# Lecture 9: Pseudorandom Numbers and Stream Ciphers

COSC362 Data and Network Security

Book 1: Chapter 8 – Book 2: Chapter 20

Spring Semester, 2021

#### Motivation

- Random values required in many cases in cryptography.
- For practical reasons, pseudorandom deterministic algorithms are used.
- Stream ciphers constructed from (pseudo)random number generators.
- Examples of stream ciphers widely deployed:
  - ▶ A5 cipher used in GSM mobile phones
  - AES in counter (CTR) mode

```
Random Numbers
DRBG
CTR_DRBG
Dual EC DRBG
```

Stream Ciphers

One Time Pad

Visual Cryptography

Prominent Stream Ciphers

A5 Cipher

**RC4** Cipher

ChaCha Cipher

Conclusion

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#### Randomness

- ▶ Defining randomness is difficult.
- ▶ What we want: any specific string of bits is exactly as random as any other string.
- Generators of random strings:
  - True random number generator (TRNG) is a physical process which outputs each valid string independently with equal probability.
  - Pseudorandom number generator (PRNG) is a deterministic algorithm which approximates a TRNG.
- Using a TRNG to provide a seed for a PRNG.

## True Random Number Generator (TRNG)

- ► NIST Special Publication 800-90B (Jan. 2016):
  - ► Framework for design and validation of TRNG algorithms, called *entropy sources*.
  - Specification of statistical tests for validating the suitability of entropy sources.
- The entropy source includes:
  - A physical noise source
  - A digitization process
  - Post-processing stages
- The output of the entropy source is any requested number of bits.
- ▶ Periodic *health test* to ensure continuing reliable operation.
- Intel introduced TRNG into Ivy Bridge processors in 2012.

## Pseudorandom Number Generator (PRNG)

- ▶ NIST Special Publication 800-90A (June 2015):
  - Recommendation of specific PRNG algorithms, named deterministic random bit generator (DRBG)
  - ▶ DRBG is based on hash functions, a specific MAC (known as HMAC) and block ciphers in counter mode.
- Each generator takes a seed as input.
- It outputs a bit string before updating its state.
- ► The seed should be updated after some number of calls.
- The seed can be obtained from a TRNG.

LDRBG

#### **Functions**

- Instantiate: setting the initial state of the DRBG using a seed.
- Generate: providing an output bit string for each request.
- Reseed: inputting a new random seed and updating the state.
- ► *Test:* checking correct operation of the other functions.
- Uninstantiate: deleting (zeroising) the state of the DRBG.

LDRBG

## Security

Security w.r.t. the ability of an attacker to distinguish reliably between its output and a truly random string:

- Backtracking resistance: an attacker who obtains the current state of the DRBG should not be able to distinguish between the output of earlier calls to the function Generate and random strings.
- ► Forward prediction resistance: an attacker who obtains the current state of the DRBG should not be able to distinguish between the output of later calls to the function Generate and random strings.

#### CTR\_DRBG

- ▶ Using a block cipher in counter (CTR) mode:
  - ▶ Recommendation: AES with 128-bit keys
- DRBG initialised with a seed whose length is equal to the key length PLUS the block length:
  - ▶ 128 + 128 = 256 for AES with 128-bit master keys
- ▶ Seed defines a key *K* and a counter value *ctr*:
  - No separate nonce as in a normal CTR mode
- CTR mode encryption is run iteratively, with no plaintext added.
- The output blocks form the CTR\_DRBG output.

## **Update Function**

- Each request to DRBG generates up to 2<sup>19</sup> bits.
- From the function Generate:
  - (K, ctr)'s state must be updated after each request by generating 2 blocks using the current key to obtain the new key and a counter.
- Updating provides backtracking resistance.
- ► Restriction: up to 2<sup>48</sup> requests to the function Generate before requiring re-seeding.
- Each re-seed provides forward prediction and backtracking resistance.

#### Random Numbers

└─Dual EC DRBG

## Dual\_EC\_DRBG

- ► From an older standard (Dec. 2012).
- Based on elliptic curve discrete logarithm problem:
  - ▶ But no security proof exists
  - ▶ And many flaws: https://blog.cryptographyengineering.com/2013/ 09/18/the-many-flaws-of-dualecdrbg/
- Much slower than other DRBGs in the standard.
- Press (Reuters) reported a secret 10 million dollar deal between NSA and RSA Security company to use Dual\_EC\_DRBG as the default PRNG in its software suite (Dec. 2013).

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#### Stream Ciphers

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Prominent Stream Ciphers
A5 Cipher
RC4 Cipher
ChaCha Cipher

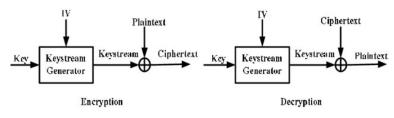
Conclusion

## Stream Ciphers

- ► Characterised by the generation of a *keystream* using a short key and an initialisation value *IV*.
- ► Each element of the keystream is used successively to encrypt 1 or more ciphertext characters.
- Usually symmetric key ciphers:
  - ▶ The sender and receiver share the same key.
  - ▶ They can generate the same keystream given the same *IV*.

## Synchronous Stream Ciphers

- ► The keystream is generated independently of the plaintext.
- ▶ Both sender and receiver need to generate the same keystream and synchronise on its usage.
- Vigenère cipher seen as a (periodic) synchronous stream cipher where each shift is defined by a key letter.
- CTR mode of operation for a block cipher is one method to generate a keystream.



## Binary Synchronous Stream Ciphers

#### For each time interval t:

- $\blacktriangleright$  Binary sequence s(t), that is the keystream
- ▶ Binary plaintext p(t)
- ▶ Binary ciphertext c(t)

Encryption:  $c(t) = p(t) \oplus s(t)$ Decryption:  $p(t) = c(t) \oplus s(t)$ 

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#### One Time Pad

- Often attributed to Vernam who made a one-time pad machine using teletype machinery in 1917 (earlier historical uses are known).
- Key is a random sequence of characters s.t. all of them are independently generated.
- Each character in the key is used ONE TIME ONLY.
- Alphabet of any length but usually:
  - A natural language alphabet
  - ► The binary alphabet {0, 1}
- Example: (non-periodic) binary synchronous stream cipher.
- Providing perfect secrecy.

## Perfect Secrecy

#### Shannon's definition:

- ▶ Message set  $\{M_1, \dots, M_k\}$ .
- ▶ Ciphertext set  $\{C_1, \dots, C_l\}$ .
- ▶  $Pr(M_i|C_j)$  is the probability that  $M_i$  is encrypted given that  $C_j$  is observed.
- $\triangleright$  In most cases, the messages  $M_i$  are NOT be equally likely.
- ▶ For all messages  $M_i$  and ciphertexts  $C_i$ :

$$Pr(M_i|C_j) = Pr(M_i)$$

## One Time Pad Using Roman Alphabet

- ▶ Plaintext characters:  $p_1, \dots, p_r$
- ▶ Ciphertext characters:  $c_1, \dots, c_r$
- ▶ Keystream: random characters  $k_1, \dots, k_r$
- ▶ Encryption:  $c_i = (p_i + k_i) \mod 26$
- ▶ Decryption:  $p_i = (c_i k_i) \mod 26$
- Ciphertext is the addition of plaintext and keystream characters, modulo 26.

## One Time Pad Perfect Secrecy

- ▶ Let a ciphertext *C<sub>i</sub>* be observed.
- Any message could have been sent, depending on the keystream.
- ▶ The probability that  $M_i$  is sent given that  $C_j$  is observed = the probability that  $M_i$  is chosen, weighted by the probability that the right keystream is chosen.
- ► Each key is chosen with equal probability.
- ▶ Conditional probability is thus  $Pr(M_i|C_j) = Pr(M_i)$

## Example

Plaintext: HELLO

Keystream: EZABD

▶ Ciphertext: LDLMR

Given the ciphertext, the plaintext can be ANY 5-letter message.

#### Vernam Binary One Time Pad

- ▶ Plaintext: binary sequence  $b_1, \dots, b_r$
- ▶ Ciphertext: binary sequence  $c_1, \dots, c_r$
- ▶ Keystream: random binary sequence  $k_1, \dots, k_r$
- ▶ Encryption:  $c_i \equiv p_i \oplus k_i$
- ▶ Decryption:  $p_i \equiv c_i \oplus k_i$
- Keystream is SAME length as plaintext.
- Providing perfect secrecy since any ciphertext is equally possible given the plaintext.
- Encryption and decryption are identical processes.

#### **Properties**

- ► Shannon showed that any cipher with perfect secrecy MUST have as many keys as there are messages.
- One time pad is the ONLY unbreakable cipher.
- Practical usage is possible for pre-assigned communications between fixed parties.
- Problem: how to deal with key management of completely random keys?
  - Key generation, key transportation, key synchronization, key destruction are ALL problematic since the keys are SO large.

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#### Visual Cryptography

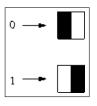
Prominent Stream Ciphers A5 Cipher RC4 Cipher ChaCha Cipher

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## Visual Cryptography

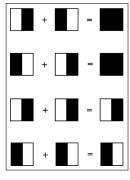
- Application of one time pad: visual cryptography splits an image into 2 shares.
- ▶ Decryption works by *overlaying* the 2 shared images.
- First proposed by Naor and Shamir in 1994.
- Simple case: monochrome images with black and white pixels.
- Many generalisations are possible.
- Each pixel is shared in a random way, similar to splitting a bit in the one time pad.
- Each share reveals NO information about the image:
  - Unconditional security as one time pad.

## **Encryption**



- Generate a one time pad P (random bit string) with length equal to the number of pixels for the image I
- ▶ Generate a share S<sub>I,1</sub> by replacing each bit in P using the sub-pixel patterns shown on the left
- ▶ Generate the other share  $S_{1,2}$  s.t.:
  - ▶ the same as  $S_{l,1}$  for all the white pixels of l
  - the opposite of  $S_{l,1}$  for all black pixels of l

## Decryption



- ► To reveal the hidden image I,  $S_{I,1}$  and  $S_{I,2}$  are overlayed
- ► Each black pixel of *I* is black in the overlay
- ► Each white pixel of *I* is half white in the overlay

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A5 Cipher

# A5 Cipher

- Binary synchronous stream cipher applied in most GSM mobile telephones.
- 3 variants:
  - ▶ A5/1 is the original algorithm defined in 1987.
  - A5/2 is a weakened version of A5/1, originally intended for deployment outside Europe, but no longer allowed under GSM standards.
  - ► A5/3, also known as KASUMI, is an algorithm for deployment in 3G mobile systems.
- Design was originally kept confidential by its designers but became public in 1994.

LA5 Cipher

# A5/1 Design

- ► A5/1 algorithm uses 3 linear feedback shift registers (LFSRs) whose output is combined.
- ▶ The 3 LFSRs are irregularly clocked:
  - ▶ The overall output is non-linear.
  - ▶ 64-bit keystream s.t. 10 bits fixed at zero.
  - The effective key length is thus 54 bits.
- Many successful attacks:
  - ▶ In 2008, Gendrullis, Novotny and Rupp reported an attack which broke A5/1 in practice in 7 hours given.

RC4 Cipher

## **RC4** Cipher

- ▶ World-based stream cipher designed by Ron Rivest in the 80s: "Ron's code #4".
- ▶ Simple, efficient for software implementation.
- Originally proprietary owned by RSA Security, but leaked in 1994.
- Widely deployed in TLS before 2013.
- Practical attacks:
  - ▶ When used in TLS protocol and in wireless WPA-TKIP due to bias in its keystream output.
- ▶ Widely believed to be too weak to use in new systems.

## ChaCha Algorithm

- ► Available in TLS ciphersuites (RFC 7905) as a possible replacement for RC4.
- Designed by D. J. Bernstein in 2008.
- Faster than AES:
  - ▶ As little as 4 cycles per byte on x86 processors.
- Combining XOR, addition modulo 2<sup>32</sup> and rotation operations over 20 rounds to produce 512 bits of keystream:
  - ► Example: add-rotate-xor (ARX) cipher.
- Using 256-bit key.

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#### Conclusion

- TRNG constructed from physical devices, used as seeds for PRNG.
- PRNG constructed from other primitives including block ciphers.
- TRNG used to make unbreakable encryption via one time pad.
- ▶ PRNG used as practical synchronous stream cipher.