0\right)$$

$I_{ambient} = (0.01, 0.02, 0.01)$

$$\begin{aligned} I_{diffuse} &= (1,1,0.9)(0.3,0.8,0.9)\left[(0.86,-0.51,0)\cdot\left(\tfrac{1}{\sqrt{2}}, \tfrac{-1}{\sqrt{2}}, 0\right)\right] \\ &= (0.3, 0.8, 0.81)\,[0.608 + 0.3605 + 0] \\ &= (0.3, 0.8, 0.81)(0.968) \\ &= (0.29, 0.774, 0.784) \end{aligned}$$

$$\begin{aligned} I_{specular} &= (1,1,0.9)\left[(0.55,-0.83,0)\cdot(0.86,-0.51,0)\right] \\ &= (1,1,0.9)(0.896)^{20} \\ &= (0.11, 0.11, 0.10) \end{aligned}$$

$$\begin{aligned} \text{Total Illumination} &= (0.01, 0.02, 0.01) + (0.29, 0.774, 0.784) + (0.11, 0.11, 0.10) \\ &= (0.41, 0.90, 0.89) \end{aligned}$$

c) For Gourad Shading, using Blinn-Phong Lighting, illumination at points B and C remains the same.

Point B

$I_{amb} = (0.01, 0.02, 0.01)$

$I_{dif} = (0.29, 0.774, 0.784)$

$I_{spec} = (0.11, 0.11, 0.1)$

$I_{total} = (0.41, 0.90, 0.89)$

Point C

$I_{amb} = (0.01, 0.02, 0.01)$

$I_{dif} = (0, 0, 0)$

$I_{spec} = (0, 0, 0)$

$I_{total} = (0.01, 0.02, 0.01)$

Point D

Since Point D is midway between point B and C, we just average out the lighting.

$I_{amb} = \frac{2}{2}(0.01, 0.02, 0.01) = (0.01, 0.02, 0.01)$

$I_{dif} = \frac{1}{2}(0.29, 0.774, 0.784) = (0.145, 0.387, 0.392)$

$I_{spec} = (0.055, 0.055, 0.05)$

$I_{total} = (0.21, 0.46, 0.45)$

d) For Phong shading, using Blinn-Phong lighting, illumination at points B and C remains the same.

Point B

$I_{amb} = (0.01, 0.02, 0.01)$

$I_{dif} = (0.29, 0.774, 0.784)$

$I_{spec} = (0.11, 0.11, 0.1)$

$I_{total} = (0.41, 0.90, 0.89)$

Point C

$I_{amb} = (0.01, 0.02, 0.01)$

$I_{dif} = (0, 0, 0)$

$I_{spec} = (0, 0, 0)$

$I_{total} = (0.01, 0.02, 0.01)$

Point D

Since point D is midway between B and C

$$\vec{N}_D = \frac{1}{2}(1.86, -0.51, 0) = (0.93, -0.255, 0)$$

$$\hat{N}_D = (0.96, -0.26, 0)$$

$$\vec{l}_D = \frac{(1, -3, 0)}{\sqrt{10}} = (0.316, -0.949, 0)$$

$$\vec{h}_D = \frac{(0.31, -0.94, 0) + (0.19, -0.98, 0)}{1.984} = (0.25, -0.967, 0)$$

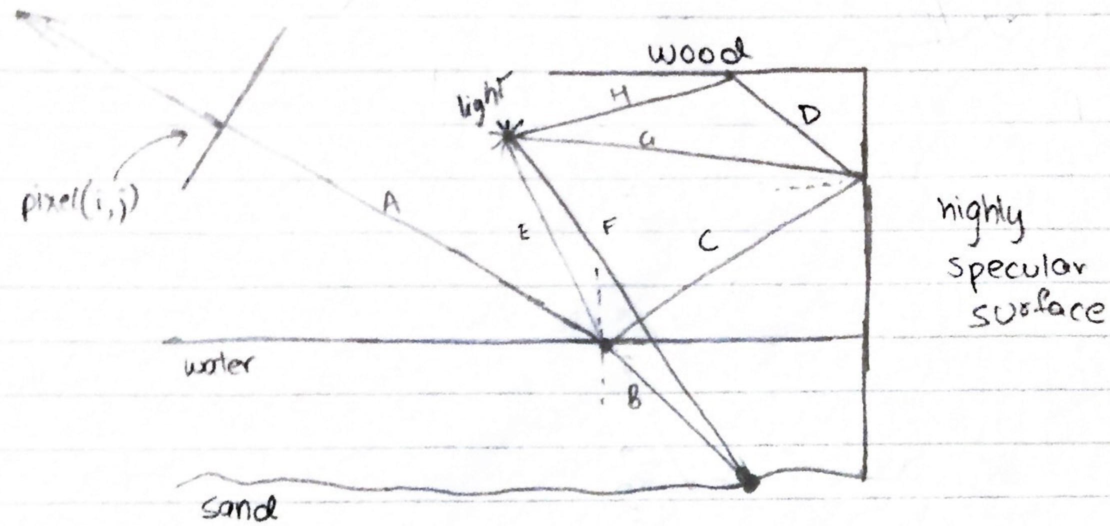
$$I_{amb} = (0.01, 0.02, 0.01)$$

$$I_{diff} = (0.3, 0.8, 0.81)[0.55] = (0.165, 0.44, 0.445)$$

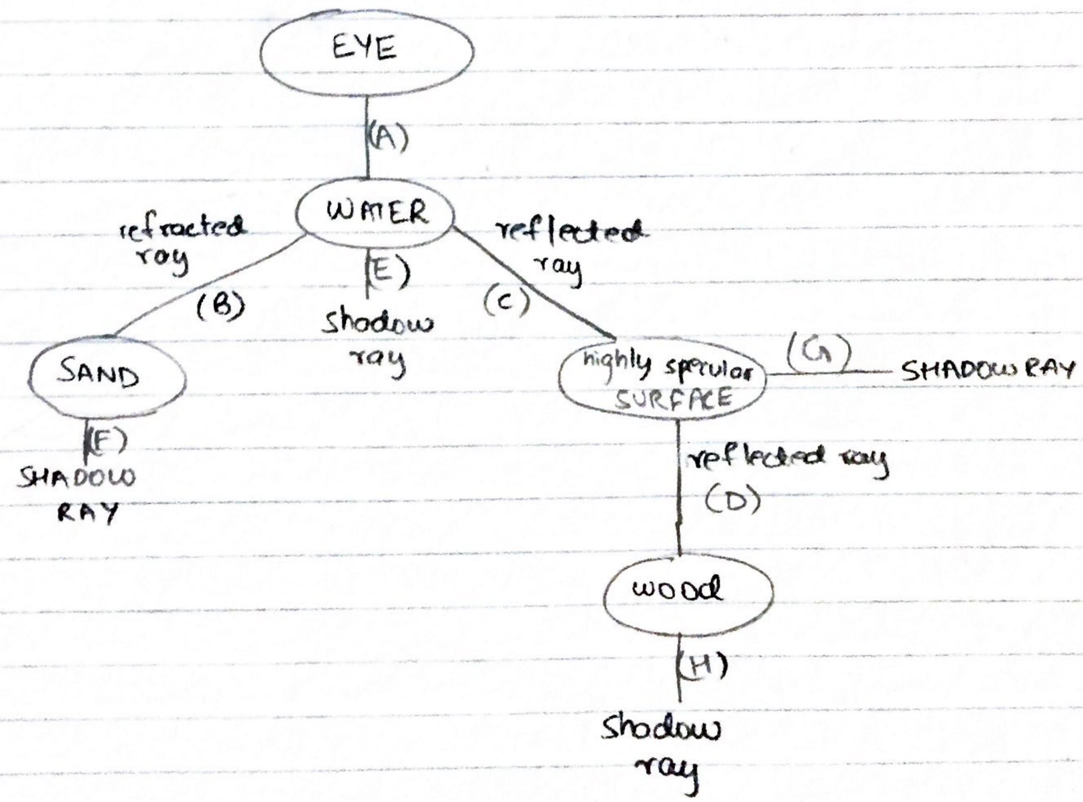
$$I_{spec} = (1, 1, 0.9)(0.489)^{20} = (0, 0, 0)$$

$$I_{total} = (0.175, 0.46, 0.455)$$

7. a)


wood
light
H
G
D
pixel(i,j)
A
E
F
C
highly specular surface
water
B
sand

b)


EYE
(A)
WATER
refracted ray
(B)
(E)
shadow ray
reflected ray
(C)
SAND
(F)
SHADOW RAY
highly specular SURFACE
(G)
SHADOW RAY
reflected ray
(D)
wood
(H)
shadow ray

8. a) $\vec{p}(t) = \vec{a}_0 + \vec{a}_1 t + \vec{a}_2 t^2 + \vec{a}_3 t^3$

3:1

where our parameter is t, and $\vec{a}_0, \vec{a}_1, \vec{a}_2, \vec{a}_3$ are coefficients.

b) $p'(t) = \vec{a}_1 + 2\vec{a}_2 t + 3\vec{a}_3 t^2$

$p''(t) = 2\vec{a}_2 + 6\vec{a}_3 t$

c) $\vec{p}(0) = \vec{a}_0 = \vec{P}_0$
$\vec{p}'(0) = \vec{a}_1 = \vec{T}_0$
$\vec{p}''(0) = 2\vec{a}_2 = \vec{A}_0 \Rightarrow \vec{a}_2 = \dfrac{\vec{A}_0}{2}$

$\vec{p}(1) = \vec{a}_0 + \vec{a}_1 + \vec{a}_2 + \vec{a}_3 = \vec{P}_1$

$\Rightarrow \vec{a}_3 = \vec{P}_1 - \vec{P}_0 - \vec{T}_0 - \dfrac{\vec{A}_0}{2}$

Basis Matrix: $\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1/2 & -1/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$