COSC367: Artificial Intelligence

This course introduces major concepts and algorithms in Artificial Intelligence. Topics include problem solving, reasoning, games, and machine learning.

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2021-05-06 16:57

Contents

Artificial Intelligence

Course Information

The course covers core topics in AI including:

- uninformed and informed graph search algorithms,
- propositional logic and forward and backward chaining algorithms,
- declarative programming with Prolog,
- the min-max and alpha-beta pruning algorithms,
- Bayesian networks and probabilistic inference algorithms,
- classification learning algorithms,
- consistency algorithms,
- local search and heuristic algorithms such as simulated annealing, and population-based algorithms such as genetic search and swarm optimisation.

Grades

Standard Computer science policy applies

- Average 50% over all assessment items
- Average at least 45% on all invigilated assessment items

Grading structure for course

- Assignments (5%)
  - Two Super Quiz's
- Quizzes (16.5%)
  - Weekly Quiz Assessments (1.5% ea)
- Lab Test (20%)
- Final Exam (58.5%)

Textbooks / Resources

- Poole, David L.1958, Mackworth, Alan K; Artificial intelligence : foundations of computational agents; Cambridge University Press, 2010.
- Russell, Stuart J, Norvig, Peter; Artificial intelligence : a modern approach; 3rd ed; Prentice Hall, 2010.

Readings

Lectures

Lecture One: Searching the State Space

What is state?

- A state is a data structure that represents a possible configuration of the world *agent and environment*
- The **state space** is the set of all possible states for that problem
- actions change the state of the world

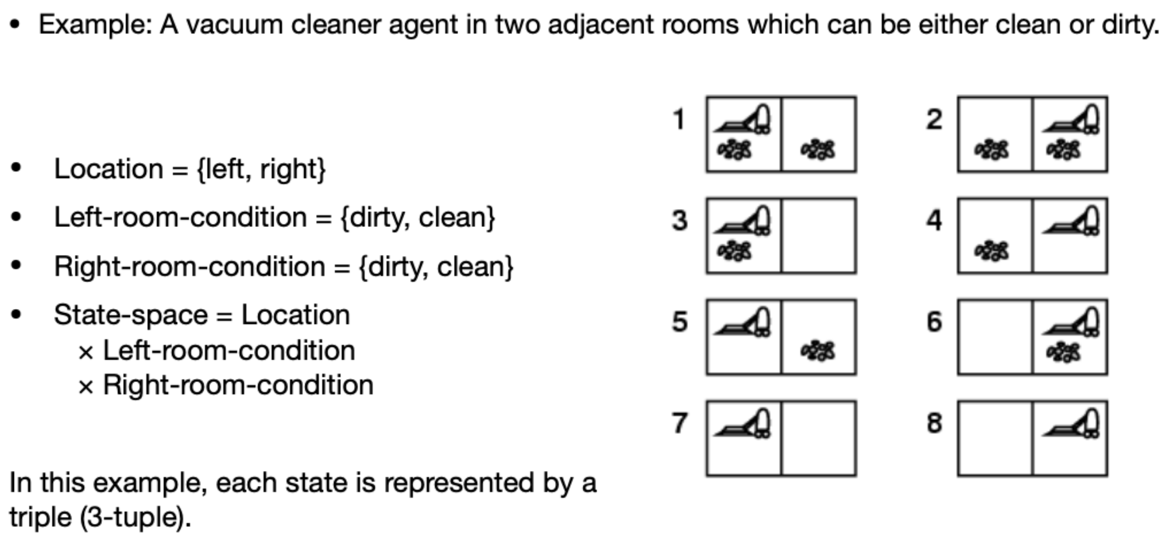


Figure 1: State space example one

State can also be represented as a graph *both directed and undirected*

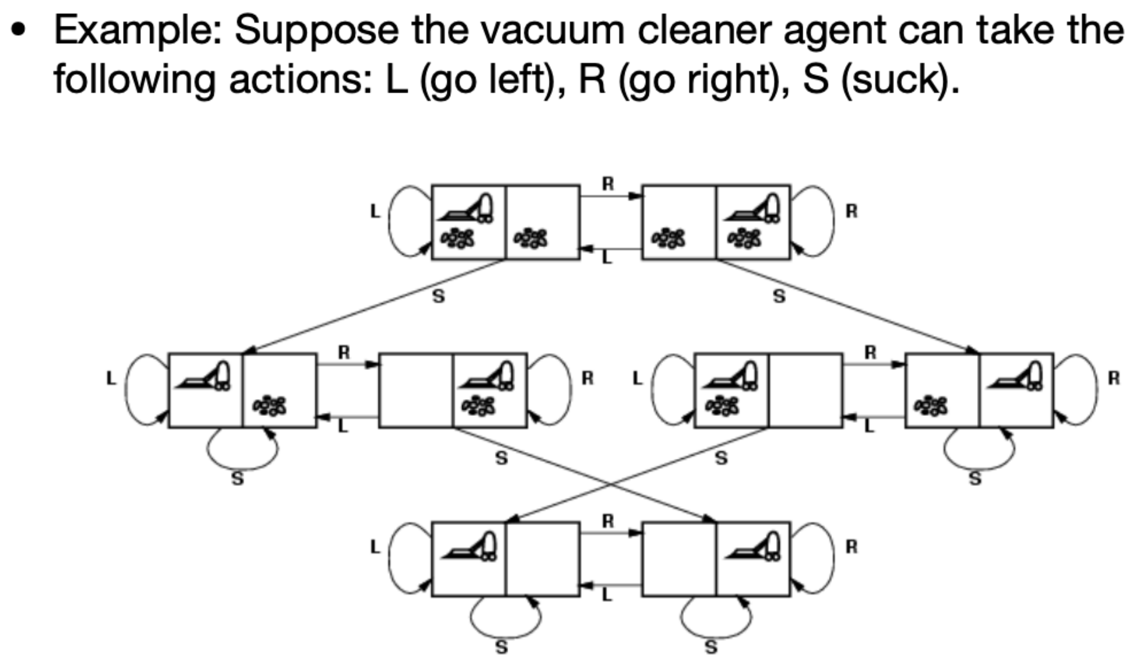


Figure 2: State space graph simplified

- Many problems in AI can be abstracted to the problem of finding a path in a directed graph
- Notation we use is **Nodes** and **arcs** for **vertices** and **edges** in a graph

Explicit vs Implicit graphs

- In **explicit graphs** nodes and arcs are readily available, they are read from the input and stored in a data structure such as an adjacency list/matrix.
  - the entire graph is in memory.
  - the complexity of algorithms are measured in the number of nodes and/or arcs.
- In **implicit graphs** a procedure `outgoing_arcs` is defined that given a node, returns a set of directed arcs that connect node to other nodes.
  - The graph is generated as needed *due to the complexity of the graphs.*
  - The complexity is measured in terms of the depth of the goal state node or *how far do we have to get into the graph to find a solution*.

Explicit graphs in quizzes

- In some exercises we use small explicit graphs to stydy the behaviour of various frontiers
- Nodes are specified in a set

- Edges are specified in a list
  - pairs of nodes, or triples of nodes (in a tuple)

Searching graphs

- We will use generic search algorithms: given a graph, start nodes, and goal nodes, incrementally explore paths from the start nodes.
- Maintain a **frontier** of paths that have been explored
  - frontier: paths that we have already explored
- As search proceeds, the frontier is updated and the graph is explored until a goal node is found.
- The order in which paths are removed and added to the frontier defines the search strategy
- A **search tree** is a tree drawn out of all the possible actions in terms of a tree.
  - How do we handle loops? *Covered in next lecture*
  - In the search tree outlined below, you can see that the *end of paths on frontier* represents a BFS relationship note this is not always the case.

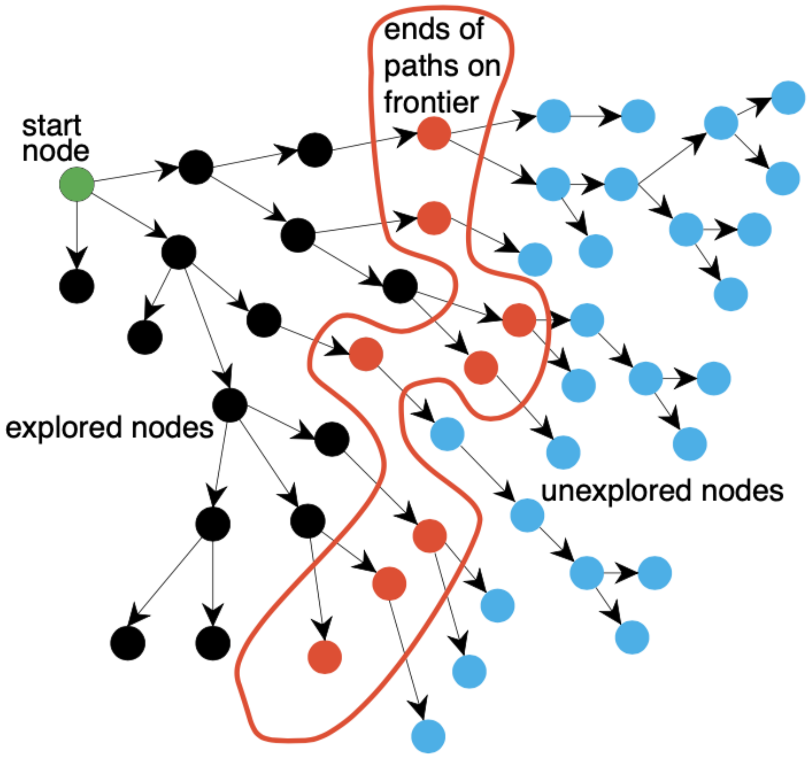


Figure 3: search tree

Generic graph search algorithm

Input: a graph,
a set of start nodes,
Boolean procedure $goal(n)$ that tests if n is a goal node
$frontier := \{\langle s \rangle : s \text{ is a start node}\}$;
while $frontier$ is not empty:
select and **remove** path $\langle n_0, \ldots, n_k \rangle$ from $frontier$;
if $goal(n_k)$
return $\langle n_0, \ldots, n_k \rangle$;
for every neighbor n of n_k
add $\langle n_0, \ldots, n_k, n \rangle$ to $frontier$;
end while

Figure 4: Generic Search

> NOTE: you will have to use what ever data structure for the seach you are using (BFS use a queue), (DFS use a stack).

In the generic algorithm, neighbours are going to use the method `outgoing_arcs`, we are given this algorithm in the form of a python module.

Depth-first search

- In order to perform DFS, the generic graph search must be used with a stack frontier *LIFO*
- If the stack is a python list, where each element is a path, and has the form [..., p, q]
  - *q* is selected and popped
  - of the algorithm continues then paths that extend *q* are pushed (appended) to the stack
  - *p* is only selected when all paths from *q* have been explored.
- As a result, at each stage the algorithm expands the deepest path
- The orange nodes in the graph below are considered the frontier nodes

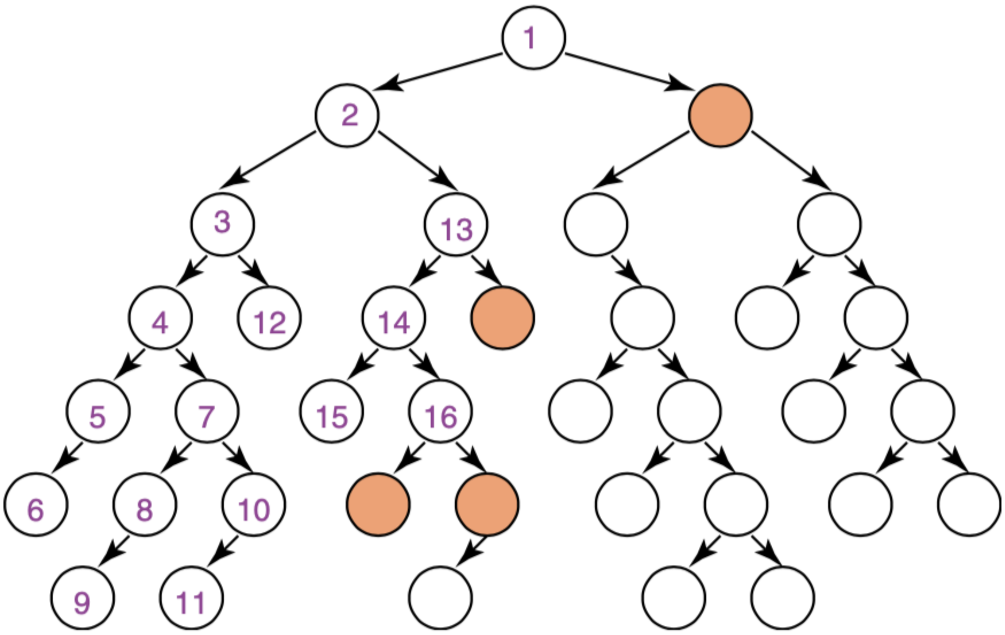


Figure 5: DFS

- DFS does not guarantee a solution without pruning, due to the fact that we can have infinite loops
- It is not guaranteed to complete if it does not use pruning

A note on complexity

Assume a finite search tree of depth *d* and branching factor of *b*:

- What is the time complexity?
  - It will be exponential: $O(b^d)$
- What is the space complexity?
  - It will be linear: $O(bd)$

How do we trace the frontier

- starting with an empty frontier we record all the calls to the frontier: to add or get a path we dedicate one line per call
- When we ask the frontier to add a path, we start the line with a + followed by the path that has been added
- When we ask for a path from the frontier we start the line with a − followed by the path being removed

- When using a priority queue, the path is followed by a comma and then the key *e.g, cost, heuristic, f-value, ...*
- The lines of the trace should match the following regular expression `^[+-][a-z]+(,\d+)?!?$`
- We stop when we **remove** a path from the trace

Given the following graph

nodes={a, b, c, d},
edge_list=[(a,b), (a,d), (a, c), (c, d)],
starting_nodes = [a],
goal_nodes = {d}

trace the frontier in depth-first search (DFS).

Answer:

```
+ a
- a
+ ab
+ ad
+ ac
- ac
+ acd
- acd
```

Figure 6: DFS trace using generic algorithm

Breath-first search

- In order to perform BFS, the generic graph search must be used with a queue frontier *FIFO*.
- If the queue is a python deque of the form [p,q,...,r], then
  - p is selected (dequeued)
  - if the algorithm continues then paths that extend *p* are enqueued *appended* to the queue after *r*
- As a result, at each state the algorithm expands the shallowest path.

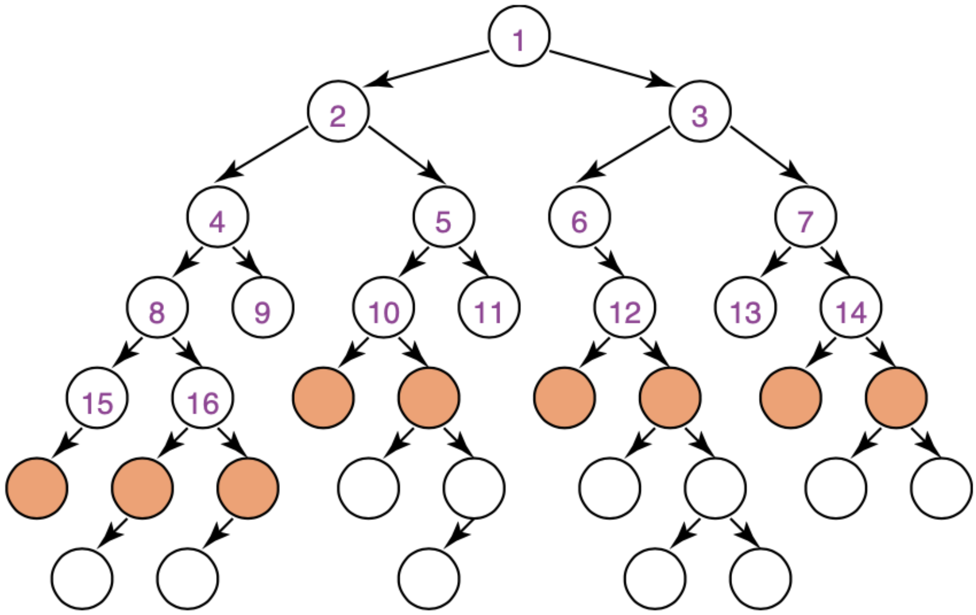


Figure 7: BFS Illustration of search tree

- BFS **does** guarantee to find a solution with the fewest arcs if there is a solution
- It will complete
- It will not halt due to some graphs having *cycles, with no pruning*

A note on complexity

> BFS has higher complexity than DFS

- What is the time complexity?
  - It will be exponential: $O(b^d)$
- What is the space complexity?
  - It will be linear: $O(b^d)$

Given the following graph

nodes={a, b, c, d},
edge_list=[(a,b), (a,d), (a, c), (c, d)],
starting_nodes = [a],
goal_nodes = {d}

trace the frontier in breadth-first search (BFS).

Answer:

```
+ a
- a
+ ab
+ ad
+ ac
- ab
- ad
```

Figure 8: BFS trace using generic algorithm

Lowest-cost-first search

- The cost of a path is the sum of the costs of its arcs
- This algorithm is very similar to Dijkstra's except modified for larger graphs
- LCFS selects a path on the frontier with the lowest cost
- The frontier is a priority queue ordered by path cost
  - A priority queue is a container in which each element has a priority *cost*
  - An element with a higher priority is always selected/removed before an element with a lower priority
  - In python we can use the `heapq` you will need to store objects in a way that these properties hold
- LCFS finds an optimal solution: a least-cost path to a goal node.
- Another name for this algorithm is *uniform-cost search*.

> NOTE: For an example of this queue, see Lecture One: 1:45 time stamp

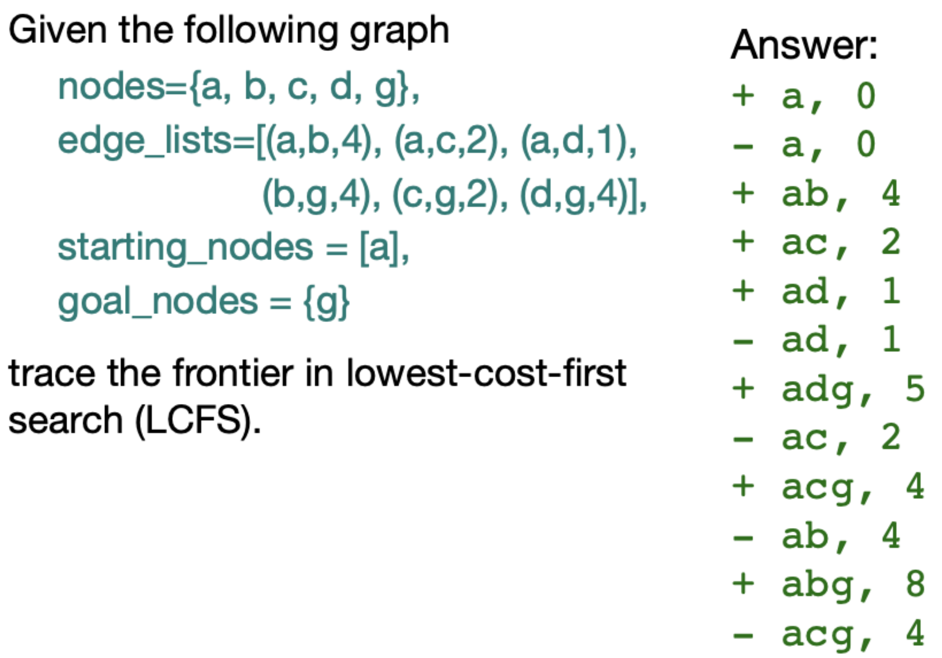


Figure 9: LCFS trace generic

Lecture Two: Searching the State Space (part two)

Pruning

- This is our method to deal with cycles and multiple paths.
- this means we can have wasted computation and cycles in our graph

> Principle: Do not expand paths to nodes that have already been expanded

Pruning Implementation

- The frontier keeps track of expanded or *closed* nodes
- When adding a new path to the frontier, it is only added if another path to the same end-node has not already been expanded, otherwise the new path is discarded (*pruned*)
- When asking for the **next path** to be returned by the frontier, a path is selected and removed but it is returned only if the end-node has not been expanded before, otherwise the path is discarded (pruned) and not returned. The selection and removal is repeated until a path is returned (or the frontier becomes empty). If a path is returned, its end-node will be remembered as an expanded node.

In frontier traces every time a path is pruned, we add an explanation mark `!` at the end of the line

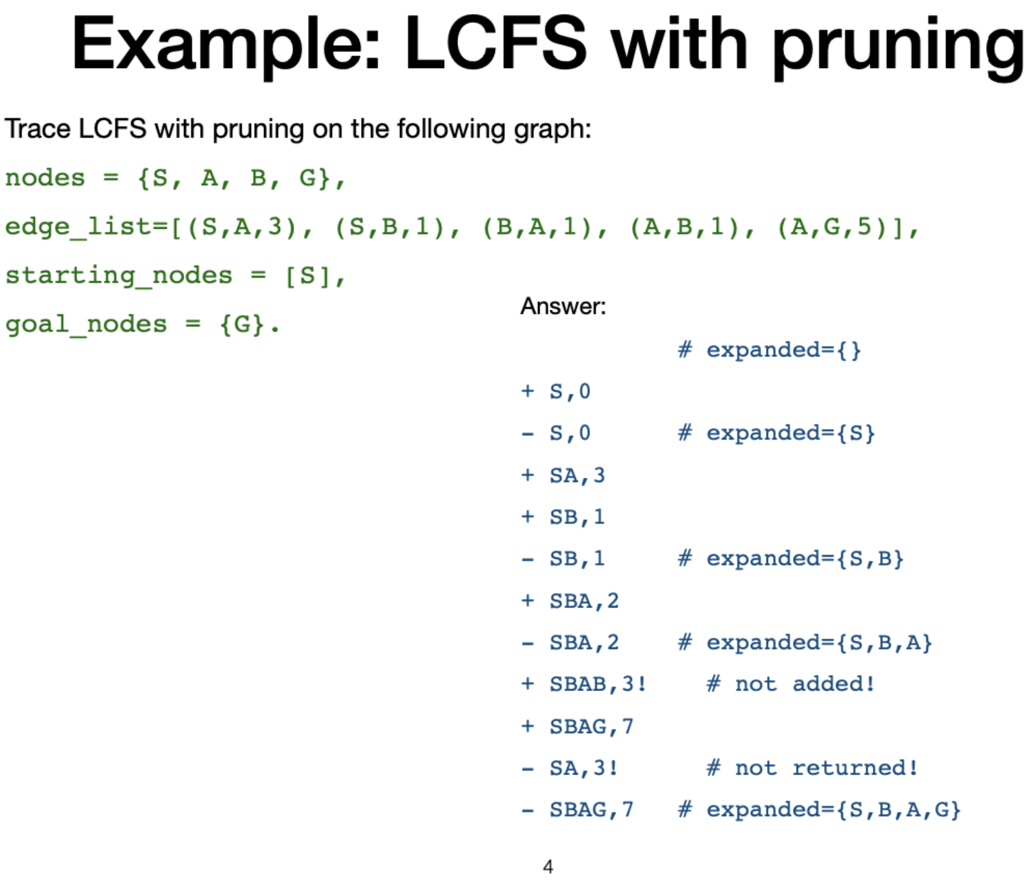


Figure 10: Example: LCFS with pruning

How does LCFS behave?

- LCFS explores increasing cost contours
  - Finds an optimal solution always
  - Explores options in every direction
  - No information about goal location

We are going to use a search heuristic, function `h()` is an estimate of the cost for the shortest path from node *n* to a goal node.

- *h* needs to be efficient to compute
- *h* can be extended to paths: $h(< n_0, ..., n_k) = h(n_k)$
- *h* is said to be admissible if and only if:

- $\forall n\ h(n) \geq 0$, *h* is non-negative and $h(n) \leq C$ where C is the optimal cost of getting from *n* to a goal node

> NOTE: We will have to come up with our own heuristic for the assignment as it depends on context.

Best-first Search

- Idea: select the path whose end is closest to a goal node according to the heuristic function.
- Best-first search is a greedy strategy that selects a path on the frontier with minimal *h*-value
- Main drawback: this does not guarentee finding an optimal solution.

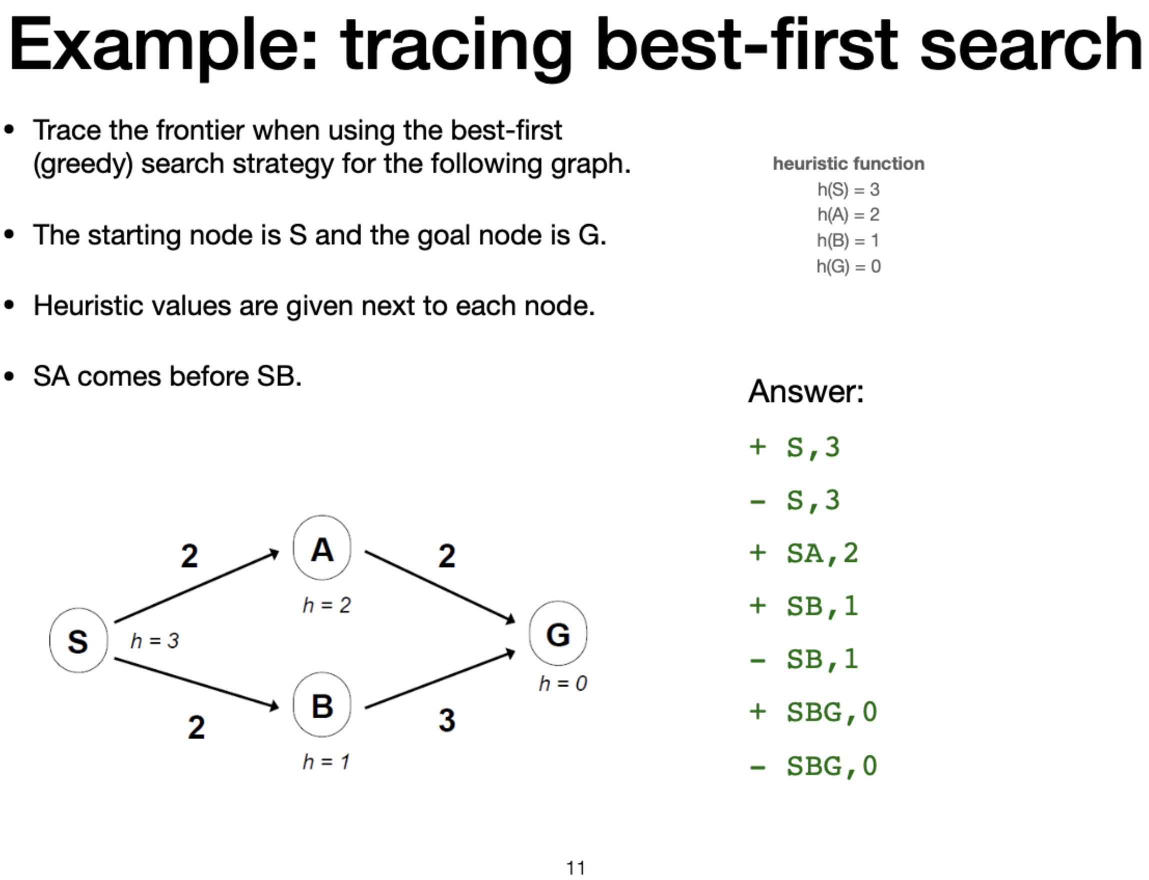


Figure 11: Tracing best-first search

A **search strategy**

Properties:

- Always finds an optimal solution as long as:

  - there is a solution
  - there is no pruning
  - the heuristic function is admissible

- Does it halt on every graph?

Idea:

- Don't be as wasteful as LCFS
- Don't be as greedy as best-first search
- Estimate the cost of paths as if they could be extended to reach a goal in the best possible way.

Evaluation function: $f(p) = cost(p) + h(n)$

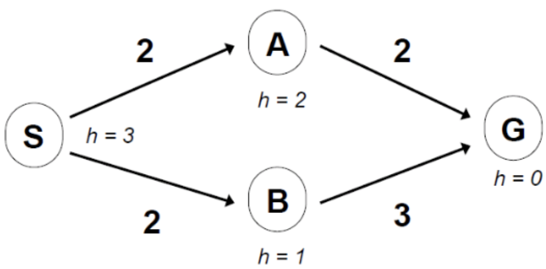
- p is a path, n is the last node on p
- $cost(p)$ = cost of path p *this is the actual cost from the starting node to node n*
- $h(n)$ = an estimate of the cost from n to goal node
- $f(p)$ = estimated total cost of path through p to goal node

The frontier is a priority queue ordered by $f(p)$

Example: tracing A* search

- Trace the frontier when using the A* search strategy for the following graph.
- The starting node is S and the goal node is G.
- Heuristic values are given next to each node.
- SA comes before SB.

heuristic function
h(S) = 3
h(A) = 2
h(B) = 1
h(G) = 0



Answer:

```
+ S,3     # 0 + 3 = 3
- S,3
+ SA,4    # 2 + 2 = 4
+ SB,3    # 2 + 1 = 3
- SB,3
+ SBG,5   # 5 + 0 = 5
- SA,4
+ SAG,4   # 4 + 0 = 4
- SAG,4
```

Note: This small example only show the inner working of A*. It does not demonstrate its advantage over LCFS.

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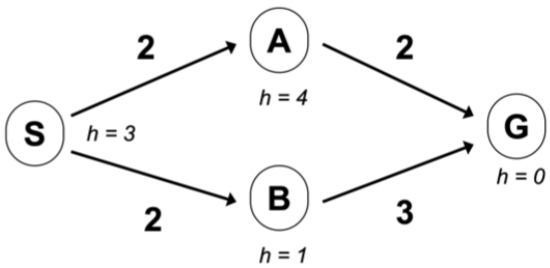
heuristic function
h(S) = 3
h(A) = 4
h(B) = 1
h(G) = 0

- Same example as the one before just assume *h(A) = 4* instead.



Answer:

```
+ S,3     # 0 + 3 = 3
- S,3
+ SA,6    # 2 + 4 = 6
+ SB,3    # 2 + 1 = 3
- SB,3
+ SBG,5   # 5 + 0 = 5
- SBG,5
```

Non-optimal solution! Why?

A*: proof of optimality

When using A* (without pruning) the first path p from a starting node to a goal node that is selected and removed from the frontier has the lowest cost.

Sketch of proof:

- Suppose to the contrary that there is another path from one of the starting nodes to a goal node with a lower cost.
- There must be a path p' on the frontier such that one of its continuations leads to the goal with a lower overall cost than p.
- Since p was removed before p':

$$f(p) \leq f(p') \implies cost(p) + h(p) \leq cost(p') + h(p') \implies cost(p) \leq cost(p') + h(p')$$

- Let c be any continuation of p' that goes to a goal node; that is, we have a path $p'c$ from a start node to a goal node. Since h is admissible, we have:

$$cost(p'c) = cost(p') + cost(c) \geq cost(p') + h(p')$$

- Thus:

$$cost(p) \leq cost(p') + h(p') \leq cost(p') + cost(c) = cost(p'c)$$

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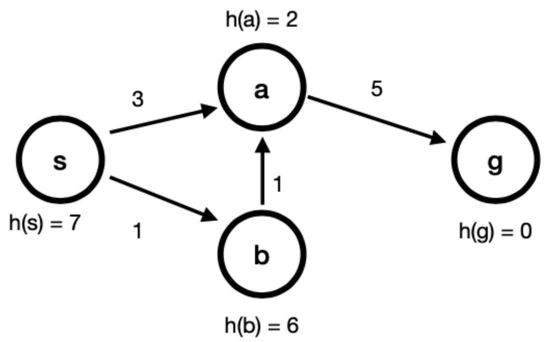
Effect of pruning on A*

Trace the frontier in A* search for the following graph, with and without pruning.

```
nodes={s, a, b, g},
estimates = {s:7, a:2, b:6, g:0},
edge_list=[(s,a,3), (s,b,1),
           (b,a,1), (a,g,5)],
starting_nodes = [s],
goal_nodes = {g}.
```



Answer <u>without</u> pruning

```
+ S, 7
- S, 7
+ SA, 5
+ SB, 7
- SA, 5
+ SAG, 8
- SB, 7
+ SBA, 4
- SBA, 4
+ SBAG, 7
- SBAG, 7
```

Answer **with** pruning

```
# expanded={}
+ S, 7
- S, 7        # expanded={S}
+ SA, 5
+ SB, 7
- SA, 5       # expanded={S,A}
+ SAG, 8
- SB, 7       # expanded={S,A,B}
+ SBA, 4!
- SAG, 8
```

Non-optimal solution!

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What went wrong when pruning A Search

- An expensive path, *sa* was expanded before a cheaper path *sba* could be discovered, because $f(sa) < f(sb)$
- Is the heuristic function *h* admissible?
  - Yes
- So what can we do?
  - We need a stronger condition than admissibility to stop this from happening

> Principle: When we are removing nodes, we are essentially saying we have found a cheaper solution, in this case, this was not true and hence why the algorithm fails, we need to use a stronger condition as outlined below

Monotonicity

A heuristic function is monotone or consistent if for every two nodes n and n' which is reachable from n:

$$h(n) \leq cost(n, n') + h(n')$$

With the monotone restriction, we have:

$$\begin{aligned} f(n') &= cost(s, n') + h(n') \\ &= cost(s, n) + cost(n, n') + h(n') \\ &\geq cost(s, n) + h(n) \\ &\geq f(n) \end{aligned}$$

How about using the actual cost as a heuristic?

- Would it be a valid heuristic?
- Would we save on nodes expanded?
- What's wrong with it?
    - It becomes as computationally expensive as it is to just do the problem

Choosing a heuristic: a trade-off between quality of estimate and work per node!

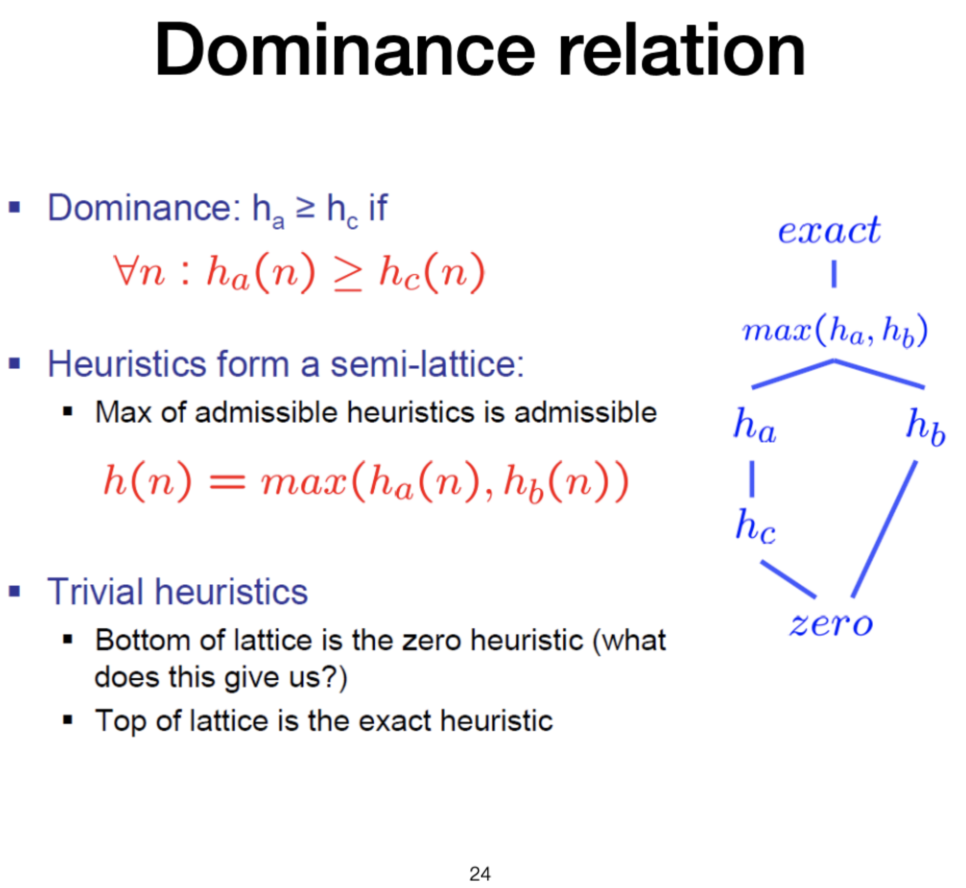
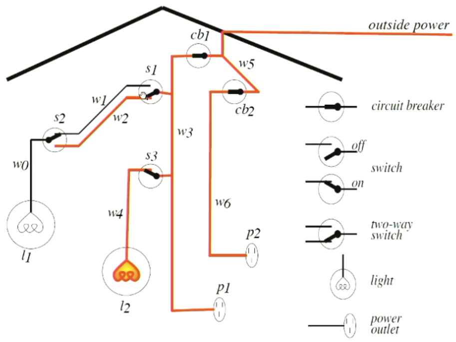
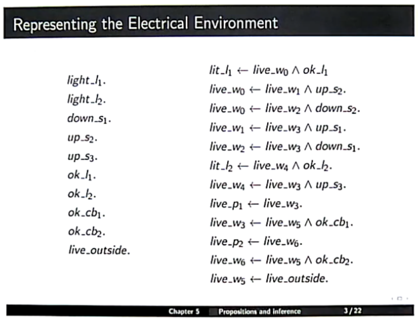


Figure 12: Dominance Relation

> Further algorithms are discussed in this segment of the and lecture *boarders onto lecture three* however this content will not be assessed in the duration of this course.

Lecture Three: Knowledge Base and Information

How to represent information in a knowledge base

Information	Knowledge Base
	

- The computer doesn't know the meaning of the symbols (logical and etc…)
- The user can interpret the symbol using their meaning
- There is no specific syntax for this, it is just what ever is readable for the user/writer

Simple language and definitions

- An **atom** is a symbol starting with a lower case letter
- A **body** is an atom or is of the form $b_1 \wedge b_2$ where b_1 and b_2 are bodies
- A **definite clause** is an atom or a rule of the form $h \leftarrow b$ where h is an atom and b is a body
- A **knowledge base** is a set of definite clauses
- An **interpretation** i assigns a truth value to each atom
- A **body** $b_1 \wedge b_2$ is true in i if b_1 is true in i and b_2 is true in i
- A **rule** $h \leftarrow b$ is false in i if b is true in i and h is false in i, the rule is true otherwise
- A **knowledge base** KB is true in i if and only if every clause in KB is true in i
- A **model** of a set of clauses is an interpretation in which all the clauses are *true*
- If KB is a set of clauses and g is a conjunction of atoms, g is a **logical consequence** of KB, this is denoted as $KB \models g$, if g is *true* in every model of KB
  - That is, $KB \models g$ if there is no interpretation in which KB is *true* and g is *false*.
- A **Proof procedure** is a -possibly non-deterministic - algorithm for deriving consequences of a knowledge
- Given a proof procedure, $KB \vdash g$ means g can be derived from knowledge base KB
- Recall $KB \models g$ means g is *true* in all models of KB
- A proof procedure is **sound** if $KB \vdash g \implies KB \models g$
- A proof procedure is **complete** if $KB \models g \implies KB \vdash g$

$$KB = \begin{cases} p \leftarrow q. \\ q. \\ r \leftarrow s. \end{cases}$$

How many interpretations?

	p	q	r	s	model of KB?
I_1	*true*	*true*	*true*	*true*	
I_2	*false*	*false*	*false*	*false*	
I_3	*true*	*true*	*false*	*false*	
I_4	*true*	*true*	*true*	*false*	
I_5	*true*	*true*	*false*	*true*	

Which of p, q, r, s logically follow from KB?

Figure 13: simple example question

Answers to the questions:

> We have four atoms {p,q,r,s}, because we have 4 atoms, there are 16 permutations in our truth table (2^4), therefore we have 16 interpretations

Bottom-up proof procedure

Rule of derivation:

if $h \leftarrow b_1 \wedge ... \wedge ...b_m$ is a clause in the knowledge base, and each b_i has been derived, then h can be derived

- This is **Forward chaining** on this clause (this rule also covers the case when $m = 0$)
- $KB \vdash g$ if $g \in C$ at the end of the below algorithmic procedure

- Tracing tutorial: 1:13:30

$C := \{\};$
repeat
  select clause "$h \leftarrow b_1 \land \ldots \land b_m$" in KB such that
    $b_i \in C$ for all i, and
    $h \notin C$;
  $C := C \cup \{h\}$
until no more clauses can be selected.

Figure 14: Bottom-up proof procedure algorithm pseudo code

Top-down proof procedure

Idea: search backward from a query to determine if it is a logical concequence of KB

An **answer clause** is of the form:

- $yes \leftarrow a_i \land ... \land a_m$

The SLD Resolution of this answer clause on atom a_i with the clause:

- $a_i \leftarrow b_1 \land ... \land b_p$
- Tracing tutorial: 1:31:00

An **answer** is an answer clause with $m = 0$. That is the answer clause $yes \leftarrow$.

A **Derivation** of query $?q_1 \land ... \land q_k$ from KB is a sequence of answer clauses $\lambda_0, \lambda_1, ...\lambda_n$

- λ_0 is the answer clause $yes \leftarrow q_1 \land ... \land q_k$
- λ_1 is obtained by resolving λ_{i-1} with a clause in KB
- λ_n is the answer

To solve the query $?q_1 \wedge \ldots \wedge q_k$:

> $ac :=$ "$yes \leftarrow q_1 \wedge \ldots \wedge q_k$"
> **repeat**
> > **select** atom a_i from the body of ac
> > **choose** clause C from KB with a_i as head
> > replace a_i in the body of ac by the body of C
>
> **until** ac is an answer.

Figure 15: Top-down proof procedure algorithm pseudo code