CSE-421

Computer Graphics:

2D Transformations

Lecture-01

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2D Transformations

“Transformations are the operations applied to geometrical description of an object to change its position, orientation, or size are called geometric transformations”.

Given a 2D object, transformation is to change the object’s

– *Position (translation)*

– *Size (scaling)*

– *Orientation (rotation)*

– *Shapes (shear)*

Point Representation

- We can use a column vector (a 2x1 matrix) to represent a 2D point.

$$\begin{vmatrix} x \\ y \end{vmatrix}$$

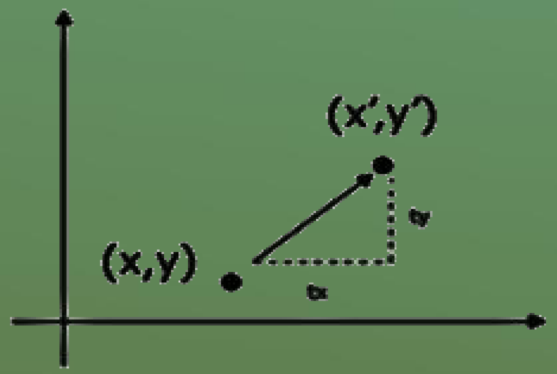
- Apply a sequence of matrix multiplications to the object vertices.

Translation

- Re-position a point along a straight line
- Given a point (x,y), and the translation distance (t_x,t_y)

The new point, P′ : (x′, y′)

$$x' = x + t_x$$
$$y' = y + t_y$$



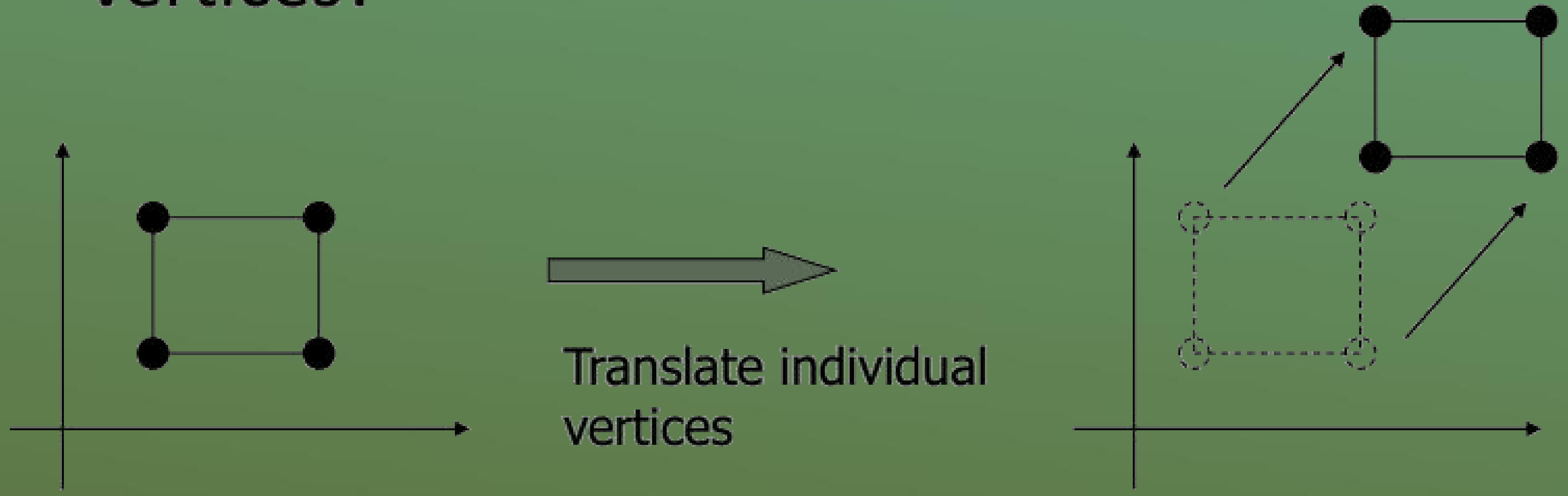
OR $P' = P + T$ where

$$[x' \quad y'] = [x \quad y] + [t_x \quad t_y] \quad \text{OR}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Translation

- How to translate an object with multiple vertices?

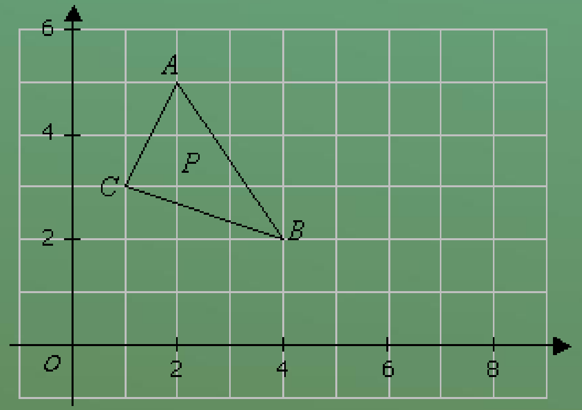


Before Translation Square= (1,1), (5,1), (5,3), (1,3)	tx=2 ty=3	AfterTranslation Square= (3,4), (7,4), (7,6), (3,6)

Translation is a process of changing the position of an object in a straight-line path from one coordinate location to another.

Example:

The triangle P is mapped onto the triangle Q by the translation $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$



a) Find the coordinates of triangle Q.
b) On the diagram, draw and label triangle Q.

Solution:

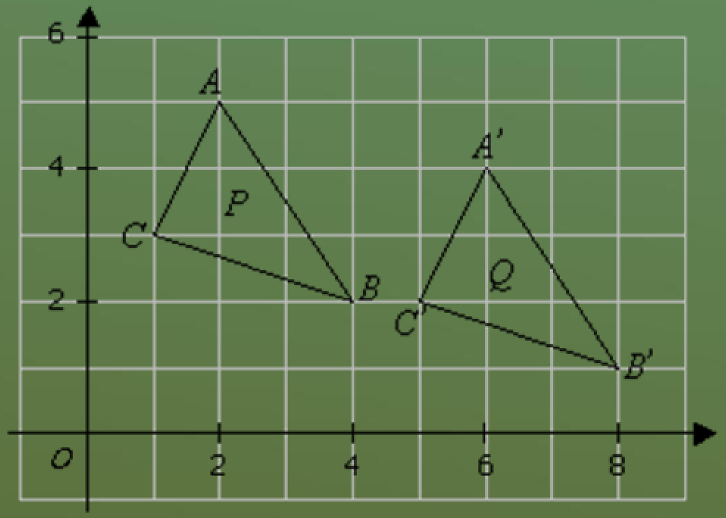
a)

$$A' = \begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} 4 \\ -1 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$$

$$B' = \begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 4 \\ -1 \end{pmatrix} = \begin{pmatrix} 8 \\ 1 \end{pmatrix}$$

$$C' = \begin{pmatrix} 1 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

b)



Translate a polygon with co-ordinates A(2,5) B(7,10) and C(10,2) by 3 units in X direction and 4 units in Y direction.

$A' = A + T$

$$= \begin{bmatrix} 2 \\ 5 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 5 \\ 9 \end{bmatrix}$$

$B' = B + T$

$$= \begin{bmatrix} 7 \\ 10 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 10 \\ 14 \end{bmatrix}$$

$C' = C + T$

$$= \begin{bmatrix} 10 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 13 \\ 6 \end{bmatrix}$$

Scaling

- A scaling transformation changes the size of an object.
- This operation can be carried out for polygons by multiplying the co-ordinates values (x , y) of each vertex by scaling factors Sx and Sy to produce the transformed co-ordinates (x' , y').

$$x' = x \cdot Sx$$

$$y' = y \cdot Sy$$

- In the matrix form

$$\begin{bmatrix} x' & y' \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix} \begin{pmatrix} Sx & 0 \\ 0 & Sy \end{pmatrix}$$

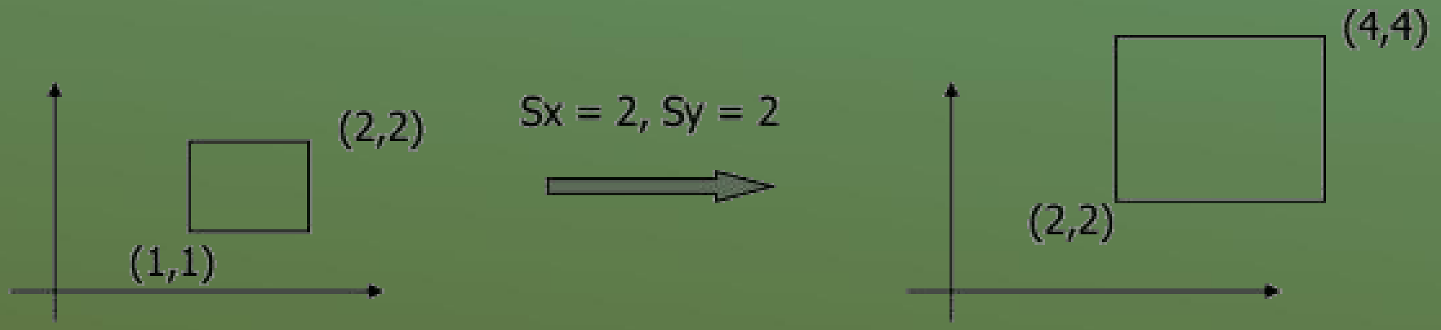
$$= P \cdot S$$

2D Scaling

Scale: Alter the size of an object by a scaling factor (Sx, Sy), i.e.

$$x' = x \cdot Sx \qquad y' = y \cdot Sy$$

$$\Rightarrow \quad \begin{vmatrix} x' \\ y' \end{vmatrix} = \begin{vmatrix} Sx & 0 \\ 0 & Sy \end{vmatrix} \begin{vmatrix} x \\ y \end{vmatrix}$$



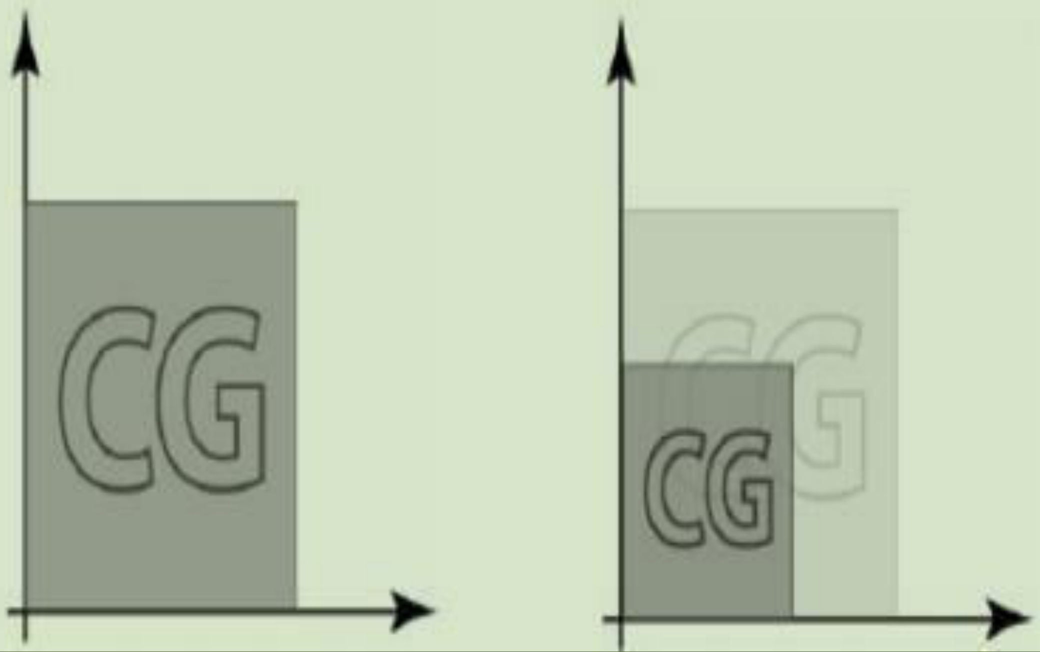
If 1<Sx & Sy <1 then Object size decreases /Point closer to origin

If Sx & Sy >1 then Object size increases/Point away from origin

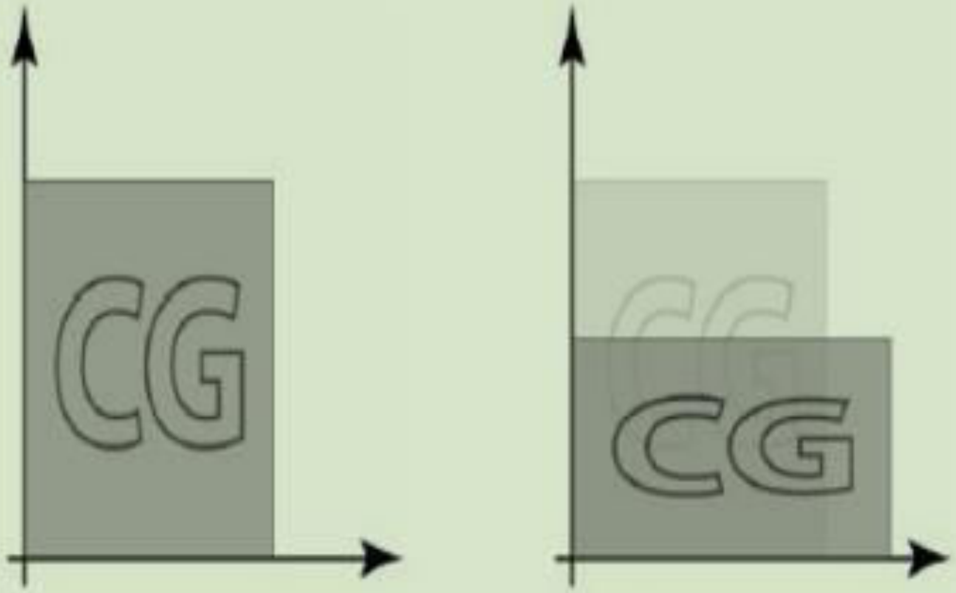
If Sx =Sy →Scaling is done uniformly, shape not changed else shape changes

Scaling

Uniform Scaling

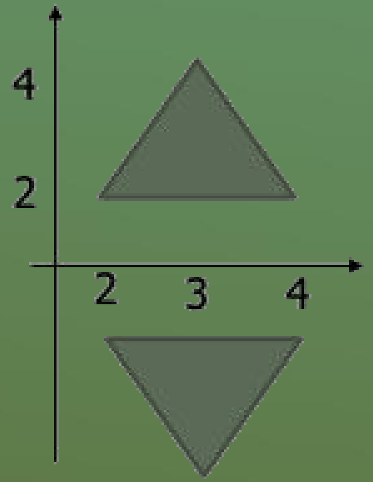


Un-uniform Scaling



Reflection

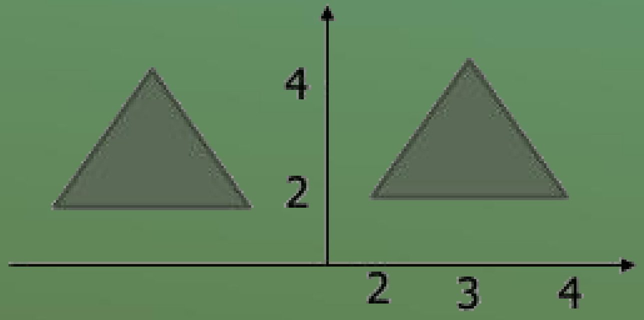
i) About x- axis
$x' = x$
$y' = -y$



Coordinates are (2,2), (4,2), (3,4)

(2,2)→ (2,-2),
(4,2)→ (4,-2)
(3,4)→ (3,-4)

i) About y- axis
$x' = -x$
$y' = y$

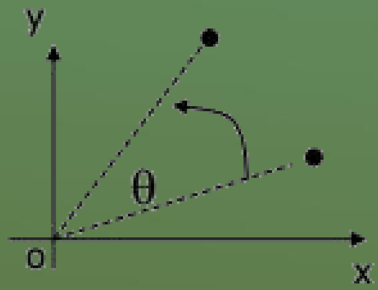


Coordinates are (2,2), (4,2), (3,4)

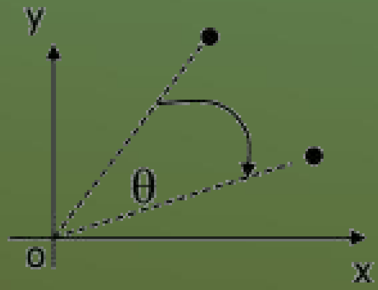
(2,2)→ (-2,2),
(4,2)→ (-4,2)
(3,4)→ (-3,4)

2D Rotation

Default rotation center: Origin (0,0)



$\theta > 0$: Rotate counter clockwise

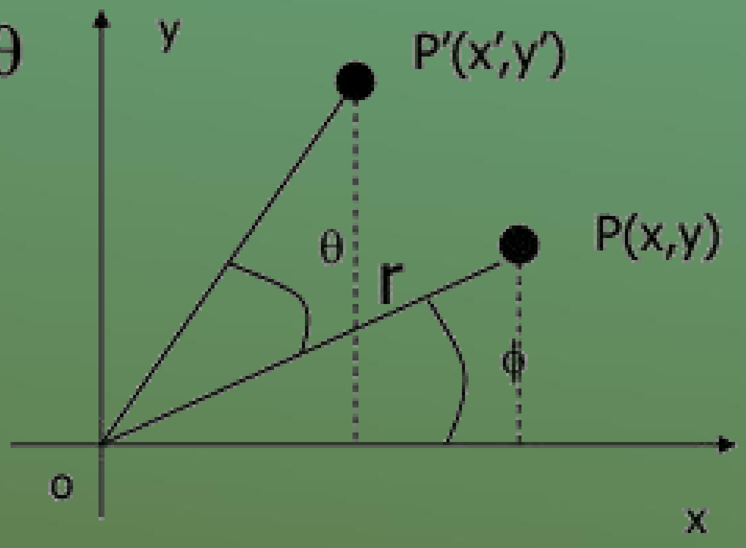


$\theta < 0$: Rotate clockwise

2D Rotation

$P(x,y)$ -> Rotate *about the origin* by θ

$$\longrightarrow P'(x', y')$$



How to compute (x', y') ?

After rotation, new angle= $(\phi + \theta)$

Now

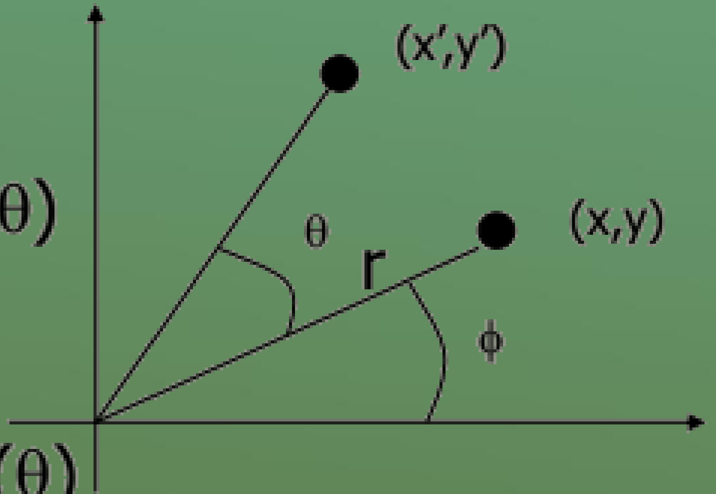
$x = r\cos(\phi)$ $\quad y = r\sin(\phi)$

$x' = r\cos(\phi + \theta)$ $\quad y' = r\sin(\phi + \theta)$

2D Rotation

$$x = r\cos(\phi) \quad y = r\sin(\phi)$$

$$x' = r\cos(\phi+\theta) \quad y' = r\sin(\phi+\theta)$$

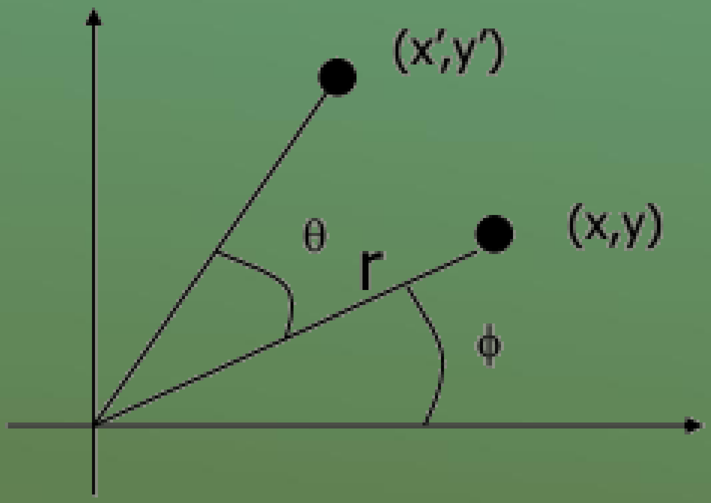


$$\begin{aligned} x' &= r\cos(\phi+\theta) \\ &= r\cos(\phi)\cos(\theta) - r\sin(\phi)\sin(\theta) \\ &= x\cos(\theta) - y\sin(\theta) \end{aligned}$$

$$\begin{aligned} y' &= r\sin(\phi+\theta) \\ &= r\sin(\phi)\cos(\theta) + r\cos(\phi)\sin(\theta) \\ &= y\cos(\theta) + x\sin(\theta) \end{aligned}$$

2D Rotation

$$x' = x\cos(\theta) - y\sin(\theta)$$
$$y' = y\cos(\theta) + x\sin(\theta)$$

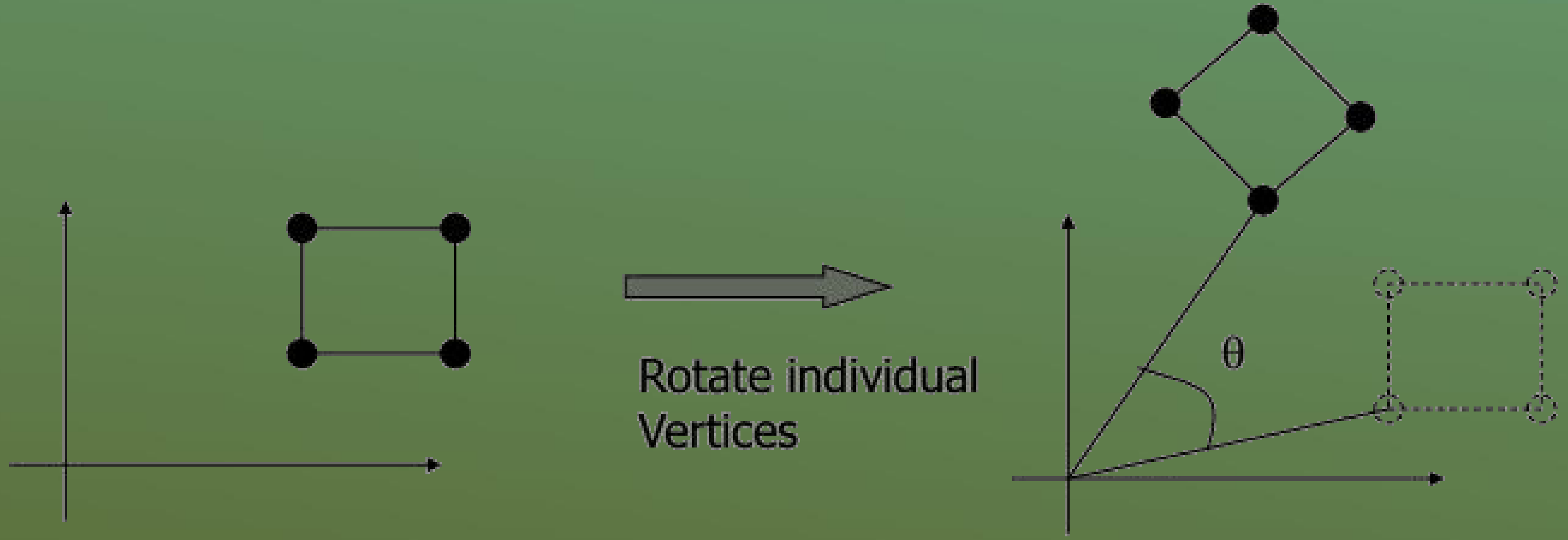


Matrix form?

$$\begin{vmatrix} x' \\ y' \end{vmatrix} = \begin{vmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{vmatrix} \begin{vmatrix} x \\ y \end{vmatrix}$$

2D Rotation

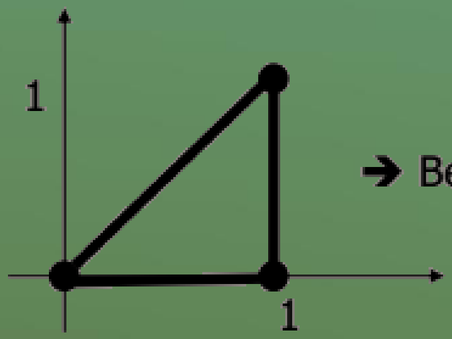
- How to rotate an object with multiple vertices?



2D Rotation

Example: Suppose a Triangle has three sides (0,0), (1,0), (1,1)

Let $\theta = 90^0$ (anticlockwise)



→ Before Rotation

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

For (0,0)

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

→ (x′, y′)=(0,0)

For (1,0)

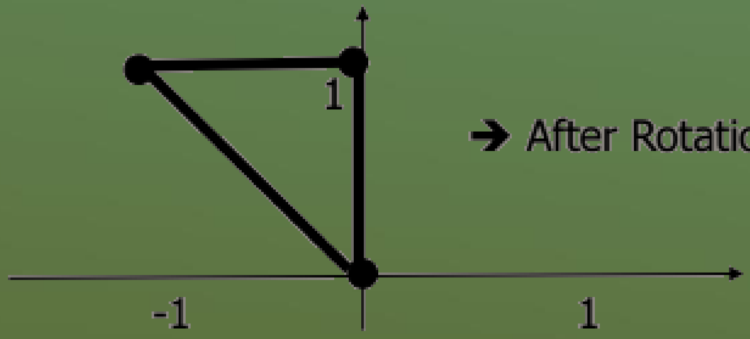
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

→ (x′, y′)=(0,1)

For (1,1)

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

→ (x′, y′)=(-1,1)



→ After Rotation

A point (4,3) is rotated counterclockwise by angle of 45.

find the rotation matrix and the resultant point.

$$R = \begin{pmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} \cos 45 & -\sin 45 \\ -\sin 45 & \cos 45 \end{pmatrix}$$

$$= \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

$$P' = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

$$= \begin{bmatrix} 4/\sqrt{2} - 3/\sqrt{2} & 4/\sqrt{2} + 3/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & 7/\sqrt{2} \end{bmatrix}$$

Shearing

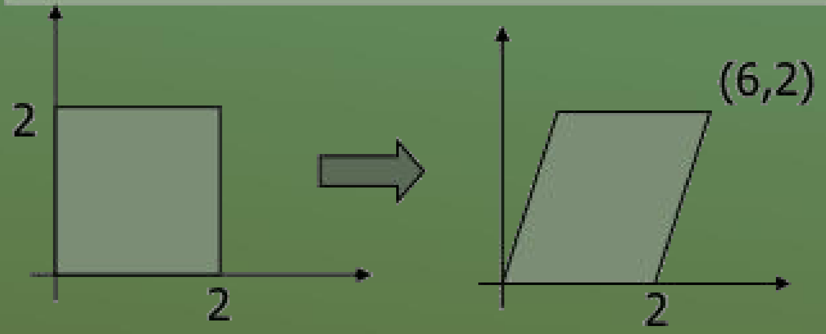
x- shear
y- share ⇐ skewing

x- shearing

$y' = y$

$x' = x + shr*y$

Y coordinates are unaffected, but x coordinates are translated linearly with y



Coordinates are (0,0), (0,2), (2,2), (2,0)

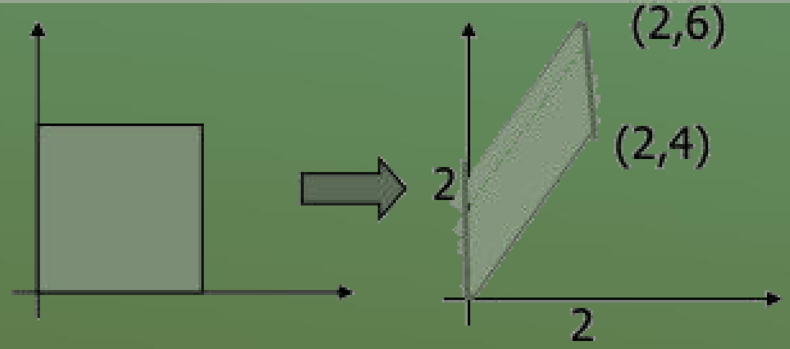
(0,0)→ (0,0),
(0,2)→ (0+2*2, 2)=(4,2)
(2,0)→ (2+2*0, 0)=(2,0)
(2,2)→ (2+2*2,2)=(6,2)

y- shearing

$x' = x$

$y' = y + shr*x$

x coordinates are unaffected, but y coordinates are translated linearly with x



Coordinates are (0,0), (0,2), (2,2), (2,0)

(0,0)→ (0,0),
(0,2)→ (0, 2+2*0)=(0,2)
(2,0)→ (2, 0+2*2)=(2,4)
(2,2)→ (2, 2+2*2)=(2,6)