

## Chapter-1 (co-ordinates)

Formula:

1. Area of a polygon with vertices  $(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4), \dots (x_n, y_n)$  is

$$\text{Area} = \frac{1}{2} \begin{vmatrix} x_1 & x_2 & x_3 & x_4 & \dots & x_n & x_1 \\ y_1 & y_2 & y_3 & y_4 & \dots & y_n & y_1 \end{vmatrix}$$

when,

Area = 0 ; the points lie on the same straight line.

2. Condition to be collinear of the points  $(x_1, y_1), (x_2, y_2), (x_3, y_3)$  is

$$\text{Area} = \frac{1}{2} \begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix} = 0$$

Note: taking all points in the Anti-clockwise direction

method-2:  $\triangle ABC = \frac{1}{2} S_{ABC}$

$$= \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

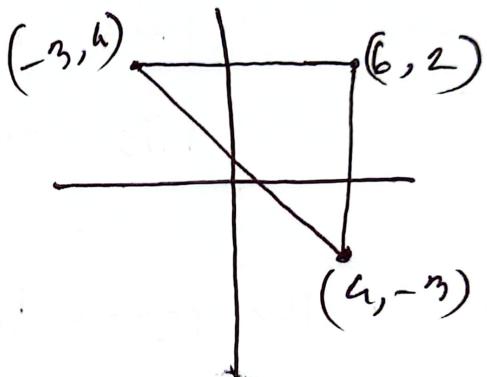
## Math (CL):

Q.2 Find the area of the triangle with vertices

(a)  $(-3, 4), (6, 2), (4, -3)$

(b)  $(a, b+c), (b, a+c), (c, a+b)$

Q.2(a): Sol: Rearranging these points in anti-clockwise direction



$$\therefore \text{Area} = \frac{1}{2} \begin{vmatrix} 6 & -3 & 4 \\ 2 & 4 & -3 \end{vmatrix}$$

$$= \frac{1}{2} \left\{ (24 + 7 - 8) - (-6 + 26 - 18) \right\}$$

$$= 4 \text{ sq. units.}$$

Q.2(b):

$$\text{Area of the given points} = \frac{1}{2} \begin{vmatrix} a & b & c & a \\ b+c & a+c & a+b & b+c \end{vmatrix}$$

$$= \frac{1}{2} \left\{ (a\sqrt{+}ac+ab+b\sqrt{+}bc+c\sqrt{)} - (b\sqrt{+}bc+ac+c\sqrt{+}ab) \right\}$$

$$= 0 \text{ sq. units.}$$

Q.2:  $(-1, 2), (2, -1), (h, 3)$  are collinear, show that

$$h = -2$$

Sol<sup>n</sup>:  $(-1, 2), (2, -1), (h, 3)$  since these points are collinear, hence area  $\equiv 0$

$$\therefore \text{Area} = \frac{1}{2} \begin{vmatrix} -1 & 2 & h & -1 \\ 2 & -1 & 3 & 2 \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{2} \left\{ (1+6+2h) - (4-h-3) \right\} = 0$$

$$\Rightarrow 3h + 6 = 0$$

$$\therefore h = -2 \quad (\text{Ans showed})$$

Q.3: For what value of  $k$  are the points  $(2, 3)$ ,  $(-4, -6)$  and  $(k, 12)$  collinear?

Sol<sup>n</sup>: since these points are collinear, area  $\equiv 0$

$$\text{Area} = \frac{1}{2} \begin{vmatrix} 2 & 4 & k & 2 \\ 3 & -6 & 12 & 3 \end{vmatrix}$$

$$\Rightarrow \frac{1}{2} \left\{ (-12 - 48 + 3k) - (-12 - 6k + 24) \right\}$$

$$\Rightarrow (-60 + 3k - 12 + 6k) = 0$$

$$\therefore k = 8 \quad (\text{Ans})$$

Q.5 If the area of the quadrilateral, whose angular points A, B, C, D taken in order are  $(2, 2)$ ,  $(-5, 6)$   $(7, -4)$  and  $(k, -2)$  be zero, find k?

Soln: since these points are quadrilateral,  
area = 0.

$$\text{Area} = \frac{1}{2} \begin{vmatrix} 2 & -5 & 7 & k & 2 \\ 2 & 6 & -4 & -2 & 2 \end{vmatrix} = 0$$

$$\text{or, } \frac{1}{2} \left\{ (6+20 - 19 + 2k) - (-10 + 42 - 4k - 2) \right\} = 0$$

$$\therefore k = 3 \text{ (Ans)}.$$

Q.7 Show that the three points  $(4, 2)$ ,  $(7, 5)$ ,  $(0, 7)$  lie on a right line.

Soln: Area of three points  $= \frac{1}{2} \begin{vmatrix} 4 & 7 & 0 & 4 \\ 2 & 5 & 7 & 2 \end{vmatrix}$

$$= \frac{1}{2} \left\{ (20 + 40) + 28 - (24 + 25 + 28) \right\}$$

$$= 0$$

Since, Area = 0, three points lie on a right line.

## Math (H.W.):

Q.4: Find the area of the quadrilateral whose angular points are....

- (a)  $(2, 2), (3, 4), (5, -2), (4, -7)$
- (b)  $(1, 2), (6, 2), (5, 3), (3, 4)$
- (c)  $(2, 4), (3, 2), (4, 1), (7, 6)$

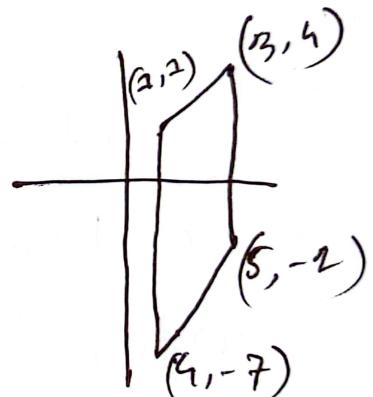
Soln.

— (a) Rearranging these points in anti-clockwise direction,

$$\text{Area} = \frac{1}{2} \begin{vmatrix} 3 & 1 & 4 & 5 & 3 \\ 4 & 2 & -7 & -2 & 4 \end{vmatrix}$$

$$= \frac{1}{2} \left\{ (13 - 7 - 8 + 20) - (4 + 4 - 15 - 6) \right\}$$

$$= 41/2 \text{ sq. units}$$

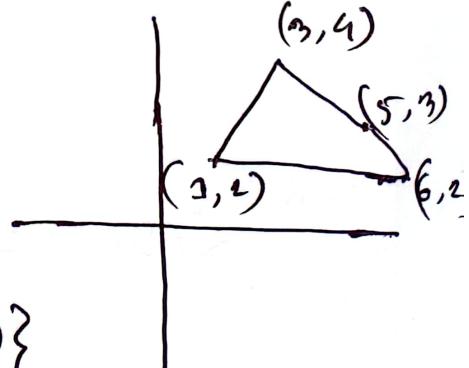


(b) Rearranging these points in anti-clockwise direction

$$\text{Area} = \frac{1}{2} \begin{vmatrix} 6 & 5 & 3 & 1 & 6 \\ 2 & 3 & 4 & 2 & 2 \end{vmatrix}$$

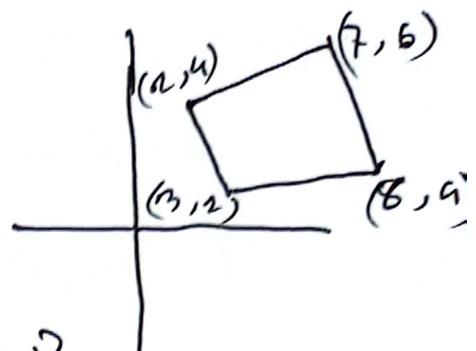
$$= \frac{1}{2} \left\{ (28 + 20 + 6 + 2) - (10 + 9 + 4 + 12) \right\}$$

$$= 17/2 \text{ sq. units.}$$



Q(c) Rearranging these points in anti-clockwise direction

$$\text{Area} = \frac{1}{2} \begin{vmatrix} 8 & 7 & 2 & 3 & 8 \\ 4 & 6 & 4 & 2 & 4 \end{vmatrix}$$



$$= \frac{1}{2} \left\{ (48 + 28 + 4 + 12) - (28 + 12 + 12 + 16) \right\}$$

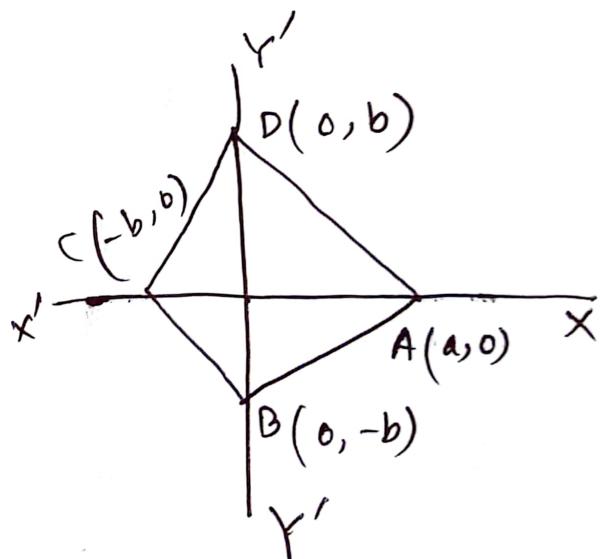
$$= 12 \text{ sq. units}$$

Q.6 Show that the area of the quadrilateral whose vertices are  $(a, 0)$ ,  $(-b, b)$ ,  $(0, -b)$ ,  $(a, b)$  is zero,  $(a, b > 0)$ . Explain the result with a diagram.

prof: Let the points be  
 $A(a, 0)$ ,  $B(0, -b)$ ,  $C(-b, 0)$  and  
according to order D will be  $(0, b)$

$$\text{Area } \triangle ABC = \frac{1}{2} \begin{vmatrix} a & 0 & -b & a \\ 0 & -b & 0 & 0 \end{vmatrix}$$

$$= \frac{1}{2} (-ab - b^2)$$



and  $\triangle ADC = \frac{1}{2} \begin{vmatrix} a & 0 & -b & a \\ 0 & b & 0 & 0 \end{vmatrix}$

$$= \frac{1}{2} (ab + b^2)$$

$$\therefore \text{Area of quadrilateral } ABCD = \triangle ABC + \triangle ADC \\ = 0 \quad (\text{showed})$$

Q.8: Find the areas of the polygons whose vertices are given

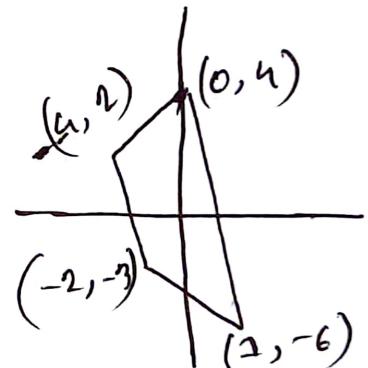
(a)  $(0, 4), (-1, -6), (-2, -3), (-4, 2)$

(b)  $(1, 5), (-2, 4), (-3, -2), (2, -3), (5, 1)$

(c)  $(1, 3), (4, 2), (5, 3), (3, 2), (2, 4)$

Soln: (a) Rearranging these points in anti-clockwise direction,

$$\text{Area} = \frac{1}{2} \begin{vmatrix} 0 & -4 & -2 & 1 & 0 \\ 4 & 2 & -3 & -6 & 4 \end{vmatrix}$$



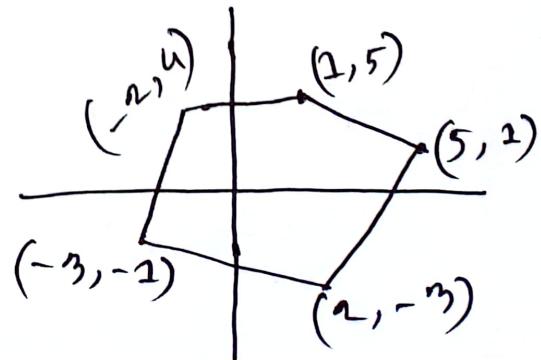
$$= \frac{1}{2} \left\{ (0+12+12+4) - (-16-4-3-0) \right\}$$

$$= 53/2 \text{ sq. units}$$

(b) Rearranging these points in anti-clockwise direction

$$\text{Area} = \frac{1}{2} \begin{vmatrix} 5 & 1 & -2 & -3 & 2 & 5 \\ 1 & 5 & 4 & -1 & -3 & 1 \end{vmatrix}$$

$$= \frac{1}{2} \left\{ (25+9+2+9+2) - \right.$$



$$(1-10-12-2-15) \}$$

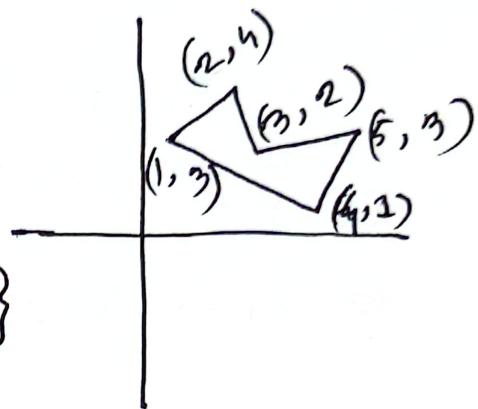
= 90 sq. units (Ans)

8(c) Rearranging these points in anti-clockwise direction

$$\text{Area} = \frac{1}{2} \begin{vmatrix} 5 & 3 & 2 & 1 & 4 & 5 \\ 3 & 2 & 4 & 3 & 1 & 3 \end{vmatrix}$$

$$= \frac{1}{2} \left\{ (20+12+6+2+12) - (9+4+4+12+5) \right\}$$

= 7/2 sq. units (Ans)



## Chapter - 2 (Polar Co-ordinates)

Formula:

$$\textcircled{1} \quad r = r \cos \theta \quad r = \sqrt{x^2 + y^2}$$

$$y = r \sin \theta \quad \theta = \tan^{-1} \left( \frac{y}{x} \right)$$

\textcircled{2} Distance between two points  $(x_1, y_1)$  and  $(x_2, y_2)$  is  $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

③ Distance between points  $A(r_1, \theta_1)$  and  $B(r_2, \theta_2)$

$$\text{is} - \sqrt{r_1^2 + r_2^2 - 2 r_1 r_2 \cos(\theta_2 - \theta_1)}$$

④ Area of the triangle with vertices  $A(r_1, \theta_1)$ ,  $B(r_2, \theta_2)$  and  $C(r_3, \theta_3)$  is -

$$\Delta ABC = \frac{1}{2} r_1 r_2 \sin(\theta_2 - \theta_1) + \frac{1}{2} r_2 r_3 \sin(\theta_3 - \theta_2)$$

$$+ \frac{1}{2} r_3 r_1 \sin(\theta_3 - \theta_1)$$

Shortcut:

$$1. (x, y) \rightarrow r = \sqrt{x^2 + y^2}, \theta = \tan^{-1} \left| \frac{y}{x} \right|$$

$$2. (-x, y) \rightarrow r = \sqrt{x^2 + y^2}, \theta = \pi - \tan^{-1} \left| \frac{y}{x} \right|$$

$$3. (-x, -y) \rightarrow r = \sqrt{x^2 + y^2}, \theta = \pi + \tan^{-1} \left| \frac{y}{x} \right|$$

$$4. (x, -y) \rightarrow r = \sqrt{x^2 + y^2}, \theta = 2\pi - \tan^{-1} \left| \frac{y}{x} \right|$$

# Conditions: of three point will form a triangle  
a, b and c

$$a+b > c$$

$$b+c > a$$

$$a+c > b$$

Q.1: Find the polar co-ordinates of the following points:

$$\text{① } (\sqrt{3}, 1), \text{ ② } (-1, 1), \text{ ③ } (\sqrt{3}, -1), \text{ extra } (-1, -1)$$

Soln.: 2(i)  $(\sqrt{3}, 1)$ , Given that,

$$x = \sqrt{3}$$

$$y = 1$$

We know,

$$r = \sqrt{x^2 + y^2} = \sqrt{(\sqrt{3})^2 + (1)^2} = 2$$

$$\text{and } \theta = \tan^{-1} \left| \frac{y}{x} \right| = \frac{\pi}{6}$$

∴ the required polar form is  $(2, \frac{\pi}{6})$

2(ii) Given that,

$$x = -1$$

$$y = 1$$

We know,

$$r = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + (1)^2} = \sqrt{2}$$

$$\text{and, } \theta = \pi - \tan^{-1} \left| \frac{y}{x} \right| = \pi - \tan^{-1} \left| \frac{1}{-1} \right| = \frac{3\pi}{4}$$

∴ the required polar form is  $(\sqrt{2}, \frac{3\pi}{4})$

2(iii) are given

We know that,

$$x = \sqrt{3},$$

$$y = -1$$

We know,

$$r = \sqrt{x^2 + y^2} = \sqrt{(\sqrt{3})^2 + (-1)^2} = 2$$

and,

$$\theta = 2\pi - \tan^{-1} \left| \frac{-1}{\sqrt{3}} \right|$$

$$= \frac{11\pi}{6}$$

$\therefore$  the required polar form is  $(2, \frac{11\pi}{6})$

2(extra)

We are given that,

$$x = -1, y = -1$$

We know that,

$$r = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$$

and,

$$\theta = \pi + \tan^{-1} \left| \frac{-1}{-1} \right|$$

$$= \frac{5\pi}{4}$$

$\therefore$  the required polar form is  $(\sqrt{2}, \frac{5\pi}{4})$

Convert into cartesian form...

(i)  $(\sqrt{2}, 5\pi/4)$

We know that,  $x = r \cos \theta$   
 $y = r \sin \theta$

$$\begin{array}{l|l} x = \sqrt{2} \times \cos 5\pi/4 & y = r \sin \theta \\ = -1 & = \sqrt{2} \times \sin 5\pi/4 \\ & = -1 \end{array}$$

∴ The required cartesian form is  $(-1, -1)$

Q.2 Find the polar distance of the co-ordinate of the points.

①  $(3, 45^\circ)$  and  $(7, 105^\circ)$

②  $(\sqrt{2}, 5\pi/4)$  and  $(2, 2\pi/3)$

Soln.: ① We are given that,

$$r_1 = 3, r_2 = 7, \theta_1 = 45^\circ, \theta_2 = 105^\circ$$

We know,

$$D = \sqrt{r_1^2 + r_2^2 - 2r_1r_2 \cos(\theta_2 - \theta_1)}$$

$$= \sqrt{3^2 + 7^2 - 2 \cdot 3 \cdot 7 \cdot \cos(105 - 45)}$$

$$= \sqrt{37} \text{ units.}$$

$\therefore$  Polar distance is  $\sqrt{37}$  units

(ii) We are given that,

$$\begin{array}{l|l} r_1 = \sqrt{2} & \theta_1 = 5\pi/4 \\ r_2 = 2 & \theta_2 = 2\pi/3 \end{array}$$

We know,

$$\text{distance} = \sqrt{r_1^2 + r_2^2 - 2r_1r_2 \cos(\theta_2 - \theta_1)}$$

$$= \sqrt{(\sqrt{2})^2 + 2^2 - 2 \cdot \sqrt{2} \cdot 2 \cdot \cos\left(\frac{2\pi}{3} - \frac{5\pi}{4}\right)}$$

$$= 2.732$$

$\therefore$  Polar distance is 2.732 units

Q.3 Determine the nature of the triangle whose vertices  $(0,0)$ ,  $(3, \frac{\pi}{2})$ ,  $(3, \frac{\pi}{6})$  and calculate its area.

Soln: We are given that,

$$A(0,0), B(3, \frac{\pi}{2}), C(3, \frac{\pi}{6})$$

$$AB = \sqrt{0^2 + 3^2 - 2 \cdot 3 \cdot 0 \cdot \cos\left(\frac{\pi}{6} - 0\right)} = 3 \text{ units}$$

$$BC = \sqrt{3^2 + 3^2 - 2 \cdot 3 \cdot 3 \cos\left(\frac{\pi}{6} - \frac{\pi}{2}\right)} = 3 \text{ units}$$

$$AC = \sqrt{3^2 + 0^2 - 2 \cdot 3 \cdot 0 \cdot \cos\left(0 - \frac{\pi}{6}\right)} = 3 \text{ units.}$$

Since,

$AB = BC = AC$ ; this is an equilateral triangle.

$$\begin{aligned} \text{Area, } &= \frac{1}{2} r_1 r_2 \sin(\theta_2 - \theta_1) + \frac{1}{2} r_2 r_3 \sin(\theta_3 - \theta_2) \\ &\quad + \frac{1}{2} r_3 r_1 \sin(\theta_1 - \theta_3) \end{aligned}$$

$$\begin{aligned} &= 0 + \frac{1}{2} \times 3 \times 3 \times \sin\left(\frac{\pi}{6} - \frac{\pi}{2}\right) + 0 \\ &= -\frac{9\sqrt{3}}{4}; \text{ Area cannot be negative} \end{aligned}$$

$$\therefore \text{Area} = \frac{9\sqrt{3}}{4} \text{ sq. units.}$$

## Math (H.W.)

Q.4 Find the areas of the triangle the polar co-ordinates of whose angular points are ...

i)  $(3, 30^\circ), (5, 150^\circ), (7, 210^\circ)$

ii)  $(a, \theta), (2a, \theta + \pi/3), (3a, \theta + 2\pi/3)$

Soln.: i) We are given that,

$$\left. \begin{array}{l} r_1 = 3 \\ r_2 = 5 \\ r_3 = 7 \end{array} \right| \left. \begin{array}{l} \theta_1 = 30^\circ \\ \theta_2 = 150^\circ \\ \theta_3 = 210^\circ \end{array} \right.$$

$$\therefore \text{Area} = \frac{1}{2} r_1 r_2 \sin(\theta_2 - \theta_1) + \frac{1}{2} r_2 r_3 \sin(\theta_3 - \theta_2)$$

$$+ \frac{1}{2} r_3 r_1 \sin(\theta_3 - \theta_1)$$

$$= \frac{1}{2} \left\{ 3 \times 5 \times \sin(150 - 30) + 5 \times 7 \times \sin(210 - 150) + 7 \times 3 \times \sin(210 - 30) \right\}$$

$$= \frac{25\sqrt{3}}{2} \text{ sq. units}$$

$$\therefore \text{Area} = \frac{25\sqrt{3}}{2} \text{ sq. units.}$$

⑪ We are given that,

$$\begin{array}{l} r_1 = a \\ r_2 = 2a \\ r_3 = 3a \end{array} \quad \left| \begin{array}{l} \theta_2 - \theta_1 = \left(\theta + \frac{\pi}{3}\right) - \theta = \frac{\pi}{3} \\ \theta_3 - \theta_2 = \left(\theta + \frac{2\pi}{3}\right) - \left(\theta + \frac{\pi}{3}\right) = \frac{\pi}{3} \\ \theta_3 - \theta_1 = \left(\theta + \frac{2\pi}{3}\right) - \theta = \frac{2\pi}{3} \end{array} \right.$$

$$\therefore \text{Area} = \frac{1}{2} \left| r_1 r_2 \sin(\theta_2 - \theta_1) + r_2 r_3 \sin(\theta_3 - \theta_2) + r_1 r_3 \sin(\theta_3 - \theta_1) \right|$$

$$= \frac{1}{2} \left\{ |axax \times \sin \frac{\pi}{3} + 2ax^3a \times \sin \left(\frac{\pi}{3}\right) + 3axax \times \sin \frac{2\pi}{3}| \right\}$$

$$= \frac{\sqrt{3}}{2 \times 2} | 2a^2 + 6a^2 + 3a^2 |$$

$$= \frac{11\sqrt{3} a^2}{4}$$

$$\therefore \text{Area} = \frac{11\sqrt{3} a^2}{4} \text{ sq. units.}$$

Q.5 change the equation to polar co-ordinates.

$$\text{i) } (x^{\sqrt{}} + y^{\sqrt{}})^{\sqrt{}} = 2a^{\sqrt{}}xy \quad \text{ii) } r^4 + r^{\sqrt{}}y^{\sqrt{}} - (r + y)^{\sqrt{}} = 0$$

Soln ① We know that,

$$\begin{array}{l|l} x = r\cos\theta & r^{\sqrt{}} = \sqrt{x^2 + y^2} \\ y = r\sin\theta & \end{array}$$

$$\therefore (r^{\sqrt{}} + y^{\sqrt{}})^{\sqrt{}} = 2a^{\sqrt{}}xy$$

$$\text{or, } r^4 = 2a^{\sqrt{}}r\cos\theta \cdot r\sin\theta \quad [\text{putting value of } x, y, r^{\sqrt{}}]$$

$$\text{or, } r^4 = r^{\sqrt{}}a^{\sqrt{}}2\sin\theta\cos\theta$$

$$\text{or, } r^{\sqrt{}} = a^{\sqrt{}}\sin 2\theta \quad (\text{Ans})$$

② We know that,

$$\begin{array}{l|l} x = r\cos\theta & r^{\sqrt{}} = \sqrt{x^2 + y^2} \\ y = r\sin\theta & \end{array}$$

$$\text{Now, } \cancel{\text{or, }} r^4 + r^{\sqrt{}}y^{\sqrt{}} - (r + y)^{\sqrt{}} = 0$$

$$\text{or, } r^4 + r^{\sqrt{}}y^{\sqrt{}} = (r + y)^{\sqrt{}}$$

$$\text{or, } r^{\sqrt{}}(r^{\sqrt{}} + y^{\sqrt{}}) = r^{\sqrt{}}(\cos\theta + \sin\theta)^{\sqrt{}}$$

$$\text{or, } r^{\checkmark} \cos \theta \cdot r^{\checkmark} = r^{\checkmark} (\cos \theta + \sin \theta)^{\checkmark}$$

$$\text{or, } r^{\checkmark} = \left( \frac{\cos \theta + \sin \theta}{\cos \theta} \right)^{\checkmark}$$

$$\text{or, } r^{\checkmark} = (1 + \tan \theta)^{\checkmark}$$

$$\therefore r = \pm (1 + \tan \theta) \quad (\text{Any})$$

### Chapter - 3 (The Straight Line)

#### Formulae:

① Equation of st. line . .

i) parallel to X axis is

$$y = \text{constant (say } b)$$

ii) parallel to Y axis is

$$x = \text{constant (say } a)$$

②  $y = mx + c$ ,  $m = \tan \theta$  → slope  
gradient  
tan

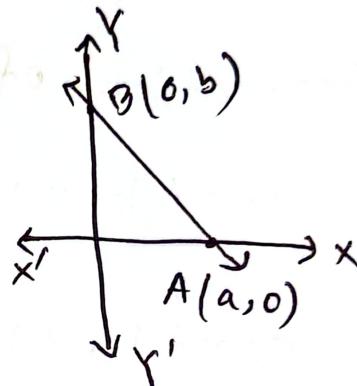
$$\textcircled{3} \quad \frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$$

$$\textcircled{4} \quad y - y_1 = m(x - x_1)$$

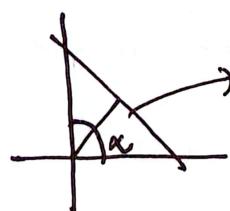
↳ Eqn of st. line passing through a point  $(x_1, y_1)$  whose slope is  $m$ .

$$\textcircled{5} \quad \frac{x}{a} + \frac{y}{b} = 1$$

↳ Intercept form  
(वर्तनी रूप समीकरण)



$$\textcircled{6} \quad x \cos \alpha + y \sin \alpha = p$$



$$x \cos \alpha + y \sin \alpha = p$$

here,  
 $p$  = perpendicular distance from origin  $O(0,0)$   
(अक्षांश)

$$\textcircled{7} \quad \frac{x - x_1}{\cos \theta} = \frac{y - y_1}{\sin \theta} = r$$

$$\textcircled{8} \quad x = x_1 + r \cos \theta, \quad y = y_1 + r \sin \theta$$

⑨ Eq<sup>n</sup> of st. line parallel to ...

$$ax+by+c=0 \text{ is } \rightarrow ax+by+c_1=0$$

⑩ Eq<sup>n</sup> of st. line perpendicular to ...

$$ax+by+c=0 \text{ is } bx-ay+c_1=0$$

\*1 Eq<sup>n</sup> of a st. line passing through the point of intersection of the given two lines ...

$$a_1x+b_1y+c_1=0 \text{ and } a_2x+b_2y+c_2=0 \text{ is}$$

$$a_1x+b_1y+c_1+k(a_2x+b_2y+c_2)=0$$

\*2 the length of perpendicular line from a point  $(x_1, y_1)$  on the line  $ax+by+c=0$  is

$$d = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

\*3 Eq<sup>n</sup> of the bisector (समद्विभाजक) of the two lines  $a_1x+b_1y+c=0$  and  $a_2x+b_2y+c_2=0$  is

$$\frac{|a_1x+b_1y+c_1|}{\sqrt{a_1^2+b_1^2}} = \pm \frac{|a_2x+b_2y+c_2|}{\sqrt{a_2^2+b_2^2}}$$

\*4 Condition for concurrency (सम्भागित) of the lines  $a_1x+b_1y+c_1=0$ ,  $a_2x+b_2y+c_2=0$  and  $a_3x+b_3y+c_3=0$  is

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

\*5 Area of a triangle formed with the three lines  $a_1x+b_1y+c_1=0$ ,  $a_2x+b_2y+c_2=0$  and  $a_3x+b_3y+c_3=0$  is

$$\frac{1}{2} \cdot \frac{\Delta}{C_1 C_2 C_3}$$

Here,

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$C_1 = \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

$$C_2 = \begin{vmatrix} a_1 & b_1 \\ a_3 & b_3 \end{vmatrix}$$

$$C_3 = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

## Math (CL):

Exp - 1: Find the Eq<sup>n</sup> to lines passing through (-5, 6) and (i) parallel (ii) perpendicular to the line  $7x - 8y = 9$

Sol<sup>n</sup>: Given line  $7x - 8y - 9 = 0 \dots (i)$

Now, Eq<sup>n</sup> of a parallel line to ..(i) is  $7x - 8y + k = 0$  which passes through the point (-5, 6) then -

$$7(-5) - 8 \times 6 + k = 0$$

$$\therefore k = 83$$

Hence (i)  $\Rightarrow 7x - 8y + 83 = 0$  (Ans)

and

(ii) Eq<sup>n</sup> of a perpendicular line to ..(i) is

$8x - 7y + k = 0 \dots (ii)$  which passes through the point (-5, 6) then -

$$8(-5) - 7 \cdot 6 + k = 0$$

$$\therefore k = 82$$

Hence, (ii)  $\Rightarrow 8x - 7y + 82 = 0$  (Ans)

(22)

Q.2 Find the equation of st. line passes through the intersection of the lines  $3x - 5y + 9 = 0$  and  $4x + 7y - 28 = 0$  and satisfies and following condition;

- (a) passes through  $(4, 2)$
- (b) Is parallel to  $2x + 3y - 5 = 0$
- (c) Is perpendicular to  $4x + 5y - 20 = 0$
- (d) whose intercepts are equal.

Soln.: Given that,

$$3x - 5y + 9 = 0$$

$$4x + 7y - 28 = 0$$

The required line:

$$3x - 5y + 9 + k(4x + 7y - 28) = 0 \quad \dots \text{(i)}$$

$$\rightarrow (3+4k)x + (-5+7k)y + 9 - 28k = 0 \quad \dots \text{(ii)}$$

(a) Since eqn (ii) passes through  $(4, 2)$ , then

$$(3+4k) \cdot 4 + (-5+7k) \cdot 2 + 9 - 28k = 0$$

$$\therefore k = -\frac{11}{2}$$

putting  $k = -\frac{17}{2}$  in eqn(ii), we get,

$$\left(-3 - \frac{49}{2}\right)x + \left(-5 - \frac{77}{2}\right)y + 7 + 154 = 0$$

$$\Rightarrow -19x - \frac{87}{2}y + 163 = 0$$

$$\therefore 38x + 87y - 326 = 0 \quad (\text{Ans})$$

(b) slope of eqn(ii),  $m_1 = -\frac{3+4k}{7k-5}$

slope of  $2x+5y-5=0$ ,  $m_2 = -\frac{2}{3}$

Since, they are parallel so,

$$m_1 = m_2$$

$$\frac{3+4k}{7k-5} = -\frac{2}{3}$$

$$\Rightarrow 9+12k = 14k - 10$$

$$\therefore k = \frac{19}{2}$$

putting  $k = \frac{19}{2}$  in eqn(ii),

$$(3+38)x + \left(\frac{7 \times 19}{2} - 5\right)y + 7 - \frac{28 \times 19}{2} = 0$$

$$\Rightarrow \frac{91}{2}x + \frac{123}{2}y - 257 = 0$$

$$\therefore 82x + 123y - 514 = 0 \quad (\text{Ans})$$

⑥ Slope of eqn (ii),  $m_2 = -\frac{3+4k}{7k-5}$

Slope of  $4x + 5y - 20$ ,  $m_2 = -\frac{4}{5}$

since, they are  $\perp$ , so  
 $m_1 \times m_2 = -1$

or  $\frac{3+4k}{7k-5} \times \frac{4}{5} = -1$

or,  $\frac{12+16k}{35k-25} = -1$

or,  $k = 13/51$

putting  $k = \frac{13}{51}$  in eqn (ii),

$$\left(3 + \frac{4 \times 13}{51}\right)x + \left(\frac{7 \times 13}{51} - 5\right)y + 9 - \frac{28 \times 13}{51} = 0$$

or,  $\frac{205}{51}x - \frac{164}{51}y + \frac{95}{51} = 0$

$\therefore 205x - 164y + 95 = 0$  (Ans)

⑦ From (ii),  
 $(3+4k)x + (7k-5)y = 28k-9$

$$\Rightarrow \frac{(3+4K)}{28K-9}x + \frac{7K-5}{28K-9}y = 1$$

$$\Rightarrow \frac{x}{\frac{28K-9}{3+4K}} + \frac{y}{\frac{28K-9}{7K-5}} = 1$$

Since intercepts are equal, So.

$$\frac{28K-9}{3+4K} = \frac{28K-9}{7K-5} \neq 0$$

$$\text{Or, } (28K-9) \left\{ \frac{1}{3+4K} - \frac{1}{7K-5} \right\} = 0$$

here,

$$28K-9 = 0$$

$$\therefore K = 9/28$$

$$\text{Or, } \frac{1}{3+4K} = \frac{1}{7K-5}$$

$$\text{Or, } K = 8/3$$

Now, putting  $K = 9/28$  and  $K = 8/3$  in eqn(ii),

$$(3 + \frac{9 \times 9}{28})x + \left( \frac{7 \times 9}{28} - 5 \right)y + 9 - \frac{28 \times 9}{28} = 0$$

$$\text{Or, } \frac{30}{7}x - \frac{21}{4}y = 0$$

$$\therefore 120n - 77y = 0 \text{ (Ans)}$$

again,

$$\left(3 + \frac{4 \times 8}{3}\right)n + \left(\frac{7 \times 8}{3} - 5\right)y + n - \frac{28 \times 8}{3} = 0$$

$$\text{or, } 41n + 41y - 197 = 0 \text{ (Ans)}$$

Ex-2 Determine the eq<sup>n</sup> of the bisectors of the angle bet<sup>n</sup> the lines  $3n - 4y + 12 = 0$  and  $12n + 5y - 30 = 0$

Sol<sup>n</sup>: Given lines:  $3n - 4y + 12 = 0 \dots \text{(i)}$   
 $12n + 5y - 30 = 0 \dots \text{(ii)}$

Eq<sup>n</sup> of bisector of the angle bet<sup>n</sup> these two lines are

$$\frac{3n - 4y + 12}{\sqrt{37}} = \pm \frac{12n + 5y - 30}{\sqrt{144 + 25}}$$

taking '+'

$$13(3n - 4y + 12) = 5(12n + 5y - 30)$$

$$\text{or, } 37n - 52y + 156 = 60n + 25y - 150$$

$$\therefore 21n + 77y = 306 \text{ (Ans)}$$

taking (-)

$$13(3n - 4y + 12) = -5(12n + 5y - 30)$$

$$\text{or, } 99n - 27y + 6 = 0$$

$$\therefore 33n - 9y + 2 \equiv 0 \text{ (Ans)}$$

Math(H.W.)

Ex. 3 Find the area of the triangle formed by the lines  $2n+y-3=0$ ,  $3n+2y-1=0$  and  $2n+3y+4=0$

Soln: We are given that,

$$2n+y-3=0$$

$$3n+2y-1=0$$

$$2n+3y+4=0$$

We know

$$\text{Area} a = \frac{1}{2} \left| \begin{matrix} 1 & 1 & -3 \\ 3 & 2 & -1 \\ 2 & 3 & 4 \end{matrix} \right|$$

here,

$$\Delta = \left| \begin{matrix} 2 & 1 & -3 \\ 3 & 2 & -1 \\ 2 & 3 & 4 \end{matrix} \right| = -7$$

$$C_1 = \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix}, \quad C_2 = \begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix}, \quad C_3 = \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix}$$

$= 5$                      $= 4$                      $= 1$

$$\therefore \text{Area} = \frac{\Delta}{C_1 C_2 C_3} \times \frac{1}{2}$$

$$= \frac{(-7)}{5 \times 4 \times 1} \times \frac{1}{2} = \frac{49}{40} \text{ sq. units. (Ans)}$$

Q.3 If  $p$  and  $p_1$  be the perpendiculars from the origin upon the st. lines whose eqn are  $x \sec \theta + y \cosec \theta = a$  and  $x \cos \theta - y \sin \theta = a \cos 2\theta$  prove that  $p^2 + p_1^2 = a^2$

Soln: Given lines:

$$x \sec \theta + y \cosec \theta - a = 0$$

$$\text{and } x \cos \theta - y \sin \theta - a \cos 2\theta = 0$$

Now, the perpendicular distance  $p$  from the origin  $(0,0)$  to the line  ~~$a \sec \theta x + b \cosec \theta y + c = 0$~~   $a \sec \theta x + b \cosec \theta y + c = 0$  is

$$P = \frac{|a \sec \theta x + b \cosec \theta y + c|}{\sqrt{a^2 + b^2}}$$

For line,  $r \sec \theta + y \cosec \theta - a = 0$

$$P = \frac{|0 \cdot \sec \theta + 0 \cdot \cosec \theta - a|}{\sqrt{\sec^2 \theta + \cosec^2 \theta}}$$

$$= \frac{|a|}{\sqrt{\frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta}}}$$

$$= \frac{|a| \sqrt{\sin^2 \theta \cos^2 \theta}}{\sqrt{1}} = a \sin \theta \cos \theta = \frac{a \sin 2\theta}{2}$$

again,

for line  $r \cos \theta - y \sin \theta - a \cos 2\theta = 0$

$$P_1 = \frac{|0 \cdot \cos \theta - 0 \cdot (-\sin \theta) - a \cos 2\theta|}{\sqrt{\cos^2 \theta + (-\sin \theta)^2}}$$

$$= \frac{a \cos 2\theta}{1} = a \cos 2\theta$$

$$\text{L.H.S.} = 4P^2 + P_1^2$$

$$= 4 \cdot \frac{a^2 \sin^2 2\theta}{4} + a^2 \cos^2 2\theta$$

$$= a^2 (\sin^2 2\theta + \cos^2 2\theta)$$

$$= a^{\checkmark}$$

= R.H.S (proved)

Q.4 Show that the perpendicular let fall from any points on the st. line  $7x - 9y + 20 = 0$  upon the two lines  $3x + 4y - 5 = 0$  and  $12x + 5y - 7 = 0$  are equal to each other.

Soln.: Given lines:

$$7x - 9y + 20 = 0 \quad \dots (i)$$

$$3x + 4y - 5 = 0 \quad \dots (ii)$$

$$12x + 5y - 7 = 0 \quad \dots (iii)$$

Here, we consider the point  $(\alpha, \beta)$  which lies on eqn (i), then

$$7\alpha - 9\beta + 20 = 0$$

$$\text{or, } \beta = \frac{7\alpha + 20}{9} \quad \dots (iv)$$

Now, the  $\perp^r$  distance of eqn (ii) from  $(\alpha, \beta)$  is

$$d_2 = \frac{|3\alpha + 4\beta - 5|}{\sqrt{3^2 + 4^2}} \quad \dots \checkmark$$

and  $\perp^r$  distance of eqn(iii) from  $(\alpha, \beta)$  is

$$d_2 = \frac{|12\alpha + 5\beta - 7|}{\sqrt{12^2 + 5^2}} \dots (\text{vi})$$

passing  $\beta = \frac{7\alpha + 20}{9}$  into ⑦

$$\begin{aligned} d_2 &= \frac{|3\alpha + 4\left(\frac{7\alpha + 20}{9}\right) - 5|}{5} \\ &= \frac{|27\alpha + 28\alpha + 40 - 45|}{5 \times 9} \end{aligned}$$

$$\therefore d_2 = \frac{|11\alpha - 1|}{9}$$

passing  $\beta = \frac{7\alpha + 20}{9}$  into ⑥

$$\begin{aligned} d_2 &= \frac{|12\alpha + 5\left(\frac{7\alpha + 20}{9}\right) - 7|}{13} \\ &= \frac{|108\alpha + 35\alpha + 50 - 63|}{13} \end{aligned}$$

$$\therefore d_2 = \frac{|11\alpha - 1|}{9}$$

Since,  $d_1 = d_2 = \frac{|11\alpha - 1|}{9}$ , hence (showed)

Q.5 Find the condition that st. lines  $y = m_1 n + a_1$  and  $y = m_2 n + a_2$  and  $y = m_3 n + a_3$  may at a point

Soln.: Given lines :

$$m_1 n - y + a_1 = 0$$

$$m_2 n - y + a_2 = 0$$

$$m_3 n - y + a_3 = 0$$

hence,

$$\Delta = \begin{vmatrix} m_1 & -1 & a_1 \\ m_2 & -1 & a_2 \\ m_3 & -1 & a_3 \end{vmatrix}$$

$$= m_1(a_2 - a_3) + m_2(a_3 - a_1) + m_3(a_1 - a_2)$$

(Ans)

Ques. 6(i) Prove that diagonals of the parallelogram formed by the four st. lines.  $\sqrt{3}n + y = 0$ ,  $\sqrt{3}y + n = 0$ ,  $\sqrt{3}n + y = 1$  and  $\sqrt{3}y + n = 1$ .

Soln.: Given lines

$$\sqrt{3}n + y = 0 \quad \dots (i)$$

$$n + \sqrt{3}y = 0 \quad \dots (ii)$$

$$\sqrt{3}n + y = 1 \quad \dots (iii)$$

$$n + \sqrt{3}y = 1 \quad \dots (iv)$$

taking, eqn(i) and eqn(ii)

$$x = 0, y = 0$$

$\therefore$  ~~the~~ intersection point is A(0,0).

taking, eqn(i) and eqn(iv)

$$x = -\frac{1}{2}, y = \frac{\sqrt{3}}{2}$$

$\therefore$  intersection point is B $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

taking eqn(iii) and eqn(ii)

$$x = \frac{\sqrt{3}}{2}, y = -\frac{1}{2}$$

$\therefore$  intersection point C $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

taking,  
eqn(iii) and eqn(iv)

$$x = \left(\frac{\sqrt{3}-1}{2}\right), y = \left(\frac{\sqrt{3}-1}{2}\right)$$

$\therefore$  intersection point is D $\left(\frac{\sqrt{3}-1}{2}, \frac{\sqrt{3}-1}{2}\right)$

$$\text{slope of } AC = m_{AC} = \frac{\frac{\sqrt{3}-1}{2} - 0}{\frac{\sqrt{3}-1}{2} - 0} = 1$$

$$\text{slope of } BD = m_{BD} = \frac{\frac{\sqrt{3}}{2} - (-\frac{1}{2})}{-\frac{1}{2} - \frac{\sqrt{3}}{2}} = -1$$

$$\therefore m_{AC} \times m_{BD} = -1 \times 1 = 1 \text{ (proved)}$$

Q 10(a) Show that the area of the triangle formed by the lines whose eqn are  $y = m_1 n + c_1$ ,  $y = m_2 n + c_2$  and  $n=0$  is  $\frac{1}{2} |c_1 - c_2| / (m_2 - m_1)$

Soln.: Given lines:  $m_1 n - y + c_1 = 0$   
 $m_2 n - y + c_2 = 0$   
 $n + 0 \cdot y + 0 = 0$

$$\Delta = \begin{vmatrix} m_1 & -1 & c_1 \\ m_2 & -1 & c_2 \\ 1 & 0 & 0 \end{vmatrix}$$

$$= 1(-c_2 + c_1) + 0 + 0$$

$$= c_1 - c_2$$

$$C_1 = \begin{vmatrix} m_1 & -1 \\ 1 & 0 \end{vmatrix}; \quad C_2 = \begin{vmatrix} m_2 & -1 \\ 1 & 0 \end{vmatrix}; \quad C_3 = \begin{vmatrix} m_1 & -1 \\ m_2 & -1 \end{vmatrix}$$

$$= 1$$

$$= 1$$

$$= m_2 - m_1$$

$$\therefore \text{Area} = \frac{1}{2} \times \frac{d}{C_1 C_2 C_3} = \frac{1}{2} \times \frac{(c_2 - c_1)}{(m_2 - m_1)}$$

∴

$$= \frac{1}{2} (c_2 - c_1) / (m_2 - m_1)$$

(showed)

Q. 10(b) Find the triangle's area formed by the lines  
 $x + y + 2 = 0$ ;  $2x - y - 3 = 0$  and  $3x + 2y - 5 = 0$

Soln: Given lines:  
 $x + y + 2 = 0$ . (i)  
 $2x - y - 3 = 0$  -- (ii)  
 $3x + 2y - 5 = 0$  . (iii)

Here,

$$D = \begin{vmatrix} 1 & 1 & 2 \\ 2 & -1 & -3 \\ 3 & 2 & -5 \end{vmatrix}$$

$$= 1(5+6) - 1(-10+7) + 2(4+3)$$

$$= 26$$

$$C_1 = \begin{vmatrix} 3 & 1 \\ 3 & 2 \end{vmatrix}; C_2 = \begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix}; C_3 = \begin{vmatrix} 2 & 1 \\ 2 & -2 \end{vmatrix}$$
$$= 7 \quad = -2 \quad = -3$$

$$\therefore \text{Area} = \frac{1}{2} \times \frac{\Delta^2}{c_1 c_2 c_3} = \frac{1}{2} \times \frac{26^2}{7 \cdot (-1) \cdot (-3)}$$

$$= \frac{338}{21} \text{ sq. units}$$

Q. 27 Prove that the three st. lines  $2x - 7y + 20 = 0$ ,  $3x - 2y - 1 = 0$ ,  $x - 12y + 21 = 0$  concur at a point

Soln.: Given lines are,

$$2x - 7y + 20 = 0 \dots (i)$$

$$3x - 2y - 1 = 0 \dots (ii)$$

$$x - 12y + 21 = 0 \dots (iii)$$

$$\text{eq}^n(i) \times 2 - \text{eq}^n(ii) \times 7$$

$$4x - 14y + 40 = 0$$

$$\begin{array}{r} (-) 2x - 14y - 7 = 0 \\ \hline (+) - 2x + 14y = 0 \\ \hline - 27y + 27 = 0 \end{array}$$

$$\therefore y = 27/17$$

putting,  $y = 27/17$  into  $\text{eq}^n(i)$ .

$$2\left(\frac{27}{17}\right) - 7y + 20 = 0$$

$$\therefore y = \frac{32}{17}$$

Now, putting  $\left(\frac{27}{17}, \frac{32}{17}\right)$  into eqn(iii)

$$\frac{27}{17} - 12\left(\frac{32}{17}\right) + 22 = 0$$

$$\text{or } 0 = 0$$

since, left side = right side, the point  $\left(\frac{27}{17}, \frac{32}{17}\right)$  satisfies the eqn(iii)

$\therefore$  the three lines concur at the point  $\left(\frac{27}{17}, \frac{32}{17}\right)$

Q.24 find the area of the triangle formed by the line  $8x+7y-43=0$ ;  $2x+5y=1$  and  $4x-3y+22=0$

Soln: Given lines;

$$8x+7y-43=0$$

$$2x+5y-1=0$$

$$4x-3y+22=0$$

Here

$$\Delta = \begin{vmatrix} 8 & 7 & -43 \\ 2 & 5 & -1 \\ 4 & -3 & 22 \end{vmatrix}$$

$$= 8(55-3) - 7(22+4) - 43(-6-20)$$

$$= 2352$$

Now,

$$C_1 = \begin{vmatrix} 2 & 5 \\ 4 & 3 \end{vmatrix}; \quad C_2 = \begin{vmatrix} 8 & 7 \\ 4 & 3 \end{vmatrix}; \quad C_3 = \begin{vmatrix} 8 & 7 \\ 2 & 5 \end{vmatrix}$$

$$= -26 \qquad \qquad \qquad = -52 \qquad \qquad \qquad = 26$$

$$\therefore \text{Area} = \frac{1}{2} \cdot \frac{\Delta}{|C_1 C_2 C_3|}$$

$$= \frac{1}{2} \times \frac{(1352)}{(-26) \cdot (-52) \cdot (26)}$$

$$= 26 \text{ sq. units.}$$