

Homework #3: Zeros and Direction

A. Zeros

Consider the system with two states, and the state-space model matrices given by

$$A = \begin{bmatrix} -6 & 1 \\ -5 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C = [1 \quad 0]$$

Find the zeros for this system

Solution

To find the zeros for the system we have to form the following determinant

$$\det \left(\begin{bmatrix} s_0 I - A & -B \\ C & D \end{bmatrix} \right) = \det \left(\begin{bmatrix} \begin{bmatrix} s_0 + 6 & -1 \\ 5 & s_0 \end{bmatrix} & \begin{bmatrix} -1 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 \end{bmatrix} & 0 \end{bmatrix} \right) = \det \left(\begin{bmatrix} s_0 + 6 & -1 & -1 \\ 5 & s_0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \right) = s_0 = 0$$
$$\rightarrow s_0 = 0$$

B. Directions

Consider the system with two states, and the state-space model matrices given by

$$A = \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C = [1 \quad 0]$$

Find the zeros and directions for this system

Solution

To find the zeros for the system we have to form the following determinant

$$\det \left(\begin{bmatrix} s_0 I - A & -B \\ C & D \end{bmatrix} \right) = \det \left(\begin{bmatrix} \begin{bmatrix} s_0 - 5 & -2 \\ -3 & s_0 - 1 \end{bmatrix} & \begin{bmatrix} -1 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 \end{bmatrix} & 0 \end{bmatrix} \right) = \det \left(\begin{bmatrix} s_0 - 5 & -2 & -1 \\ -3 & s_0 - 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \right)$$
$$= s_0 - 1 = 0 \rightarrow s_0 = 1$$

Now to find the direction, we need to solve the following problem

$$\begin{bmatrix} s_0 - 5 & -2 & -1 \\ -3 & s_0 - 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{01} \\ x_{02} \\ u_0 \end{bmatrix} = \begin{bmatrix} -4 & -2 & -1 \\ -3 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{01} \\ x_{02} \\ u_0 \end{bmatrix} = 0 \rightarrow \begin{cases} -4x_{01} - 2x_{02} - u_0 = 0 \\ -3x_{01} = 0 \\ x_{01} = 0 \end{cases}$$
$$\rightarrow \begin{cases} x_{01} = 0 \\ x_{01} = 0 \\ x_{02} = \frac{-u_0}{2} \end{cases} \rightarrow \text{we choose } x_{02} = 1 \rightarrow u_0 = -2 \rightarrow \begin{cases} X_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ u_0 = -2e^{1t} \end{cases}$$