CM146, Winter 2018
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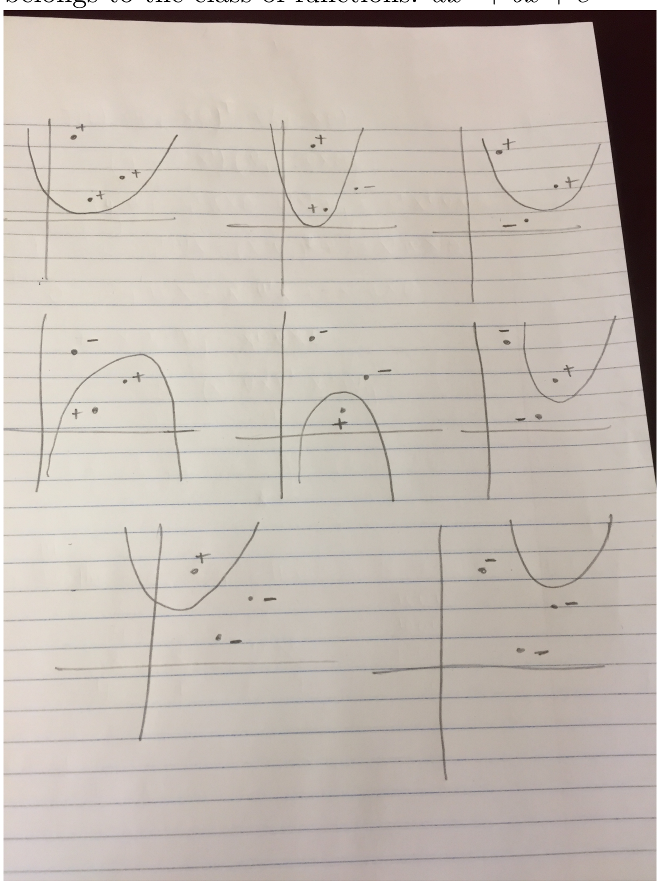
1 Problem 1

(a) Problem 1a **Solution:**
The VC dimension of the given function is 3 The following images show that for any labeling of 3 points, we can find a decision boundary that belongs to the class of functions: $ax^2 + bx + c$



This means that VC dimension ≥ 3
However, consider 4 points given to us. Out of these 4 points, one point will have the largest x-axis value, one will have the smallest x-axis value, one will have the largest y-axis value and one will have the

smallest y-axis value. Since the function only finds decision boundaries that are either upward or downward facing parabolas, a labeling where the points with the following labelling cannot be satisfied

i. largest x-value point: +

ii. smallest x-value point: +

iii. largest y-value point: -

iv. smallest y-value point: -

Hence, VC dimension < 4 and VC Dimension ≥ 3
Thus, VC dimension $= 3$

2 Problem 2

Solution:

Assume that $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$

$$K_\beta(x, z) = (1 + \beta x \cdot z)^3$$

$$\begin{aligned} K_\beta(x, z) =& x_1^3 \beta^3 z_1^3 + x_2^3 \beta^3 z_2^3 + 3x_1 x_2^2 \beta^3 z_1 z_2^2 + 3x_1^2 x_2 \beta^3 z_1^2 z_2 + 3x_1^2 \beta^2 z_1^2 + 3x_2^2 \beta^2 z_2^2 + \\ & 6x_1 x_2 \beta^2 z_1 z_2 + 3x_1 \beta z_1 + 3x_2 \beta z_2 + 1 \end{aligned}$$

$$\Rightarrow \phi_\beta(\boldsymbol{x}) = \begin{bmatrix} 1 \\ \sqrt{3}\sqrt{\beta} x_1 \\ \sqrt{3}\sqrt{\beta} x_2 \\ \sqrt{6}\beta x_1 x_2 \\ \sqrt{3}\beta x_1^2 \\ \sqrt{3}\beta x_2^2 \\ \sqrt{3}\beta^{3/2} x_1^2 x_2 \\ \sqrt{3}\beta^{3/2} x_1 x_2^2 \\ \beta^{3/2} x_1^3 \\ \beta^{3/2} x_2^3 \end{bmatrix}$$

The kernel function K_β is similar to the $K(x, z) = (1 + x \cdot z)^3$, when $\beta = 1$
The only similarity when $\beta \neq 1$, is the constant 1, which is the first feature of this higher dimensional space.
The role of beta is to provide a weighing factor. The beta factor, scales each feature differently, as shown above. For richer/more complex features, if $\beta > 1$, it weighs the higher order features more in the kernel, and thus they have a larger contribution in the inner product. If $\beta < 1$, it weighs the lower order features more, and the higher order features less. If $\beta = 1$, it weighs all the features in the kernel equally.

3 Problem 3

(a) **Solution:**

Consider $w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$

$$w^T x_1 = w_1 + w_2$$

$$w^T x_2 = w_1$$

Using the values of the labels, we get the inequalities:

$$w_1 + w_2 \geq 1$$

$$w_1 \leq -1 \Rightarrow w_1^2 \geq 1$$

$$w_2 \geq 2 \Rightarrow w_2^2 \geq 4$$

Since we're trying to minimize $\frac{1}{2}||w||^2$, we can instead minimize $w_1^2 + w_2^2$
Hence, $w_1 = -1, w_2 = 2$

$$w^* = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

(b) **Solution:**

If we allow a non-zero offset, the best separation would pass through the center, and the projection from each of the lines to the separating hyperplane would be perpendicular to it. This corresponds to:

$$w^* = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, b^* = -1$$

4 Problem 4

4.1

(d) **Solution:**
The number of unique words in the dataset is 1811.
Hence, the dimensionality of the dataset is 1811.

4.2

(b) Its beneficial to maintain class proportions across folds, because this gives us a better picture of the actual dataset in the sense that the actual proportions of the dataset are reflected. This enables us to have a model that generalizes well.

(d) **Solution:**

C	accuracy	F1-score	AUROC
10^{-3}	0.7089	0.8297	0.8105
10^{-2}	0.7107	0.8306	0.8111
10^{-1}	0.8060	0.8755	0.8575
10^{0}	0.8146	0.8749	0.8712
10^{1}	0.8182	0.8766	0.8696
10^{2}	0.8182	0.8766	0.8696
best C	10	10	1

For the accuracy measurement, we can see that the performance improves as C increases. However, we can see that it slowly approaches the value 0.8182, and will probably not increase much for larger C, and will probably decrease for more C.
For the F1-Score measurement, the C value continues to increase till C = 10, and then becomes fairly constant, similar to the accuracy measurement. However, the performance according to this metric is better than the performance measured using accuracy. (This does not say anything about which metric is better though).
For the AUROC metric, the C value first increases till C=1, and then

decreases after that. The performance measured is comparable to the performance measured using F1-score.

4.3

(a) From the above results, we choose the following c values:

i. Accuracy: C = 10
ii. F1-Score: C = 10
iii. AUROC: C = 1

(c) The results on the test set are as follows:

i. Accuracy: 0.7429
ii. F1-Score: 0.4375
iii. AUROC: 0.7405