MECH-3221 Control Theory

Homework 7

Instructions

- Make sure the name and student number for this homework are yours, if not contact your course instructor immediately.
- This evaluation covers **material from the seventh week of classes**.
- Note that each student has a different version, so do not try to copy from one another as it would cost you both mark and risk of plagiarism.
- If asked, write all the steps involved and all the equations used. Final answer ≠ full mark!
- This evaluation **is not** strictly multiple-choice
- Be cautious of the **time cues**.
- If this is not a strictly multiple-choice evaluation
    a) For qualitative questions, write down the key points, illustrate key concepts, and be concise.
    b) Make sure to sectionalize your answers referring to question elements and put your final answer for each section in a box.
    c) You need to either print this document, complete writing your solution and scan the material back to PDF and upload it or use a tablet or any other device that allows you to write on PDF files, save it and upload it. If neither is possible, you can only scan your solution pages and upload. For multiple choice questions, on your answer sheet, mention the question number and your choice for the question.
    d) The filename to upload must follow the "Lastname_firstname_XX.pdf" where XX is the last 2 digits of your student number and your name as shown on top of this page.
- All submissions must be electronic, no other submission format is accepted.
- Late submission is not accepted and will get a mark of ZERO.

Evaluation

Questions are graded based on the rubrics

Question 1 [4 marks] [20 minutes] [LO. 4]

A simplified, linear representation of an electrohydraulic actuator (EHA) consists of a power amplifier, a solenoid model, and a mechanical valve model. The linear I/O equations for each subsystem are

Power amplifier:	$0.287\dot{e}_0 + e_0 = 24.4e_{in}(t)$
Solenoid actuator:	$0.436f + f = 6.6e_0$
Spool valve:	$1.11\ddot{z} + 9.2\dot{z} + 1213z = f$

where $e_{in}(t)$ is the low-power voltage input to the amplifier, e_0 is the amplifier voltage output, f is the output force of the solenoid actuator (in N), and z is the position of the spool-valve mass m (in m). If the voltage input is a step function $e_{in}(t) = 0.6U(t)V$ which is a step with magnitude of $0.6\ V$ at $t = 0$ and the amplifier output is initially zero and the valve is initially at static equilibrium ($z = 0$).

a) If the EHA system described above is to be shown using the following diagram, find the transfer functions TF_1, TF_2 and TF_3 in Figure 1. [0.6 *mark*]

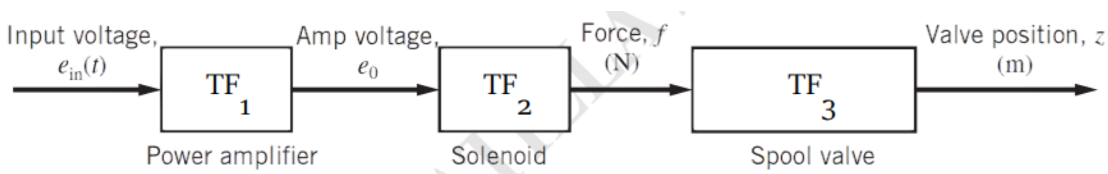


Figure 1 overall EHA diagram with transfer functions

b) Accurately sketch the response of the power amplifier output. You must calculate the values for the settling time and steady-state voltage for this sub-system and **hand-draw** the corresponding plot using this information. You plot must have proper x and y axes, labels, units, grids. [*1 mark*]

c) Compute the steady-state position of the spool valve. You must calculate the overall gain for the system to be able to calculate the steady-state position of the spool. [0.2 *mark*]

d) Accurately sketch the response of the spool-valve position $z(t)$. Label all important response characteristics on your sketch. You must calculate the values for the undamped natural frequency, damping ratio, peak time, maximum overshoot, maximum position, settling time, period of oscillation, and the number of cycles to steady-state for this sub-system and **hand-draw** the corresponding plot using this information. You plot must have proper x and y axes, labels, units, grids. [2.2 *mark*]

You are required to show all the steps involved in finding your final answer, even the smallest details. Make sure in your final answer, all numerical values are substituted and equations are simplified to the simplest form.

Solution

Provide your step by step solution here. Note that only providing the correct final answer does not guarantee a full mark for the question!

a)

$$TF_1 = \frac{24.4}{0.287s + 1}\ ;\ TF_2 = \frac{6.6}{0.436s + 1}\ ;\ TF_3 = \frac{1}{1.11s^2 + 9.2s + 1213}$$

b) Power Amplifier characteristic equation:

$$\tau r + 1 = 0.287r + 1 = 0$$

Characteristic root:

$$r = -\frac{1}{\tau} = -\frac{1}{0.287} = -3.484$$

Natural Response:

$$y_H(t) = ce^{-3.484t}$$

Settling Time:

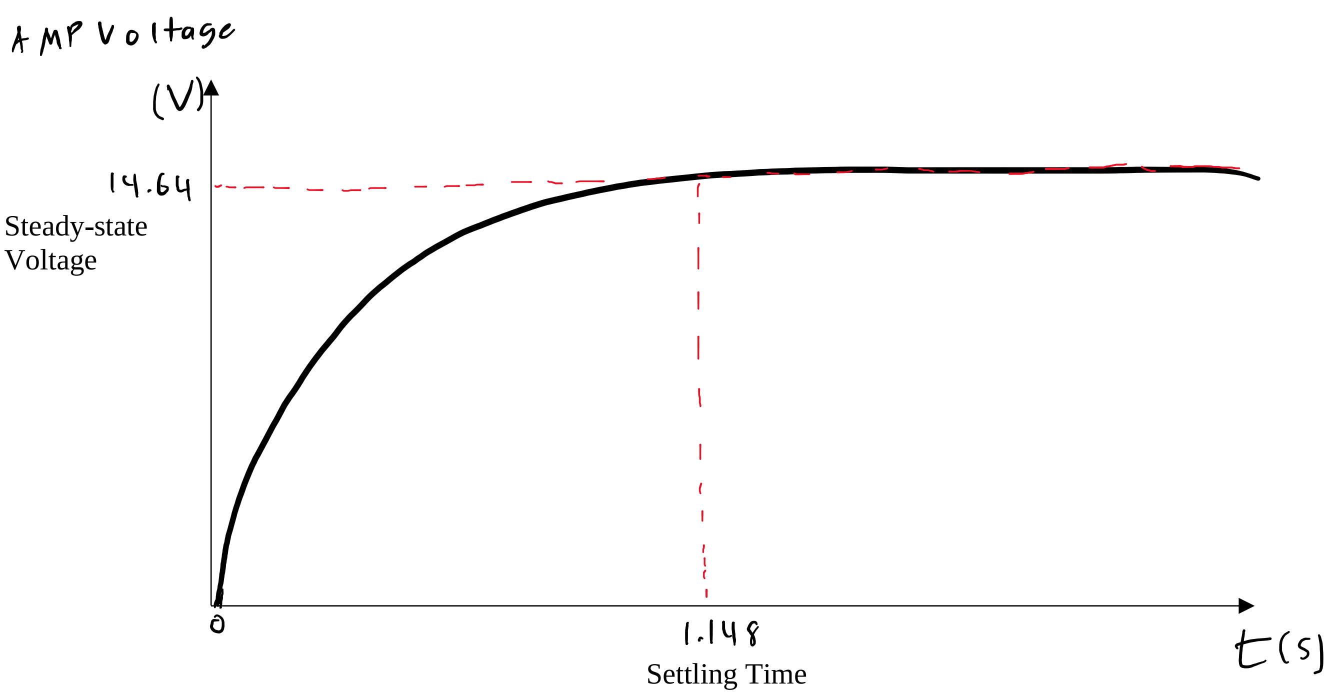
$$t_s \approx 4\tau = 1.148\ sec$$

Steady-state voltage:

$$y_{ss} = AU = (24.4)(0.6) = 14.64\ V$$

Initial Voltage

$$y_0 = 0.6\ V$$



c)

$$TF_{overall}(s) = TF_1 \cdot TF_2 \cdot TF_3$$

$$TF_{overall}(s) = \frac{4026}{25 \cdot (\frac{3472413s^4}{25E6} + \frac{4884361s^3}{25E5} + \frac{39886679s^2}{25E4} + \frac{886199s}{1000} + 1213)}$$

$$TF_{overall}(s = 0) = \frac{4026}{25 \cdot 1213} \cong 0.132762$$

$$z_{ss} = TF_{overall}(s = 0) \cdot U = (0.132762)(0.6) = 0.079657\ m$$

d) Undamped natural frequency:

$$\omega_n = \sqrt{\frac{1213}{1.11}} = 33.06$$

Damping ratio:

$$\zeta = \frac{9.2/1.11}{2\omega_n} = 0.1254$$

Peak time:

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.09579\ sec$$

Max Overshoot:

$$M_{OS} = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} = 0.672 = 67.2\%$$

Max position:

$$z_{max} = z_{ss}(1 + M_{OS}) = (0.079657)(1 + 0.658) = 0.1332\ m$$

Settling time:

$$t_s = \frac{4}{\zeta\omega_n} = 0.965\ sec$$

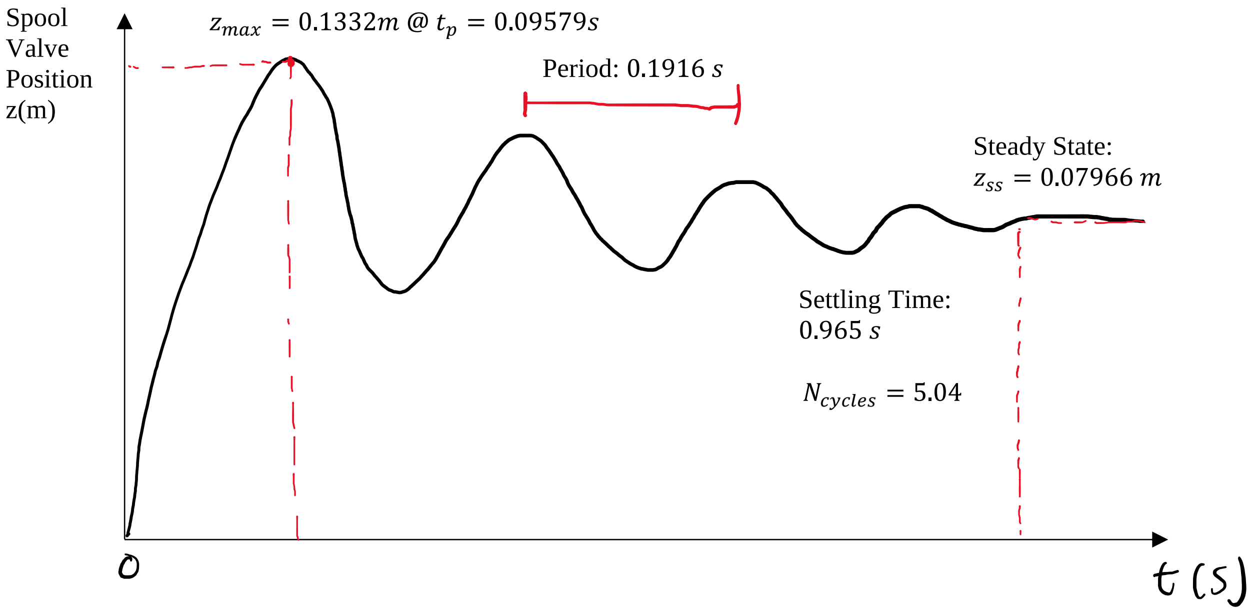
Period of Oscillation:

$$T_{period} = \frac{2\pi}{\omega_n\sqrt{1-\zeta^2}} = 2t_p = 0.1916\ sec$$

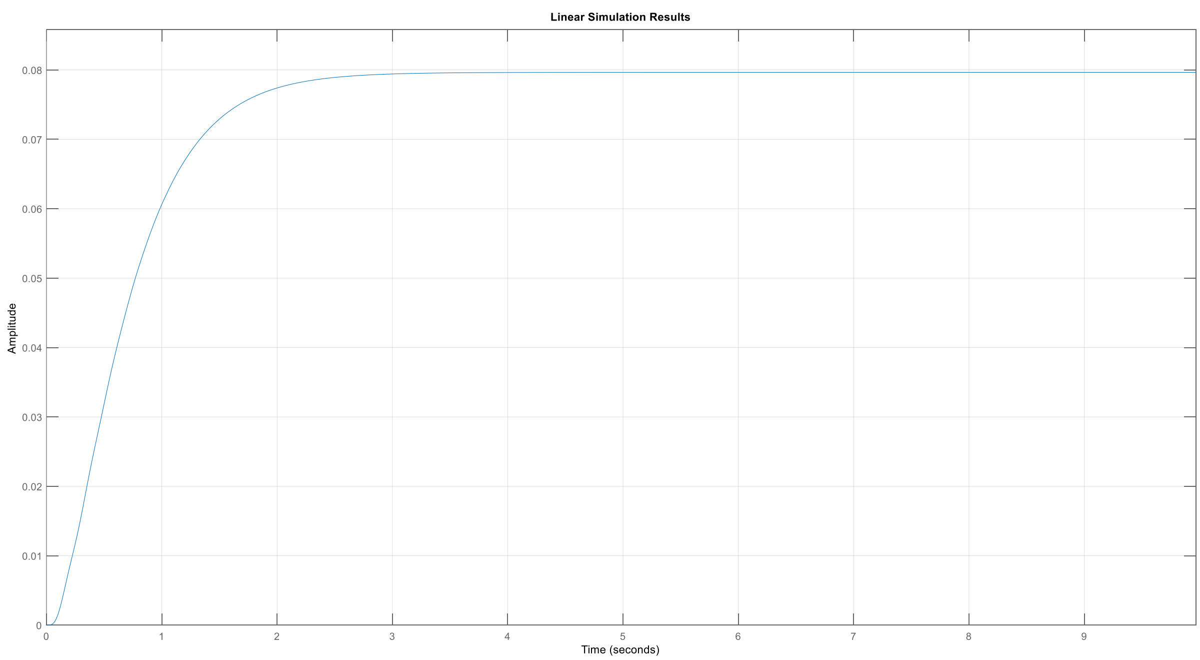
Number of cycles:

$$N_{cycles} = \frac{2\sqrt{1-\zeta^2}}{\pi\zeta} \cong 5.04$$

Hand Sketch using calculated characteristics from lectures:



MATLAB code and step response from entire EHA system's transfer function:

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%Control Theory HW7 - Atilla Saadat
syms s
a = (24.4/(0.287*s+1)) * (6.6/(0.436*s+1)) * (1/(1.11*s^2+9.2*s+1213)) ;
pretty(expand(a));
sys = tf([4026],25*[3472413/25000000 4884361/2500000 39886679/250000 886199/100
0 1213]);
t = 0:0.001:10;
u = 0.6*ones(size(t));
damp(sys);
lsim(sys,u,t);
grid on
```