Chapter-1 (co-ordinates)

Formula:

1. Area of a polygon with vertices (x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4), ... (x_n, y_n) is

$$\text{Area} = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & \cdots & x_n & x_1 \\ y_1 & y_2 & y_3 & y_4 & \cdots & y_n & y_1 \end{vmatrix}$$

when,
Area $= 0$; the points lie on the same straight line.

2. Condition to be collinear of the points (x_1, y_1), (x_2, y_2), (x_3, y_3) is

$$\text{Area} = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix} = 0$$

Note: taking all points in the Anti-clockwise direction

method-2: $\Delta ABC = \frac{1}{2}\, SABC$

$$= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

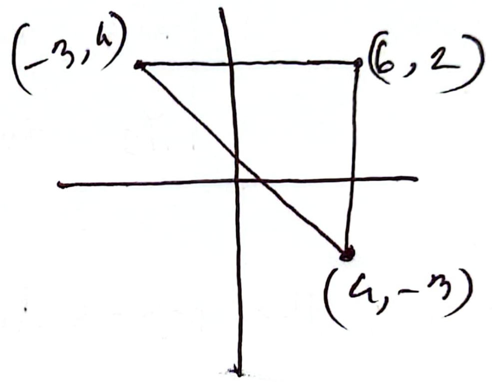
Math (CL):

Q.1 Find the area of the triangle with vertices

(a) $(-3,4), (6,2), (4,-3)$

(b) $(a, b+c), (b, a+c), (c, a+b)$

Q.1(a): Soln: Rearranging these points in anti-clockwise direction



$$\therefore \text{Area} = \frac{1}{2}\begin{vmatrix} 6 & -3 & 4 & 6 \\ 2 & 4 & -3 & 2 \end{vmatrix}$$

$$= \frac{1}{2}\{(24+9-8)-(-6+16-18)\}$$

$$= 49/2 \text{ sq. units.}$$

Q.1(b): Area of the given points $= \frac{1}{2}\begin{vmatrix} a & b & c & a \\ b+c & a+c & a+b & b+c \end{vmatrix}$

$$= \frac{1}{2}\{(a^2+ac+ab+b^2+bc+c^2)-(b^2+bc+ac+c^2+a^2+ab)\}$$

$$= 0 \text{ sq. units.}$$

Q.2: $(-1,2),(2,-1),(h,3)$ are collinear, show that $h=-2$

Sol^n: $(-1,2),(2,-1),(h,3)$ since these points are collinear, hence area $=0$

$$\therefore \text{Area} = \frac{1}{2}\begin{vmatrix} -1 & 2 & h & -1 \\ 2 & -1 & 3 & 2 \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{2}\{(1+6+2h)-(4-h-3)\} = 0$$

$$\Rightarrow 3h+6 = 0$$

$$\therefore h=-2 \text{ (~~Ans~~ showed)}$$

Q.3: For what value of K are the points $(2,3)$, $(-4,-6)$ and $(K,12)$ collinear?

Sol^n: Since these points are collinear, area $=0$

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 2 & 4 & K & 2 \\ 3 & -6 & 12 & 3 \end{vmatrix}$$

$$\Rightarrow \frac{1}{2}\{(-12-48+3K)-(-12-\cancel{8}6K+24)\}$$

$$\Rightarrow (-60+3K-12+6K) = 0$$

$$\therefore K = 8 \text{ (Ans)}$$

Q.5 If the area of the quadrilateral, whose angular points A, B, C, D taken in order are $(1,2)$, $(-5,6)$, $(7,-4)$ and $(K,-2)$ be zero, find K?

Soln:

since these points are quadrilateral,

area $= 0$.

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 1 & -5 & 7 & K & 1 \\ 2 & 6 & -4 & -2 & 2 \end{vmatrix} = 0$$

$$\text{or, } \frac{1}{2}\left\{(6+20-14+2k)-(-10+42-4K-2)\right\} = 0$$

$\therefore K = 3$ (Ans).

Q.7 Show that the three points $(4,2)$, $(7,5)$, $(9,7)$ lie on a right line.

Soln: Area of three points $= \frac{1}{2}\begin{vmatrix} 4 & 7 & 9 & 4 \\ 2 & 5 & 7 & 2 \end{vmatrix}$

$$= \frac{1}{2}\left\{(20+49+18)-(14+45+28)\right\}$$

$$= 0$$

Since, Area $= 0$, three points lie on a right line.

Math (H.W.):

Q.4: Find the area of the quadrilateral whose angular points are....

(a) $(1,1), (3,4), (5,-2), (4,-7)$

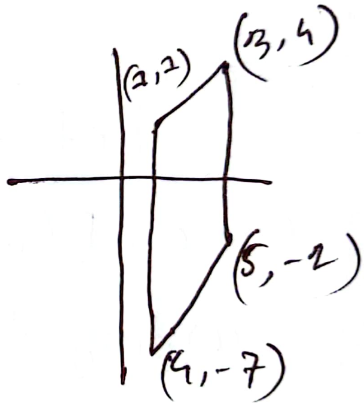
(b) $(1,2), (6,2), (5,3), (3,4)$

(c) $(2,4), (3,2), (6,4), (7,6)$

Soln.:

(a) Rearranging these points in anti-clockwise direction,

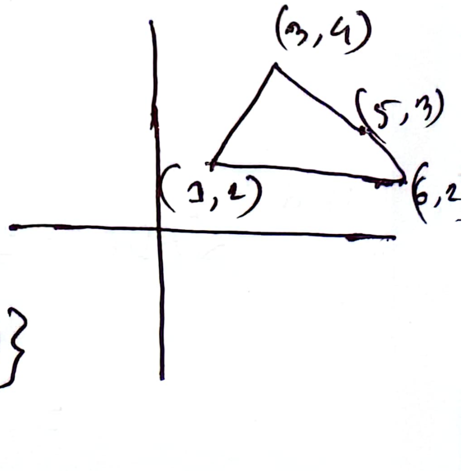
$$\text{Area} = \frac{1}{2}\begin{vmatrix} 3 & 1 & 4 & 5 & 3 \\ 4 & 1 & -7 & -2 & 4 \end{vmatrix}$$



$$= \frac{1}{2}\{(3-7-8+20)-(4+4-35-6)\}$$

$$= 41/2 \text{ sq. units}$$

4(b) Rearranging these points in anti-clockwise direction

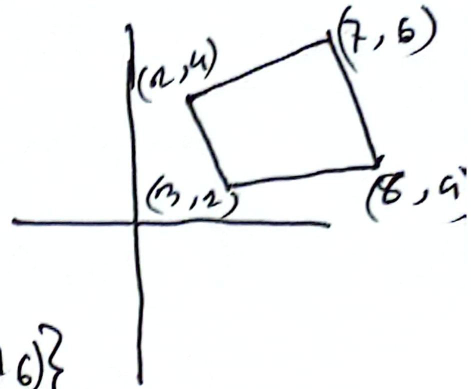


$$\text{Area} = \frac{1}{2}\begin{vmatrix} 6 & 5 & 3 & 1 & 6 \\ 2 & 3 & 4 & 2 & 2 \end{vmatrix}$$

$$= \frac{1}{2}\{(18+20+6+2)-(10+9+4+12)\}$$

$$= 11/2 \text{ sq. units.}$$

4(c) Rearranging these points in anti-clockwise direction



$$\text{Area} = \frac{1}{2}\begin{vmatrix} 8 & 7 & 2 & 3 & 8 \\ 4 & 6 & 4 & 2 & 4 \end{vmatrix}$$

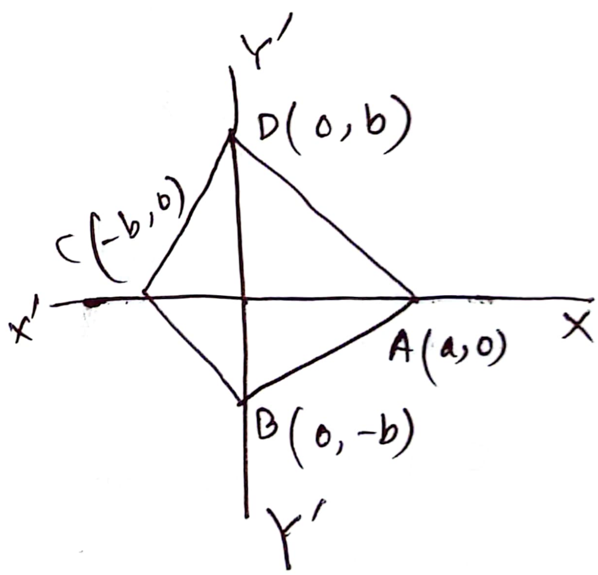
$$= \frac{1}{2}\{(48+28+4+12)-(28+12+12+16)\}$$

$= 12$ sq. units

Q.6 Show that the area of the quadrilateral whose vertices are $(a,0), (-b,0), (0,-b)$ is zero, $(a,b>0)$. Explain the result with a diagram.

prof: Let the points be

$A(a,0)$, $B(0,-b)$, $C(-b,0)$ and according to order D will be $(0,b)$



$$\text{Area } \triangle ABC = \frac{1}{2}\begin{vmatrix} a & 0 & -b & a \\ 0 & -b & 0 & 0 \end{vmatrix}$$

$$= \frac{1}{2}(-ab-b^2)$$

and

$$\triangle ADC = \frac{1}{2}\begin{vmatrix} a & 0 & -b & a \\ 0 & b & 0 & 0 \end{vmatrix}$$

$$= \frac{1}{2}(ab+b^2)$$

$\therefore$ Area of quadrilateral $ABCD = \triangle ABC + \triangle ADC$

$= 0$ (showed)

Q.8: Find the areas of the polygons whose vertices are given

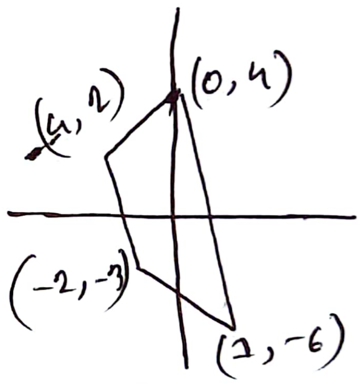
(a) $(0,4), (1,-6), (-2,-3), (-4,2)$

(b) $(1,5), (-2,4), (-3,-1), (2,-3), (5,1)$

(c) $(1,3), (4,1), (5,3), (3,2), (2,4)$

Soln: 8(a) Rearranging these points in anti-clockwise direction,

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 0 & -4 & -2 & 1 & 0 \\ 4 & 2 & -3 & -6 & 4 \end{vmatrix}$$

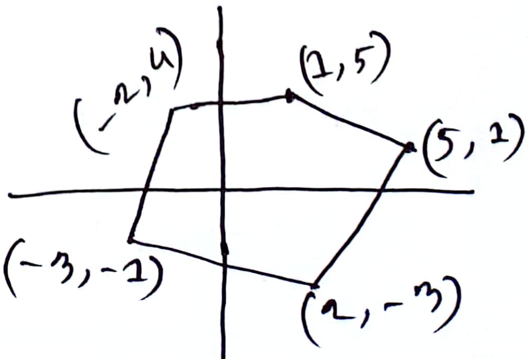


$$= \frac{1}{2}\{(0+12+12+4)-(-16-4-3-0)\}$$

$$= 51/2 \text{ sq. units}$$

8(b) Rearranging these points in anti-clockwise direction

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 5 & 1 & -2 & -3 & 2 & 5 \\ 1 & 5 & 4 & -1 & -3 & 1 \end{vmatrix}$$



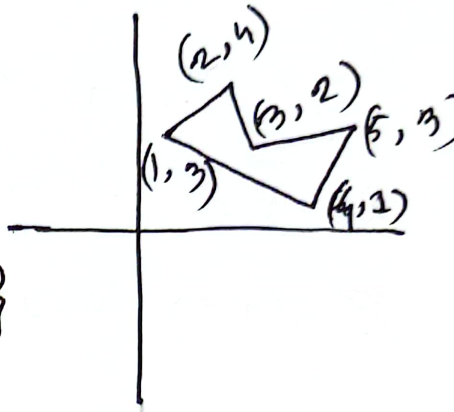
$$= \frac{1}{2}\{(25+4+2+9+2) -$$

$(1-10-12-2-15)\}$

$= 40$ sq. units (Ans)

8(c) Rearranging these points in anti-clockwise direction

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 5 & 3 & 2 & 1 & 4 & 5 \\ 3 & 2 & 4 & 3 & 1 & 3 \end{vmatrix}$$



$$= \frac{1}{2}\{(10+12+6+1+12)-(9+4+4+12+5)\}$$

$= 7/2$ sq. units (Ans)

Chapter - 2 (Polar Co-ordinates)

Formula:

① $x = r\cos\theta$ $\qquad r = \sqrt{x^2+y^2}$

$y = r\sin\theta$ $\qquad \theta = \tan^{-1}\left(\frac{y}{x}\right)$

② Distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2-x_1)^2-(y_2-y_1)^2}$

③ Distance between points $A(r_1, \theta_1)$ and $B(r_2, \theta_2)$

is — $\sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$

④ Area of the triangle with vertices $A(r_1, \theta_1)$, $B(r_2, \theta_2)$ and $C(r_3, \theta_3)$ is —

$$\triangle ABC = \frac{1}{2} r_1r_2\sin(\theta_2 - \theta_1) + \frac{1}{2} r_2r_3\sin(\theta_3 - \theta_2)$$
$$+ \frac{1}{2} r_3r_1\sin(\theta_3 - \theta_1)$$

Shortcut:

1. $(x, y) \rightarrow r = \sqrt{x^2 + y^2}$, $\theta = \tan^{-1}\left|\frac{y}{x}\right|$

2. $(-x, y) \rightarrow r = \sqrt{x^2 + y^2}$, $\theta = \pi - \tan^{-1}\left|\frac{y}{x}\right|$

3. $(-x, -y) \rightarrow r = \sqrt{x^2 + y^2}$, $\theta = \pi + \tan^{-1}\left|\frac{y}{x}\right|$

4. $(x, -y) \rightarrow r = \sqrt{x^2 + y^2}$, $\theta = 2\pi - \tan^{-1}\left|\frac{y}{x}\right|$

Condition: of three point will form a triangle

a, b and c

$a + b > c$

$b + c > a$

$a + c > b$

Q.1 : Find the polar co-ordinates of the following points:

(i) $(\sqrt{3}, 1)$, (ii) $(-1, 1)$, (iii) $(\sqrt{3}, -1)$, extra → $(-1, -1)$

Solⁿ: 1(i) $(\sqrt{3}, 1)$, Given that,

$$x = \sqrt{3}$$
$$y = 1$$

We know,

$$r = \sqrt{x^2+y^2} = \sqrt{(\sqrt{3})^2+(1)^2} = 2$$

and $\theta = \tan^{-1}\left|\frac{1}{\sqrt{3}}\right| = \pi/6$

∴ The required polar form is $(2, \pi/6)$

1(ii) Given that,

$$x = -1$$
$$y = 1$$

We know,

$$r = \sqrt{x^2+y^2} = \sqrt{(-1)^2+(1)^2} = \sqrt{2}$$

and, $\theta = \pi - \tan^{-1}\left|\frac{y}{x}\right| = \pi - \tan^{-1}\left|\frac{1}{-1}\right| = 3\pi/4$

∴ The required polar form is $\left(\sqrt{2}, \frac{3\pi}{4}\right)$

1(iii) are given
We ~~know~~ that,

$$x = \sqrt{3},$$

$$y = -1$$

we know,

$$r = \sqrt{x^2 + y} = \sqrt{(\sqrt{3})^2 + (-1)^2} = 2$$

and, $$\theta = 2\pi - \tan^{-1}\left|\frac{-1}{\sqrt{3}}\right|$$

$$= 11\pi/6$$

$\therefore$ The required polar form is $(2, 11\pi/6)$

1(extra)

We are given that,

$$x = -1, \; y = -1$$

We know that,

$$r = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$$

and,
$$\theta = \pi + \tan^{-1}\left|\frac{-1}{-1}\right|$$

$$= 5\pi/4$$

$\therefore$ The required polar form is $\left(\sqrt{2}, \frac{5\pi}{4}\right)$

Convert into cartesian form...

(i) $(\sqrt{2}, 5\pi/4)$

We know that, $x = r\cos\theta$

$y = r\sin\theta$

$x = \sqrt{2} \times \cos 5\pi/4$

$= -1$

$y = r\sin\theta$

$= \sqrt{2} \times \sin 5\pi/4$

$= -1$

$\therefore$ The required cartesian form is $(-1, -1)$

Q.2 Find the polar distance of the co-ordinate of the points.

(i) $(3, 45^\circ)$ and $(7, 105^\circ)$

(ii) $(\sqrt{2}, 5\pi/4)$ and $(2, 2\pi/3)$

Soln: (i) We are give that,

$r_1 = 3, \ r_2 = 7, \ \theta_1 = 45^\circ, \ \theta = 105^\circ$

We know,

$$D = \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos(\theta_2 - \theta_1)}$$

$$= \sqrt{3^2 + 7^2 - 2.3.7.\cos(105 - 45)}$$

$= \sqrt{37}$ units.

$\therefore$ Polar distance is $\sqrt{37}$ units

(ii) We are given that,

$$r_1 = \sqrt{2} \quad \theta_1 = 5\pi/4$$
$$r_2 = 2 \quad \theta_2 = 2\pi/3$$

We know,

$$\text{distance} = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$$

$$= \sqrt{(\sqrt{2})^2 + (2)^2 - 2.\sqrt{2}.2.\cos\left(\frac{2\pi}{3} - \frac{5\pi}{4}\right)}$$

$$= 2.732$$

$\therefore$ Polar distance is 2.732 units

Q.3 Determine the nature of the triangle whose vertices $(0,0)$, $(3, \frac{\pi}{2})$, $(3, \frac{\pi}{6})$ and calculate it's area.

Soln: We are given that,

$$A(0,0),\ B(3, \tfrac{\pi}{2}),\ C(3, \tfrac{\pi}{6})$$

$$AB = \sqrt{0^2 + 3^2 - 2.3.0.\cos\left(\frac{\pi}{6} - 0\right)} = 3 \text{ units}$$

$$BC = \sqrt{3^2 + 3^2 - 2.3.3\cos\left(\frac{\pi}{6} - \frac{\pi}{2}\right)} = 3 \text{units}$$

$$AC = \sqrt{3^2 + 0^2 - 2.3.0.\cos\left(0 - \frac{\pi}{6}\right)} = 3 \text{ units.}$$

Since,

$AB = BC = AC$; this is an equilateral triangle.

$$\text{Area}, = \frac{1}{2} r_1 r_2 \sin(\theta_2 - \theta_1) + \frac{1}{2} r_2 r_3 \sin(\theta_3 - \theta_2)$$

$$+ \frac{1}{2} r_3 r_1 \sin(\theta_3 - \theta_1)$$

$$= 0 + \frac{1}{2} \times 3 \times 3 \times \sin\left(\frac{\pi}{6} - \frac{\pi}{2}\right) + 0$$

$$= -\frac{9\sqrt{3}}{\cancel{8}4}$$; Area cannot be negative

$$\therefore \text{ Area} = \frac{9\sqrt{3}}{4} \text{ sq. units.}$$

Math (H.W.)

Q.4 Find the areas of the triangle the polar co-ordinates of whose angular points are...

i) $(3, 30^\circ)$, $(5, 150^\circ)$, $(7, 210^\circ)$

ii) (a, θ), $(2a, \theta + \pi/3)$, $(3a, \theta + 2\pi/3)$

Sol^n: i) We are given that,

$$\begin{array}{l|l} r_1 = 3 & \theta_1 = 30^\circ \\ r_2 = 5 & \theta_2 = 150^\circ \\ r_3 = 7 & \theta_3 = 210^\circ \end{array}$$

$\therefore$ Area $= \frac{1}{2} r_1 r_2 \sin(\theta_2 - \theta_1) + \frac{1}{2} r_2 r_3 \sin(\theta_3 - \theta_2)$

$+ \frac{1}{2} r_3 r_1 \sin(\theta_3 - \theta_1)$

$= \frac{1}{2} \{ 3 \times 5 \times \sin(150 - 30) + 5 \times 7 \times \sin(210 - 150)$

$+ 7 \times 3 \times \sin(210 - 30^\circ) \}$

$= \frac{25\sqrt{3}}{2}$ sq. units

$\therefore$ Area $= \frac{25\sqrt{3}}{2}$ sq units.

(11) We are given that,

$r_1 = a$

$r_2 = 2a$

$r_3 = 3a$

$\theta_2 - \theta_1 = \left(\theta + \frac{\pi}{3}\right) - \theta = \frac{\pi}{3}$

$\theta_3 - \theta_2 = \left(\theta + \frac{2\pi}{3}\right) - \left(\theta + \frac{\pi}{3}\right) = \frac{\pi}{3}$

$\theta_3 - \theta_1 = \left(\theta + \frac{2\pi}{3}\right) - \theta = \frac{2\pi}{3}$

$$\therefore \text{Area} = \frac{1}{2}\left| r_1 r_2 \sin(\theta_2 - \theta_1) + r_2 r_3 \sin(\theta_3 - \theta_2) + r_1 r_3 \sin(\theta_3 - \theta_1) \right|$$

$$= \frac{1}{2}\left\{\left| a \times 2a \times \sin\frac{\pi}{3} + 2a \times 3a \times \sin\left(\frac{\pi}{3}\right) + 3a \times a \times \sin\frac{2\pi}{3} \right|\right\}$$

$$= \frac{\sqrt{3}}{2 \times 2}\left| 2a^2 + 6a^2 + 3a^2 \right|$$

$$= \frac{11\sqrt{3}\, a^2}{4}$$

$$\therefore \text{Area} = \frac{11\sqrt{3}\, a^2}{4} \text{ sq. units.}$$

Q.5 Change the equation to polar co-ordinates.

i) $(x^2+y^2)^2 = 2a^2xy$ (ii) $x^4+x^2y^2-(x+y)^2=0$

Sol^n (i) We know that,

$x = r\cos\theta$ | $r^2 = x^2+y^2$
$y = r\sin\theta$ |

$\therefore (x^2+y^2)^2 = 2a^2xy$

or, $r^4 = 2a^2 r\cos\theta \cdot r\sin\theta$ [putting value of x, y, r^2]

or, $r^4 = r^2a^2 2\sin\theta\cos\theta$

or, $r^2 = a^2\sin 2\theta$ (Ans)

(ii) We know that,

$x = r\cos\theta$ | $r^2 = x^2+y^2$
$y = r\sin\theta$ |

Now, $\cancel{x^4}$ $x^4+x^2y^2-(x+y)^2=0$

or, $x^4+x^2y^2 = (x+y)^2$

or, $x^2(x^2+y^2) = r^2(\cos\theta+\sin\theta)^2$

$$\text{or, } r^2\cos^2\theta \cdot r^2 = r^2(\cos\theta + \sin\theta)^2$$

$$\text{or, } r^2 = \left(\frac{\cos\theta + \sin\theta}{\cos\theta}\right)^2$$

$$\text{or, } r^2 = (1 + \tan\theta)^2$$

$$\therefore r = \pm(1 + \tan\theta) \quad \text{(Ans)}$$

Chapter - 3 (The Straigh Line)

Formulae:

① Equation of st. line..

i) parallel to X axis is

$y = \text{constant (say } b)$

ii) parallel to Y axis is

$x = \text{constant (say } a)$

② $y = mx + c$, $\boxed{m = \tan\theta}$ — slope / gradient / ঢাল

③ $$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1}$$

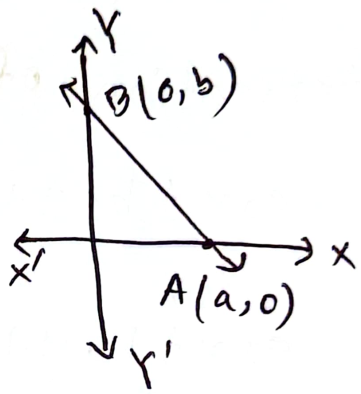
④ $$y-y_1 = m(x-x_1)$$

↳ Eqⁿ of st. line passing through a point (x_1, y_1) whose slope is m.

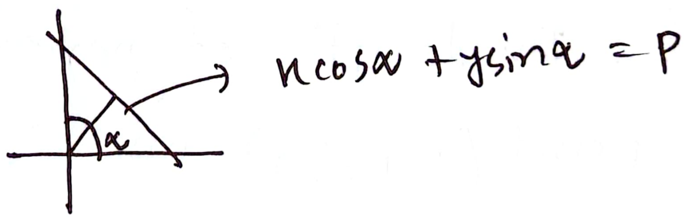
⑤ $$\frac{x}{a} + \frac{y}{b} = 1$$

↳ Intercept from

(অক্ষের খণ্ডিতাংশের সমী.)



⑥ $$x\cos\alpha + y\sin\alpha = p$$



here, p = perpendicular distance from origin $O(0,0)$ (মূলবিন্দু)

⑦ $$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$$

⑧ $$x = x_1 + r\cos\theta, \quad y = y_1 + r\sin\theta$$

(9) Eq^n of st. line parallel to ...

$$ax + by + c = 0 \text{ is } \rightarrow ax + by + c_1 = 0$$

(10) Eq^n of st. line perpendicular to ...

$$ax + by + c = 0 \text{ is } bx - ay + c_1 = 0$$

(*1) Eq^n of a st. line passing through the point of intersection of the given two lines ...

$$a_1x + b_1y + c_1 = 0 \text{ and } a_2x + b_2y + c_2 = 0 \text{ is}$$

$$a_1x + b_1y + c_1 + K(a_2x + b_2y + c_2) = 0$$

(*2) The length of perpendicular line from a point (x_1, y_1) on the line $ax + by + c = 0$ is

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

(*3) Eq^n of the bisector (সমদ্বিখণ্ডকরেখা) of the two lines $a_1x + b_1y + c = 0$ and $a_2x + b_2y + c_2 = 0$

is

$$\frac{|a_1x+b_2y+c_1|}{\sqrt{a_1^2+b_1^2}} = \pm\frac{|a_2x+b_2y+c_2|}{\sqrt{a_2^2+b_2^2}}$$

*4 Condition for concurrency (সমবিন্দুগামী) of the lines $a_1x+b_1y+c_1=0$, $a_2x+b_2y+c_2=0$ and $a_3x+b_3y+c_3=0$ is

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

*5 Area of a triangle formed with the three lines $a_1x+b_1y+c_1=0$, $a_2x+b_2y+c_2=0$ and $a_3x+b_3y+c_3=0$ is

$$\frac{1}{2}\cdot\frac{\Delta^2}{C_1C_2C_3}$$

Here,

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$C_1 = \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

$$C_2 = \begin{vmatrix} a_1 & b_1 \\ a_3 & b_3 \end{vmatrix}$$

$$C_3 = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

Math (CL):

Exp -1: Find the eqⁿ to lines passing through $(-5,6)$ and (i) parallel (ii) perpendicular to the line $7x-8y=9$

Solⁿ: Given line $7x-8y-9=0$ ...(i)

Now, Eqⁿ of a parallel line to ...(i) is $7x-8y+k=0$ which passes through the point $(-5,6)$ then -

$$7(-5)-8\times6+k=0$$

$$\therefore K=83$$

Hence (ii) $\Rightarrow 7x-8y+83=0$ (Ans)

and

(ii) Eqⁿ of a perpendicular line to ...(i) is

$8x-7y+K=0$ ...(ii) which passes through the point $(-5,6)$ then -

$$8(-5)-7.6+K=0$$

$$\therefore K=82$$

Hence, (ii) $\Rightarrow 8x-7y+82=0$ (Ans)

Q.1 Find the equation of st. line passes through the intersection of the lines $3x-5y+9=0$ and $4x+7y-28=0$ and satisfies and following condition;

ⓐ passes through $(4,2)$

ⓑ Is parallel to $2x+3y-5=0$

ⓒ Is perpendicular to $4x+5y-20=0$

ⓓ whose intercepts are equal.

Sol^n: Given that,

$$3x-5y+9=0$$
$$4x+7y-28=0$$

The required line:

$$3x-5y+9+k(4x+7y-28)=0 \quad \cdots(i)$$

$$\Rightarrow (3+4k)x+(-5+7k)y+9-28k=0 \quad \cdots(ii)$$

ⓐ Since eq^n (ii) passes through $(4,2)$, then

$$(3+4k).4+(-5+7k).2+9-28k=0$$

$$\therefore k=-11/2$$

putting $K = -11/2$ in eq^n (ii), we get,

$$\left(-3 - \frac{44}{2}\right)x + \left(-5 - 77/2\right)y + 7 + 154 = 0$$

$$\Rightarrow -19x - \frac{87}{2}y + 163 = 0$$

$$\therefore 38x + 87y - 326 = 0 \text{ (Ans)}$$

(b) slope of eq^n (ii), $m_1 = -\dfrac{3+4K}{7K-5}$

slope of $2x+5y-5=0$, $m_2 = -\dfrac{2}{3}$

Since, they are parallel so,

$$m_1 = m_2$$

$$\frac{3+4K}{7K-5} = \frac{2}{3}$$

$$\Rightarrow 9 + 12K = 14K - 10$$

$$\therefore K = 19/2$$

putting $K = 19/2$ in eq^n (ii),

$$(3 + 38)x + \left(\frac{7\times 19}{2} - 5\right)y + 7 - \frac{28\times 19}{2} = 0$$

$$\Rightarrow 41x + \frac{123}{2}y - 257 = 0$$

$$\therefore 82x + 123y - 514 = 0 \text{ (Ans)}$$

(c) slope of eq^n(ii), $m_1 = -\dfrac{3+4K}{7K-5}$

slope of $4x+5y-20$, $m_2 = -\dfrac{4}{5}$

since, they are $\perp^r$, so

$$m_1 \times m_2 = -1$$

$$\text{or, } \frac{3+4K}{7K-5} \times \frac{4}{5} = -1$$

$$\text{or, } \frac{12+16K}{35K-25} = -1$$

$$\text{or, } K = 13/51$$

putting $K = \dfrac{13}{51}$ in eq^n (ii),

$$\left(3+\frac{4\times 13}{51}\right)x + \left(\frac{7\times 13}{51} - 5\right)y + 9 - \frac{28\times 13}{51} = 0$$

$$\text{or, } \frac{205}{51}x - \frac{164}{51}y + \frac{95}{51} = 0$$

$\therefore$ $205x - 164y + 95 = 0$ (Ans)

(d) From (ii),

$$(3+4k)x + (7K-5)y = 28K-9$$

$$\Rightarrow \frac{(3+4K)}{28K-9}x + \frac{7K-5}{28K-9}y = 1$$

$$\Rightarrow \frac{x}{\frac{28K-9}{3+4K}} + \frac{y}{\frac{28K-9}{7K-9}} = 1$$

Since intercepts are equal, So.

$$\frac{28K-9}{3+4K} = \frac{28K-9}{7K-5} = 0$$

$$\text{or, } (28K-9)\left\{\frac{1}{3+4K} - \frac{1}{7K-5}\right\} = 0$$

here,

$28K-9=0$

$\therefore K = 9/28$

or,

$$\frac{1}{3+4K} = \frac{1}{7K-5}$$

or, $K = 8/3$

Now, putting $K = 9/28$ and $K = 8/3$ in eq^n(ii),

$$\left(3 + \frac{4\times9}{28}\right)x + \left(\frac{7\times9}{28} - 5\right)y + 9 - \frac{28\times9}{28} = 0$$

$$\text{or, } \frac{30}{7}x - \frac{11}{4}y = 0$$

$\therefore\ 120x - 77y = 0$ (Ans)

again,

$$\left(3 + \frac{4\times 8}{13}\right)x + \left(\frac{7\times 8}{3} - 5\right)y + 7 - \frac{28\times 8}{13} = 0$$

or, $41x + 41y - 197 = 0$ (Ans)

Ex-2 Determine the eq^n of the bisectors of the angle bet^n the lines $3x-4y+12=0$ and $12x+5y-30=0$

Sol^n: Given lines:

$$3x - 4y + 12 = 0 \quad \cdots (i)$$
$$12x + 5y - 30 = 0 \quad \cdots (ii)$$

Eq^n of bisector of the angle bet^n these two lines are

$$\frac{3x - 4y + 12}{\sqrt{9+16}} = \pm \frac{12x + 5y - 30}{\sqrt{144 + 25}}$$

taking '+'

$$13(3x - 4y + 12) = 5(12x + 5y - 30)$$

or, $39x - 52y + 156 = 60x + 25y - 150$

$\therefore\ 21x + 77y = 306$ (Ans)

taking (-)

$$13(3x-4y+12) = -5(12x+5y-30)$$

or, $99x - 27y + 6 = 0$

$\therefore 33x - 9y + 2 = 0$ (Ans)

Math (H.W.)

Ex. 3 Find the area of the triangle formed by the lines $2x+y-3=0$, $3x+2y-1=0$ and $2x+3y+4=0$

Sol^n: We are given that,

$$2x+y-3=0$$

$$3x+2y-1=0$$

$$2x+3y+4=0$$

We know

$$\text{Area} = \frac{1}{2}\left|\frac{\Delta^2}{C_1C_2C_3}\right|$$

here,

$$\Delta = \begin{vmatrix} 2 & 1 & -3 \\ 3 & 2 & -1 \\ 2 & 3 & 4 \end{vmatrix} = -7$$

$$C_1 = \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix}, \quad C_2 = \begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix}, \quad C_3 = \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix}$$

$$= 5 \qquad\qquad = 4 \qquad\qquad = 1$$

$$\therefore \text{Area} = \frac{\Delta^2}{C_1 C_2 C_3} \times \frac{1}{2}$$

$$= \frac{(-7)^2}{5 \times 4 \times 1} \times \frac{1}{2} = \frac{49}{40} \text{ sq. units. (Ans)}$$

Q.3 If p and p_1 be the perpendiculars form the origin upon ~~the~~ st. lines whose eqn are $x\sec\theta + y\operatorname{cosec}\theta = a$ and $x\cos\theta - y\sin\theta = a\cos 2\theta$ prove that $4p^2 + p_1^2 = a^2$

Soln: Given lines:

$$x\sec\theta + y\operatorname{cosec}\theta - a = 0$$

$$\text{and } x\cos\theta - y\sin\theta - a\cos 2\theta = 0$$

Now, the perpendicular distance p from the origin $(0,0)$ to the line ~~$ax+by$~~ $ax+by+c=0$ is

$$p = \frac{|ax+by+c|}{\sqrt{a^2+b^2}}$$

For line, $x\sec\theta + y\csc\theta - a = 0$

$$p = \frac{|0.\sec\theta + 0.\csc\theta - a|}{\sqrt{\sec^2\theta + \csc^2\theta}}$$

$$= \frac{|a|}{\sqrt{\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}}}$$

$$= \frac{a\sqrt{\sin^2\theta\cos^2\theta}}{\sqrt{1}} = a\sin\theta\cos\theta = \frac{a\sin 2\theta}{2}$$

again,

for line $x\cos\theta - y\sin\theta - a\cos 2\theta = 0$

$$p_1 = \frac{|0.\cos\theta - 0.(-\sin\theta) - a\cos 2\theta|}{\sqrt{\cos^2\theta + (-\sin\theta)^2}}$$

$$= \frac{a\cos 2\theta}{1} = a\cos 2\theta$$

$$\text{L.H.S.} = 4p^2 + p_1^2$$

$$= 4.\frac{a^2\sin^2 2\theta}{4} + a^2\cos^2 2\theta$$

$$= a^2(\sin^2 2\theta + \cos^2 2\theta)$$

$= a^2$

$= R.H.S$ (proved)

Q.4 Show that the perpendicular let fall from any points on the st. line $7x-9y+10=0$ upon the two lines $3x+4y-5=0$ and $12x+5y-7=0$ are equal to each other.

Soln: Given lines:

$$7x-9y+10=0 \quad \text{---(i)}$$

$$3x+4y-5=0 \quad \text{---(ii)}$$

$$12x+5y-7=0 \quad \text{----(iii)}$$

Here, we consider the point (α, β) which lies on eqn (i), then

$$7\alpha - 9\beta + 10 = 0$$

$$\text{or, } \beta = \frac{7\alpha+10}{9} \quad \text{---(iv)}$$

Now, the $\perp^r$ distance of eqn (ii) from (α, β) is

$$d_1 = \frac{|3\alpha+4\beta-5|}{\sqrt{3^2+4^2}} \quad \text{--} ✓$$

and $\perp^r$ distance of eq^n (iii) from (α, β) is

$$d_2 = \frac{|12\alpha + 5\beta - 7|}{\sqrt{12^2 + 5^2}} \quad \ldots (vi)$$

passing $\beta = \dfrac{7\alpha + 10}{9}$ into (v)

$$d_1 = \frac{\left|3\alpha + 4\left(\frac{7\alpha + 10}{9}\right) - 5\right|}{5}$$

$$= \frac{|27\alpha + 28\alpha + 40 - 45|}{5 \times 9}$$

$$\therefore d_1 = \frac{|11\alpha - 1|}{9}$$

passing $\beta = \dfrac{7\alpha + 10}{9}$ into (vi)

$$d_2 = \frac{\left|12\alpha + 5\left(\frac{7\alpha + 10}{9}\right) - 7\right|}{13}$$

$$= \frac{|108\alpha + 35\alpha + 50 - 63|}{13}$$

$$\therefore d_2 = \frac{|11\alpha - 1|}{9}$$

Since, $d_1 = d_2 = \dfrac{|11\alpha - 1|}{9}$, hence (showed)

Q.5 Find the condition that st. lines $y = m_1x + a_1$ and $y = m_2x + a_2$ and $y = m_3x + a_3$ may at a point

Solⁿ: Given lines :

$$m_1x - y + a_1 = 0$$

$$m_2x - y + a_2 = 0$$

$$m_3x - y + a_3 = 0$$

here,
$$\Delta = \begin{vmatrix} m_1 & -1 & a_1 \\ m_2 & -1 & a_2 \\ m_3 & -1 & a_3 \end{vmatrix}$$

$$= m_1(a_2 - a_3) + m_2(a_3 - a_1) + m_3(a_1 - a_2)$$
(Ans)

Ques. 6(i) Prove that diagonals of the parallelogram formed by the four st. lines. $\sqrt{3}x + y = 0$, $\sqrt{3}y + x = 0$, $\sqrt{3}x + y = 1$ and $\sqrt{3}y + x = 1$.

Solⁿ: Given lines

$$\sqrt{3}x + y = 0 \quad \cdots (i)$$

$$x + \sqrt{3}y = 0 \quad \cdots (ii)$$

$$\sqrt{3}x + y = 1 \quad \cdots (iii)$$

$$x + \sqrt{3}y = 1 \quad \cdots (iv)$$

taking, eq^n (i) and eq^n (ii)

$$x = 0, \quad y = 0$$

$\therefore$ ~~A~~ ~~insertio~~ intersection point is $A(0,0)$.

taking, eq^n (i) and eq^n (iv)

$$x = -\frac{1}{2}, \quad y = \frac{\sqrt{3}}{2}$$

$\therefore$ intersection point is $B\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

taking eq^n (iii) and eq^n (ii)

$$x = \frac{\sqrt{3}}{2}, \quad y = -\frac{1}{2}$$

$\therefore$ intersection point $C\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

taking,

eq^n (iii) and eq^n (iv)

$$x = \left(\frac{\sqrt{3}-1}{2}\right), \quad y = \left(\frac{\sqrt{3}-1}{2}\right)$$

$\therefore$ intersection point is $D\left(\frac{\sqrt{3}-1}{2}, \frac{\sqrt{3}-1}{2}\right)$

slope of $AC = m_{AC} = \dfrac{\frac{\sqrt{3}-1}{2} - 0}{\frac{\sqrt{3}-1}{2} - 0} = 1$

slope of $BD = m_{BD} = \dfrac{\sqrt{3}/2 - (-\frac{1}{2})}{-\frac{1}{2} - \sqrt{3}/2} = -1$

$\therefore m_{AC} \times m_{BD} = -1 \times 1 = 1$ (proved)

Q 10(a) Show that the area of the triangle formed by the lines whose eq^n are $y = m_1x + c_1$, $y = m_2x +$ and $x = 0$ is $\frac{1}{2}(c_1 - c_2)^2/(m_2 - m_1)$

Sol^n: Given lines: $m_1x - y + c_1 = 0$

$m_2x - y + c_2 = 0$

$x + 0.y + 0 = 0$

Now, $\Delta = \begin{vmatrix} m_1 & -1 & c_1 \\ m_2 & -1 & c_2 \\ 1 & 0 & 0 \end{vmatrix}$

$= 1(-c_2 + c_1) + 0 + 0$

$= c_1 - c_2$

$C_1 = \begin{vmatrix} m_2 & -1 \\ 1 & 0 \end{vmatrix}$; $C_2 = \begin{vmatrix} m_1 & -1 \\ 1 & 0 \end{vmatrix}$; $C_3 = \begin{vmatrix} m_1 & -1 \\ m_2 & -1 \end{vmatrix}$

$= 1$ $\qquad = 1 \qquad = m_2 - m_1$

$\therefore$ Area $= \frac{1}{2} \times \frac{\Delta^2}{C_1 C_2 C_3} = \frac{1}{2} \times \frac{(c_1 - c_2)^2}{(m_2 - m_1)}$

$= \frac{1}{2} (c_1 - c_2)^2 / (m_2 - m_1)$ sq. uni

(showed)

Q.10(b) Find the triangle's area formed by the lines $x + y + 2 = 0$; $2x - y - 3 = 0$ and $3x + 2y - 5 = 0$

Soln: Given lines:

$x + y + 2 = 0 \cdots (i)$

$2x - y - 3 = 0 \cdots (ii)$

$3x + 2y - 5 = 0 \cdots (iii)$

Here,

$$\Delta = \begin{vmatrix} 1 & 1 & 2 \\ 2 & -1 & -3 \\ 3 & 2 & -5 \end{vmatrix}$$

$= 1(5+6) - 1(-10+9) + 2(4+3)$

$= 26$

$C_1 = \begin{vmatrix} 3 & -1 \\ 3 & 2 \end{vmatrix}$; $C_2 = \begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix}$; $C_3 = \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix}$

$= 7 \qquad = -1 \qquad = -3$

$\therefore$ Area $= \frac{1}{2} \times \frac{\Delta^2}{C_1 C_2 C_3} = \frac{1}{2} \times \frac{26^2}{7 \cdot (-1) \cdot (-3)}$

$= \frac{338}{21}$ sq. units

Q.17 Prove that the three st. lines $2x - 7y + 10 = 0$, $3x - 2y - 1 = 0$, $x - 12y + 21 = 0$ concur at a point

Soln: Given lines are,

$2x - 7y + 10 = 0 \cdots (i)$

$3x - 2y - 1 = 0 \cdots (ii)$

$x - 12y + 21 = 0 \cdots (iii)$

eqn (i) $\times 2$ − eqn (ii) $\times 7$

$$\begin{array}{r} 4x - 14y + 20 = 0 \\ \underset{(-)}{}21x \underset{(+)}{-} 14y \underset{(+)}{-} 7 = 0 \\ \hline -17x + 27 = 0 \end{array}$$

$\therefore x = 27/17$

putting, $x = 27/17$ into eqn (i).

$2\left(\frac{27}{17}\right) - 7y + 10 = 0$

$\therefore y = \frac{32}{17}$

Now, putting $\left(\frac{27}{17}, \frac{32}{17}\right)$ into eq^n(iii)

$$\frac{27}{17} - 12\left(\frac{32}{17}\right) + 21 = 0$$

or $\quad 0 = 0$

Since, left side = right side, the point $\left(\frac{27}{17}, \frac{32}{17}\right)$ satisfies the eq^n (iii)

$\therefore$ The three lines concur at the point $\left(\frac{27}{17}, \frac{32}{17}\right)$

Q.24 Find the area of the triangle formed by the line $8x+7y-43=0$; $2x+5y=1$ and $4x-3y+11=0$

Sol^n: Given lines;

$$8x+7y-43=0$$
$$2x+5y-1=0$$
$$4x-3y+11=0$$

Here

$$\Delta = \begin{vmatrix} 8 & 7 & -43 \\ 2 & 5 & -1 \\ 4 & -3 & 11 \end{vmatrix}$$

$$= 8(55-3) - 7(22+4) - 43(-6-20)$$

$$= 1352$$

Now,

$$C_1 = \begin{vmatrix} 2 & 5 \\ 4 & -3 \end{vmatrix} ; \quad C_2 = \begin{vmatrix} 8 & 7 \\ 4 & -3 \end{vmatrix} ; \quad C_3 = \begin{vmatrix} 8 & 7 \\ 2 & 5 \end{vmatrix}$$

$$= -26 \qquad = -52 \qquad = 26$$

$$\therefore \text{Area} = \frac{1}{2} \cdot \frac{\Delta^2}{C_1 C_2 C_3}$$

$$= \frac{1}{2} \times \frac{(1352)^2}{(-26)\cdot(-52)\cdot(26)}$$

$= 26$ sq. units.

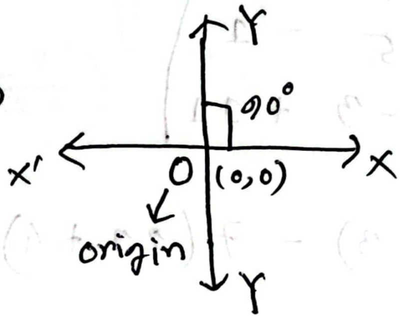
$= 26$ squre units.

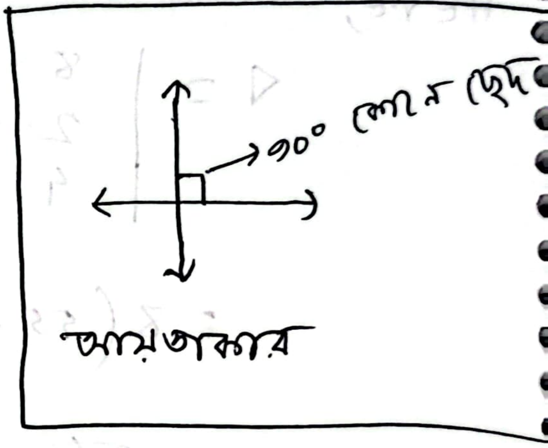
Chapter-4 (Change of axes)

Lecture-7
02.07.2024

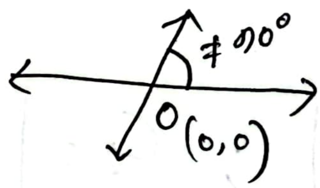
* Co-ordinates systems

- Rectangular co-ordinates system →





- Oblique system →



Formula:

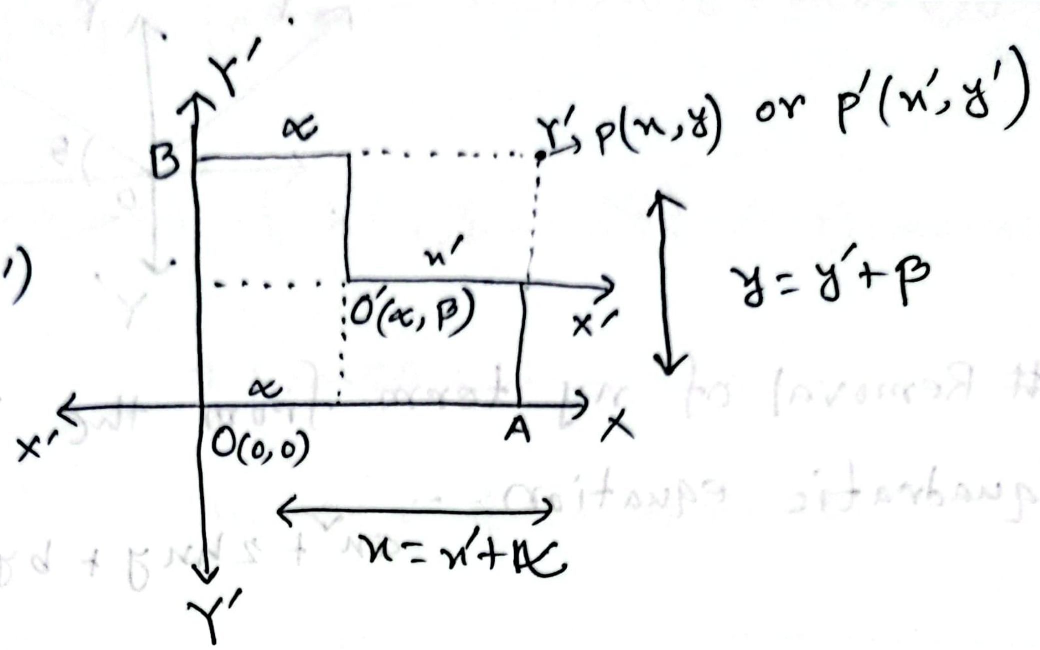
change of origin (Translation ~~of~~ of axes)

$$x = x' + \alpha$$

$$y = y' + \beta$$

In $XOY \rightarrow p(x,y)$

$X'O'Y' \rightarrow P'(x',y')$



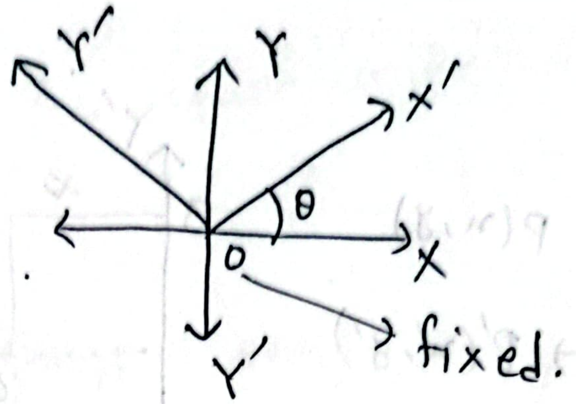
Formulae:

① Change in the co-ordinate of a point when the origin is shifted to another point $O'(\alpha, \beta)$ is

$$\left.\begin{aligned} x' &= x - \alpha \\ y' &= y - \beta \end{aligned}\right\} \rightarrow \left.\begin{aligned} x &= x' + \alpha \\ y &= y' + \beta \end{aligned}\right]$$ change of origin when axes are fixed.

② The change in the co-ordinate of a point when the direction of axes is turned through an angle θ is

$$\left.\begin{aligned} x &= x'\cos\theta - y'\sin\theta \\ y &= x'\sin\theta + y'\cos\theta \end{aligned}\right]$$ Rotation of axes when origin is fixed



Removal of xy term from the homogeneous ~~quadra~~ quadratic equation...

$$ax^2 + 2hxy + by^2 = 0 \quad \cdots (i)$$

for x,y term removal we use...

$$\left.\begin{aligned} x &= x'\cos\theta - y'\sin\theta \\ y &= x'\sin\theta + y'\cos\theta \end{aligned}\right\} \cdots (ii)$$

substitute eq^n(ii) in eq^n(i) we have,

$$a(x'\cos\theta - y'\sin\theta)^2 + 2h(x'\cos\theta - y'\sin\theta)(x'\sin\theta + y'\cos\theta) + b(x'\sin\theta + y'\cos\theta)^2 = 0$$

$$\Rightarrow a(x'^2\cos^2\theta - 2x'y'\sin\theta\cos\theta + y'^2\sin^2\theta) + 2h(x'^2\sin\theta\cos\theta - x'y'\sin^2\theta + x'y'\cos^2\theta - y'^2\sin\theta\cos\theta) + b(x'^2\sin^2\theta + 2x'y'\sin\theta\cos\theta + y'^2\cos^2\theta) = 0$$

$$\Rightarrow (a\cos^2\theta + 2h\sin\theta\cos\theta + b\sin^2\theta)x'^2 + 2\{h(\cos^2\theta - \sin^2\theta)$$

$-(a-b)\sin\theta\cos\theta\}x'y' + (a\sin^2\theta - 2h\sin\theta\cos\theta + h\cos^2\theta)y'^2 = 0$

Now,

$x'y'$ will be removed if, $h(\cos^2\theta - \sin^2\theta) - (a-b)\sin\theta\cos\theta = 0$

$$\Rightarrow h\cos 2\theta = (a-b)\sin\theta\cos\theta$$

$$\therefore h\cos 2\theta = \frac{1}{2}(a-b)\sin 2\theta$$

and,

$$\tan 2\theta = \frac{2h}{a-b}$$

$$2\theta = \tan^{-1}\left(\frac{2h}{a-b}\right)$$

$$\therefore \theta = \frac{1}{2}\tan^{-1}\left(\frac{2h}{a-b}\right)$$

* Find the condition to remove x', y' homogeneous quadratic eq^n
* What is the required condition to remove the xy term from homogeneous quadratic eq^n

Chap-4 Lecture-8

05.02.2024

page-28

Exercise-iv(a)

~~Ex. 10~~ Transform the eq^n $17x^2 + 18xy - 7y^2 - 16x - 32y - 18 = 0$ to one in which there is no term involving x, y and xy both sets of axes being rectangular.

$\underline{Sol^n}$: Given,

$$17x^2+18xy-7y^2-16x-32y-18=0 \quad \cdots (i)$$

Put $x=x_1+\alpha$ and $y=y_1+\beta$ in $eq^n(i)$

$$17(x_1+\alpha)^2+18(x_1+\alpha)(y_1+\beta)-7(y_1+\beta)^2-16(x_1+\alpha)-32(y_1+\beta)-18=0$$

$$\Rightarrow 17(x_1^2+2x_1\alpha+\alpha^2)+18(x_1+\alpha)(y_1+\beta)-7(y_1^2+2y_1\beta+\beta^2)-16(x_1+\alpha)-32(y_1+\beta)-18=0$$

$$\Rightarrow 17x_1^2+34x_1\alpha+17\alpha^2+18x_1y_1+18x_1\beta+18\alpha y_1+18\alpha\beta-7y_1^2+14y_1\beta+7\beta^2-16x_1+16\alpha-32y_1+32\beta-18=0$$

$$\Rightarrow 17x_1^2+18x_1y_1-7y_1^2+(34\alpha+18\beta-16)x_1+(18\alpha-14\beta-32)y_1+17\alpha^2+18\alpha\beta-7\beta^2-16\alpha-32\beta-18=0 \quad \cdots (ii)$$

To vanish, x_1y_1 term from $eq^n(ii)$, we have

$$34\alpha+18\beta-16=0 \quad \cdots (iii)$$

$$18\alpha-14\beta-32=0 \quad \cdots (iv)$$

Note

(i) $x=x_1+\alpha$, $y=y_1+\beta$ দ্বারা xy term remove করা যায়

(ii) $\theta=\frac{1}{2}\tan^{-1}\frac{2h}{a-b}$ দ্বারা xy term remove করা যায়

solving (iii) and (iv), we get,

$$(\alpha, \beta) = (1, -1)$$

putting these value of $(\alpha, \beta) \equiv (1, -1)$ in eq^n(ii), we get,

$$17x_1^2 + 18x_1y_1 - 7y_1^2 - 10 = 0 \quad \cdots (v)$$

Removing suffixes, we obtain,

$$17x^2 + 18xy - 7y^2 - 10 = 0 \quad \cdots (vi)$$

which is the eq^n (iv) excluding xy term.

<u>2nd part:</u> To remove xy term from eq^n(vi) we have to set, $\theta = \frac{1}{2}\tan^{-1}\frac{2h}{a-b}$

$a = 17$

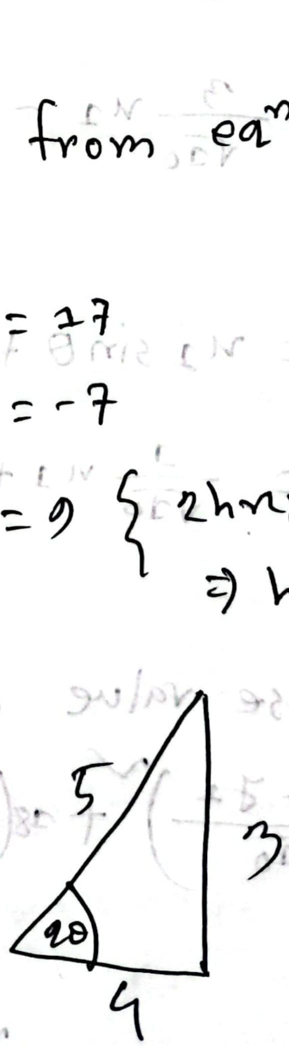
$b = -7$

$h = 9 \left\{ 2hxy = 2.9.xy \Rightarrow h = 9 \right\}$

$$\Rightarrow \theta = \frac{1}{2}\tan^{-1}\frac{2\times 9}{17+7}$$

$$\Rightarrow 2\theta = \tan^{-1} 3/4$$

$$\therefore \tan 2\theta = \frac{3}{4}$$



$$\therefore \sin 2\theta = \frac{3}{5}$$

$$\Rightarrow 2\sin\theta\cos\theta = \frac{3}{5}$$

$\Rightarrow \sin\theta = \dfrac{3}{10\cos\theta}$

$\Rightarrow \sin\theta = \dfrac{3}{10 \times \dfrac{3}{\sqrt{10}}}$

$\therefore \sin\theta = \dfrac{1}{\sqrt{10}}$

and,

$\cos 2\theta = \dfrac{4}{5}$

$\Rightarrow \cos^2\theta - \sin^2\theta = \dfrac{4}{5}$

$\Rightarrow \cos^2\theta - 1 + \cos^2\theta = \dfrac{4}{5}$

$\Rightarrow 2\cos^2\theta = \dfrac{9}{5}$

$\therefore \cos\theta = \dfrac{3}{\sqrt{10}}$

we know,

$$x = x_1\cos\theta - y_1\sin\theta$$

$$= \frac{3}{\sqrt{10}}x_1 - \frac{1}{\sqrt{10}}y_1 = \frac{3x_1 - y_1}{\sqrt{10}}$$

$$y = x_1\sin\theta + y_1\cos\theta$$

$$= \frac{1}{\sqrt{10}}x_1 + \frac{3}{\sqrt{10}}y_1 = \frac{x_1 + 3y_1}{\sqrt{10}}$$

putting these value of x, y in eq^n (vi) we get,

$$17\left(\frac{3x_1 - y_1}{\sqrt{10}}\right)^2 + 18\left(\frac{3x_1 - y_1}{\sqrt{10}}\right)\left(\frac{x_1 + 3y_1}{\sqrt{10}}\right) - 7\left(\frac{x_1+3y_1}{\sqrt{10}}\right)^2 - 10 = 0$$

or, $17(3x_1 - y_1)^2 + 18(3x_1 - y_1)(x_1 + 3y_1) - \cancel{7(x_1+3y_1)^2}$

$-7(x_1+3y_1)^2 - 100 = 0$

$\Rightarrow 200x_1^2 - 100y_1^2 - 100 = 0$

$\Rightarrow 2x_1^2 - y_1^2 = 1$

(Removing suffixes we obtain).

$\therefore 2x^2 - y^2 = 1$ (which is the required eqn)

MAT 1241

Lecture - 7

07.7.2024

page - 328

Ex. 7 If the direction is turned through an angle $30°$ and the origin remains unchanged then find the Indentity and sketch it.

Soln: Given that, $x^2 + 2\sqrt{3}xy - y^2 - 2a^2 = 0$ ---(i)

We know,

$$x = x_1 \cos 30° - y_1 \sin 30°$$

$$= \frac{\sqrt{3}}{2}x_1 - \frac{y_1}{2} = \frac{\sqrt{3}x_1 - y_1}{2}$$

$$y = x_1 \sin 30° - y_1 \cos 30°$$

$$= \frac{x_1}{2} + \frac{\sqrt{3}}{2}y_1 = \frac{x_1 + \sqrt{3}y_1}{2}$$

putting these values in eq^n(i), we got

$$\left(\frac{\sqrt{3}x_1 - y_1}{2}\right)^2 + 2\sqrt{3}\left(\frac{\sqrt{3}x_1 - y_1}{2}\right)\left(\frac{x_1 + \sqrt{3}y_1}{2}\right) - \left(\frac{x_1 + \sqrt{3}y_1}{2}\right)^2 - (2a)^2 = 0$$

$$\Rightarrow 3x_1^2 - 2\sqrt{3}x_1y_1 + y_1^2 + 2\sqrt{3}(\sqrt{3}x_1^2 - x_1y_1 + 3x_1y_1 - \sqrt{3}y_1^2) - (x_1^2 + 2\sqrt{3}x_1y_1 + 3y_1^2) - 8a^2 = 0$$

$$\Rightarrow 3x_1^2 + y_1^2 + 6x_1^2 - 6y_1^2 - x_1^2 - 3y_1^2 - 8a^2 = 0$$

$$\Rightarrow 8(x_1^2 - y_1^2 - a^2) = 0$$

$\therefore x_1^2 - y_1^2 - a^2 = 0$, Removing suffixes,

$$\boxed{x^2 - y^2 = a^2}$$

which is required transformed eq^n...

সূ

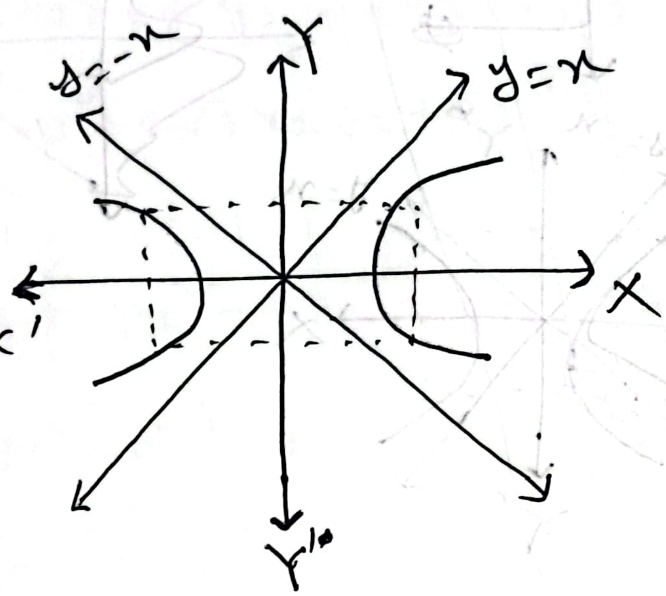
Ellipse (উপবৃত্ত) : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Hyperbola (অধিবৃত্ত) : $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Rectangular Hyperbola : $x^2 - y^2 = a^2$

2nd Part: Here, $x^2 - y^2 = a^2$ is the eqn of rectangular hyperbola

sketch: $x^2 - y^2 = 0 \Rightarrow \boxed{y = \pm x} \rightarrow$ eqn of asymplote (অসীমতট)



H.W. Exercise – iv → Math (1, 2, 3)

H.W.

Q.1 Transform to parallel axes through the new origin of the eqn.

a) Origin(1, -2), $2x^2 + y^2 - 4x + 4y = 0$

b) Origin(3, 1), $x^2 - 6x + 2y^2 + 7 = 0$

Sol^n: a) We know that,

$$x = x' + \alpha$$
$$y = y' + \beta$$

where (x', y') are the new co-ordinates with respect to the shifted origin, and (α, β) are the co-ordinates of the new origin.

We are given that,
$$2x^2 + y^2 - 4x + 4y = 0 \quad \cdots (1)$$

with origin$(1, -2)$, we get,
$$x = x' + 1$$
$$y = y' - 2$$

putting value of ~~(x, y)~~ x and y in eq^n (i),

$$2(x'+1)^2 + (y'-2)^2 - 4(x'+1) + 4(y'-2) = 0$$

or, $2(x'^2 + 2x' + 1) + (y'^2 - 4y' + 4) - 4(x'+1) + 4(y'-2) =$

or, $2x'^2 + 4x' + 2 + y'^2 - 4y' + 4 - 4x' - 4 + 4y' - 8 = 0$

or, $2x'^2 + y'^2 - 6 = 0$

$\therefore$ $2x'^2 + y'^2 = 6$

removing suffixes,

the required eqⁿ is $2x^2+y^2=6$

(b) We know that,

$$x = x' + \alpha$$

$$y = y' + \beta$$

where (x', y') are the new co-ordinates with respect to the shifted origin and (α, β) are the co-ordinates of the new origin.

given eqⁿ, $\quad x^2-6x+2y^2+7=0 \quad \ldots \text{(i)}$

with the origin at $(3,1)$, we get,

$$x = x'+3 \quad \text{and} \quad y = y'+1$$

putting x and y value in eqⁿ (i)

$$(x'+3)^2 - 6(x'+3) + 2(y'+1)^2 + 7 = 0$$

$$\Rightarrow x'^2+6x'+9-6x'-18+2y'^2+4y'+2+7=0$$

or, $\quad x'^2+2y'^2+4y'=0$

removing suffixes we get,

$$x^2 + 2y^2 + 4y = 0$$ which is the required eqn

Q.2 Transform to axes inclined at $45°$ to the original axes the equations.

i) $x^2 - y^2 = a^2$

ii) $x^2 - y^2 - 2\sqrt{2}x - 10\sqrt{2}y + 2 = 0$

Soln: i) Given eqn:

$$x^2 - y^2 = a^2 \quad \cdots (i)$$

We know that,

$$x = x'\cos\theta - y'\sin\theta$$
$$y = x'\sin\theta + y'\cos\theta$$

Given that, rotating the co-ordinat by $45°$ $\therefore \theta = 45°$

$\therefore$ $x = x'\cos 45° - y'\sin 45°$

$$= \frac{x'}{\sqrt{2}} - \frac{y'}{\sqrt{2}}$$

$$= \frac{x' - y'}{\sqrt{2}}$$

$y = x'\sin 45° + y'\cos 45°$

$$= \frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}$$

$$= \frac{x' + y'}{\sqrt{2}}$$

Now, putting $x = \frac{x' - y'}{\sqrt{2}}$ and $y = \frac{x' + y'}{\sqrt{2}}$ into eqn (i) we have,

$$\left(\frac{x'-y'}{\sqrt{2}}\right)^2-\left(\frac{x'+y'}{\sqrt{2}}\right)^2=a^2$$

or, $\frac{(x'-y')^2}{2}-\frac{(x'+y')^2}{2}=a^2$

or, $\frac{x'^2-2x'y'+y'^2}{2}-\frac{x'^2+2x'y'+y'^2}{2}=a^2$

or, $(x'^2-2x'y'+y'^2)-(x'^2+2x'y'+y'^2)=2a^2$

or, $-4x'y'-2a^2=0$ $\therefore 2x'y'=a^2$

or, $-2(2x'y'-a^2)=0$

removing suffixes, we get,

$2xy=a^2$ which is required eqn

(ii) Given that,

$x^2-y^2-2\sqrt{2}x-10\sqrt{2}y+2=0$ ...(i)

rotating the co-ordinates by $45°$ ~~$\therefore \theta=45°$~~

$\therefore \theta=45°$

we know that,

$$x = x'\cos 45° - y'\sin 45° = \frac{x'}{\sqrt{2}} - \frac{y'}{\sqrt{2}}$$

$$\therefore x = \frac{x'-y'}{\sqrt{2}}$$

$$y = x'\sin 45° + y'\cos 45° = \frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}$$

$$\therefore y = \frac{x'+y'}{\sqrt{2}}$$

putting $x = \frac{x'-y'}{\sqrt{2}}$ and $y = \frac{x'+y'}{\sqrt{2}}$ into eq^n (i),

we get,

$$\left(\frac{x'-y'}{\sqrt{2}}\right)^2 - \left(\frac{x'+y'}{\sqrt{2}}\right)^2 - 2\cdot\sqrt{2}\cdot\frac{x'-y'}{\sqrt{2}} - 10\cdot\sqrt{2}\cdot\frac{x'+y'}{\sqrt{2}} + 2 = 0$$

or, $\frac{x'^2 - 2x'y' + y'^2}{2} - \frac{x'^2 + 2x'y' + y'^2}{2} - 2x' + 2y' - 10x' - 10y' + 2 = 0$

or, $\frac{x'^2 - 2x'y' + y'^2 - x'^2 - 2x'y' - y'^2}{2} - 2x' + 2y' - 10x' - 10y' + 2 = 0$

or, $-2x'y' - 12x' - 8y' + 2 = 0$

or, $x'y' + 6x' + 4y' = 1$

removing suffixes we get,

$xy + 6x + 4y = 1$ which is required equation.

Q.3 Remove the first degree terms in each eq^n

a) $3x^2-4y^2-6x-8y-10=0$

b) $2x^2+5y^2-12x+10y-7=0$

c) $3x^2-4y^2+6x+24y-135=$

Sol^n: a) Given: $3x^2-4y^2-6x-8y-10=0$ - (i)

in eq^n (i) completing square we get,

$$3(x^2-2x)-4(y^2+2y)-10=0$$

$$\text{or, } 3\{(x-1)^2-1\}-4\{(y+1)^2-1\}-10=0$$

$$\text{or, } 3(x-1)^2-4(y+1)^2=9 \quad \text{..(ii)}$$

put $x-1=x'$ and $y+1=y'$ in eq^n (ii), we get

$3x'^2-4y'^2=9$; ~~this is satisfied only by $x'=0, y'=0$~~

(Ans)

c) Given:

$$3x^2-4y^2+6x+24y-135=0 \quad \text{...(i)}$$

in eq^n(i) completing square we get,

$$3(x^2+2x)-4(y^2-6y)-135=0$$

$$\text{or, } 3\{(x+1)^2-1\}-4\{(y-3)^2-9\}-135=0$$

$$\text{or, } 3(x+1)^2-3-4(y-3)^2+36-135=0$$

$$\text{or, } 3(x+1)^2-4(y-3)^2=102 \quad \text{..(ii)}$$

put, $x+2 = x'$, $y-3 = y'$ in eq^n(ii), we get,

$$3x'^2 - 4y'^2 = 102 \text{ (Ans)}$$

b) Given: $2x^2+5y^2-12x+10y-7=0$ ...(i)

in eq^n(i) completing square,

$$2(x^2-6x) + 5(y^2+5y) - 7 = 0$$

or, $2\{(x-3)^2-9\} + 5\{(y+1)^2-1\} - 7 = 0$

or, $2(x-3)^2 - 18 + 5(y+1)^2 - 5 - 7 = 0$

or, $2(x-3)^2 + 5(y+1)^2 = 30$ ...(ii)

~~in~~ put $x-3 = x'$, $y+1 = y'$ in eq^n(ii), we get,

$$2x'^2 + 5y' = 30 \text{ (Ans)}$$

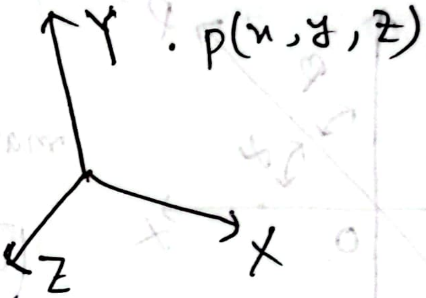
Solid Geometry

Lecture-10
10.09.2024

Rectangular co-ordinates

Formulae:

1. Position of a point in space is



2. Distance bet^n two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

3. Mid point of $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is

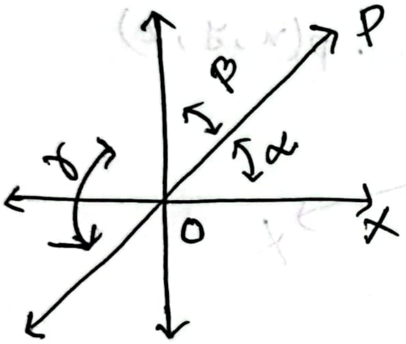
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

4. ভরকেন্দ্র (centroid) : $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$

Q1 Define direction cosines and direction ratios

⇒ **Direction cosines:** If a line OP makes angles α, β, γ with the positive direction of axes X, Y and Z respectively then $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are said to be direction cosines (d, c's) of the line OP

direction cosines



mathematically we write,

$$\boxed{\cos\alpha = l}, \boxed{\cos\beta = m}, \boxed{\cos\gamma = n}$$

⇒ **Define Ratios:** Any three numbers a, b and c which are proportional to the direction cosines l, m, n of a given line OP, then three numbers are said to be direction ratios of OP

Here, we can write

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c} = \frac{\sqrt{l^2+m^2+c^2}}{\sqrt{a^2+b^2+c^2}}$$

$$\therefore \frac{l}{a} = \frac{m}{b} = \frac{n}{c} = \pm\frac{1}{\sqrt{a^2+b^2+c^2}}$$

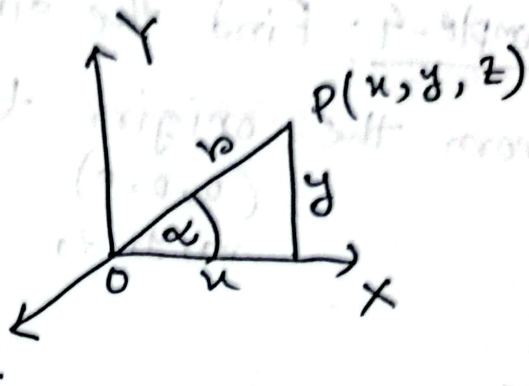
$l \propto a \Rightarrow \frac{l}{a} = K$

$m \propto b \Rightarrow \frac{m}{b} = K$

$n \propto c \Rightarrow \frac{en}{c} = K$

Relation between the direction cosines: (page-20 (17))

proof: Let $p(x, y, z)$ be any point in the ~~plane~~ space whose direction cosines are l, m, n.



Here,
$$\cos\alpha = l, \quad \cos\beta = m, \quad \cos\gamma = n$$

and, $OP = r = \sqrt{x^2+y^2+z^2}$ → দুই বিন্দুগামী স্থানাঙ্কের মধ্যবর্তী দূরত্ব ...

From figure,
$$\cos\alpha = \frac{x}{r}$$
$$\therefore x = lr \quad \cdots (i)$$

Similarly,
$$y = mr \quad \cdots (ii)$$
$$z = nr \quad \cdots (iii)$$

applying, $(i)^2 + (ii)^2 + (iii)^2$, we get,
$$x^2+y^2+z^2 = r^2(l^2+m^2+n^2)$$
or, $r^2 = r^2(l^2+m^2+n^2)$ $[\because r^2 = x^2+y^2+z^2$

$\therefore l^2+m^2+n^2 = 1$, which is required relation bet^n l, m, n.

(page-13) Solid geometry

Example-4: Find the direction cosines of the line drawn from the origin $(0,0,0)$ x_1, y_1, z_1 to the point $(-6, 2, 3)$ x_2, y_2, z_2

Soln: Let the co-ordinate of origin be $O(0,0,0)$

$$\therefore l = \cos\alpha = \frac{x_2 - x_1}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}}$$

$$= \frac{-6}{7}$$

$$l = \pm\frac{a}{\sqrt{a^2+b^2+c^2}}$$

$a \to (x_2-x_1)$, $b \to (y_2-y_1)$, $c \to (z_2-z_1)$

Similarly,

$$\therefore m = \cos\beta = \frac{2}{7}$$

$$n = \cos\gamma = \frac{3}{7}$$

$$m = \pm\frac{b}{\sqrt{a^2+b^2+c^2}}$$

$$n = \pm\frac{c}{\sqrt{a^2+b^2+c^2}}$$

Example-5: Find the direction cosines of the line which is equally inclined to the axes.

Soln: If the line makes angles α, β, γ with the axes then $\cos\alpha = \cos\beta = \cos\gamma$

$$\therefore \alpha = \beta = \gamma$$

Let the inclination of the line with the co-ordination es axes be α, β and γ. According to ~~qe~~ question,

$$\cos\alpha = \cos\beta = \cos\gamma \Rightarrow \alpha = \beta = \gamma \text{ (given)}$$

$$\therefore \frac{l}{1} = \frac{m}{1} = \frac{n}{1} = \pm \frac{\sqrt{l^2+m^2+n^2}}{\sqrt{1+1+1}} = \pm \frac{1}{\sqrt{3}} \quad \left[\because l^2+m^2+n^2 = 1\right]$$

$$\therefore l = \pm \frac{1}{\sqrt{3}}\ ; \ m = \pm \frac{1}{\sqrt{3}}\ ; \ n = \pm \frac{1}{\sqrt{3}}$$

$\therefore$ d, c's are $\left(\pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}\right)$

Formula : # Angle betn two lines whose direction cosines are (l_1, m_1, n_1) and (l_2, m_2, n_2) is

$$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$$

page-24 (exercise) Q.8 Find the cosines of the angle between two directed lines having direction cosines :

(a) $\frac{4}{9}, -8/9, -1/9\ ;\ -1/3, -2/3, -2/3$

(b) $\frac{1}{\sqrt{3}}, -1/\sqrt{3}, 1/\sqrt{3}\ ;\ 1/2, \sqrt{(6/4)}, -1/2$

Solⁿ: let,

(a) $l_1 = \frac{4}{9}$, $m_1 = -\frac{8}{9}$, $n_1 = -\frac{1}{9}$

$l_2 = -\frac{1}{3}$, $m_2 = -\frac{2}{3}$, $n_2 = -\frac{2}{3}$

We know that,

$$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2 \quad \cdots (1)$$

$$\therefore \cos\theta = \frac{4}{9} \times -\frac{1}{3} + \frac{-8}{9} \times \frac{-2}{3} + \frac{-1}{9} \times \frac{-2}{3}$$

$$= -\frac{4}{27} + \frac{16}{27} + \frac{2}{27}$$

$$= 14/27$$

$$\boxed{\therefore \theta = \cos^{-1} 14/27 \text{ (Ans)}}$$

(b) let, $l_1 = \frac{1}{\sqrt{3}}$, $m_1 = -\frac{1}{\sqrt{3}}$, $n_1 = \frac{1}{\sqrt{3}}$

$l_2 = \frac{1}{2}$, $m_2 = \sqrt{(6/4)}$, $n_2 = -\frac{1}{2}$

we know that,

$$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$$

$$\therefore \cos\theta = \frac{1}{\sqrt{3}} \times \frac{1}{2} + \frac{-1}{\sqrt{3}} \times \frac{\sqrt{6}}{\sqrt{4}} + \frac{1}{\sqrt{3}} \times \frac{-1}{2}$$

$$= -\frac{1}{\sqrt{2}} = \cancel{\sqrt{2}}$$

$\therefore \theta = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

Chapter: Plane (solid geometry)

① Eqn of plane $ax + by + cz + d = 0$

② Eqn of a plane passing through (x_1, y_1, z_1)

is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

③ Eqn of plane passes through three non-collinear point (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3)

is $\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = 0$ $\boxed{\text{or, } \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1}$

Lecture-11
23.07.2024

⊛ Angle betⁿ two planes $ax+by+cz+d=0$ and $a_1x+b_1y+c_1z+d_1=0$,

$$\cos\theta = \frac{aa_1+bb_1+cc_1}{\sqrt{a^2+b^2+c^2}\cdot\sqrt{a_1^2+b_1^2+c_1^2}}$$

Example-4: Given that,

$$2x-y+z=6$$
$$x+y+2z=7$$

Here,
$a=2,\ b=-1,\ c=1$
$a_1=1,\ b_1=1,\ c_1=2$

we know,

$$\cos\theta = \frac{2\times1+1(-1)+2\times1}{\sqrt{6}\times\sqrt{6}}$$
$$=\frac{1}{2}$$

$\therefore \cos\theta=\frac{1}{2} \Rightarrow \theta=\cos^{-1}\left(\frac{1}{2}\right)$
$= \pi/3 = 60^\circ$ (Ans)

Example-6 Find the eq^n of the plane through $(4,0,1)$ and parallel to the plane $4x+3y-12z+6=0$

Sol^n: Given plane,

$$4x+3y-12z+6=0$$

it's parallel plane,

$$4x+3y-12z+K=0 \quad \text{--- (i)}$$

which passes through $(4,0,1)$ then

$$\therefore 16+0-12+K=0$$

$$\therefore K=-4$$

putting $K=-4$ in eq^n (i) we get,

$4x+3y-12z-4=0$ which is the required eq^n.

Example: 1 Find the ~~eqn~~ eq^n of the plane through the points $(2,3,1)$, $(1,1,3)$, and $(2,2,3)$. Find also the perpendicular distance from the point ~~6~~ $(5,6,7)$ to this plane:

Sol^n: The required plane is

$$\begin{vmatrix} x & y & z & 1 \\ 2 & 3 & 1 & 1 \\ 1 & 1 & 3 & 1 \\ 2 & 2 & 3 & 1 \end{vmatrix} = 0$$

or, $+x\begin{vmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 2 & 3 & 1 \end{vmatrix} - y\begin{vmatrix} 2 & 1 & 1 \\ 1 & 3 & 1 \\ 2 & 3 & 1 \end{vmatrix} + z\begin{vmatrix} 2 & 3 & 1 \\ 1 & 1 & 1 \\ 2 & 2 & 1 \end{vmatrix} - 1\begin{vmatrix} 2 & 3 & 1 \\ 1 & 1 & 3 \\ 2 & 2 & 3 \end{vmatrix}$

or, $-2x + 2y + z - 3 = 0$

$\therefore \; 2x - 2y \cancel{+} - z + 3 = 0 \; \cdots$ (i) (Ans)

Perpendicular distance from the point $(5, 6, 7)$ to eq^n (i) is,

$$p = \frac{2.5 - 2.6 - 7 + 3}{\sqrt{2^2 + (-2)^2 + (-1)^2}} = -2 \; \cancel{(Ans)}$$

since distance cannot be negative. So,

p. distance $= 2$

Example - 5 Find the eq^n of the plane through the point $(4,-2,1)$ and parallel to the plane whose direction numbers are $7, 2, -3$.

Sol^n: The eq^n to the plane through $(4,-2,1)$ is

$$a(x-4)+b(y+2)+c(z-1)=0 \quad \cdots (i)$$

in eq^n(i) co-efficients a, b, c are proportional to $7, 2, -3$. $\therefore$ eq^n of required plane is,

$$7K(x-4)+2K(y+2)-3K(z-1)=0 \quad \cdots (ii)$$

in eq^n(ii) K is constant,

$$\therefore 7(x-4)+2(y+2)-3(z-3)=0$$

$$\text{or, } 7x-28+2y+4-3z+9=0$$

$$\therefore 7x+2y-3z-15=0 \quad \text{(Ans)}$$

Example - 2: Show the four points $(0,-1,-1)$, $(4,5,1)$, $(3,9,4)$ and $(-4,4,4)$ lie on a plane.

The four points are complanar if the determinant is zero.

$$\therefore \begin{vmatrix} 0 & -1 & -1 & 1 \\ 4 & 5 & 1 & 1 \\ 3 & 9 & 4 & 1 \\ -4 & 4 & 4 & 1 \end{vmatrix}$$

$$= +0 \begin{vmatrix} 5 & 1 & 1 \\ 9 & 4 & 1 \\ 4 & 4 & 1 \end{vmatrix} + 1 \begin{vmatrix} 4 & 1 & 1 \\ 3 & 4 & 1 \\ -4 & 4 & 1 \end{vmatrix} - 1 \begin{vmatrix} 4 & 5 & 1 \\ 3 & 9 & 1 \\ -4 & 4 & 1 \end{vmatrix} - 1 \begin{vmatrix} 4 & 5 & 1 \\ 3 & 9 & 4 \\ -4 & 4 & 4 \end{vmatrix}$$

$= 21 - 33 + 12$

$= 0$

Since determinant $= 0$, the four points are complanar.