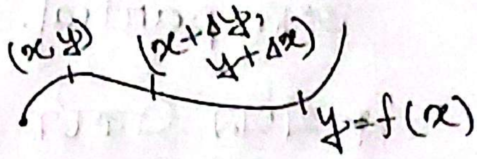
$$V = \frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$



$$y = f(x)$$

$$y + \Delta y = f(x + \Delta x)$$

$$\Delta y = f(x+\Delta x) - f(x)$$

$$\lim_{\Delta x \to 0} V = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{h}$$

Any equation is said to be differential equation if its involved one dependent varia and one or more than independent variabl and also derivative.

(i) $\frac{d^2y}{dx^2} + y = 0$

(ii) $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{d^3y}{dx^3}\right) + y = 0$

(iii) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

(iv) $ax^2 + bx^2 + c = 0$

* অধীন চলক একটা স্বাধীন চলক অনেকগুলো তখন partial.

* যখন একটা অধীন একটা স্বাধীনতা তখন ordinary.

* সর্বোচ্চ বার ডেরিভেটিভই হবে তার অর্ডার

$$\frac{d^2y}{dx^2} + y = 0$$

~~অর্ড~~ order = 2

* সবচে সর্বোচ্চ অর্ডারের ওপর যত পাওয়ার থাকবে সেটিই ডিগ্রী।

$$\left(\frac{d^3y}{dx^3}\right)^{10} + \left(\frac{d^5y}{dx^5}\right)^2 + y = 0$$

order = 5 ; degree = 2 .

Q. $y = a\sin x + b\cos x$

$= a\cos x - b\sin x$

$$\frac{d^2y}{dx^2} = -(a\sin x + b\cos x) \; ; \quad \frac{d^2y}{dx^2} + y = 0$$

Create a differential equation of the curve $y = a\sin x + b\cos x$. where a, b. are arbitary constant.

$\Rightarrow y = a + bx + cx^2$

যতবার arbitary constant ততবার derivate

create a differential eqn of the curve, where

① a is an arbitary constant

(ii) b " " " "

(iii) c " " " "

(iv) ab " " " "

(v) bc " " " "

(vi) ca " " " "

(vii) abc " " " "

① $y = a + bx + cx^2$

$$\frac{dy}{dx} = b + 2cx$$

$\boxed{\text{(1-5) page book compulsarry}}$

Lecturrens

3. Given that,

$$c(y+c)^2 = x^3 \quad \text{———} (1)$$

$$\Rightarrow c.2(y+c)\left(\frac{dy}{dx}+0\right) = 3x^2$$

$$\Rightarrow c.2(y+c)\,y_1 = 3x^2 \quad \text{———} (ii)$$

(1) ÷ (ii) ⇒

$$\cancel{c(y+c)^2 = x^3 \quad \text{———} (1)}$$

$$\cancel{\Rightarrow c.2(y+c)\left(\frac{dy}{dx}+0\right) = 3x^2}$$

$$\cancel{\Rightarrow c.2(y+c)\,y_1 = 3x^2 \quad \text{———} (ii)}$$

$$\frac{c(y+c)^2}{2c(y+c)y_1} = \frac{\cancel{x^3}^{x}}{3\cancel{x^2}}$$

$$\Rightarrow y+c = \frac{2xy_1}{3} \quad \text{———} (iii)$$

$$y+c = \frac{2xy_1}{3}$$

$$\Rightarrow c = \frac{2xy_1}{3} - y$$

$$\Rightarrow \frac{2xy_1}{3} - y\left(\frac{2xy_1}{3}\right)^2 = x^3$$

$$\therefore \left(\frac{2xy_1}{3} - y\right)\frac{4x^2y_1^2}{9} = x^3$$

using,

$$\left(\frac{2xy_1}{3} - y\right)\frac{4x^2y_1^2}{9} = x^3$$

$$\Rightarrow \frac{8x^3y_1^3}{27} - \frac{4x^2yy_1^2}{9} = x^3$$

$$\therefore 8x^3y_1^3 - 12x^2yy_1^2 = 27x^3$$

4\. Standard $(x-h)^2+(y+k)^2=r^2$

General $x^2+y^2+2gx+2fy+c=0$

we know that,

the General eqn of circle is

$$x^2+y^2+2gx+2fy+c=0 \text{ —— } ①$$

Origin দিয়ে pass করলে constant $=0$

$$(x,y)=(0,0)$$

① এ বসালে $c=0$

* Since equation —— ① passes through origin we have $c=0$.

* Since the centre of equation ① lies on x-axis then we have, $f=0$ then eqn becomes

$$x^2+y^2+2gx=0 \text{ —— } (ii)$$

10 — (11) করতে হবে (page-05)

$$y = ae^{5x} + be^{10x} + ce^{-2x}$$

$$\Rightarrow ye^{2x} = ae^{7x} + be^{12x} + c$$

$$\Rightarrow y e^{2x}.e + e^{2x}.y_1 = 7ae^{7x} + 12b e^{12x} + 0$$

$$\Rightarrow (2y + y_1) \cdot e^{-5x} = 7a + 12be^{5x}$$

$$\Rightarrow (2y + y_1) \cdot e^{-5x} \cdot (-5) + e^{-5x}(2y_1 + y_2) = 0 + 12be^{5x}.5$$

$$\Rightarrow e^{-5x}(-10y - 5y_1 - 12y_1 + y_2) = 60be^{5x}$$

$$\Rightarrow e^{-10x}(y_2 - 3y_1 + 10y) = 60b$$

$$\Rightarrow e^{-10x}(y_3 - 3y_2 - 10y_1) + (y_2 - 3y_1 - 10y)e^{-10x}(-1 = 0$$

$$\Rightarrow e^{-10x}(y_3 - 3y_2 - 10y_1 - 10y_2) + 30y_1 + 100y = 0$$

Therefore,

$$y^3 - 13y_2 + 20y_1 + 100y = 0$$

required.

First order, first degree differential equation,

$$M + N\frac{dy}{dx} = 0$$

or, $M\,dx + N\,dy = 0$

This eqn. is said to be first order and first degree differentiation eqn where: M and N are function of x and y or constant.

* $M = f_1(x) \cdot f_2(x)$
* $N = g_1(x) \cdot g_2(y)$

$$f_1 \cdot f_2 \cdot dx + g_1 g_2\, dy = 0$$

$$\therefore \frac{f_1}{g_1}dx + \frac{g_2}{f_2}dy = 0$$

$$2xy\,dy + y^2 dy = 0$$

$$\Rightarrow 2x\,dx + y\,dy = 0 \quad [\because y \text{ দ্বারা ভাগ}]$$

Integrating,

$$-\int 2x\,dx + \int y\,dy = 0$$

$$\Rightarrow 2\cdot\frac{x^2}{2} + \frac{y^2}{2} = A$$

$$\Rightarrow 2x^2 + y^2 = 2A$$

$$\boxed{\therefore 2x^2 + y^2 = C}$$

Q. $(1-x^2-y^2-x^2y^2)dx + x\,dy = 0$

$$\Rightarrow \{(1-x^2) - y^2(1-x^2)\}dx + x\,dy = 0$$

$$\Rightarrow (1-x^2)(1-y^2)\,dx + x\,dy = 0$$

$$\Rightarrow \frac{(1-x^2)\cancel{(1-y^2)}}{x\cancel{(1-y^2)}}dx + \frac{\cancel{x}}{\cancel{x}(1-y^2)}dy = 0$$ [$(1-y^2)$ এবং x দ্বারা ভাগ]

$$\Rightarrow \frac{1-x^2}{x}dx + \frac{1}{1-y^2}dy = 0$$

$$\Rightarrow \left(\frac{1}{x} - x\right)dx + \frac{1}{1^2-y^2}dy = 0$$

Integrating,

$\therefore \ln x - \frac{x^2}{2} + \frac{1}{2}\ln\left|\frac{1+y}{1-y}\right| = A.$

Q. $\sec^2 x \cdot \tan y \, dx + \sec^2 y \, dy = 0$

$\Rightarrow \sec^2 x \, dx + \frac{\sec^2 y}{\tan y} dy = 0$ [$\because$ $\tan y$ দ্বারা ভাগ]

Integrating, $\int \sec^2 x \, dx + \frac{\sec^2 x}{\tan y} dy = 0$

$\therefore \tan x + \int \frac{\sec^2 y}{\tan y} dy = 0$

let,
$z = \tan y$
$dz = \sec^2 y \, dy$

$\Rightarrow \tan x + \int \frac{dz}{z} = 0$

$\Rightarrow \tan x + \int \frac{1}{z} dz = 0$

$\Rightarrow \tan x + \ln(\tan y) = A$

(Ans:)

Q. $(3x+5y+10)^2 dx + dy = 0$

$\Rightarrow (3x+5y+10)^2 + \frac{dy}{dx} = 0$ ——— (i) [dx দ্বারা ভাগ]

Let,
$v = 3x+5y+10$ ——— (ii)

$$\therefore \frac{dv}{dx} = \frac{d}{dx}(3x + 5y + 10)$$

$$= 3 + 5\frac{dy}{dx}$$

$$\Rightarrow \frac{dv}{dx} - 3 = 5\frac{dy}{dx}$$

$$\therefore \frac{1}{5}\left(\frac{dv}{dx} - 3\right) = \frac{dy}{dx} \quad \text{——— (iii)}$$

using that eqn (ii) and (iii) in eqn (i)

$$v^2 + \frac{1}{5}\left(\frac{dv}{dx} - 3\right) = 0$$

[∵ 5 দ্বারা গুণ]

$$\Rightarrow 5v^2 + \frac{dv}{dx} - 3 = 0$$

$$\Rightarrow \frac{dv}{dx} = 3 - 5v^2$$

$$\therefore dx = \frac{dv}{3 - 5v^2}$$

Integration.

$$\int \frac{dv}{3 - 5v^2} = \int dx$$

$$= \int \frac{1}{3-5v^2} dv = \int dx$$

$$= \int \frac{1}{(\sqrt{3})^2 - (\sqrt{5}v)^2} dv = \int dx$$

$$= \frac{1}{2\sqrt{3}} \ln \left| \frac{\sqrt{3} + \sqrt{5}v}{\sqrt{3} - \sqrt{5}v} \right| = x + A$$

$$= \frac{1}{2\sqrt{3}} \ln \left| \frac{\sqrt{3} + \sqrt{5}(3x+5y+10)}{\sqrt{3} - \sqrt{5}(3x+5y+10)} \right| = x + A$$

Formula

* $\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}$
* $\int \frac{1}{a^2-x^2} dx = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right|$
* $\int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right|$
* $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1} \frac{a}{x}$
* $\int \frac{1}{\sqrt{x^2+a^2}} dx = \ln \left(x + \sqrt{x^2+a^2} \right)$

Lecture 05

Homogenius differential equation

☐ A function is said to be homogenius function of degree n if the power of every term of this function is equal to n

Q.. $(x^2+y^2)dx + xy\,dy = 0$

$\Rightarrow x^2+y^2+xy\frac{dy}{dx} = 0$

$\Rightarrow xy\frac{dy}{dx} = -(x^2+y^2)$

$\therefore \frac{dy}{dx} = -\frac{(x^2+y^2)}{xy}$ —— (i)

which is Homogenius differential eqn.

Let, $y = \cancel{v}xv$ —— (i)

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(vx)$

$= v\frac{d}{dx}(x) + x\frac{d}{dx}(v)$

$= v + x\frac{dv}{dx}$ —— (iii)

using (ii) and (iii) in eqn (i),

$$v + x\frac{dv}{dx} = \cancel{\ldots}$$

$$= \frac{-x^2 + y^2}{xy}$$

$$= \frac{-x^2 + (vx)^2}{x \cdot vx}$$

$$= \frac{-x^2 + v^2x^2}{x^2 v}$$

$$\Rightarrow x\left(\frac{dv}{dx}\right) = \frac{\cancel{x^2}(-1 + v^2)}{\cancel{x^2} \cdot v} - v$$

$$= \frac{-1 + v^2}{v} - v$$

$$= -\left(\frac{1 + v^2 + v^2}{v}\right)$$

$$\Rightarrow \frac{x}{dx} dv = -\frac{1 + 2v^2}{v}$$

$$\Rightarrow \frac{v}{1 + 2v^2} dv = -\frac{dx}{x}$$

$$\therefore -\frac{dx}{x} = \frac{v}{1 + 2v^2} dv$$

Integrating,

$$\int \frac{v}{1+2v^2}\,dv = -\int \frac{dx}{x}$$

Let,

$z = 1+2v^2$

$dz = 4v\,dv$

$\frac{1}{4}dz = v\,dv$

$$\Rightarrow \int \frac{\frac{1}{4}dz}{z} = -\ln x + A$$

$$\Rightarrow \frac{1}{4}\ln z + \ln x = A$$

$$\Rightarrow \ln(1+2v^2) + 4\ln x = 4A$$

$$\Rightarrow \ln(1+2v^2) + \ln x^4 = 4A$$

$$\Rightarrow \ln\{(1+2v^2)\cdot x^4\} = 4A$$

$$\Rightarrow \left(1+2\frac{y^2}{x^2}\right)\cdot x^4 = e^{4A}$$

$$\Rightarrow (x^2+2y^2)\cdot x^2 = e^{4A}$$

$$\boxed{\because c = e^{4A}}$$

$$\therefore (x^2+2y^2)\cdot x^2 = c$$