SINGLE-SOURCE SHORTEST PATHS PROBLEM

MOTIVATION

$G(V, E, W)$ → weight

V → Vertices

E → edges

Two ALGO

DIJKSTRA + edges

$O(V \lg V + E)$

$E = O(V^2)$

BELLMAN-FORD

-/+ edges

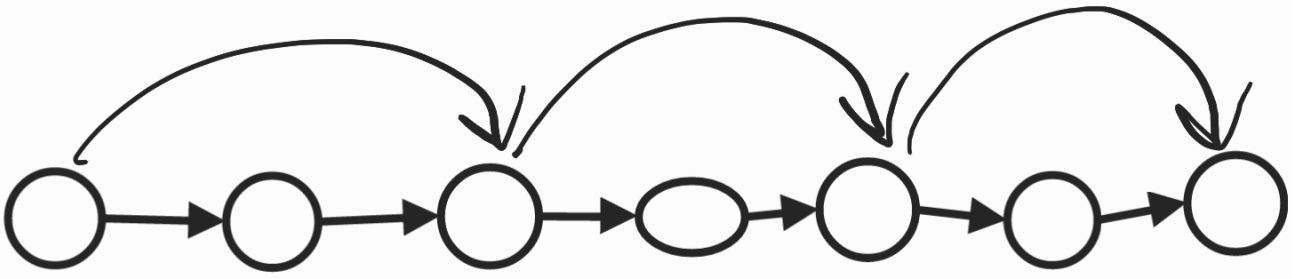
$O(VE)$

path $p = \langle v_0, v_1 \ldots v_k \rangle$

$(v_i, v_{i+1}) \in E$ for $0 \le i < k$

$$w(p) = \sum_{i=0}^{k-1} w(v_i, v_{i+1})$$

... find p with minimum weight ...



Weighted graphs

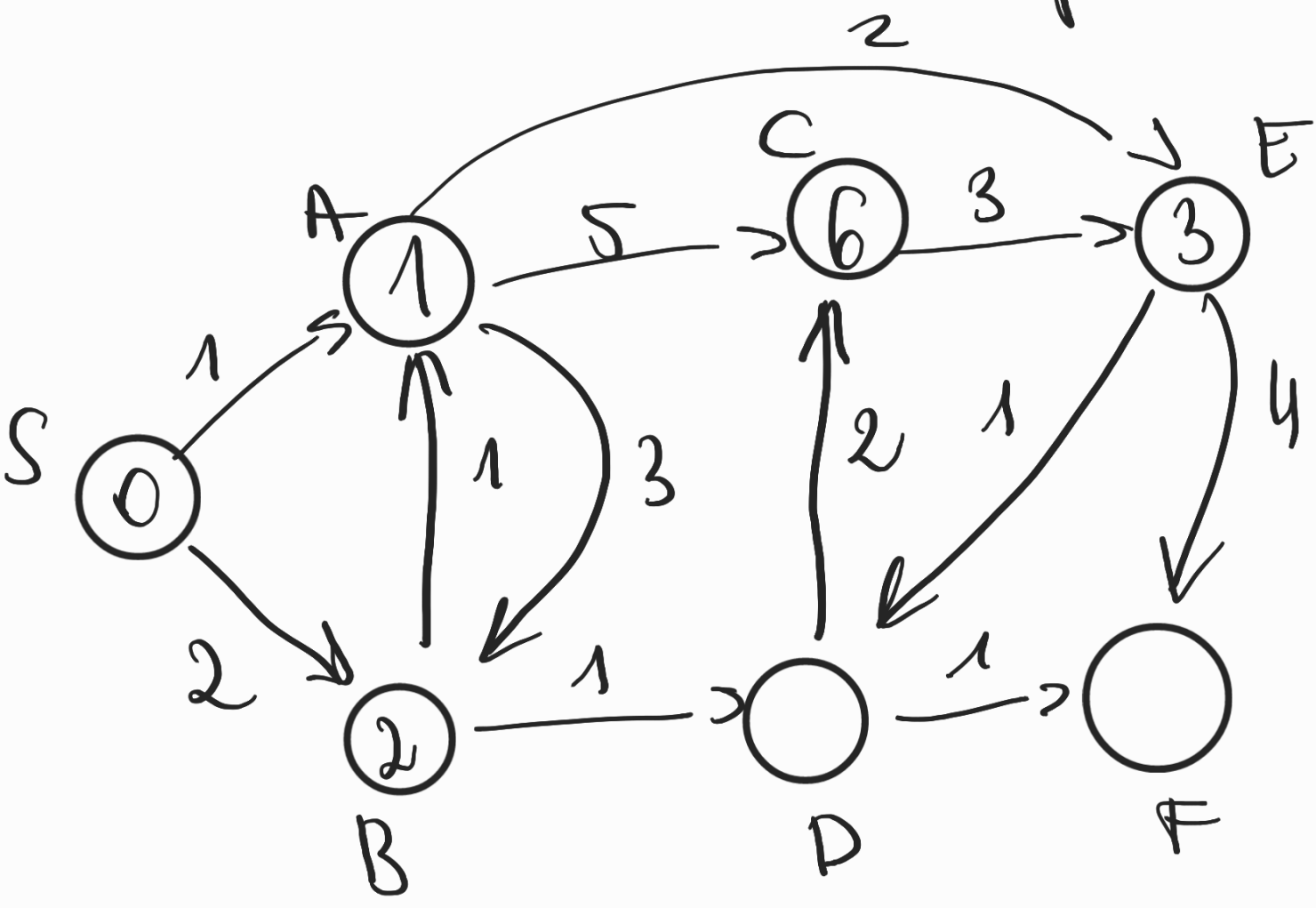
$v_0 \xrightarrow{p} v_k$ (v_0) is a path from v_0 to v_0 of weight 0

Shortest path weight from u to v as

$$\delta(u,v) = \begin{cases} \min\{w(p) : u \xrightarrow{p} v\} \\ \infty \end{cases}$$

otherwise

Any such path



General structure

$d[v] \leftarrow \infty$

$\pi[u] \leftarrow NIL$

Initialize for $u \in V$

$d[s] \leftarrow 0$

Repeat | select edge (u,v)

if $d[v] > d[u] + w(u,v)$

$d[v] = d[u] + w(u,v)$

$\pi[v] \leftarrow u$

until all edges have $d[v] \leq d[u] + w(u,v)$