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# CS 174A (Assignment 1, Part 1)

1\. a) Co-ordinate System A

$$\vec{p} = O_A + 2\vec{i}_A - \vec{j}_A$$

System B

$$\vec{p} = O_B + 3\vec{i}_B + \vec{j}_B$$

System C

$$\vec{p} = O_C - 3\vec{i}_C + \frac{7}{3}\vec{j}_C$$

b) 2 points are required to describe a vector.

System A

$$\vec{v} = 2\vec{i}_A - \vec{j}_A$$

System B

$$\vec{v} = -2\vec{i}_B - 1.5\vec{j}_B$$

System C

$$\vec{v} = \vec{i}_C$$

c) A co-ordinate frame is represented using basis vectors and an origin.

d) $M_A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ $M_B = \begin{bmatrix} -1 & 0 & 6 \\ -1 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ $M_C = \begin{bmatrix} 2 & 3 & 2 \\ -1 & -3 & 5 \\ 0 & 0 & 1 \end{bmatrix}$

e) $\vec{p}_A = (2,-1)$ $\vec{p}_B = (3,1)$ $\vec{p}_C = (-3, 7/3)$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 6 \\ -1 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 & 2 \\ -1 & -3 & 5 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -3 \\ 7/3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

Each of these multiplications give $\vec{p} = (3,1)$, which are the co-ordinates of $\vec{p}$ in the world frame.

2. $M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

3. modelMatrix = modelMatrix * Scale(1, 1, 2);
modelMatrix = modelMatrix * Translate(1, 1, 1);

4. $[0.5 \;\; 2.5 \;\; 2 \;\; 1]^T$

5. At point A,

$M_A = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

At point B,

$M_B = \begin{bmatrix} 0 & 0 & -1 & 2 \\ 0 & 1 & 0 & 3 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

At point C,

$M_C = \begin{bmatrix} 0 & 0 & -1 & 2 \\ 0 & 0.5 & 0 & 3.5 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

At point D,

$M_D = \begin{bmatrix} 0 & 0 & -1 & 2 \\ 0 & 1 & 0 & 3 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

6. Let the equation of the line be

$$y = x\tan\theta + b$$

Steps to reflect:

1. Translate by $(0, -b)$
2. Rotate by $-\theta$
3. Reflect around x-axis
4. Rotate by $\theta$
5. Translate by $(0, b)$

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -b \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\theta & \sin 2\theta & -b\sin 2\theta \\ \sin 2\theta & -\cos 2\theta & b\cos 2\theta \\ 0 & 0 & 1 \end{bmatrix}$$

OpenGL shader code:

```
modelMatrix.setAsIdentity();
modelMatrix = modelMatrix * Translate(0,b,0);
modelMatrix = modelMatrix * RotateZ(theta);
modelMatrix = modelMatrix * Scale(1,-1,1);
modelMatrix = modelMatrix * RotateZ(-theta);
modelMatrix = modelMatrix * Translate(0,-b,0);
```

7. Sequence of Commands:

a)
```
modelMatrix = modelMatrix * Scale(2,1,1);
modelMatrix = modelMatrix * Translate(1,1,0);
modelMatrix = modelMatrix * RotateZ(90); drawL();
```

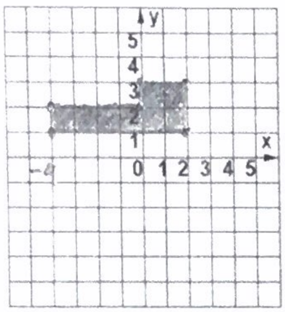
b)
```
modelMatrix = modelMatrix * RotateZ(90);
modelMatrix = modelMatrix * Scale(2,1,1);
modelMatrix = modelMatrix * Scale(-1,1,1); drawL();
```

c)
```
modelMatrix = modelMatrix * RotateZ(90);
modelMatrix = modelMatrix * Translate(1,1,0);
modelMatrix = modelMatrix * Scale(-1,1,1); drawL();
```

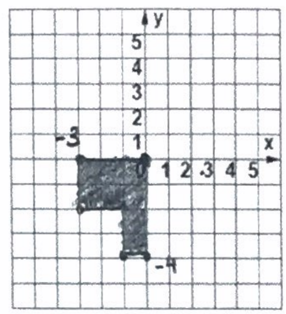
d)
```
modelMatrix = modelMatrix * Scale(-1,1,1);
modelMatrix = modelMatrix * RotateZ(180);
modelMatrix = modelMatrix * Scale(2,1,1);
modelMatrix = modelMatrix * Scale(-1,1,1); drawL();
```

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

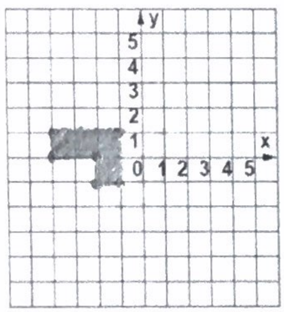
a) L' = ABC L



b) L' = CAD L



c) L' = CBD L



d) L' = DCCAD L