MECH-3221 Control Theory

Homework 2

Instructions

- Make sure the name and student number for this homework are yours, if not contact your course instructor immediately.
- This evaluation covers **material from the second week of classes**.
- Note that each student has a different version, so do not try to copy from one another as it would cost you both mark and risk of plagiarism.
- If asked, write all the steps involved and all the equations used. Final answer ≠ full mark!
- This evaluation **is not** strictly multiple-choice
- Be cautious of the **time cues**.
- If this is not a strictly multiple-choice evaluation
    a) For qualitative questions, write down the key points, illustrate key concepts, and be concise.
    b) Make sure to sectionalize your answers referring to question elements and put your final answer for each section in a box.
    c) You need to either print this document, complete writing your solution and scan the material back to PDF and upload it or use a tablet or any other device that allows you to write on PDF files, save it and upload it. If neither is possible, you can only scan your solution pages and upload. For multiple choice questions, on your answer sheet, mention the question number and your choice for the question.
    d) The filename to upload must follow the "Lastname_firstname_XX.pdf" where XX is the last 2 digits of your student number and your name as shown on top of this page.
- All submissions must be electronic, no other submission format is accepted.
- Late submission is not accepted and will get a mark of ZERO.

Evaluation

Questions are graded based on the rubrics

Question 1 [4 marks] [20 minutes] [LO. 1]

Figure 1 shows an electrical system. Derive the mathematical model in terms of the appropriate dynamic variables. The source for this circuit provides the input voltage $e_{in}(t)$. The rest of the elements in the circuit have the following properties: $R_1 = 26.9\,\Omega$, $R_2 = 4.7\,\Omega$, and $R_3 = 0.3\,\Omega$. $C = 11.2\,F$ and $L = 5.9\,H$. Make sure in your final answer, all numerical values are substituted and equations are simplified to the simplest form. Note that the unit for a resistance is ohms or Ω. The unit for the capacitance is farads or F, and the unit for the inductance is henries or H in an electrical circuit.

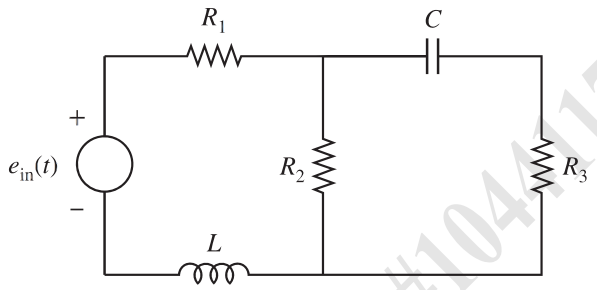
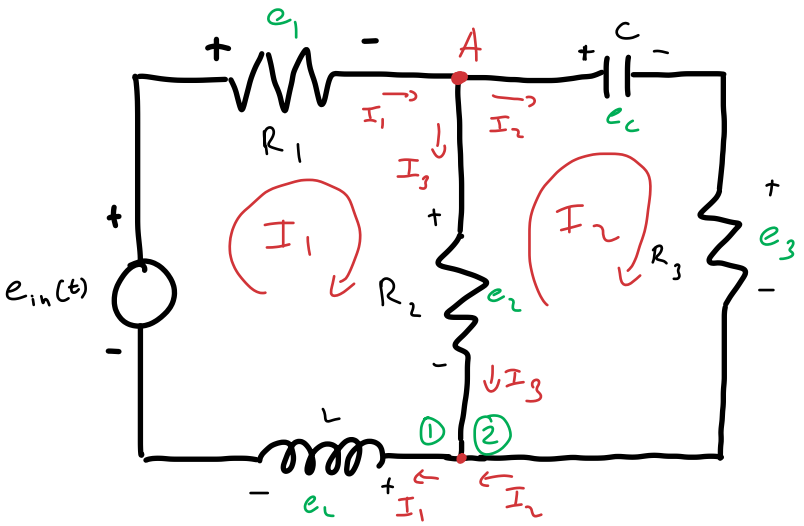


Figure 1 schematic of the electrical circuit

Solution

Provide your step by step solution here. Note that only providing the correct final answer does not guarantee a full mark for the question!



$R_1 = 26.9\ \Omega$
$R_2 = 4.7\ \Omega$
$R_3 = 0.3\ \Omega$
$C = 11.2\ F$
$L = 5.9\ H$

① $C\dot{e}_c = I_2$
② $L\dot{I}_1 = e_L$
} 1st order ODEs

KCL @ A : $I_1 = I_2 + I_3$

KVL @ ① : $e_{in}(t) - e_1 - e_2 - e_L = 0$

$\Rightarrow e_{in}(t) - I_1 R_1 - I_3 R_2 - e_L = 0$

KVL @ ② : $-e_c - e_3 + e_2 = 0$

$\Rightarrow -e_c - I_2 R_3 + I_3 R_2 = 0$ sub ① $\Rightarrow -e_c - (C\dot{e}_c) R_3 + I_3 R_2 = 0$

$\therefore I_3 = \dfrac{e_c + C\dot{e}_c R_3}{R_2}$ ②

sub KCL @ A & sub ②

$e_{in}(t) - (I_2 + I_3) R_1 - I_3 R_2 - L\dot{I}_1 = 0$

$\Rightarrow e_{in}(t) - I_2 R_1 - I_3 R_1 - I_3 R_2 - L\dot{I}_1 = 0$

$\Rightarrow e_{in}(t) - (R_1 + R_2) I_3 - (C\dot{e}_c) R_1 - L\dot{I}_1 = 0$ sub ①

$e_{in}(t) - (R_1 + R_2)\left(\dfrac{e_c + C\dot{e}_c R_3}{R_2}\right) - C\dot{e}_c R_1 - L\dot{I}_1 = 0$

$e_{in}(t) - \dfrac{R_1 + R_2}{R_2} e_c - \dfrac{(R_1 + R_2) C\dot{e}_c R_3}{R_2} - C\dot{e}_c R_1 - L\dot{I}_1 = 0$

$e_{in}(t) - \left(\dfrac{R_1 + R_2}{R_2}\right) e_c - \dfrac{R_1 R_3}{R_2} C\dot{e}_c - \dfrac{R_2 R_3}{R_2} C\dot{e}_c - C\dot{e}_c R_1 - L\dot{I}_1 = 0$

$\therefore \boxed{e_{in}(t) - \left(\dfrac{R_1 + R_2}{R_2}\right) e_c - \left(\dfrac{R_1 R_3}{R_2} + R_3 + R_1\right) C\dot{e}_c - L\dot{I}_1 = 0}$

Numerical

$\boxed{e_{in}(t) - 6.72\, e_c - 323.87\, \dot{e}_c - 5.9\, \dot{I}_1 = 0}$

