

Abstract

- A. NU Industries
- B. Optimization (Most in Scarce world)
- C. Goal Profit Maximization
- D. Constraints
- E. Relaxation of Binding Constraints - "What if"
- F. Keywords

Introduction and Problem Review

NU Industries

Over the years businesses have continued to evolve their logistics management and supply chain management as they aim to achieve their market competitive advantage over their competitors with geographically dispersed customers. Companies benefit in meeting their respective industry market demands when manufactured products can easily get into the supply chain when products, suppliers and customers are geographically close (Supply Chain Dive, 2022). With more and more geographically dispersed customers and product suppliers, the ability of companies to deliver the products to customers is challenged by Transportation costs, Marketing costs, Cost of Inventory, the Cost of labor (regular time and overtime), and Cost of raw materials which impact profitability and increase business expenses.

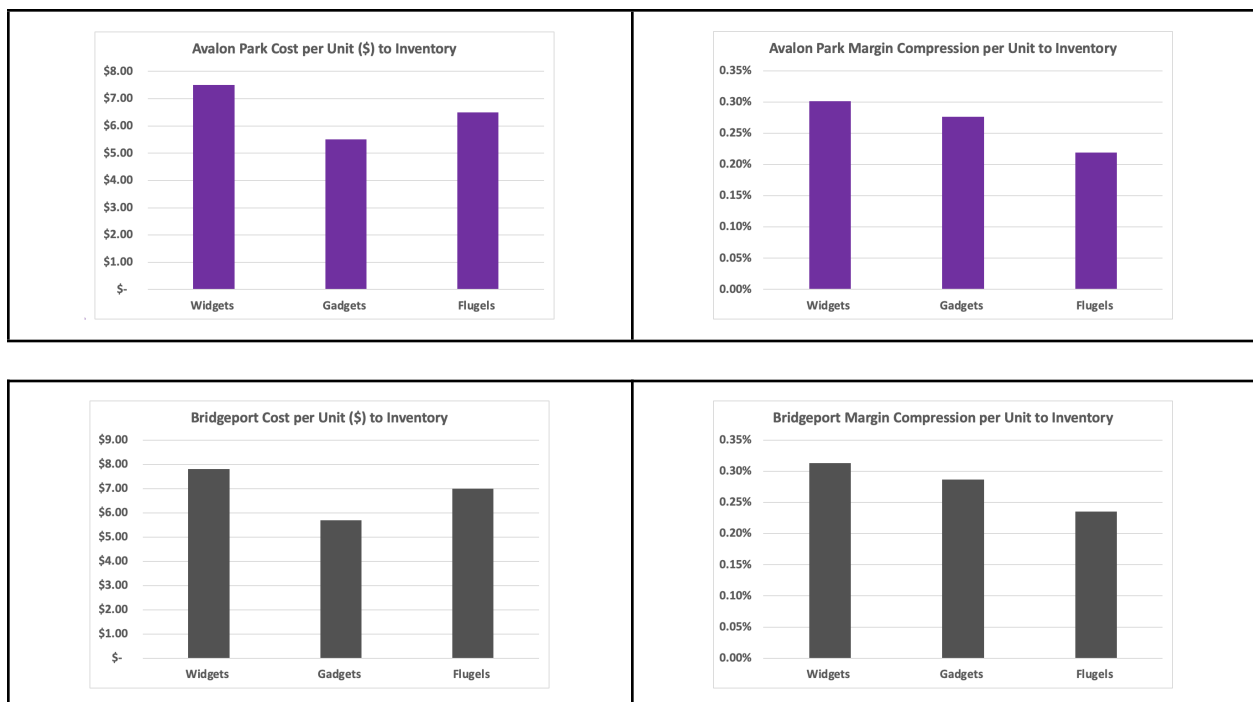
In this paper, we review operations management and logistics management challenges and opportunities presented for the NU Industries company. This business manufactures three products while looking to maximize sales profits and reduce costs based on operational constraints. NU Industries operates as a wholesaler outside of Evanston, Illinois with two manufacturing plants on the cities' Southside (Avalon Park) and Westside (Bridgeport). The manufacturing plants produce three products: Widgets, Gadgets, and Flugels and have the ability to store at adjacent space to the factory. The finished products are shipped to the Distribution Center for final distribution to the customers. Five periods of production are to be scheduled.

The inventory space(s) are not owned by NU industries and thus incur a cost to store. As the inventory space is on-site the inventory locations do not entail incremental cost

to transport. The cost to inventory ranges from roughly \$5 a unit to \$8 per unit. The max allowed, by business rule, is 120 units in inventory at any one time (70 at the Avalon Park Location and 50 in Bridgeport). This rule was created to maintain a just in time model to minimize exposure to demand swings. The owner subjectively set a policy to have less than a 10 day's demand in inventory at any one time. This was implemented as a ratio of minimum total demand experienced in any one period (410 units) relative to aggregate inventory.

Management is unsure if this is the best course of action as the inventory costs per unit on a relative basis to its selling price range from roughly 20 basis points to 30 basis points. Given the opportunity cost of missing out on sales because of a lack of inventory when the gross margin encumbrance is small, this optimization appears to be something they would like to study further.

Charts 1-4 – Avalon and Bridgeport Plant Cost and Margin Compression Per Unit (\$) to Inventory



The manufacturing plants are not identical. The differentiator between the plants is fourfold, one is the labor pool with which it operates, two is the relative efficiency of production, three is the cost of raw materials, and four is the transportation costs to the distribution centers. There are a lot of back and forth, with no dominant operational advantage by either plant.

A significant cost in production is labor. Avalon Park (A) has a dramatically smaller labor force to pull from with roughly 50% less hours available per period than Bridgeport (B). Although Avalon Park has a much smaller labor pool, the cost structure is roughly equivalent to Bridgeport. In periods 3+, Avalon Park is 5% cheaper per hour which could

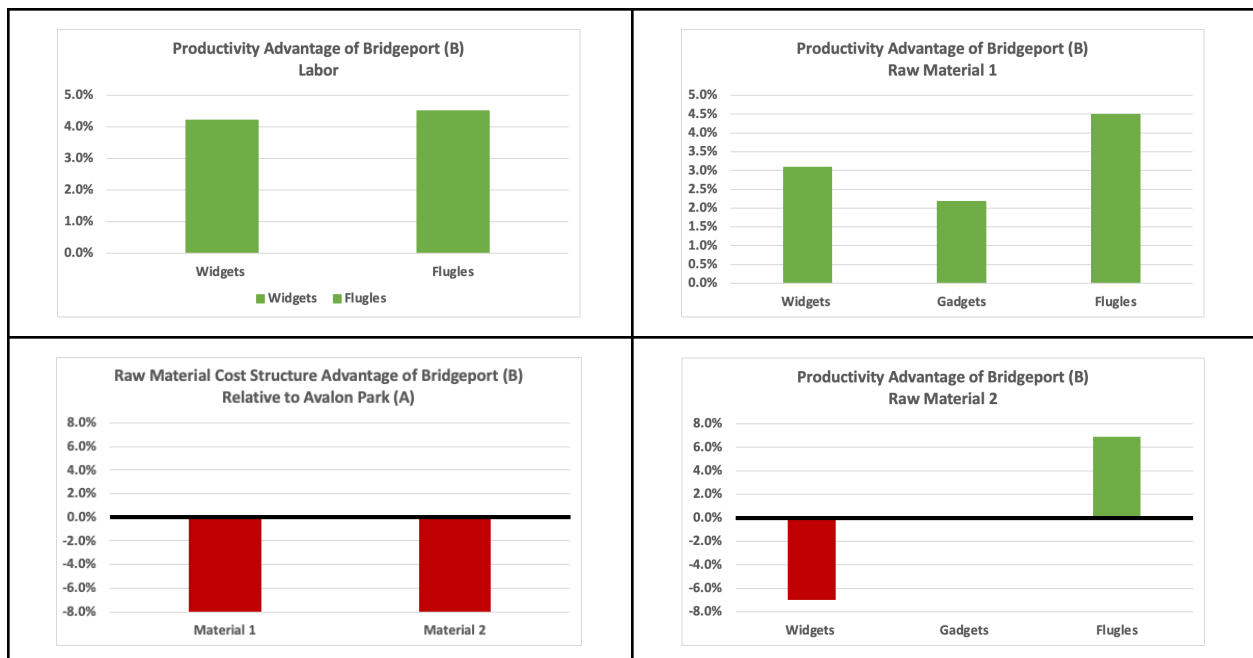
iron itself out over time as it is cheaper for a reason, about 4-5 % less productive per unit produced.

Charts 5-6 – Labor Access and Labor Cost Structure



A manufacturing constraint and significant component in the production of products are the raw materials and their efficiency by which they are used. For raw material efficiency, Bridgeport (B) is measurably more efficient than Avalon Park (A), except in the production of widgets. Cost structures are also different between the manufacturing sites for raw materials. Whatever gains in efficiency are offset by the raw material cost structure, whereby Bridgeport (B) is at a measurable disadvantage.

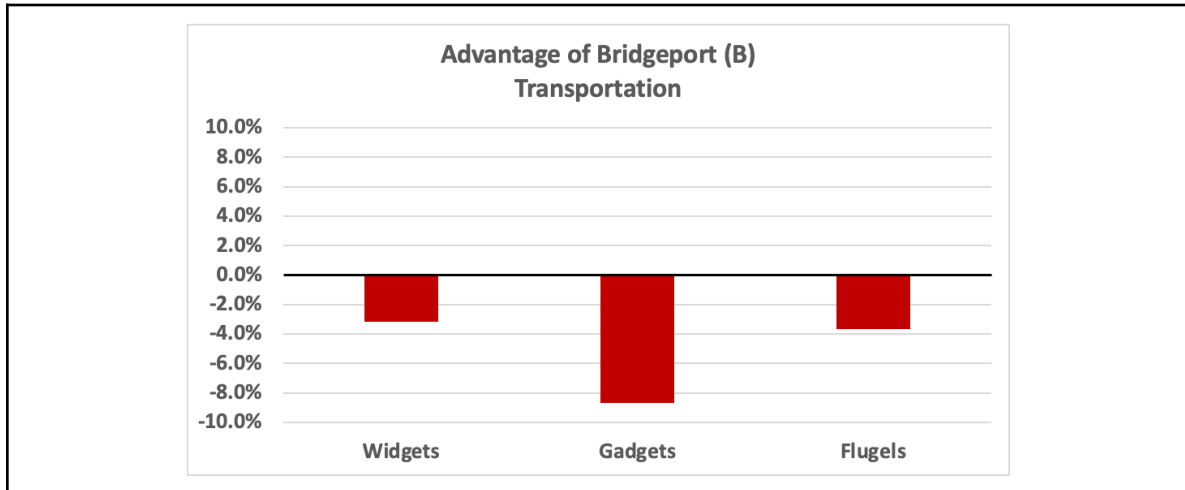
Charts 7-10 – Productivity Advantage of Avalon and Bridgeport Plants



For transportation, the business model is for NU industries is to ship to distribution nodes where customers (retailers) arrive and transact with NU industries. In this case,

Bridgeport (B) is at a disadvantage on the order of 3% to 9% to the Avalon Park (A) cost structure.

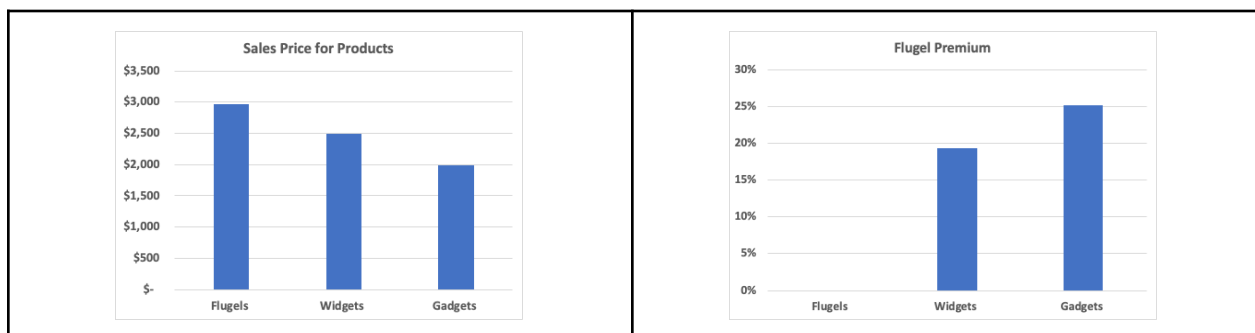
Charts 11 – Transportation Advantage of Bridgeport (B)

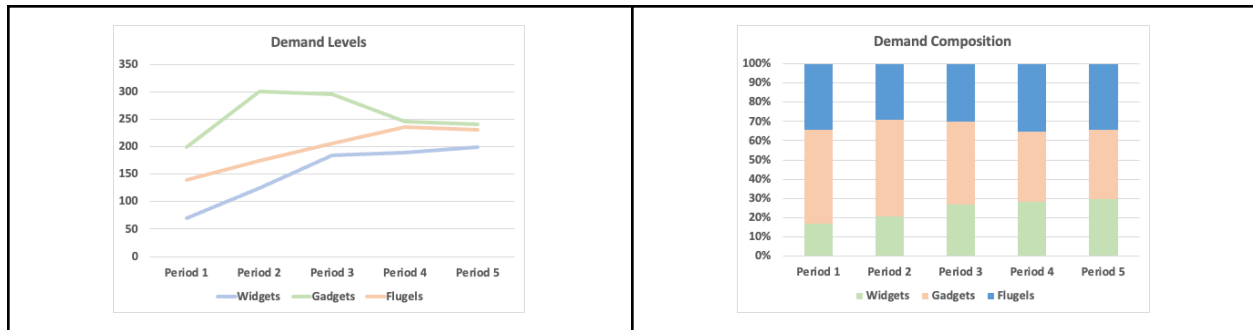


Marketplace

NU industries focus is on three different products: widgets, gadgets, and Flugels. The product that garners the highest price is Flugels by roughly 20% to widgets and 25% to gadgets. The products appear to be regulated in price and potentially quantity, given their inelasticity whereby price does not vary from high demand to low demand periods. Additionally, increased marketing spend unearths demand in a known and constant fashion. Base demand, while it does vary over the time, is known five periods in advance with certainty. Very few non-regulated industries would exhibit this.

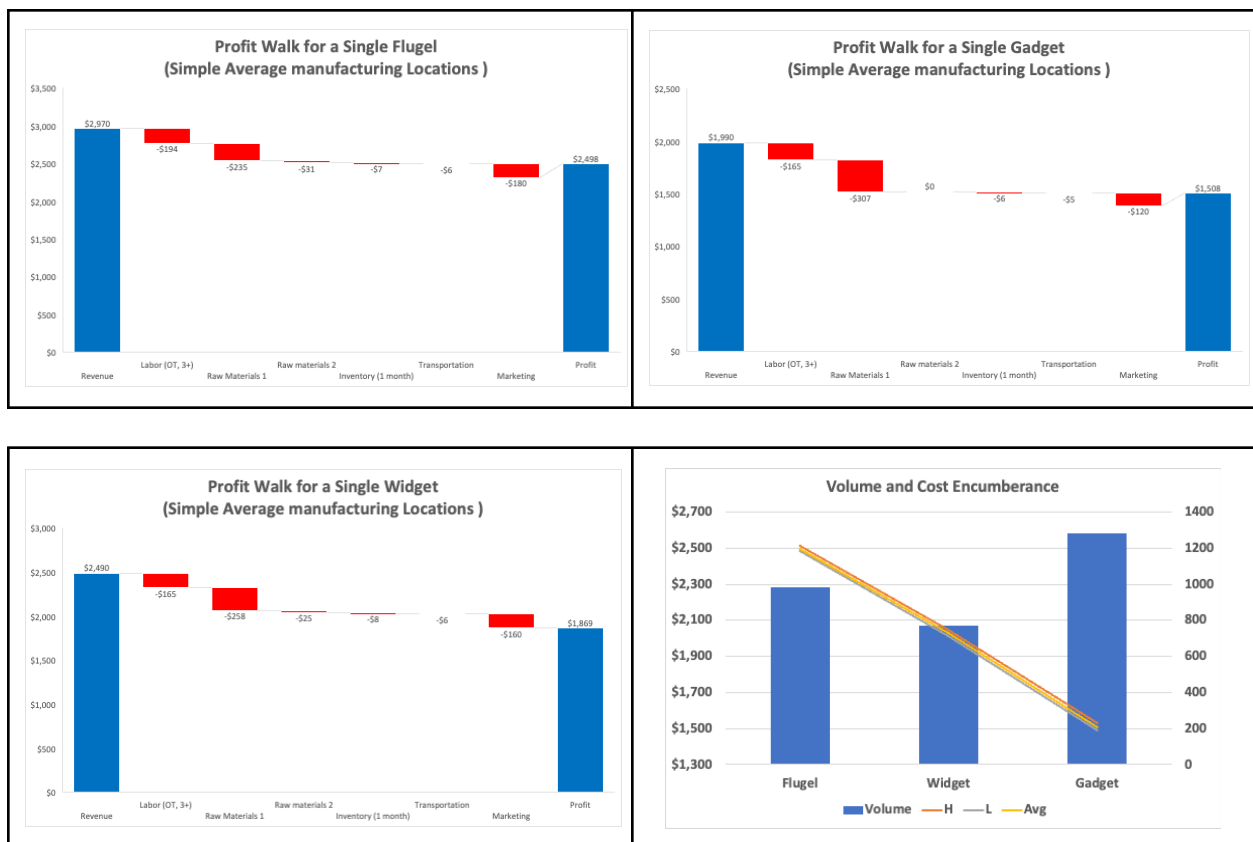
Charts 12-15 – Product Sales Prices, Flugel Premium, Demand Levels and Demand Composition





Profit is variable across locations but there appears to be plenty of it. Flugels have the largest gross margin, on average, with widgets second and gadgets last. The spread between the high manufacturing location, the low manufacturing location and the simple average of the two appears pretty tight. The majority of the volume is in the gadget product which does have the lowest gross margin.

Charts 16-19 – Profit Walks Per Product and Volume/Cost Encumbrance



Literature Review

The NU Industries company is faced with management challenges that are for the most part common production planning and supply chain problems faced by companies. With the improvements in computational models' speed, it's now easier to process scientific and mathematical models with a larger number of variables. Businesses have leveraged the use of computer models to more accurately represent operational data that closely resemble variables in sales prediction models instead of verbal statements (Vazsonyi, 1954). The role of electronic computing machine models continues to be key for developing models that better help predict future state of business problems from a managerial perspective.

One of the challenges faced by NU Industries are inventory level problems when manufacturing products. Companies have to address production planning and inventory policies given limited resources and market constraints. With business line managers conflicting inventory priorities, one of the areas of focus for companies is to determine the levels of inventory to keep at hand to meet market demands. As businesses grow more and more complex, the use of mathematical solves instead of mere intuition have become more prevalent for companies to maintain healthy levels of product inventory. Mathematical model approaches help uncover hidden costs such as inventory capital costs, raw material investment costs and set-up costs associated with the inventory policy (Magee, 1976). Thus, it's important to have a good understanding of movement of inventories based on storage and transportation costs required to maintain a certain level of product inventory. The average amount of movement inventory can be determined from the mathematical expression of $I = S \times T$; where S represents the average sales rate, T the transit time for transporting the product from a manufacturing plant to their storage facility given rate of sales. An optimization model must consider I or the movement inventory needed to keep enough inventory available for sale. In the NU industries problem set up, we considered a constraint that any unsold products at the end of the period must be stored in inventory, which has an associated cost and limit. Moreover, inventory does not exceed the available space. This could not be done in the variable declaration because the inventory is for the combination of all products. NU Industries company had to set inventory limitations based on the Plant combined constraint as shown in the table below.

Table 1 – Inventory Limit and Inventory Cost By Plant and Product

Plant	Product	Inventory Limit	Inventory Cost
Plant A	Flugels	70	6.5
Plant A	Gadgets	70	5.5
Plant A	Widgets	70	7.5

Plant B	Flugels	50	7
Plant B	Gadgets	50	5.7
Plant B	Widgets	50	7.8

In our computational model, we built constraints that the inventory does not exceed the available space. This could not be done in the variable declaration because the inventory is for the combination of all products. For a particular plant in a particular time period, this constraint is shown below.

$$\sum_{items} \text{Number of Products in inventory} \leq \text{Inventory Limit}$$

The NU Industries Optimization problem determines the marketing, production, distribution, and inventory strategy that maximizes profit. When planning future market behaviors of production and inventory in fluctuating markets, it is important to leverage realistic constraints set by changing market conditions (Dano et al., 1958). We leverage these constraints to better represent that there is no inventory at the start of the first period and there should be no inventory at the end of the planning horizon. All other plant overhead is assumed to be constant and it is ignored for this analysis. Thus, we set up the following constraints to simulate specific market conditions for analysis such as Marketing Budget Constraint, Raw Material Constraint, Labor Constraint, and Inventory Constraints. Further details of these constraints can be found in the following section.

Methodology

To optimize the production plan for NU Industries, we constructed two independent linear programming problems. The first used the Python package Pulp to invoke the GLPK linear program solver. The second was created in Excel to validate the result from the Python optimization. All of the results and output discussed in this paper are derived from the Python version, which will be outlined in this paper.

The Python program was divided into four main sections: problem set up, variable initialization, constraints, and the objective function.

Problem set up included initializing the pulp linear program object, as well as set up the constants detailing the problem description. There were many constants including material costs, labor cost, demand requirements, and inventory limitations. To better organize the input values, many of the constants were stored in an input Excel file that

was parsed by the Python program. Others were entered directly into the Python code as hard-coded values.

The next major section of the optimization program is initializing the linear program variables. These variables were divided among the major categories they were describing. The production variables count the number of products produced in each plant, in each period. These variables provide the main production plan after optimization is complete. The marketing variables represent the number of additional unit sales driven for each product in periods one through five. The variables for period one are included for code simplicity, and are constrained to equal zero. The transportation variables track the count of each product transported to the distribution center from each plant in each period. While not strictly necessary, these variables provide a straightforward way to calculate sales in each period. The labor variables count the labor hours for each period in each plant. Regular hours are stored in separate variables from overtime hours. The regular hours are also capped at their maximum value in the variable initialization. Materials variables tabulate the pounds of material shipped to a particular plant in a particular period. Finally, inventory variables count the number of products of each kind stored at a particular plant to the next period. The inventory variables include twelve dummy variables that are constrained to equal zero. These dummy variables store the number of products stored in inventory from period zero to period one and the products stored in inventory from period five to period six. In addition to making the coding for one of the constraints more consistent and clear, this also provides an intuitive value in the output to remind the end-user that inventory is assumed to be zero at the beginning and end of the production plan.

In total, 151 linear programming variables were used for optimization. Of the 151, fifteen were dummy variables constrained to equal zero. In addition, the 30 transportation variables and the 20 material variables are not strictly required, but are included as accounting variables. Technically, each of their values could be calculated based on the production variables and the constants in the problem set up. Despite this, they were included to make the output of the optimization easier to interpret.

Once all linear programming variables were created, constraints were set up to ensure they met the requirements of the NU Industries situation. Once again, the constraints were split up into the major categories of marketing, materials, labor, inventory, and production.

The only marketing constraint ensured that the total marketing spend stayed within the \$70,000 budget. Because the marketing variables have the number of products sold due to marketing as the units, we must convert this to the dollars spent on marketing. The marketing constraint used is:

$$\sum_{i \in \text{items}, t \in \text{time periods}} (\text{Additional units of } i \text{ in } t) \times (\text{Marketing cost of } i) \leq 70000$$

Two materials constraints were required. First, we needed to ensure that our material purchase plan did not call for more materials than the supplier can provide us in a single period. For each material and period, the material supply limit is:

$$\text{Lbs material to A} + \text{Lbs material to B} \leq \text{Supplier limit}$$

The second material constraint enforces the raw material requirements of the production plan. These constraints calculate the amount of material shipped to each plant in a period and make sure it is greater than or equal to the amount needed for production. One constraint must be created for each plant and material. As an equation, this is:

$$\text{Lbs material to plant} - \left(\sum_{i \in \text{items}} \text{Lbs to produce } i \times \text{Units of item } i \right) \geq 0$$

A very similar constraint was created for labor hours. Because the limit on the number of regular labor hours available was accounted for in the variable initialization, the only labor hours constraint needed is that the total number of labor hours planned must be enough to meet production plans. This constraint was created for each plant in each period. As an equation, this is:

$$\left(\sum_{\forall l \in \text{labor types}} \text{Hours of } l \right) - \left(\sum_{i \in \text{items}} i \text{ produced} \times \text{Hours to produce } i \right) \geq 0$$

Each plant has limited inventory, and the limit applies to the sum of all products stored. One constraint is needed for each plant in each period to ensure that the capacity for the storage area is not exceeded. As equations, this is:

$$\sum_{\text{items}} \text{Number of items in inventory} \leq \text{Inventory Limit}$$

One additional constraint is needed to ensure that the inventory variable has logical values. In any period at any plant, the following equation must be true for each product i:

$$\text{Starting Inventory} + \text{Produced} - \text{Shipped} - \text{Ending Inventory} = 0$$

The final set of constraints which must be added are demand constraints. These constraints ensure that the production plan meets the demand from customers. They account for the base demand and the demand created from marketing. Without these constraints, the linear programming model could create a plan that produces more products than the market will purchase or specialize one kind of product and fail to meet our commitment to our customers for other products. The constraint is a logical equality which equates the amount of products transported from each plant (a proxy for sales) to

the base demand combined with the marketing demand. One constraint must be created for each product in each period. As an equation this is:

$$\text{Transported A} + \text{Transported B} - \text{Base demand} - \text{Marketing demand} = 0$$

Now that all of the constraints are in place, we have all the requirements for a valid linear program, except for the objective function. The objective function for this problem is quite complicated, so it was created in sections similarly to the variable initialization and the constraints. The overall formula for the objective function is:

$$\text{Maximize: Revenue} - \text{Marketing} - \text{Inventory} - \text{Labor} - \text{Materials} - \text{Transportation}$$

To construct this objective function, each linear programming variable needs to be converted to dollars. For revenue, marketing, inventory, and transportation, this can be accomplished by multiplying the number of units by the associated sales price or unit cost. Labor and materials are converted to dollars by multiplying by the \$/hour and \$/pound cost respectively. In Python, the revenue or cost generated by every linear programming variable was iteratively appended to a list where costs were given a negative value. The final equation was simply a sum of the list, which was optimized by GLPK.

With all of these constraints, variables, and the objective function, the linear program will provide an optimal production plan. The resulting problem output provides production plan by period, the number of labor hours scheduled, materials purchased, marketing sales drive, products transported, and units in inventory. These variables represent many key aspects of the business and should provide valuable information for production planning.

Baseline Results

NU Industries is looking to maximize its profit. To do so, the company has begun a ground up study of its business from demand components to supply components. By testing the wiggle room within the current architecture and then testing a relaxation of its current architecture, the company better understands the profit implications on the firm. The study is being conducted by an external party NU Advanced Consulting Services and will begin by optimizing the “today”, which then restricts the focus of the analysis to the cost structure of the equation. First, the team built the construct for analysis, objective function (detailed in section Methodology section), to model the business and the impacts to profit, and how profits could be maximized. In the first stage of analysis, the team dove into the reduction of costs by building to demand in the most efficient manner given the known demands and constraints on the business today. The demands on the business are provided by NU Industries Sales Department for production requirements that must be met due to contracts during the planning horizon. The

variable of demand is the marketing spend allotted for the five periods, which will generate incremental demand for the products based on their profit and the system's constraint thresholds. In the baseline scenario, the marketing spend was \$70k.

Table 2 – NU Industries Contract Production Requirements

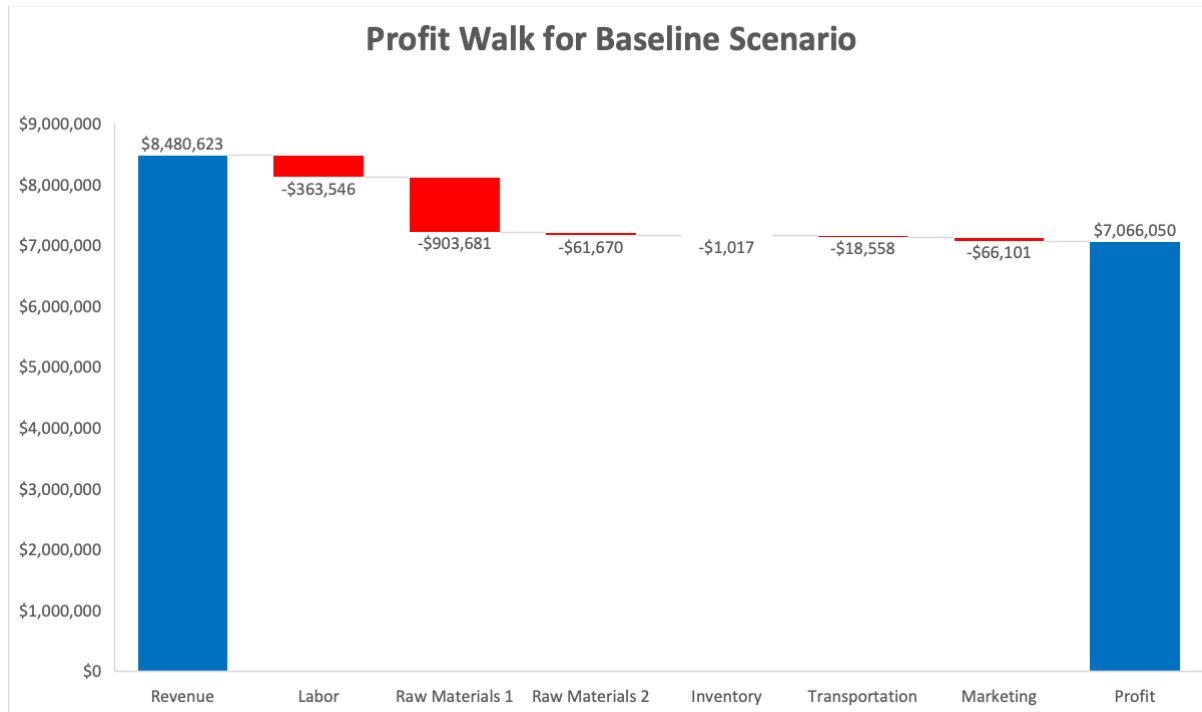
Product	Period 1	Period 2	Period 3	Period 4	Period 5
Widgets	70	125	185	190	200
Gadgets	200	300	295	245	240
Flugels	140	175	205	235	230

The cost composition to NU industries generates a series of constraints that are detailed in the heterogeneity of inventory carrying costs, labor resource and cost differentials, material resource and cost variances, and logistic cost disparities (NU Industries section).

The findings from the baseline scenarios seared in the intuition that this was production and inventory control problem would hinge on the availability of resources and limitation of resources. The demand constraint was limited to the known demand and profit was not encumbered as production increased. Of note, in future analysis the team will study the allocation of more marketing budget to drive additional demand. The cost constraints were fixed for the most part as their escalation for higher quantities produced was minimal and margins were healthy.

The implication of the optimizing solution was for a profit of \$7.1M (Chart 20) on \$8.5M of revenue. Of note, \$80k in marketing spend generated \$1.1M in incremental revenue, this will be a “what if” generating assumption the team will leverage in the future. The implications of the solution are broad based on the use of inventory, labor, material, and logistics. The implications on the factors of production and logistics are detailed in the appendix for the reader.

Chart 20 – Profit Walk for Baseline Scenario



Sensitivity Analysis

Several alternative scenarios were considered to increase NU Industries profits. Two of the most promising operational changes we found were increasing the amount of Material 1 available from the supplier in tandem with an increased marketing budget, and increasing the inventory capacity.

Scenario 1D: Increasing Material 1 Purchase Limit and Marketing Budget

After running the original optimization model configuration, we inspected the sensitivity analysis produced by GLPK. We observed Raw Material 1 is the limiting resource in periods two through five. In addition, despite marketing being a profitable expenditure, our marketing budget was not a binding constraint as material had run out before the marketing budget could be used in full. If we could procure more of Raw Material 1 from the supplier, NU Industries would drive more demand through marketing to make more profit. We used the sensitivity analysis to determine appropriate values to iteratively increase the purchase limit of Material 1 and the marketing budget until we found some third bottleneck resource. After a few reoptimization due to increases outside the bounds of the sensitivity analysis, we found an increase in profit of \$259,501.50.

Table 3 – Scenario 1D Description

Scenario	Description	Maximum Profit
0 - Original Problem	Original problem setup. We only have 140,000 lbs of Material 1 to purchase every period and \$70,000 to spend on marketing.	\$7,066,036.94
1D	Increased marketing budget to 85k and increased Material 1 purchase limit by 8000 lbs.	\$7,325,538.44

This 3.6% increase in profit is significant when considering the potentially small operational change of a 5.7% increase in purchases of Raw Material 1 and a 21% increase in marketing budget. The marketing budget constraint is binding in this scenario. For each additional dollar spent up to \$86,002 total, NU Industries can expect an increase in profit of \$13.08. We observed that in Scenario 1D, Material 2 was a limiting resource in periods four and five, which never occurred when less Material 1 was available.

Table 4 – Scenario 1D Materials By Period

Material Type	Period 1	Period 2	Period 3	Period 4	Period 5
Material 1	105,302	148,000	147,014	145,292	144,576
Material 2	3,272	4,672	4,548	5,000	5,000

Scenario 2: Increasing Inventory Capacity

During analysis of several other scenarios, it became clear that material resources and regular labor hours are generally at capacity in periods two through five, but are underutilized in the first period. Furthermore, the optimal production plan consistently used the maximum inventory in the first period to take advantage of some of this excess capacity in the first period. We theorized that increasing inventory availability could allow for a more efficient production schedule that utilizes the “use it or lose it” resources imposed by the regular labor hours limit and the Material 1 purchase limit. We

also expected that increasing warehousing space in the short term would be much easier than adjusting the size of the labor force or renegotiating a purchase contract.

To test our theory, we increase the inventory capacity by 100 units at each facility. All other constraints and costs remained the same, including the cost of inventory. After reoptimization, we found that production was noticeably more evenly spread between periods than in the original problem. As a result, overtime hours were completely eliminated from the production plan. The purchase limit for material one was only reached in period three, whereas in the original production plan it was reached in each of periods two through five. In addition, the unlocking of unused capacity allowed for the full marketing budget to be utilized.

Table 5 – Scenario 2 Material Utilization

Material Type	Period 1	Period 2	Period 3	Period 4	Period 5
Material 1	135,108	127,711	140,000	137,448	134,609
Material 2	3,759	4,947	4,034	4,552	4,414

Table 6 – Scenario 2 Inventory between Periods

Facility	Product	Period 1 to 2	Period 2 to 3	Period 3 to 4	Period 4 to 5
A	Flugels	0	0	0	0
A	Gadgets	115.14	0	0	0
A	Widgets	0	0	0	0
B	Flugels	150	0	0	0
B	Gadgets	0	0	0	0
B	Widgets	0	0	0	0

Despite the increased cost of inventory and marketing, the more efficient production schedule increased profits by \$65,557.37 to \$7,131,594.37. The additional inventory freed up so much unused capacity that Material 1 is no longer a bottleneck resource and production is not limited by demand.

Table 7 – Scenario 2 Description

Scenario	Description	Profit
----------	-------------	--------

0 - Original Problem	No change - original problem setup. Plant A has an inventory capacity of 70 and plant B has an inventory capacity of 50.	\$7,066,036.94
2	Original problem, except both plants have capacity to store 100 additional products.	\$7,131,594.37

Our next budgetary increase would be to increase the marketing budget to profitably create more demand. The current shadow price for the marketing budget constraint indicates that NU Industries can expect to make an additional \$13.21 for each additional marketing dollar spent up to a budget of \$72,834. If NU Industries does decide to increase their available inventory in the first period, they should certainly increase the marketing by \$2,834 as well.

Discussion/Conclusion

- A. Learnings
- B. Areas for future research
 - 1. Labor Negotiations
 - 2. Market Concentration and Price Elasticities

Discussion/Conclusion

References

Cliff Ragsdale. 2017. *Spreadsheet Modeling & Decision Analysis: A Practical Introduction to Business Analytics* (8th. ed.). South-Western College Publishing.

Garland, Max. "Lowe's Expands Use of Market Delivery Model for Large Products." *Supply Chain Dive*, November 22, 2022.
<https://www.supplychaindive.com/news/lowes-expands-market-delivery-model-regions-products/637055/>

Hillier, Frederick S., and Gerald S. Liebermann. 2021. *Introduction to Operations Research* (eleventh ed.). New York: McGraw-Hill.

Magee, John F. "Guides to Inventory Policy." *Business Logistics*, 1976, 92–137.
https://doi.org/10.1007/978-1-4613-4379-0_8.

Sven Dano, Ernst Lykke Jensen, (1958) Letter to the Editor—Production and Inventory Planning in a Fluctuating Market. Operations Research 6(2):293-295. <https://doi.org/10.1287/opre.6.2.293>

Vazsonyi, Andrew (1954) The Use of Mathematics in Production and Inventory Control. Management Science 1(1):70-85. <https://doi.org/10.1287/mnsc.1.1.70>

Appendix: Variable Generation

See attached python code file for variable generating code

Appendix: Python Code

See attached python code file

Appendix: Supplementary Tables

Table 8 – Raw Materials By Product and Plant

Product	Plant	Material 1	Material 2
Widgets	A	194	8.6
Gadgets	A	230	0
Flugels	A	178	11.6
Widgets	B	188	9.2
Gadgets	B	225	0
Flugels	B	170	10.8

Table 9 – Labor Hours By Product and Plant

Product	Plant	Labor Hours
Flugels	A	11.1

Gadgets	A	7.1
Widgets	A	9.5
Flugels	B	10.6
Gadgets	B	7.8
Widgets	B	9.1

Table 10 – Transportation Costs By Product and Plant

Product	Plant	Transportation Costs
Flugels	A	\$5.50
Gadgets	A	\$4.60
Widgets	A	\$6.30
Flugels	B	\$5.70
Gadgets	B	\$5.00
Widgets	B	\$6.50

Table 11 – Inventory Costs By Product and Plant

Plant	Product	Inventory Cost
Flugels	A	\$6.50
Gadgets	A	\$5.50
Widgets	A	\$7.50
Flugels	B	\$7.00
Gadgets	B	\$5.70

Widgets	B	\$7.80
----------------	---	--------

Table 12 – Cost of Marketing Additional Units

Product	Cost of Additional Unit
Flugels	\$180
Gadgets	\$120
Widgets	\$160