Date:

Question No 1 :

a) $$\lim_{(x,y)\to(2,1)} \frac{x^2 - 2xy}{x^2 - 4y^2}$$

Let

$$x^2 = 4y^2$$
$$y = x/2$$

$$\lim_{(x,x/2)\to(2,1)} \frac{x^2 - 2x(x/2)}{x^2 - 4(x/2)^2}$$

$$= \frac{(2)^2 - 2(2)(1)}{(2)^2 - 4(1)} = 0$$

b) $$\lim_{(x,y)\to(0,0)} \frac{x-4y}{6y+7x}$$

$$6y = 7x$$
$$y = 7x/6$$

$$\lim_{(x\to0)} \frac{x - 4(7x/6)}{7x + 7x} =$$

$$\lim_{x\to0} \frac{-11x}{52x}$$

$$= \frac{-11}{52}$$

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c) $$\lim_{(x,y)\to(0,0)} \frac{x^2 - y^6}{xy^3}$$

$y = x$

$$\lim_{x\to 0} \frac{x^2 - x^6}{xy^3} = \frac{\cancel{x^2}(1 - x^4)}{x^4}$$

$$\lim_{x\to 0} \frac{1}{x^2} - \frac{\cancel{x^4}}{\cancel{x^2}}$$

$= 0$

d) $$\lim_{(x,y,z)\to(-1,0,4)} \frac{x^3 - ze^{2y}}{6x + 2y - 3z}$$

$$\frac{(-1)^3 - (4)e^{2(0)}}{6(-1) + 2(0) - 3(4)} = \frac{-1-4}{-6-12} = \frac{\cancel{-}5}{\cancel{-}18}$$

$= 5/18$

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Question No 2 :

(a) $f(x,y) = \cos(x/y)$ in $V = (3, -4)$

Solution

$$f\nabla = \frac{\partial}{\partial x}\left(\cos(x/y)\right)i + \frac{\partial}{\partial y}\left(\cos(x/y)\right)j$$

$$= -\frac{1}{y}\sin\frac{x}{y}\, i + \frac{x}{y^2}\sin\frac{x}{y}\, j$$

unit vector of v =

$$\frac{3i \mp 4j}{\sqrt{9+16}} \Rightarrow \frac{3}{5}i \mp \frac{4}{5}j$$

$$D_{\bar{u}} f = -\frac{3}{5y}\sin(x/y) \mp \frac{4x}{5y^2}\sin(x/y)$$

$$D_{\vec{u}} f = -\frac{1}{5y}\sin(x/y)\left(3 + \frac{4}{y}x\right)$$

b) $f(x,y,z) = x^2y^3 - 4xz$; $\vec{v} = (-1, 2, 0)$

$$\nabla f = 2xy^3 i - 4z\, i + 3xy^2 j - 4x\hat{k}$$

$$\hat{v} = \frac{-i + 2j}{\sqrt{5}} = \frac{-1}{\sqrt{5}}i + \frac{2}{\sqrt{5}}j$$

$$D_{\bar{u}} f = -\frac{2}{\sqrt{5}}xy^3 i + \frac{6}{\sqrt{5}}xy^2\hat{j}$$

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Question No 3 :-

$$f(x, y, z) = 4x - y^2 e^{3xz} \text{ at } (3, -1, 0)$$

& $\vec{v} = (-1, 4, 2)$

Solution

$$\nabla f = \left(4 - y^2 e^{3xz}(3z)\right)i + -2e^{3xz}y\,j + -y^2 e^{3xz}(3x)\,k$$

Putting values

$$\nabla f|_{(3,-1,0)} = \left(4 - (-1)^2 e^{3(3)(0)}(3(0))\right)i - 2e^{3(3)(0)}(-1)\,j - (-1)^2 e^{3(3)(0)}(3(3))k$$

$$= 4i + 2j - 9k$$

$$\vec{v} = (-1, 4, 2)$$

$$\hat{v} = \frac{-i + 4j + 2k}{\sqrt{21}} = -\frac{1}{\sqrt{21}}i + \frac{4}{\sqrt{21}}j + \frac{2}{\sqrt{21}}k$$

$$D_{\vec{v}}f = \frac{-4}{\sqrt{21}} + \frac{8}{\sqrt{21}} - \frac{18}{\sqrt{21}}$$

$$= \frac{-14}{\sqrt{21}}$$

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Question No 4:

a) $f(x, y) = \sqrt{x^2+y^3}$ at $(-2, 3)$

$\nabla f = \frac{1}{2}(x^2+y^3)^{-1/2}(2x)\,i + \frac{1}{2}(x^2+y^3)^{-1/2}(3y^2)\,j$

$\nabla f(-2,3) = \frac{1}{2}(4+9)^{-1/2}(-2) + \frac{1}{2}(4+27)^{-1/2}(3(9))\,j$

$\frac{-2}{\sqrt{31}}\,i + \frac{27}{2\sqrt{31}}\,j$

b)

$f(x, y, z) = e^{2x}\cos(y-2z)$ at $(4, -2, 0)$

$\nabla f = (e^{2x} . 2\cos(y-2z))\,i + e^{2x}(-\sin(y-2z)(1))\,j + e^{2x}(-\sin(y-2z)(-2))\,K$

$\nabla f \cdot (4, -2, 0) = e^{4x} \cdot 2\cos(-2-0)\,i + e^{4x}(-\sin(-2))\,j + e^{4x}(-\sin(-2)(-2))$

$= e^{4x}(-2\cos 2\, i - \sin(-2)\,j + 2\sin(-2)\,K)$

Question No 5

$\vec{F} = x^2y\,\hat{i} - (z^3 - 3x)\,\hat{j} + 4y^3\,\vec{K}$

$\nabla = 2xy\,i \quad - 0j$

Dir $f = \nabla f \cdot f$

$= (2xy\,i) \cdot (x^2y\,i - (z^3 - 3x)\,j + 4y^3 K)$

$= 2x^3y^2$

Curle = $\nabla f \times f$

$$= \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & -(z^3 - 3x) & 4y^2 \end{vmatrix}$$

$$= (8y + 3z^3)\hat{i} - (0-0)\hat{j} + (-(-3) - x^2)\hat{k}$$

$$= (8y + 3z^2)\hat{i} + (3 - x^2)\hat{k}$$

b

$$\vec{F} = (2x + 2z^2)\hat{i} + \frac{x^3y^2}{z}\hat{j} - (z - 7x)\hat{k}$$

div = $\nabla f \cdot f$

$$= \left(\frac{d}{dx}\hat{i} + \frac{d}{dy}\hat{j} + \frac{d}{dz}\hat{k}\right) \cdot \left((2x + 2z^2)\hat{i} + \frac{x^3}{z}y^2\hat{j} - (z - 7x)\hat{k}\right)$$

$$= 2 + \frac{2x^2}{z}y - 1$$

$$= 1 + \frac{2x^3}{z}y$$

Curl = $\nabla f \times f$

$$= \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3x + 9z^2 & \frac{x^3y^2}{z} & -(z - 7x) \end{vmatrix}$$

$$= \frac{x^3y^2}{z^2}\hat{i} - (7 - 4z)\hat{j} + \left(\frac{3x^2y^2}{z}\right)\hat{k}$$

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Question No 6 :-

a) $\vec{F} = \left(4x^2 + \frac{3x^2 y}{z^2}\right)\hat{i} + \left(8xy + \frac{x^3}{z^2}\right)\hat{j} + \left(11 - \frac{2x^3 y}{z^3}\right)\hat{k}$

The vector field is conservative if & only if

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, $\frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}$, $\frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}$

$\vec{F} = \left(4y^2 + \frac{3x^2}{z^2} y\right)\hat{i} + \left(8xy + \frac{x^3}{z^2}\right)\hat{j} + \left(11 - \frac{2x^3 y}{z^3}\right)\hat{k}$

$\frac{\partial M}{\partial y} = 8y + \frac{3x^2}{z}$, $\frac{\partial N}{\partial x} = 8y - \frac{3x^2}{z^2}$

$\frac{\partial N}{\partial z} = \cancel{x^3 z^{-2}} \quad -\frac{2x^3}{z^3}$

$\frac{\partial P}{\partial y} = -\frac{2x^3}{z^3}$

$\frac{\partial M}{\partial z} = 4x^2 + \frac{3x^2 y}{z^2} = 3x^2 y(-2)z^{-3} = -\frac{6x^2 y}{z^3}$

$\frac{\partial P}{\partial x} = \frac{-6x^2 y}{z^3}$

Hence Conservative.

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Question No 7

a) $z = x^2 - w$, $x = t^3 + 7$
$y = \cos(2t)$, $w = 4t$

$\frac{dz}{dt} = ?$

$$\frac{dz}{dt} = \frac{dz}{dx} \cdot \frac{dx}{dt} + \frac{dz}{dy} \cdot \frac{dy}{dt} + \frac{dz}{dw} \cdot \frac{dw}{dt}$$

$$\frac{dz}{dt} = \frac{2x}{y^4}, \quad \frac{dz}{dy} = \frac{x^2 - w}{y^4}$$

$$\frac{dx}{dt} = 3t^2 \qquad \frac{dz}{dy} = \frac{(x^2 - w)(-4)}{y^{-8}}$$

$$\frac{dy}{dt} = -2\sin 2t$$

$$\frac{dz}{dw} = \frac{-1}{y^4} \qquad \frac{dw}{dt} = 4$$

$$\frac{dz}{dt} = \frac{2x}{y^4} \cdot 3t^2 + \frac{(-4)(x^2 - w}{y^5} \cdot -2\sin 2t + \left(\frac{-1}{y^4}\right) \cdot 4$$

$$= \frac{6x^2 t^2}{y^4} + \frac{8(x^2 - w)\sin 2t}{y^1} - \frac{4w}{y^4}$$

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b) $z = x^3 y^4 - 2y \quad , \quad y = \sin(x^2)$

$\frac{dz}{dx} = ?$

$\frac{dz}{dx} = \frac{dz}{dy} \frac{dy}{dx}$

$\frac{dz}{dy} = \frac{d}{dy}\left(x^2 y^4 - 2y\right)$

$= 4x^2y^3 - 2$

$\frac{dy}{dx} = \frac{d}{dx}\left(\sin x^2\right)$

$= 2x \cos x^2$

$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$

$= (4x^2y^3 - 2)(2x\cos x^2)$

$= 8x^3y^3\cos x^2 - 4x\cos x^2$

c $\quad x^2y^4 - 3 = \sin(xy)$

$d/dx\left(x^2y^4 - 3\right) = d/dx\left(\sin(xy)\right)$

$2xy^4 + 4x^2y^3 \frac{dy}{dx} = y\cos xy$

$\frac{dy}{dx} = \left(\frac{\cos(xy) - 2x^2y^3}{4x^2y^2}\right)$