Contents

Artificial Intelligence **2**

Course Information 2

Textbooks / Resources 2

Readings 3

Lectures 3

Lecture One: Searching the State Space 3

Lecture Two: Searching the State Space (part two) 9

Lecture Three: Knowledge Base and Information 17

Lecture Four: Declarative Programming (Part One) 21

Lecture Five: Declarative Programming (Part Two) 23

Lecture Six: Local and Global Search (Optimisation) 25

Lecture Seven: Belief Networks 30

Artificial Intelligence

Course Information

The course covers core topics in AI including:

- uninformed and informed graph search algorithms,
- propositional logic and forward and backward chaining algorithms,
- declarative programming with Prolog,
- the min-max and alpha-beta pruning algorithms,
- Bayesian networks and probabilistic inference algorithms,
- classification learning algorithms,
- consistency algorithms,
- local search and heuristic algorithms such as simulated annealing, and population-based algorithms such as genetic search and swarm optimisation.

Grades

Standard Computer science policy applies

- Average 50% over all assessment items
- Average at least 45% on all invigilated assessment items

Grading structure for course

- Assignments (5%)
    - Two Super Quiz's
- Quizzes (16.5%)
    - Weekly Quiz Assessments (1.5% ea)
- Lab Test (20%)
- Final Exam (58.5%)

Textbooks / Resources

- Poole, David L.1958, Mackworth, Alan K; Artificial intelligence : foundations of computational agents; Cambridge University Press, 2010.
- Russell, Stuart J, Norvig, Peter; Artificial intelligence : a modern approach; 3rd ed; Prentice Hall, 2010.

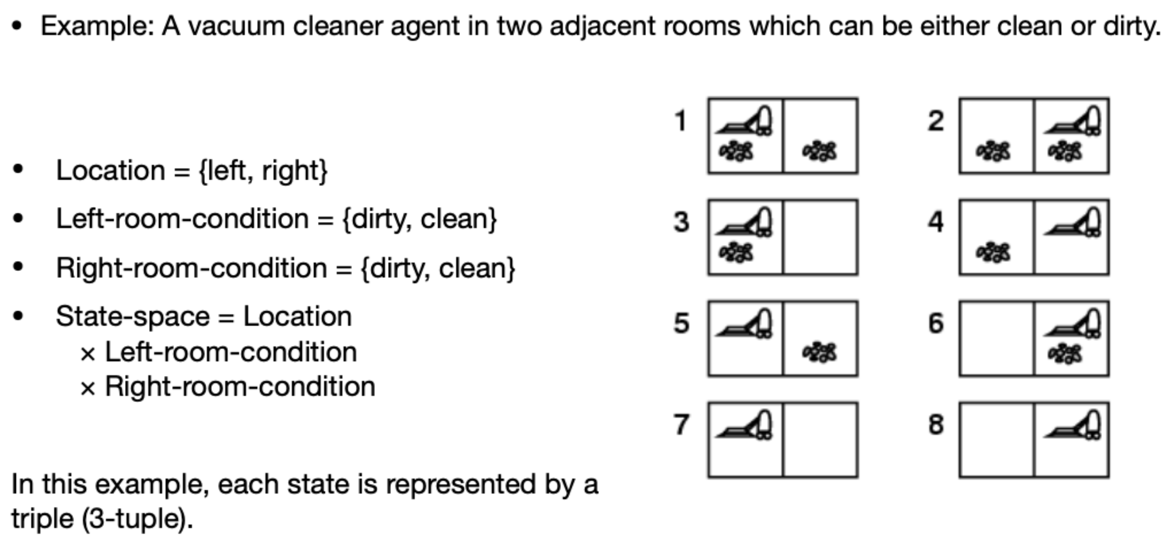
Readings

Lectures

Lecture One: Searching the State Space

What is state?

- A state is a data structure that represents a possible configuration of the world *agent and environment*
- The **state space** is the set of all possible states for that problem
- actions change the state of the world



- Example: A vacuum cleaner agent in two adjacent rooms which can be either clean or dirty.

- Location = {left, right}
- Left-room-condition = {dirty, clean}
- Right-room-condition = {dirty, clean}
- State-space = Location
  × Left-room-condition
  × Right-room-condition

In this example, each state is represented by a triple (3-tuple).

Figure 1: State space example one

State can also be represented as a graph *both directed and undirected*

- Example: Suppose the vacuum cleaner agent can take the following actions: L (go left), R (go right), S (suck).

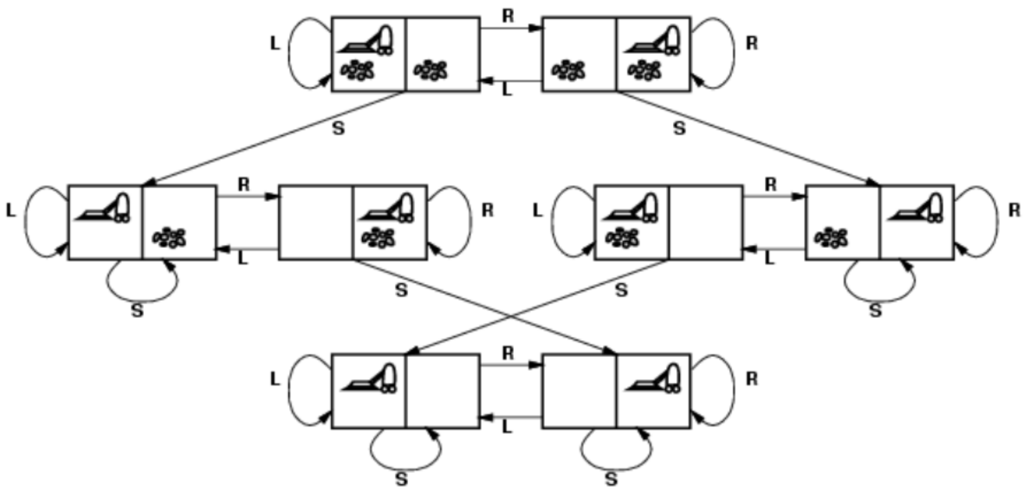


Figure 2: State space graph simplified

- Many problems in AI can be abstracted to the problem of finding a path in a directed graph
- Notation we use is **Nodes** and **arcs** for **vertices** and **edges** in a graph

Explicit vs Implicit graphs

- In **explicit graphs** nodes and arcs are readily available, they are read from the input and stored in a data structure such as an adjacency list/matrix.
  - the entire graph is in memory.
  - the complexity of algorithms are measured in the number of nodes and/or arcs.
- In **implicit graphs** a procedure `outgoing_arcs` is defined that given a node, returns a set of directed arcs that connect node to other nodes.
  - The graph is generated as needed *due to the complexity of the graphs.*
  - The complexity is measured in terms of the depth of the goal state node or *how far do we have to get into the graph to find a solution*.

Explicit graphs in quizzes

- In some exercises we use small explicit graphs to stydy the behaviour of various frontiers
- Nodes are specified in a set

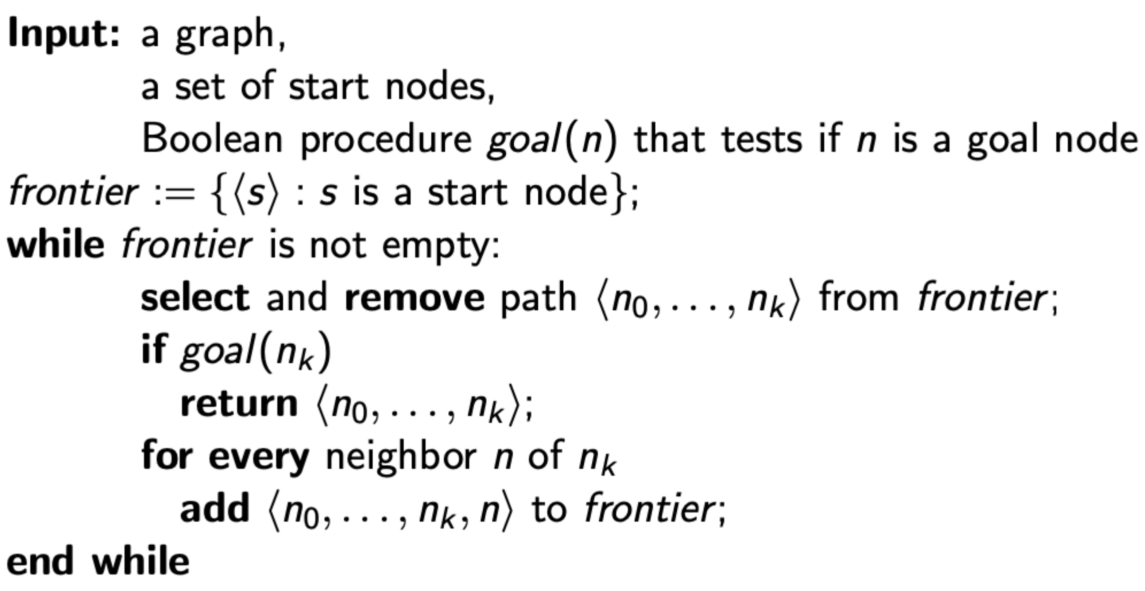
- Edges are specified in a list
    - pairs of nodes, or triples of nodes (in a tuple)

Searching graphs

- We will use generic search algorithms: given a graph, start nodes, and goal nodes, incrementally explore paths from the start nodes.
- Maintain a **frontier** of paths that have been explored
    - frontier: paths that we have already explored
- As search proceeds, the frontier is updated and the graph is explored until a goal node is found.
- The order in which paths are removed and added to the frontier defines the search strategy
- A **search tree** is a tree drawn out of all the possible actions in terms of a tree.
    - How do we handle loops? *Covered in next lecture*
    - In the search tree outlined below, you can see that the *end of paths on frontier* represents a BFS relationship note this is not always the case.

search tree

Generic graph search algorithm



```
Input: a graph,
       a set of start nodes,
       Boolean procedure $goal(n)$ that tests if $n$ is a goal node
$frontier := \{\langle s \rangle : s$ is a start node$\}$;
while $frontier$ is not empty:
       select and remove path $\langle n_0, \ldots, n_k \rangle$ from $frontier$;
       if $goal(n_k)$
         return $\langle n_0, \ldots, n_k \rangle$;
       for every neighbor $n$ of $n_k$
         add $\langle n_0, \ldots, n_k, n \rangle$ to $frontier$;
end while
```

Figure 3: Generic Search

> NOTE: you will have to use what ever data structure for the seach you are using (BFS use a queue), (DFS use a stack).

In the generic algorithm, neighbours are going to use the method `outgoing_arcs`, we are given this algorithm in the form of a python module.

Depth-first search

- In order to perform DFS, the generic graph search must be used with a stack frontier *LIFO*
- If the stack is a python list, where each element is a path, and has the form [..., p, q]
  - *q* is selected and popped
  - of the algorithm continues then paths that extend *q* are pushed (appended) to the stack
  - *p* is only selected when all paths from *q* have been explored.
- As a result, at each stage the algorithm expands the deepest path
- The orange nodes in the graph below are considered the frontier nodes

DFS

- DFS does not guarantee a solution without pruning, due to the fact that we can have infinite loops
- It is not guaranteed to complete if it does not use pruning

A note on complexity

Assume a finite search tree of depth *d* and branching factor of *b*:

- What is the time complexity?
  - It will be exponential: $O(b^d)$
- What is the space complexity?
  - It will be linear: $O(bd)$

How do we trace the frontier

- starting with an empty frontier we record all the calls to the frontier: to add or get a path we dedicate one line per call
- When we ask the frontier to add a path, we start the line with a + followed by the path that has been added
- When we ask for a path from the frontier we start the line with a − followed by the path being removed
- When using a priority queue, the path is followed by a comma and then the key *e.g, cost, heuristic, f-value, …*
- The lines of the trace should match the following regular expression `^[+-][a-z]+(,\d+)?!?$`
- We stop when we **remove** a path from the trace

DFS trace using generic algorithm

Breath-first search

- In order to perform BFS, the generic graph search must be used with a queue frontier *FIFO*.
- If the queue is a python deque of the form [p,q,…,r], then
  - p is selected (dequeued)
  - if the algorithm continues then paths that extend *p* are enqueued *appended* to the queue after *r*
- As a result, at each state the algorithm expands the shallowest path.

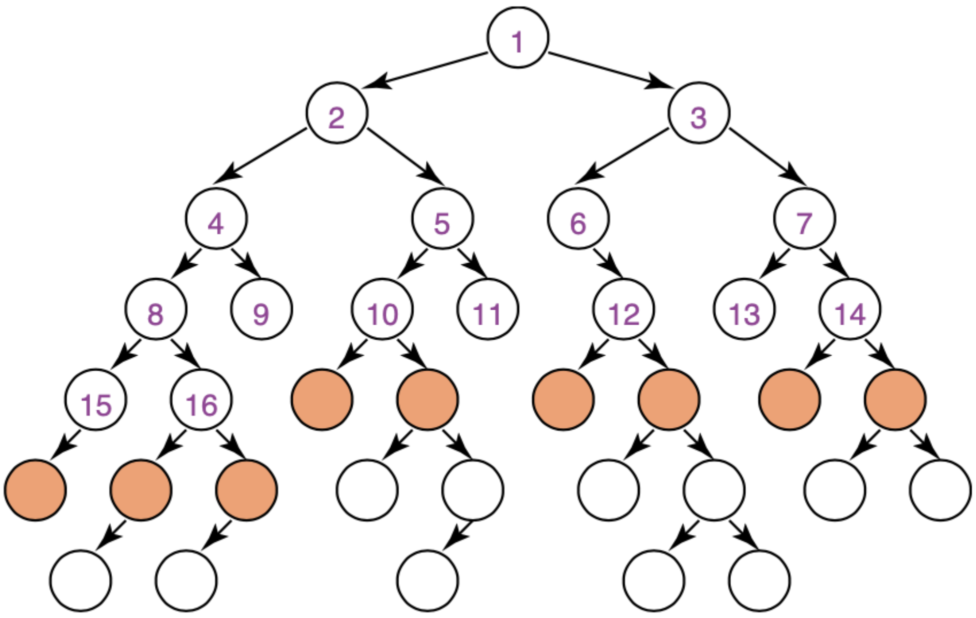


Figure 4: BFS Illustration of search tree

- BFS **does** guarantee to find a solution with the fewest arcs if there is a solution
- It will complete
- It will not halt due to some graphs having *cycles, with no pruning*

A note on complexity

> BFS has higher complexity than DFS

- What is the time complexity?
  - It will be exponential: $O(b^d)$
- What is the space complexity?
  - It will be linear: $O(b^d)$

BFS trace using generic algorithm

Lowest-cost-first search

- The cost of a path is the sum of the costs of its arcs
- This algorithm is very similar to Dijkstra's except modified for larger graphs
- LCFS selects a path on the frontier with the lowest cost
- The frontier is a priority queue ordered by path cost

  - A priority queue is a container in which each element has a priority *cost*
  - An element with a higher priority is always selected/removed before an element with a lower priority
  - In python we can use the `heapq` you will need to store objects in a way that these properties hold

- LCFS finds an optimal solution: a least-cost path to a goal node.
- Another name for this algorithm is *uniform-cost search*.

> NOTE: For an example of this queue, see Lecture One: 1:45 time stamp

LCFS trace generic

Lecture Two: Searching the State Space (part two)

Pruning

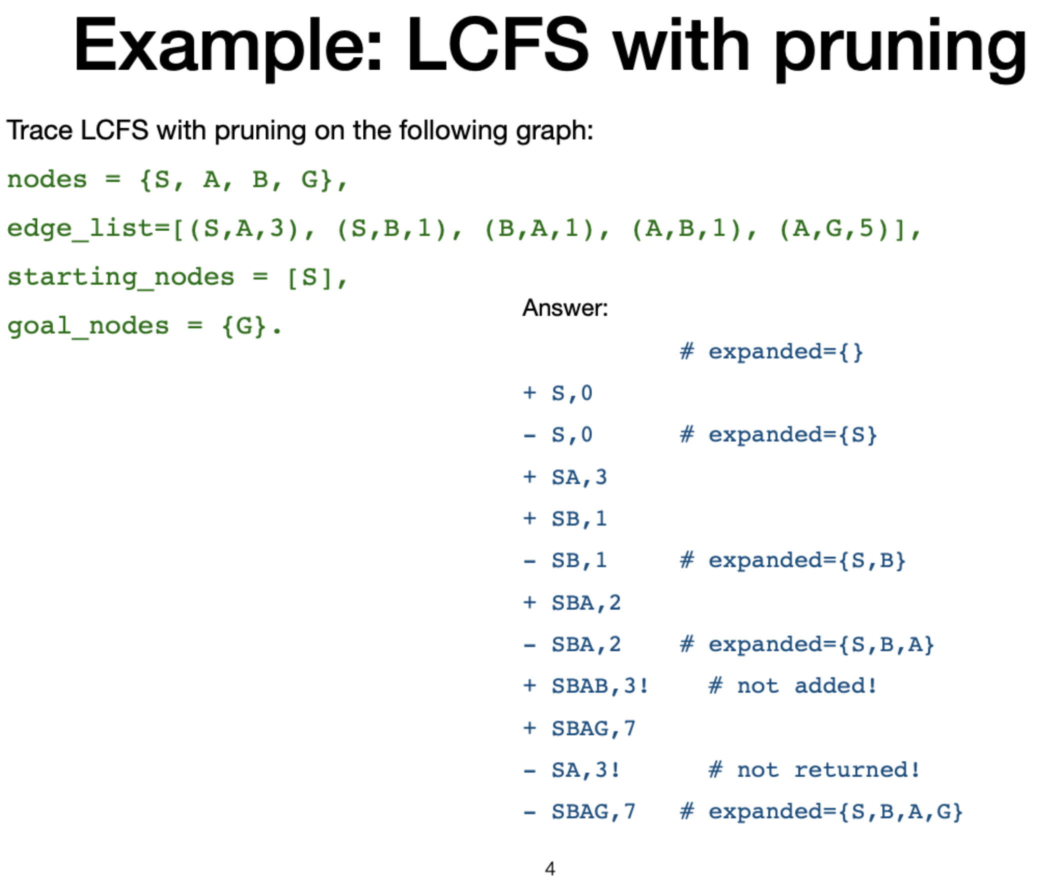
- This is our method to deal with cycles and multiple paths.
- this means we can have wasted computation and cycles in our graph

> Principle: Do not expand paths to nodes that have already been expanded

Pruning Implementation

- The frontier keeps track of expanded or *closed* nodes
- When adding a new path to the frontier, it is only added if another path to the same end-node has not already been expanded, otherwise the new path is discarded (*pruned*)
- When asking for the **next path** to be returned by the frontier, a path is selected and removed but it is returned only if the end-node has not been expanded before, otherwise the path is discarded (pruned) and not returned. The selection and removal is repeated until a path is returned (or the frontier becomes empty). If a path is returned, its end-node will be remembered as an expanded node.

In frontier traces every time a path is pruned, we add an explanation mark `!` at the end of the line



Example: LCFS with pruning

Trace LCFS with pruning on the following graph:

```
nodes = {S, A, B, G},
edge_list=[(S,A,3), (S,B,1), (B,A,1), (A,B,1), (A,G,5)],
starting_nodes = [S],
goal_nodes = {G}.
```

Answer:

```
                # expanded={}
+ S,0
- S,0           # expanded={S}
+ SA,3
+ SB,1
- SB,1          # expanded={S,B}
+ SBA,2
- SBA,2         # expanded={S,B,A}
+ SBAB,3!         # not added!
+ SBAG,7
- SA,3!           # not returned!
- SBAG,7        # expanded={S,B,A,G}
```

4

Figure 5: Example: LCFS with pruning

How does LCFS behave?

- LCFS explores increasing cost contours
  - Finds an optimal solution always
  - Explores options in every direction
  - No information about goal location

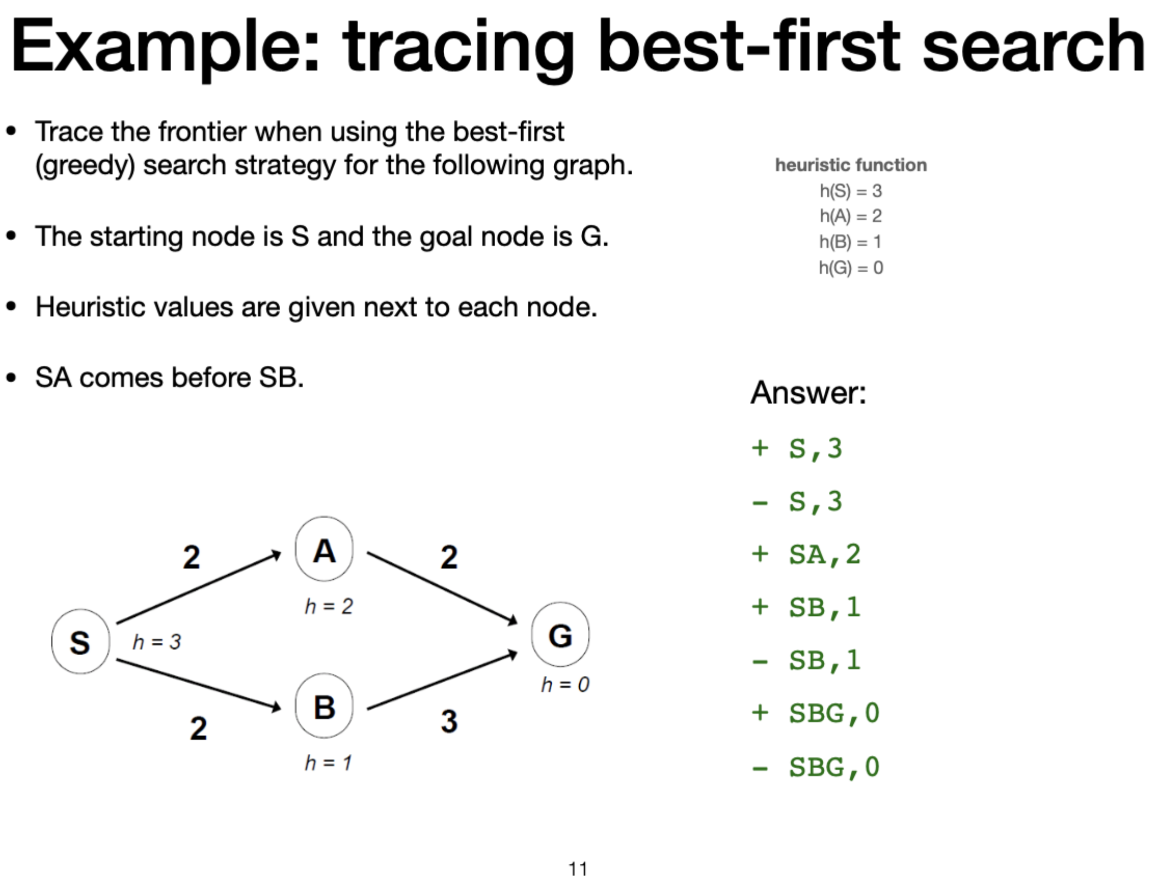
We are going to use a search heuristic, function `h()` is an estimate of the cost for the shortest path from node *n* to a goal node.

- *h* needs to be efficient to compute
- *h* can be extended to paths: $h(< n_0, ..., n_k) = h(n_k)$
- *h* is said to be admissible if and only if:
  - $\forall n\; h(n) \geq 0$, *h* is non-negative and $h(n) \leq C$ where C is the optimal cost of getting from *n* to a goal node

> NOTE: We will have to come up with our own heuristic for the assignment as it depends on context.

Best-first Search

- Idea: select the path whose end is closest to a goal node according to the heuristic function.
- Best-first search is a greedy strategy that selects a path on the frontier with minimal *h*-value
- Main drawback: this does not guarentee finding an optimal solution.



Example: tracing best-first search

- Trace the frontier when using the best-first (greedy) search strategy for the following graph.
- The starting node is S and the goal node is G.
- Heuristic values are given next to each node.
- SA comes before SB.

heuristic function

h(S) = 3
h(A) = 2
h(B) = 1
h(G) = 0

S
h = 3
A
h = 2
B
h = 1
G
h = 0
2
2
2
3

Answer:

```
+ S,3
- S,3
+ SA,2
+ SB,1
- SB,1
+ SBG,0
- SBG,0
```

11

Figure 6: Tracing best-first search

A search strategy

Properties:

- Always finds an optimal solution as long as:
  - there is a solution
  - there is no pruning

    - the heuristic function is admissible

- Does it halt on every graph?

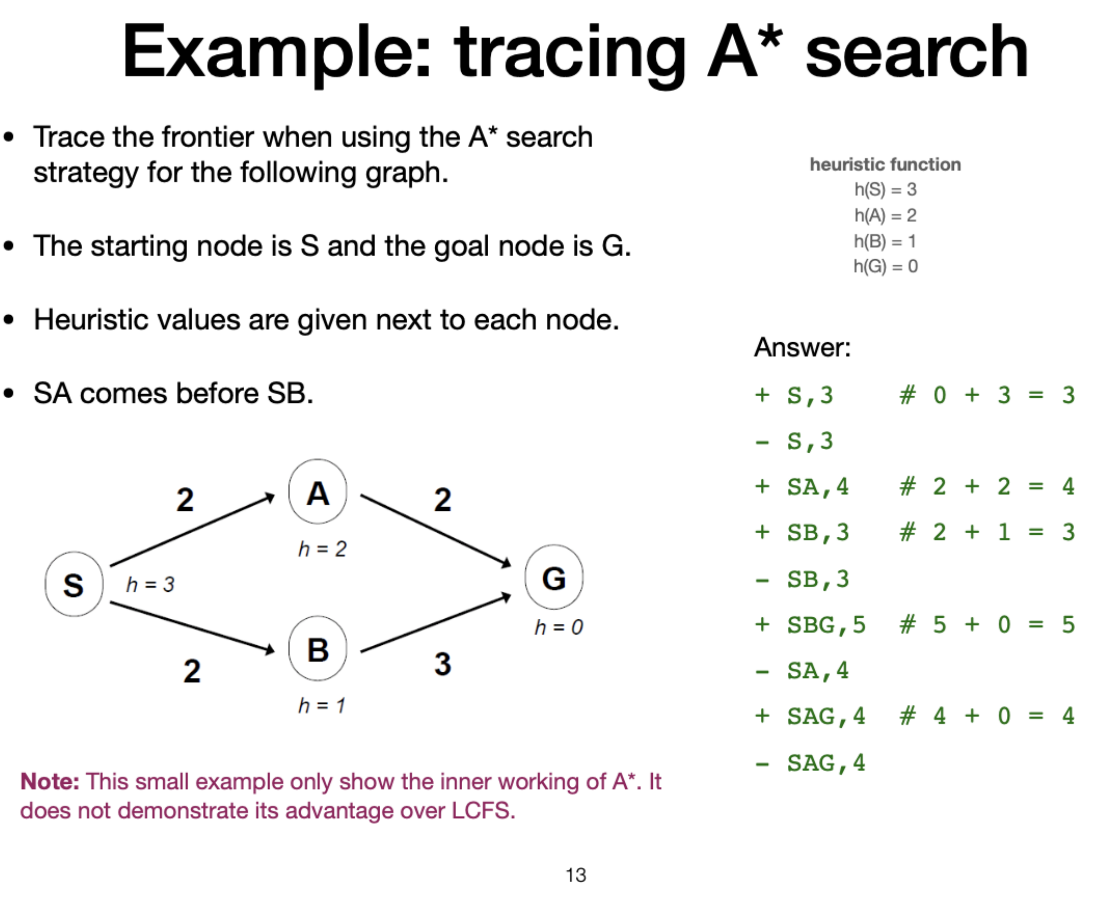
Idea:

- Don't be as wasteful as LCFS
- Don't be as greedy as best-first search
- Estimate the cost of paths as if they could be extended to reach a goal in the best possible way.

Evaluation function: $f(p) = cost(p) + h(n)$

- p is a path, n is the last node on p
- $cost(p)$ = cost of path p *this is the actual cost from the starting node to node n*
- $h(n)$ = an estimate of the cost from n to goal node
- $f(p)$ = estimated total cost of path through p to goal node

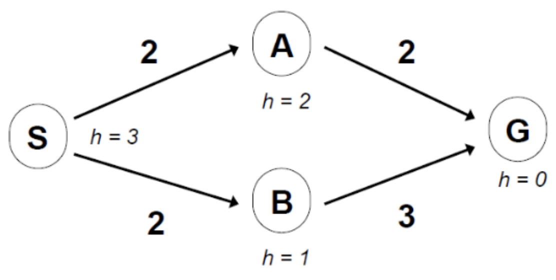
The frontier is a priority queue ordered by $f(p)$



Example: tracing A* search

- Trace the frontier when using the A* search strategy for the following graph.
- The starting node is S and the goal node is G.
- Heuristic values are given next to each node.
- SA comes before SB.

heuristic function
h(S) = 3
h(A) = 2
h(B) = 1
h(G) = 0



Note: This small example only show the inner working of A*. It does not demonstrate its advantage over LCFS.

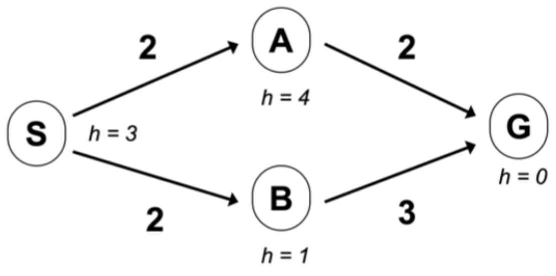
Answer:

```
+ S,3     # 0 + 3 = 3
- S,3
+ SA,4    # 2 + 2 = 4
+ SB,3    # 2 + 1 = 3
- SB,3
+ SBG,5   # 5 + 0 = 5
- SA,4
+ SAG,4   # 4 + 0 = 4
- SAG,4
```

13

heuristic function
h(S) = 3
h(A) = 4
h(B) = 1
h(G) = 0

- Same example as the one before just assume *h(A) = 4* instead.



Answer:

```
+ S,3     # 0 + 3 = 3
- S,3
+ SA,6    # 2 + 4 = 6
+ SB,3    # 2 + 1 = 3
- SB,3
+ SBG,5   # 5 + 0 = 5
- SBG,5
```

Non-optimal solution! Why?

A*: proof of optimality

When using A* (without pruning) the first path *p* from a starting node to a goal node that is selected and removed from the frontier has the lowest cost.

Sketch of proof:

- Suppose to the contrary that there is another path from one of the starting nodes to a goal node with a lower cost.
- There must be a path *p*' on the frontier such that one of its continuations leads to the goal with a lower overall cost than *p*.
- Since *p* was removed before *p*':

$$f(p) \leq f(p') \implies cost(p) + h(p) \leq cost(p') + h(p') \implies cost(p) \leq cost(p') + h(p')$$

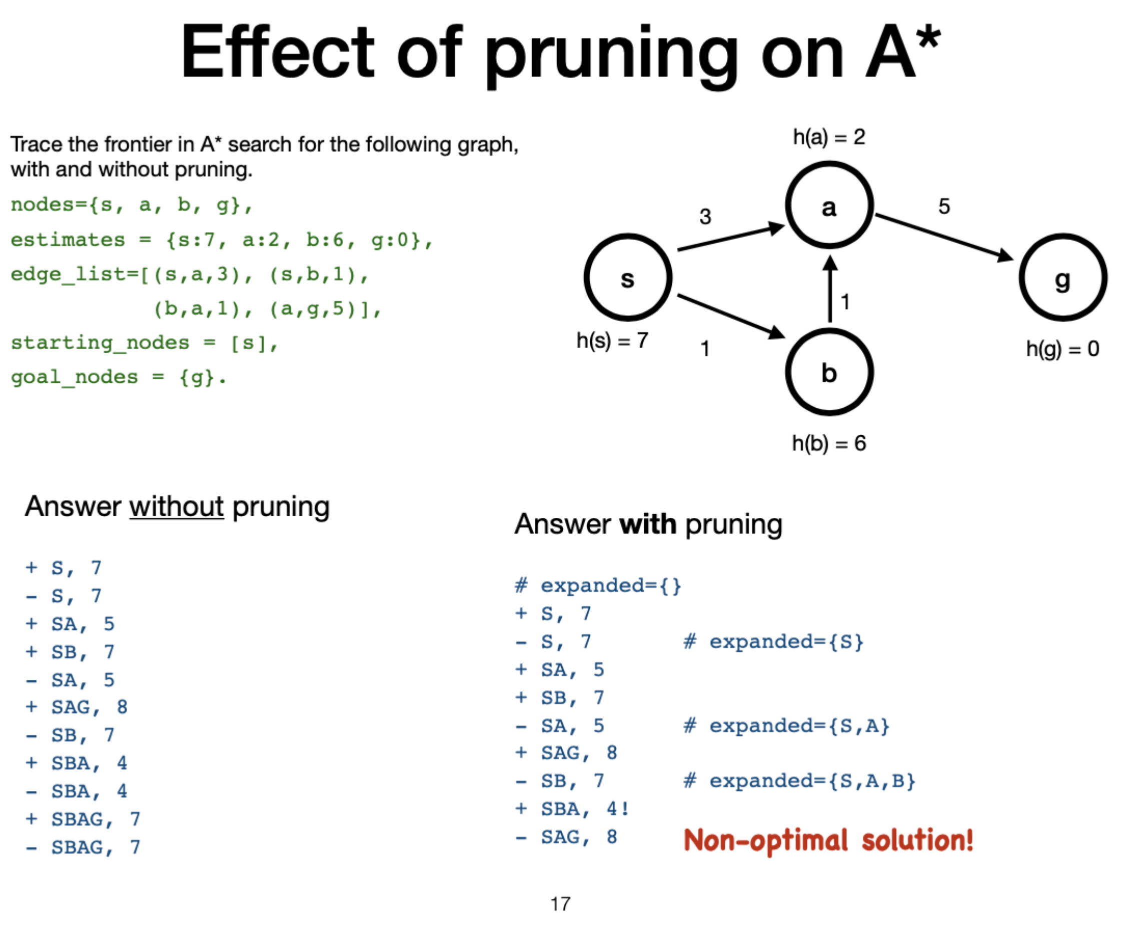
- Let *c* be any continuation of *p*' that goes to a goal node; that is, we have a path *p*'*c* from a start node to a goal node. Since *h* is admissible, we have:

$$cost(p'c) = cost(p') + cost(c) \geq cost(p') + h(p')$$

- Thus:

$$cost(p) \leq cost(p') + h(p') \leq cost(p') + cost(c) = cost(p'c)$$

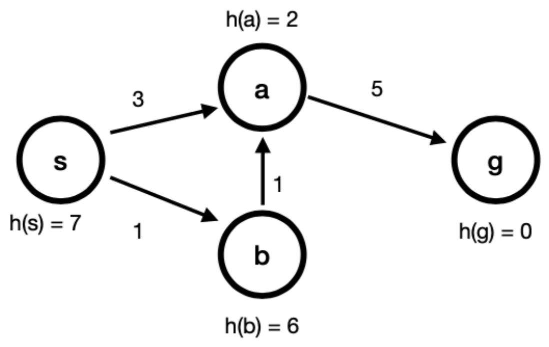
16



Effect of pruning on A*

Trace the frontier in A* search for the following graph, with and without pruning.

```
nodes={s, a, b, g},
estimates = {s:7, a:2, b:6, g:0},
edge_list=[(s,a,3), (s,b,1),
           (b,a,1), (a,g,5)],
starting_nodes = [s],
goal_nodes = {g}.
```



Answer without pruning

```
+ S, 7
- S, 7
+ SA, 5
+ SB, 7
- SA, 5
+ SAG, 8
- SB, 7
+ SBA, 4
- SBA, 4
+ SBAG, 7
- SBAG, 7
```

Answer **with** pruning

```
# expanded={}
+ S, 7
- S, 7          # expanded={S}
+ SA, 5
+ SB, 7
- SA, 5         # expanded={S,A}
+ SAG, 8
- SB, 7         # expanded={S,A,B}
+ SBA, 4!
- SAG, 8
```

Non-optimal solution!

17

What went wrong when pruning A Search

- An expensive path, *sa* was expanded before a cheaper path *sba* could be discovered, because $f(sa) < f(sb)$
- Is the heuristic function *h* admissible?
  - Yes
- So what can we do?
  - We need a stronger condition than admissibility to stop this from happening

> Principle: When we are removing nodes, we are essentially saying we have found a cheaper solution, in this case, this was not true and hence why the algorithm fails, we need to use a stronger condition as outlined below

Monotonicity

A heuristic function is monotone or consistent if for every two nodes n and n' which is reachable from n:

$$h(n) \leq cost(n, n') + h(n')$$

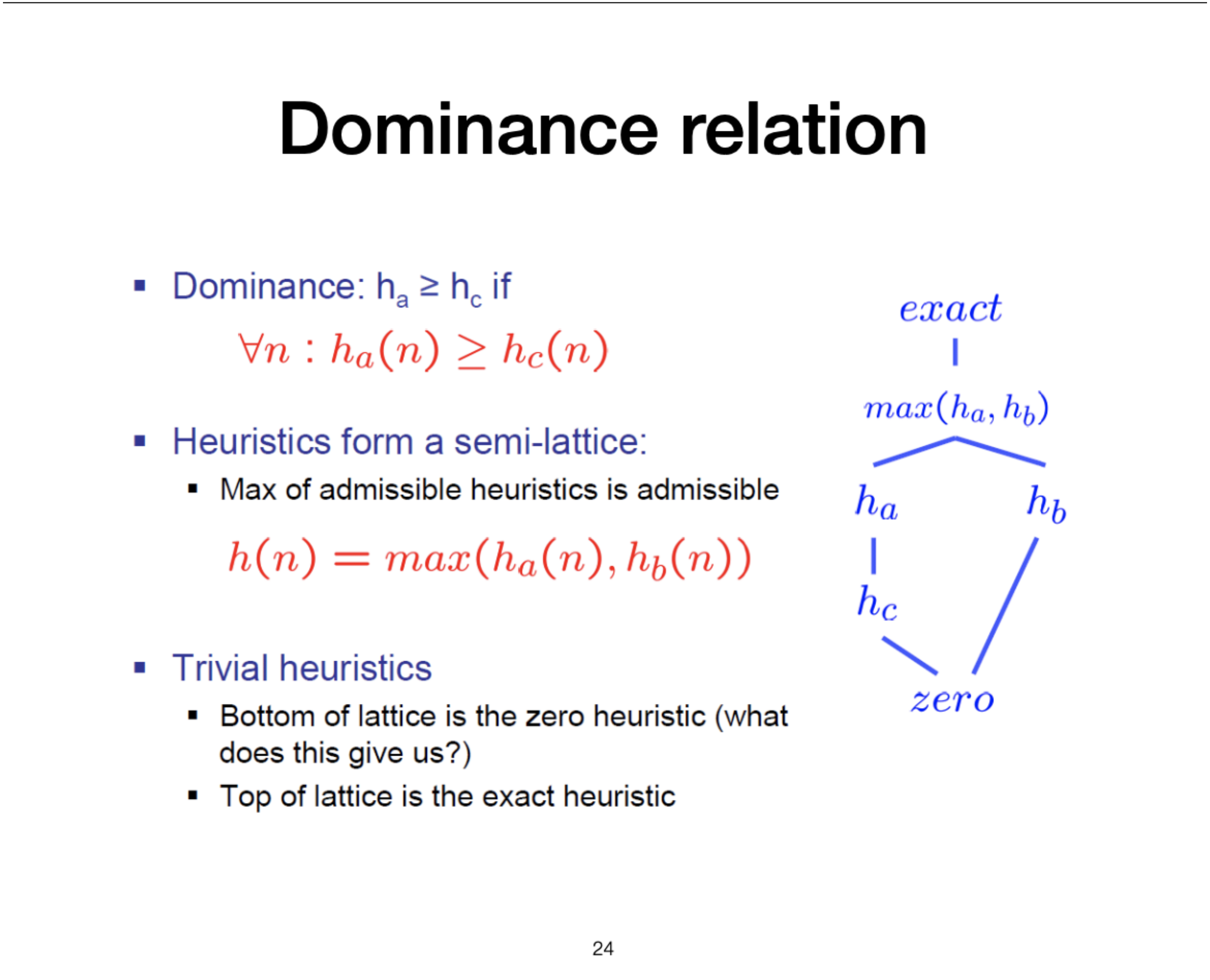
With the monotone restriction, we have:

$$\begin{aligned} f(n') &= cost(s, n') + h(n') \\ &= cost(s, n) + cost(n, n') + h(n') \\ &\geq cost(s, n) + h(n) \\ &\geq f(n) \end{aligned}$$

How about using the actual cost as a heuristic?

- Would it be a valid heuristic?
- Would we save on nodes expanded?
- What's wrong with it?
    - It becomes as computationally expensive as it is to just do the problem

Choosing a heuristic: a trade-off between quality of estimate and work per node!



Dominance relation

- Dominance: $h_a \geq h_c$ if

$$\forall n : h_a(n) \geq h_c(n)$$

- Heuristics form a semi-lattice:
  - Max of admissible heuristics is admissible

$$h(n) = max(h_a(n), h_b(n))$$

- Trivial heuristics
  - Bottom of lattice is the zero heuristic (what does this give us?)
  - Top of lattice is the exact heuristic

$$\begin{tikzcd}
 & exact \arrow[d, dash] & \\
 & max(h_a, h_b) \arrow[dl, dash] \arrow[dr, dash] & \\
h_a \arrow[d, dash] & & h_b \arrow[ddl, dash] \\
h_c \arrow[dr, dash] & & \\
 & zero &
\end{tikzcd}$$

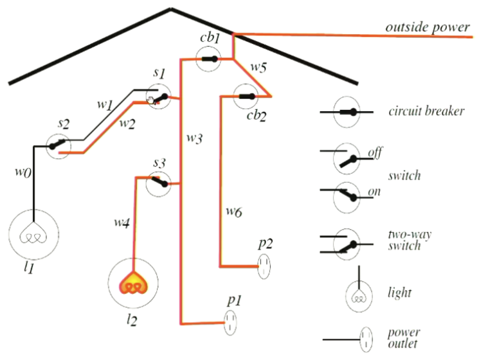
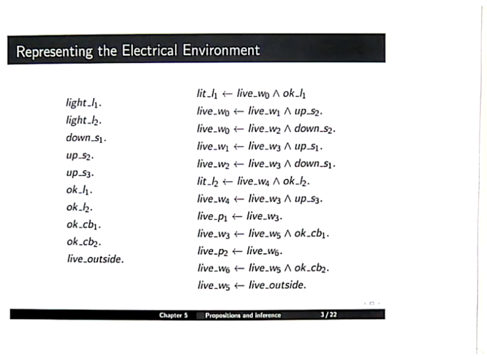
24

Figure 7: Dominance Relation

> Further algorithms are discussed in this segment of the and lecture *boarders onto lecture three* however this content will not be assessed in the duration of this course.

Lecture Three: Knowledge Base and Information

How to represent information in a knowledge base

Information	Knowledge Base
	

outside power
cb1
w5
s1
w1
w2
s2
cb2
w3
s3
w0
w4
w6
p2
l1
l2
p1
circuit breaker
off
switch
on
two-way switch
light
power outlet

Representing the Electrical Environment

$light\_l_1.$
$light\_l_2.$
$down\_s_1.$
$up\_s_2.$
$up\_s_3.$
$ok\_l_1.$
$ok\_l_2.$
$ok\_cb_1.$
$ok\_cb_2.$
$live\_outside.$

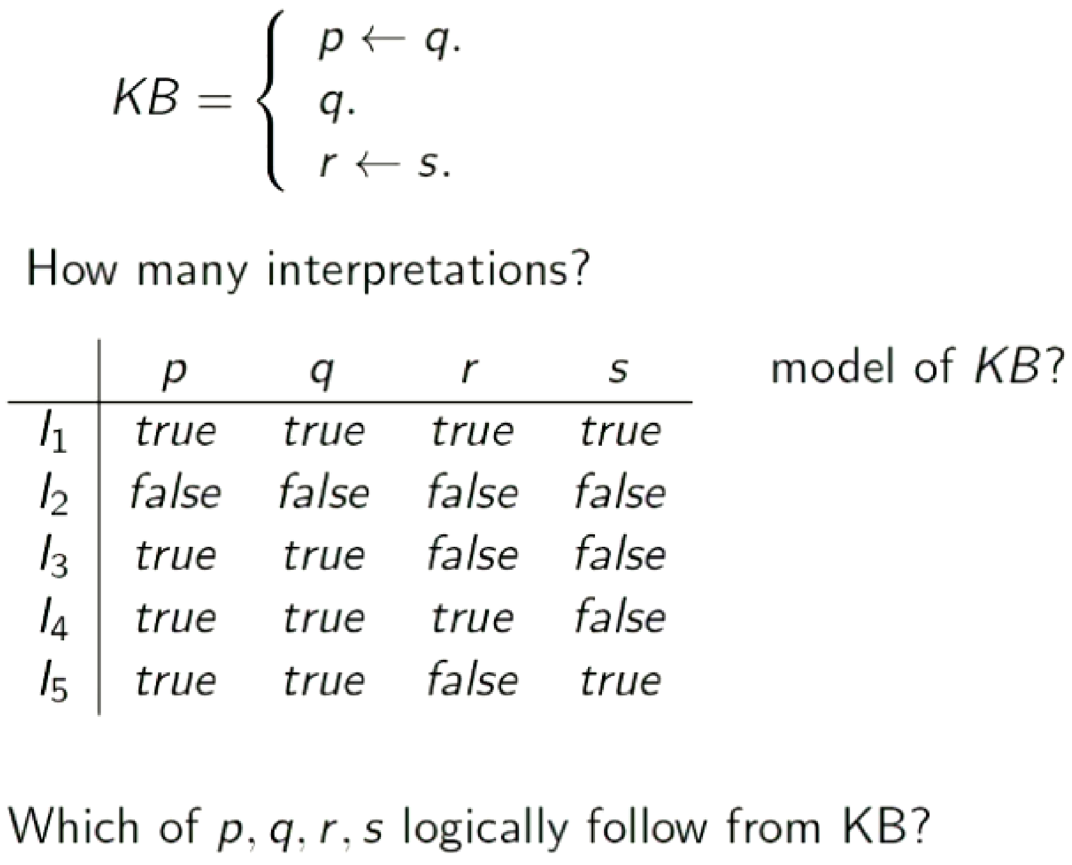
$lit\_l_1 \leftarrow live\_w_0 \land ok\_l_1$
$live\_w_0 \leftarrow live\_w_1 \land up\_s_2.$
$live\_w_0 \leftarrow live\_w_2 \land down\_s_2.$
$live\_w_1 \leftarrow live\_w_3 \land up\_s_1.$
$live\_w_2 \leftarrow live\_w_3 \land down\_s_1.$
$lit\_l_2 \leftarrow live\_w_4 \land ok\_l_2.$
$live\_w_4 \leftarrow live\_w_3 \land up\_s_3.$
$live\_p_1 \leftarrow live\_w_3.$
$live\_w_3 \leftarrow live\_w_5 \land ok\_cb_1.$
$live\_p_2 \leftarrow live\_w_6.$
$live\_w_6 \leftarrow live\_w_5 \land ok\_cb_2.$
$live\_w_5 \leftarrow live\_outside.$

Chapter 5 Propositions and Inference 3 / 22

- The computer doesn't know the meaning of the symbols (logical and etc…)
- The user can interpret the symbol using their meaning
- There is no specific syntax for this, it is just what ever is readable for the user/writer

Simple language and definitions

- An **atom** is a symbol starting with a lower case letter
- A **body** is an atom or is of the form $b_1 \land b_2$ where b_1 and b_2 are bodies
- A **definite clause** is an atom or a rule of the form $h \leftarrow b$ where h is an atom and b is a body
- A **knowledge base** is a set of definite clauses
- An **interpretation** i assigns a truth value to each atom
- A **body** $b_1 \land b_2$ is true in i if b_1 is true in i and b_2 is true in i
- A **rule** $h \leftarrow b$ is false in i if b is true in i and h is false in i, the rule is true otherwise
- A **knowledge base** KB is true in i if and only if every clause in KB is true in i
- A **model** of a set of clauses is an interpretation in which all the clauses are *true*
- If KB is a set of clauses and g is a conjunction of atoms, g is a **logical consequence** of KB, this is denoted as $KB \models g$, if g is *true* in every model of KB
  - That is, $KB \models g$ if there is no interpretation in which KB is *true* and g is *false*.
- A **Proof procedure** is a -possibly non-deterministic - algorithm for deriving consequences of a knowledge
- Given a proof procedure, $KB \vdash g$ means g can be derived from knowledge base KB
- Recall $KB \models g$ means g is *true* in all models of KB
- A proof procedure is **sound** if $KB \vdash g \implies KB \models g$
- A proof procedure is **complete** if $KB \models g \implies KB \vdash g$



$$KB = \begin{cases} p \leftarrow q. \\ q. \\ r \leftarrow s. \end{cases}$$

How many interpretations?

	p	q	r	s	model of KB?
I_1	*true*	*true*	*true*	*true*	
I_2	*false*	*false*	*false*	*false*	
I_3	*true*	*true*	*false*	*false*	
I_4	*true*	*true*	*true*	*false*	
I_5	*true*	*true*	*false*	*true*	

Which of p, q, r, s logically follow from KB?

Figure 8: simple example question

Answers to the questions:

> We have four atoms {p,q,r,s}, because we have 4 atoms, there are 16 permutations in our truth table (2^4), therefore we have 16 interpretations

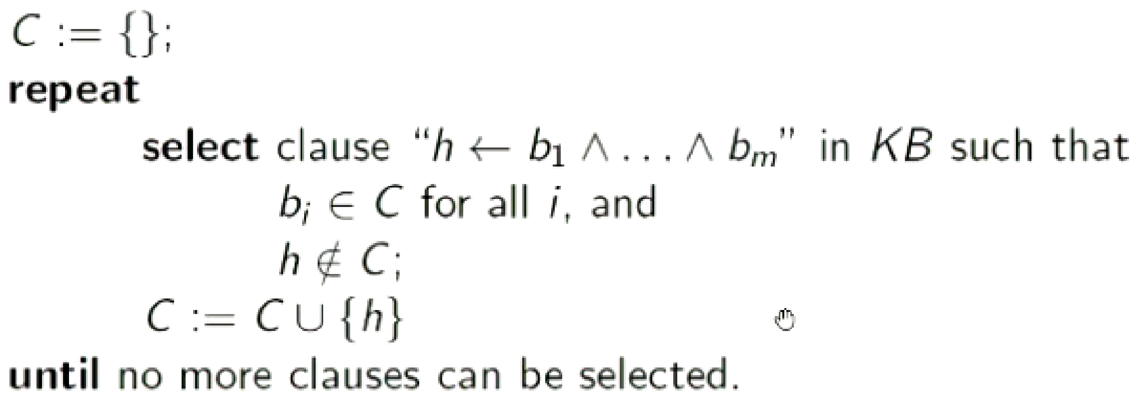
Bottom-up proof procedure

Rule of derivation:

if $h \leftarrow b_1 \wedge ... \wedge ...b_m$ is a clause in the knowledge base, and each b_i has been derived, then h can be derived

- This is **Forward chaining** on this clause (this rule also covers the case when $m = 0$)
- $KB \vdash g$ if $g \in C$ at the end of the below algorithmic procedure

- Tracing tutorial: 1:13:30



```
$C := \{\}$;
repeat
        select clause "$h \leftarrow b_1 \wedge \ldots \wedge b_m$" in $KB$ such that
                $b_i \in C$ for all $i$, and
                $h \notin C$;
        $C := C \cup \{h\}$
until no more clauses can be selected.
```

Figure 9: Bottom-up proof procedure algorithm pseudo code

Top-down proof procedure

Idea: search backward from a query to determine if it is a logical concequence of KB

An **answer clause** is of the form:

- $yes \leftarrow a_i \wedge ... \wedge a_m$

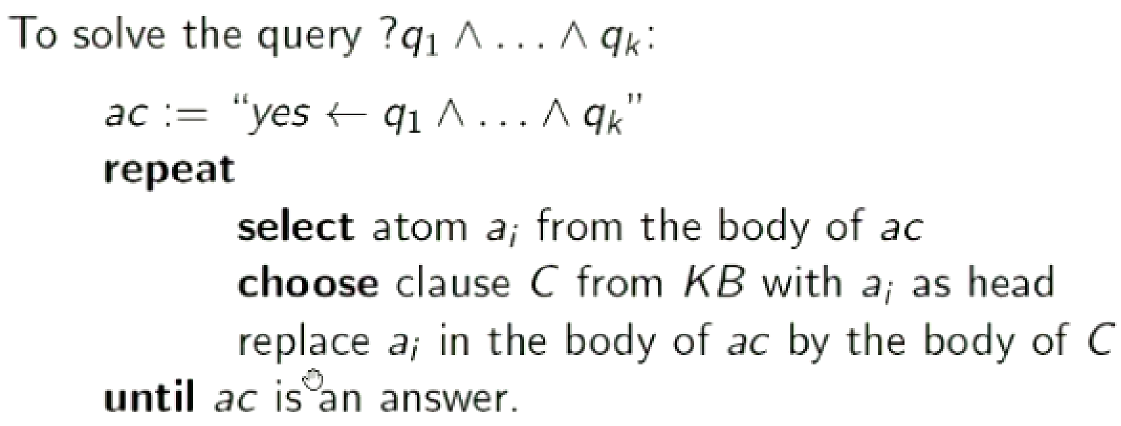
The SLD Resolution of this answer clause on atom a_i with the clause:

- $a_i \leftarrow b_1 \wedge ... \wedge b_p$
- Tracing tutorial: 1:31:00

An **answer** is an answer clause with $m = 0$. That is the answer clause $yes \leftarrow$.

A **Derivation** of query $?q_1 \wedge ... \wedge q_k$ from KB is a sequence of answer clauses $\lambda_0, \lambda_1, ...\lambda_n$

- λ_0 is the answer clause $yes \leftarrow q_1 \wedge ... \wedge q_k$
- λ_1 is obtained by resolving λ_{i-1} with a clause in KB
- λ_n is the answer



```
To solve the query $?q_1 \wedge \ldots \wedge q_k$:
    $ac :=$ "$yes \leftarrow q_1 \wedge \ldots \wedge q_k$"
    repeat
        select atom $a_i$ from the body of $ac$
        choose clause $C$ from $KB$ with $a_i$ as head
        replace $a_i$ in the body of $ac$ by the body of $C$
    until $ac$ is an answer.
```

Figure 10: Top-down proof procedure algorithm pseudo code

> There is more information on SLD Resolution at the end of this lecture, this will be needed in the assignment

Lecture Four: Declarative Programming (Part One)

What is declarative programming?

Declarative programming is the use of mathematical logic to describe the logic of computation without describing its control flow

- Knowledge bases and queries in propositional logic are made up of propositions and connectives
- Predicate logic adds the notion of *predicates* and *variables*
- We take a non-theoretical approach to predicate logic by introducing *declarative programming*
- useful for: expert systems, diagnostics, machine learning, parsing text, theorem proving, …

Datalog

- Prolog is a declarative programming language and stand for PROgramming in LOGic
- we only look at a sybset of the language which is equal to Datalog
- Think declaratively, not procedurally
- High level, interpreted language
- We will have a file that contains a knowledge base, and we will have an interpreter where we can ask queries

Here is an example of a knowledge base in Datalog:

```
woman(mia)
woman(jody)
woman(yolanda)
playesAirGuitar(yolanda)
```

Here is how we may query data using the interpreter:

```
$ woman(mia)
yes
```

> Further examples of this are in the slides of lecture four

Operators

- Implication :-
- Conjunction: , (AND)
- Disjunction ; (OR)
- We will later talk about how to simulate the (NOT) operator

Interpreter Operands and rules:

- Variables: X, Y, Z, Cam, AnythingThatStartswithUppercase
  - Acts as a `wildcard` to match with when querying
- Order of arguments matters
- `Arity` is important
- Unification/matching:
  - Two terms unify or match if they are the same term or if the contain variables that can be uniformly instanciated with terms in such a way that the resulting terms are equal (this is how we query)
  - Example: $l(s(g), Z) = k(X, t(Y))$

With only Unification we can do some programming

```
vertical(line(point(X,Y), point(X,Z))
horizontal(line(point(X,Y), point(Z,Y))
```

Proof Search

- Prolog has a specific way of answering queries
  - Search knowledge base from top to bottom
  - Processes clauses from left to right

    - Backtracking to recover from bad choices

- Further examples using prolog: 1:10:00

Recursive Programming

```
child(anna, bridget)
child(bridget, caroline)
child(caroline, donna)
child(donna, emily)
decendent(X,Y):-child(X,Y)
decendent(X,Y):-child(X,Z), decendent(Z,Y)
```

If we make the following query with the above knowledge base, we get a positive response

```
$- decendent(anna, donna)
yes
```

Lecture Five: Declarative Programming (Part Two)

Lists in Prolog

- A list is a finite sequence of elements
- List elements are enclosed in square brackets
- we can think of non-empty lits as a head and tail
    - Head is first item
    - Tail is the rest of the list
- Empty list has no head or tail
- Here are some examples of lists in prolog

```
[mia, vincent, jules, yolanda]
[mia, robber(honeybunny), X, 2, mia]
[]
```

Pipe Operand

- Can be used for creating a list
- Example:

```
[head|tail] = [mia, vincent, jules, yolanda].
Head = mia
tail = [vincent, jules, yolanda].
```

- We can have anonymous variables denoted with the `_`

- These do not get recorded and assigned to variables

```
[_,X2,_,X4|_] = [mia, vincent, jules, yolanda].
X2 = vincent
X4 = Jody
```

Defining Members of a list

- One of the most basic things we would like to know is whether something is an element of a list or not
- So let's write a predicate that when given a term X and a list L, tells us whether $X \in L$
- We can define member as the following:

```
member(X,[X,_]).
member(X,_),T]):-member(X,T).
```

Defining Append

- We can define an important predixate, append whose arguements are all lists
- Declaratively, append(L1,L2,L3) is true if list L3 is the result of concat L1, L2
- Recursive definition,
  - Base case: appending the empty list to any list produces the same list
  - The recursive step says that when concatenating non-empty list $[H|T]$ with list L, the result is a list with head H and the result of concatenating T and L

Definition:

```
append([],L,L).
append([H|L1],L2,[H|L3]):-append(L1,L2,L3).
```

Expected Output:

```
$- append([a,b,c],[d,e,f], Z).
$- Z = [a,b,c,d,e,f].
yes
```

Sublist

- Now it is very easy to write a predicate that finds sub-lists of lists
- The sub-lists of a list L are simply the prefixes of suffixes of `L`
- Checks if a list is a subset of another list

```
sublist(Sub,List):-suffix(Suffix,List),prefix(Sub,Suffix).
```

Reversing a list

- Recursive definition
    1. If we reverse the empty list, we obtain the empty list
    2. If we reverse the list $[H|T]$, we end up with the list obtained by reversing T
    3. This solution works, but is extremely inefficient, *Quadratic time*

```
reverse([], []).
reverse([H|T],R) :- reverse(T,RT), append(RT,[H],R).
```

- Here is a much more efficient solution:
- We can use an accumulator (list to append the reverse to) in order to make this faster

```
accReverse([],L,L).
accReverse([H,T],Acc,Rev):-accReverse(T,[H|Acc],Rev).

reverse(L1,L2):-accReverse(L1,[],L2). # Wrapper for accReverse function
```

> The above is a more efficient solution

Negation as Failure

- We need to use the cut operator (!) to suppress backtracking
- The fail predicate always fails
- They can be combined to get a negation as failure

```
neg(Goal):-Goal,!,fail.
neg(Goal).
```

Lecture Six: Local and Global Search (Optimisation)

Optimisation Problems

Given:

- A set of variables and their domains; and
- An objective function (aka a cost function),

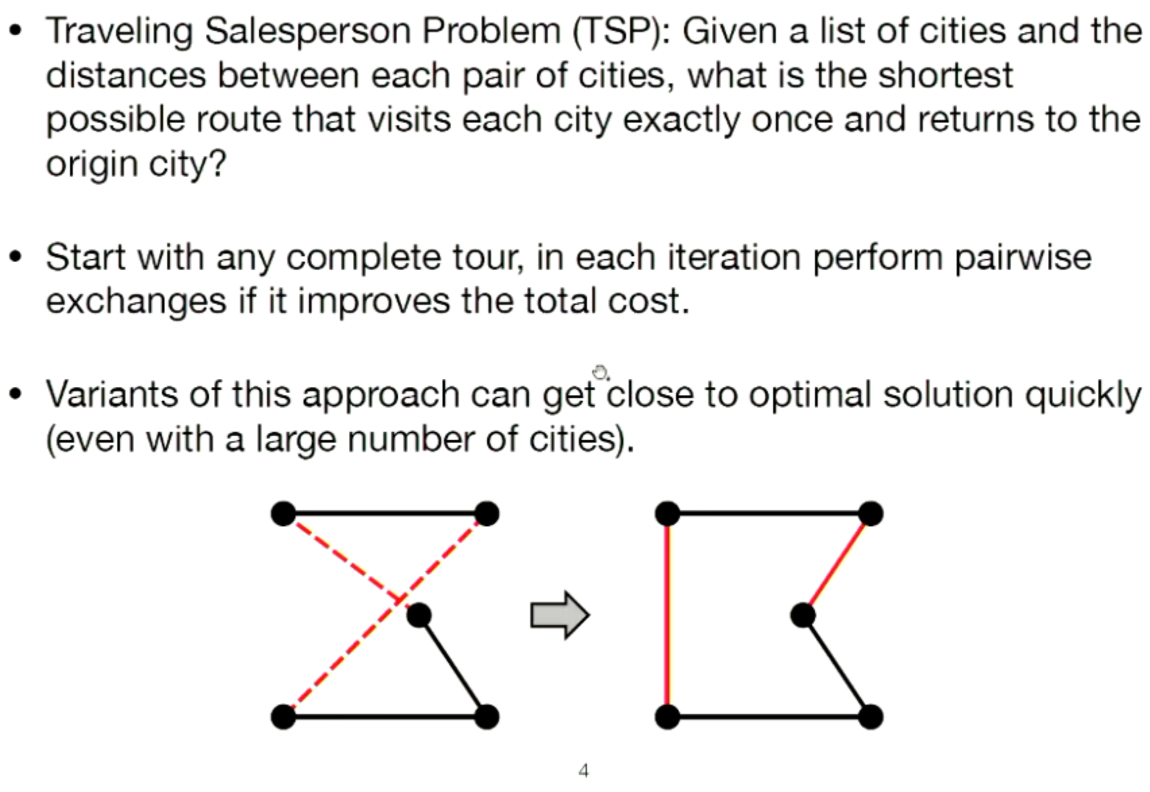
Find an assignment (of each value to each variable) that optimises (max or min) the value of the objective function.

- Optimisation usually involves searching
- CSP's and optimisation problems can be converted (reduced) to each other
- There are special algorithms for certain kind of optimisation problems (linear programming, convex optimisation)

- In this lecture we will look at two families of algorithms (local and global)

Local Search for Optimisation

- A **Local search** algorithm is an iterative algorithm that keeps a single current state and in each iteration tries to improve it by moving to one of its neighbouring states.
- Two key aspects to decide:
  - Neighbourhood: which states are the neighbours of a given state
  - Movement: which neighbouring state should the algorithm go to
- Asearch algorithm is considered to be greedy if it always moves to the best neighbour. Two variants happen to have special names:
  - *Hill climbing*: for maximisation
  - *Greedy descent*: for minimisation



- Traveling Salesperson Problem (TSP): Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?
- Start with any complete tour, in each iteration perform pairwise exchanges if it improves the total cost.
- Variants of this approach can get close to optimal solution quickly (even with a large number of cities).

4

Figure 11: Example of local search

Local search for CSP's:

- A constrained satisfaction problem can be reduced to an optimisation problem

- Given an assignment, a conflict is an unsatisfied constraint
- Heuristic function: the number of conflicts produced by an assignment
- Optimisation problem: find an assignment that minimises this heuristic function

Local search for CSP's in neighbourhood:

- Neighbours of a given state can be defined in many ways
    - All possible assignments except the current one
    - Select a variable that appears in any conflict, neighbours are assignment in which that variable takes a different value from its domain
    - Select a variables in the current assignment that participates in the most number of conflicts. Neighbours are assignments in which that variable takes a different value from its domain
    - n-queens example (35:00)

Global Search

Parallel search:

- A total assignment is called an **individual**
- Idea: maintain a population of k individuals instead of one
- At every stage, update each individual in the population
- Like k restarts, but uses k times the minimum number of steps
- A basic form of global search

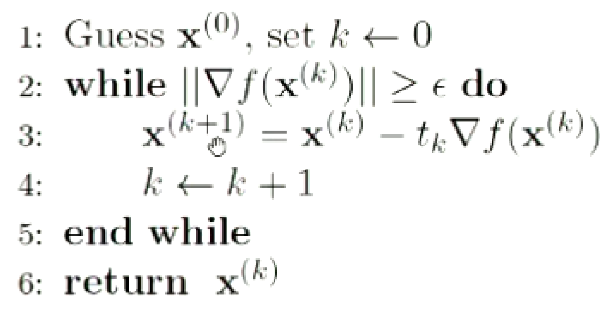
Simulating Annealing:

- Pick a variable at random and a new value at random
- If it is an improvement, adopt it
- If it isn't an improvement adopt it probabilistically depending on a temperature parameter, T
- Temperature can be reduced

Temperature	1-worse	2-worse	3-worse
10	0.91	0.81	0.74
1	0.37	0.14	0.05
0.25	0.02	0.003	0.00005
0.1	0.00005	0	0

Gradient Descent

- A widely used local search algorithm in numeric optimisation (machine learning)
- Used when the variagles are numeric and continous
- The objective function must be differentiable (mostly)



```
1: Guess $\mathbf{x}^{(0)}$, set $k \leftarrow 0$
2: while $||\nabla f(\mathbf{x}^{(k)})|| \geq \epsilon$ do
3:     $\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} - t_k \nabla f(\mathbf{x}^{(k)})$
4:     $k \leftarrow k + 1$
5: end while
6: return $\mathbf{x}^{(k)}$
```

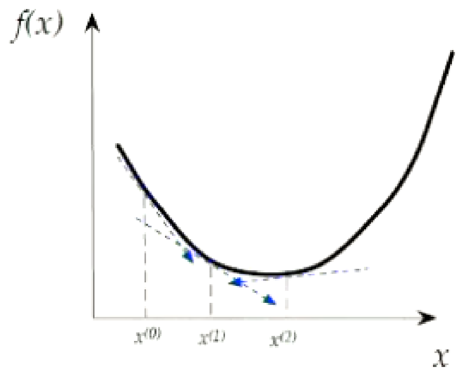


Figure 12: Gradient descent algorithm

Genetic Algorithms

- Genetic algorithms and the whole family of evolutionary algorithms are inspired by natural selection
- They are in the global search family
- The algorithm maintains a population of individuals which evolves over time
- We can make an individual to represent anything we want
- A fitness function is needed. The function takes an individual as input and returns a numeric number indicating how good/bad the individual is
- A mechanism is needed to create the initial population
- A mechanism is needed to `evolve` the current population to the next one. This involves the following mechanisms:
  - Selection: decide which individuals survive or can reproduce
  - Crossover: given a number of parent individuals, create a number of children
  - Mutation: make some random changes to individuals
  - How this works (1:20:00)

Roulette Wheel selection: Example

- Sum the fitness of all individuals, call it T
- Generate a random number N between 1 and T
- Return individual whose fitness added to the running total is equal to or larger than N

- Chance to be selected is exactly proportional to fitness
- Individual: [1,2,3,4,5,6]
- Fitness: [8,2,17,7,4,11]
- Running total: [8,10,27,34,38,49]
- $N: 23$
- selected: 3

Tournament Selection

- Choose n individuals randomly; the fittest one is selected as a parent
- n is the `size` of the tournament
- By changing the size, selection pressure can be adjusted

Crossover

Often individuals are represented as a sequence (tuple) of values. With this representation, cross over can be performed very easily.

- Generate 1,2, or a number of random *crossover points*
- Split the parents at these points
- Create offspring's by exchanging alternative segments

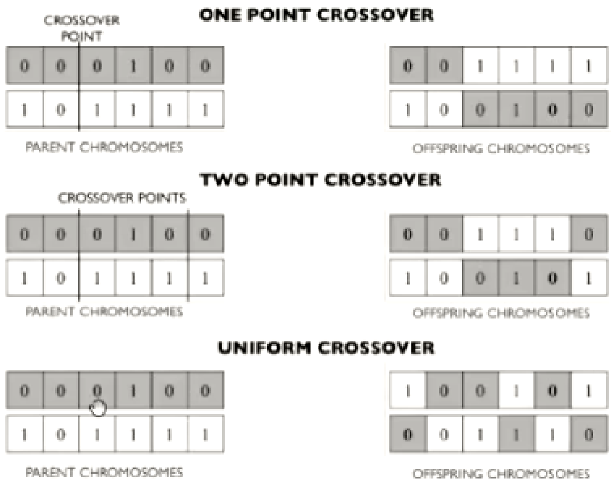


Figure 13: Crossover Example

Mutation

With sequential representation (tuples), mutation is performed by selecting one or more random locations (indices) and changing the values at those locations to some random values (from the domain).

Mutation vs Crossover

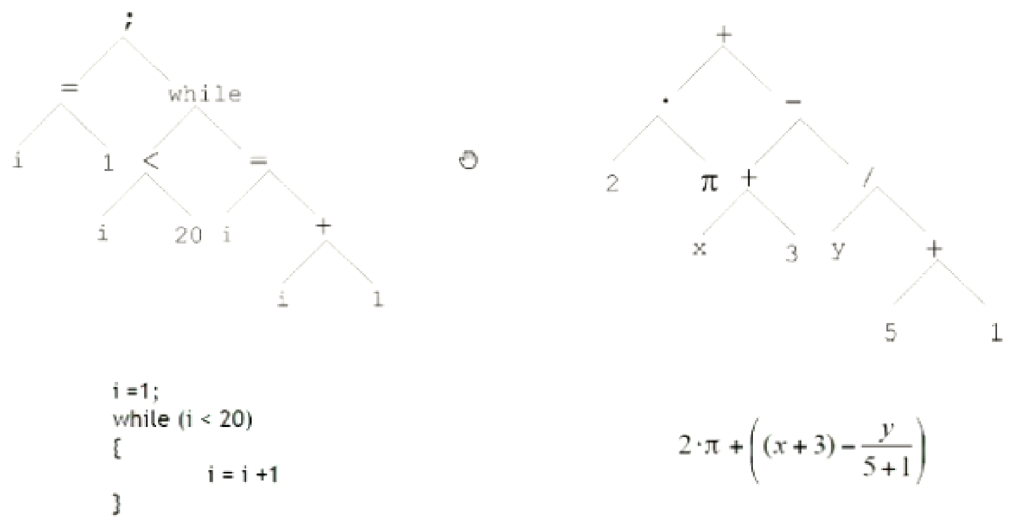
- Purpose of crossover: combining somewhat good candidates in the hope of producing better children
- Purpose of mutation: bringing diversity
- Its good to have both
- Mutation-only-EA is possible, crossover-only-EA would not work

Fitness landscapes

- EA's are known to be able to handle relatively challenging fitness landscapes
- *see lecture: sep 14 for more information, note to self: go over this again before final*

Tree representation

- Individuals can have more sophisticated structure
- The following shows two example trees representing statements and expressions



```
i =1;
while (i < 20)
{
        i = i +1
}
```

$$2 \cdot \pi + \left((x+3) - \frac{y}{5+1} \right)$$

Figure 14: Tree representation example

Lecture Seven: Belief Networks

What are belief networks about?

- Long answer short **Probabilities**

- Reasons for uncertainty and randomness

Random Variables

- A random variable is some aspect of the world which we have uncertainty
    - R = Is it raining?
    - D = How long will it take to drive to work?
    - L = Where am I?
- We denote random variables with capital letters
- Each random variable has a domain
    - $R \in \{True, False\}$ as an example

Probability distributions

- Unobserved random variables have distributions
- A distribution is a TABLE of probabilities of values
- A probability is a single number

Joint distributions

A *joint distribution* over a set of random variables is a map of assignments to real values

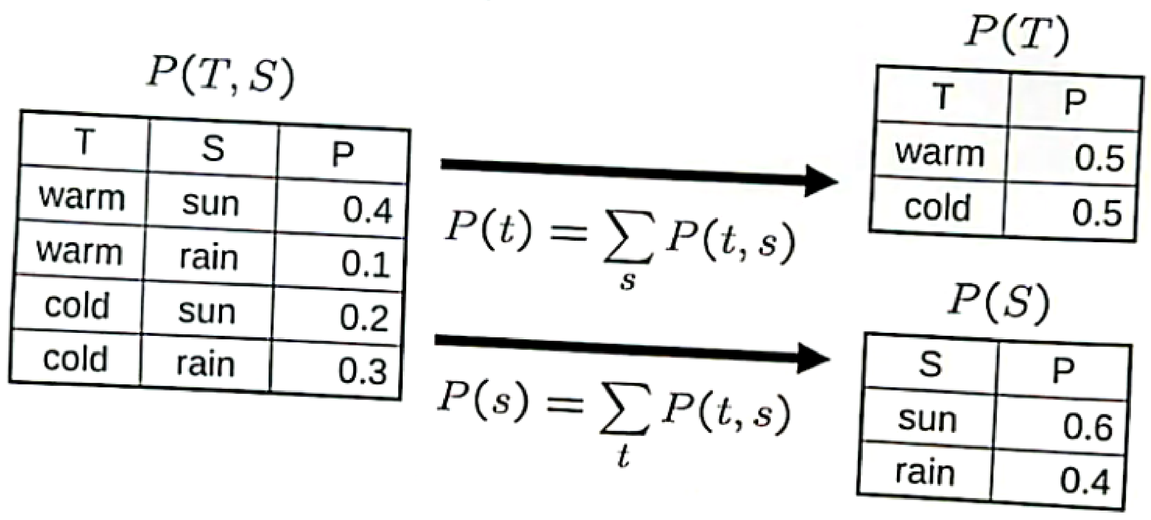
Events

- A set of assignments
- From a joint distribution we can calculate the probability of any event

Marginalization

- Marginalization (or summing out) is *projecting* a joint distribution to a sub-distribution over subset of variables.

$$P(X_1 = x_1) = \sum_{x2} P(X_1 = x_1, X_2 = x_2)$$



$P(T, S)$

T	S	P
warm	sun	0.4
warm	rain	0.1
cold	sun	0.2
cold	rain	0.3

$$P(t) = \sum_s P(t, s)$$

$$P(s) = \sum_t P(t, s)$$

$P(T)$

T	P
warm	0.5
cold	0.5

$P(S)$

S	P
sun	0.6
rain	0.4

Conditional Probabilities

- A conditional probability is the probability of an event given another event, *center of a venn diagram*

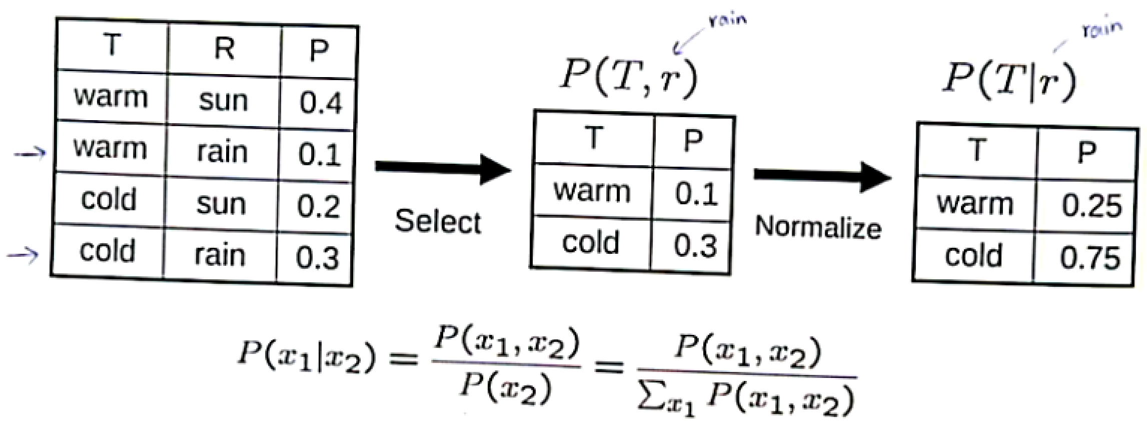
$$P(a|b) = \frac{P(a,b)}{P(b)}$$

Conditional Distributions

- Conditional distributions are probability distributions over some variables given fixed values of others.

Normalization Trick

- A trick to get the whole conditional distribution at once
  - Select the joint probabilities matching the evidence
  - Normalize the selection *divide by total sum so they sum to one*



T	R	P
warm	sun	0.4
warm	rain	0.1
cold	sun	0.2
cold	rain	0.3

$P(T, r)$

T	P
warm	0.1
cold	0.3

$P(T|r)$

T	P
warm	0.25
cold	0.75

$$P(x_1|x_2) = \frac{P(x_1, x_2)}{P(x_2)} = \frac{P(x_1, x_2)}{\sum_{x_1} P(x_1, x_2)}$$

Figure 15: Normalization trick example

The product rule

- Sometimes joint P(X,Y) is easy to get
- Sometimes easier to get conditional P(X|Y)

Defined by the following:

$$P(x|y) = \frac{P(x,y)}{P(y)} \equiv P(x,y) = P(x|y) \times P(y)$$

More generally we can write any joint distribution as incremental product of conditional distributions.

$$P(x_1, x_2, x_3) = P(x_1) \times P(x_2|X_1) \times P(x_3|x_1, x_2)$$

$$P(x_1, x_2, ..., x_n) = \prod_i P(x_i|X_1 ... x_{i-1})$$

Probabilistic Inference

- Probabilistic inference: compute a desired probability from other known probabilities *conditional from joint*
- We generally compute conditional probabilities
  - P(on time | no report accidents) = 0.9
  - These represent the agent's *beliefs* given the evidence

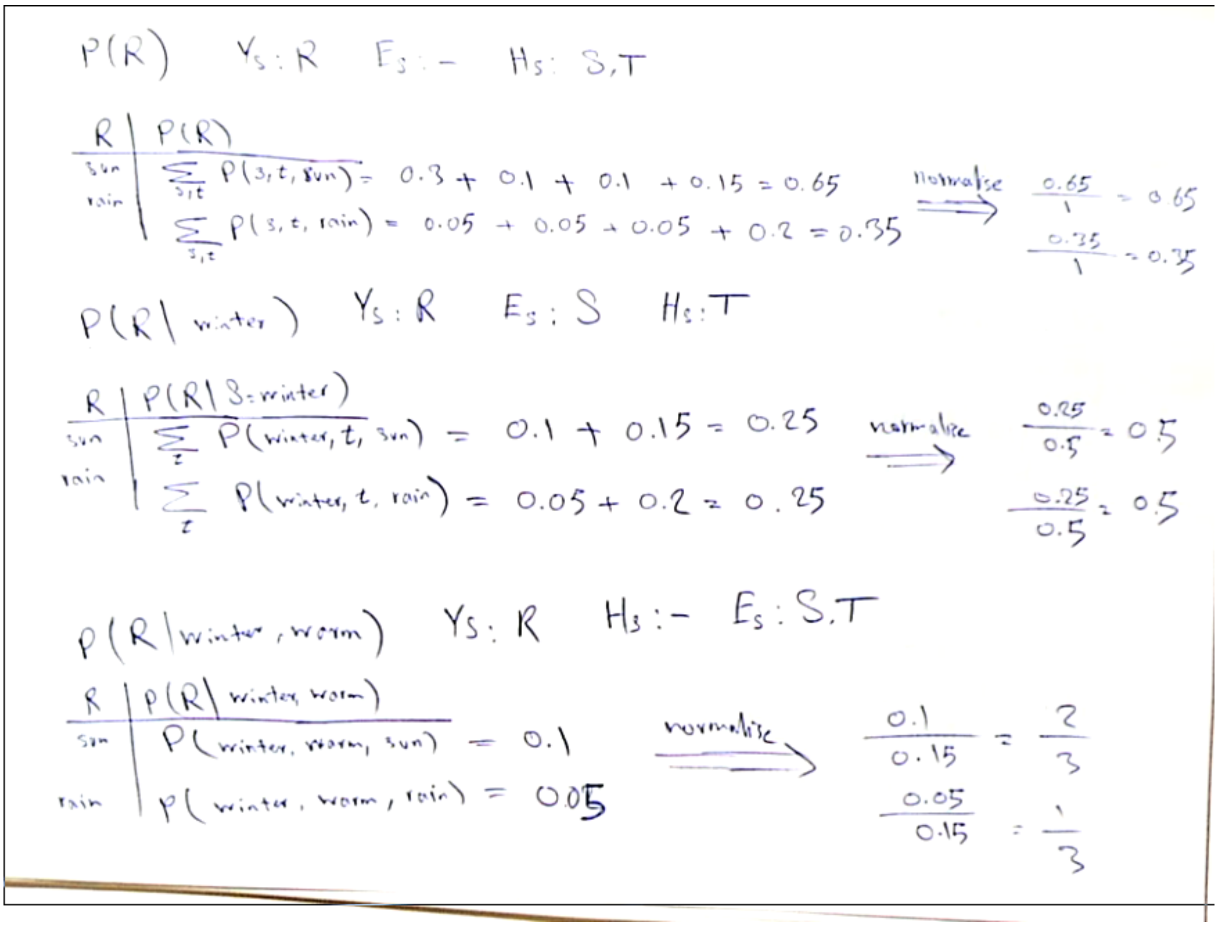
- Probabilities change with new evidence:
  - P(on time | no accidents, 5 am) = 0.95
  - P(on time | no accidents, 5 am, raining) = 0.80
  - Observing new evidence causes beliefs to be updated

Inference by Enumeration:

- we want $P(Y_1...Y_m|e_1...e_k)$
  - Evidence variables $(E_1...E_k) = (e_1...e_k)$
  - Query variables: $Y_1...Y_m$
  - Hidden variables: $H_1...H_r$
- First, select the entries consistent with the evidence
- Second, sum out H:

$$P(Y_1...Y_m, e_1...e_k) = \sum_{h1...hr} P(Y_1...Y_m, h_1...h_r, e_1...e_k) = \{X_1, X_2, ..., X_n\}$$

- Finally normalize the remaining entries
- Obvious problems
  - Worst-case time complexity $O(d^n)$
  - Space complexity $O(d^n)$ to store the joint distribution



$P(R)$ $\quad Y_s: R \quad E_s: - \quad H_s: S,T$

R	$P(R)$		
sun	$\sum_{s,t} P(s,t,sun) = 0.3 + 0.1 + 0.1 + 0.15 = 0.65$	$\xrightarrow{\text{normalise}}$	$\frac{0.65}{1} = 0.65$
rain	$\sum_{s,t} P(s,t,rain) = 0.05 + 0.05 + 0.05 + 0.2 = 0.35$		$\frac{0.35}{1} = 0.35$

$P(R \mid winter)$ $\quad Y_s: R \quad E_s: S \quad H_s: T$

R	$P(R \mid S{=}winter)$		
sun	$\sum_{t} P(winter,t,sun) = 0.1 + 0.15 = 0.25$	$\xrightarrow{\text{normalize}}$	$\frac{0.25}{0.5} = 0.5$
rain	$\sum_{t} P(winter,t,rain) = 0.05 + 0.2 = 0.25$		$\frac{0.25}{0.5} = 0.5$

$P(R \mid winter, warm)$ $\quad Y_s: R \quad H_s: - \quad E_s: S,T$

R	$P(R \mid winter, warm)$		
sun	$P(winter, warm, sun) = 0.1$	$\xrightarrow{\text{normalize}}$	$\frac{0.1}{0.15} = \frac{2}{3}$
rain	$P(winter, warm, rain) = 0.05$		$\frac{0.05}{0.15} = \frac{1}{3}$

Figure 16: Example

Complexity of Models

- Engineers and designers are interested in simple and compact models
  - Simple models are easier to build
  - Simple models are easier to explain
  - Compact models take less space
  - Ususally implies more efficient computational time

If a probabilistic model has multiple distributions, the number of its free parameters is the sum of the number of free parameters of the tables/distributions.

Independence

- Two variables are independent if $P(x, y) = P(x)P(y)$
  - This says that their joint distribution factors into a product of two simpler distributions

- We can use independence as a modelling assumption
  - Independence can be simplify assumptions

Conditional Independence

- Absolute/Unconditional independence is very rare
- Conditional independence:
  - $\forall x, y, z : P(x, y|z) = P(x|z)P(y|z)$
  - $\forall x, y, z : P(x|y, z) = P(x|z)$