MECH-3221 Control Theory

Homework 1

Instructions

- Make sure the name and student number for this homework are yours, if not contact your course instructor immediately.
- This evaluation covers **material from the first week of classes**.
- Note that each student has a different version, so do not try to copy from one another as it would cost you both mark and risk of plagiarism.
- If asked, write all the steps involved and all the equations used. Final answer ≠ full mark!
- This evaluation **is not** strictly multiple-choice
- Be cautious of the **time cues**.
- If this is not a strictly multiple-choice evaluation
    a) For qualitative questions, write down the key points, illustrate key concepts, and be concise.
    b) Make sure to sectionalize your answers referring to question elements and put your final answer for each section in a box.
    c) You need to either print this document, complete writing your solution and scan the material back to PDF and upload it or use a tablet or any other device that allows you to write on PDF files, save it and upload it. If neither is possible, you can only scan your solution pages and upload. For multiple choice questions, on your answer sheet, mention the question number and your choice for the question.
    d) The filename to upload must follow the "Lastname_firstname_XX.pdf" where XX is the last 2 digits of your student number and your name as shown on top of this page.
- All submissions must be electronic, no other submission format is accepted.
- Late submission is not accepted and will get a mark of ZERO.

Evaluation

Questions are graded based on the rubrics

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Question 1 [4 marks] [20 minutes] [LO. 1]

Figure 1 shows a lumped-parameter representation of a concept for a MEMS "tuning fork gyroscope" for measuring angular velocity. Displacements x_1, x_2, and x_3 are absolute positions of the respective masses and are measured from their static equilibrium positions where there is no deflection in the springs $k_1 = 2.9\ \frac{N}{m}$, $k_2 = 3.1\ \frac{N}{m}$, and $k_3 = 0.9\ \frac{N}{m}$. The system friction is represented by three ideal lumped dashpot elements $b_1 = 0.5\ \frac{N}{m/s}$, $b_2 = 3.2\ \frac{N}{m/s}$, and $b_3 = 1.1\ \frac{N}{m/s}$. Two external forces $f_1 = 176\ N$ and $f_2 = 160\ N$ are applied to masses $m_1 = 5\ kg$ and $m_2 = 19\ kg$, respectively. Derive the mathematical model of the MEMS gyroscope. Assume $x_1 > x_3$ (spring k_1 is compressed), $x_3 > x_2$ (spring k_2 is compressed) and positive direction is to the right.

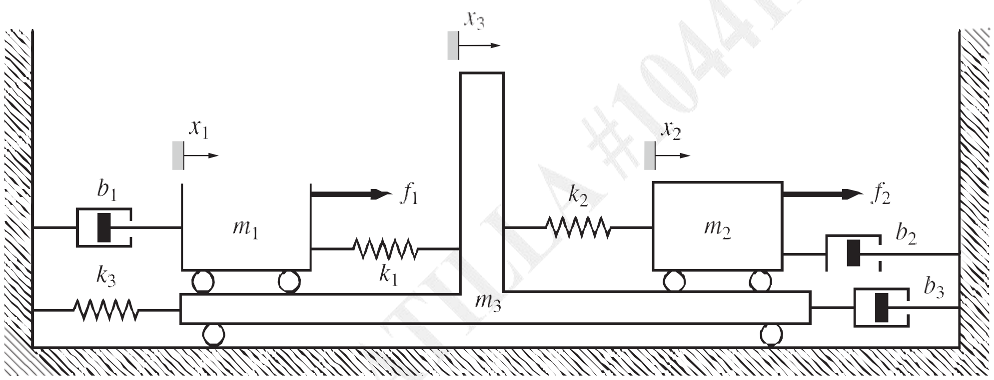

Figure 1 schematic of the tuning fork gyroscope system

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Solution

Provide your step by step solution here. Note that only providing the correct final answer does not guarantee a full mark for the question!

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Mech-3221 Control Theory - Homework 1 - Atilla Saadat

(1) RTF: Mathematical model of MEMS Gyro

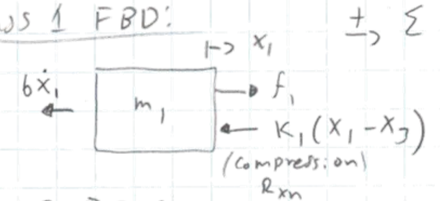
$x_1 > x_3$, $x_3 > x_2$ $\quad k_1 = 2.9\ N/m \quad k_2 = 3.1\ N/m \quad k_3 = 0.9\ N/m$

$b_1 = 0.5\ N/m/s \quad b_2 = 3.2\ N/m/s \quad b_3 = 1.1\ N/m/s$

$f_1 = 176\ N \quad f_2 = 160\ N \quad m_1 = 5\ kg \quad m_2 = 19\ kg$

Mass 1 FBD:

$x_1 > x_3$

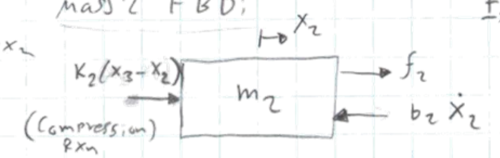


$$\overset{+}{\rightarrow} \sum F = -k_1(x_1 - x_3) + f_1 - b\dot{x} = m\ddot{x}$$

$$(1)\quad m_1\ddot{x}_1 + b_1\dot{x}_1 + k_1(x_1 - x_3) = f_1$$

Mass 2 FBD:

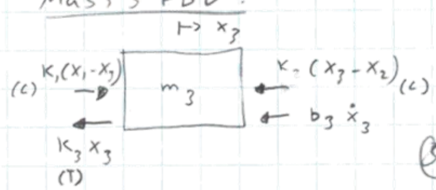
$x_3 > x_2$



$$\overset{+}{\rightarrow} \sum F = f_2 - b_2\dot{x}_2 + k_2(x_3 - x_2) = m_2\ddot{x}_2$$

$$(2)\quad m_2\ddot{x}_2 + b_2\dot{x}_2 - k_2(x_3 - x_2) = f_2$$

Mass 3 FBD:



$$\overset{+}{\rightarrow} \sum F = k_1(x_1 - x_3) - k_2(x_3 - x_2) - k_3x_3 - b_3\dot{x}_3 = m_3\ddot{x}_3$$

$$(3)\quad m_3\ddot{x}_3 + b_3\dot{x}_3 - k_1(x_1 - x_3) - k_2(x_3 - x_2) - k_3x_3$$

System equation (1 + 2 + 3)

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} b_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & b_3 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} + \begin{bmatrix} k_1 & 0 & -k_1 \\ 0 & k_2 & -k_2 \\ -k_1 & k_2 & (k_1 - k_3 - k_2) \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}$$

w/ Numerical values

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 19 & 0 \\ 0 & 0 & m_3 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 3.2 & 0 \\ 0 & 0 & 1.1 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} + \begin{bmatrix} 2.9 & 0 & -2.9 \\ 0 & 3.1 & -3.1 \\ -2.9 & 3.1 & -1.1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 176 \\ 160 \\ f_3 \end{bmatrix}$$

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