# OPTICS: Ordering Points To Identify the Clustering Structure

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Presented by Chris Mueller

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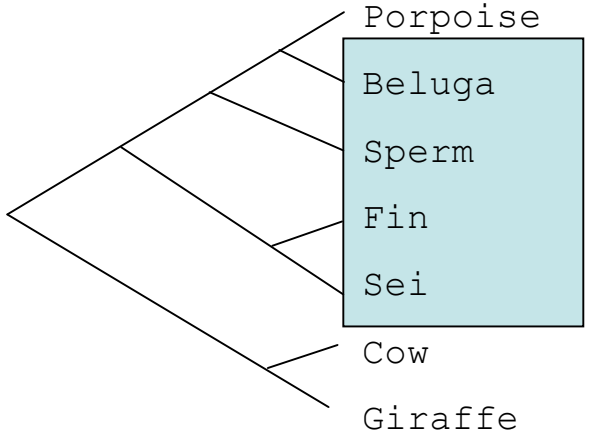
## Clustering

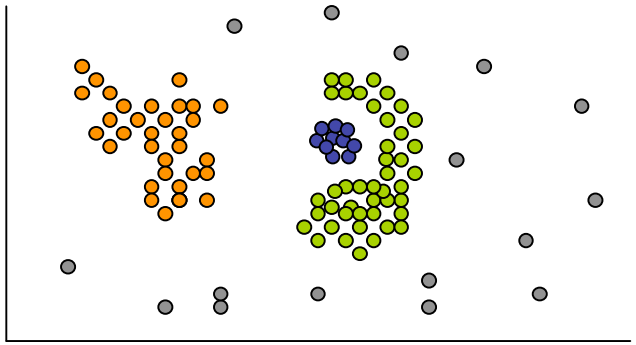
Goal: Group objects into meaningful subclasses as part of an exploratory process to insight into data or as a preprocessing step for other algorithms.

## Clustering Strategies

- Hierarchical
- Partitioning
  - k-means
  - **Density Based**

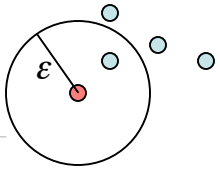



Density Based clustering requires a distance metric between points and works well on high dimensional data and data that forms irregular clusters.

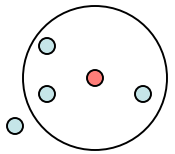


# DBSCAN: Density Based Clustering

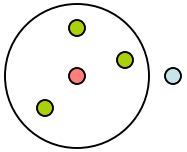
An object *p* is in the **ε-neighborhood** of *q* if the distance from *p* to *q* is less than ε.



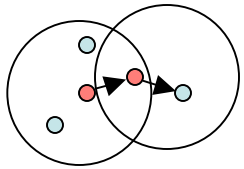

A **core object** has at least *MinPts* in its ε-neighborhood.



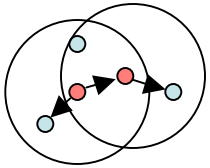

An object *p* is **directly density-reachable** from object *q* if *q* is a core object and *p* is in the ε-neighborhood of *q*.



An object *p* is **density-reachable** from object *q* if there is a chain of objects $p_1, \ldots, p_n$, where $p_1 = q$ and $p_n = p$ such that $p_{i+1}$ is directly density reachable from $p_i$.

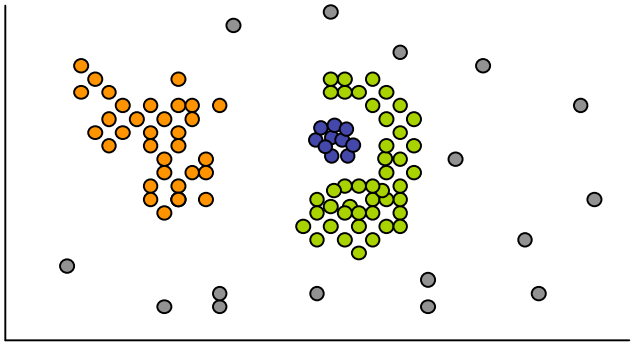


An object *p* is **density-connected** to object *q* if there is an object *o* such that both *p* and *q* are **density-reachable** from *o*.



A **cluster** is a set of density-connected objects which is maximal with respect to density-reachability.

**Noise** is the set of objects not contained in any cluster.



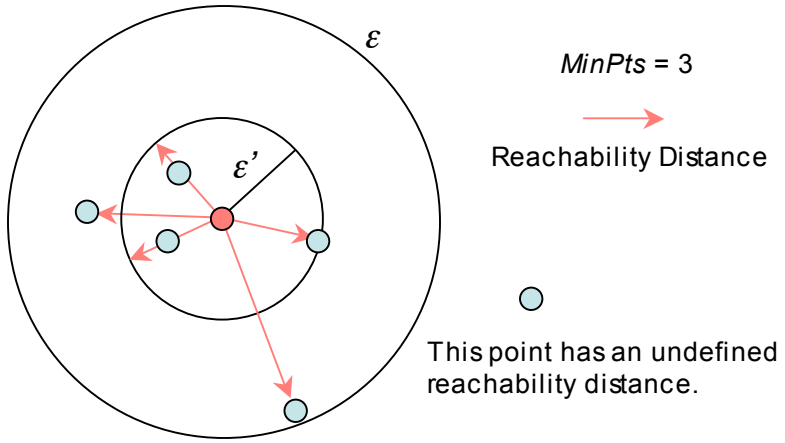
# OPTICS: Density-Based Cluster Ordering

OPTICS generalizes DB clustering by creating an ordering of the points that allows the extraction of clusters with arbitrary values for $\varepsilon$.

The **generating-distance** $\varepsilon$ is the largest distance considered for clusters. Clusters can be extracted for all $\varepsilon_i$ such that $0 \le \varepsilon_i \le \varepsilon$.

The **core-distance** is the smallest distance $\varepsilon'$ between *p* and an object in its $\varepsilon$-neighborhood such that *p* would be a core object.

The **reachability-distance** of *p* is the smallest distance such that *p* is density-reachable from a core object *o*.



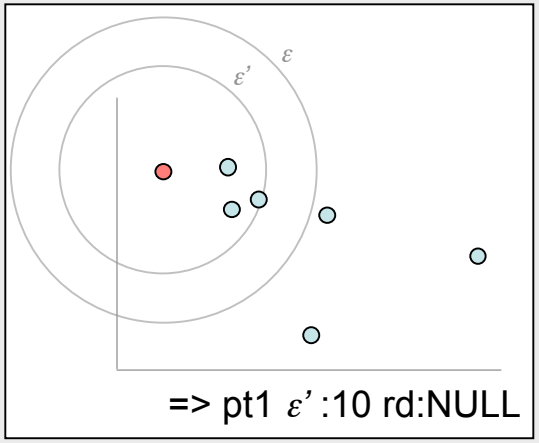

```
1  OPTICS(Objects, e, MinPts, OrderFile):
2    for each unprocessed obj in objects:
3      neighbors = Objects.getNeighbors(obj, e)
4      obj.setCoreDistance(neighbors, e, MinPts)
5      OrderFile.write(obj)
6      if obj.coreDistance != NULL:
7        orderSeeds.update(neighbors, obj)
8        for obj in orderSeeds:
9          neighbors = Objects.getNeighbors(obj, e)
10         obj.setCoreDistance(neighbors, e, MinPts)
11         OrderFile.write(obj)
12         if obj.coreDistance != NULL:
13           orderSeeds.update(neighbors, obj)
```

```
1  OrderSeeds::update(neighbors, centerObj):
2    d = centerObj.coreDistance
3    for each unprocessed obj in neighbors:
4      newRdist = max(d, dist(obj, centerObj))
5      if obj.reachability == NULL:
6        obj.reachability = newRdist
7        insert(obj, newRdist)
8      elif newRdist < obj.reachability:
9        obj.reachability = newRdist
10       decrease(obj, newRdist)
```

Get Neighbors, Calc Core Distance, Save Current Object

Calc/Update Reachability Distances

Update Processing Order

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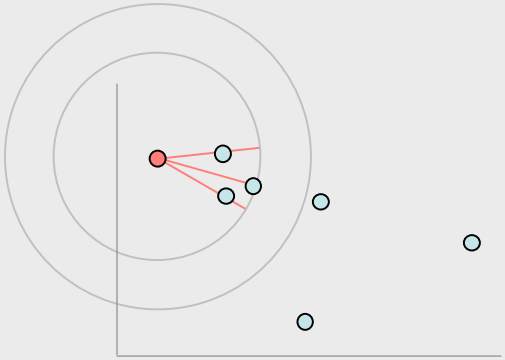
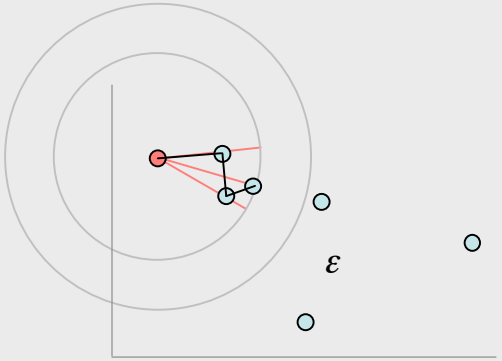
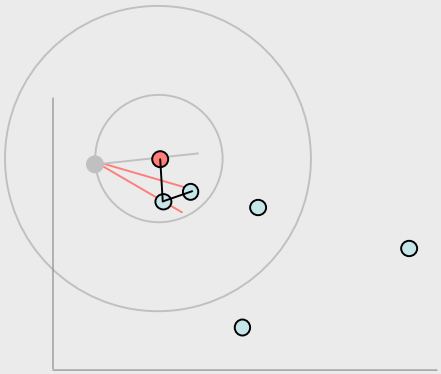





ε'



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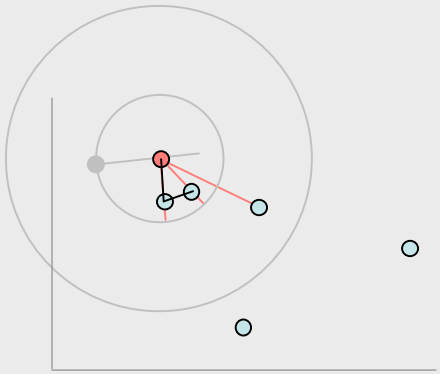

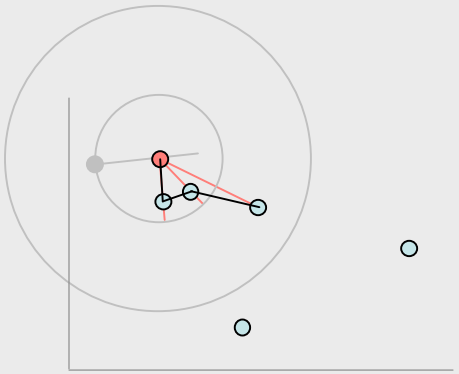

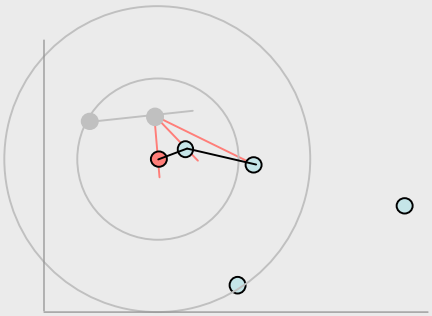

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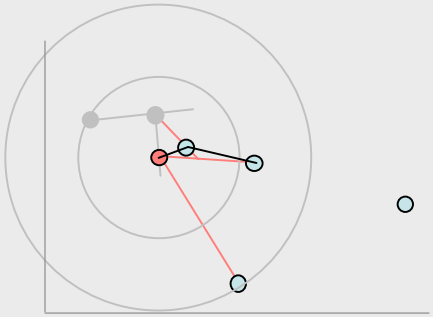

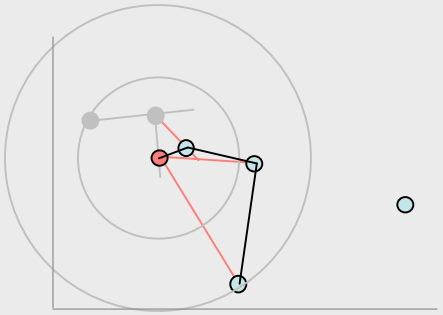

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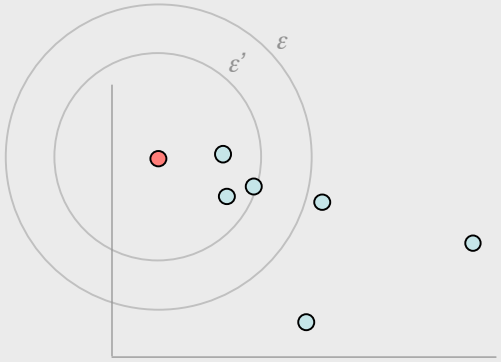


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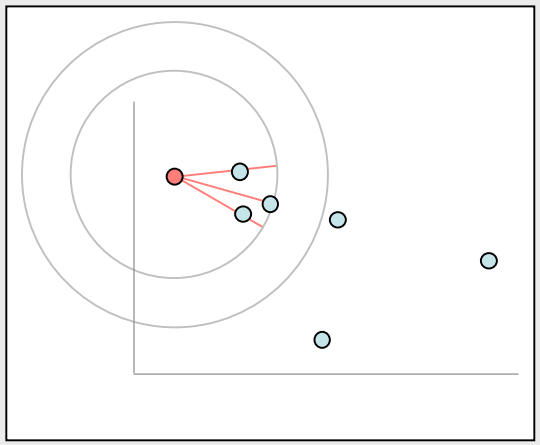

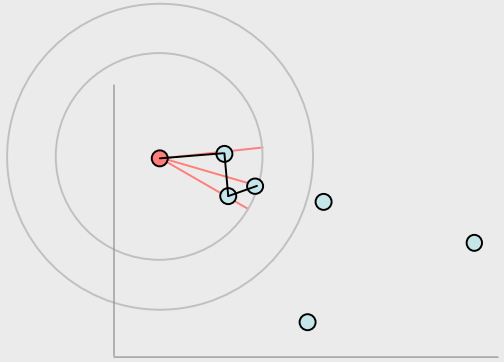

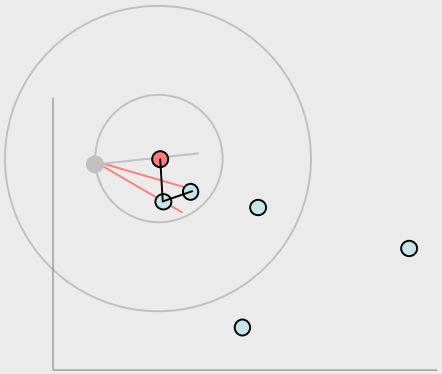

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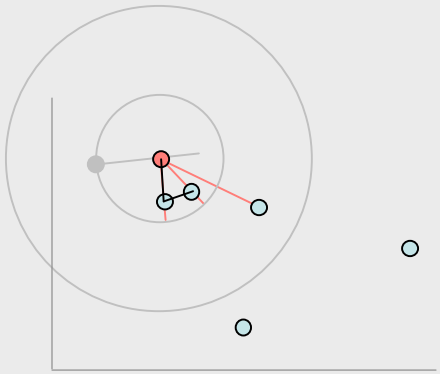

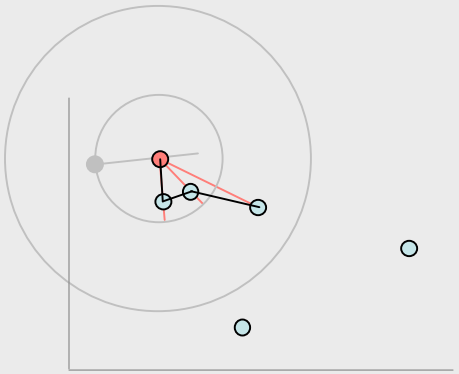

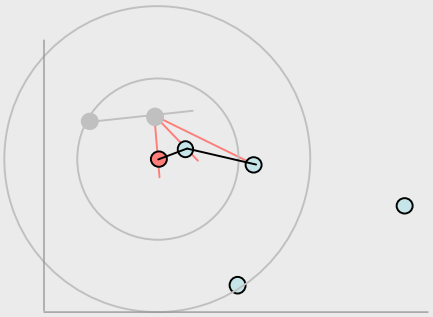

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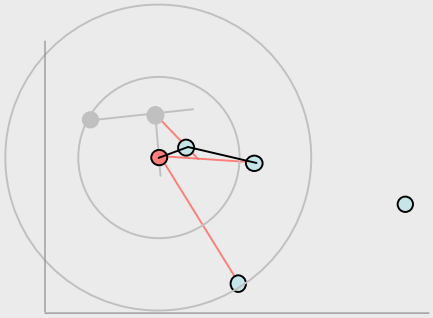

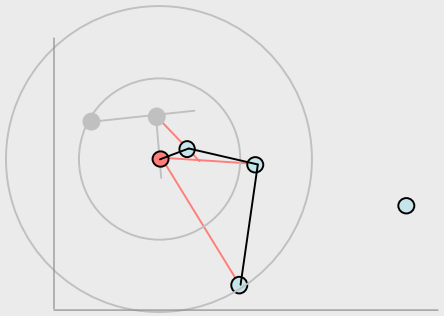

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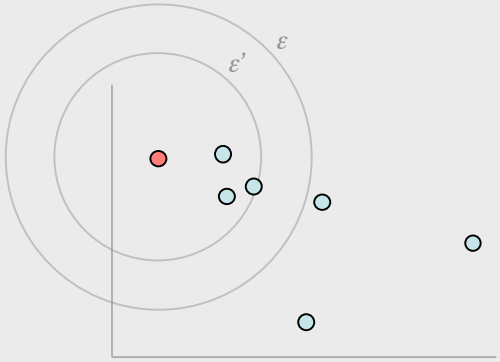


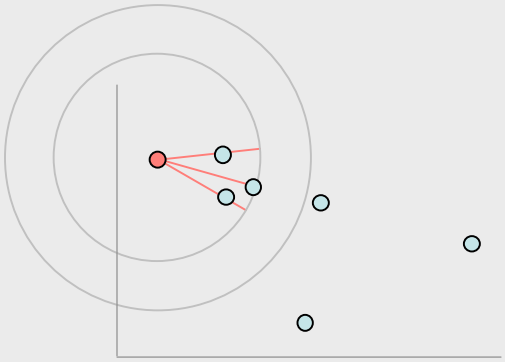

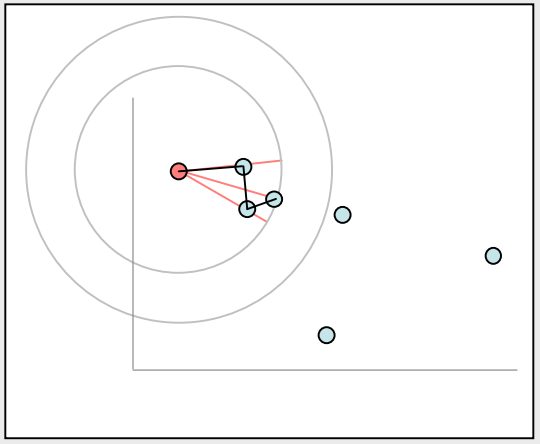

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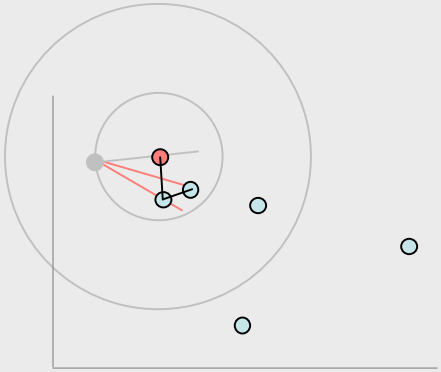

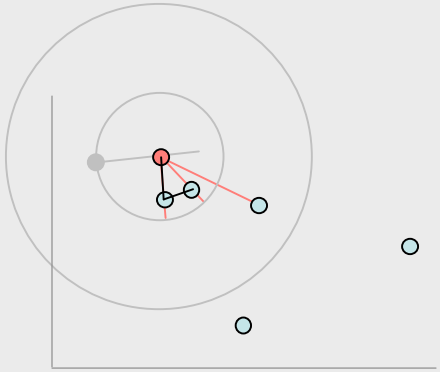

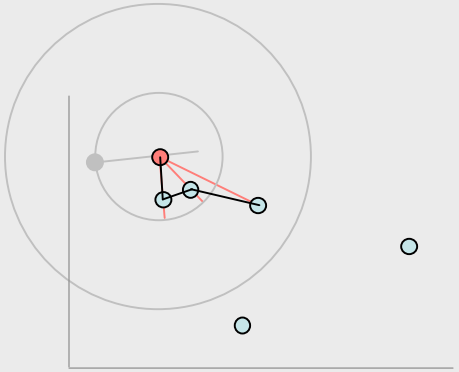

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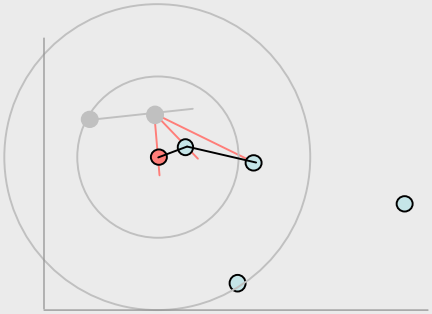

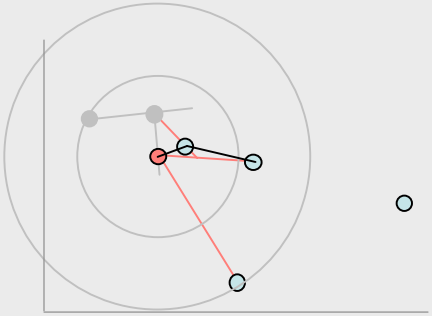

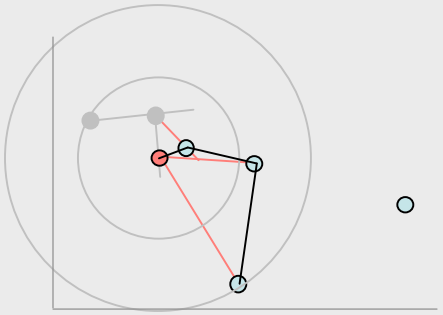

=> pt3 7 5

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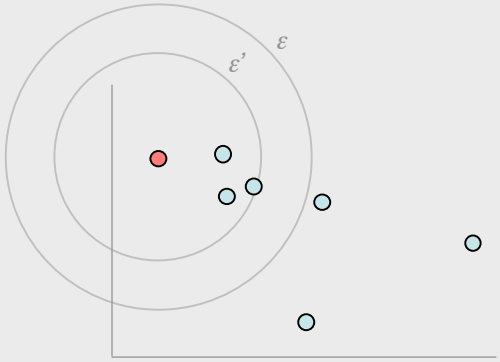


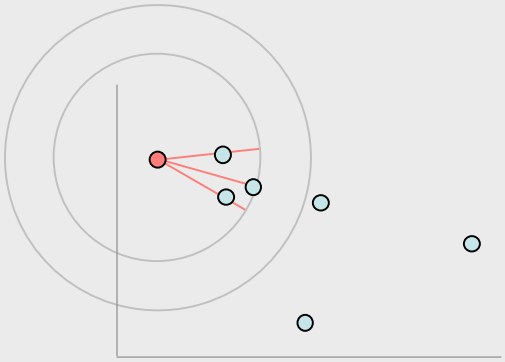

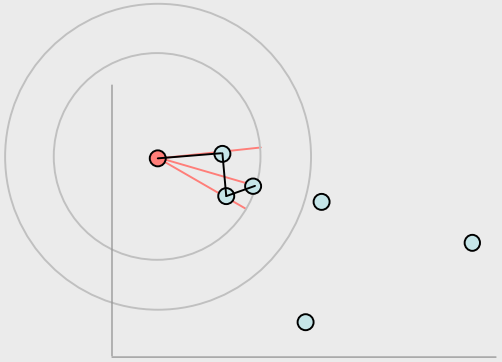

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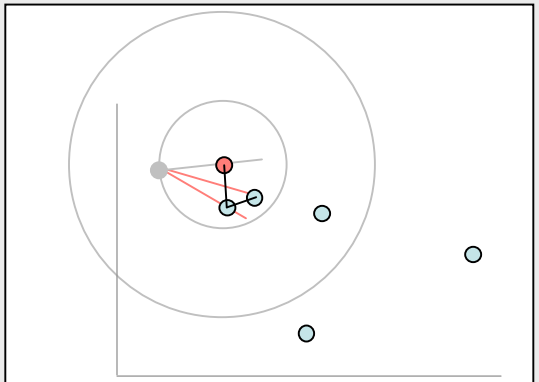
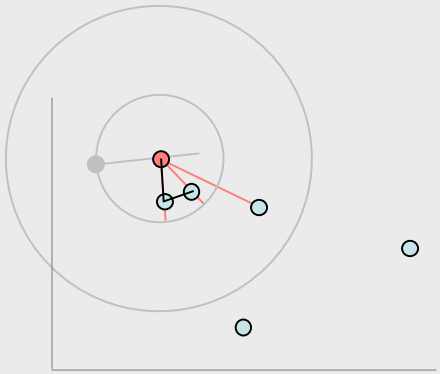
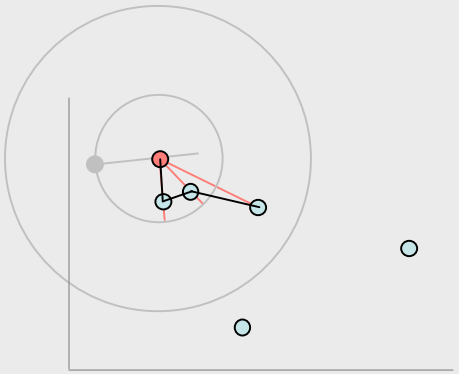
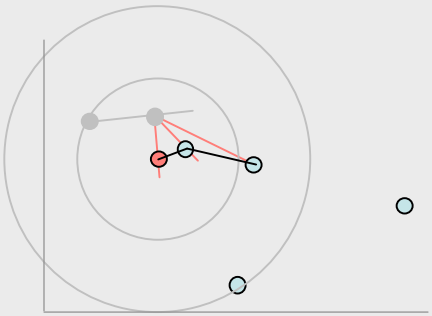


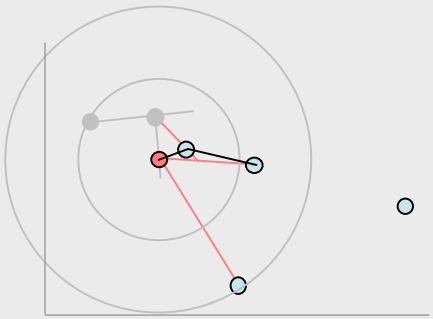
=> pt2 5 10

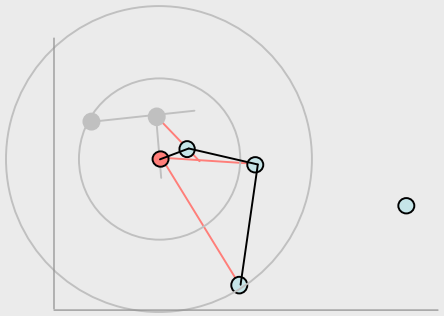






=> pt3 7 5





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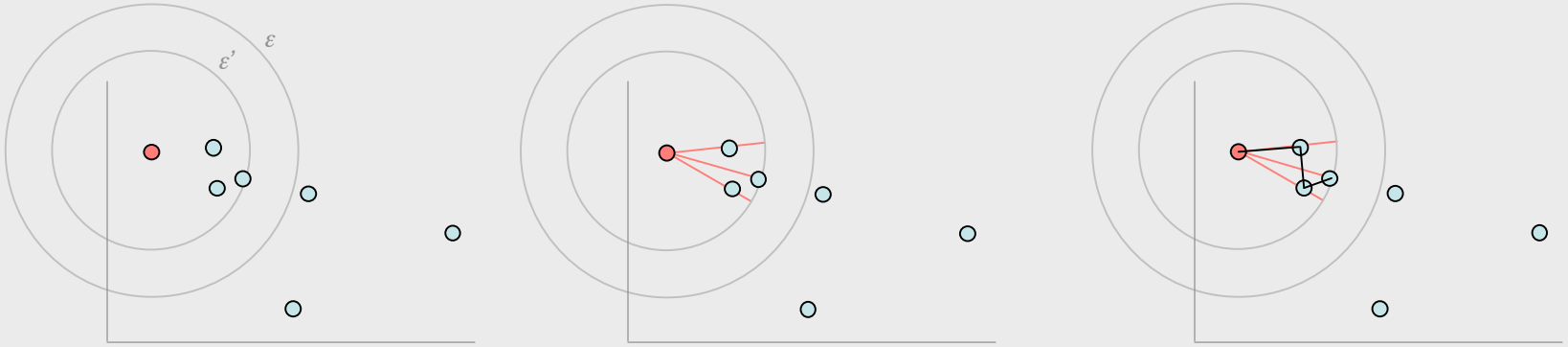


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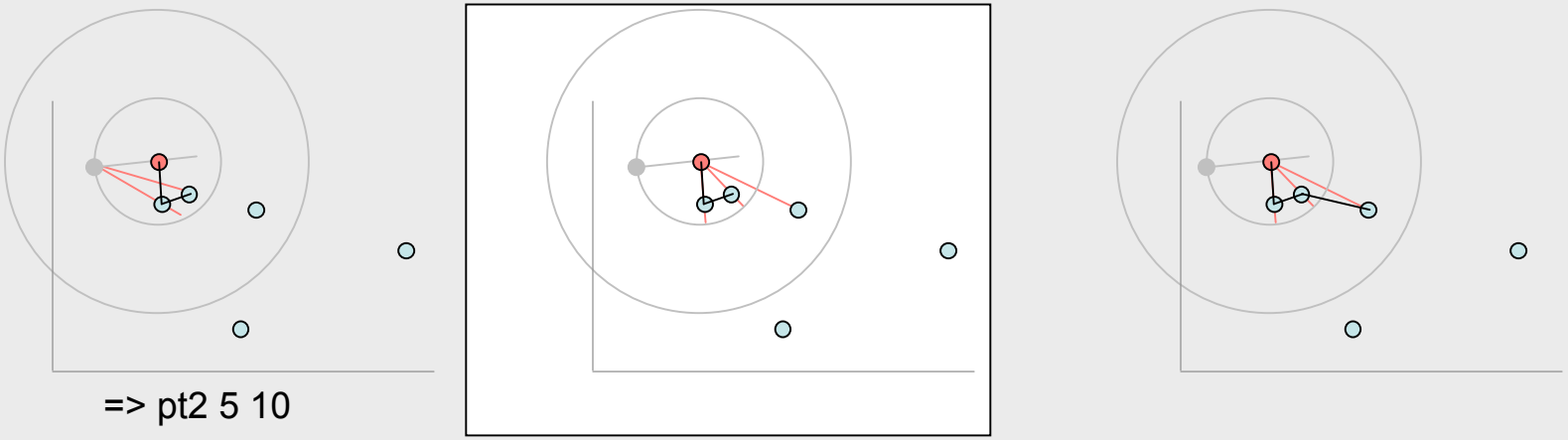

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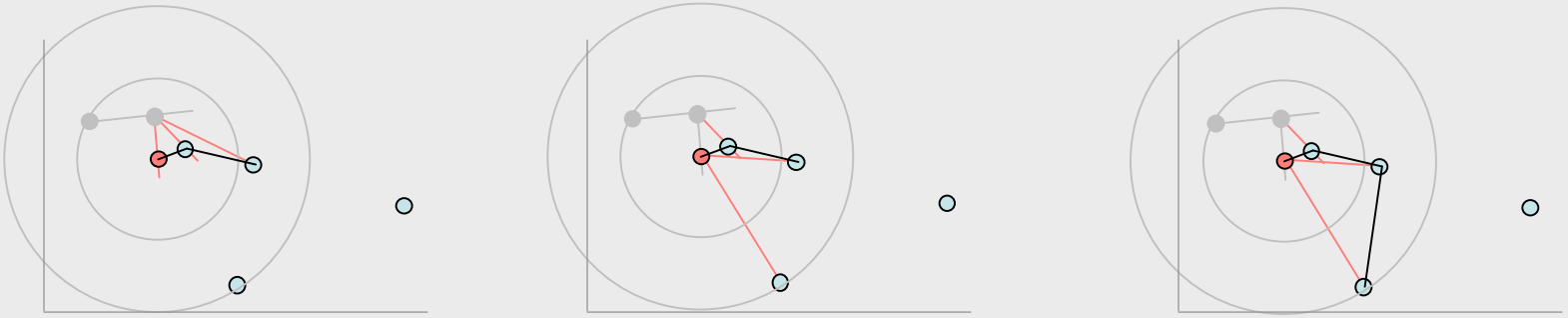

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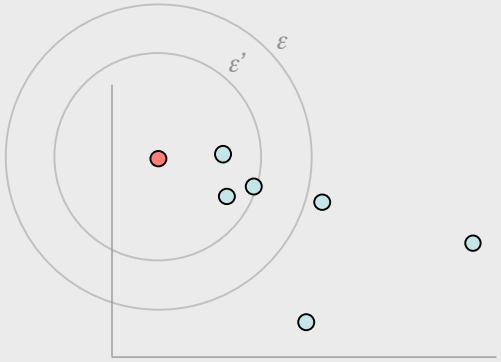


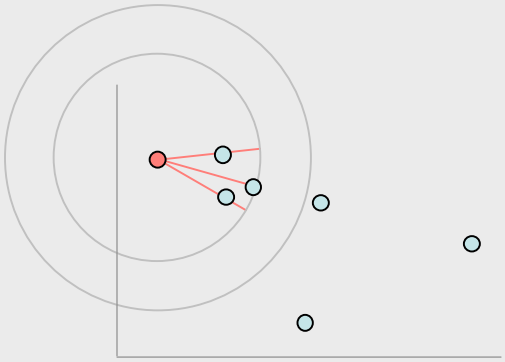

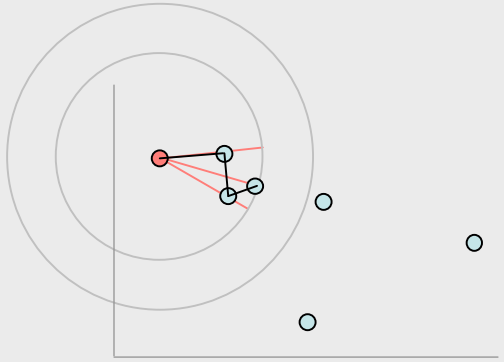

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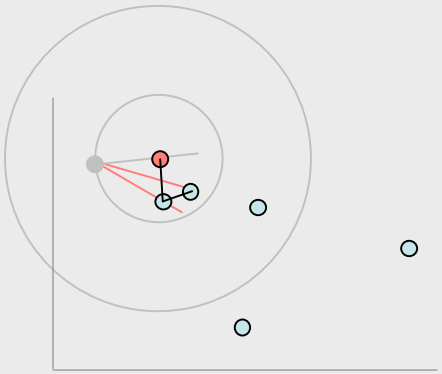

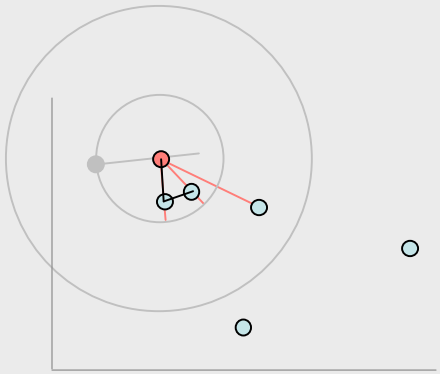

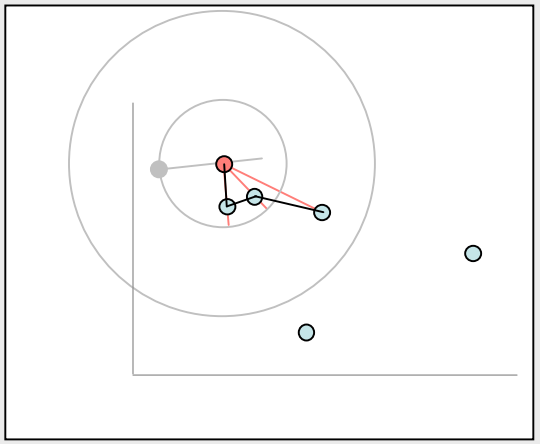

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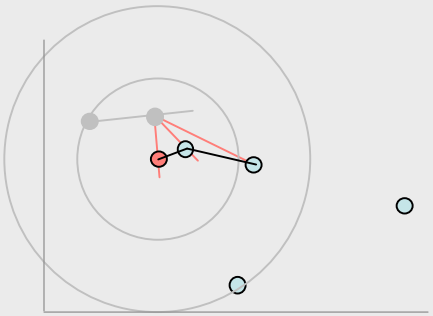

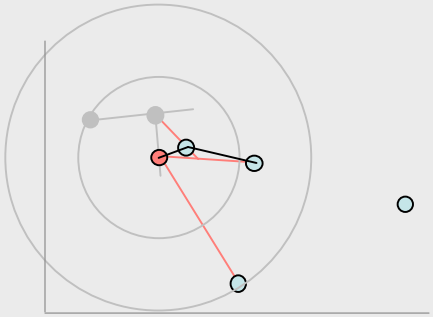

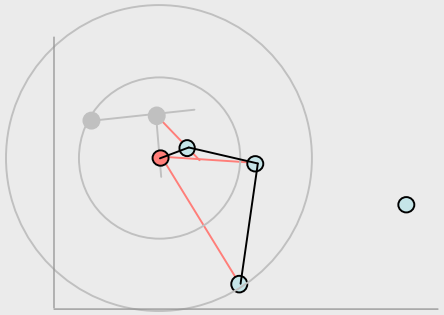

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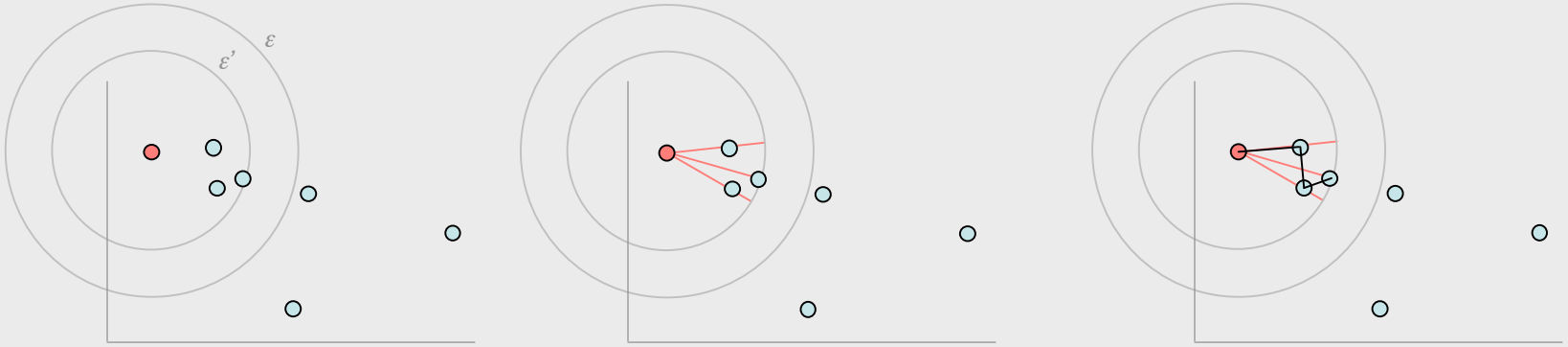
...

Get Neighbors, Calc Core Distance, Save Current Object

Calc/Update Reachability Distances

Update Processing Order

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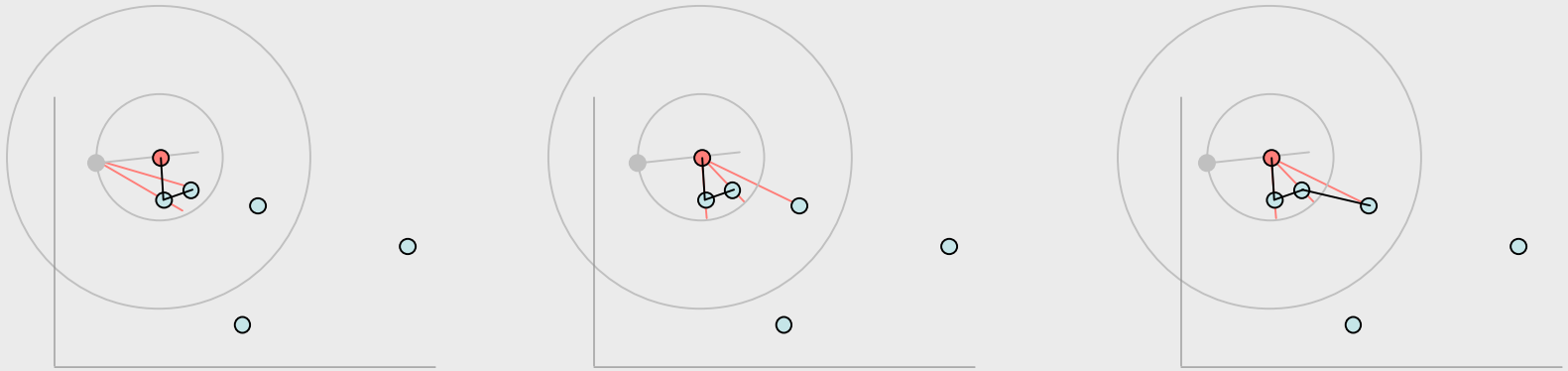

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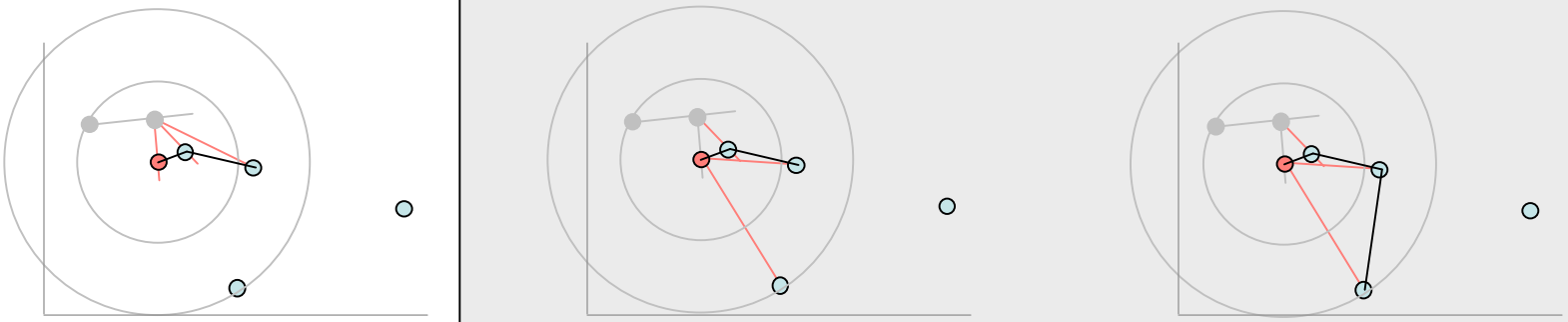

=> pt3 7 5

...

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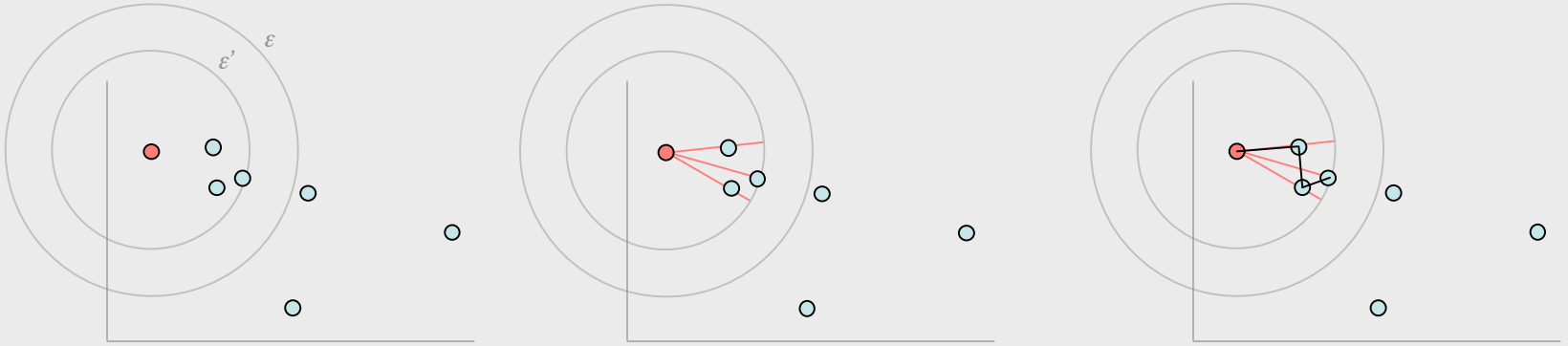


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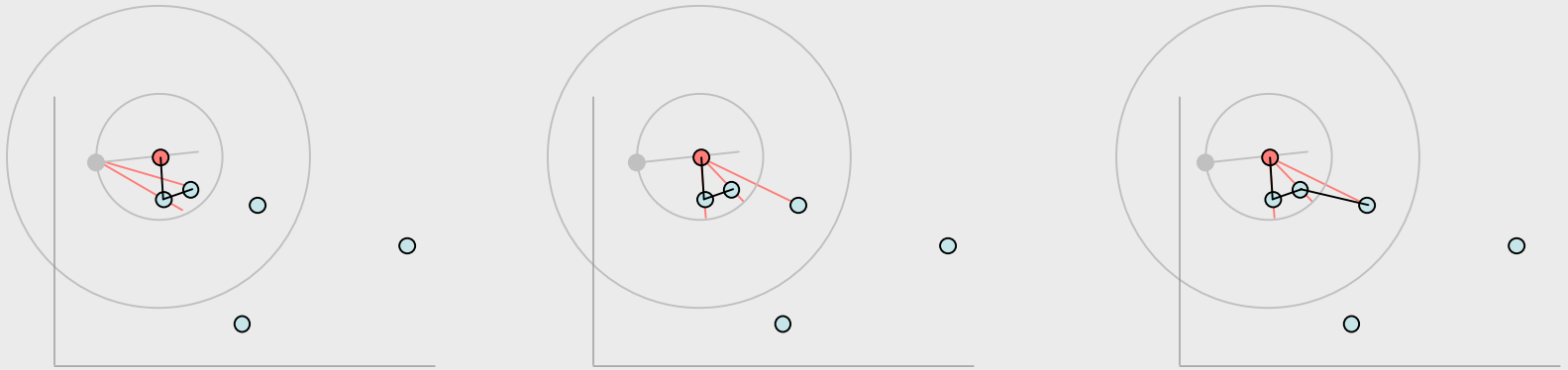

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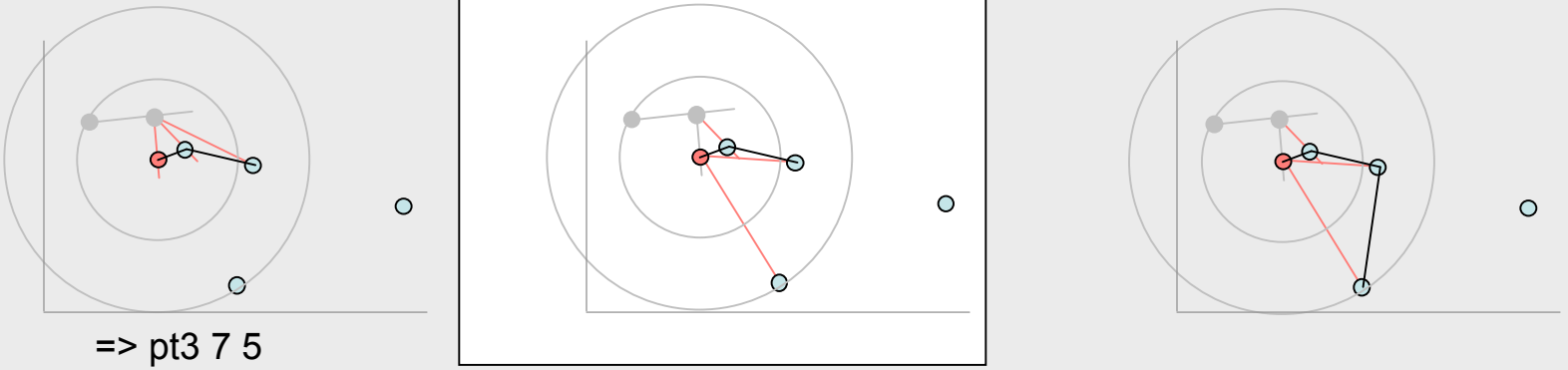

=> pt3 7 5

...

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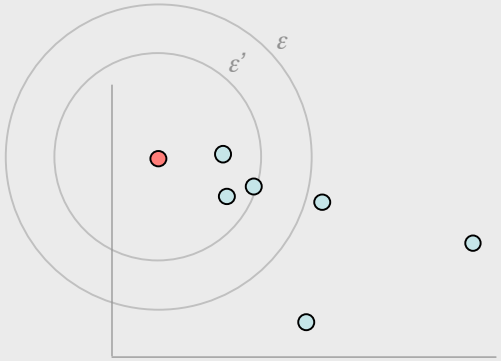


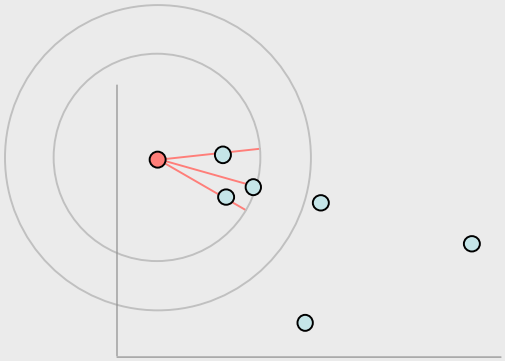

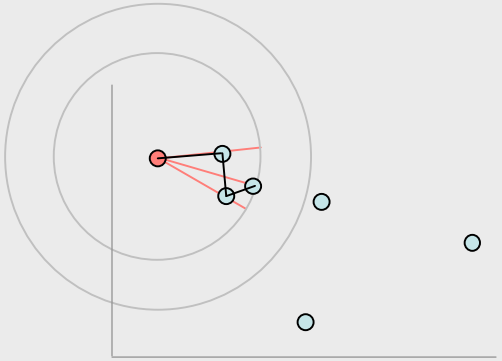

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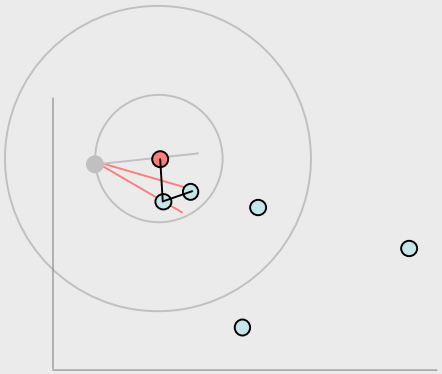

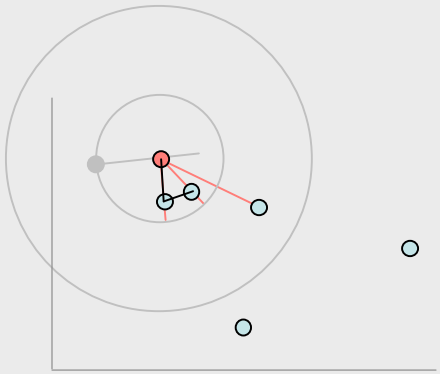

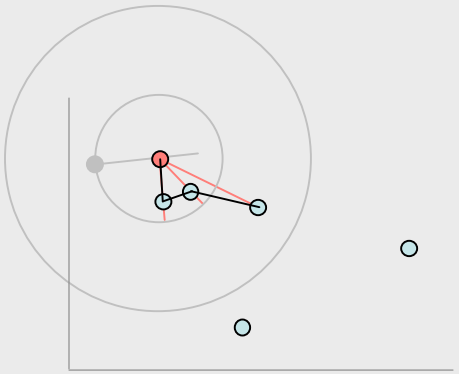

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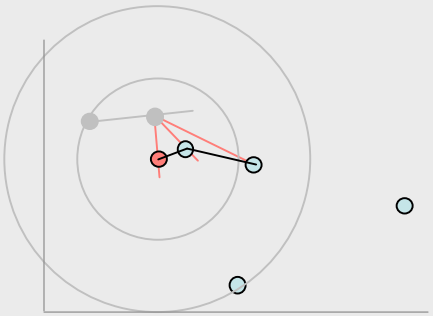
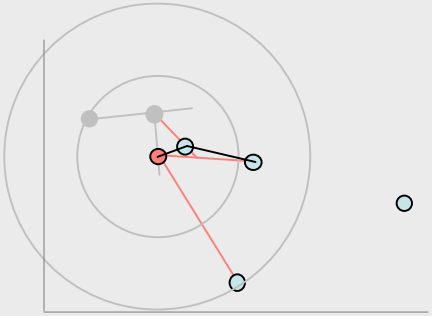
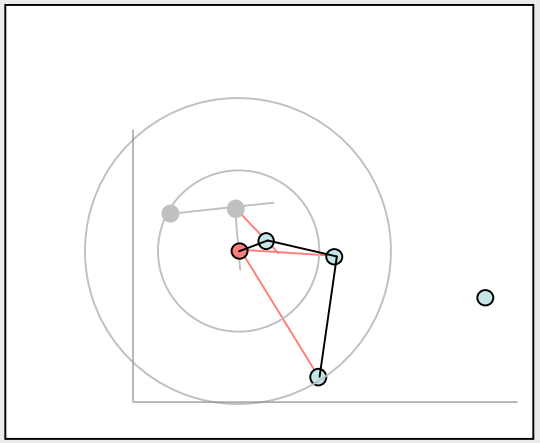






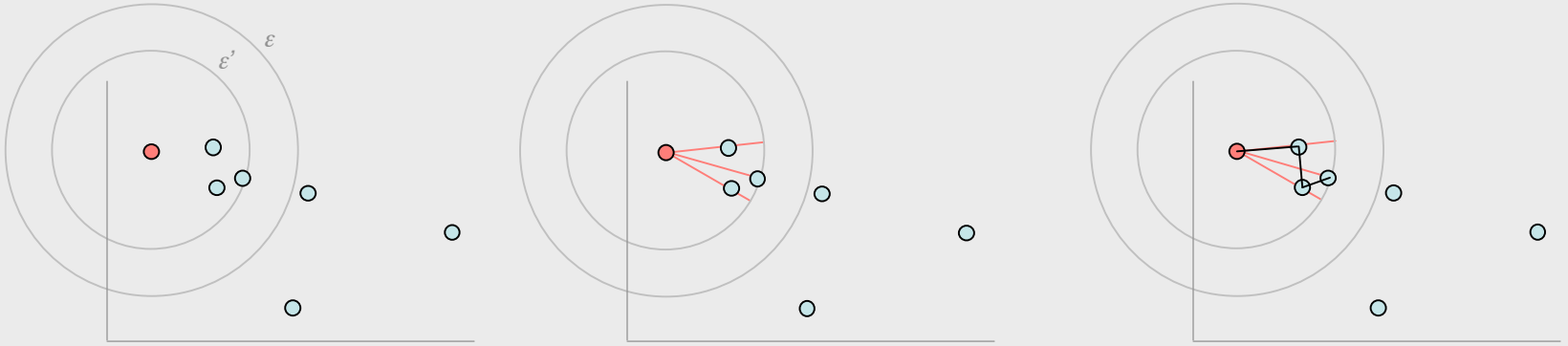
=> pt3 7 5

...

Get Neighbors, Calc Core Distance, Save Current Object

Calc/Update Reachability Distances

Update Processing Order

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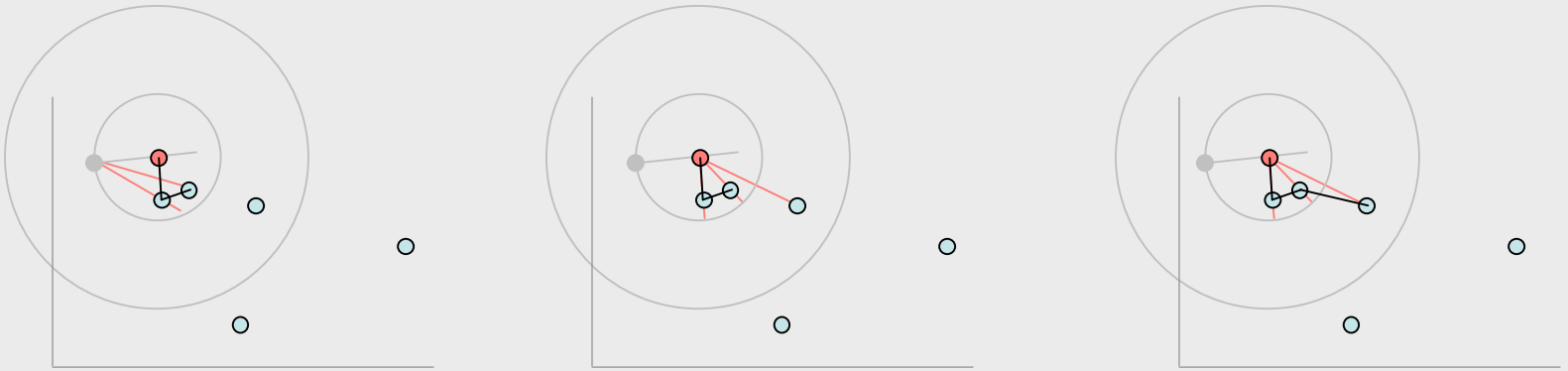

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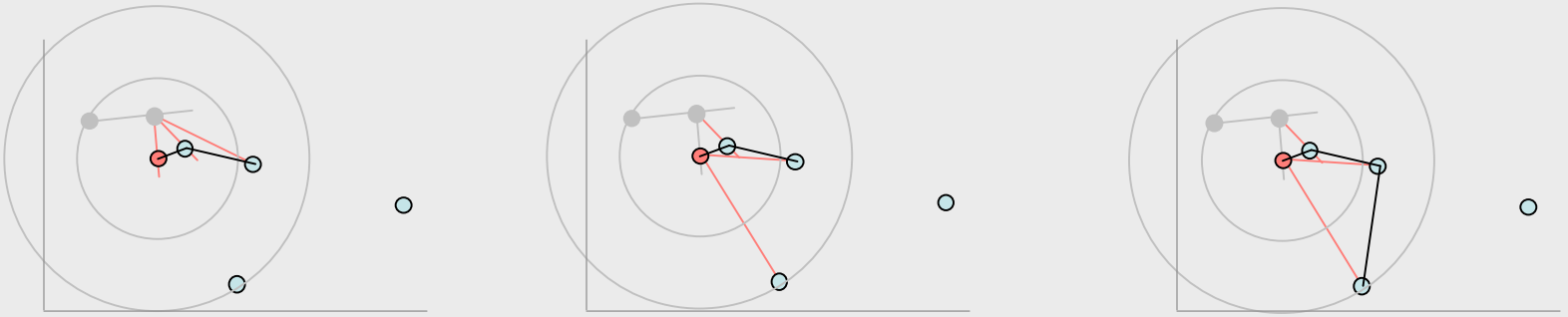
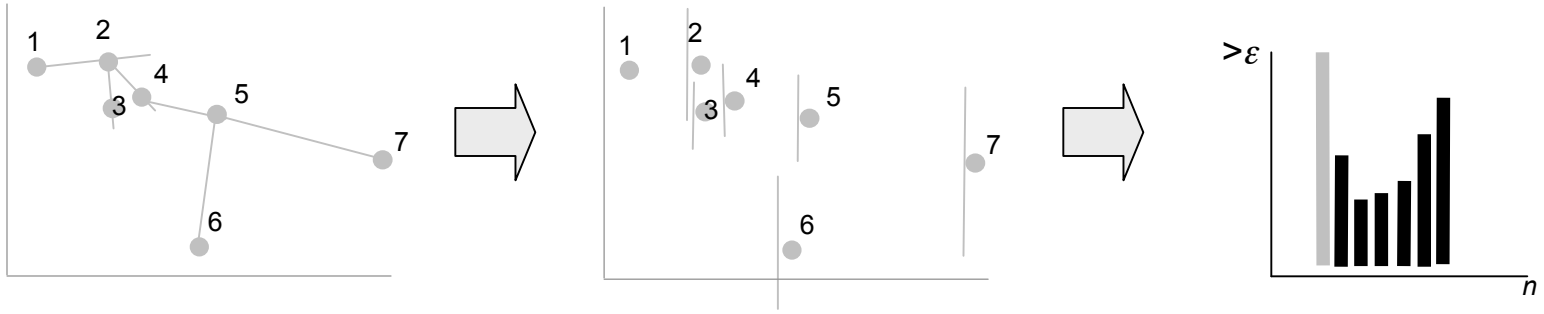


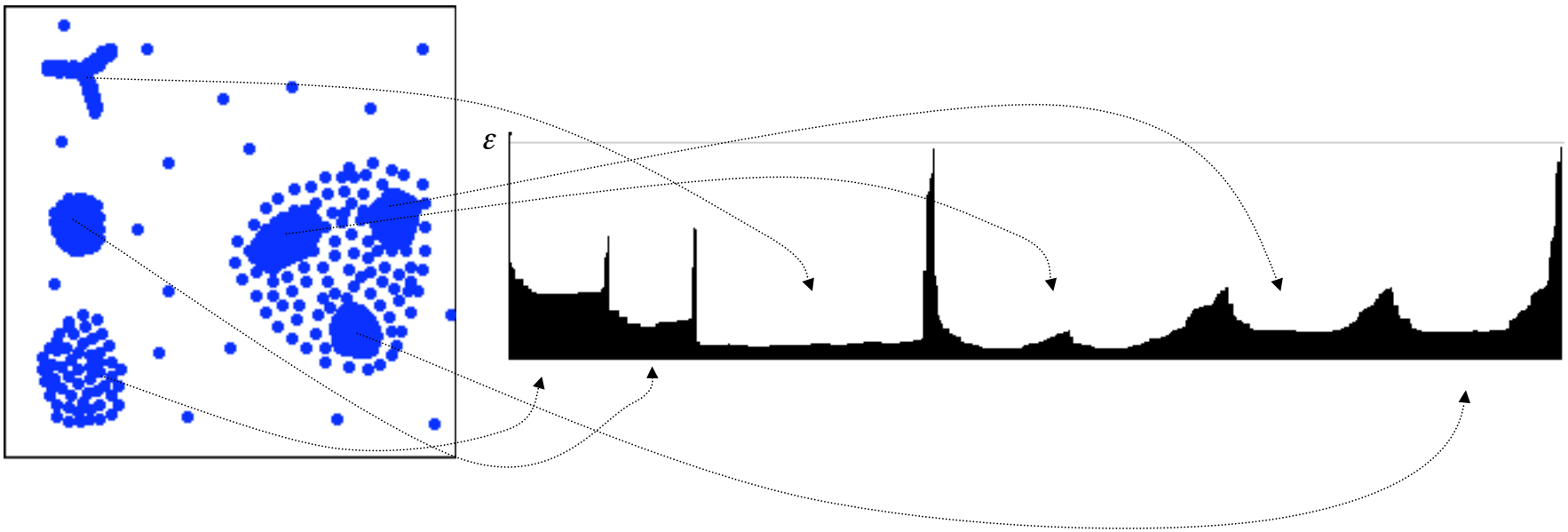
=> pt3 7 5

...

# Reachability Plots

A **reachability plot** is a bar chart that shows each object's reachability distance in the order the object was processed. These plots clearly show the cluster structure of the data.

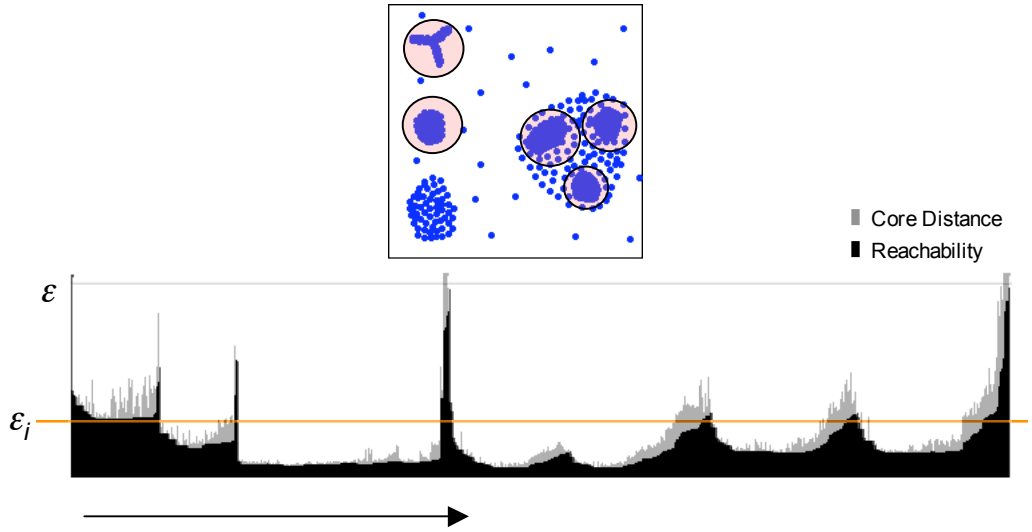


# Automatic Cluster Extraction

## Retrieving DBSCAN clusters

```
1 ExtractDBSCAN(OrderedPoints, ei, MinPts):
2  clusterId = NOISE
3  for each obj in OrderedPoints:
4    if obj.reachability > ei:
5      if obj.coreDistance <= ei:
6        clusterId = nextId(clusterId)
7        obj.clusterId = clusterId
8      else:
9        obj.clusterId = NOISE
10   else:
11     obj.clusterId = clusterId
```

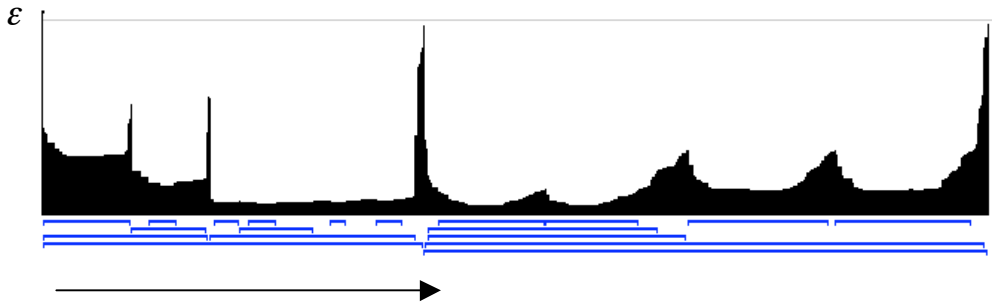



## Extracting hierarchical clusters

A **steep upward point** is a point that is *t*% lower that its successor. A **steep downward point** is similarly defined.

A **steep upward area** is a region from [*s, e*] such that *s* and *e* are both steep upward points, each successive point is at least as high as its predecessors, and the region does not contain more than *MinPts* successive points that are not steep upward.

A **cluster**:
- Starts with a steep downward area
- Ends with a steep upward area
- Contains at least *MinPts*
- The reachability values in the cluster are at least *t*% lower than the first point in the cluster.




```
1 HierachicalCluster(objects):
2  for each index:
3    if start of down area D:
4      add D to steep down areas
5      index = end of D
6    elif start of steep up area U:
7      index = end of U
8     for each steep down area D:
9       if D and U form a cluster:
10        add [start(D), end(U)] to set of clusters
```

# References

[DBSCAN] Ester M., Kriegel H.-P., Sander J., Xu X.: “A DensityBased Algorithm for Discovering Clusters in Large Spatial Databases with Noise”, Proc. 2nd Int. Conf. on Knowledge Discovery and Data Mining, Portland, OR, AAAI Press, 1996, pp.226-231.

[OPTICS] Ankerst, M., Breunig, M., Kreigel, H.-P., and Sander, J. 1999. OPTICS: Ordering points to identify clustering structure. In Proceedings of the ACM SIGMOD Conference, 49-60, Philadelphia, PA.