CS177 Homework 2: Expectation & Conditional Independence

Atif Ahmad

October 16, 2017

Question 1:

a) $E[X] = \binom{3}{0} \times (0 working) \times (3 failures) + \binom{3}{1} \times (1 working) \times (2 failures) + \binom{3}{2} \times (2 working) \times (1 failure)$
$= 3(1-p)p^2 + 3(1-p)^2 p$
$= 3(1-p)p[p + (1-p)]$
$= 3(1-p)p(1)$
$= 3(1-p)p$
$P(p) = P(X = 2) = \binom{3}{2} p^2(1-p) = 3p^2(1-p)$

b) $E[X] = \binom{5}{0} \times (0 working) \times (5 failures) + \binom{5}{1} \times (1 working) \times (4 failures) + \binom{5}{2} \times (2 working) \times (3 failures) + \binom{5}{3} \times (3 working) \times (2 failures)$
$= 5(1-p)p^4 + \frac{5!}{2!\,3!}(1-p)^2 p^3 + \frac{5!}{3!\,2!}(1-p)^3 p^2$
$= 5(1-p)p^4 + 10(1-p)^2 p^3 + 10(1-p)^3 p^2$
$= 5(1-p)p^2[p^2 + 2(1-p)p + 2(1-p)^2]$
$= 5(1-p)p^2[p^2 + 2(1-p)[p + (1-p)]]$
$= 5(1-p)p^2(p^2 + 2 - 2p)$
$= (5-5p)p^4 + 2(5-5p)p^2 - (5-5p)(2p^3)$
$= 5p^4 - 5p^5 + 10p^2 - 10p^3 - 10p^3 + 10p^4$
$= 15p^4 - 5p^5 + 10p^2 - 20p^3$
$= 10p^2 - 20p^3 + 15p^4 - 5p^5$
$= 5p^2(2 - 4p + 3p^2 - p^3)$
$P(p) = P(X = 3) = \binom{5}{3}(1-p)^3 p^2 = 10(1-p)^3 p^2$

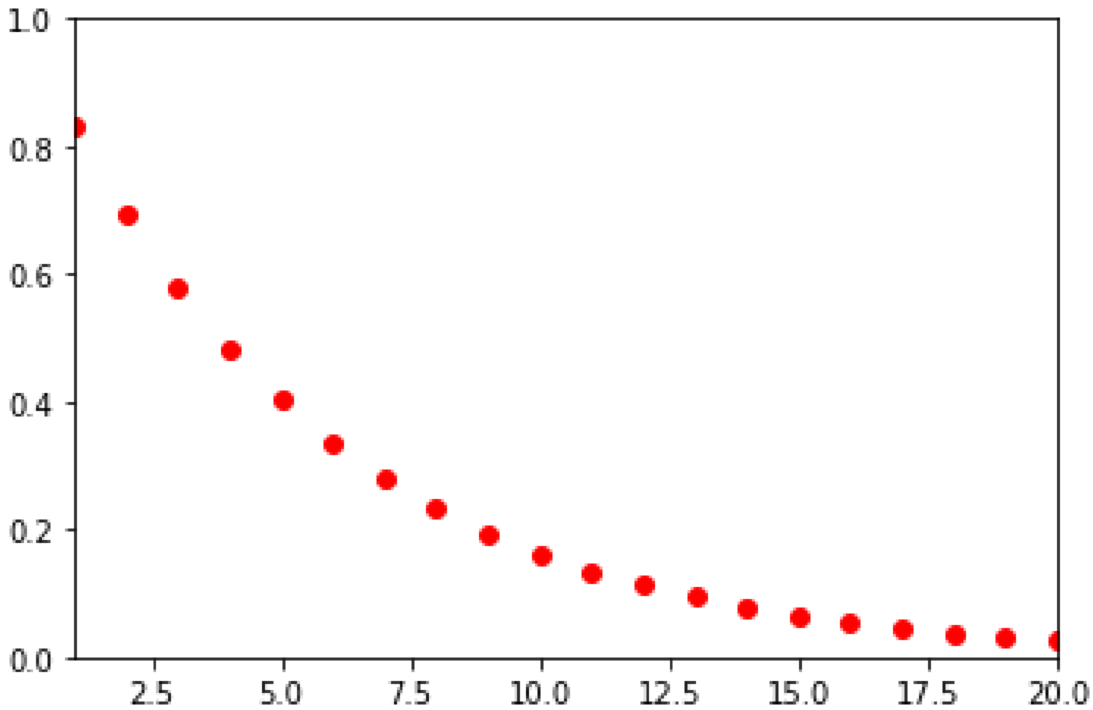
c) $(a.)P(0.1) = 3(0.1)^2(1-0.1) = 3(0.1)^2(0.9) = 0.027$
$(b.)P(0.1) = 10(1-0.1)^3(0.1)^2 = 10(0.9)^3(0.1)^2 = 0.0729$
Part (b.) is more reliable.

d) $(a.)P(0.6) = 3(0.6)^2(1-0.6) = 3(0.36)(0.4) = 0.432$

$(b.)P(0.6) = 10(1-0.6)^3(0.6)^2 = 10(0.4)^3(0.36) = 10(0.064)(0.36) = 0.2304$

Part (a.) is more reliable.

2a Plot.png



Question 2:

a) $P(\text{Getting a 6}) = \frac{1}{6}$

$E[X] = \binom{n}{0}$ ·(n not 6's)· (0 6's) + $\binom{n}{1}$ ·(n - 1 not 6's)· (1 6's)

$+ \cdots + \binom{n}{n}$ ·(0 not 6's)· (n 6's)

n = 1: $E[X] = 0 \cdot \frac{1}{6} +$ (v) $\cdot \frac{5}{6}$

n = 2: $E[X] = \binom{2}{0}$(v + w) $(\frac{5}{6})^2$

n = 3: $E[X] = \binom{3}{0}$(v + w + z) $(\frac{5}{6})^3$

n = 4: $E[X] = \binom{3}{0}$(v + w + z + x) $(\frac{5}{6})^3$

.

.

.

For n, $E[X] =$ C$(\frac{5}{6})^3$, for some C > 0.

The smallest n that maximizes the expected payoff is n = 1.

b) $E[X] = \binom{10}{5}$ $(\frac{5}{6})(\frac{4}{5})(\frac{3}{4})(\frac{2}{3})(\frac{1}{2})$

$= \frac{10!}{5!\,5!}(\frac{1}{6})$

$= 252(\frac{1}{6})$

$= 42$

Question 3:

a) p(C = 1) = pXYC(X = 0, Y = 0, C = 1) + pXYC(X = 0, Y = 1, C = 1) + pXYC(X = 1, Y = 0, C= 1) + pXYC(X = 1, Y = 1, C = 1)

= 0.0 + 0.1 + 0.05 + 0.25 = 0.4

b) $p(C = 0 \mid X = 1, Y = 0) = \frac{pXYC(X=1,Y=0,C=0)}{pxY(X=1,Y=0)} = \frac{0.2}{0.2+0.05} = 0.8$

c) p(X = 0, Y = 0) = pXYC(X = 0, Y = 0, C = 0) + pXYC(X = 0, Y = 0, C = 1) = 0.1 + 0.0 = 0.1

d) $p(C = 0 \mid X = 0) = \frac{p(C=0,X=0}{p(X=0} = \frac{0.1+0.2}{0.1+0.2+0.0+0.1} = \frac{0.3}{0.4} = 0.75$

e) p(X, Y) = 0.1 + 0.2 + 0.2 + 0.1 + 0.0 + 0.1 + 0.05 + 0.25 = 1

p(X) = 0.1 + 0.2 + 0.2 + 0.1 + 0.0 + 0.1 + 0.05 + 0.25 = 1

p(Y) = 0.1 + 0.2 + 0.2 + 0.1 + 0.0 + 0.1 + 0.05 + 0.25 = 1

p(X)p(Y) = (1)(1) = 1 = p(X, Y)

Yes. p(X, Y) = p(X)p(Y)

f) $p(X, Y \mid C) = \frac{p(X,Y,C)}{p(X,Y)} = \frac{0.1+0.2+0.2+0.1+0.0+0.1+0.05+0.25}{0.1+0.2+0.2+0.1+0.0+0.1+0.05+0.25} = 1$

$p(X \mid C) = \frac{p(X,C)}{p(X)} = \frac{0.1+0.2+0.2+0.1+0.0+0.1+0.05+0.25}{0.1+0.2+0.2+0.1+0.0+0.1+0.05+0.25} = 1$

$p(Y \mid C) = \frac{p(Y,C)}{p(Y)} = \frac{0.1+0.2+0.2+0.1+0.0+0.1+0.05+0.25}{0.1+0.2+0.2+0.1+0.0+0.1+0.05+0.25} = 1$

$p(X \mid C)p(Y \mid C) = (1)(1) = 1 = p(X, Y \mid C)$

Yes. $p(X, Y \mid C) = p(X \mid C)p(Y \mid C)$

Question 4:

a) $P(Yi = S \mid Xi) > P(Yi = H \mid Xi)$

$(P(Xi \mid Yi = S)P(Yi = S))/P(Xi) > (P(Xi \mid Yi = H)P(Yi = H))/P(Xi)$

Since $P(Xi) > 0$, we can ignore the denominator.

$P(Xi \mid Yi = S)P(Yi = S) > P(Xi \mid Yi = H)P(Yi = H)$

By Equation (1),

$P(Xi \mid Yi = S)(0.5) > P(Xi \mid Yi = H)(0.5)$

$P(Xi \mid Yi = S) > P(Xi \mid Yi = H)$

b) See code.

c) See code.

d) See code.

e) See code.

f) See code.

g) See code.