Homework #10: Feedforward DOFB Servo

Feedforward DOFB Servo

1. Consider a system with the state space model

$$
\begin{aligned}
\dot{x} &= 8.75x + u + w \\
y &= -2x + v
\end{aligned}
$$

where w and v are process and sensor noise, respectively. Develop a feedforward DOFB servo controller for this system by working through the questions below. Use an optimal regulator with $Q/R = 1/\rho$ and an optimal estimator with $R_{ww}/R_{vv} = 1/\rho_e$. The integration error penalty is E.

a. Determine the values of ρ and E which yield the following solution to the algebraic Riccati equation for the LQR problem:

$$\overline{P}_{lqr} = \begin{bmatrix} 2 & 1 \\ 1 & 5.625 \end{bmatrix}$$

b. Which does this LQR design penalize more heavily: state errors or control effort? Explain your answer.

c. Determine the regulator gains K and K_I.

d. Show that the regulator poles are $\lambda_{reg} = -2.21, -9.04$ (rounded to 3 significant figures).

e. For an estimator pole of $\lambda_{est} = -20$, determine the estimator gain L.

f. Evaluate if the estimator is fast enough. Explain the criterion you use to assess the speed of the estimator.

g. Show that for this estimator pole location, $\rho_e = 0.012367$.

h. Does this LQE design have greater uncertainty for the model or for the measurements? Explain your answer.

i. If $\alpha = 0.7$, determine the full controller state space model in the form

$$
\begin{aligned}
\dot{\breve{\vec{x}}} &= A_{\breve{c}}\breve{\vec{x}} + B_{\breve{c},y}y + B_{\breve{c},r}r \\
u &= f\left(\breve{\vec{x}}, r\right)
\end{aligned}
$$

including the matrices $A_{\breve{c}}$, $B_{\breve{c},y}$, $B_{\breve{c},r}$, and the full form of the output u. r is a reference input.

j. Sketch the control architecture for the entire system. Be sure to label all the signals.

k. Determine the closed-loop system matrices A_{cl}, B_{cl}, C_{cl}.

l. Determine the closed-loop poles.

m. What is the purpose of adding the feedforward term to the DOFB servo configuration?

Solution:

a) $\bar{P} = \begin{bmatrix} 2 & 1 \\ 1 & 5.625 \end{bmatrix}$

$$\bar{Q} = \begin{bmatrix} Q & 0 \\ 0 & E \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & E \end{bmatrix}$$

$$\bar{P}\bar{A} - \bar{P}\bar{B}R^{-1}\bar{B}^T\bar{P} + \bar{Q} + \bar{A}^T\bar{P} = 0$$

$$A = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} = \begin{bmatrix} 8.75 & 0 \\ 2 & 0 \end{bmatrix}$$

$$\bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 5.625 \end{bmatrix}\begin{bmatrix} 8.75 & 0 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 1 & 5.625 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}\frac{1}{\rho}\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 5.625 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & E \end{bmatrix}$$

$$+ \begin{bmatrix} 8.75 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 5.625 \end{bmatrix} = 0$$

$$\begin{bmatrix} 19.5 & 0 \\ 20 & 0 \end{bmatrix} - \frac{1}{\rho}\begin{bmatrix} 2 \\ 1 \end{bmatrix}\begin{bmatrix} 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & E \end{bmatrix} + \begin{bmatrix} 19.5 & 20 \\ 0 & 0 \end{bmatrix} = 0$$

$$\begin{bmatrix} 39 & 20 \\ 20 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & E \end{bmatrix} - \frac{1}{\rho}\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} = 0 \to \begin{cases} 20 - \dfrac{2}{\rho} = 0 \to \boxed{\rho = \dfrac{1}{10}} \\ E = \dfrac{1}{\rho} \to \boxed{E = 10} \end{cases}$$

b) State error are penalized more heavily because $Q > R$

c) $\bar{K} = \begin{bmatrix} K & K_I \end{bmatrix} = R^{-1}\bar{B}^T\bar{P} = (10)\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 5.625 \end{bmatrix} = (10)\begin{bmatrix} 2 & 1 \end{bmatrix} = \begin{bmatrix} 20 & 10 \end{bmatrix} \to \begin{cases} K = 20 \\ K_I = 10 \end{cases}$

d) $A_{reg} = \bar{A} - \bar{B}\bar{K} = \begin{bmatrix} 8.75 & 0 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} 20 & 10 \end{bmatrix} = \begin{bmatrix} -11.25 & -10 \\ 2 & 0 \end{bmatrix}$

$$\left|\lambda_{reg}I - A_{reg}\right| = 0 \to \begin{vmatrix} \lambda + 11.25 & 10 \\ -2 & \lambda \end{vmatrix} = 0 \to \lambda^2 + 11.25\lambda + 20 = 0$$

Using given λ:

$$(-2.21)^2 + 11.25(-2.21) + 20 \approx 0 \;\; \to checks$$

$$(-9.04)^2 + 11.25(-9.04) + 20 \approx 0 \to checks$$

e) $\lambda_{est} = -20$

$$\lambda_{est} = A - LC \rightarrow LC = A - \lambda_{est} \rightarrow L = \frac{A - \lambda_{est}}{C} = \frac{8.75 - (-20)}{-2} \rightarrow \boxed{L = -14.375}$$

f) $\lambda_{est} < 2(\min(\lambda_{reg})) \rightarrow OK \rightarrow -20 < -9.04$

g) $0 = AQ_e + Q_e A^T + BR_{ww}B^T - Q_e C^T R_{vv}^{-1} C Q_e$

$$0 = (2)(8.75)Q_e + 1 - \frac{4}{\rho_e} Q_e^2$$

Since $L = Q_e C^T R_{vv}^{-1} = Q_e \frac{(-2)}{\rho_e} \rightarrow Q_e = -\frac{L\rho_e}{2} = \frac{14.375}{2}\rho_e$

$$0 = (2)(8.75)\left(\frac{14.375}{2}\right)\rho_e + 1 - \left(\frac{14.375}{2}\right)^2 (4)\frac{\rho_e^2}{\rho_e}$$

$$0 = 125.78125\rho_e + 1 - 206.640625\rho_e$$

$\rightarrow \rho_e = 0.012367$ Q.E.D. ("Quod Erat Demonstrandum"="that which was to be demonstrated")

h) $R_{ww} > R_{vv} \rightarrow$ greater uncertainty in the model

i) $\alpha = 0.7 \rightarrow R = -\alpha C^T = -0.7(-2) = -1.4$

$$A_{\check{c}} = \begin{bmatrix} A - LC - BK & -BK_I \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} -40 & -20 \\ 0 & 0 \end{bmatrix}$$

$$B_{\check{c},y} = \begin{bmatrix} L \\ -I \end{bmatrix} = \begin{bmatrix} -14.375 \\ -1 \end{bmatrix}$$

$$B_{\check{c},r} = \begin{bmatrix} -BKR \\ I \end{bmatrix} = \begin{bmatrix} -28 \\ 1 \end{bmatrix}$$

$$u = -\bar{K}\check{\vec{x}} - KRr$$

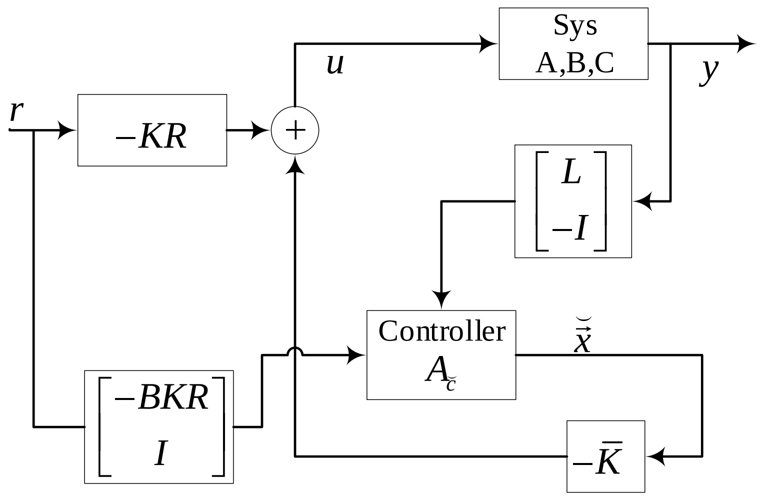
$$\check{\vec{x}} = \begin{bmatrix} \vec{x}_c \\ x_I \end{bmatrix}$$

$$\boxed{u = -[20 \quad 10]\check{\vec{x}} - 28r}$$

$$\dot{\check{x}} = A_{\check{c}}\check{x} + B_{\check{c},y}y + B_{\check{c},r}r$$

$$\boxed{\dot{\check{x}} = \begin{bmatrix} -40 & -20 \\ 0 & 0 \end{bmatrix}\check{x} + \begin{bmatrix} -14.375 \\ -1 \end{bmatrix}y + \begin{bmatrix} -28 \\ 1 \end{bmatrix}r}$$

j)



k)

$$A_{cl} = \begin{bmatrix} A & -B\bar{K} \\ \begin{bmatrix} LC \\ -C \end{bmatrix} & A_{\check{c}} \end{bmatrix} = \begin{bmatrix} 8.75 & -20 & -10 \\ 28.75 & -40 & -20 \\ 2 & 0 & 0 \end{bmatrix}$$

$$B_{cl} = \begin{bmatrix} -BKR \\ \begin{bmatrix} -BKR \\ I \end{bmatrix} \end{bmatrix} = \begin{bmatrix} -28 \\ -28 \\ 1 \end{bmatrix}$$

$$C_{cl} = \begin{bmatrix} C & 0 & 0 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 \end{bmatrix}$$

l)

$$\lambda_{cl} = \lambda_{reg}, \lambda_{est}: \begin{cases} -2.21 \\ -9.04 \\ -20 \end{cases} \rightarrow \text{from separation principle}$$

m) Purpose is to speed up transient response by letting system estimates “see” r, not only the integration error.