Assignment 1 (35 pts)

Due February 25 2021

1. (**25pts**) Implement the particle swarm optimization (PSO) algorithm for general constrained optimization problems and apply your code to the portfolio of optimization problems described in the "Case Studies" document on Quercus. Here are some guidelines:

   - Use the quadratic penalty function approach to enforce the constraints.
   - Use the 2D bump test function (**P4**) to tune the parameters of your algorithm.
   - Compare the performance of static and dynamic quadratic penalty function approaches for the 2D bump test function. Based on these studies, you can select the approach that worked best for the other constrained test problems.
   - Since PSO is a stochastic optimization algorithm, carry out *at least* **10** independent runs for each parameter setting. In your report, include a plot containing the convergence trends for the independent runs. Report the best solution obtained over these runs in a table.

2. (**5pts**) Consider a generalized linear model of the form

$$\widehat{f}(\mathbf{x}, \mathbf{w}) = w_0 + \sum_{i=1}^{M-1} w_i \phi_i(\mathbf{x}),$$

   where $\mathbf{x} \in \mathbb{R}^D$, $w_i, i = 0, 1, 2, \ldots, M-1$ are the undetermined weights of the model, and $\phi_i : \mathbb{R}^D \to \mathbb{R}$ are known basis functions.

   Given a regression dataset $\{(\mathbf{x}^{(i)}, y^{(i)})\}_{i=1}^N$, we seek to estimate the weights by minimizing the ℓ_2 regularized weighted least-squares error function

$$\sum_{i=1}^{N} c_i \; (\widehat{f}(\mathbf{x}^{(i)}, \mathbf{w}) - y^{(i)})^2 + \lambda_i \sum_{i=1}^{M} w_i^2,$$

   where $\lambda_i \in \mathbb{R}$ are user-defined regularization parameters, and $c_i = 1, 2, \ldots, N$ denote a set of user defined weights. Derive a system of linear algebraic equations to be solved for the weights $(w_i, i = 0, 1, 2, \ldots, M-1)$ of the generalized linear model.

3. (**5pts**) Consider the constrained optimization problem

$$\min_{\mathbf{x}\in\mathbb{R}^n} \ f(\mathbf{x}),$$

subject to the inequality constraints $g_j(\mathbf{x}) \leq 0, j = 1, 2, \ldots, m$.

While solving this optimization problem, it was found that the feasible set is empty and no solution that satisfies all the constraints could be found. One way to resolve this problem would be to rewrite the constraints as $g_j(\mathbf{x}) \leq \delta_j, j = 1, 2, \ldots, m$, where $\delta_j \geq 0$ can be thought of as a constraint relaxation parameter.

- Your goal is to relax the constraints (i.e., find the smallest possible values of $\delta_j, j = 1, 2, \ldots, m$) such that the feasible set is non-empty. Can you pose this problem as an optimization problem?

Bonus question (5pts): Apply your PSO code to the bump test function (**P4**) with $n = 50$. Report the best solution you obtain and the corresponding value of $\mathbf{x}^*$ in a file appended to your assignment submission. The bonus marks will be awarded using a relative scaling system with the best solution in the class (f^{best}) being awarded 5pts. The points awarded to other submissions with an objective function value of f' will be $f^{best}/f' \times 5$.

Submission guidelines:

- Your assignment must be submitted in pdf format to Quercus. Reports submitted in other formats such as Microsoft Word are not acceptable.
- You are permitted to use Python for this assignment. Please mention the Python version you are using in your report.
- Submit MATLAB/Python codes archived in tar, zip or gzip format along with input files used to generate results. Do NOT use proprietary data compression formats such as rar. Please verify the integrity of your tar/zip/gzip file before uploading.
- Include a README file that describes how the codes must be run.
- Clearly explain all working steps in your answers.
- All submissions must comply with the AER1415 Collaboration Policy posted on Blackboard.

Marking guidelines:

- Q1: Marks for Q1 will be assigned as follows:
  - Code organization and clarity [2pts]

  - PSO results for case studies [23pts]: Marks will be split in the following fashion for the case studies: **P1** (20%), **P2** (20%), **P3** (15%), **P4** (20%), **P5** (25%)

- Q2: Clearly explain all working steps and define any new variables that you introduce in your formulation

- Q3: You are expected mathematically describe your approach to this problem. Clearly define all variables and explain all working steps.

Suggested format of report:

  - PSO algorithm implementation and testing
    * Algorithm description
    * Parameter tuning studies
    * Results for case studies (including formulation of **P5**)
    * Conclusions
  - Your answer to Q2
  - Your answer to Q3