CS177 Homework 3: Expectation & Empirical Distributions

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Question 1:

a) pX(2) = $\binom{n}{2}(\frac{1}{2})^{2-1}$

Let X(n) = the number of edges as a function of the number of nodes, n.

$E[X(n)]$ = n $\binom{n}{2}$ $(\frac{1}{2})$

$E[X(10)] = 10\ \binom{10}{2}\ (\frac{1}{2}) = 10\ (\frac{10!}{2!,8!})\ (\frac{1}{2}) = 10\ (\frac{10\cdot 9\cdot 8!}{2!,8!})\ (\frac{1}{2}) = 225$

b) pX(3) = $\binom{n}{3}(\frac{1}{2})^{3-1}$

Let X(n) = the number of edges as a function of the number of nodes, n.

$E[X(n)]$ = n $\binom{n}{3}$ $(\frac{1}{4})$

$E[X(10)] = 10\ \binom{10}{3}\ (\frac{1}{4}) = 10\ (\frac{10\cdot 9\cdot 8\cdot 7!}{3!,7!})\ (\frac{1}{4}) = 10\ (\frac{10\cdot 9\cdot 8}{3\cdot 2\cdot 1})\ (\frac{1}{4}) = 30$

c) pX(k >= 3) = $\binom{n}{3}(\frac{1}{2})^{3-1} + \binom{n}{4}(\frac{1}{2})^{4-1} + \binom{n}{5}(\frac{1}{2})^{5-1} + \cdots + \binom{n}{10}(\frac{1}{2})^{10-1}$

Let X(n) = the number of edges as a function of the number of nodes, n.

$E[X(n)]$ = n$[\binom{n}{3}(\frac{1}{2})^2 + \binom{n}{4}(\frac{1}{2})^3 + \binom{n}{5}(\frac{1}{2})^4 + \cdots + \binom{n}{10}(\frac{1}{2})^9]$

$E[X(10)] = 10[\binom{10}{3}(\frac{1}{2})^2 + \binom{10}{4}(\frac{1}{2})^3 + \binom{10}{5}(\frac{1}{2})^4 + \binom{10}{6}(\frac{1}{2})^5 + \binom{10}{7}(\frac{1}{2})^6 + \binom{10}{8}(\frac{1}{2})^7 + \binom{10}{9}(\frac{1}{2})^8$
$+ \binom{10}{10}(\frac{1}{2})^9]$

$= 10[30 + \frac{210}{8} + \frac{252}{16} + \frac{210}{32} + \frac{120}{64} + \frac{45}{128} + \frac{10}{256} + \frac{1}{512}]$

$= 80.8301$

Question 2:

a) $10 = \frac{1}{\theta}$

$\theta = \frac{1}{10} = 0.1$

E[X] $= \int_0^4 t(0.1)(0.1)e^{-0.1t}dt$

$=$ [-t$(0.1)e^{-0.1t}\Big|_0^4$] - $(0.1)\int_0^4 -e^{-0.1t}dt$

$=$ [-4$(0.1)e^{-0.1(4)}$ + $(0)(0.1)e^{-0.1(0)}$] + $0.1[\frac{-1}{0.1}e^{-0.1t}\Big|_0^4]$

$=$ -4$(0.1)e^{-0.4}$ + $(0.1)(\frac{-1}{0.1}e^{-0.4}$ + $\frac{1}{0.1}e^{-0.1(0)})$

$=$ -4$(0.1)e^{-0.4}$ + (0.1)(-10$e^{-0.4}$ + $\frac{1}{0.1}$)

$=$ -14$(0.1)e^{-0.4}$ + (0.1)10

$= 0.615519$

b) $15 = \frac{1}{\theta}$

$\theta = \frac{1}{15}$

$P(0 <= X <= 4) = \int_0^4 (1/15)e^{(-1/15)t}dt$

$= (1/15)\int_0^4 e^{(-1/15)t}dt$

$=$ (1/15)[-15$e^{-(1/15)t}\Big|_0^4$]

$=$ (1/15) [-15$e^{-4/15}$ + 15]

$= 0.234072$

c) $10 = \frac{1}{\theta}$

$\theta = \frac{1}{10} = 0.1$

$P(0 <= X <= 1) = \int_0^1 (0.1)e^{-0.1t}dt$

$= 0.1\int_0^1 e^{-0.1t}dt$

$=$ 0.1[(-1/0.1)$e^{-0.1t}\Big|_0^1$]

$=$ 0.1((-1/0.1)$e^{-0.1}$ + (1/0.1))

$=$ -$e^{-0.1}$ + 1

$= 0.095163$

d) 10 - 6 $= 4 = \frac{1}{\theta}$

$\theta = \frac{1}{4} = 0.25$

$P(0 <= X <= 1) = \int_0^1 (0.25)e^{-0.25t}dt$

$= 0.25\int_0^1 e^{-0.25t}dt$

$= 0.25[(\text{-}1/0.25)e^{-0.25t}\Big|_0^1]$

$= 0.25((\text{-}1/0.25)e^{-0.25} + (1/0.25))$

$= \text{-}e^{-0.25} + 1$

$= 0.221199$

Question 3:

a) $P(X_1 > X_2) = 1 - P(X_1 \leq X_2) = 1 - \int_{X_1}^{X_2} \frac{1}{100-0} dx$

$= 1 - \frac{1}{100} \int_{X_1}^{X_2} dx = \frac{1}{100}(X_2 - X_1) = \frac{100 - X_2 + X_1}{100}$

b) $P(X_1 > X_2, X_1 > X_3) = 1 - P(X_1 \leq X_2, X_1 \leq X_3) = 1 - P(X_1 \leq X_2)P(X_1 \leq X_3)$

$= 1 - (\int_{X_1}^{X_2} \frac{1}{100} dx)(\int_{X_1}^{X_3} \frac{1}{100} dx) = 1 - (\frac{X_2 - X_1}{100})(\frac{X_3 - X_1}{100}) = 1 - \frac{(X_2 - X_1)(X_3 - X_1)}{10{,}000}$

$= \frac{10{,}000 - (X_2 - X_1)(X_3 - X_1)}{10{,}000}$

c) $P(X_1 > X_2 \mid X_1 > X_3) = 1 - P(X_1 \leq X_2 \mid X_1 \leq X_3) = 1 - \frac{P(X_1 \leq X_2, X_1 \leq X_3)}{P(X_1 \leq X_3)}$

$= 1 - \frac{P(X_1 \leq X_2)P(X_1 \leq X_3)}{P(X_1 \leq X_3)} = 1 - P(X_1 \leq X_2) = 1 - \int_{X_1}^{X_2} \frac{1}{100} dx$

$= 1 - \frac{1}{100} \int_{X_1}^{X_2} dx = 1 - \frac{X_2 - X_1}{100} = \frac{100 - X_2 + X_1}{100}$

d) $P(X_1 > X_2 \mid X_2 > X_3) = 1 - P(X_1 \leq X_2 \mid X_2 \leq X_3) = 1 - \frac{P(X_1 \leq X_2, X_2 \leq X_3)}{P(X_2 \leq X_3)}$

$= 1 - \frac{P(X_1 \leq X_2)P(X_2 \leq X_3)}{P(X_2 \leq X_3)} = 1 - P(X_1 \leq X_2) = 1 - \int_{X_1}^{X_2} \frac{1}{100} dx$

$= 1 - \frac{1}{100} \int_{X_1}^{X_2} dx = 1 - \frac{X_2 - X_1}{100} = \frac{100 - X_2 + X_1}{100}$

e) $P(N > n) = 1 - P(N \leq n) = 1 - \int_{N}^{n} \frac{1}{100} dx$

$= 1 - \frac{1}{100}(\text{n - N}) = \frac{100 - n + N}{100}$

f) $\text{E[N]} = \int_0^N (\frac{1}{100} x(x - X_1)) dx = \frac{1}{100} \int_0^N x(x - X_1) dx$

$= \frac{1}{100} \int_0^N (x^2 - X_1 x) dx = \frac{1}{100} [\frac{x^3}{3} \Big|_0^N] - X_1 [\frac{x^2}{2} \Big|_0^N]$

$= \frac{1}{100}(\frac{N^3}{3}) - X_1(\frac{N^2}{2}) = \frac{N^3 - 150N^2 X_1}{300}$

$= \frac{N^2(N - 150X_1)}{300}$

Question 4:

a) sum = 0

```
for i in range(0, n):
    value = S[i] * S[i]
    sum = sum + value
expectSSquared = sum/n
meanSSquared = meanS * meanS
VarS = expectSSquared - meanSSquared
sum = 0
for i in range(0, n):
    value = T[i] * T[i]
    sum = sum + value
expectTSquared = sum/n
meanTSquared = meanT * meanT
VarT = expectTSquared - meanTSquared
print 'Var[S] = ', VarS
print 'Var[T] = ', VarT
```

Var[S] = 1.29793889045

Var[T] = 184.143814879

b) See code.

s1bar = 68.0

s2bar = 136.0

s3bar = 204.0

t1bar = 110.0

t2bar = 178.0

t3bar = 246.0

c) See code.

pXY(x,y) = 0.958187292764

pX(x) = 0.0468883578777

pY(y) = 0.953111642122

d) See code.

pX(x)*pY(y) = 0.0446898397732

They are not independent because $p_X(x) \cdot p_Y(y) \neq p_{XY}$(x,y).