CM146, Fall 2017
Problem Set 1: Atibhav Mittal (ID:804598987)
Due Jan 30, 2017

1 Problem 1

(a) Problem 1a

Solution:
Without splitting the data even once, we have two possible cases $y = 1$ or $y = 0$

If we predict $y = 1$ always, number of mistakes $= 2^{n-3}$ If we predict $y = 0$ always, number of mistakes $= 2^n - 2^{n-3} = 7 \cdot 2^{n-3}$

Hence, the best 1-leaf decision tree will make at least 2^{n-3} mistakes

(b) Problem 1b

Solution:
No there is no such split that reduces the number of mistakes by at least 1. Since our target function is $X_1 \vee X_2 \vee X_3$, the best case would be to have the first internal node, split based on one of these indicators. Because of symmetry, it doesn't matter which indicator we choose. So let's say the first decision node split based on X_1

If $X_1 = 1$, we predict $y = 1$ If $X_1 = 0$, we can either predict 0 or 1. In case we predict y = 1, number of mistakes $= 2^{n-3}$ In case we predict y = 0, number of mistakes $= 2^{n-1} - 2^{n-3} = 3 \cdot 2^{n-3}$

Hence, the minimum number of mistakes we'll make is still 2^{n-3}

(c) Problem 1c **Solution:**

$$Entropy(S) = -\left(\frac{2^n - 2^{n-3}}{2^n}\right)\log\left(\frac{2^n - 2^{n-3}}{2^n}\right) - \left(\frac{2^{n-3}}{2^n}\right)\log\left(\frac{2^{n-3}}{2^n}\right)$$

$$Entropy(S) = \frac{4.4}{8} = 0.55$$

(d) Problem 1d **Solution:** Yes, a split based on X_1 would reduce the entropy.

$\text{Entropy}(X_1 = 1) = -0\log(0) - 1\log(1) = 0$
$\text{Entropy}(X_1 = 0) = -\left(\frac{2^{n-1}-2^{n-3}}{2^{n-1}}\right)\log\left(\frac{2^{n-1}-2^{n-3}}{2^{n-1}}\right) - \left(\frac{2^{n-3}}{2^{n-1}}\right)\log\left(\frac{2^{n-3}}{2^{n-1}}\right)$
$\text{Entropy}(X_1 = 0) = 0.812$
$\text{Conditional Entropy} = \frac{2^{n-1}}{2^n}(0) + \frac{2^{n-1}}{2^n}(0.812) = 0.406$

Hence, this split reduces the entropy by $0.55 - 0.406 = 0.144$

2 Problem 2

Solution:

Entropy Before the split $= B\left(\frac{p}{p+n}\right)$

$$= -\left(\frac{p}{p+n}\right)\log\left(\frac{p}{p+n}\right) - \left(\frac{n}{p+n}\right)\log\left(\frac{n}{p+n}\right)$$

Entropy After the Split $= \sum_{i=1}^{k} \frac{p_k+n_k}{p+n}$ Entropy(S_k)

Entropy After the Split $= \frac{p_k+n_k}{p+n}$ (Entropy $(S_k)) \sum_{i=1}^{k} 1$
Entropy After the Split $= k \cdot \frac{p_k+n_k}{p+n}$ (Entropy (S_k))
Entropy After the Split $= \frac{k\cdot(p_k+n_k)}{p+n}$ (Entropy $(S_k)) = \frac{p+n}{p+n}$ (Entropy (S_k))

Entropy After the Split = Entropy (S_k)

Entropy$(S_k) = -\left(\frac{p_k}{p_k+n_k}\right)\log\left(\frac{p_k}{p_k+n_k}\right) - \left(\frac{n_k}{p_k+n_k}\right)\log\left(\frac{n_k}{p_k+n_k}\right)$

The fraction $\frac{p_k}{p_k+n_k} = \frac{p}{p+n}$ (Since $p = kp_k$ and $n = kn_k$)
Entropy$(S_k) = = -\left(\frac{p}{p+n}\right)\log\left(\frac{p}{p+n}\right) - \left(\frac{n}{p+n}\right)\log\left(\frac{n}{p+n}\right)$

This is equal to the Entropy Before the split.

Information Gain $= Entropy_{before} - Entropy_{after} = 0$
Hence, proved

3 Problem 3

(a) **Solution:**
The value of k = 1 minimizes the training error. Since every point is a neighbor of itself, the training error = 0

(b) **Solution:**
Using large values of k might be bad because there is no clear clustering of the positive and negative points, i.e. they are mixed well. Because of this, the negative points towards the bottom-right of the graph will be classified as positive incorrectly, and same with the negative points at the top left
Using really small values might be bad because for instance, k = 1, each point is classified as itself, which leads to overfitting. Furthermore other smaller values of k such as 2 or 3 will also increase the training error as the points aren't clustered together.

(c) **Solution:**
k = 5 minimizes the leave-one-out cross-validation error
The error in that case would be the 4 points that are in the top left or the bottom right.

4 Problem 4

Solution: Solution to problem 4

(a) i. Age: The survived category based on age is as expected since the number of people aged 20-40 will be higher than the rest. However, we can see that a larger percentage of infants and old people survived.

ii. Embarked: The graph shows that majority of the people boarded the ship at the port encoded as 2. A larger percentage of people who boarded the ship at port 0 survived, so it is possible that people boarding the ship at port 0 had a better chance for survival. But for port 0 and port 1, the trends are as expected where majority of the people did not survive.

iii. Fare: This graph shows that people who paid higher fares had a better chance of surviving. It also shows that a majority of the people paid lower fares, so the inference that higher fare corresponds to better chance of survival may be incorrect.

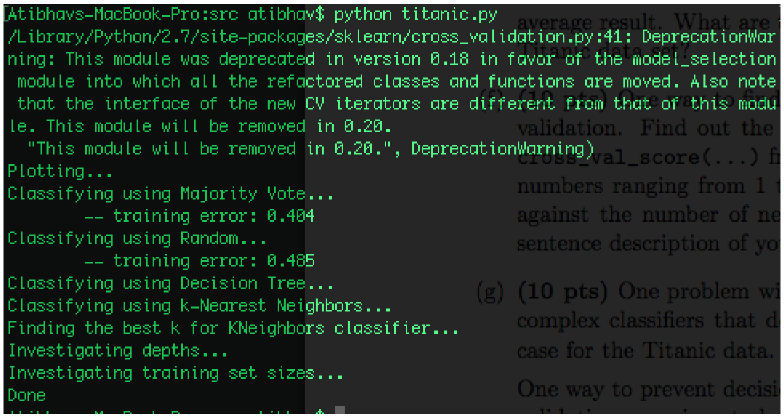
iv. PClass: The PClass graph shows that people who were in a better class of the ship had a better chance of survival. This could be due to earlier evacuation of people in first/second class compared to people in the third class.

v. Parch: This graph shows that most of the people didn't travel with their parents/children Those who did, had a better chance of survival because they were probably let off the ship first.

vi. Sex: The histogram generated shows that a majority of the female passengers survived. Hence, a female on Titanic would have a better chance of survival

vii. Sibsp: The graph shows that the number of people traveling with their siblings/spouses is pretty low and people traveling with only 1 sibling/spouse had a better chance of survival.

(b) **Solution:**



(c) **Solution:**
The training error using a DecisionTreeClassifier is: 0.014

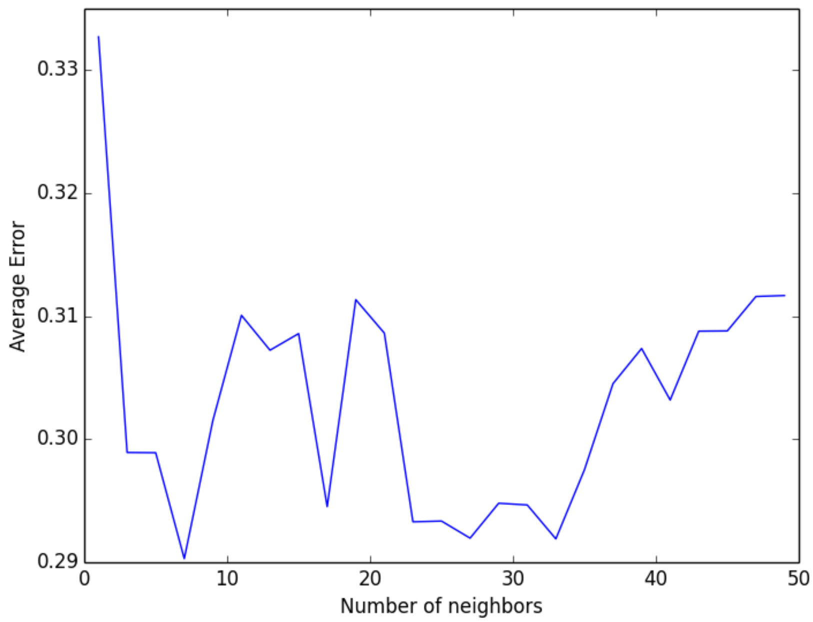
(d) **Solution:**

i. Training Error (k = 3): 0.167
ii. Training Error (k = 5): 0.201
iii. Training Error (k = 7): 0.240

(e) **Solution:**

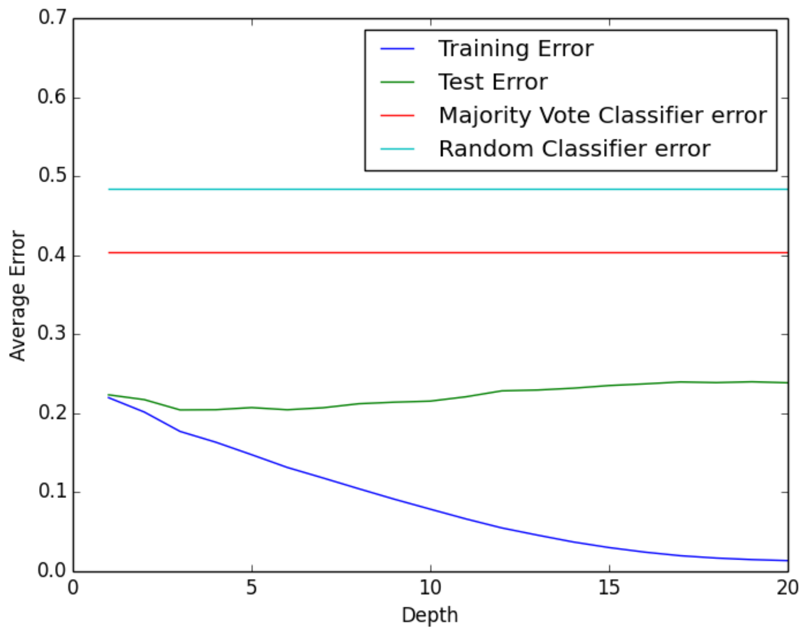
Classifier	Train Error	Test Error
Majority Vote Classifier	0.404	0.407
Random Classifier	0.489	0.486
Decision Tree Classifier	0.012	0.241
K-Neighbors Classifier	0.213	0.315

(f) **Solution:**



From the above graph, the validation error is minimum at k = 7
We can see that initially for low values of k, the error is high as expected. At k = 7, this error goes to its minimum and then increases after (k > 7) due to overfitting.

(g) **Solution:**



From the above graph we can see that the best depth limit to avoid overfitting is 6 that the test error is at its lowest. As depth limit $\rightarrow$ 10, the training error goes down but the test error increases. This is due to overfitting.

(h) **Solution:**

For both the Decision Tree and K-Nearest Neighbors classifier, the error is less than that of the baseline classifiers we used. For the Decision tree, the training error is quite low, and stays almost constant for most of the training data, whereas the test data has the lowest error for 80% of the training data available. For the K-Nearest Classifiers, the training error decreases, which means that the algorithm is learning from the training data, however the test data starts to increase after 50% of the training data, which could possibly be attributed to overfitting.