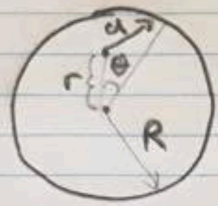
Part a) :



Mass density $\rho = \frac{M}{2\pi R} \Rightarrow dM = \rho R \, d\theta = \frac{M}{2\pi} d\theta$

$$\Phi = \int_{\text{mass}} -\frac{G\,dM}{a} = -\frac{GM}{2\pi} \int_0^{2\pi} \frac{d\theta}{(r^2+R^2-2rR\cos\theta)^{1/2}} = -\frac{GM}{2\pi R} \int_0^{2\pi} \frac{d\theta}{(1+(\frac{r}{R})^2-2(\frac{r}{R})\cos\theta)^{1/2}}$$

$$= -\frac{GM}{2\pi R} \int_0^{2\pi} \left(\left[1 - \frac{1}{2}x^2 + x\cos\theta\right] + \frac{3}{8}\left[x^2 - 2x\cos\theta\right]^2 + \cdots \right) d\theta \qquad (\text{Letting } x = r/R)$$

$$= -\frac{GM}{2\pi R} \int_0^{2\pi} \left[1 - \frac{1}{2}x^2 + x\cos\theta + \frac{3}{8}x^4 - \frac{3}{2}x^3\cos\theta + \frac{3}{2}x^2\cos^2\theta + \cdots \right] d\theta$$

$$= -\frac{GM}{2\pi R} \left[2\pi - \pi x^2 + \frac{6\pi}{8}x^4 + \frac{3\pi}{2}x^2 + \cdots + \cdots \right]$$

$$= -\frac{GM}{R} \left(1 + \frac{1}{4}x^2 + \frac{3}{8}x^4 + \cdots \right)$$

$$\boxed{= -\frac{GM}{R} \left(1 + \frac{r^2}{4R^2} + \frac{3r^4}{8R^4} + \cdots \right)}$$