

Dijkstra

Relax(u, v, w)

if $d[v] > d[u] + w(u, v)$

$d[v] = d[u] + w(u, v)$

$\pi[v] = u$

Relaxation is safe

Lemma: The relaxation operation maintains the invariant that $d[v] \geq \delta(s, v)$ for all $v \in V$

Proof. By induction on the number of steps By induction

$d[u] \geq \delta(s, u)$. By Δ -inequality

$\delta(s, v) \leq \delta(s, u) + \delta(u, v)$

Dijkstra (G, w, s)

$d[s] = 0$ \swarrow Initialize (G, s); $S \leftarrow \emptyset$; $Q \leftarrow V[G]$
while $Q \neq \emptyset$

$u \leftarrow \text{extract_min}(Q) // u \text{ from } Q$

$S \leftarrow S \cup \{u\}$

for each vertex $v \in \text{Adj}[u]$
relax(u, v, w)

