

Adversarial Search

CSE 4617: Artificial Intelligence



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AI in Games



- 1950: First computer player
- 1994: Computer beats the best human player
- 2007: Checkers is solved !



AI in Games

Me after playing a game
with Stockish 15



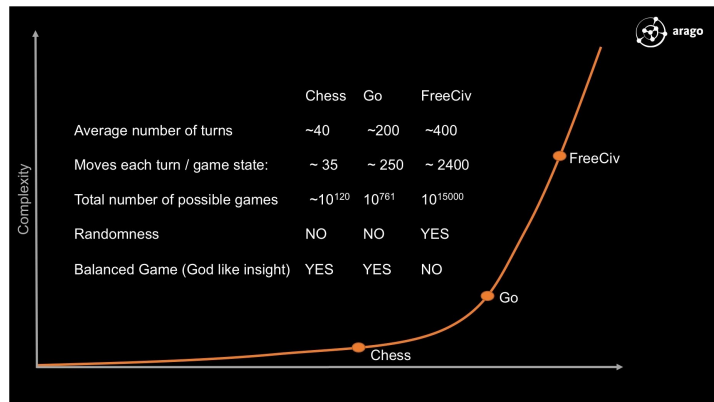
- 1997: Deep Blue defeats human champion
- Chess is still unsolved

"Inevitably the machines must win, but there is still a long way to go before a human on his or her best day is unable to defeat the best computer."

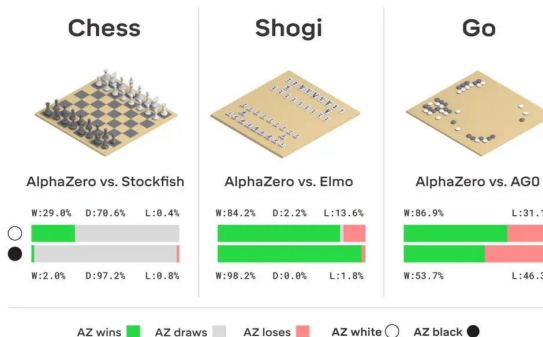
— Garry Kasparov



AI in Games



- 2010s: Human champions are now starting to be challenged by machines
- 2016: Alpha GO defeats human champion
- 2017: Introduced alpha zero



Who would win?



The world's most acclaimed chess computer. With access to the worlds combined chess experience developed over centuries and the ability to evaluate 70 million positions per second.

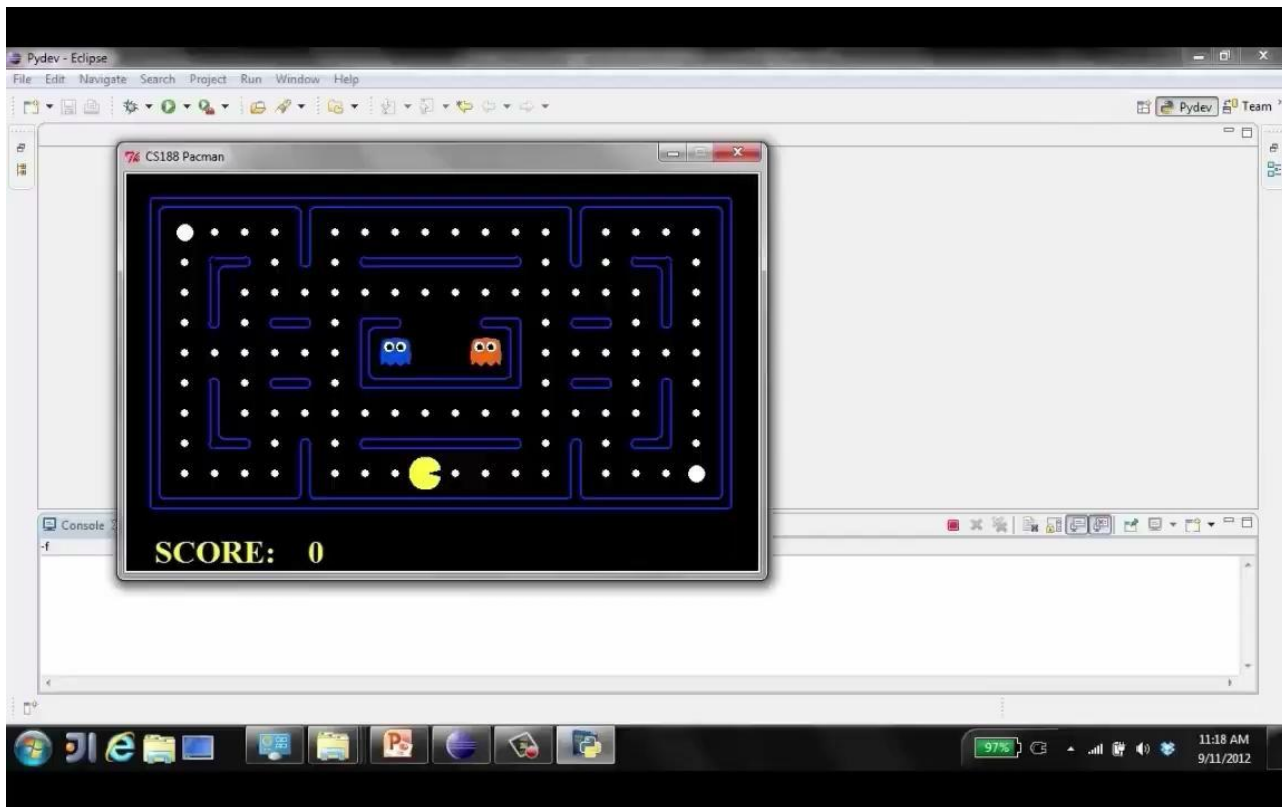


one nully boi
with 4 hours of
preperation

AI in Games



AI in Games



Types of Games

Types of Games

- Deterministic or Stochastic?
- One, Two or more players?
- Zero sum game?
- Perfect information of states?

Want algorithms for calculating a **strategy (policy)** which recommends a move from each state

Solved Game → Can force a draw if both players play optimally

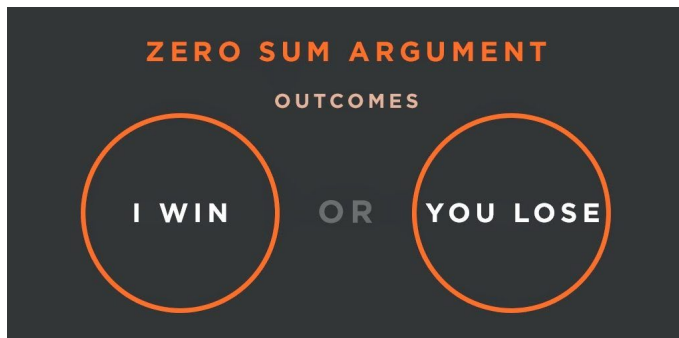
When the homie about to win and you been plotting for a minute



Types of Games

Zero-sum Games:

- Agents have opposite utilities (opposite goals)
- Lets us think of a single value that one maximizes and the other minimizes
- Adversarial, pure competition



General Games:

- Agents have independent utilities
- Cooperation, indifference, competition, and more are all possible
- More later on non-zero-sum games



Formalizing Deterministic Games

- States $\rightarrow S$ (Start state = s_o)
- Players $\rightarrow P = \{1, 2, \dots, N\}$ (Usually takes turns)
- Actions $\rightarrow A$ (Similar to search)
 - May depend on the player/state
- Transition Function \rightarrow Similar to successor function
- Terminal Test $\rightarrow S = \{true, false\}$
 - Example terminal states: Win, Lose, Draw
- Terminal Utilities \rightarrow Assign certain rewards/scores to all outcomes of a game
- Solution for a player is to find a **policy**

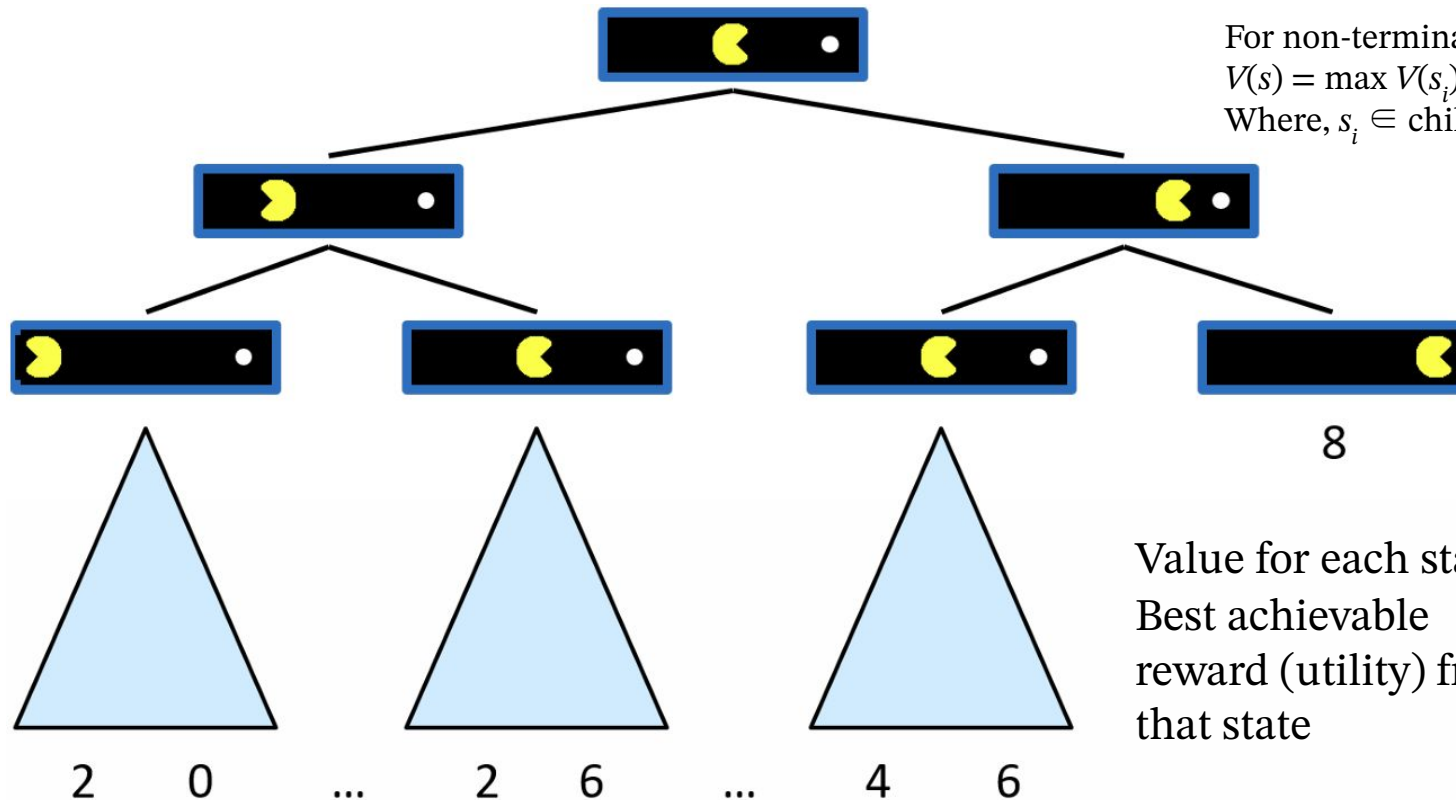


Adversarial Search

Single-Agent Tree

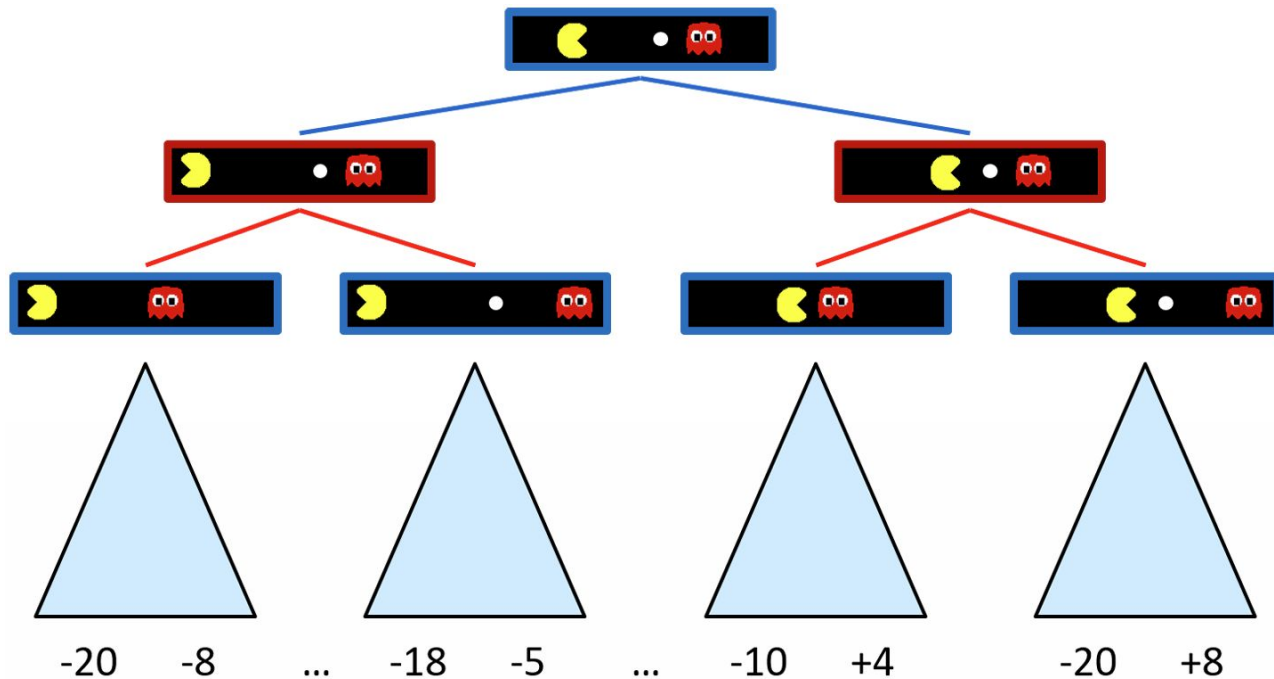
For terminal states:
 $V(s) = \text{known value}$

For non-terminal states:
 $V(s) = \max V(s_i)$
Where, $s_i \in \text{children}(s)$

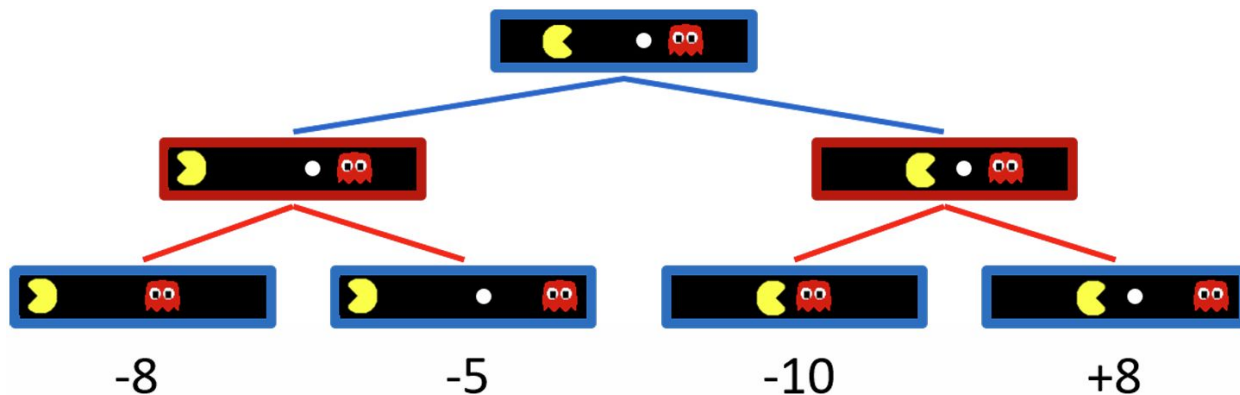


Value for each state:
Best achievable
reward (utility) from
that state

Adversarial Search Tree



Adversarial Search Tree



For terminal states:

$V(s) = \text{known value}$

States under opponent's control:

$V(s) = \min V(s_i)$

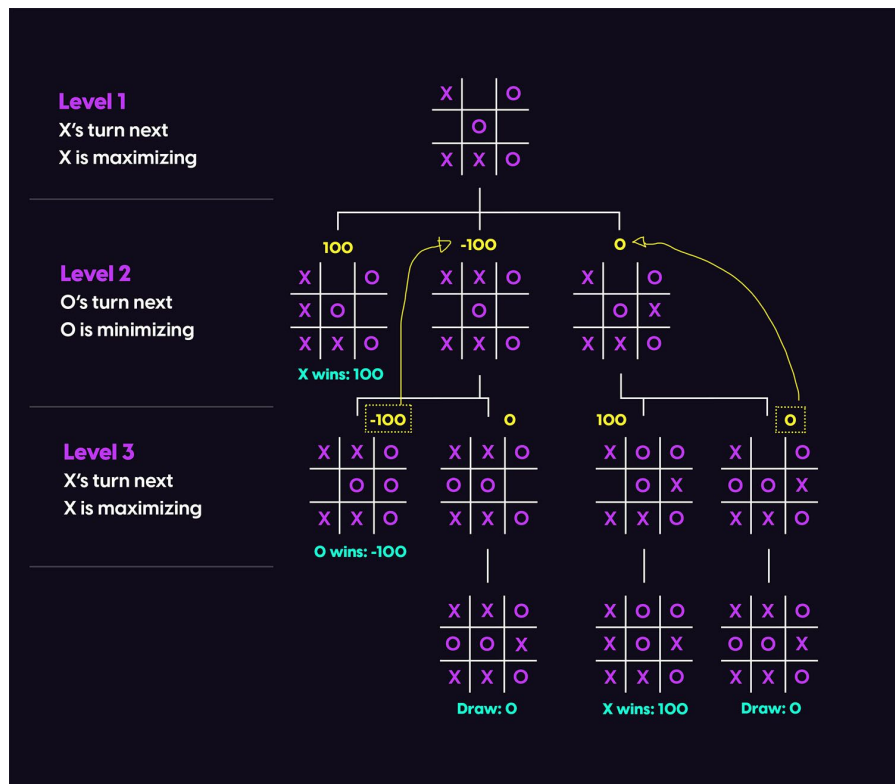
Where, $s_i \in \text{children}(s)$

States under agent's control:

$V(s) = \max V(s_i)$

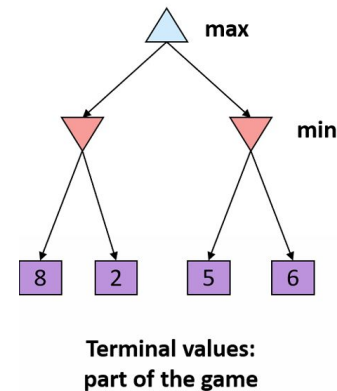
Where, $s_i \in \text{children}(s)$

Adversarial Search



Deterministic, Zero-sum games:

- Tic-tac-toe, chess, checkers...
- One player maximizes result
- The other minimizes result



Minimax Search:

- A state-space search tree
- Players alternate turns
- Compute each node's minimax value → The best achievable utility against a rational (optimal) adversary

Minimax Algorithm

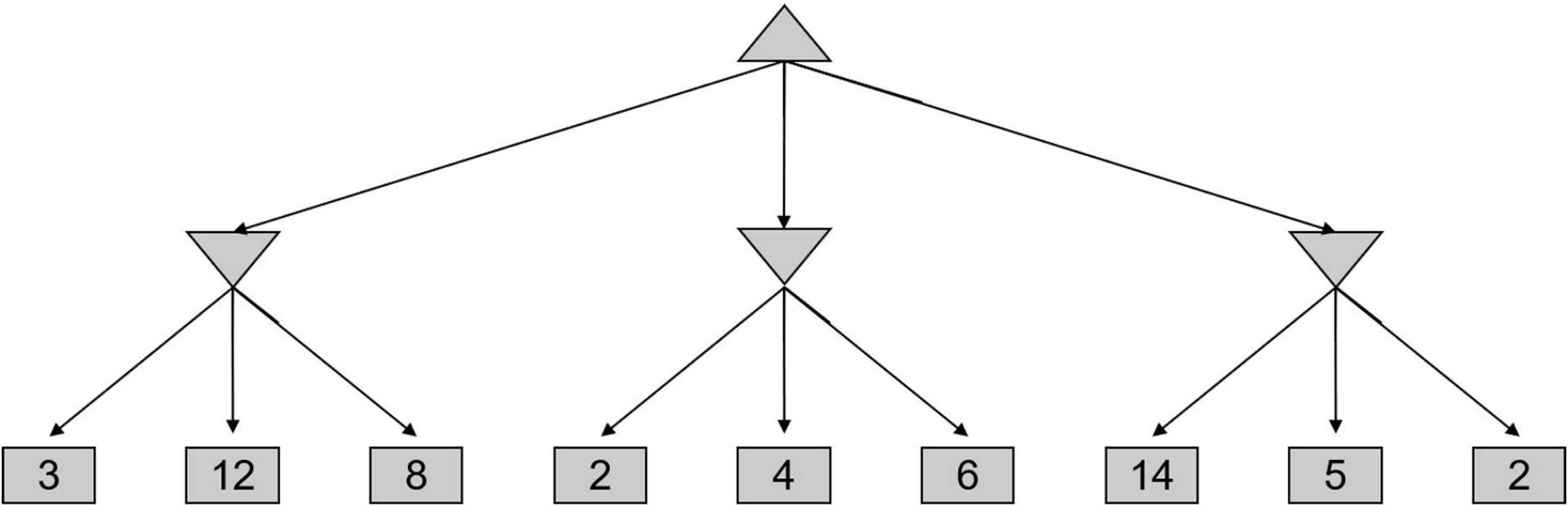
```
VALUE (state) → returns utility value of the state
  if state is TERMINAL: then return utility
  if state is MAX-AGENT: then return MAX-VALUE (state)
  if state is MIN-AGENT: then return MIN-VALUE (state)
```

```
MAX-VALUE (state) → returns utility value of a state
  v ←  $-\infty$ 
  for successor in state:
    v = max (v, VALUE (successor))
  return v
```

```
MIN-VALUE (state) → returns utility value of a state
  v ←  $+\infty$ 
  for successor in state:
    v = min (v, VALUE (successor))
  return v
```

What is the
complexity?

Minimax Algorithm



Formalizing Deterministic Games

What is the time & space complexity of minimax?

- Similar to DFS
- Time complexity: $O(b^m)$
- Space complexity: $O(bm)$

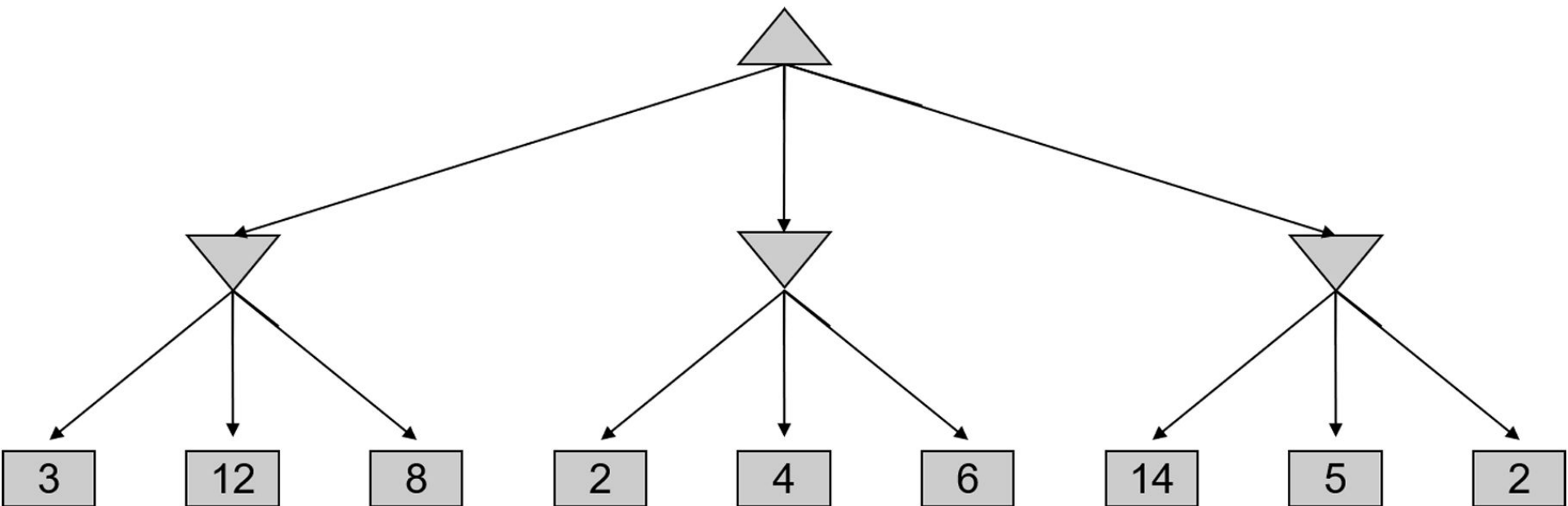
Here, $b \rightarrow$ branching factor $m \rightarrow$ depth

For chess, $b \approx 35$ and $m \approx 100 \rightarrow$ Finding exact utilities from terminal states is impossible



Game Tree Pruning

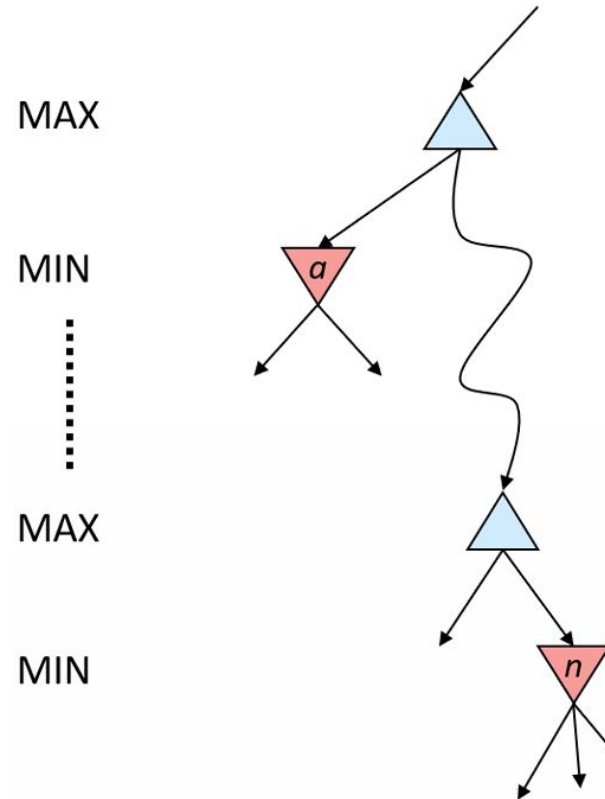
Game Tree Pruning



Alpha-Beta Pruning

From the perspective of MIN:

- Computing the MIN-VALUE at some node n
- Looping over the children of n
- N 's estimate of the children's minimum is always dropping
- Let α be the best value that MAX can get at any choice point along the current path from the root
- If n becomes lower than α , MAX will avoid it, so stop considering n 's other children.



Alpha-Beta Pruning

```
VALUE (state,  $\alpha$ ,  $\beta$ )  $\rightarrow$  returns utility value of the state
  if state is TERMINAL: then return utility
  if state is MAX-AGENT: then return MAX-VALUE (state,  $\alpha$ ,  $\beta$ )
  if state is MIN-AGENT: then return MIN-VALUE (state,  $\alpha$ ,  $\beta$ )
```

```
MAX-VALUE (state,  $\alpha$ ,  $\beta$ )  $\rightarrow$  returns utility value of a state
   $v \leftarrow -\infty$ 
  for successor in state:
     $v = \max (v, \text{VALUE} (\text{successor}, \alpha, \beta))$ 
    if  $v \geq \beta$ : then return  $v$ 
     $\alpha = \max (\alpha, v)$ 
  return  $v$ 
```

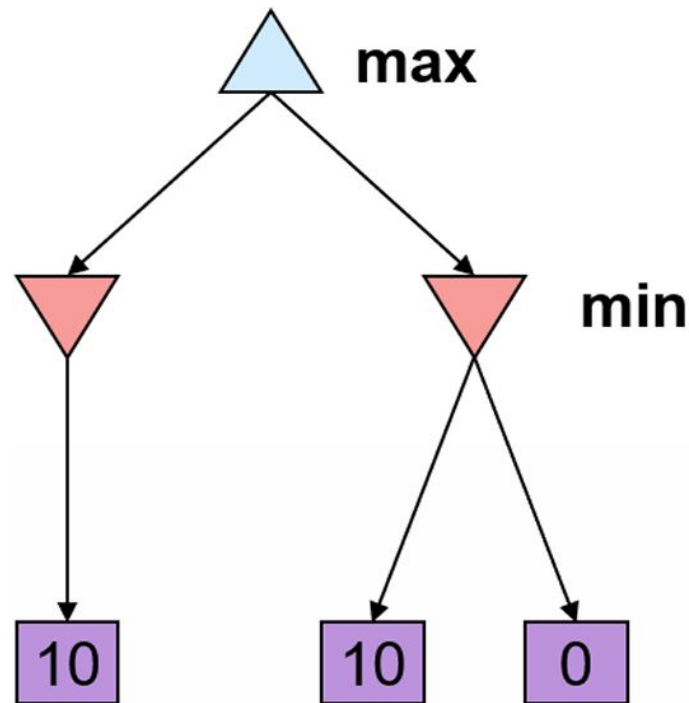
$\alpha \rightarrow$ MAX's best option
on path to root

$\beta \rightarrow$ MIN's best option
on path to root

```
MIN-VALUE (state,  $\alpha$ ,  $\beta$ )  $\rightarrow$  returns utility value of a state
   $v \leftarrow +\infty$ 
  for successor in state:
     $v = \min (v, \text{VALUE} (\text{successor}, \alpha, \beta))$ 
    if  $v \leq \alpha$ : then return  $v$ 
     $\beta = \min (\beta, v)$ 
  return  $v$ 
```

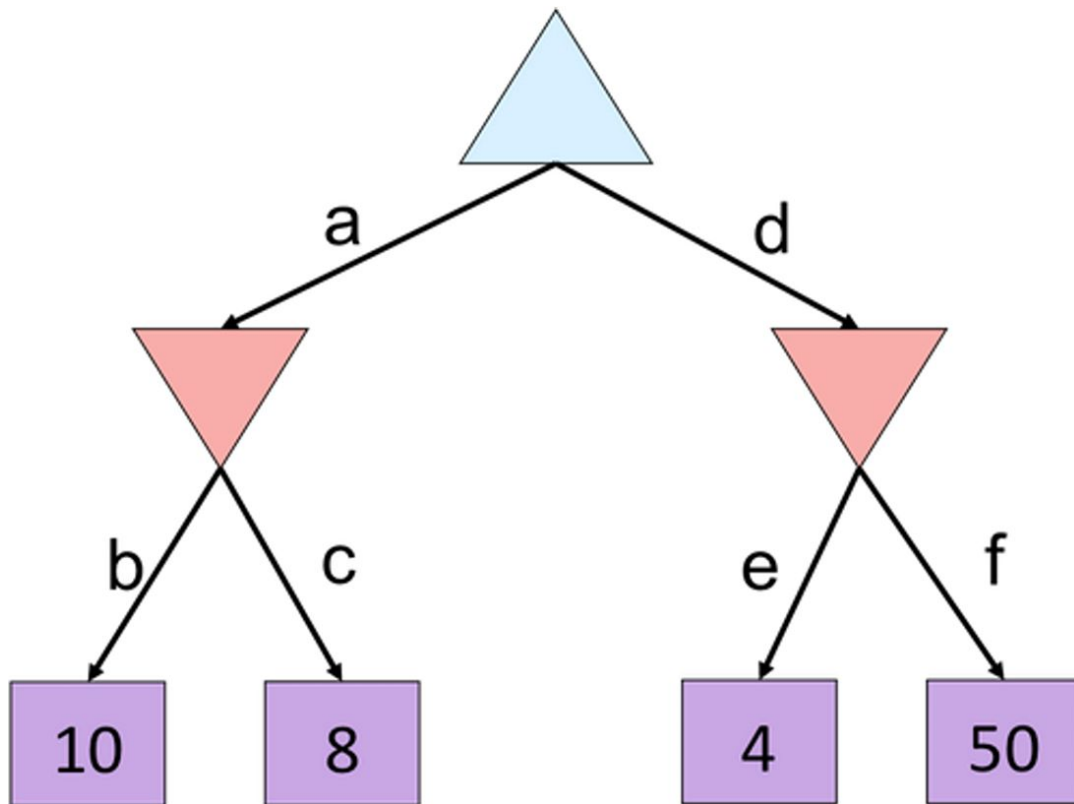
Alpha-Beta Pruning

- Pruning has **no effect** on the minimax value at the root → Why?
- Values of the intermediate nodes may be wrong
 - Prune only on inequality → Not the best
 - Keep track of which one was first
- Good child ordering improves effectiveness
 - Explore good options before bad ones
- With perfect ordering:
 - Time complexity: $O(b^{m/2})$
 - Solvable depth doubles
 - Still no enoch for chess
 - This is an example of meta-reasoning

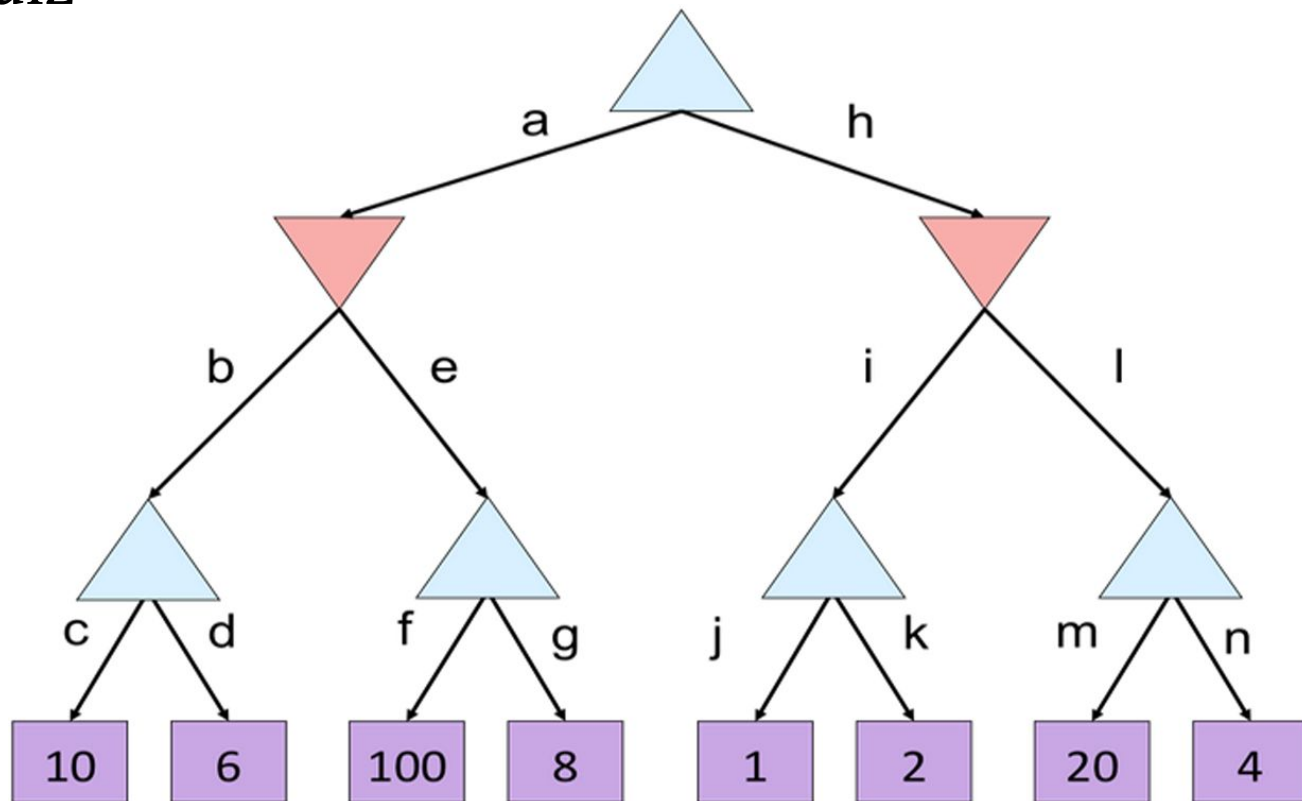


Mini Quiz

Mini Quiz



Mini Quiz



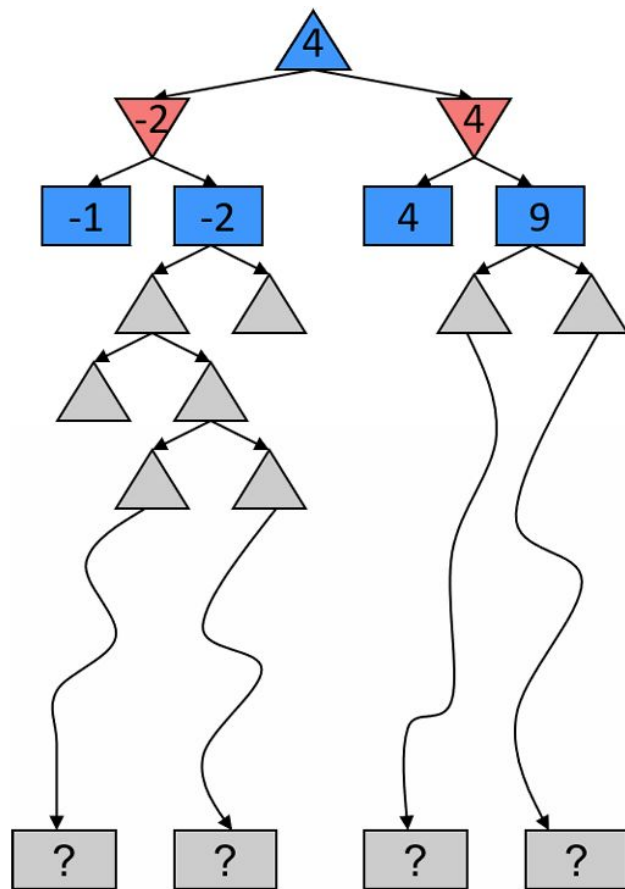
Resource Limits

Resource Limits

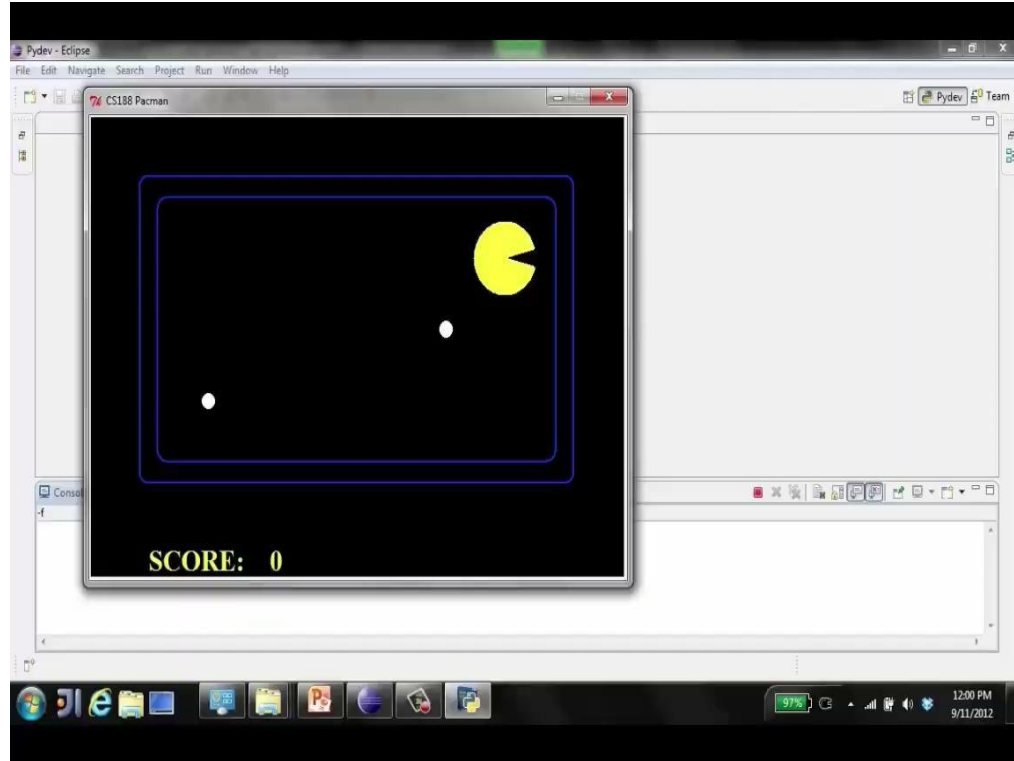
In demanding games like chess, we can not search to the leaves!

Depth-limited Search:

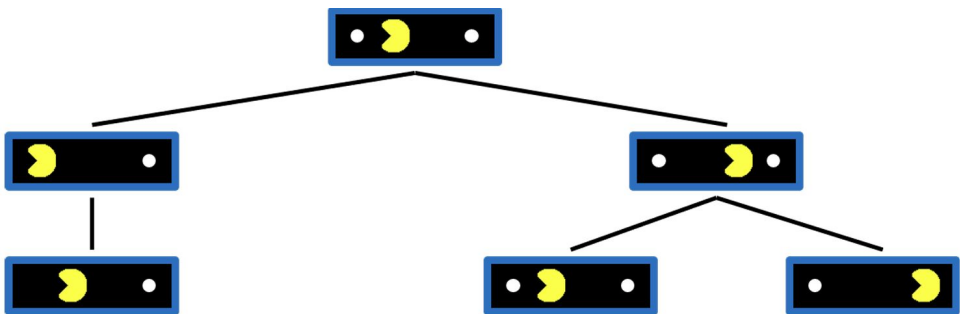
- Search only to a limited depth in the tree
- Replace terminal utilities with an evaluation function for non-terminal positions
- Guarantee of optimal play is gone
- The more depth you check, the better your moves become
- Use iterative deepening or a better outcome



Resource Limits



Adversarial Search

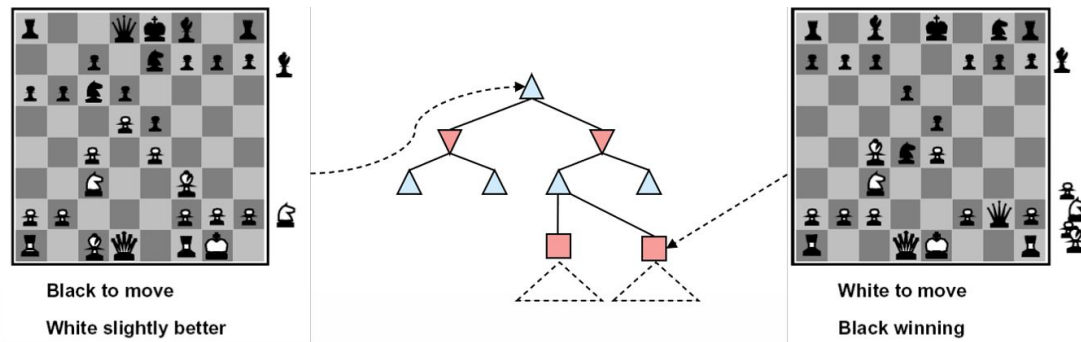


Dangers of replanning:

- He knows his score will go up by eating the dot now (west, east)
- He knows his score will go up just as much by eating the dot later (east, west)
- There are no point-scoring opportunities after eating the dot (within the limited depth)
- Therefore, waiting seems just as good as eating: he may go east, then back west in the next round of replanning!

Evaluation Function → Used to score non-terminals in depth-limited search

Evaluation Function



- Ideally → returns the actual minimax value of the position
- In practice → typically weighted linear sum of features:
 - $\text{Eval}(s) = w_1 f_1(s) + w_2 f_2(s) + \dots$
- Evaluation functions are always imperfect
- The deeper in the tree the evaluation function is buried, the less the quality of the evaluation function matters
- An important example of the tradeoff between complexity of features and complexity of computation

Additional Resources

- [Why is It Difficult to Make Good AI for Games?](#)
- [AlphaGo - The Movie](#)
- [Training AI to Play Pokemon with Reinforcement Learning](#)
- [The Evaluation Function in Board Games](#)

Thank you