Question No 1 :-

$\vec{OA} = 4i + 4j + k$
$\vec{OB} = -4i + 3j - 4k$
$\vec{OC} = 4i - j - 2k$

a) Equation of plane ABC

Since the points A, B & C lies in the plane, So

$\vec{AB} = (-8, -1, -5)$
$= -8i - j - 5k$

$\vec{AC} = (0, -5, -3)$
$= -5j - 3k$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ -8 & -1 & -5 \\ 0 & -5 & -3 \end{vmatrix}$$

$\vec{AB} \times \vec{AC} = -22i - 24j + 40k$

Using the point-normal form

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

Where $n = (a, b, c) = (-22, -24, 40)$

$-22(x-4) - 24(y-4) + 40(z-1) = 0$

$-22x - 24y + 40z = -144$

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b) Perpendicular Distance

As we have to find the distance between point $O(0,0,0)$ and plane ABC ($-22x + -24y + 40z + 144 = 0$)

As $$D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2+b^2+c^2}}$$

Subsituting Values

$$D = \frac{144}{\sqrt{(-22)^2+(-24)^2+(40)^2}}$$

$$D = \frac{144}{\sqrt{2660}} = \frac{144}{2\sqrt{665}}$$

$$D = \frac{72}{\sqrt{665}} = 2.79$$

c)

$$D = 2i + 3j - 3k$$

Line OD: $r = a + \lambda b$

$$r = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ -3 \end{pmatrix}$$

as some values of λ

$$\begin{pmatrix} 2\lambda \\ 3\lambda \\ -3\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \\ 4 \end{pmatrix} = -8$$

$$= 2\lambda + 12\lambda - 12\lambda = -8$$

$$\lambda = -4$$

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$$r = \begin{bmatrix} 2(-4) \\ 2(-4) \\ -3(-4) \end{bmatrix} = (-8, -12, 12)$$

Question No 2

$A = (7i + 4j - k),\ B(11i + 3j),\ C(2i + 6j + 3k)$

$D(2i + 7j + \lambda k)$

a) $AB = 4i - 1j + k$

$CD = 0i - j + (3 - \lambda)k$

L_1 : line $AB = OA + \lambda AB$

L_2 : line $CD = OC + \mu CD$

$$L_1 : \vec{r_1} = \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix}$$

$$L_2 : \vec{r_2} = \begin{bmatrix} 2 \\ 6 \\ -3 \end{bmatrix} + \mu \begin{bmatrix} 0 \\ -1 \\ 3-\lambda \end{bmatrix}$$

$$D = \frac{(b_1 \times b_2) \cdot (a_2 - a_1)}{|b_1 \times b_2|}$$

$$b_1 \times b_2 = \begin{vmatrix} i & j & k \\ 4 & -1 & 1 \\ 0 & -1 & 3-\lambda \end{vmatrix}$$

$$b_1 \times b_2 = i(-3 + \lambda) - j(12 + 4\lambda) + k(-4)$$

$$|b_1 \times b_2| = \sqrt{\lambda^2 + 9 - 2\lambda + 144 + 16\lambda^2 - 24\lambda + 16}$$

$$= \sqrt{17\lambda^2 - 26\lambda + 169}$$

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$$a_2 - a_1 = \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} - \begin{bmatrix} 2 \\ 6 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ -2 \\ 2 \end{bmatrix}$$

$$3 = \frac{(-3+\lambda)5 + 2(12-4\lambda) - 2(4)}{\sqrt{17\lambda^2 - 26\lambda + 169}}$$

$$9(17\lambda^2 - 26\lambda + 169) = (-15 + 5\lambda + 24 - 8\lambda - 8)^2$$
$$9(17\lambda^2 - 26\lambda + 169) = (7\lambda + 24 - 23)^2$$
$$9(17\lambda^2 - 26\lambda + 169) = (7\lambda - 1)^2$$
$$= 49\lambda^2 + 1 - 24\lambda$$

Then $\lambda^2 - 5\lambda + 4 = 0$

b)

Let $\bar{\pi}_1$ be plane ABD when $\lambda = 1$
Let $\bar{\pi}_2$ be plane ABD when $\lambda = 4$

i) Write equ of $\bar{\pi}_1$ in form $r = a + sb + tc$
ii) Write equ of π_2 in form $ax + by + cz = d$

plane ABD when $\lambda = 1$

So $D = 2i + 7j + k$

$$AB \times AD = \begin{vmatrix} i & j & k \\ 4 & -1 & 1 \\ -5 & 3 & 2 \end{vmatrix}$$

$$= i(-2-3) - j(8+5) + k(12-5)$$
$$= -5i - 13j + 7k$$

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Equ = $-5(2) \cdot -13(7)s + 7(1)t$

$= -10 - 91s + 7t$

For plane $\bar{\pi}_2$, when $\lambda = 4$

$$AB \times AD = \begin{vmatrix} i & j & k \\ 4 & -1 & 1 \\ 5 & 3 & 5 \end{vmatrix}$$

$= i(-5-3) - j(20-5) + k(12+5)$

$= -8i - 15j + 17k$

Equ $= -8(x-11) - 15(y-3) + 17(z-0)$

$= -8x + 88 - 15y + 45 + 17z$

$= 8x + 15y - 17z - 132$

c) Angle b/w $\bar{\pi}_1$ & $\bar{\pi}_2$

$$\theta = \cos^{-1}\left(\frac{|n_1 \cdot n_2|}{|n_1|\,|n_2|}\right)$$

$n_1 \cdot n_2 = 4 + 195 + 119$

$$= \cos^{-1}\left(\frac{318}{\sqrt{25+169+119}\ \sqrt{64+225+289}}\right)$$

$$= \cos^{-1}\left(\frac{318}{374.56}\right)$$

$\theta = 31.89$

Question No 3

a) Find the value of t

$L_1 = t\hat{i} + \hat{j}$ $\quad -2i - j$

$L_2 = i + k\,k$ $\quad -2i + k$

The shortest distance between line is $\sqrt{21}$

$\vec{r_1} = OA + \lambda AB$

$\vec{r_2} = OA + u AB$

$L_1 : \vec{r_1} = \begin{bmatrix} t \\ 1 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$

$L_2 : \vec{r_2} = \begin{bmatrix} 0 \\ 1 \\ t \end{bmatrix} + u \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$

$$D = \frac{(b_1 \times b_2) \cdot (a_2 - a_1)}{|b_1 \cdot b_2|}$$

$$b_1 \times b_2 = \begin{vmatrix} i & j & k \\ -2 & 1 & 0 \\ 0 & -2 & 1 \end{vmatrix}$$

$= i(1-0) - j(-2) + k(4)$

$= i + 2j + 4k$

$a_2 - a_1 = -ti + kt$

$$\sqrt{21} = \frac{(i + 2j + 4k) \cdot (-ti + 0j + kt)}{\sqrt{21}}$$

$21 = -t + 4t$

$t = 7$

(b) $\bar{\pi}_1 = \vec{r} = OA_1 + AB\lambda + \mu AC$

$= 7i + j + \lambda(-2i - j) + \mu(-2j + k)$

c)

$5x - 6y + 7x = 0$

$\theta = ?$ b/w l_2 & $\bar{\pi}_2$

n_1 of line : $(0, -2, 1)$

n_2 of $\bar{\pi}_2 = (5, -6, 7)$

$$\theta = \cos^{-1} \frac{|n_1 \cdot n_2|}{|n_1| \, |n_2|}$$

$$= \cos^{-1} \frac{(0 + 12 + 7)}{(\sqrt{4+1})(\sqrt{25+36+49})}$$

$$= \cos^{-1}\left(\frac{19}{23.43}\right)$$

$\theta = 35.813$

(d) $\overline{\Pi_1 n} = \begin{bmatrix} 7 \\ 1 \\ 0 \end{bmatrix}$, $\pi_2 = \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$

$$\theta = \cos^{-1}\left[\frac{35 - 6 + 0}{\sqrt{49+1}\,\sqrt{25+36+49}}\right]$$

$$= \cos^{-1}\left(\frac{29}{\sqrt{50}\,\sqrt{110}}\right)$$

$\theta = 67.09°$

Question No 5

(a) $P = (-2, -1)$, $Q = (-6, -3)$ are end point of diameter of circle. Find equation of circle.

As equation of circle is

$$(x-h)^2 + (y-k)^2 = r^2$$

$$h = \frac{-2-6}{2} \quad , \quad k = \frac{-1-3}{2}$$

$$h = -4 \quad , \quad k = -2$$

$$(x+4)^2 + (y+2)^2 = r^2 \quad \Rightarrow (1)$$

For r we will ~~sb~~ substitute a point P in ~~@~~ equation (1)

$$(-2+4)^2 + (-1+2)^2 = r^2$$

$$(2)^2 + (1)^2 = r^2$$

$$4 + 1 = r^2$$

$$\sqrt{5} = r$$

Put this value in equation (1)

$$(x+4)^2 + (y+2)^2 = (\sqrt{5})^2$$

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b) Circle pass through (4,0) & (0,2) & center at y axis

Radius = ?

Standard form

$$(x-h)^2 + (y-k)^2 = r^2 \quad \Rightarrow \quad (1)$$

As circle is centered at y axis so h will be 0

Substituting both points in eq (1) respectively

$$(4)^2 + k^2 = r^2 \quad \Rightarrow \quad (3)$$

$$(2-k)^2 = r^2$$

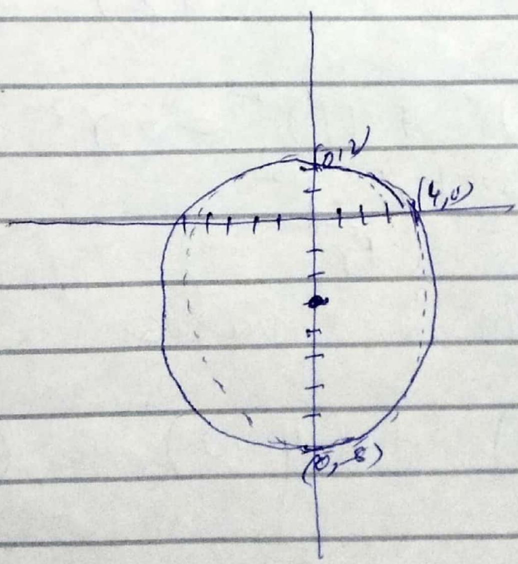
$$4 - 4k + k^2 = r^2 \quad \Rightarrow \quad (4)$$

As eq (3) & (4) are equal to r^2 so comparing both equations

$$16 + \cancel{k^2} = 4 - 4k + \cancel{k^2}$$

$$-4k = 12$$

$$k = -3$$



For radius

$$(4)^2 + (3)^2 = r^2$$

$$16 + 9 = r^2$$

$$25 = r^2$$

$$r = 5$$

c) Equation of parabola $y^2 = 100x$

Find directrix

Standard form of Parabola

$$y^2 = 4ax$$

$$y^2 = 4(25)x$$

Comparing this

$a = 25$

Directrix $= -a = -25$

(d) Equation of axis of parabola

$$x^2 = 24y$$

$$x^2 = 4(6)y$$

This parabola will be along y axis Eq will opens Upward.

e) $\left(\frac{x^2}{25}\right)^2 + \left(\frac{y}{16}\right)^2 = 1$

$$\left(x/a\right)^2 + \left(y/b\right)^2 = 1$$

$a = 5$, $b = 4$

$c = \sqrt{a^2 - b^2}$

$= \sqrt{25 - 16}$

$= \pm 3$

$F_1 = (-3, 0)$, $F_2 (3, 0)$

lenght $= 2a \Rightarrow 2(5) = 10$

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