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Date

A6:- To show that the average degree $\langle k \rangle$ of an undirected & unweighted graph equals $\frac{2L}{N}$, we can see that :-

- In an undirected graph, each edge corresponds 2 to the sum of the degrees because an edge connects 2 vertices. Therefore the sum of degrees of all vertices is twice the no. of edges (L)

- Mathematically lets denote k_i as the degree of vertex i. Then the sum of degrees $\sum_{i=1}^{N} k_i$ is equal to $2L$, when L is the no. of edges, & N is the no. of vertices.

- Average degree is defined as sum of degrees of all vertices divided by total no. of vertices N.

$$\langle k \rangle = \frac{\sum_{i=1}^{N} k_i}{N} = \frac{2L}{N}$$

A7:- a) Suppose adj mat = A, vector = K

A vector K whose elements are degrees k_i of all nodes would be :-

$$K = A \times [1, 1, \ldots, 1]'$$

It will be the sum of elements of ith row.

Neelgagan

No.

Date

The degree k_i of a node in an undirected graph is the no. of edges connected to it. In terms of adjacency matrix, it is the sum of elements in the i^{th} row. We can find it by multiplying the adjacency matrix A by a vector of ones, because matrix multiplication essentially sums to 1.

$$K = A \times \overbrace{[1, 1, \ldots, 1]}^{n}{}^{T}$$

b) The total no. of links (edges) in an undirected graph is half the sum of the degrees of all nodes, because each edge is counted twice when summing the degrees.

(Dot product)

$$L = \frac{1}{2} \cdot \left(A \times \overbrace{[1, 1, \ldots, 1]}^{n}\right) \cdot 1$$

c) A triangle in the graph corresponds to a set of 3 nodes where each node is connected to the other two. In terms of adjacency matrix, this corresponds to a set of 3 nodes i, j, k such that $A_{ij} = A_{jk} = A_{ki} = 1$. The trace of A^3 gives us the no. of such sets, because in the product A^3, the (i,i) element is the no. of paths ~~lengths~~ of lengths 3 that start & ends at node i, and a triangle is such a path. Since each triangle is counted 6 times

$$T = \frac{1}{6} \operatorname{trace}(A^3)$$

d) The ith element of K_{nn} is the sum of degrees of neighbours of node i. To find this, we first find the degree of each node as in part a. Then we use the adjacency matrix to sum these degrees for the neighbours of each node.

$$K_{nn} = A \cdot K$$

e) To find the sum of the degrees of the second neighbours we have to look 2 steps away in the adj. matrix. This can be computed as the square of the adjacency matrix multiplied by the degree vector.

$$K_{nnn} = A^2 \cdot K$$