Assignment-1

Varendra University

Department of Computer Science and Engineering

3rd Semester, Examination: Spring-2025

Course Code: **MAT 2141**

Course Title: Differential Equations

Submission Time: Before 07.02.2025 **Section: A-E** **Marks: 10**

(Answer all of the following questions)

All part of each question must be answered sequentially.

1. If the differential equation is $xlnx\frac{dy}{dx} + y = x^2$ where $y(1) = 2$. Evaluate the following cases **2.5**

i. The general solution of the given differential equation.

ii. The particular solution of the given differential equation.

iii. Express the dependent variable in terms of independent variable from particular solution.

2. Solve the following linear equations $x(1 - x^2)dy + (2x^{2y} - y - ax^3)dx = 0$. **1.5**

3. Examine the following equations can be reduced to linear form and solve them $(ylogx - 1)ydx = xdy$. **1.5**

4. Test the following equations can be reduced to linear form and solve them $y + 2\frac{dy}{dx} = y^3(x - 1)$. **1.5**

5. Solve the following equations $(x^2 - 4xy - 2y^2)dx + (y^2 - 4xy - 2x^2)dy = 0$. **1.5**

6. Solve the following equations $cosx(cosx - sinasiny)dx + cosy(cosy - sinasinx)dy = 0$. **1.5**

1. If the differential equation is $x\ln x \frac{dy}{dx} + y = x^2$ where $y(1) = 2$. Evaluate the following cases.

i) The general sol^n of the given differential equation

ii) the particular solution of the given differential equation

iii) Express the dependent variable in terms of independent variable from particular solution.

Answer to - 1

i) Given that,

$$x\ln x \frac{dy}{dx} + y = x^2 \quad \text{......(i)}$$

Dividing $x\ln x$ from equation (i)

$$\frac{dy}{dx} + \frac{1}{x\ln x} y = \frac{x}{\ln x} \quad \text{...(ii)}$$

comparing eq^n (2) with,

$$\frac{dy}{dx} + Py = Q$$

where, $P = \frac{1}{x\ln x}$, $Q = \frac{x}{\ln x}$

Integrating factor,

$$IF = e^{\int P(x)\,dx}$$

$$= e^{\int \frac{1}{x\ln x}\,dx}$$

$$= e^{\int \frac{1}{x} \cdot \frac{1}{\ln x}\,dx}$$

$$= \ln x$$

General equation,

$$y(IF) = \int (I.F)\,Q\,dx + c$$

$$y \ln x = \int \left(\ln x \cdot \frac{x}{\ln x}\right) dx + c$$

$$y \ln x = \int x\,dx + c$$

$y \ln x = \frac{1}{2} x^2 + c$, which is required General solution.

ii) From (i) we find general solution is $y \ln x = \frac{1}{2}x^2 + c$

According to question,

$$y(1) = 2$$

So, particular solution will be

$$2 \ln 1 = \frac{1}{2} \cdot 1 + c$$

$$\Rightarrow \frac{1}{2} + c = 0$$

$$\therefore c = -\frac{1}{2}$$

So, $y \ln x = \frac{1}{2}x^2 - \frac{1}{2}$

$\Rightarrow 2y\ln x = x^2 - 1$

$\therefore 2y\ln x = x^2 - 1$, which is particular solution.

iii) From equation (ii) we will find the particular solution is $2y\ln x = x^2 - 1$

$$\therefore y = \frac{x^2 - 1}{2\ln x} \quad [\text{Dividing by } 2\ln x]$$

which is required the dependent variable y in terms of independent variable x from particular solution.

2. Solve the following linear equation...

$$x(1-x^2)dy + (2x^2y - y - ax^3)dx = 0$$

Answer to - 2

Given that,

$$x(1-x^2)dy + (2x^2y - y - ax^3)dx = 0$$

First rewrite the equation in standard form

$$M(x,y)dx + N(x,y)dy = 0$$

Here,

$$M = 2x^{2y} - y - ax^3$$

$$N = x(1-x^2)$$

For the equation to be exact the condition

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$ mun hold.

compute $\frac{\partial m}{\partial y}$, $\frac{\partial m}{\partial y} = \partial/\partial y\,(2x^{2y} - y - ax^3)$

$$= 2x^{2y}.\ \ln(x).\ 2^{-1}$$

$$= 4x^{2y}\ln(x) - 1$$

compute, $\frac{\partial N}{\partial x} = \partial/\partial x\,(x(1-x^2))$

$$= 1 - 3x^2$$

Since $\frac{\partial m}{\partial y} \neq \frac{\partial N}{\partial x}$ $\therefore$ The equation is not exact.

3. Examine the following eauations an be neduced to linear form and solve them $(y\log x - 1)y\,dx = x\,dy$.

Answer to - 3

Given that, $(y \log x - 1)\,y\,dx = x\,dy$

$$\Rightarrow \frac{(y\log x-1)y}{x} = \frac{dy}{dx}$$

$$\therefore \frac{dy}{dx} = \frac{y\log x - y}{x}$$

This is a non-linear differential equation.

Let, $v = y^{-1}$ then

$$\therefore y = \frac{1}{v}, \quad \frac{dy}{dx} = -\frac{1}{v^2}\frac{dv}{dx}$$

The original equation,

$$-\frac{1}{v^2}\frac{dv}{dx} = \frac{-\left(\frac{1}{v}\right)^2 \log x - \frac{1}{v}}{x}$$

Multiply by $-v^2$

$$\frac{dv}{dx} = \frac{\log x}{x} + \frac{v}{x}$$

$$\frac{dv}{dx} - \frac{v}{x} = -\frac{\log x}{x}$$

This is a new linear differential equation in v

$$\frac{dv}{dx} - \frac{1}{x}v = -\frac{\log x}{x}$$

Integrating factor, $u(x)$

$$u(x) = e^{\int -\frac{1}{x}dx} = e^{-\log x} = \frac{1}{x}$$

multiply by $\mu(x)$

$$\frac{1}{x}\frac{dy}{dx} - \frac{1}{x^2}v = \frac{-\log x}{x^2}$$

The left hand side in the derivative of $\frac{v}{x}$

$$d/dx\left(-\frac{v}{x}\right) = \frac{-\log x}{x^2}$$

Integrate both sides,

$$\frac{v}{x} = \int \frac{-\log x}{x^2}\,dx$$

using integration by parts

Let $u = \log x, \quad du = \frac{1}{x}dx$

$$dv = -\frac{1}{x^2}dx, \quad v = \frac{1}{x}$$

Then,

$$\int \frac{-\log x}{x^2}\,dx = \frac{\log x}{x} - \int \frac{1}{x^2}\,dx$$

$$= \frac{\log x}{x} + \frac{1}{x} + c$$

Thus, $\frac{v}{x} = \frac{\log x}{x} + \frac{1}{x} + c$

so $v = \log x + 1 + cx$

$$v = y^{-1}$$

$$y^{-1} = \log x + 1 + cx$$

$$y = \frac{1}{\log x + 1 + cx}$$

$$\therefore y = \frac{1}{\log x + 1 + cx}$$

This is the general solution of the given differen-tial equation.

4. Test the following equations can be reduced to Linear form and solve them.

$$y + 2\frac{dy}{dx} = y^3(x-1)$$

Answer to - 4

Given that,

$$y + 2\frac{dy}{dx} = y^3(x-1)$$

A differential equation is linear if it can be written in the form $\frac{dy}{dx} + P(x)y = Q(x)$

Given equation,

$$2\frac{dy}{dx} = y^3(x-1) - y$$

$$\frac{dy}{dx} = \frac{y^3(x-1) - y}{2}$$

Since, the equation contains y^3. It is not linear

However it is a Bernoullis equation of line form

$$\frac{dy}{dx} + P(x)\,y = Q(x)\,y^n$$

Comparing

$$P(x) = -\frac{1}{2} \qquad Q(x) = \frac{x-1}{2}$$

$$n = 3$$

Since $n \neq 1$ we can reduce it to a linear form.
using the Bernaulli's substitution

$$v = y^{1-n} = y^{1-3} = y^{-2}$$

Differentiating both sides,

$$\frac{dv}{dx} = -2y^{-3}\frac{dy}{dx} = -2y^{-3}\cdot\frac{y^3(x-1)-y}{2}$$

or, $$\frac{dv}{dx} = \cancel{2}-y^{-3}\left(y^3(x-1)-y\right)$$

or, $$\frac{dv}{dx} = -(x-1) + y^{-2}$$

since, $v = y^{-2}$, we substitute

$$\frac{dv}{dx} + (x-1) = y^{-2}$$

$$\frac{dv}{dx} - v = -(x-1) \quad [\because v = y^{-2}]$$

which is the linear differential equation.

The standard form is

$$\frac{dv}{dx} + P(x)v = Q(x)$$

where $P(x) = -1$ and $Q(x) = -(x-1)$

The integrating factor (IF) is;

$$IF = e^{\int -1\,dx} = e^{-x}$$

multiplying the entire equation by e^{-x}

$$e^{-x}\frac{dv}{dx} - e^{x}v = -(x-1)\,e^{-x}$$

Recognizing the left hand side as the derivative of ve^{-x}: $d/dx(ve^{-x}) = -(x-1)\,e^{x}$

Integrating, $ve^{-x} = \int -(x-1)\,e^{-x}\,dx$

using integration by parts,

$$(u = x-1,\ dv = e^{-x}dx)$$

$$\int (x-1)\,e^{-x}dx = -(x-1)\,e^{x} - \int -e^{-x}\,dx$$

$$= -(x-1)\,e^{-x} + e^{-x}$$

$$= \cancel{e^{-x}}(-(x-2)\,e^{-x}$$

so, $ve^{-x} = (x-2)\,e^{-x} + c$

$\Rightarrow v = (x-2) + c$

$\therefore v = (x-2) + c$

$\Rightarrow y^{-2} = (x-2) + c \quad [\because v = y^{-2}]$

$$\Rightarrow y^2 = \frac{1}{(x-2) + ce^x}$$

$$\therefore y = \pm \sqrt{\frac{1}{(x-2) + ce^x}}$$

which is the general solution of the given equation.

5. Solve the follwing equation $(x^2 - 4xy - 2y^2)dx + (y^2 - 4xy - 2x^2)\,dy = 0$

Answer to - 5

Given that,

$$(x^2 - 4xy - 2y^2)dx + (y^2 - 4x - 2x^2)dy = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-x^2 - 4xy - 2y^2}{y^2 - 4xy - 2x^2} \cdots (i)$$

Equation(i) is a homogenius equation.

Let, $y = vx$

$$\frac{dy}{dx} = 1 + x\frac{dv}{dx}$$

putting value in equation(i)

$$1 + x\frac{dv}{dx} = \frac{-x^2 - 4vx^2 - 2v^2x^2}{v^2x^2 - 4vx^2 - 2x^2}$$

$$\Rightarrow 1 + x\frac{dv}{dx} = \frac{-1 - 4v - 2v^2}{v^2 - 4v - 2}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{-1 - 4v - 2v^2 - v^2 + 4v + 2}{v^2 - 4v - 2}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{3 - 3v^2}{v^2 - 4v - 2}$$

$$\Rightarrow \frac{1}{x}dx = -\frac{1}{3} \times \frac{v^2 - 4v - 2}{1 + v^2}dv$$

$$\Rightarrow \int \frac{1}{x}dx = -\frac{1}{3}\int \frac{v^2 - 4v - 2}{v^2 - 1}dv$$

$$\Rightarrow \int \frac{1}{x}dx = -\frac{1}{3}\int\left(\frac{-4v + 1}{v^2 - 1}\right)dv$$

$$\Rightarrow \int \frac{1}{x}dx = -\frac{1}{3}\int -\frac{4v + 1}{v^2 - 1}dv - \frac{1}{3}\int 1\,dv$$

$$\Rightarrow \ln x + c = -\frac{1}{3}2\ln(v^2 - 1) - \frac{1}{6}\ln\frac{1 - v}{1 + v} + \frac{1}{3}v$$

$$\Rightarrow 6\ln x + 6c = 4\ln(v^2 - 1) - \ln\frac{1 - v}{1 + v} + 2v$$

$$\therefore A = 4\ln\left(\frac{y^2}{x^2}-1\right) - \ln \frac{1-\frac{y}{x}}{1+\frac{y}{x}} + 2\frac{y}{x} - 6\ln x$$

6 Solve the following equations $\cos x(\cos x - \sin a \sin y)\,dx + \cos y(\cos y - \sin a \sin x)\,dy = 0$

Answer to - 6

Given that,

$$\cos x(\cos x - \sin a \sin y)\,dx + \cos y(\cos y - \sin a \sin x)\,dy = 0 \quad \dots (i)$$

comparing eauation(i), with

$$M dx + N dy = 0$$

Here,

$$M = \cos x(\cos x - \sin a \sin y)$$
$$= \cos^2 x - \sin a \cos x \sin y$$

$$N = \cos y(\cos y - \sin a \sin x)$$
$$= \cos^2 y - \sin a \cos y \sin x$$

$$\therefore \frac{\partial M}{\partial y} = -\sin a \cos x \cos y$$

$$\frac{\partial N}{\partial x} = -\sin a \cos x \cos y$$

Therefore $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, so the given differential equation is exact.

Now intregating M with respect to x, keeping y is a constant, then we have

$$\int M dx = \int (\cos x - \sin a \sin y \cos x) dx$$

$$= \int \cos^2 x dx - \int \sin a \sin y \cos x dx$$

$$= \frac{x}{2} + \frac{1}{4}\sin 2x - \sin a \sin y \sin x$$

In 'N' terms free from x is $\cos^2 y$ intregretting it with respect to y.

$$\int N dy = \int \cos^2 y \, dy$$

$$= \frac{y}{2} + \frac{1}{4}\sin^2 y$$

General eq^n is $\frac{x}{2} + \frac{1}{4}\sin 2x - \sin a \sin y \sin x$

$$+ y/2 + \frac{1}{4}\sin 2y = 0$$

$\therefore$ $2x + \sin 2x - 4 \sin a \sin y \sin x + 2y + \sin 2y = 0$

(Ans)