MECH-3221 Control Theory

Homework 3

Instructions

- Make sure the name and student number for this homework are yours, if not contact your course instructor immediately.
- This evaluation covers **material from the third week of classes**.
- Note that each student has a different version, so do not try to copy from one another as it would cost you both mark and risk of plagiarism.
- If asked, write all the steps involved and all the equations used. Final answer ≠ full mark!
- This evaluation **is not** strictly multiple-choice
- Be cautious of the **time cues**.
- If this is not a strictly multiple-choice evaluation
    a) For qualitative questions, write down the key points, illustrate key concepts, and be concise.
    b) Make sure to sectionalize your answers referring to question elements and put your final answer for each section in a box.
    c) You need to either print this document, complete writing your solution and scan the material back to PDF and upload it or use a tablet or any other device that allows you to write on PDF files, save it and upload it. If neither is possible, you can only scan your solution pages and upload. For multiple choice questions, on your answer sheet, mention the question number and your choice for the question.
    d) The filename to upload must follow the “Lastname_firstname_XX.pdf” where XX is the last 2 digits of your student number and your name as shown on top of this page.
- All submissions must be electronic, no other submission format is accepted.
- Late submission is not accepted and will get a mark of ZERO.

Evaluation

Questions are graded based on the rubrics

Question 1 [4 marks] [20 minutes] [LO. 1]

A simple model of a heat exchanger is shown in Figure 1. Steam enters the chamber with temperature $T_{in,1} = 151.9\ °K$ and mass-flow rate $w_{in,1}$ and leaves the chamber with temperature $T_{out,1} = T_1$ and mass-flow rate $w_{out,1} = w_{in,1} = w_1 = 45.2\ Kg/s$. The temperature of the steam in the chamber is T_1, which is equal to the temperature of the outgoing steam. Cold water flows through copper tubes that enter the chamber with temperature $T_{in,2} = 85.8\ °K$ and mass-flow rate $w_{in,2} = w_2 = 31.4\ Kg/s$. The temperature of the water inside the chamber is T_2, which is equal to the temperature of the outgoing hot water. Heat is transferred from the steam to the water through the copper tubes, which have thermal resistance $R_2 = 35\ K/W$, and the hot water leaves the chamber with the same mass-flow rate as the incoming cold water. Thermal insulation $R_1 = 39\ K/W$ surrounds the heat exchanger and provides a thermal barrier with the ambient temperature $T_a = 120.3\ °K$. Consider the thermal capacitance of steam chamber to be $C_1 = 10.7\ J/K$ and the thermal capacitance of water in tubes to be $C_2 = 15.2\ J/K$. Also, consider the specific heat at constant pressure for steam to be $c_{p,1} = 1.872\ kJ/KgK$ and the specific heat at constant pressure for water to be $c_{p,2} = 4.18\ kJ/KgK$. Derive the complete mathematical model of the thermal system. Make sure in your final answer, all numerical values are substituted and equations are simplified to the simplest form.

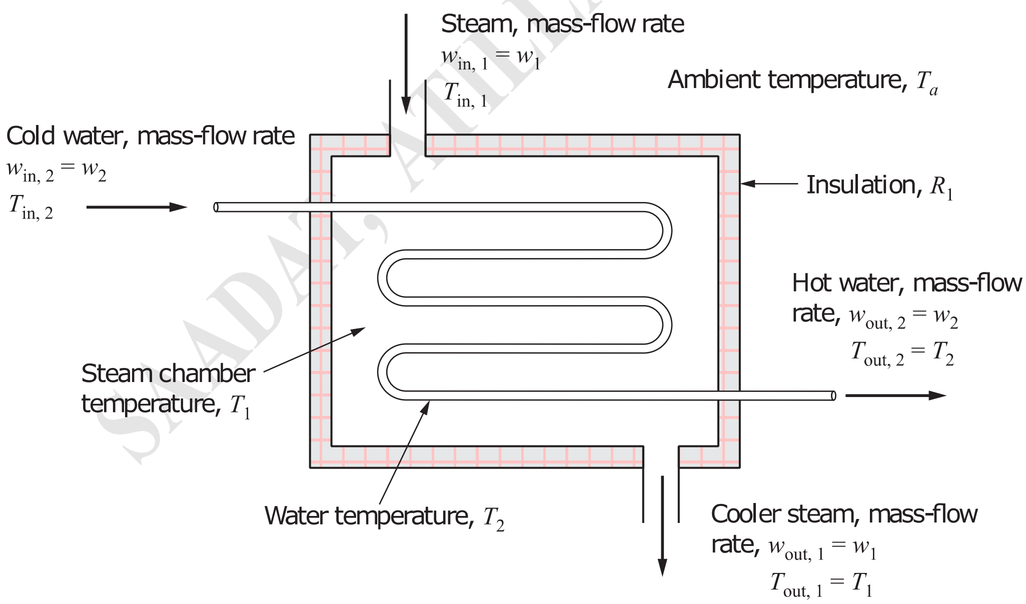
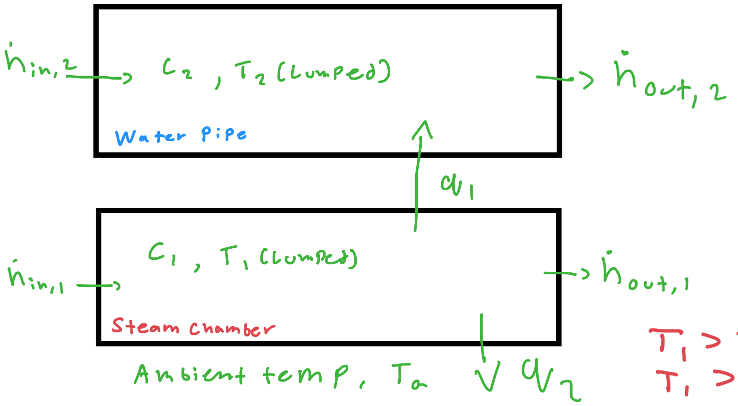


Figure 1 schematic of the heat exchanger

Solution

Provide your step by step solution here. Note that only providing the correct final answer does not guarantee a full mark for the question!

FBD:



$T_{in,1} = 151.9\ K$
$T_{in,2} = 85.8\ K$
$T_{out,1} = T_1$
$w_{out,1} = w_{in,1} = w_1 = 45.2\ kg/s$
$T_{out,2} = T_2$
$w_{in,2} = w_2 = 31.4\ kg/s$
$R_2 = 35\ K/W$
$R_1 = 39\ K/W$
$T_a = 120.3\ K$
$C_1 = 10.7\ J/K$
$C_2 = 15.2\ K/K$
$c_{p,1} = 1872\ J/kg\,K$
$c_{p,2} = 4180\ J/kg\,K$

$T_1 > T_a$
$T_1 > T_2$

Steam Chamber: $C_1 \dot{T}_1 = \dot{h}_{in,1} - \dot{h}_{out,1} - q_1 - q_2$

Water pipe: $C_2 \dot{T}_2 = \dot{h}_{in,2} - \dot{h}_{out,2} + q_1$

q_1: Steam -> Tube
q_2: steam -> ambient

Assumptions:
- Neglect Radiation, Friction effects
- Neglect tube and insulation interactions

Heat Flow & Enthalpy Rates:

$$q_1 = \frac{T_1 - T_2}{R_2} \qquad q_2 = \frac{T_1 - T_a}{R_1} \qquad \dot{h}_i = \dot{m}_i c_{p_i} T_i$$

$$\therefore\ C_1 \dot{T}_1 = \dot{m}_1 c_{p,1} T_{in,1} - \dot{m}_1 c_{p_1} T_1 - \frac{T_1 - T_2}{R_2} - \frac{T_1 - T_a}{R_1}$$

$$C_2 \dot{T}_2 = \dot{m}_2 c_{p,2} T_{in,2} - \dot{m}_2 c_{p_2} T_2 + \frac{T_1 - T_2}{R_2}$$

$$\Rightarrow \boxed{\begin{aligned} & R_1 R_2 C_1 \dot{T}_1 + (R_1 R_2 \dot{m}_1 c_{p_1} + R_1 + R_2)\, T_1 - R_1 T_2 = R_1 R_2 \dot{m}_1 c_{p_1} T_{in,1} + R_2 T_a \\ & R_2 C_2 \dot{T}_2 + R_2 \dot{m}_2 c_{p_2} T_2 - T_1 + T_2 = \dot{m}_2 c_{p,2} T_{in,2} R_2 \end{aligned}}$$

Numerical Result:

$$\boxed{\begin{aligned} & 14.6 \times 10^3\, \dot{T}_1 + 115.5 \times 10^6\, T_1 - 39\, T_2 = 1.75 \times 10^{10} \\ & 532\, \dot{T}_2 + 131 \times 10^3\, T_2 - T_1 = 567.4 \times 10^6 \end{aligned}}$$

