MECH-3221 Control Theory

Homework 9

Instructions

- Make sure the name and student number for this homework are yours, if not contact your course instructor immediately.
- This evaluation covers **material from the ninth week of classes**.
- Note that each student has a different version, so do not try to copy from one another as it would cost you both mark and risk of plagiarism.
- If asked, write all the steps involved and all the equations used. Final answer $\neq$ full mark!
- This evaluation **is not** strictly multiple-choice
- Be cautious of the **time cues**.
- If this is not a strictly multiple-choice evaluation
  a) For qualitative questions, write down the key points, illustrate key concepts, and be concise.
  b) Make sure to sectionalize your answers referring to question elements and put your final answer for each section in a box.
  c) You need to either print this document, complete writing your solution and scan the material back to PDF and upload it or use a tablet or any other device that allows you to write on PDF files, save it and upload it. If neither is possible, you can only scan your solution pages and upload. For multiple choice questions, on your answer sheet, mention the question number and your choice for the question.
  d) The filename to upload must follow the "Lastname_firstname_XX.pdf" where XX is the last 2 digits of your student number and your name as shown on top of this page.
- All submissions must be electronic, no other submission format is accepted.
- Late submission is not accepted and will get a mark of ZERO.

Evaluation

Questions are graded based on the rubrics

Question 1 [4 marks] [20 minutes] [LO. 4]

A vibration isolation system for a 1-DOF mechanical system is shown below. Displacement of the mass x is measured from the static equilibrium position and the system parameters are $m = 0.3\ kg$, $k = 10\ N/m$, $b_1 = 4.3\ Ns/m$, and $b_2 = 0.5\ Ns/m$.

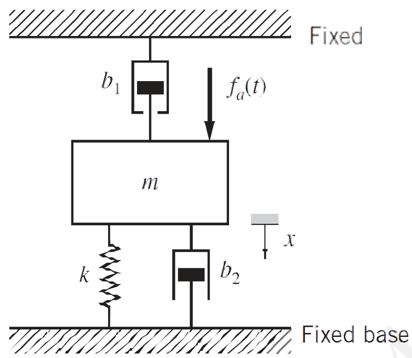


Figure 1 schematic of the vibration isolation system

a) Derive the mathematical model of the system. Make sure you have the FBD and all equations and signs are properly showcased. [$0.8 marks$]

b) Use the system equation found in part (a) to compute the frequency response $x_{ss}(t)$ if the input force is $f_a(t) = 8\sin(6t)\ N$. Note that to do this, you have to find the transfer function and sinusoidal transfer function for the system. [$2\ marks$]

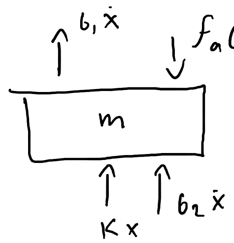
c) Determine the input frequency ω that results in the largest steady-state amplitude of the mass displacement. [$1.2 marks$]

You are required to show all the steps involved in finding your final answer, even the smallest details. Make sure in your final answer, all numerical values are substituted and equations are simplified to the simplest form.

Solution

Provide your step by step solution here. Note that only providing the correct final answer does not guarantee a full mark for the question!

a)



$$\downarrow x \quad \uparrow + \sum F_x = m\ddot{x} = -b_1\dot{x} - b_2\dot{x} - kx + f_a(t)$$

$$m\ddot{x} + (b_1 + b_2)\dot{x} + kx = f_a(t)$$

$$\Rightarrow \quad 0.3\ddot{x} + 4.8\dot{x} + 10x = 8\sin(6t)$$

b) $\mathcal{L}(0.3\ddot{x}) = 0.3(s^2 X(s) - s\cancel{X(0)} - \cancel{\dot{X}(0)}) = 0 \qquad s^2 X(s)$

$\mathcal{L}(4.8\dot{x}) = 4.8(sX(s) - \cancel{X(0)}) = 4.8sX(s$

$\mathcal{L}(10x) = 10X(s)$

$$G(s) = \frac{1}{0.3s^2 + 4.8s + 10} = \frac{X(s)}{F_a(s)}$$

$$x_{ss}(t) = |G(jw)|\, 8\sin(6t + \phi)$$

$$G(jw) = \frac{1}{10 - 0.3w^2 + j4.8w} \qquad w = 6 \text{ rad/s}$$

$$= \frac{1}{-0.8 + 28.8j}$$

$$|G(jw)| = \frac{\sqrt{1^2 + 0^2}}{\sqrt{0.8^2 + 28.8^2}} = 0.03471$$

$$\phi = \tan^{-1}\left(\frac{0}{1}\right) - \tan^{-1}\left(\frac{28.8}{-0.8}\right) = 0 - (-1.543)$$
$$= 1.543 \text{ rad}$$

$$X_{ss} = 0.03471 \sin(6t + 1.543)$$

c) $\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{10}{0.3}} = 5.77$ rad/s

$$\zeta = \frac{a_1}{2\sqrt{a_0}} = \frac{4.8/0.3}{2\sqrt{10/0.3}} = 1.386$$

$$\omega_r = \omega_n \sqrt{1 - 2\zeta^2}$$

$2\zeta^2 > 1$

$\therefore$ imaginary resonant Frequency

Because $\zeta\,(1.386) > 0.7071$

a resonant Peak does not exist and there is no resonant frequency resulting in a maximum amplitude