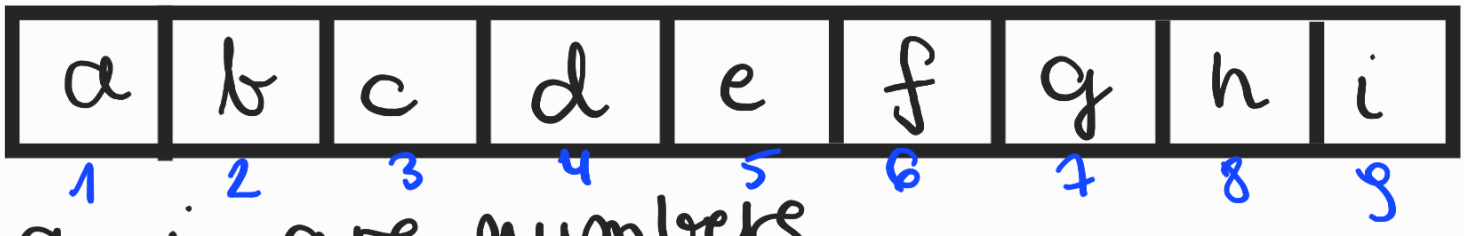
ALGORITHMS THINKING, PEAK FINDING

PEAK FINDING

One-dimensional



a - i are numbers

- Position 2 is a peek if and only if $b \geq a$ & $b \geq c$
- Positin 9 is a peek if $i \geq h$

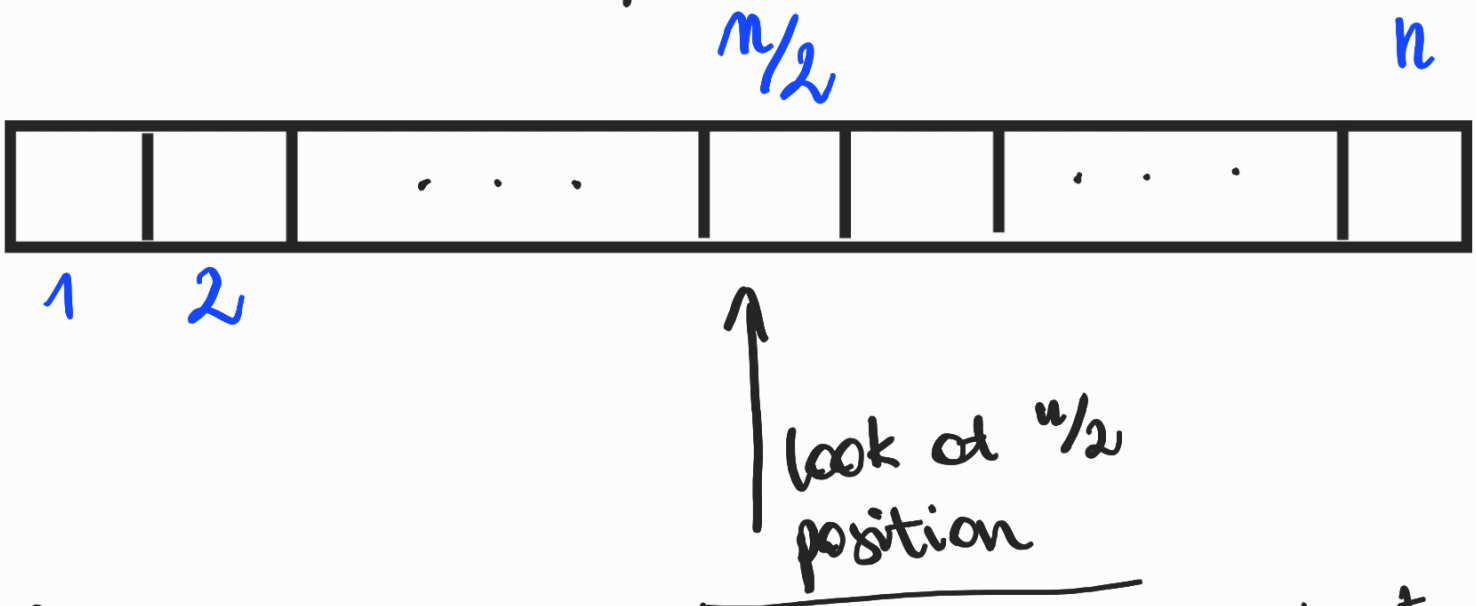
PROBLEM: Find a peek if it exists.

Straightforward algorithm

Start from left $n/2$ n

look at n/2 elements.
Worst-case complexity $\Theta(n)$

Divide & conquer



if $a[n/2] < a[n/2-1]$ then only look at left half $(1 \ldots n/2-1)$ to look a peek.

Else if $a[n/2] < a[n/2+1]$ then ...

$n/2+1 \ldots n$ for a peak

Else n/2 position is a peak

(Argument that algorithms is correct)

DIAGRAM?

$T(n) = T(n/2) + \Theta(1)$

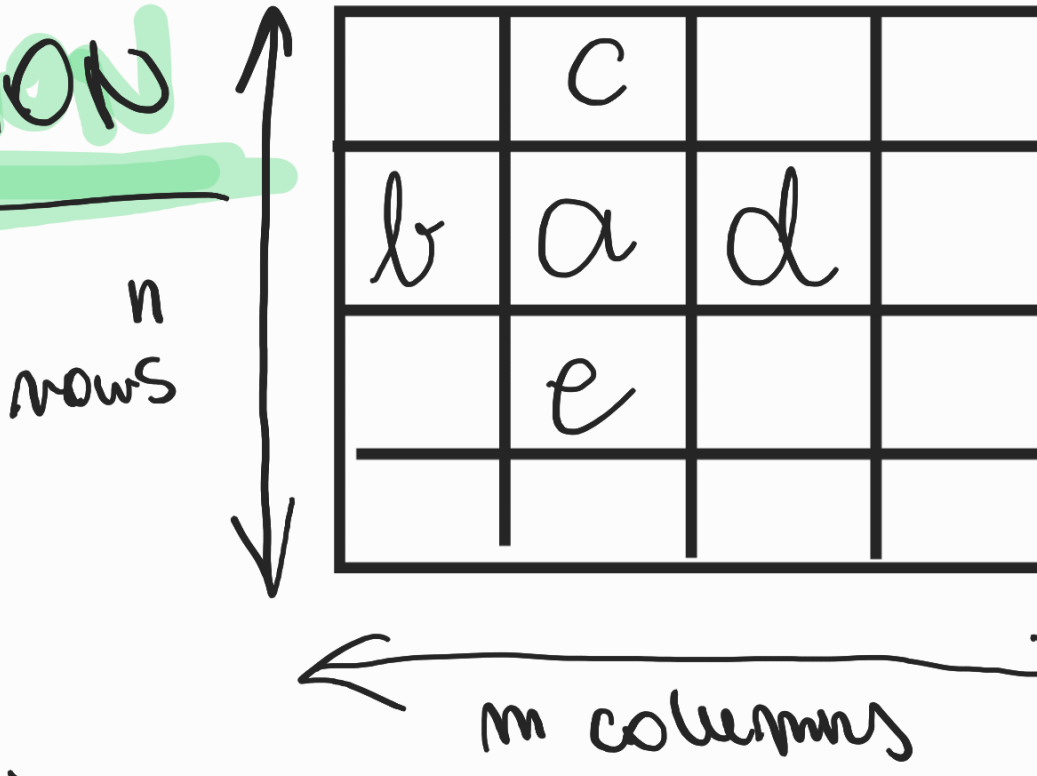
↑ work algo does on input of size n

base case: $T(1) = \Theta(1)$

$T(n) = \underbrace{\Theta(1) + \Theta(1) + \ldots + \Theta(1)}_{\lg_2 n \text{ times}}$

$= \Theta(\lg n)$

2D VERSION



Peak is when

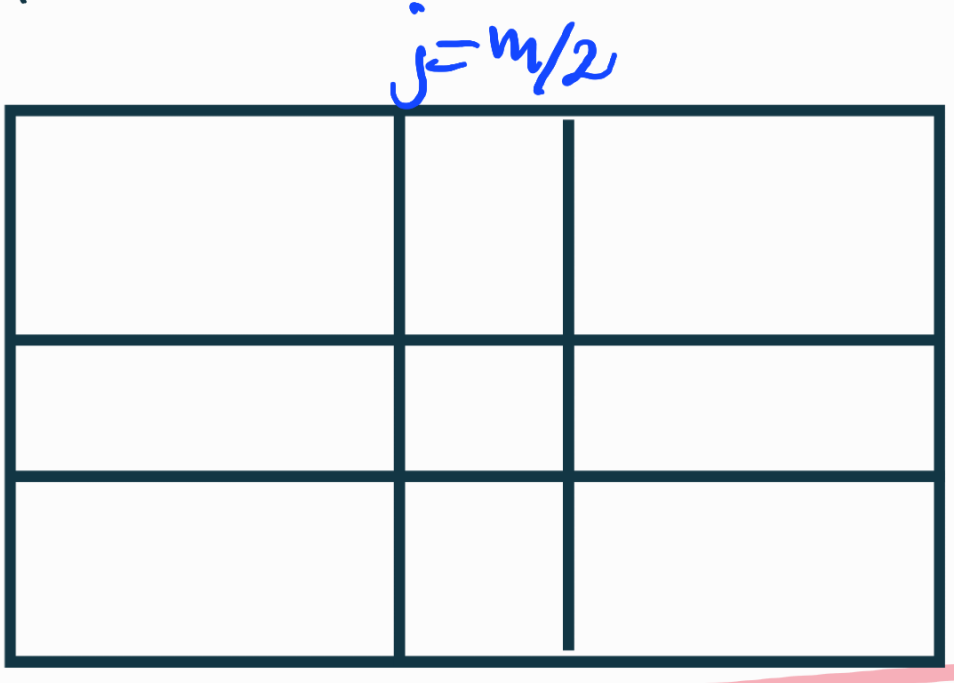
$a \geq b, a \geq d,$

$a \geq c, a \geq e$

Greedy Ascent algorithm:

14	15	12	
15	9	11	17
16	17	19	20

$\Theta(nm)$ complexity, when $m=n$ then $\Theta(n^2)$



Pick middle column $j = m/2$

Find a 1D-peak at (i, j)

Use (i,j) as a start to find a 1D-peak on row i

INCORRECT

Problem is: 2D peak may not exist on row i

Attempt #2

Pick middle column $j = m/2$

Find global max on column j at (i,j) compare $(i,j-1)$, (i,j), $(i,j+1)$.

Pick left columns if $(i,j-1) > (i,j)$ (similarly for right).

If $(i,j) \geq (i,j-1), (i,j+1) \Rightarrow$

$\Rightarrow (i,j)$ is 2-D peak

Solve the new problem with half the numbers of columns.

When you have a single column, find the global max ← DONE.

$$T(n,m) = T(n, m/2) + \Theta(n)$$

$$T(n,1) = \Theta(n)$$

$$T(n,m) = \underbrace{\Theta(n) + \ldots + \Theta(n)}_{\log_2 m \text{ times}}$$

$\Theta(n)$ ↑ max

$$= \Theta(n \cdot \log_2 n)$$