MECH-3221 Control Theory

Homework 5

Instructions

- Make sure the name and student number for this homework are yours, if not contact your course instructor immediately.
- This evaluation covers **material from the fifth week of classes**.
- Note that each student has a different version, so do not try to copy from one another as it would cost you both mark and risk of plagiarism.
- If asked, write all the steps involved and all the equations used. Final answer ≠ full mark!
- This evaluation **is not** strictly multiple-choice
- Be cautious of the **time cues**.
- If this is not a strictly multiple-choice evaluation
  a) For qualitative questions, write down the key points, illustrate key concepts, and be concise.
  b) Make sure to sectionalize your answers referring to question elements and put your final answer for each section in a box.
  c) You need to either print this document, complete writing your solution and scan the material back to PDF and upload it or use a tablet or any other device that allows you to write on PDF files, save it and upload it. If neither is possible, you can only scan your solution pages and upload. For multiple choice questions, on your answer sheet, mention the question number and your choice for the question.
  d) The filename to upload must follow the "Lastname_firstname_XX.pdf" where XX is the last 2 digits of your student number and your name as shown on top of this page.
- All submissions must be electronic, no other submission format is accepted.
- Late submission is not accepted and will get a mark of ZERO.

Evaluation

Questions are graded based on the rubrics

Question 1 [4 marks] [20 minutes] [LO. 2]

Figure 1 shows the optical disk drive as a simplified two-mass mechanical system. The pick-up head (PUH) mass $m_1 = 0.49\ kg$ is connected to the cart with stiffness $k_1 = 4.1\ \frac{N}{m}$ and friction coefficient $b_1 = 0.1\ \frac{N}{m/s}$, while the chassis and cart are lumped into mass $m_2 = 0.25\ kg$. A series of rubber mounts connects the chassis mass m_2 to the frame and these mounts have lumped stiffness $k_2 = 4.6\ \frac{N}{m}$ and friction coefficient $b_2 = 0.7\ \frac{N}{m/s}$. The mounts are used to suppress transmitted vibrations from the motion of the frame. The absolute displacements (as measured from the static equilibrium position) of the PUH and chassis/cart are z_1 and z_2, respectively. The absolute displacement of the frame is $z_{in}(t)$. The modeling equations for this system can be obtained as

$$m_1\ddot{z}_1 + b_1(\dot{z}_1 - \dot{z}_2) + k_1(z_1 - z_2) = 0$$
$$m_2\ddot{z}_2 + b_2\dot{z}_2 + b_1(\dot{z}_2 - \dot{z}_1) + k_1(z_2 - z_1) = b_2\dot{z}_{in} + k_2 z_{in}$$

$+ k_2 z_2$

Having these equations:

a) Derive an I/O equation, using the D-operator, of the optical disk drive system with pick-up head mass displacement z_1 as the system output and frame displacement $z_{in}(t)$ as the input.

b) Derive the transfer function of the optical disk drive system where pick-up head mass displacement z_1 is the system output and frame displacement $z_{in}(t)$ is the input.

You are required to show all the steps involved in finding your final answer even the smallest details. Make sure in your final answer, all numerical values are substituted and equations are simplified to the simplest form.

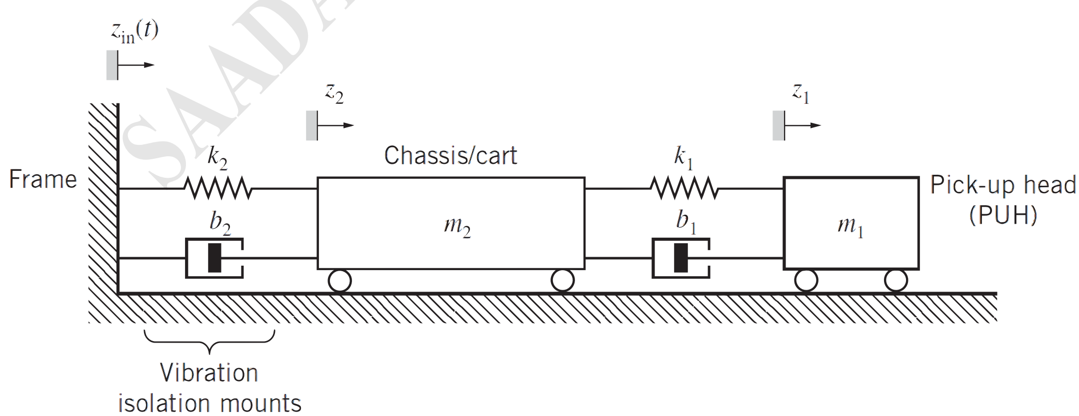

Figure 1 schematic of the simplified equivalent optical disk drive

Solution

Provide your step by step solution here. Note that only providing the correct final answer does not guarantee a full mark for the question!

$y = z_1$ $u = z_{in}(t)$

a) $m_1 \ddot{y} + b_1(\dot{y} - \dot{z}_2) + k_1(y - z_2) = 0$

$m_2 \ddot{z}_2 + b_2\dot{z}_2 + b_1(\dot{z}_2 - \dot{y}) + k_1(z_2 - y) + k_2 z_2 = b_2\dot{u} + k_2 u$

D-operator

$\Rightarrow m_1 D^2 y + b_1 D y - b_1 D z_2 + k_1(y - z_2) = 0$ (1)

$m_2 D^2 z_2 + b_2 D z_2 + b_1 D z_2 - b_1 D y + k_1(z_2 - y) + k_2 z_2 = b_2 D u + k_2 u$ (2)

Expand (1) for z_2:

$$z_2 = \frac{m_1 D^2 y + b_1 D y + k_1 y}{b_1 D + k_1} \quad (3)$$

Sub (3) into (2):
using MATLAB

$$k_1 k_2 y - k_1 k_2 u - D^2 b_1 b_2 u + D^2 b_1 b_2 y + D^3 b_1 m_1 y + D^3 b_1 m_2 y + D^3 b_2 m_1 y + D^2 k_1 m_1 y + D^2 k_1 m_2 y + D^2 k_2 m_1 y + D^4 m_1 m_2 y - D b_1 k_2 u - D b_2 k_1 u + D b_1 k_2 y + D b_2 k_1 y = 0$$

Rearrange and sub $D^k \to y^{(k)}$

$$\Rightarrow m_1 m_2 y^{(4)} + \underbrace{(b_1 m_1 + b_1 m_2 + b_2 m_1)}_{=\alpha}\dddot{y} + \underbrace{(b_1 b_2 + k_1 m_1 + k_1 m_2 + k_2 m_1)}_{=\beta}\ddot{y} + \underbrace{(b_1 k_2 + b_2 k_1)}_{=\gamma}\dot{y}$$

$$+ k_1 k_2 y = b_1 b_2 \ddot{u} + (b_1 k_2 + b_2 k_1)\dot{u} + k_1 k_2 u$$

where $y = z_1$
$u = z_{in}(t)$

Numerical: $\boxed{0.1225\, y^{(4)} + 0.4170\, \dddot{y} + 5.3580\, \ddot{y} + 3.33\, \dot{y} + 18.86\, y = 0.07\, \ddot{u} + 3.33\, \dot{u} + 18.86\, u}$

b) $$\frac{Y(t)}{U(t)} = \frac{b_1 b_2 D^2 + (b_1 k_2 + b_2 k_1) D + k_1 k_2}{m_1 m_2 D^4 + \alpha D^3 + \beta D^2 + \gamma D + k_1 k_2}$$

Replace D with s

$$\boxed{= \frac{0.07 s^2 + 3.33 s + 18.86}{0.1225 s^4 + 0.4170 s^3 + 5.358 s^2 + 3.33 s + 18.86}}$$

```
%Q1 I/O solver
syms m1
syms m2
syms D
syms y
syms b1
syms b2
syms k1
syms k2
syms u

%((m1*D^2*y + b1*D*y + k1*y)/(b1*D + k1))

XX = m2*D^2*(m1*D^2*y + b1*D*y + k1*y)...
    + b2*D*(m1*D^2*y + b1*D*y + k1*y)...
    + b1*D*(m1*D^2*y + b1*D*y + k1*y)...
    - b1*D*y*(b1*D + k1)...
    + k1*(m1*D^2*y + b1*D*y + k1*y)...
    - k1*y*(b1*D + k1)...
    + k2*(m1*D^2*y + b1*D*y + k1*y)...
    - b2*D*u*(b1*D + k1)...
    - k2*u*(b1*D + k1);

sympref('AbbreviateOutput',false);
pretty(expand(XX))
```