Homework Assignment (Problem Set) 3: Adam Tierney

1. An engineer at Fertilizer Company has synthesized a sensational new fertilizer made of just two interchangeable basic raw materials. The company wants to take advantage of this opportunity and produce as much as possible of the new fertilizer. The company currently has $40,000 to buy raw materials at a unit price of $8000 and $5000 per unit, respectively. When amounts x_1 and x_2 of the basic raw materials are combined, a quantity q of fertilizer results given by: $q = 4x_1 + 2x_2 - 0.5x_1^2 - 0.25x_2^2$

Part A: Formulate as a constrained nonlinear program. Clearly indicate the variables, objective function, and constraints.

Variables:
x_1 = raw material 1
x_2 = raw material 2

Objective Function:
Max $q = 4x_1 + 2x_2 - 0.5x_1^2 - 0.25x_2^2$

Constraints:
$8{,}000(4x_1 - 0.5x_1^2) + 5{,}000(2x_2 - 0.25x_2^2) <= 40{,}000$
$x_1, x_2 >= 0$

Part B: Solve the Program (provide exact values for all variables and the optimal objective function).

	x1	x2			
Variables	3.1578947	2.9473684			
Obj.	7.6454294	3.7229917	11.368421		
Const.	8000	5000	40000	<=	40000

Variables (rounded to the nearest hundredth)
Raw Material 1 = 3.16
Raw Material 2 = 2.95

Optimal Objective Value = 11.39

2. The area of a triangle with sides of length a, b, and c is $\sqrt{s(s-a)(s-b)(s-c)}$, where s is half the perimeter of the triangle. We have 60 feet of fence and want to fence a triangular-shaped area.

Part A: Formulate the problem as a constrained nonlinear program that will enable us to maximize the area of the fenced area, with constraints. Clearly indicate the variables, objective function, and constraints.

Max: $\sqrt{s(s-a)(s-b)(s-c)}$

Variables:

s = half of the perimeter
a = length of side a
b = length of side b
c = length of side c

Constraints:

s <= 30
a + b + c <= 60
a - (b+c) <= 0
b - (a+c) <= 0
c - (a+b) <= 0
s – (a+b+c)/2 = 0
s,a,b,c > 0

Hint: *The length of a side of a triangle must be less than or equal to the sum of the lengths of the other two sides.*

Part B: Solve the Program (provide exact values for all variables and the optimal objective function).

Max:							
$\sqrt{s(s-a)(s-b)(s-c)}$							
ST							
s / 2 <= 60							
a + b + c <= 60		s	a	b	c		Objective
a - (b+c) <= 0		30	20	20	20		173.2
b - (a+c) <= 0							
c - (a+b) <= 0		LEFT SIDE	RIGHT SIDE				
s – (a+b+c)/2 = 0	Constraint 1	30	30	<=			
s,a,b,c > 0	Constraint 2	60	60	<=			
	Constraint 3	-20	0	<=			
	Constraint 4	-20	0	<=			
	Constraint 5	-20	0	<=			
	Constraint 6	-8E-07	0	=			

Optimal Value = 173.2 square feet

s = 30
a = 20
b = 20
c = 20

3. The Tiny Toy Company makes three types of new toys: the tiny tank, the tiny truck, and the tiny turtle. Plastic used in one unit of each is 1.5, 2.0 and 1.0 pounds, respectively. Rubber for one unit of each toy is 0.5, 0.5, and 1.0 pounds, respectively. Also, each tank uses 0.3 pounds of metal and the truck uses 0.6 pounds of metal during production. The average weekly availability for plastic is 16,000 pounds, 9,000 pounds of metal, and 5,000 pounds of rubber. It takes two hours of labor to make one tank, two hours for one truck, and one hour for a turtle. The company allows no more than 40 hours a week for production (priority #1). Finally, the cost of manufacturing one tank is \$7, 1 truck is \$5 and 1 turtle is \$4; a target budget of \$164,000 is initially used as a guideline for the company to follow.

a) Minimize over-utilization of the weekly available supply of materials used in making the toys and place twice as much emphasis on the plastic (priority #2)

b) Minimize the under and over-utilization of the budget. Maximize available labor hour usage (priority #3).

<u>Formulate</u> the above decision problem as a single linear goal program. Clearly identify your achievement vector (i.e., hierarchy of priority levels for the goals). Do not solve.

Variables:

x_1 = # of Tiny Tanks
x_2 = # of Tiny Trucks
x_3 = # of Tiny Turtles

Priority 1:	$2x_1 + 2x_2 + 1x_3 \leq 40$
Priority 2:	$1.5x_1 + 2.0x_2 + 1.0x_3 \leq 16000$
Priority 3:	$0.3x_1 + 0.6x_2 \leq 9000$
Priority 4:	$0.5x_1 + 0.5x_2 + 1.0x_3 \leq 5000$
Priority 5:	$7x_1 + 5x_2 + 4x_3 \leq 164000$

$x_1, x_2, x_3 \geq 0$

$2x_1 + 2x_2 + 1x_3 + \eta_1 - \rho_1 = 40$
$1.5x_1 + 2.0x_2 + 1.0x_3 + \eta_2 - \rho_2 = 16000$
$0.3x_1 + 0.6x_2 + \eta_3 - \rho_3 = 9000$
$0.5x_1 + 0.5x_2 + 1.0x_3 + \eta_4 - \rho_4 = 5000$
$7x_1 + 5x_2 + 4x_3 + \eta_5 - \rho_5 = 164000$
$x_1, x_2, x_3 \geq 0$ and $\eta_i, \rho_i \geq 0$ for all i

$$\text{Lex Min} \begin{bmatrix} p_1 \\ 2p_2 + p_3 + p_4 \\ p_5 + p_1 \end{bmatrix}$$

4. XYZ Company is planning an advertising campaign for its new product. The media considered are television and radio. Rated exposures per thousand dollars of advertising expenditure are 10,000 for TV and 7,500 for radio. Management has agreed that the campaign cannot be judged successful if total exposures are under 750,000. The campaign would be viewed as superbly successful if 1 million exposures occurred. In addition, the company has realized that the two most important audiences for its product are persons 18 to 21 years of age and persons 25 to 30 years of age. The following table estimates the number of individuals in the two age groups expected to be exposed to advertisements per \$ 1,000 of expenditures:

Exposures per \$1000

Age	Television	Radio
18-21	2,500	3,000
25-30	3,000	1,500

Management has rank ordered five goals it wishes to achieve, arranged from highest to lowest priorities.

a) Achieve total exposures of at least 750,000 persons.
b) Avoid expenditures of more than $100,000.
c) Avoid expenditures of more than $70,000 for television advertisements.
d) Achieve at least 1 million total exposures.
e) Reach at least 250,000 persons in each of the two age groups, 18-21 and 25-30 years. In addition, management realizes and wishes to account for the fact that the purchasing power of the 25-30 age group is twice that of the 18-21 age group.

<u>Formulate</u> the above decision problem as a single linear goal program. Clearly identify your achievement vector (i.e., hierarchy of priority levels for the goals). Do not solve.

Variables:

x_1 = # of exposures for TV
x_2 = # of exposures for Radio

Goal 1:	$x_1 + x_2 \geq 750000$
Goal 2:	$10x_1 + 7.5x_2 \leq 100000$
Goal 3:	$10x_1 \leq 70000$
Goal 4:	$x_1 + x_2 \geq 1000000$
Goal 5:	$2.5x_1 + 3x_2 \geq 250000$
Goal 6:	$3x_1 + 1.5x_2 \geq 250000$

$x_1, x_2 \geq 0$

$x_1 + x_2 + \eta_1 - \rho_1 = 750000$
$10x_1 + 7.5x_2 + \eta_2 - \rho_2 = 100000$
$10x_1 + \eta_3 - \rho_3 = 70000$
$x_1 + x_2 + 1.0x_3 + \eta_4 - \rho_4 = 1000000$
$2.5x_1 + 3x_2 + \eta_5 - \rho_5 = 250000$
$3x_1 + 1.5x_2 + \eta_6 - \rho_6 = 250000$

$x_1, x_2 \geq 0$ and $\eta_i, \rho_i \geq 0$ for all i

$$\text{Lex Min} \begin{bmatrix} n_1 \\ p_2 \\ p_3 \\ n_4 \\ n_5 + 2n_6 \end{bmatrix}$$

5. A large food chain owns a number of pharmacies that operate in a variety of settings. Some are situated in small towns and are open for only 8 hours a day, 5 days per week. Others are located in shopping malls and are open for longer hours. The analysts on the corporate staff would like to develop a model to show how a store's revenues depend on the number of hours that it is open. They have collected the following information from a sample of stores.

Hours of Operation	Average Revenue ($)
40	5958
44	6662
48	6004
48	6011
60	7250
70	8632
72	6964
90	11097
100	9107
168	11498

a) Use a linear function (e.g., y = ax + b; where a and b are parameters to optimize) to represent the relationship between revenue and operating hours and find the values of the parameters using the nonlinear solver that provide the **best fit** to the given data. What revenue does your model predict for 120 hours?

We find that the predicted revenue would be $10,075. Our values are a = 47 and b = 4435.

```
In [17]: x = df['Hours of Operation']
         y = df['Average Revenue ($)']
```

```
In [30]: # Import optimize module
         from scipy import optimize

         def nonlinear_f(x, a, b):
             return  a *(x) + b

         # Fit a linear model:
         solution, _ = optimize.curve_fit(nonlinear_f, x, y, method='lm')

         print("The estimated solution is: ", solution)

         The estimated solution is:  [  47.07048984 4435.0837515 ]
```

```
In [33]: 47*120 + 4435

Out[33]: 10075
```

b) Suggest a two-parameter nonlinear model (e.g., $y = ax^b$; where a and b are parameters to optimize) for the same relationship and find the parameters using the Nonlinear Solver that provide the **best fit**. What revenue does your model predict for 120 hours? Which if the models in (a) and (b) do you prefer and why?

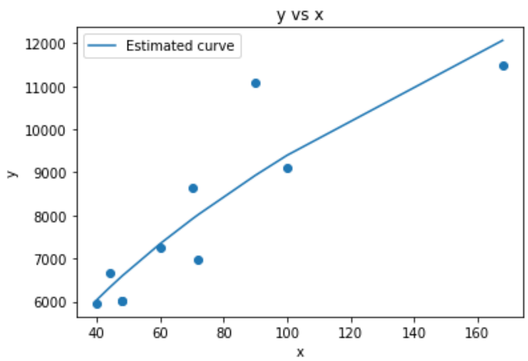
The revenue predicted in this model is $10,173 with a = 1022 and b = 0.48. I think I prefer the second model as it appears to better describe the data. This is a smaller sample size so there might be overfitting occurring here, but off of visual representation, it accounts for the lower left hand side of the chart to move closer to the x axis.

In [43]:
```
# Import optimize module
from scipy import optimize

def nonlinear_f(x, a, b):
    return  a *((x)** b)

# Fit a linear model:
solution, _ = optimize.curve_fit(nonlinear_f, x, y, method='lm')

print("The estimated solution is: ", solution)
```

```
The estimated solution is:  [1.02203428e+03 4.81785291e-01]
```

In [45]:
```
1022*120**.48
```

Out[45]:
```
10173.207292300187
```



Your solutions for (a) and (b) should contain a detailed spreadsheet model (where the decision variables, parameters, objective function and constraints are identified and explained), as well as answers to the questions posed. You may use Microsoft Excel, Python, or R to solve.