

# Linear Differential Eqn

Lecture 09

$$a_0 y + a_1 \frac{dy}{dx} + a_2 \frac{d^2y}{dx^2} + \dots + a_{n-1} \frac{d^{n-1}y}{dx^{n-1}} + a_n y = X$$

Main Form:

$$a_0 \frac{dy}{dx^n} + a_1 \frac{d^{n-1}y}{dx^{n-1}} + a_2 \frac{d^{n-2}y}{dx^{n-2}} + \dots + a_{n-1} \frac{dy}{dx} + a_n y = X$$

Note: Capital 'X' এর function

For example:

$$\frac{d^2y}{dx^2} - 5 \frac{dy}{dx} + 6y = 0$$

Let,  $y = e^{mx}$ , be a trial soln.

$$\therefore \frac{dy}{dx} = me^{mx}$$

$$\therefore \frac{d^2y}{dx^2} = m^2 e^{mx}$$

$$0 = P_0 + P_1 D + P_2 D^2 - Q_0 e^{mx}$$

$$0 = (1 - 5D + 6D^2) y$$

$$0 = 1 - 5D + 6D^2$$

So,

$$m^2 e^{mx} - 5m e^{mx} + 6e^{mx} = 0 \quad \text{or } m^2 - 5m + 6 = 0$$

$$\Rightarrow e^{mx}(m^2 - 5m + 6) = 0$$

$$\Rightarrow m^2 - 5m + 6 = 0 \Rightarrow \text{auxiliary eqn.}$$

$$\Rightarrow m^2 - 3m - 2m + 6 = 0$$

$$\Rightarrow (m-3)(m-2) = 0 \quad \therefore m = 2, 3$$

∴ General soln,  $\psi = C_1 e^{2x} + C_2 e^{3x}$

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Note: order যত হবে orbital constant তে  
রয়ে  $\psi = C_1 e^{2x} + C_2 e^{3x}$

Note:  $\frac{dy}{dx} = y_1 = \psi = D\psi$

$$D = \frac{d}{dx} (D - 1) = 0$$

short form:  $O = \psi_0 + \frac{k_0}{\psi_0} - \frac{\psi_0}{k_0}$

$$\frac{d^2\psi}{dx^2} - 5 \frac{dy}{dx} + 6\psi = 0$$

$$\Rightarrow D^2\psi - 5D\psi + 6\psi = 0$$

$$\Rightarrow \psi(D^2 - 5D + 6) = 0$$

$$\therefore D^2 - 5D + 6 = 0$$

$$\therefore D = 2, 3$$

$$O = \omega_m \psi_0 + \omega_m \omega_m \psi_0^2 - \omega_m \omega_m$$

$$O = (\omega + m\omega - m\omega) \omega_m$$

so  $O = \omega + m\omega - m\omega$

$$O = \omega + m\omega - m\omega - m\omega$$

$$E.S = m$$

$$O = (\omega - m)(\omega - m)$$

Case.01:  $D = m_1, m_2, m_3, m_4$   $D = p^2(1 - q)$   $\therefore$

$$Y = C_1 e^{m_1 x} + C_2 e^{m_2 x} + C_3 e^{m_3 x} + C_4 e^{m_4 x}$$
$$D = p^2(1 - q) \therefore E.A$$

Main form:

Case.02:  $D = m, m, m$

$$Y = (C_1 + C_2 x + C_3 x^2) e^{mx} + (C_4 + C_5 x + C_6 x^2) e^{-mx}$$

More complicated function.

Case.03:  $D = \alpha + i\beta$

$$Y = e^{\alpha x} (C_1 \cos \beta x + C_2 \sin \beta x)$$
$$D = p^2(1 - q) \therefore (E - D) \cdot (E + D - q) = 0$$

$$\begin{cases} D = \alpha + i\beta \\ D = \alpha - i\beta \end{cases} \quad \begin{cases} D - qD - D + q = 0 \\ (D + q)(1 - (E - D)) = 0 \end{cases} \quad \begin{cases} D = q \\ D = E \end{cases}$$

$$D = 3, 3e^3, 3e^{-3}, T, q$$

$$Y = (C_1 e^{(q+T)x} + C_2 e^{qx}) + (C_3 e^{(q-T)x} + C_4 e^{-qx}) + (C_5 e^{qx} + C_6 e^{-qx})$$

$$Q \cdot (D-4)^5 Y = 0 \quad [case. 02]$$

$$A.E, \therefore (D-4)^5 = 0$$

$$\therefore D = 4, 4, 4, 4, 4$$

$$\therefore Y = (C_1 + C_2 x + C_3 x^2 + C_4 x^3 + C_5 x^4) e^{4x}$$

$$Q \cdot (D-3)^4 \cdot (D^2 - 7D + 6) Y = 0 \quad [case. 02, case. 03]$$

$$\therefore (D-3)^4 \cdot (D^2 - 7D + 6) = 0 \quad [A.E]$$

$$\therefore (D-3)^4 = 0$$

$$\Rightarrow D-3=0$$

$$\therefore D=3$$

$$D^2 - 7D + 6 = 0$$

$$\Rightarrow D^2 - 6D - D + 6 = 0$$

$$\Rightarrow (D-6) \cdot (D-1) = 0$$

$$\therefore D=6, 1$$

$$\therefore D = 3, 3, 3, 3, 1, 6$$

$$\therefore Y = (C_1 + C_2 x + C_3 x^2 + C_4 x^3) e^{3x} + C_5 e^x + C_6 e^{6x}$$

$$Q. (D^2 - \sqrt{7}D + 4)Y = 0 \quad P = B(0, 147) \quad [\text{case 03}]$$

$$A.E, D^2 - \sqrt{7}D + 4 = 0 \quad 0 = D^2 - \sqrt{7}D + 4 < 0, \text{ J.A.}$$

$$\therefore D = \frac{\sqrt{7} \pm \sqrt{7 - 4 \cdot 1 \cdot 4}}{2 \cdot 1} \quad P.D = \sqrt{7} \pm \sqrt{3} \quad D = \frac{\sqrt{7} \pm \sqrt{3}}{2}$$

$$= \frac{\sqrt{7}}{2} \pm \frac{3i}{2} \quad \left(\text{as } 2^2 > 3^2\right) \quad \frac{i\sqrt{3}}{2} \pm \frac{\sqrt{3}i}{2} = i\sqrt{3}$$

$$\therefore Y = a e^{(\frac{\sqrt{7}}{2} + \frac{3i}{2})x} + b e^{(\frac{\sqrt{7}}{2} - \frac{3i}{2})x} \quad \text{as } i \pm x =$$

$$= a \cdot e^{\frac{\sqrt{7}}{2}x} \left( e^{3i\frac{x}{2}} + b \cdot e^{-\frac{3i}{2}x} \right) e^{-\frac{3i}{2}x}$$

$$= e^{\frac{\sqrt{7}}{2}x} \left\{ a \cdot (e^{3i\frac{x}{2}} + b \cdot e^{-3i\frac{x}{2}}) \right\}$$

$$0 = Y(D - Q)(D + Q)(P_D - P_Q) \cdot Q$$

$$0 = (D - Q)(D + Q)(P_D - P_Q) \cdot Q$$

$$Q = P_D - P_Q$$

$$Q = \{P_D\} - \{P_Q\}$$

$$Q = (P_D - P_Q)(P_D + P_Q)$$

$$P_D - P_Q = Q$$

$$Q = P_D - P_Q$$

$$Q = P_D - P_Q$$

$$Q. (D^2 - \sqrt{7}D + 4) Y = 0 \quad [case\ 03]$$

$$A.E. \Rightarrow D^2 - \sqrt{7}D + 4 = 0$$

$$\Rightarrow D = \frac{\sqrt{7} \pm \sqrt{7 - 4 \cdot 4}}{2 \cdot 1}$$

$$\therefore D = \frac{\sqrt{7}}{2} \pm \frac{3i}{2}$$

$$= \alpha \pm i\beta$$

$$= e^{\sqrt{7}/2 x} (C_1 \cos \frac{3}{2}x + C_2 \sin \frac{3}{2}x)$$

$$Q. (D^4 - \alpha^4)(D^3 - 6D^2 + 11D - 6) Y = 0$$

[case 01, 02, 03]

$$A.E. (D^4 - \alpha^4)(D^3 - 6D^2 + 11D - 6) = 0$$

Then,

$$D^4 - \alpha^4 = 0$$

$$\Rightarrow (D^2)^2 - (\alpha^2)^2 = 0$$

$$\Rightarrow (D^2 + \alpha^2)(D^2 - \alpha^2) = 0$$

$$So, D^2 = -\alpha^2$$

$$\Rightarrow D = \sqrt{-\alpha^2}$$

$$\therefore D = \pm i\alpha$$

again,

$$D^2 - \alpha^2 = 0$$

$$\Rightarrow D^2 = \alpha^2$$

$$\Rightarrow D = \sqrt{\alpha^2}$$

$$\therefore D = \pm \alpha$$

Again,

$$D^3 - 6D^2 + 11D - 6 = 0$$

$$\Rightarrow D^3 - 3D^2 - 3D^2 + 2D + 9D - 6 = 0$$

$$\Rightarrow D^2(D-3) - 3D(D-3) + 2(D-3) = 0$$

$$\Rightarrow (D-3)(D^2 - 3D + 2) = 0$$

$$\Rightarrow (D-3)(D^2 - 2D + D + 2) = 0$$

$$\Rightarrow (D-3)\{(D-2) - 1(D-2)\} = 0$$

$$\Rightarrow (D-3)(D-2)(D-1) = 0$$

$$\therefore D = 1, 2, 3$$

$$\therefore D = 1, 2, 3, \pm a, \pm ai.$$

$$\therefore y = C_1 \cdot e^x + C_2 \cdot e^{2x} + C_3 \cdot e^{3x} + C_4 \cdot e^{ax} + C_5 \cdot e^{-ax}$$

$$+ e^{ax} (C_6 \cdot \cos ax + C_7 \cdot \sin ax)$$

যদি Right hand side O এন্ডে প্রযুক্তি - যাইতে

$$O = e^{-D} + D^2 e^{-D} + D^3 e^{-D}$$

case 01:

D  $\rightarrow$  Derivative

$$O = (e - D) e^{-D} + (e - D) D e^{-D} + (e - D) D^2 e^{-D}$$

$\checkmark$  D  $\rightarrow$  Integral

$$O = (2 + D e^{-D} - D^2 e^{-D}) (e - D)$$

case 02:

P. I = Particular Integral

C. F = Complementary Function.

$$O = (1 - D)(2 + D)(e - D)$$

$$\therefore P. I = \frac{1}{f(D)} e^{2x}$$

$$= \frac{1}{f(\alpha)} e^{\alpha x} \quad [f(\alpha) \neq 0]$$

$e, e, 1 = 0, \dots$

$e, e, 1 = 0, \dots$

Note:

a = constant.

case 03:

$$P. I = \frac{1}{f(D)} \sin 2x$$

$$= \frac{1}{f(-\alpha^2)} \sin \alpha x$$

$$Q. (D^2 - 2D + 6) y = e^{2x}$$

মনে করি,

Right hand side ০ আছে.

$$A.E. \Rightarrow D^2 - 2D + 6 = 0$$

$$\therefore C.F. = C_1 e^x + C_2 e^{6x}$$

Then,

$$P.I. = \frac{1}{D^2 - 2D + 6} e^{2x}$$

$$= \frac{1}{2^2 - 2D + 6} e^{2x}$$

$$= \frac{1}{-4} e^{2x}$$

$$\therefore Y = C.F. + P.I.$$

$$= C_1 e^x + C_2 e^{6x} - \frac{1}{4} e^{2x}$$

Note: P.I. এর মুক্তি মনে রাখ, যা C.F. এর মুক্তি আছে

তাই P.I. = 0 রয়ে

$$Q. (D^2 - 7D + 6) y = e^{6x}$$

$$A.E \Rightarrow D^2 - 7D + 6 = 0$$

$$\therefore D = 1, 6.$$

$$\therefore C.f = C_1 e^{1x} + C_2 e^{6x}$$

$$\therefore P.I = \frac{1}{D^2 - 7D + 6} e^{6x}, D = 7, 0, \therefore H.O$$

$$= \frac{x}{2D - 7} e^{6x}$$

$$= \frac{x}{2 \times 6 - 7} e^{6x}$$

$$= \frac{x}{5} e^{6x}$$

when, case 02 failed.

$$\therefore Y = C.f + P.I$$

$$= C_1 e^x + C_2 e^{6x} + \frac{x}{5} e^{6x}$$

Note: যদি নিচে মান বয়ানে ০ হয়, তবে উপরে x  
নিলাম আর  $\frac{dy}{dx}$  derivative করলাম। অতবার ০  
আমরে প্রথম নিবে উপরে মোর নিচে derivative.

$$Q \cdot (D^2 - 2D + 6) Y = \sin 2x$$

on L.H.S.

001

P.I.

$$\therefore A.E \Rightarrow D^2 - 2D + 6 = 0$$

$$\therefore D = 1, 6.$$

$$\therefore C.f = C_1 e^x + C_2 e^{6x}$$

$$\begin{aligned}
 \therefore P.I &= \frac{1}{D^2 - 2D + 6} \sin 2x \\
 &= \frac{1}{(-2)^2 - 2.D + 6} \sin 2x \\
 &= \frac{1}{2 - 2D} \sin 2x \\
 &= \frac{(2 + 2D)}{(2 - 2D)(2 + 2D)} \sin 2x \\
 &= \frac{2 + 2D}{4 - 4^2 D^2} \sin 2x \\
 &= \frac{2 \sin 2x + 2 \cos 2x \cdot 2}{4 - 4^2 (-2)} \\
 &= \frac{2(\sin 2x + 2 \cos 2x)}{200}
 \end{aligned}$$

$$= \frac{\sin 2x + 7\cos 2x}{100} \quad \text{since } \beta/(D+dx) = \beta/(D+D)$$

$$\therefore y_1 = C_1 \cdot e^x + C_2 \cdot e^{6x} + \frac{\sin 2x + 7\cos 2x}{100}$$

$$R_0 D_{12} + R_1 D_1 = 2 \cdot D$$

$$R_0 D_{12} = \frac{L}{D^2 - XD + D}$$

$$R_0 D_{12} = \frac{L}{D^2 - XD + D} =$$

$$R_0 D_{12} = \frac{L}{X - D}$$

$$R_0 D_{12} = \frac{(DX + D)}{(DX + D)(DX - D)} =$$

$$R_0 D_{12} = \frac{DX + D}{D^2 - X^2}$$

$$\frac{R_0 D_{12} \cos X + R_0 D_{12} \sin X}{(DX + D)(DX - D)} =$$

$$(R_0 \cos X + R_0 \sin X) \frac{1}{D}$$

0.001