We want to solve the equation $ax + by = d$, where $d = \gcd(a, b)$ which is known because we have a, b.

We know, $\gcd(a, b) = \gcd(b, a \bmod b)$.

$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{gcd(a,b)}$
99	78	$\gcd(78, 21)$
78	21	$\gcd(21, 15)$
21	15	$\gcd(15, 6)$
15	6	$\gcd(6, 3)$
6	3	$\gcd(3, 0)$
3	0	$\gcd(3,0) = 3 = \gcd(99,78)$

So we can derive an initial solution of the following form:

$$bx_0 + (a \bmod b)y_0 = \gcd(b, a \bmod b) = g = \gcd(a, b) \dots (1)$$

Since at the base of recursion $b = 0$ and $\gcd(a, 0) = a$, so in the above equation $x_0 = 1$ and $y_0 = 0$ which satisfies $a.1 + 0 = a$ from our example:

$$3x_0 + (6 \bmod 3)y_0 = \gcd(3, 6 \bmod 3) = 3$$

Since, $6 \bmod 3 = 0$, setting $x_0 = 1$ and $y_0 = 0$ satisfies the above equation.

But we want to solve for $ax + by = g$ using the initial solution $x_0 = 1,\ y_0 = 0$.

Rearranging (1) as the following we get

$bx_0 + \left(a - \left\lfloor \frac{a}{b} \right\rfloor b\right) y_0 = g\ [a \bmod b = a - \left\lfloor \frac{a}{b} \right\rfloor b]$

$$bx_0 + ay_0 - b \left\lfloor \frac{a}{b} \right\rfloor y_0 = g$$

$$ay_0 + b\left(x_0 - y_0 \left\lfloor \frac{a}{b} \right\rfloor\right) = g$$

Now we have our desired form of the problem, here $x = y_0$ and $y = x_0 - y_0 \left\lfloor \frac{a}{b} \right\rfloor$.

$\boldsymbol{a}$	$\boldsymbol{b}$	$\lfloor \boldsymbol{a/b} \rfloor$	$\boldsymbol{g}$	$\boldsymbol{x}$	$\boldsymbol{y}$
99	78	1	3	-11	14
78	21	3	3	3	-11
21	15	1	3	-2	3
15	6	2	3	1	-2
6	3	2	3	0	1
3	0	$-$	3	1	0

So the solution is $99(-11) + 78(14) = 3$