NOTE TO MARKER: See physical copy of assignment 8 for all written work for parts a) and b)

Contents

- PART A
- PART B
- PART C
- PART D

Seting up the ODE solver

```
warning('off','all')
type particlemotion
```

```
function df = particlemotion(t,f)

df = zeros(2,1);
df(1) = f(2); % represents y(x)
df(2) = (1+(f(2)).^2)/(4-2*f(1)); %represents y'(x)
```

PART A

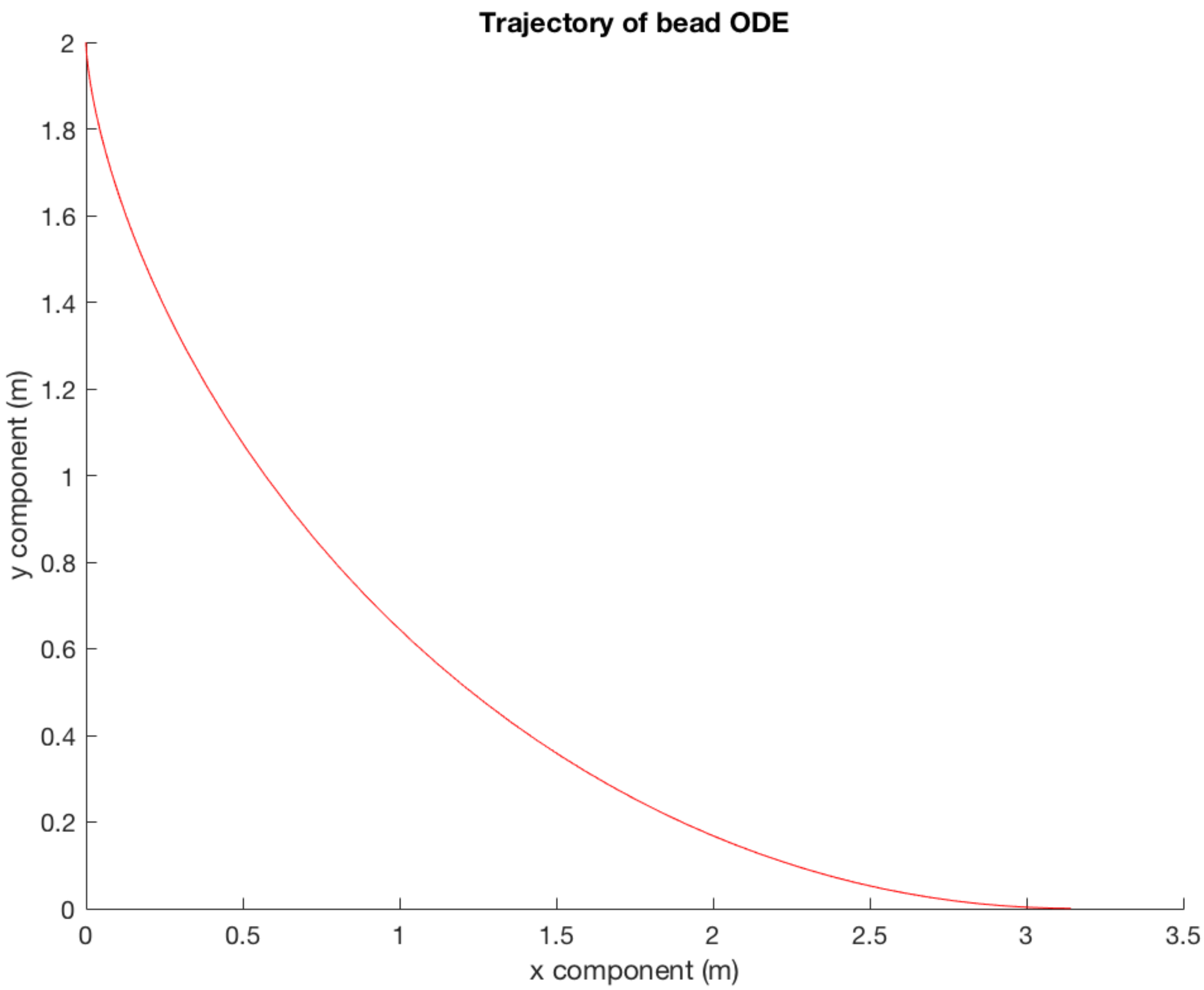
We begin by creating the initial conditions for the bead. Note that we require a large negative starting slope

```
initialconditions=[1.999999,-1390];
x=[0,pi];

[x,sol]=ode45(@particlemotion,x,initialconditions);


theta=linspace(0,pi);
xcycloid=theta-sin(theta);
ycycloid=1+cos(theta);
```

This is a graph of our solution to the ODE

Graph 1

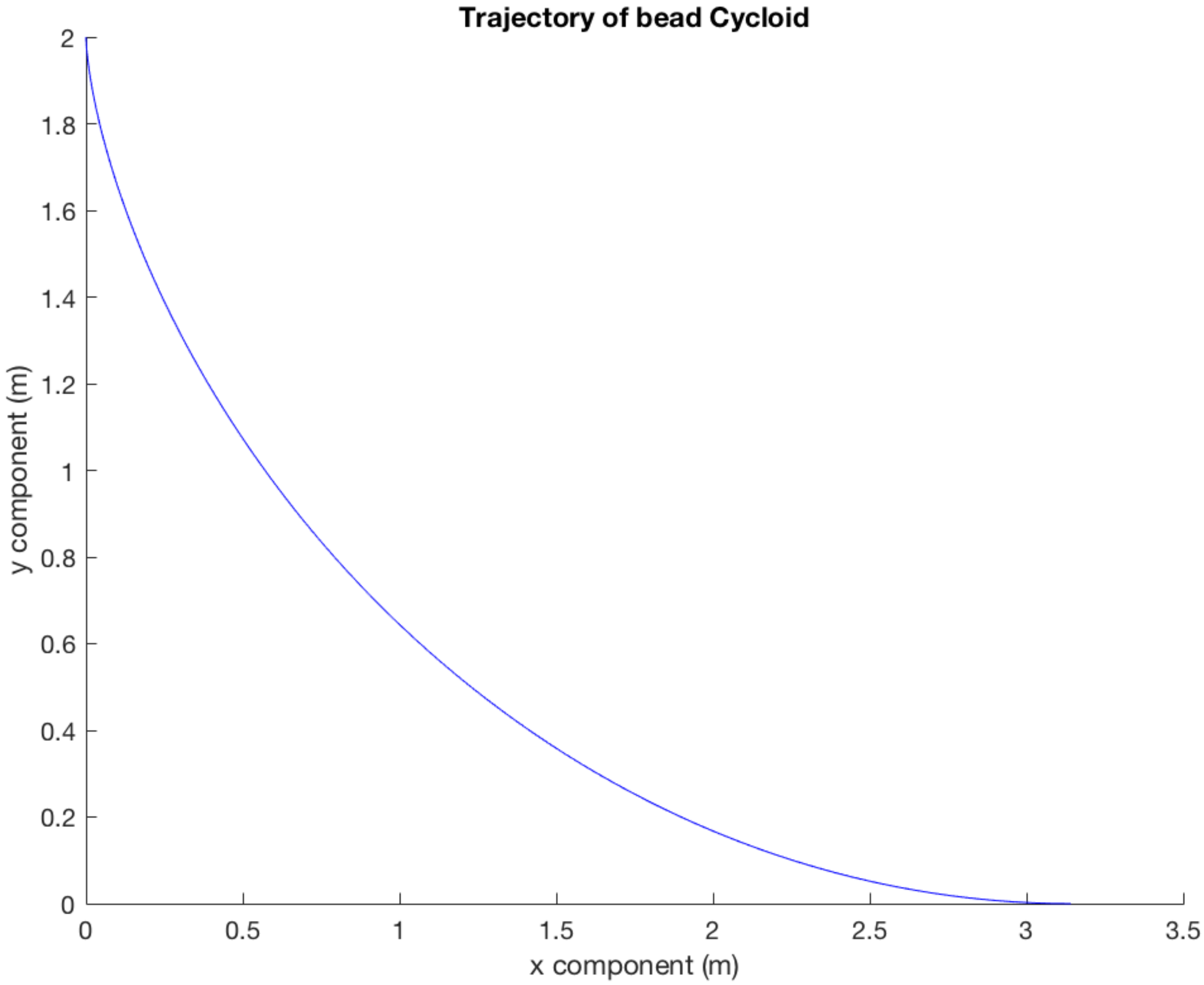
```
figure('name','Solution to ODE','NumberTitle','on');
hold on;
title('Trajectory of bead ODE')
ylabel('y component (m)')
xlabel('x component (m)')
plot(x,sol(:,1),'r'); %particle 1
hold off;
```



This is a graph of the cycloid

Graph 2

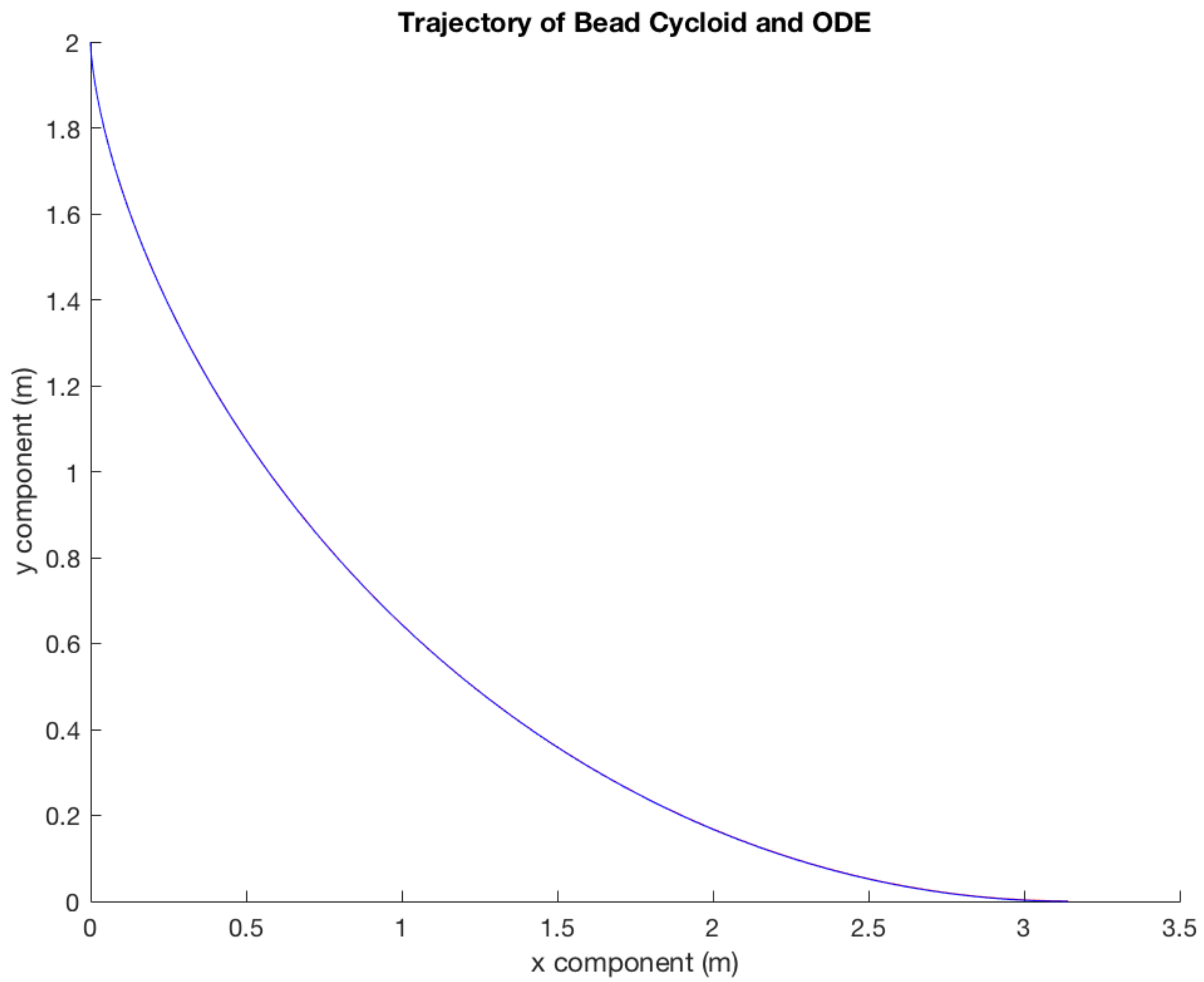
```
figure('name','Cycloid','NumberTitle','on');
hold on;
title('Trajectory of bead Cycloid')
ylabel('y component (m)')
xlabel('x component (m)')
plot(xcycloid,ycycloid,'b')
hold off;
```



Note that they are basically identical when we plot them over top of eachother.

Graph 3

```
figure('name','Solution and Cycloid','NumberTitle','on');
hold on;
title('Trajectory of Bead Cycloid and ODE')
ylabel('y component (m)')
xlabel('x component (m)')
plot(x,sol(:,1),'r'); %particle 1
plot(xcycloid,ycycloid,'b')
hold off;
```



PART B

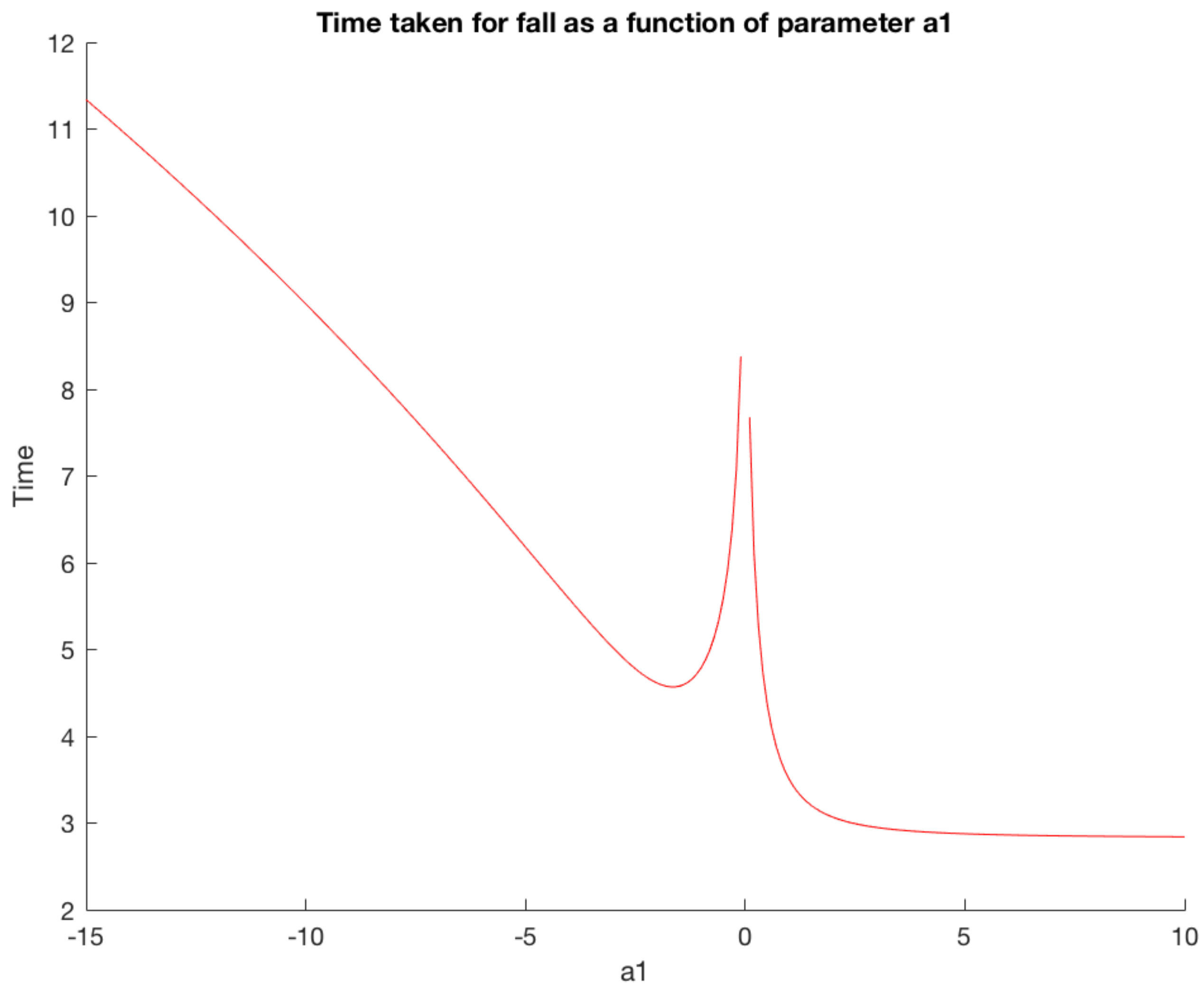
We define a2 as a function of a1 (we write a1 as 'a' in this code).

```
a2 = @(a) -((2+pi*a)/(pi^2));
f = @(a,x) ((1+(a+2*a2(a).*x).^2)./(-a.*x-a2(a).*x.^2)).^(0.5);
a = -15:.1:10;
g = 0*a;
for k = 1:numel(a)
    fab = @(x) f(a(k),x);
    g(k) = quadgk(fab,0,pi);
end
```

This graph plots the fall time as a function of a

Graph 4

```
figure('name','Value of Time for Various a1','NumberTitle','on');
hold on;
title('Time taken for fall as a function of parameter a1')
ylabel('Time')
xlabel('a1')
plot(a,g,'r');
hold off;
```

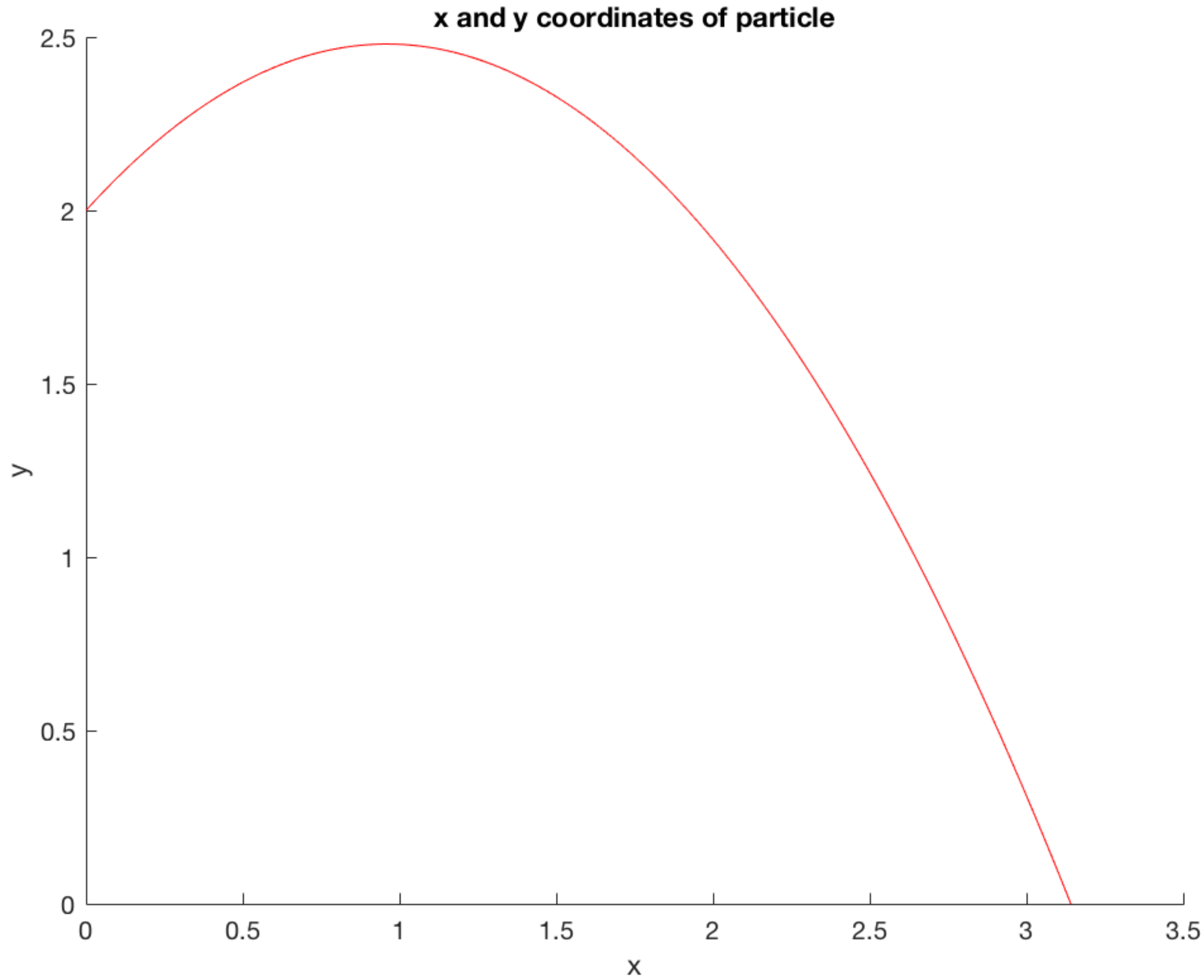


This seems to imply we want to make a as large as possible. From the graph below we show that if a>0 (for an example of a=1) then the particle begins by moving upward which can't happen if the bead is falling down.

```
x=linspace(0,pi);
y=@(x) 2+(1)*x+a2(1).*x.^2;
```

Graph 5

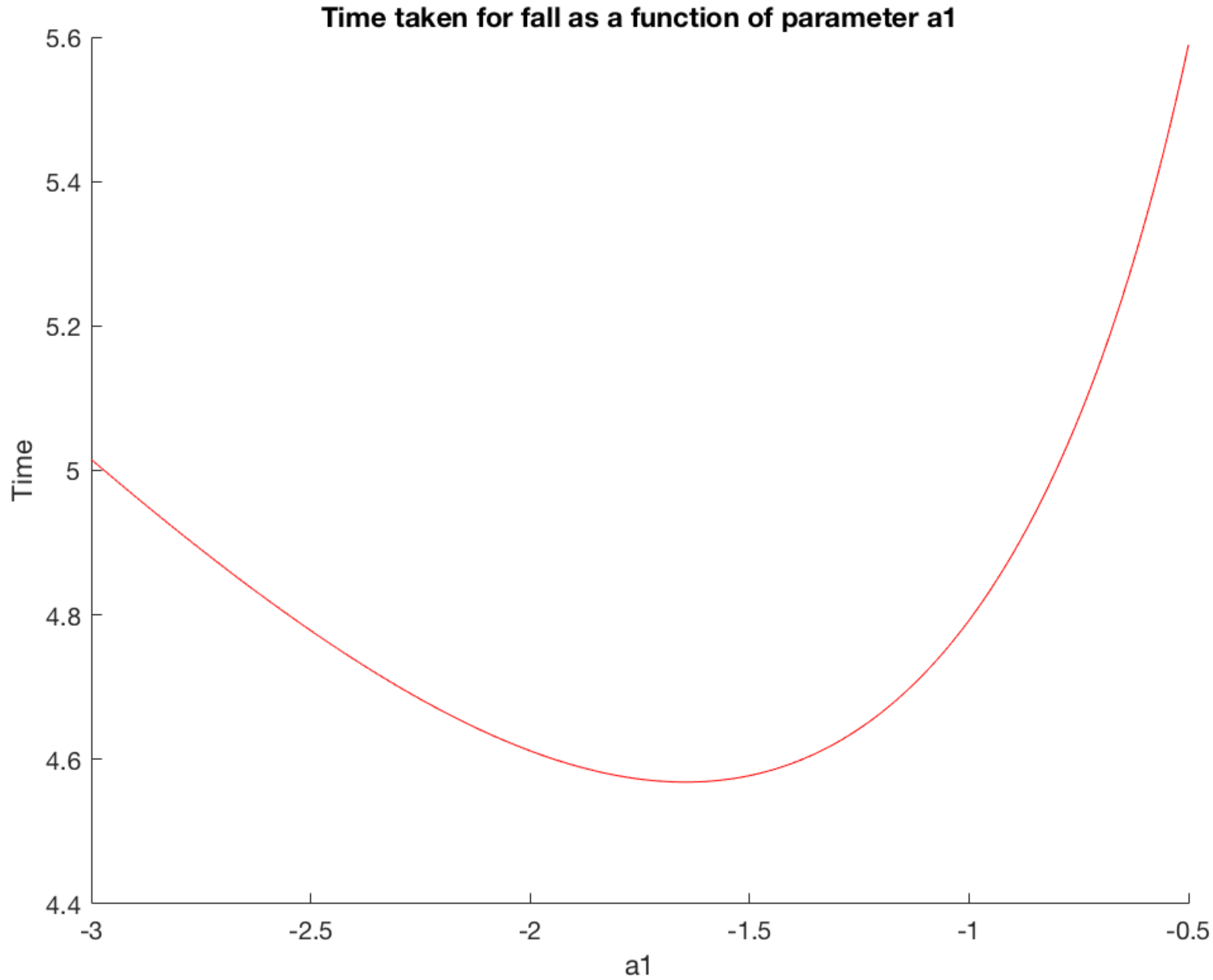
```
figure('name','Value of Time for Various a1','NumberTitle','on');
hold on;
title('x and y coordinates of particle')
ylabel('y')
xlabel('x');
plot(x,y(x),'r');
hold off;
```



It follows that a<0. We choose the value of a<0 that minimizes the motion. We thus examine the following graph time as a function of 'a' more closely.

```
a = -3:.01:-0.5;
g = 0*a;
for k = 1:numel(a)
    fab = @(x) f(a(k),x);
    g(k) = quadgk(fab,0,pi);
end
```

```
%Graph 6
figure('name','Value of Time for Various a1','NumberTitle','on');
hold on;
title('Time taken for fall as a function of parameter a1')
ylabel('Time')
xlabel('a1')
plot(a,g,'r');
hold off;
```



This is minimum at approximately a=-1.65. Hence we choose this value. It follows that in the question we have a0=2, a1=-1.65 and a2=0.323

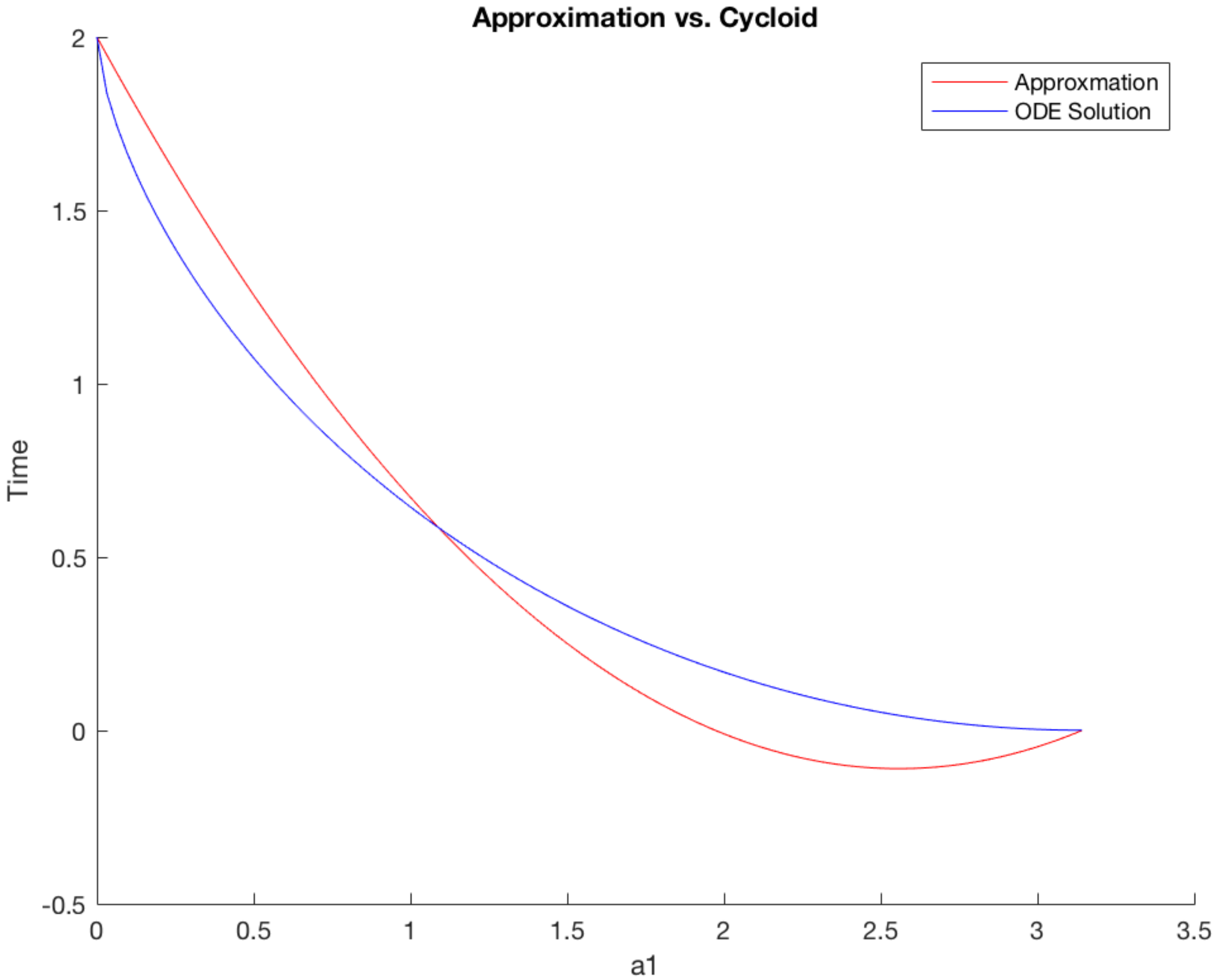
PART C

Observing graph 4 for negative values of a we estimate that the minimum fall time is 4.5686/(2g)^(1/2) which is approximately equal to 1.03 seconds.

PART D

Graph 7

```
figure('name','Value of Time for Various a1','NumberTitle','on');
hold on;
title('Approximation vs. Cycloid')
ylabel('Time')
xlabel('a1')
y=@(x) 2+(-1.65)*x+a2(-1.65).*x.^2;
plot(x,y(x),'r');
[x,sol]=ode45(@particlemotion,x,initialconditions);
plot(x,sol(:,1),'b');
legend('Approxmation','ODE Solution')
hold off;
```



Clearly the two solution agree to a significnt amount, note that the approximate trajectory falls slower at first.

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