Contents

Problem One - Conversion to standard form

Question One

$$
\begin{aligned}
\text{min}_{[x]} \quad & -9x_1 + 9x_2 + 9x_3 - 4x_4 - 4x_5 + 4x_6 \\
\text{s.t} \quad & -8x_1 + 6x_2 + 8x_3 - 9x_4 - 2x_5 + 4x_6 \geq -1 \\
& -x_1 + 3x_2 + 6x_4 + 3x_6 \leq -7 \\
& 8x_1 + 7x_2 - 9x_3 - 5x_4 - 3x_5 - 8x_6 \leq -6 \\
& 3x_1 + 3x_2 + 2x_3 - 8x_4 + 7x_5 + 6x_6 = -1 \\
& x_1 \geq 0, x_2 \geq 0, x_4 \geq 0, x_6 \geq 0
\end{aligned}
$$

First we need to remove negative constants from the right hand side of each equation the constraints:

$$
\begin{aligned}
\text{min}_{[x]} \quad & -9x_1 + 9x_2 + 9x_3 - 4x_4 - 4x_5 + 4x_6 \\
\text{s.t} \quad & 8x_1 - 6x_2 - 8x_3 + 9x_4 + 2x_5 - 4x_6 \leq 1 \\
& x_1 - 3x_2 - 6x_4 - 3x_6 \geq 7 \\
& -8x_1 - 7x_2 + 9x_3 + 5x_4 + 3x_5 + 8x_6 \geq 6 \\
& -3x_1 - 3x_2 - 2x_3 + 8x_4 - 7x_5 - 6x_6 = 1 \\
& x_1 \geq 0, x_2 \geq 0, x_4 \geq 0, x_6 \geq 0
\end{aligned}
$$

The next step to translating this equation to standard form is to remove the inequalities. In this linear program, there are three inequalities, so we need to create three slack variables $s_1, s_2, s + 3$, we need to include these in the objective function and use them to equalise constraints.

$$
\begin{aligned}
\text{min}_{[x]} \quad & -9x_1 + 9x_2 + 9x_3 - 4x_4 - 4x_5 + 4x_6 + s_1 - s_2 - s_3 \\
\text{s.t} \quad & 8x_1 - 6x_2 - 8x_3 + 9x_4 + 2x_5 - 4x_6 + s_1 = 1 \\
& x_1 - 3x_2 - 6x_4 - 3x_6 - s_2 = 7 \\
& -8x_1 - 7x_2 + 9x_3 + 5x_4 + 3x_5 + 8x_6 - s_3 = 6 \\
& -3x_1 - 3x_2 - 2x_3 + 8x_4 - 7x_5 - 6x_6 = 1 \\
& x_1 \geq 0, x_2 \geq 0, x_4 \geq 0, x_6 \geq 0, s_1 \geq 0, s_2 \geq 0, s_3 \geq 0
\end{aligned}
$$

The last step to put this into standard form, is to remove negative variables from the linear program, by assigning two non-negative values to any unconstrained values, we can grantee that the resulting variables will be non-negative. The result of this process is as follows:

$$
\begin{aligned}
\min_{[x]} \quad & -9x_1 + 9x_2 + 9(x_3' - x_3'') - 4x_4 - 4(x_5' - x_5'') + 4x_6 + s_1 - s_2 - s_3 \\
\text{s.t} \quad & 8x_1 - 6x_2 - 8(x_3' - x_3'') + 9x_4 + 2(x_5', x_5'') - 4x_6 + s_1 = 1 \\
& x_1 - 3x_2 - 6x_4 - 3x_6 - s_2 = 7 \\
& -8x_1 - 7x_2 + 9(x_3' - x_3'') + 5x_4 + 3(x_5' - x_5'') + 8x_6 - s_3 = 6 \\
& -3x_1 - 3x_2 - 2(x_3' - x_3'') + 8x_4 - 7(x_5' - x_5'') - 6x_6 = 1 \\
& x_1 \geq 0, x_2 \geq 0, x_4 \geq 0, x_6 \geq 0, s_1 \geq 0, s_2 \geq 0, s_3 \geq 0, x_3' \geq 0, x_3'' \geq 0, x_5' \geq 0, x_5'' \geq 0
\end{aligned}
$$

The linear program above is now in standard form.

Question Two

$$\begin{aligned}\min_{[x]} \quad & 5x_1 - 8x_2 - 5x_3 + 9x_4 - x_5 + 6x_6 \\ \text{s.t} \quad & -2x_1 - 4x_2 + 3x_3 - 4x_4 - x_5 \geq 2 \\ & -8x_1 - 10x_2 + x_3 + 3x_4 - 2x_5 + 9x_6 \leq 9 \\ & 3x_1 + 4x_2 - 3x_3 + 4x_4 - 3x_5 \leq -5 \\ & -7x_2 + 2x_3 + 8x_4 - x_5 + 8x_6 \leq 4 \\ & x_1 \geq 0, x_3 \geq 0, x_5 \geq 0, x_6 \geq 0\end{aligned}$$

First we need to remove negative constants from the right hand side of each equation the constraints:

$$\begin{aligned}\min_{[x]} \quad & 5x_1 - 8x_2 - 5x_3 + 9x_4 - x_5 + 6x_6 \\ \text{s.t} \quad & -2x_1 - 4x_2 + 3x_3 - 4x_4 - x_5 \geq 2 \\ & -8x_1 - 10x_2 + x_3 + 3x_4 - 2x_5 + 9x_6 \leq 9 \\ & -3x_1 - 4x_2 + 3x_3 - 4x_4 + 3x_5 \geq 5 \\ & -7x_2 + 2x_3 + 8x_4 - x_5 + 8x_6 \leq 4 \\ & x_1 \geq 0, x_3 \geq 0, x_5 \geq 0, x_6 \geq 0\end{aligned}$$

The next step to translating this equation to standard form is to remove the inequalities. In this linear program, there are four inequalities, so we need to create three slack variables s_1, s_2, s_3, s_4, we need to include these in the objective function and use them to equalise constraints.

$$\begin{aligned}\min_{[x]} \quad & 5x_1 - 8x_2 - 5x_3 + 9x_4 - x_5 + 6x_6 - s_1 + s_2 - s_3 + s_4 \\ \text{s.t} \quad & -2x_1 - 4x_2 + 3x_3 - 4x_4 - x_5 - s_1 = 2 \\ & -8x_1 - 10x_2 + x_3 + 3x_4 - 2x_5 + 9x_6 + s_2 = 9 \\ & -3x_1 - 4x_2 + 3x_3 - 4x_4 + 3x_5 - s_3 = 5 \\ & -7x_2 + 2x_3 + 8x_4 - x_5 + 8x_6 + s_4 = 4 \\ & x_1 \geq 0, x_3 \geq 0, x_5 \geq 0, x_6 \geq 0, s_1 \geq 0, s_2 \geq 0, s_3 \geq 0, s_4 \geq 0\end{aligned}$$

The last step to put this into standard form, is to remove negative variables from the linear program, by assigning two non-negative values to any unconstrained values, we can grantee that the resulting variables will be non-negative. The result of this process is as follows:

$$
\begin{aligned}
\text{min}_{[x]} \quad & 5x_1 - 8(x_2' - x_2'') - 5x_3 + 9(x_4' = x_4'') - x_5 + 6x_6 - s_1 + s_2 - s_3 + s_4 \\
\text{s.t} \quad & -2x_1 - 4(x_2' - x_2'') + 3x_3 - 4(x_4' = x_4'') - x_5 - s_1 = 2 \\
& -8x_1 - 10(x_2' - x_2'') + x_3 + 3(x_4' = x_4'') - 2x_5 + 9x_6 + s_2 = 9 \\
& -3x_1 - 4(x_2' - x_2'') + 3x_3 - 4(x_4' = x_4'') + 3x_5 - s_3 = 5 \\
& -7(x_2' - x_2'') + 2x_3 + 8(x_4' = x_4'') - x_5 + 8x_6 + s_4 = 4 \\
& x_1 \geq 0, x_3 \geq 0, x_5 \geq 0, x_6 \geq 0, s_1 \geq 0, s_2 \geq 0, s_3 \geq 0, s_4 \geq 0, x_2' \geq 0, x_2'' \geq 0, x_4' \geq 0, x_4'' \geq 0
\end{aligned}
$$

The linear program above is now in standard form.

Question Three

$$\begin{aligned}
\min_{[x]} \quad & -2x_1 - 10x_2 + 8x_3 + 2x_4 - 3x_5 + 2x_6 \\
\text{s.t} \quad & -10x_1 + 6x_2 - 6x_3 - 7x_4 - 4x_5 - 7x_6 \geq 3 \\
& -6x_1 - 4x_2 + 5x_3 - 2x_4 - x_5 - 2x_6 = 2 \\
& -7x_1 + x_2 - 9x_4 - 6x_5 - 8x_6 = 3 \\
& 9x_1 + 3x_2 - x_3 - 6x_4 + x_5 \geq -2 \\
& x_1 \geq 0, x_2 \geq 0, x_5 \geq 0, x_6 \geq 0
\end{aligned}$$

First we need to remove negative constants from the right hand side of each equation the constraints:

$$\begin{aligned}
\min_{[x]} \quad & -2x_1 - 10x_2 + 8x_3 + 2x_4 - 3x_5 + 2x_6 \\
\text{s.t} \quad & -10x_1 + 6x_2 - 6x_3 - 7x_4 - 4x_5 - 7x_6 \geq 3 \\
& -6x_1 - 4x_2 + 5x_3 - 2x_4 - x_5 - 2x_6 = 2 \\
& -7x_1 + x_2 - 9x_4 - 6x_5 - 8x_6 = 3 \\
& -9x_1 - 3x_2 + x_3 + 6x_4 - x_5 \leq 2 \\
& x_1 \geq 0, x_2 \geq 0, x_5 \geq 0, x_6 \geq 0
\end{aligned}$$

The next step to translating this equation to standard form is to remove the inequalities within our main constraints (excluding non-negative). In this linear program, there are four inequalities, so we need to create three slack variables s_1, s_2, we need to include these in the objective function and use them to equalise constraints.

$$\begin{aligned}
\min_{[x]} \quad & -2x_1 - 10x_2 + 8x_3 + 2x_4 - 3x_5 + 2x_6 - s_1 + s_2 \\
\text{s.t} \quad & -10x_1 + 6x_2 - 6x_3 - 7x_4 - 4x_5 - 7x_6 - s_1 = 3 \\
& -6x_1 - 4x_2 + 5x_3 - 2x_4 - x_5 - 2x_6 = 2 \\
& -7x_1 + x_2 - 9x_4 - 6x_5 - 8x_6 = 3 \\
& -9x_1 - 3x_2 + x_3 + 6x_4 - x_5 + s_2 = 2 \\
& x_1 \geq 0, x_2 \geq 0, x_5 \geq 0, x_6 \geq 0, s_1 \geq 0, s_2 \geq 0
\end{aligned}$$

The last step to put this into standard form, is to remove negative variables from the linear program, by assigning two non-negative values to any unconstrained values, we can grantee that the resulting variables will be non-negative. The result of this process is as follows:

$$
\begin{aligned}
\min_{[x]} \quad & -2x_1 - 10x_2 + 8(x_3' - x_3'') + 2(x_4' - x_4'') - 3x_5 + 2x_6 - s_1 + s_2 \\
\text{s.t} \quad & -10x_1 + 6x_2 - 6(x_3' - x_3'') - 7(x_4' - x_4'') - 4x_5 - 7x_6 - s_1 = 3 \\
& -6x_1 - 4x_2 + 5(x_3' - x_3'') - 2(x_4' - x_4'') - x_5 - 2x_6 = 2 \\
& -7x_1 + x_2 - 9(x_4' - x_4'') - 6x_5 - 8x_6 = 3 \\
& -9x_1 - 3x_2 + x_3' - x_3'' + 6(x_4' - x_4'') - x_5 + s_2 = 2 \\
& x_1 \geq 0, x_2 \geq 0, x_5 \geq 0, x_6 \geq 0, s_1 \geq 0, s_2 \geq 0, x_3' \geq 0, x_3'' \geq 0, x_4' \geq 0, x_4'' \geq 0
\end{aligned}
$$

The linear program above is now in standard form.

Question Four

$$\begin{aligned}
\text{min}_{[x]} \quad & -4x_1 - 3x_2 - 2x_3 - 3x_4 + 9x_5 - 5x_6 \\
\text{s.t} \quad & -10x_1 - 3x_2 - 4x_3 - 5x_4 - 8x_6 \geq -7 \\
& 1x_1 + 3x_2 + 4x_3 - 8x_4 + 3x_5 = 2 \\
& -6x_1 - 10x_2 + x_3 - 3x_5 + 8x_6 \geq -4 \\
& x_1 \geq 0, x_4 \geq 0, x_5 \geq 0, x_6 \geq 0
\end{aligned}$$

First we need to remove negative constants from the right hand side of each equation the constraints:

$$\begin{aligned}
\text{min}_{[x]} \quad & -4x_1 - 3x_2 - 2x_3 - 3x_4 + 9x_5 - 5x_6 \\
\text{s.t} \quad & 10x_1 + 3x_2 + 4x_3 + 5x_4 + 8x_6 \leq 7 \\
& 1x_1 + 3x_2 + 4x_3 - 8x_4 + 3x_5 = 2 \\
& 6x_1 + 10x_2 - x_3 + 3x_4 - 8x_6 \leq 4 \\
& x_1 \geq 0, x_4 \geq 0, x_5 \geq 0, x_6 \geq 0
\end{aligned}$$

The next step to translating this equation to standard form is to remove the inequalities. In this linear program, there are two inequalities, so we need to create three slack variables s_1, s_2, we need to include these in the objective function and use them to equalise constraints.

$$\begin{aligned}
\text{min}_{[x]} \quad & -4x_1 - 3x_2 - 2x_3 - 3x_4 + 9x_5 - 5x_6 - s_1 + s_2 \\
\text{s.t} \quad & 10x_1 + 3x_2 + 4x_3 + 5x_4 + 8x_6 - s_1 = 7 \\
& 1x_1 + 3x_2 + 4x_3 - 8x_4 + 3x_5 = 2 \\
& 6x_1 + 10x_2 - x_3 + 3x_4 - 8x_6 + s_2 = 4 \\
& x_1 \geq 0, x_4 \geq 0, x_5 \geq 0, x_6 \geq 0, s_1 \geq 0, s_2 \geq 0
\end{aligned}$$

The last step to put this into standard form, is to remove negative variables from the linear program, by assigning two non-negative values to any unconstrained values, we can grantee that the resulting variables will be non-negative. The result of this process is as follows:

$$\begin{aligned}
\text{min}_{[x]} \quad & -4x_1 - 3(x_2' - x_2'') - 2(x_3' - x_3'') - 3x_4 + 9x_5 - 5x_6 - s_1 + s_2 \\
\text{s.t} \quad & 10x_1 + 3(x_2' - x_2'') + 4(x_3' - x_3'') + 5x_4 + 8x_6 - s_1 = 7 \\
& 1x_1 + 3(x_2' - x_2'') + 4(x_3' - x_3'') - 8x_4 + 3x_5 = 2 \\
& 6x_1 + 10(x_2' - x_2'') - (x_3' - x_3'') + 3x_4 - 8x_6 + s_2 = 4 \\
& x_1 \geq 0, x_4 \geq 0, x_5 \geq 0, x_6 \geq 0, s_1 \geq 0, s_2 \geq 0, x_2' \geq 0, x_2'' \geq 0, x_3' \geq 0, x_3'' \geq 0
\end{aligned}$$

The linear program above is now in standard form.

Problem Two - Simplex Algorithm

Question One

$$\begin{aligned}
\text{min}_{[x]} \quad & 14x_1 + 8x_2 + 10x_3 + 14x_4 \\
\text{s.t} \quad & 11x_1 + 7x_2 + 6x_3 + 10x_4 = 7 \\
& 14x_1 + 2x_2 + 5x_3 + 6x_4 = 6 \\
& 10x_1 + 5x_2 + 12x_3 + 6x_4 = 7
\end{aligned}$$

We need to form an auxiliary problem in order to find a basic feasible solution to the problem above This is formed below.

Auxiliary function

$$\begin{aligned}
\text{min}_{[x]} \quad & s_1 + s_2 + s_3 \\
\text{s.t} \quad & 11x_1 + 7x_2 + 6x_3 + 10x_4 + s_1 = 7 \\
& 14x_1 + 2x_2 + 5x_3 + 6x_4 + s_2 = 6 \\
& 10x_1 + 5x_2 + 12x_3 + 6x_4 + s_3 = 7 \\
& x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, s_1 \geq 0, s_2 \geq 0, s_3 \geq 0
\end{aligned}$$

Initialise tableau for auxiliary problem

$$\begin{array}{ccccccc|c}
x_1 & x_2 & x_3 & x_4 & s_1 & s_2 & s_3 & b \\
11 & 7 & 6 & 10 & 1 & 0 & 0 & 7 \\
14 & 2 & 5 & 6 & 0 & 1 & 0 & 6 \\
10 & 5 & 12 & 6 & 0 & 0 & 1 & 7 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 0
\end{array}$$

We plug in $x_i = 0$ for all $1 \leq i \leq m$ which gives us the obvious solution, that a feasible solution to the auxiliary linear programing problem is when $b_i = x_i$, this gives us our initial feasible solution where the following conditions are met:

$$x_1 = 0, x_2 = 0, x_3 = 0, x_4 = 0, s_1 = 7, s_2 = 6, s_3 = 7$$

Now we can start the simplex method to minimise the auxiliary problem. *As the tableu above contains $z = 0$ in the bottom left corner, we know that the original Linear programming problem has a basic feasible solution.*

Simplex method on Auxiliary Problem

First we need to clean out the initial tableau, this will result in $s_1 = s_2 = s_3 = 0$ in last row of the tableau, we can achieve this by preforming arithmetic on the rows of the tableau.

$$
\begin{aligned}
R_4 - R_1 &\rightarrow R_4 \\
R_4 - R_2 &\rightarrow R_4 \\
R_4 - R_3 &\rightarrow R_4
\end{aligned}
$$

The resulting tableau of the above operations is as follows:

$$
\begin{array}{ccccccc|c}
x_1 & x_2 & x_3 & x_4 & s_1 & s_2 & s_3 & b \\
11 & 7 & 6 & 10 & 1 & 0 & 0 & 7 \\
14 & 2 & 5 & 6 & 0 & 1 & 0 & 6 \\
10 & 5 & 12 & 6 & 0 & 0 & 1 & 7 \\
-35 & -14 & -23 & -22 & 0 & 0 & 0 & -20
\end{array}
$$

We can find our pivot point by finding the largest negative and then checking the ratio $\frac{b_i}{y_i}$ for each value i within the column *q*, the element in *q* with the lowest ratio will become the new pivot point for the proceeding operations:

Applying the pivot

Calculating the rows operations

$$
\begin{aligned}
\frac{1}{14}R_2 &\rightarrow R_2 \\
R_1 - 11R_2 &\rightarrow R_1 \\
R_3 - 10R_2 &\rightarrow R_3 \\
R_1 + 35R_2 &\rightarrow R_4
\end{aligned}
$$

After these calculations, the new tableau is as follows:

$$
\begin{array}{ccccccc|c}
x_1 & x_2 & x_3 & x_4 & s_1 & s_2 & s_3 & b \\
0 & \frac{38}{7} & \frac{29}{14} & \frac{37}{7} & 1 & -\frac{11}{14} & 0 & \frac{16}{7} \\
1 & \frac{1}{7} & \frac{5}{14} & \frac{3}{7} & 0 & \frac{1}{14} & 0 & \frac{3}{7} \\
0 & \frac{25}{7} & \frac{59}{7} & \frac{12}{5} & 0 & -\frac{5}{7} & 1 & \frac{19}{7} \\
0 & -9 & -\frac{21}{2} & -7 & 0 & \frac{5}{2} & 0 & -5
\end{array}
$$

Because we still have negative cost values, we must repeat the pivot process, this time as the most negative value is $-\frac{21}{2}$, we calculate the ratios and find that the lowest is $\frac{19}{59}$.

We then calculate the next pivot

Calculating row operations:

$$\frac{R_3}{\frac{59}{7}} \to R_3$$
$$R_1 - \frac{29}{14}R_3 \to R_1$$
$$R_2 - \frac{5}{14}R_3 \to R_2$$
$$R_4 + \frac{21}{2}R_3 \to R_4$$

After these calculations, the new tableau is as follows:

$$\begin{array}{ccccccc|c}
x_1 & x_2 & x_3 & x_4 & s_1 & s_2 & s_3 & b \\
0 & \frac{537}{118} & 0 & \frac{287}{59} & 1 & -\frac{36}{59} & -\frac{29}{118} & \frac{191}{118} \\
1 & -\frac{1}{118} & 0 & \frac{21}{59} & 0 & \frac{6}{59} & -\frac{5}{118} & \frac{37}{118} \\
0 & \frac{25}{59} & 1 & \frac{12}{59} & 0 & -\frac{5}{59} & \frac{7}{59} & \frac{19}{59} \\
0 & -\frac{537}{118} & 0 & -\frac{287}{59} & 0 & \frac{95}{59} & \frac{147}{188} & -\frac{191}{118}
\end{array}$$

Because we still have negative cost values, we must repeat the pivot process, this time as the most negative value is $-\frac{287}{59}$, we calculate the ratios and find that the lowest is $\frac{191}{574}$.

We then calculate the next pivot

Calculating row operations:

$$\frac{R_1}{\frac{287}{59}} \to R_1$$
$$R_2 - \frac{21}{59}R_1 \to R_2$$
$$R_3 - \frac{12}{59}R_1 \to R_3$$
$$R_4 + \frac{287}{59}R_1 \to R_4$$

After these calculations, the final tableau is as follows:

$$
\begin{array}{ccccccc|c}
x_1 & x_2 & x_3 & x_4 & s_1 & s_2 & s_3 & b \\
0 & \frac{537}{574} & 0 & 1 & \frac{59}{287} & -\frac{36}{287} & -\frac{29}{574} & \frac{191}{574} \\
1 & -\frac{14}{41} & 0 & 0 & -\frac{3}{41} & \frac{6}{41} & -\frac{1}{41} & \frac{8}{41} \\
0 & \frac{67}{287} & 1 & 0 & -\frac{12}{287} & -\frac{17}{287} & \frac{37}{287} & \frac{73}{287} \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 0
\end{array}
$$

Now as we have a basic feasible solution where: must remove the auxiliary variables s_1, s_2 and s_3 from the basis and have a basic feasible solution where:

$$x_1 = \frac{8}{41}, x_2 = 0, x_3 = \frac{73}{287}, x_4 = \frac{191}{574}$$

We can use this as a starting initial solution for our original problem. For this we set up the new tableau:

$$
\begin{array}{cccc|c}
x_1 & x_2 & x_3 & x_4 & b \\
0 & \frac{537}{574} & 0 & 1 & \frac{191}{574} \\
1 & -\frac{14}{41} & 0 & 0 & \frac{8}{41} \\
0 & \frac{67}{287} & 1 & 0 & \frac{73}{287} \\
14 & 8 & 10 & 14 & 0
\end{array}
$$

Once we have cleared the tableau and applied one more pivot, the resulting tableau is as follows:

$$
\begin{array}{cccc|c}
x_1 & x_2 & x_3 & x_4 & b \\
0 & 1 & 0 & 0 & \frac{574}{537} \\
1 & 0 & 0 & 0 & \frac{8}{41} \\
0 & 0 & 1 & 0 & \frac{73}{287} \\
0 & 0 & 0 & 0 & -\frac{1276}{719}
\end{array}
$$

As this has strictly non-negative relative costs anymore, we have found the optimal solution where:

$$x_1 = \frac{170}{537}, x_2 = \frac{191}{537}, x_3 = \frac{92}{537}, x_4 = 0$$

Question Two

$$\begin{aligned} \text{min}_{[x]} \quad & 0x_1 + 12x_2 + 10x_3 + 6x_4 \\ \text{s.t} \quad & 9x_1 + 14x_2 + 1x_3 + 11x_4 = 14 \\ & 9x_1 + 0x_2 + 3x_3 + 6x_4 = 10 \\ & 8x_1 + 5x_2 + 9x_3 + 8x_4 = 12 \end{aligned}$$

We need to form an auxiliary problem in order to find a basic feasible solution to the problem above This is formed below.

Auxiliary function

$$\begin{aligned} \text{min}_{[x]} \quad & s_1 + s_2 + s_3 \\ \text{s.t} \quad & 9x_1 + 14x_2 + 1x_3 + 11x_4 + s_1 = 14 \\ & 9x_1 + 0x_2 + 3x_3 + 6x_4 + s_2 = 10 \\ & 8x_1 + 5x_2 + 9x_3 + 8x_4 + s_3 = 12 \\ & x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, s_1 \geq 0, s_2 \geq 0, s_3 \geq 0 \end{aligned}$$

Initialise tableau for auxiliary problem

x_1	x_2	x_3	x_4	s_1	s_2	s_3	b
9	14	1	11	1	0	0	14
9	0	3	6	0	1	0	10
8	6	9	8	0	0	1	12
0	0	0	0	1	1	1	0

We plug in $x_i = 0$ for all $1 \leq i \leq m$ which gives us the obvious solution, that a feasible solution to the auxiliary linear programing problem is when $b_i = x_i$, this gives us our initial feasible solution where the following conditions are met:

$$x_1 = 0, x_2 = 0, x_3 = 0, x_4 = 0, s_1 = 7, s_2 = 6, s_3 = 7$$

This gives us the initial tableau:

x_1	x_2	x_3	x_4	s_1	s_2	s_3	b
9	14	1	11	1	0	0	14
9	0	3	6	0	1	0	10
8	6	9	8	0	0	1	12
0	0	0	0	1	1	1	0

Simplex method on Auxiliary Problem

First we need to clean out the initial tableau, this will result in $s_1 = s_2 = s_3 = 0$ in last row of the tableau, we can achieve this by preforming arithmetic on the rows of the tableau.

$$R_4 - R_1 \rightarrow R_4$$
$$R_4 - R_2 \rightarrow R_4$$
$$R_4 - R_3 \rightarrow R_4$$

The resulting tableau of the above operations is as follows:

x_1	x_2	x_3	x_4	s_1	s_2	s_3	b
9	14	1	11	1	0	0	14
9	0	3	6	0	1	0	10
8	6	9	8	0	0	1	12
-26	-20	-13	-25	1	1	1	-36

We can find our pivot point by finding the largest negative and then checking the ratio $\frac{b_i}{y_i}$ for each value i within the column *q*, the element in *q* with the lowest ratio will become the new pivot point for the proceeding operations:

Applying the pivot

Calculating the rows operations

$$\frac{1}{9}R_2 \rightarrow R_2$$
$$R_1 - 9R_2 \rightarrow R_1$$
$$R_3 - 8R_2 \rightarrow R_3$$
$$R_4 + 26R_2 \rightarrow R_4$$

After these calculations, the new tableau is as follows:

$$
\begin{array}{ccccccc|c}
x_1 & x_2 & x_3 & x_4 & s_1 & s_2 & s_3 & b \\
0 & 14 & -2 & 5 & 1 & -1 & 0 & 4 \\
1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & \frac{1}{9} & 0 & \frac{10}{9} \\
0 & 6 & \frac{19}{3} & \frac{8}{3} & 0 & -\frac{8}{9} & 1 & \frac{28}{9} \\
0 & -20 & -\frac{13}{3} & -\frac{23}{3} & 1 & \frac{35}{9} & 1 & -\frac{88}{63}
\end{array}
$$

Because we still have negative relative cost values, we must repeat the pivot process, this time as the most negative value is -20, we calculate the ratios and find that the lowest is $\frac{2}{7}$.

We then calculate the next pivot

Calculating row operations:

$$
\begin{aligned}
\frac{1}{14}R_1 &\to R_2 \\
R_3 - 6R_1 &\to R_3 \\
R_4 + 20R_2 &\to R_4
\end{aligned}
$$

The resulting tableau of the above operations is as follows:

$$
\begin{array}{ccccccc|c}
x_1 & x_2 & x_3 & x_4 & s_1 & s_2 & s_3 & b \\
0 & 1 & -\frac{1}{7} & \frac{5}{14} & \frac{1}{14} & -\frac{1}{14} & 0 & \frac{2}{7} \\
1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & \frac{1}{9} & 0 & \frac{10}{9} \\
0 & 0 & \frac{151}{21} & \frac{11}{21} & -\frac{3}{7} & -\frac{29}{63} & 1 & \frac{88}{63} \\
0 & 0 & -\frac{151}{21} & -\frac{11}{21} & \frac{17}{7} & \frac{155}{63} & 1 & -\frac{88}{63}
\end{array}
$$

Because we still have negative relative cost values, we must repeat the pivot process, this time as the most negative value is $-\frac{151}{21}$, we calculate the ratios and find that the lowest is $\frac{88}{453}$.

We then calculate the next pivot

Calculating row operations:

$$\frac{R_3}{\frac{151}{21}} \rightarrow R_3$$
$$R_1 + \frac{1}{7}R_3 \rightarrow R_1$$
$$R_2 - \frac{1}{3}R_3 \rightarrow R_2$$
$$R_4 + \frac{151}{21}R_3 \rightarrow R_4$$

The final tableau is calculated from the above operations, and is as follows:

$$\begin{array}{ccccccc|c}
x_1 & x_2 & x_3 & x_4 & s_1 & s_2 & s_3 & b \\
0 & 1 & 0 & \frac{111}{302} & \frac{19}{302} & -\frac{73}{906} & \frac{3}{151} & \frac{142}{453} \\
1 & 0 & 0 & \frac{97}{151} & \frac{3}{161} & \frac{20}{151} & -\frac{7}{151} & \frac{158}{151} \\
0 & 0 & 1 & \frac{11}{151} & -\frac{9}{151} & -\frac{29}{453} & \frac{21}{151} & \frac{88}{63} \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 0
\end{array}$$

Now as we have a basic feasible solution where: must remove the auxiliary variables s_1, s_2 and s_3 from the basis and have a basic feasible solution where:

$$x_1 = \frac{158}{151}, x_2 = \frac{142}{453}, x_3 = \frac{88}{453}, x_4 = 0$$

We can use this as a starting initial solution for our original problem. For this we set up the new tableau:

$$\begin{array}{cccc|c}
x_1 & x_2 & x_3 & x_4 & b \\
0 & 1 & 0 & \frac{111}{302} & \frac{142}{453} \\
1 & 0 & 0 & \frac{97}{151} & \frac{158}{151} \\
0 & 0 & 1 & \frac{11}{151} & \frac{88}{63} \\
0 & 12 & 10 & 6 & 0
\end{array}$$

We then clean out the tableau by applying row operations as seen previously:

$$R_4 - 12R_1 \to R_4$$
$$R_4 - 10R_2 \to R_4$$

The final resulting tableau is as follows:

$$\begin{array}{cccc|c}
x_1 & x_2 & x_3 & x_4 & b \\
0 & 1 & 0 & \frac{111}{302} & \frac{142}{453} \\
1 & 0 & 0 & \frac{97}{151} & \frac{158}{151} \\
0 & 0 & 1 & \frac{11}{151} & \frac{88}{63} \\
0 & 0 & 0 & \frac{130}{151} & -\frac{2584}{453}
\end{array}$$

We now can use this resulting table to find the minimal solutions to the linear program.

$$x_1 = \frac{142}{453}, x_2 = \frac{158}{151}, x_3 = \frac{88}{453}, x_4 = 0$$

Question Three

$$\begin{aligned}
\text{min}_{[x]} \quad & 3x_1 + 0x_2 + 0x_3 + 3x_4 \\
\text{s.t} \quad & 14x_1 + 10x_2 + 0x_3 + 2x_4 = 7 \\
& 0x_1 + 0x_2 + 2x_3 + 14x_4 = 2 \\
& 11x_1 + 3x_2 + 13x_3 + 13x_4 = 10
\end{aligned}$$

We need to form an auxiliary problem in order to find a basic feasible solution to the problem above This is formed below.

Auxiliary function

$$\begin{aligned}
\text{min}_{[x]} \quad & s_1 + s_2 + s_3 \\
\text{s.t} \quad & 14x_1 + 10x_2 + 0x_3 + 2x_4 s_1 = 7 \\
& 0x_1 + 0x_2 + 2x_3 + 14x_4 + s_2 = 2 \\
& 11x_1 + 3x_2 + 13x_3 + 13x_4 + s_3 = 10 \\
& x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, s_1 \geq 0, s_2 \geq 0, s_3 \geq 0
\end{aligned}$$

Initialise tableau for auxiliary problem

We plug in $x_i = 0$ for all $1 \leq i \leq m$ which gives us the obvious solution, that a feasible solution to the auxiliary linear programing problem is when $b_i = x_i$, this gives us our initial feasible solution where the following conditions are met:

$$x_1 = 0, x_2 = 0, x_3 = 0, x_4 = 0, s_1 = 7, s_2 = 6, s_3 = 7$$

$$\begin{array}{ccccccc|c}
x_1 & x_2 & x_3 & x_4 & s_1 & s_2 & s_3 & b \\
14 & 10 & 0 & 2 & 1 & 0 & 0 & 7 \\
0 & 0 & 2 & 14 & 0 & 1 & 0 & 2 \\
11 & 3 & 13 & 13 & 0 & 0 & 1 & 10 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 0
\end{array}$$

Simplex method on Auxiliary Problem

First we need to clean out the initial tableau, this will result in $s_1 = s_2 = s_3 = 0$ in last row of the tableau, we can achieve this by preforming arithmetic on the rows of the tableau.

$$R_4 - R_1 \rightarrow R_4$$
$$R_4 - R_2 \rightarrow R_4$$
$$R_4 - R_3 \rightarrow R_4$$

The resulting tableau of the above operations is as follows:

$$\begin{array}{ccccccc|c}
x_1 & x_2 & x_3 & x_4 & s_1 & s_2 & s_3 & b \\
14 & 10 & 0 & 2 & 1 & 0 & 0 & 7 \\
0 & 0 & 2 & 14 & 0 & 1 & 0 & 2 \\
11 & 3 & 13 & 13 & 0 & 0 & 1 & 10 \\
-25 & -13 & -15 & -29 & 0 & 0 & 0 & -19
\end{array}$$

We can find our pivot point by finding the largest negative and then checking the ratio $\frac{b_i}{y_i}$ for each value i within the column *q*, the element in *q* with the lowest ratio will become the new pivot point for the proceeding operations:

In this case we consider x_4 due to R_4 containing the largest negative relative cost value, we then calculate the ratios, and take the smallest one of $\frac{1}{7}$.

Applying the pivot

Calculating the rows operations

$$\begin{aligned} \frac{1}{14}R_2 &\to R_2 \\ R_1 - 2R_2 &\to R_1 \\ R_3 - 13R_2 &\to R_3 \\ R_4 + 29R_2 &\to R_4 \end{aligned}$$

After these calculations, the new tableau is as follows:

$$\begin{array}{ccccccc|c} x_1 & x_2 & x_3 & x_4 & s_1 & s_2 & s_3 & b \\ 14 & 10 & -\frac{2}{7} & 0 & 1 & -\frac{1}{7} & 0 & \frac{47}{7} \\ 0 & 0 & \frac{1}{7} & 1 & 0 & \frac{1}{14} & 0 & \frac{1}{7} \\ 11 & 3 & \frac{78}{7} & 0 & 0 & -\frac{13}{14} & 1 & \frac{57}{7} \\ -25 & -13 & -\frac{76}{7} & 0 & 0 & \frac{29}{14} & 0 & -\frac{104}{7} \end{array}$$

Because we still have negative relative cost values, we must repeat the pivot process, this time as the most negative value is -25, we calculate the ratios and find that the lowest is $\frac{47}{98}$.

We then calculate the next pivot

Calculating row operations:

$$\begin{aligned} \frac{1}{14}R_1 &\to R_1 \\ R_3 - 11R_1 &\to R_3 \\ R_4 + 25R_1 &\to R_4 \end{aligned}$$

The resulting tableau of the above operations is as follows:

$$\begin{array}{ccccccc|c}
x_1 & x_2 & x_3 & x_4 & s_1 & s_2 & s_3 & b \\
1 & \frac{5}{7} & -\frac{1}{49} & 0 & \frac{1}{14} & -\frac{1}{98} & 0 & \frac{47}{98} \\
0 & 0 & \frac{1}{7} & 1 & 0 & \frac{1}{14} & 0 & \frac{1}{7} \\
0 & -\frac{34}{7} & \frac{557}{49} & 0 & -\frac{11}{14} & -\frac{40}{49} & 1 & \frac{281}{98} \\
0 & \frac{34}{7} & -\frac{49}{557} & 0 & \frac{25}{14} & \frac{89}{49} & 0 & -\frac{281}{98}
\end{array}$$

Because we still have negative relative cost values, we must repeat the pivot process, this time as the most negative value is $-\frac{557}{49}$, we calculate the ratios and find that the lowest non-negative ratio is $\frac{28}{111}$.

We then calculate the next pivot

Calculating row operations:

$$\begin{aligned}
\frac{49}{557}R_3 &\rightarrow R_3 \\
R_1 + \frac{1}{49}R_3 &\rightarrow R_1 \\
R_2 - \frac{1}{7}R_3 &\rightarrow R_2 \\
R_4 + \frac{557}{49}R_3 &\rightarrow R_4
\end{aligned}$$

The resulting tableau of the above operations is as follows:

$$\begin{array}{ccccccc|c}
x_1 & x_2 & x_3 & x_4 & s_1 & s_2 & s_3 & b \\
1 & \frac{393}{557} & 0 & 0 & \frac{39}{557} & -\frac{10}{857} & \frac{1}{557} & \frac{270}{557} \\
0 & \frac{34}{557} & 0 & 1 & \frac{7}{709} & \frac{62}{759} & -\frac{7}{557} & \frac{36}{337} \\
0 & -\frac{238}{557} & 1 & 0 & -\frac{62}{897} & -\frac{40}{557} & \frac{49}{557} & \frac{28}{111} \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 0
\end{array}$$

Now that we have a Basic feasible solution to the auxiliary problem, we can use this as a starting initial solution for our original problem. For this we set up the new tableau:

$$\begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & b \\ 1 & \frac{393}{557} & 0 & 0 & \frac{270}{557} \\ 0 & \frac{34}{557} & 0 & 1 & \frac{36}{337} \\ 0 & -\frac{238}{557} & 1 & 0 & \frac{28}{111} \\ 3 & 0 & 0 & 3 & 0 \end{array}$$

We then clean out the tableau by applying row operations as seen previously:

$$R_4 - 3R_1 \to R_4$$
$$R_4 - 3R_2 \to R_4$$

The resulting tableau is as follows:

$$\begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & b \\ 1 & \frac{393}{557} & 0 & 0 & \frac{270}{557} \\ 0 & \frac{34}{557} & 0 & 1 & \frac{36}{337} \\ 0 & -\frac{238}{557} & 1 & 0 & \frac{28}{111} \\ 0 & -\frac{1281}{557} & 0 & 0 & -\frac{1276}{719} \end{array}$$

Notice that there are still some negative relative costs within the tableau, this means that we are not done. We must make another pivot to remove negative relative costs.

We apply the appropriate row operations:

$$R_1 - \frac{557}{393}R_1 \to R_1$$
$$R_2 - \frac{34}{557}R_1 \to R_2$$
$$R_3 + \frac{238}{557}R_1 \to R_3$$
$$R_4 + \frac{1281}{557}R_1 \to R_4$$

The final resulting table is as follows:

$$\begin{array}{cccc|c}
x_1 & x_2 & x_3 & x_4 & b \\
\frac{557}{393} & 1 & 0 & 0 & \frac{90}{131} \\
-\frac{34}{393} & 0 & 0 & 1 & \frac{17}{262} \\
\frac{238}{393} & 0 & 1 & 0 & \frac{143}{262} \\
\frac{427}{131} & 0 & 0 & 0 & -\frac{51}{262}
\end{array}$$

We now can use this resulting table to find the minimal solutions to the linear program.

$$x_1 = 0, x_2 = \frac{90}{131}, x_3 = \frac{143}{262}, x_4 = \frac{17}{262}$$

Question Four

First we need to remove negative constants from the right hand side of each equation the constraints:

$$\begin{aligned}
\text{min}_{[x]} \quad & 12x_1 + 4x_2 + 1x_3 + 1x_4 \\
\text{s.t} \quad & 11x_1 + 12x_2 + 7x_3 + 9x_4 = 12 \\
& 5x_1 + 5x_2 + 10x_3 + 11x_4 = 14 \\
& 2x_1 + 12x_2 + 13x_3 + 6x_4 = 9
\end{aligned}$$

We need to form an auxiliary problem in order to find a basic feasible solution to the problem above This is formed below.

Auxiliary function

$$\begin{aligned}
\text{min}_{[x]} \quad & s_1 + s_2 + s_3 \\
\text{s.t} \quad & 11x_1 + 12x_2 + 7x_3 + 9x_4 + s_1 = 12 \\
& 5x_1 + 5x_2 + 10x_3 + 11x_4 + s_2 = 14 \\
& 2x_1 + 12x_2 + 13x_3 + 6x_4 + s_3 = 9 \\
& x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, s_1 \geq 0, s_2 \geq 0, s_3 \geq 0
\end{aligned}$$

We plug in $x_i = 0$ for all $1 \leq i \leq m$ which gives us an obvious solution, that a feasible solution to the auxiliary linear programing problem is when $b_i = x_i$.

$$\begin{array}{ccccccc|c}
x_1 & x_2 & x_3 & x_4 & s_1 & s_2 & s_3 & b \\
11 & 12 & 7 & 9 & 1 & 0 & 0 & 12 \\
5 & 5 & 10 & 11 & 0 & 1 & 0 & 14 \\
2 & 12 & 13 & 6 & 0 & 0 & 1 & 9 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 0
\end{array}$$

Simplex method on Auxiliary Problem

First we need to clean out the initial tableau, this will result in $s_1 = s_2 = s_3 = 0$ in last row of the tableau, we can achieve this by preforming arithmetic on the rows of the tableau.

$$R_4 - R_1 \to R_4$$
$$R_4 - R_2 \to R_4$$
$$R_4 - R_3 \to R_4$$

Giving us our initial tableau to start the simplex algorithm

$$\begin{array}{ccccccc|c}
x_1 & x_2 & x_3 & x_4 & s_1 & s_2 & s_3 & b \\
11 & 12 & 7 & 9 & 1 & 0 & 0 & 12 \\
5 & 5 & 10 & 11 & 0 & 1 & 0 & 14 \\
2 & 12 & 13 & 6 & 0 & 0 & 1 & 9 \\
-23 & -29 & -30 & -26 & 0 & 0 & 0 & -35
\end{array}$$

We can find our pivot point by finding the largest negative and then checking the ratio $\frac{b_i}{y_i}$ for each value i within the column *q*, the element in *q* with the lowest ratio will become the new pivot point for the proceeding operations:

In this case we consider x_3 due to R_4 containing the largest negative relative cost value, we then calculate the ratios, of each and every x_3 and take the smallest ratio of $\frac{9}{13}$, therefore $R_{3,3}$ becomes our new pivot point.

Applying the pivot

Calculating row operations:

$$
\begin{aligned}
\frac{1}{13}R_3 &\rightarrow R_3 \\
R_1 - 7R_3 &\rightarrow R_1 \\
R_2 - 10R_3 &\rightarrow R_2 \\
R_4 + 30R_3 &\rightarrow R_4
\end{aligned}
$$

The resulting tableau of the above operations is as follows:

$$
\begin{array}{ccccccc|c}
x_1 & x_2 & x_3 & x_4 & s_1 & s_2 & s_3 & b \\
\frac{129}{13} & \frac{72}{13} & 0 & \frac{68}{13} & 1 & 0 & -\frac{7}{13} & \frac{93}{13} \\
\frac{45}{13} & -\frac{49}{13} & 0 & \frac{83}{13} & 0 & 1 & -\frac{10}{13} & \frac{92}{13} \\
\frac{2}{13} & \frac{12}{13} & 1 & \frac{6}{13} & 0 & 0 & \frac{1}{13} & \frac{9}{13} \\
-\frac{174}{13} & -\frac{17}{13} & 0 & -\frac{158}{13} & 0 & 0 & -\frac{43}{13} & -\frac{185}{13}
\end{array}
$$

As we still have negative relative cost values, we must continue the process and apply another pivot, we do this the same way as before, by choosing the most negative relative cost value, and finding the smallest ratio of $\frac{b_i}{y_i}$.

Applying the next pivot

Calculating row operations:

$$
\begin{aligned}
\frac{13}{45}R_2 &\rightarrow R_2 \\
R_1 - \frac{129}{13}R_2 &\rightarrow R_1 \\
R_3 - \frac{2}{13}R_2 &\rightarrow R_3 \\
R_4 + \frac{174}{13}R_2 &\rightarrow R_4
\end{aligned}
$$

The resulting tableau of the above operations is as follows:

$$\begin{array}{ccccccc|c}
x_1 & x_2 & x_3 & x_4 & s_1 & s_2 & s_3 & b \\
0 & \frac{53}{3} & 0 & -\frac{188}{13} & 1 & -\frac{43}{15} & \frac{5}{3} & -\frac{197}{15} \\
1 & -\frac{11}{9} & 0 & \frac{83}{13} & 0 & \frac{13}{45} & -\frac{2}{9} & \frac{92}{13} \\
0 & \frac{12}{13} & 1 & \frac{6}{13} & 0 & 0 & \frac{1}{13} & \frac{9}{13} \\
0 & -\frac{53}{3} & 0 & \frac{188}{15} & 1 & \frac{73}{15} & \frac{1}{3} & \frac{197}{15}
\end{array}$$

Because we still have a negative cost value of $-\frac{53}{3}$, we must make another pivot on the lowest ratio element of $\frac{b_i}{y_i}$. In this case with a ratio of $\frac{197}{265}$.

Applying the next pivot

Calculating row operations:

$$\begin{aligned}
\frac{3}{53}R_1 &\rightarrow R_1 \\
R_2 + \frac{11}{9}R_1 &\rightarrow R_2 \\
R_3 - \frac{12}{13}R_1 &\rightarrow R_3 \\
R_4 + \frac{53}{3}R_1 &\rightarrow R_4
\end{aligned}$$

The final resulting tableau of the above operations is as follows:

$$\begin{array}{ccccccc|c}
x_1 & x_2 & x_3 & x_4 & s_1 & s_2 & s_3 & b \\
0 & 1 & 0 & -\frac{188}{265} & \frac{3}{53} & -\frac{43}{265} & \frac{5}{53} & -\frac{197}{265} \\
1 & 0 & 0 & \frac{259}{265} & \frac{11}{159} & \frac{24}{265} & -\frac{17}{159} & \frac{301}{265} \\
0 & 0 & 1 & \frac{211}{189} & -\frac{36}{689} & \frac{34}{227} & -\frac{7}{689} & \frac{1322}{959} \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 0
\end{array}$$

Now that we have a Basic feasible solution to the auxiliary problem, we can use this as a starting initial solution for our original problem. For this we set up the new tableau:

Solving the original posed problem

We setup our initial tableau for the original problem

$$
\begin{array}{cccc|c}
x_1 & x_2 & x_3 & x_4 & b \\
0 & 1 & 0 & -\frac{188}{265} & -\frac{197}{265} \\
1 & 0 & 0 & \frac{259}{265} & \frac{301}{265} \\
0 & 0 & 1 & \frac{211}{189} & \frac{1322}{959} \\
12 & 4 & 1 & 1 & 0
\end{array}
$$

We must clean out the tableau by doing the following row operations:

$$
R_4 - 4R_1 \to R_4
$$
$$
R_4 - 12R_2 \to R_4
$$
$$
R_4 - 1R_3 \to R_4
$$

This gives us the following tableau:

$$
\begin{array}{cccc|c}
x_1 & x_2 & x_3 & x_4 & b \\
0 & 1 & 0 & -\frac{188}{265} & -\frac{197}{265} \\
1 & 0 & 0 & \frac{259}{265} & \frac{301}{265} \\
0 & 0 & 1 & \frac{211}{189} & \frac{1322}{959} \\
0 & 0 & 0 & \frac{2585}{287} & \frac{5825}{484}
\end{array}
$$

As we still have a negative relative cost value, we must make another pivot on $x_{4,1}$. After the appropriate row operations are done, the resulting tableau is as follows:

$$
\begin{array}{cccc|c}
x_1 & x_2 & x_3 & x_4 & b \\
\frac{188}{259} & 1 & 0 & 0 & \frac{3}{37} \\
\frac{305}{259} & 0 & 0 & 1 & \frac{43}{37} \\
-\frac{827}{724} & 0 & 1 & 0 & \frac{3}{37} \\
\frac{2949}{320} & 0 & 0 & 0 & -\frac{58}{37}
\end{array}
$$

As we have no negative relative cost values, this is our final tableau.

We now can use this resulting table to find the minimal solutions to the linear program.

$$x_1 = 0, x_2 = \frac{3}{37}, x_3 = \frac{3}{37}, x_4 = -\frac{58}{37}$$