## WZORY NA POCHODNĄ

1. $(c)' = 0$
2. $(x^n)' = n \cdot x^{n-1}$
3. $(x)' = 1$
4. $\left(\frac{a}{x}\right)' = -\frac{a}{x^2}$
5. $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$
6. $(a^x)' = a^x \ln a$
7. $(e^x)' = e^x$
8. $(\log_a x)' = \frac{1}{x \cdot \ln a}$
9. $(\ln x)' = \frac{1}{x}$
10. $(\sin x)' = \cos x$
11. $(\cos x)' = -\sin x$
12. $(\mathrm{tg}\, x)' = \frac{1}{\cos^2 x}$
13. $(\mathrm{ctg}\, x)' = -\frac{1}{\sin^2 x}$
14. $(\arcsin)' = \frac{1}{\sqrt{1-x^2}}$
15. $(\arccos)' = -\frac{1}{\sqrt{1-x^2}}$
16. $(\mathrm{arctg})' = \frac{1}{x^2+1}$
17. $(\mathrm{arcctg})' = -\frac{1}{x^2+1}$

## WŁAŚCIWOŚCI POCHODNYCH

1. $[f(x)+g(x)]' = f'(x) + g'(x)$
2. $[f(x)-g(x)]' = f'(x) - g'(x)$
3. $[a \cdot f(x)]' = a \cdot f'(x)$
4. $[f(x) \cdot g(x)]' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$
5. $\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2}$

$\sqrt[b]{x^a} = x^{\frac{a}{b}}$

$\frac{1}{x^a} = x^{-a}$

$a^b = e^{b \cdot \ln a}$

## GRANICE

$\left[\frac{A}{0}\right] = \pm\infty$

$\left[\frac{A}{\pm\infty}\right] = 0$

$\ln 0 \to -\infty$

$\ln 1 = 0$

$\ln e = 1$

$\ln \infty \to \infty$

$\lim a \cdot f(x) = a \cdot \lim f(x)$

$\lim e^{f(x)} = e^{\lim f(x)}$

$\log_e x = \ln x$

## SYMBOLE NIEOZNACZONE

$\left[\frac{0}{0}\right]$

$\left[\frac{\infty}{\infty}\right]$

$[0 \cdot \infty]$

$[\infty - \infty]$

$[1^\infty]$

$[\infty^0]$

$[0^0]$

$$(a^2-b^2)=(a-b)(a+b)$$

$$(x^4+1)=(x^2-\sqrt{2}x+1)(x^2+\sqrt{2}x+1)$$

$$(a^3-b^3)=(a-b)(a^2+ab+b^2)$$

$$(a^3+b^3)=(a+b)(a^2-ab+b^2)$$

$$(a^n-1^n)=(a-1)(a^{n-1}+a^{n-2}+\dots+a+1)$$

$$\cos\alpha-\cos\beta=-2\cdot\sin\frac{\alpha+\beta}{2}\cdot\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha+\cos\beta=2\cdot\cos\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha-\sin\beta=2\cdot\sin\frac{\alpha-\beta}{2}\cdot\cos\frac{\alpha+\beta}{2}$$

$$\sin\alpha+\sin\beta=2\cdot\sin\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}$$

$$\sin(x+y)=\sin x\cdot\cos y+\cos x\cdot\sin y$$

$$\cos(x+y)=\cos x\cdot\cos y-\sin x\cdot\sin y$$

$$\sin(x-y)=\sin x\cdot\cos y-\cos x\cdot\sin y$$

$$\cos(x-y)=\cos x\cdot\cos y+\sin x\cdot\sin y$$

$$\cos 2x=\cos^2 x-\sin^2 x$$

$$\sin 2x=2\sin x\cdot\cos x$$

$$\lim_{\square\to\infty}\left(1+\frac{a}{\square}\right)^{\square}=e^a$$

$$\lim_{\square\to 0}\frac{\ln(1+\square)}{\square}=1$$

$$\lim_{\square\to 0}\frac{\log_a(1+\square)}{\square}=\frac{1}{\ln a}=\log_a e$$

$$\lim_{\square\to 0}\frac{e^{\square}-1}{\square}=1$$

$$\lim_{\square\to 0}\frac{a^{\square}-1}{a}=\ln a$$

$$\lim_{x\to 0}\frac{1-\cos x}{x}=\frac{1}{2}$$

1) $\int \frac{dx}{x} = \int \frac{1}{x} \cdot dx = \int x^{-1} \cdot dx = \ln|x| + C$

2) $\int a \, dx = ax + C$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

3) $\int x \cdot dx = \frac{1}{2} x^2 + C$

$= \int x^n dx = \frac{1}{n+1} x^{n+1} + C$

$n \neq 1$

4) $\int a^x dx = \frac{a^x}{\ln a} + C$

5) $\int e^x dx = e^x + C$

6) $\int \sin x \, dx = -\cos x + C$

7) $\int \cos x \, dx = \sin x + C$

8) $\int \operatorname{tg} x \, dx = -\ln|\cos x| + C$

9) $\int \operatorname{ctg} x \cdot dx = \ln|\sin x| + C$

10) $\int \frac{dx}{\cos^2 x} = \operatorname{tg} x + C$

11) $\int \frac{dx}{\sin^2 x} = -\operatorname{ctg} x + C$

12) $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C$

13) $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + C$

14) $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$

15) $\int \frac{dx}{\sqrt{x^2 + q}} = \ln\left|x + \sqrt{x^2 + q}\right| + C$

$\int_a^\infty f(x)\,dx = \lim_{\varepsilon\to\infty}\int_a^\varepsilon f(x)\,dx = \dots$

$\int_{-\infty}^b f(x)\,dx = \lim_{\varepsilon\to-\infty}\int_\varepsilon^b f(x)\,dx = \dots$

$\int_b^e f(x)\,dx = \lim_{\varepsilon\to b^+}\int_\varepsilon^e f(x)\,dx = \dots$

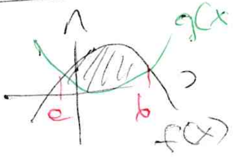
$b^+ \to$ b bo nie sprawdzić dziedzinie

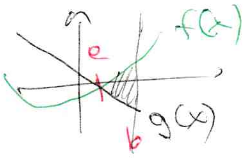
$\int_{-\infty}^\infty f(x)\,dx = \int_{-\infty}^a f(x)\,dx + \int_b^\infty f(x)\,dx = \lim_{\varepsilon\to-\infty}\int_\varepsilon^a f(x)\,dx + \lim_{\varepsilon\to\infty}\int_b^\varepsilon f(x)\,dx = \dots$

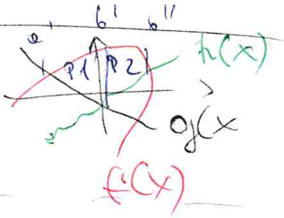
## POLE OBSZARU

$P=\int_a^b [f(x)-g(x)]\,dx$

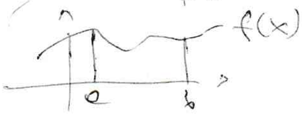
ograniczone z góry — z dołu



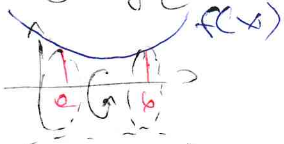




## PŁ. ŁUKU



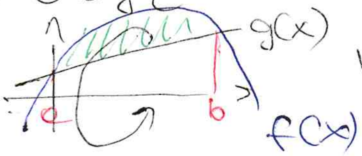

$L=\int_a^b\sqrt{1+[f'(x)]^2}\,dx$

## OBJĘTOŚĆ I



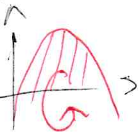

$V=\pi\int_a^b[f^2(x)]\,dx$

## OBJĘTOŚĆ II




$V=\pi\int_a^b[f^2(x)-g^2(x)]\,dx$

## POLE POWIERZCHNI



$P_p=2\pi\int_a^b f(x)\sqrt{1+[f'(x)]^2}\,dx$ dla $f(x)>0$

aryt $S_n=\frac{a_1+a_n}{2}\cdot n$

geo $S_n=a_1\cdot\frac{1-q^n}{1-q}$ | $\frac{a_1}{1-q}$

$\cos 2x=\cos^2x-\sin^2x$

$\sin 2x = 2\cdot\sin x\cdot\cos x$

$(a+b)^3=a^3+3a^2b+3ab^2+b^3$

$(a-b)^3=a^3-3a^2b+3ab^2-b^3$

$a^3+b^3=(a+b)(a^2-ab+b^2)$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

$(a^n-1)=(a-1)(a^{n-1}+a^{n-2}+\dots+a+1)$

$f'(0)-\lim_{x\to0}f(x)$

$(y-y_0)=f'(x_0)(x-x_0)$

$\cos\alpha-\cos\beta=-2\cdot\sin\frac{\alpha+\beta}{2}\cdot\sin\frac{\alpha-\beta}{2}$

$\cos\alpha+\cos\beta=2\cdot\cos\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}$

$\sin\alpha-\sin\beta=2\cdot\sin\frac{\alpha-\beta}{2}\cdot\cos\frac{\alpha+\beta}{2}$

$\sin\alpha+\sin\beta=2\cdot\sin\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}$

$\sin(x+y)=\sin x\cos y+\cos x\cdot\sin y$

$\cos(x+y)=\cos x\cdot\cos y-\sin x\sin y$

$\sin(x-y)=\sin x\cdot\cos y-\cos x\sin y$

$\cos(x-y)=\cos x\cdot\cos y+\sin x\cdot\sin y$

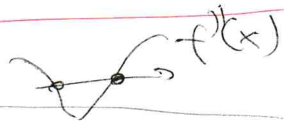
Zbadaj zbieżność jednostajną z pozbądź się n

1) $f(x)$ to $\lim_{n\to\infty} f_n(x)$

2) $|f_n(x)-f(x)|=\dots$

3) podstaw pod x coś innego

punkty przegięcia




## RIEMANN:

1) dostajesz granicę → zmień do szeregu

2) pozbądź się n i pod to coś masz np $k/n$ podstaw x (tak aby we wzorze były tylko liczby i x)

3) podstaw do całki $\int_{k=0n\frac{0n}{n}=0}^{k/n\,(n\to\infty)}$

## POCHODNA:

$\lim_{\Delta x\to0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$

## MACLAURIN ($x_0=0$)

$T=\sum\frac{f^{(n)}(x_0)}{n!}(x-x_0)^n$

1) policz pochodne i $f(0)$

2) znajdź wzór $f^{(n)}x$ i podstaw $x=0$ $f^{(n)}(0)$

## TAYLOR

$T=f(x_0)+\frac{f'(x_0)}{1!}\cdot(x-x_0)+\frac{f''(x_0)}{2!}(x-x_0)^2+\dots$

POCHODNA

1) pochodne

CAŁKOWANIE po x

SZEREGU (do n) + kryterium co pochodnej $|\;| < 1$

co trzeba pokazać na końcu wyniku zależy

$(c)' = 0$

$(x)' = 1$

$(x^n)' = n x^{n-1}$

$\left(\frac{a}{x}\right)' = -\frac{a}{x^2}$

$(a^x)' = a^x \ln a$

$(e^x)' = e^x$

$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$

$(\ln x)' = \frac{1}{x}$

$(\log_a x)' = \frac{1}{x \cdot \ln a}$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$

$(\operatorname{arctg} x)' = \frac{1}{x^2+1}$

$(\operatorname{arcctg} x)' = -\frac{1}{x^2+1}$

$\int a\,dx = ax + C$

$\int x^n dx = \frac{1}{n+1} x^{n+1} + C$

$\int \frac{1}{x} dx = \ln|x| + C$

$\int a^x dx = \frac{a^x}{\ln a}$

$\int e^x dx = e^x$

$\int \sin x\,dx = -\cos x + C$

$\int \cos x\,dx = \sin x + C$

$\int \operatorname{tg} x\,dx = -\ln|\cos x| + C$

$\int \operatorname{ctg} x\,dx = \ln|\sin x| + C$

$\int \frac{dx}{\cos^2 x} = \operatorname{tg} x + C$

$\int \frac{dx}{\sin^2 x} = -\operatorname{ctg} x + C$

$\int \frac{dx}{x^2+a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C$

$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + C$

$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$

$\int \frac{dx}{\sqrt{x^2+q}} = \ln\left|x + \sqrt{x^2+q}\right| + C$

D'ALAMB. $\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$ $< 1$ zb, $> 1$ rozb

CAUCHY $\lim_{n\to\infty} \sqrt[n]{a_n}$ $< 1$ zb, $> 1$ rozb

| | $0°$ | $30$ | $45°$ | $60°$ | $90°$ |
|---|---|---|---|---|---|
| | $0$ | $\pi/6$ | $\pi/4$ | $\pi/3$ | $\pi/2$ |
| sin | $0$ | $1/2$ | $\sqrt{2}/2$ | $\sqrt{3}/2$ | $1$ |
| cos | $1$ | $\sqrt{3}/2$ | $\sqrt{2}/2$ | $1/2$ | $0$ |
| tg | $0$ | $\sqrt{3}/3$ | $1$ | $\sqrt{3}$ | $\infty$ |
| ctg | $\infty$ | $\sqrt{3}$ | $1$ | $\sqrt{3}/3$ | $0$ |

L =

M =

L ≥ M → podziel $\frac{L}{M}$ - - - > Czy da się rozłożyć M?

L > M → Czy da się rozłożyć M?

NIE → Czy → STAŁA → KONIEC; → 1° NIE $a\left[\left(x+\frac{b}{2a}\right) - \frac{\Delta}{4a^2}\right]$ L przyrównaj M!

TAK → $a(x-x_1)(x-x_2)$ → KONIEC

$z = x + iy$

$|z| = \sqrt{x^2+y^2}$

$z = |z|(\cos\varphi + i\sin\varphi)$

$z^n = |z|^n(\cos n\varphi + i \sin n\varphi)$

$\sqrt[n]{z} = \sqrt[n]{|z|}\left(\cos\frac{\varphi+2k\pi}{n} + i\sin\frac{2k\pi+\varphi}{n}\right)$

$k = 0, 1, \dots n-1$

$\cos\varphi = \frac{x}{|z|}$

$\sin\varphi = \frac{y}{|z|}$

$\frac{1}{n^\alpha}$ $\alpha > 1$ zb. $\alpha \le 1$ rozb

$f(x) = \frac{a}{x-p} + q$

$|a_n|$ TAK → kryt. dowo → zb absolutnie; NIE → nie wiem → TAK $\lim_{n\to\infty} a_n = 0$ → $a_{n+1} \le a_n$ → zb.; N → rozb.

$a^b = e^{b \ln a}$

$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$

$\lim_{x\to\infty} \left(1 + \frac{a}{\square}\right)^{\square} = e^a$

$\lim_{\square\to 0} \frac{\ln(1+\square)}{\square} = 1$

$\lim_{\square\to 0} \frac{\log_a(1+\square)}{\square} = \log_a e$

$\lim_{\square\to 0} \frac{a^{\square}-1}{\square} = \ln a$

$\lim_{\square\to 0} \frac{e^{\square}-1}{\square} = 1$.

| | I | II | III | IV |
|---|---|---|---|---|
| cos | + | − | − | + |
| sin | + | + | − | − |

Znajdź min i max na prz. $[y;x]$

1) $f(y) = \dots$, $f(x) = \dots$ $|x-1|$! $x = 1$, $x = -1$

2) $f'(x) = 0$ w tych pkt 3) wybierz max min

fun. ma pochodną w pkt $x_0$

fun jest różniczkowalna

1) $\lim_{x\to x_0^-} f(x) = \lim_{x\to x_0^+} f(x) = f(x_0)$ } ciągłość

2) $\lim_{x\to x_0^-} f'(x) = \lim_{x\to x_0^+} f'(x)$

pkt. przegięcia

$f''(x) > 0$ wypukła

$f''(x) < 0$ wklęsła

ASYMPTOTY

1 $\lim_{x\to x_0^+} f(x) = \pm\infty$

$\lim_{x\to\pm\infty} \frac{f(x)}{x} = a$ jeśli $a = 0$ to

2° $\lim_{x\to\pm\infty} (f(x) - ax) = b$