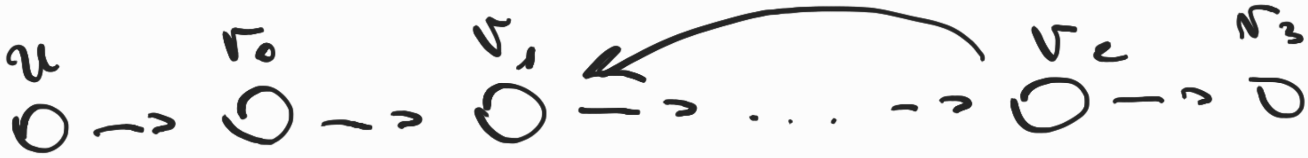
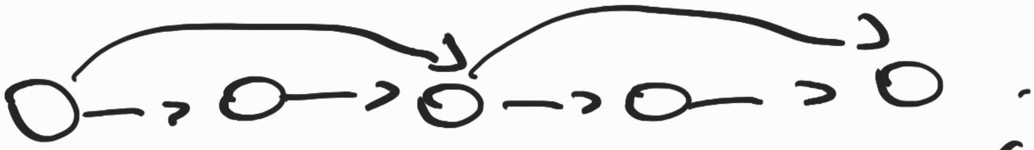
BELLMAN - FORD



$\delta(u, v_1) \to$ undef

Initialize for $v \in V$
$d[s] \leftarrow 0$
Repeat select edge [somehow]
Relax edge(u, v, w)
until you can't relax any more

1) Complexity could be exp time (even for + edge)



n nodes $\quad O(2^{n/2})$

2) might not even terminate if there is a - cycle reachable from the source

Bellman - Ford(G, w, s)
Initialize$()$
$O(VE)$ for $i = 1$ to $|V-1|$

for each edge $(u,v) \in E$

Relax (u,v,w)

check

for each edge $(u,v) \in E$

if $d[v] > d[u] + w(u,v)$

then report $-$cycle exists

Relax (u,v,w)

if $d[v] > d[u] + w(u,v)$

$d[v] = d[u] + w(u,v)$

$\pi[v] = u$

<u>Theorem</u>: If $G=(V,E)$ contain no $-$weight cycles then after B-F executes $d[v] = \delta(s,v)$ for all $v \in V$.

<u>Corollary</u>: If a value $d[v]$ fails to converge after $|V|-1$ passes, there exists a $-$ wt cycle reachable from s.

$v = v_k$

$p = v_0, v_1 \dots v_k$

$k \leq |V| - 1$ else we have cycle

Let $v \in V$. $p = \langle v_0, v_1 \dots v_k \rangle$ $v_0 = s$ +. $v_k = v$.

Thus path is a shortest path wit min # edges.

No - cycles $\Rightarrow$ p is simple, $\Rightarrow k \leq |V|-1$

After 1 pass then E, we have $d[v_1] = \delta(s, v_1)$, because we will relax edge (v_0, v_1).

After 2 passes, $d[v_2] = \delta(s, v_2)$ because in 2nd pass we will have (v_1, v_2)

$\vdots$

After k passes $d[v_k] = \delta(s, v_k)$

$|V|-1$ passes $\Rightarrow$ all reachable vertices have δ values