CS177 Homework 4: Continuous Variables & Conditioning

Atif Ahmad

October 30, 2017

Question 1:

a) $P(X \leq x) = \phi(\frac{x-\mu}{\sigma})$

$P(X \geq 19) = 1 - P(X \leq 19) = 1 - \phi(\frac{19-15}{2.5}) = 1 - \phi(1.6)$

from scipy.stats import norm

value = norm.cdf(1.6)

probability = 1 - value

print probability

= 0.0547992916996

b) $P(12 \leq X \leq 18) = P(X \leq 18) - P(X \leq 12) = \phi(\frac{18-15}{2.5}) - \phi(\frac{12-15}{2.5}) = \phi(1.2) - \phi(-1.2)$

$= \phi(1.2) - (1 - \phi(1.2))$

value = norm.cdf(1.2, 15, 2.5)

probability = value - (1 - value)

print probability

= 0.769860659557

c) $0.01 = P(X \geq x) = 1 - P(X \leq x) = 1 - \phi(\frac{x-15}{2.5})$

$-0.99 = -\phi(\frac{x-15}{2.5})$

$0.99 = \phi(\frac{x-15}{2.5})$

Looked at Z - Score.

Z = 2.33

$2.33 = \frac{x-15}{2.5}$

5.825 = x - 15

$0.99 = P(X \leq x) = \phi(-\frac{x-15}{2.5})$

-5.825 = x - 15

x = 9.175 seconds

d) $\mu = (\frac{1}{5})15 = 3$ seconds

$\sigma = (\frac{1}{5})2.5 = 0.5$ seconds

Question 2:

a) $F_{X|G}(x \mid G=1) = \int_0^{90} cx dx = c[\frac{x^2}{2}\Big|_0^{90}] = c(\frac{90^2}{2}) = c(\frac{8{,}100}{2}) = c(4,050) = 1$

$c = \frac{1}{4{,}050}$

$F_{X|G}(x \mid G=1) = \int_{\frac{1}{4{,}050}} x^2 dx = \frac{x^2}{2(4{,}050)} = \frac{x^2}{8{,}100}$

$F_{X|G}(x \mid G=1) =$

$$\begin{cases} \frac{x^2}{8{,}100} & \text{if } 0 \leq x \leq 90 \\ 0 & \text{otherwise} \end{cases}$$

b) $E[X \mid G=1] = \int_0^{90} \frac{1}{4{,}050} x^2 dx = [\frac{x^3}{3(4{,}050)}\Big|_0^{90}] = \frac{90^3}{12{,}150} = 60$

$Var[X \mid G=1] = E[X^2 \mid G=1] - (E[X \mid G=1])^2$

$E[X^2 \mid G=1] = \int_0^{90} \frac{1}{4{,}050} x^3 dx = [\frac{x^4}{4(4{,}050)}\Big|_0^{90}] = \frac{90^4}{16{,}200} = 4,050$

$(E[X])^2 = (60)^2 = 3,600$

$Var[X \mid G=1] = 4,050 - 3,600 = 450$

c) $E[X] = 0.9(E[X \mid G=1]) + 0.1(E[X \mid G=0]) = 0.9(60) + 0.1(90) = 54 + 9 = 63$

$Var[X] = 0.9(Var[X \mid G=1]) + 0.1(Var[X \mid G=0]) = 0.9(450) + 0.1(0) = 450$

d) $0.8 = \int_0^x \frac{1}{4{,}050} t dt = \frac{1}{4{,}050}[\frac{t^2}{2}\Big|_0^{90}] = \frac{x^2}{2}(\frac{1}{4{,}050})$

$6480 = x^2$

x = 80.49845 minutes

e) $P(G=0) = 1 - P(G=1) =$ 1 - $\int_0^{45} \frac{1}{4{,}050} x dx =$ 1 - $\frac{1}{4{,}050}[\frac{x^2}{2}\Big|_0^{45}] =$ 1 - $\frac{2{,}025}{8{,}100} = \frac{6{,}075}{8{,}100} = 0.75$

Question 3:

pdf: $\lambda e^{-\lambda x}$
cdf: 1 - $e^{-\lambda x}$
mean: λ^{-1}
variance: λ^{-2}
standard deviation: λ^{-1}

Y = 0: correctly manufactured
mean = 1

Y = 1: malfunctions
mean = 50

H_0 = correctly manufactured
H_1 = malfunctions

a) $p_H(0) = 0.5$

$p_H(1) = 0.5$

$\lambda_{10} = \lambda_{01} = 1$

$\frac{p_{X|H}(x|1)}{p_{X|H}(x|0)} \geq 1$

$\frac{50e^{-50x}}{e^{-x}} \geq 1$

$50e^{-49x} \geq 1$

b) $p_H(0) = 0.99$

$p_H(1) = 0.01$

$\lambda_{10} = \lambda_{01} = 1$

$\frac{p_{X|H}(x|1)}{p_{X|H}(x|0)} \geq \frac{0.99}{0.01}$

$\frac{p_{X|H}(x|1)}{p_{X|H}(x|0)} \geq 99$

$\frac{50e^{-50x}}{e^{-x}} \geq 99$

$50e^{-49x} \geq 99$

Question 4:

a) $logf_{X|Y}(x_i \mid y_i = 1) = log[\prod_{j=1}^{M} \frac{1}{\sqrt{2\pi\sigma_{1j}^2}} e^{-\frac{(x_{ij}-\mu_{1j})^2}{2\sigma_{1j}^2}}$

$$= log(\prod_{j=1}^{M} \frac{1}{\sqrt{2\pi\sigma_{1j}^2}}) + log(e^{-\frac{(x_{ij}-\mu_{1j})^2}{2\sigma_{1j}^2}})$$

$$= \sum_{j=1}^{M} log(\frac{1}{\sqrt{2\pi\sigma_{1j}^2}}) - \frac{(x_{ij}-\mu_{1j})^2}{2\sigma_{1j}^2} log\ e$$

$$= \sum_{j=1}^{M} log((2\pi\sigma_{1j}^2)^{-\frac{1}{2}}) - \frac{(x_{ij}-\mu_{1j})^2}{2\sigma_{1j}^2}$$

$$= \sum_{j=1}^{M} (-\frac{1}{2} log((2\pi\sigma_{1j}^2))) - \frac{(x_{ij}-\mu_{1j})^2}{2\sigma_{1j}^2}$$

$$= -\frac{1}{2} \sum_{j=1}^{M} (log(2\pi) + log(\sigma_{1j}^2))) - \frac{(x_{ij}-\mu_{1j})^2}{2\sigma_{1j}^2}$$

$$= -\frac{1}{2} \sum_{j=1}^{M} log(2\pi) - \frac{1}{2} \sum_{j=1}^{M} (2\ log(\sigma_{1j})) - \frac{(x_{ij}-\mu_{1j})^2}{2\sigma_{1j}^2}$$

$$= -\frac{1}{2} log\ 2\pi(M) - \sum_{j=1}^{M} log\ \sigma_{1j} - \frac{(x_{ij}-\mu_{1j})^2}{2\sigma_{1j}^2}$$

$logf_{X|Y}(x_i \mid y_i = 0) = log[\prod_{j=1}^{M} \frac{1}{\sqrt{2\pi\sigma_{0j}^2}} e^{-\frac{(x_{ij}-\mu_{0j})^2}{2\sigma_{0j}^2}}$

$$= log(\prod_{j=1}^{M} \frac{1}{\sqrt{2\pi\sigma_{0j}^2}}) + log(e^{-\frac{(x_{ij}-\mu_{0j})^2}{2\sigma_{0j}^2}})$$

$$= \sum_{j=1}^{M} log(\frac{1}{\sqrt{2\pi\sigma_{0j}^2}}) - \frac{(x_{ij}-\mu_{0j})^2}{2\sigma_{0j}^2} log\ e$$

$$= \sum_{j=1}^{M} log((2\pi\sigma_{0j}^2)^{-\frac{1}{2}}) - \frac{(x_{ij}-\mu_{0j})^2}{2\sigma_{0j}^2}$$

$$= \sum_{j=1}^{M} (-\frac{1}{2} log((2\pi\sigma_{0j}^2))) - \frac{(x_{ij}-\mu_{0j})^2}{2\sigma_{0j}^2}$$

$$= -\frac{1}{2}\sum_{j=1}^{M}(log(2\pi) + log(\sigma_{0j}^2))) - \frac{(x_{ij} - \mu_{0j})^2}{2\sigma_{0j}^2}$$

$$= -\frac{1}{2}\sum_{j=1}^{M} log(2\pi) - \frac{1}{2}\sum_{j=1}^{M}(2\ log(\sigma_{0j})) - \frac{(x_{ij} - \mu_{0j})^2}{2\sigma_{0j}^2}$$

$$= -\frac{1}{2}log\ 2\pi(M) - \sum_{j=1}^{M} log\ \sigma_{0j} - \frac{(x_{ij} - \mu_{0j})^2}{2\sigma_{0j}^2}$$

b) See code.

```
pY0 = 0.5
pY1 = 0.5
variance0 = 1
variance1 = 1
fXcondY0 = np.zeros(M)
fXcondY1 = np.zeros(M)
for i in range(0, M):
    fXcondY0[i] = -(M / 2) * math.log (2 * math.pi) - (math.pow(trainFeat[i][i] - muhat[0][i],
2) / 2 * variance0)
    fXcondY1[i] = -(M / 2) * math.log (2 * math.pi) - (math.pow(trainFeat[i][i] - muhat[1][i],
2) / 2 * variance1)
    if(math.log(pY1) + fXcondY1[i] > math.log(pY0) + fXcondY0[i]):
        yHat[i] = 1
    else:
        yHat[i] = 0
```

Test accuracy: 0.916667

Test false alarms: 31

Test missed detections: 23

c) See code.

Test accuracy: 0.879630

Test false alarms: 56

Test missed detections: 22

d) See code.

Test accuracy: 0.879630

Test false alarms: 56

Test missed detections: 22