# Reinforcement Learning II

CSE 4617: Artificial Intelligence

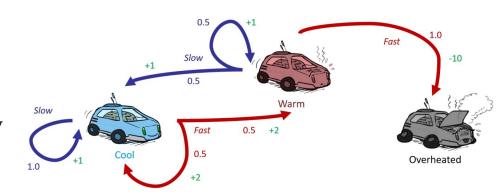


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# Formalizing RL Problems

- Still assume an MDP
  - $\circ$  A set of states  $s \in S$
  - $\circ$  A set of actions per state A
  - $\circ$  A probability model T(s, a, s')
  - $\circ$  A reward function R(s, a, s')
- Still trying to find the most optimal policy  $\pi^*$
- We do not know T(s, a, s') and R(s, a, s')
- Must actually try out actions and states to learn



### MDP vs RL

### Known MDP: Offline Solution

| Goal | Technique |
|------|-----------|
| Goal | recrimque |

Compute V\*, Q\*,  $\pi$ \* Value / policy iteration

Evaluate a fixed policy  $\pi$  Policy evaluation

### Unknown MDP: Model-Based

### Goal Technique

Compute V\*, Q\*,  $\pi$ \* VI/PI on approx. MDP

Evaluate a fixed policy  $\pi$  PE on approx. MDP

#### Unknown MDP: Model-Free

Goal Technique

Compute V\*, Q\*,  $\pi$ \* Q-learning

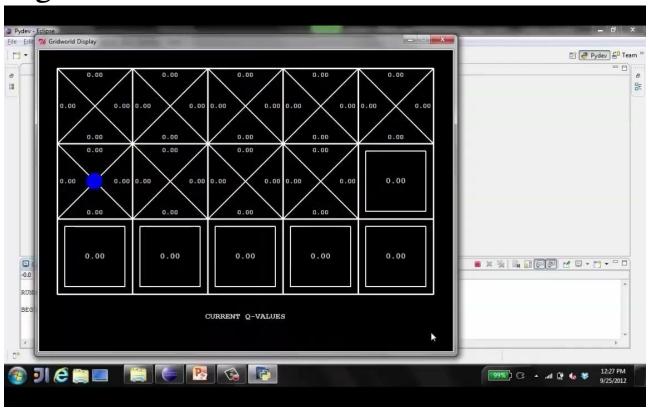
Evaluate a fixed policy  $\pi$  Value Learning

# Q-Learning

### Learn Q(s, a) as you go

- Receive a sample (s, a, s', r)
- Consider the old estimate Q(s, a)
- New sample estimate  $\rightarrow sample = R(s, a, s') + \gamma \max_{a'} Q(s', a')$
- Incorporate the new estimate into the running average
  - $\circ \quad Q(s, a) \leftarrow (1 \alpha)Q(s, a) + \alpha (sample)$

## Q-Learning



# **Q-Learning Properties**

- Q-learning converges to optimal policy → even if you're acting suboptimally!
- This is called off-policy learning

### Cons:

- You have to explore enough
- You have to eventually make the learning rate  $(\alpha)$  small enough
- But not decrease the learning rate  $(\alpha)$  too quickly

### How to Explore?

### Schemes for forcing exploration

- Random action (ε-greedy):
  - Every time step, flip a coin
  - $\circ$  With a very small probability ( $\epsilon$ ), stop following the established policy and act randomly
  - $\circ$  With a large probability (1- $\varepsilon$ ), keep following the established policy
  - Setting ε to a fixed value ensures sufficient exploration but the agent will not get a chance to use what it has learned
  - Gradually decrease ε

### **Exploration Functions**

Explore areas whose **badness/goodness** is not (yet) established, eventually stop exploring

- Takes a value estimate u and a visit count n, and returns an optimistic utility: f(u, n) = u + k/(n+1)
- Modified Q-update:

$$Q(s,a) \leftarrow_{\alpha} R(s,a,s') + \gamma \max_{a'} f(Q(s',a'), N(s',a'))$$



# Regret

# Regret



- Even if you learn the optimal policy, you still make mistakes along the way!
- Regret is the difference between the total expected rewards and optimal expected rewards
- Gives a notion of how quickly an agent learns
- Minimizing regret means you will optimally learn how to be optimal!
- Random exploration and exploration functions both end up optimal, but random exploration has higher regret

# Approximate Q-Learning

# Generalizing Across States



- Simple Q-Learning keeps a table of all q-values →
   Or in a dictionary!
- In realistic situations, we cannot possibly learn about every single state! → Too many states!
  - Too many states to visit them all in training
  - Too many states to hold the q-tables in memory
- A better idea is to generalize to unseen states
  - Learn about a small number of states
  - Generalize the experience to novel yet similar states
  - This idea can be borrowed from machine learning

### Generalizing Across States

Let's say we discover through experience that this state is bad:

In naïve q-learning, we know nothing about this state:

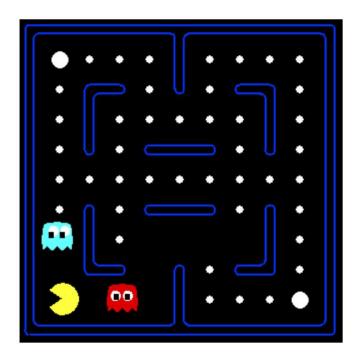
Or even this one!







### Feature-based Representations



- Describe a state using a vector of features
- Features are functions from states to real numbers (often [0, 1]) that capture important properties of the state
- They can be hand crafted → Similar to evaluation functions of a state
  - Distance to closest ghost
  - Distance to closest food
  - Number of ghosts
  - $\circ$  1 / (distance to food)<sup>2</sup>
  - Is Pacman in a tunnel?  $(0/1) \rightarrow \text{True/False}$
- Or these vectors can be learned → Deep Learning

### Linear Value Functions

Using a feature representation, we can write a q-function (or value function) for any state using a few weights:

$$V(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \dots + w_n f_n(s,a)$$

- Q-learning will learn the weights  $(w_1, w_2 \dots w_n)$  to approximate the values of the states
- If the number of features are not adequate, your agent will not be able to tell states apart, even though they are very different from each other

## Approximate Q-Learning

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \dots + w_n f_n(s,a)$$

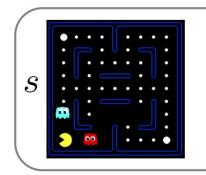
Q-Learning with linear Q-functions:

$$\begin{aligned} & \text{transition } = (s, a, r, s') \\ & \text{difference} = \left[r + \gamma \max_{a'} Q(s', a')\right] - Q(s, a) \\ & Q(s, a) \leftarrow Q(s, a) + \alpha \text{ [difference]} \end{aligned} \quad \begin{aligned} & \text{Exact Q's} \\ & w_i \leftarrow w_i + \alpha \text{ [difference]} f_i(s, a) \end{aligned} \quad \text{Approximate Q's} \end{aligned}$$

• If something unexpectedly bad happens, blame the features that were on: disprefer all states with that state's features

### Approximate Q-Learning

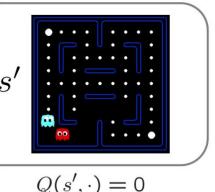
$$Q(s, a) = 4.0 f_{DOT}(s, a) - 1.0 f_{GST}(s, a)$$



$$f_{DOT}(s, NORTH) = 0.5$$

$$f_{GST}(s, NORTH) = 1.0$$

a = NORTH r = -500



$$Q(s, NORTH) = +1$$

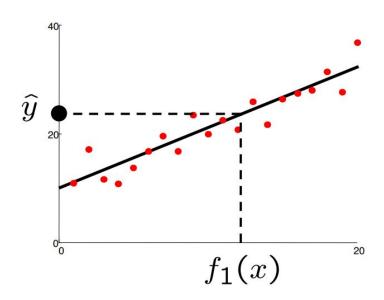
$$r + \gamma \max_{a'} Q(s', a') = -500 + 0$$

$$w_{DOT} \leftarrow 4.0 + \alpha [-501] \ 0.5$$
  
 $w_{GST} \leftarrow -1.0 + \alpha [-501] \ 1.0$ 

difference = 
$$-501$$

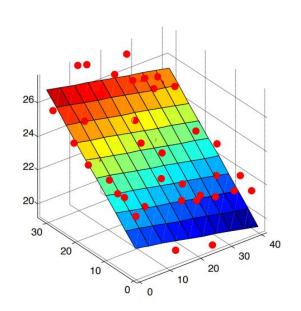
# Least Squares Approximation

### Linear Regression



### Prediction:

$$\hat{y} = w_0 + w_1 f_1(x)$$

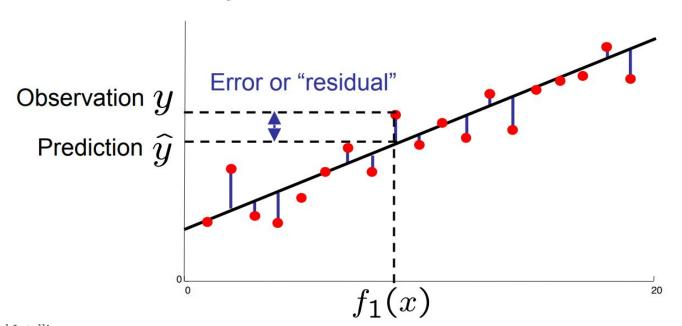


### Prediction:

$$\hat{y}_i = w_0 + w_1 f_1(x) + w_2 f_2(x)$$

### Linear Regression

total error = 
$$\sum_{i} (y_i - \hat{y}_i)^2 = \sum_{i} \left( y_i - \sum_{k} w_k f_k(x_i) \right)^2$$



# Linear Regression

$$\operatorname{error}(w) = \frac{1}{2} \left( y - \sum_{k} w_{k} f_{k}(x) \right)^{2}$$

$$\frac{\partial \operatorname{error}(w)}{\partial w_{m}} = -\left( y - \sum_{k} w_{k} f_{k}(x) \right) f_{m}(x)$$

$$w_{m} \leftarrow w_{m} + \alpha \left( y - \sum_{k} w_{k} f_{k}(x) \right) f_{m}(x)$$

$$w_{m} \leftarrow w_{m} + \alpha \left[ r + \gamma \max_{k} Q(s', a') - Q(s, a) \right] f_{m}(s, a)$$

"target"

"prediction"

Error or "residual"

# Thank you