EGZAMIN Z ANALIZY NUMERYCZNEJ (L)
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Pierwszy termin

**Pracuj samodzielnie!!!**

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Numer części: 2 Numer zadania: 4

| -3 | -2 | 0 | 2 | 3 |
|---|---|---|---|---|
| 4 | 1 | 2 | 1 | 4 |

iloczyn skalarny $\langle f,g\rangle = \sum_{i=-3}^{-2} f(i)g(i) + \sum_{i=0}^{0} f(i)g(i) + \sum_{i=2}^{3} f(i)g(i)$

$P_0(x) = 1$

$P_1(x) = x - c_1$

$P_k(x) = (x-c_k)P_{k-1}(x) - d_k P_{k-2}(x)$

| | $x_0$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $\sum$ |
|---|---|---|---|---|---|---|
| $x_i$ | -3 | -2 | 0 | 2 | 3 | 0 |
| $x_i^2$ | 9 | 4 | 0 | 4 | 9 | 26 |
| $x_i^3$ | -27 | -8 | 0 | 8 | 27 | 0 |

$P_0(x) = 1$

$P_1(x) = x - \frac{\langle xP_0, P_0\rangle}{\langle P_0, P_0\rangle} = x - \frac{\sum_{i=0}^{4} x_i \cdot 1}{\sum_{i=0}^{4} 1} = x - \frac{0}{5} = x$

$c_k = \frac{\langle xP_{k-1}, P_{k-1}\rangle}{\langle P_{k-1}, P_{k-1}\rangle}$

$d_k = \frac{\langle P_{k-1}, P_{k-1}\rangle}{\langle P_{k-2}, P_{k-2}\rangle}$

$\langle P_k, P_k\rangle = \sum_{k=0}^{4} P_k(x)P_k(x)$

$P_2(x) = (x-c_2)P_1(x) - d_2 P_0(x) = \left(x - \frac{\langle xP_1, P_1\rangle}{\langle P_1, P_1\rangle}\right)\cdot x - \frac{\langle P_1, P_1\rangle}{\langle P_0, P_0\rangle}\cdot 1 = \left(x - \frac{\sum_{i=0}^{4} x_i^3}{\sum_{i=0}^{4} x_i^2}\right)\cdot x - \frac{\sum_{i=0}^{4} x_i^2}{\sum_{i=0}^{4} 1} =$

$= \left(x - \frac{0}{26}\right)\cdot x - \frac{26}{5} = x^2 - \frac{26}{5} = x^2 - 5\frac{1}{5}$

$w_2^* = \sum_{k=0}^{4} a_k P_k(x)$

$\downarrow$

$a_k = \frac{\langle f, P_k\rangle}{\langle P_k, P_k\rangle}$

$a_0 = \frac{\langle f, P_0\rangle}{\langle P_0, P_0\rangle} = \frac{\sum_{i=0}^{4} f(x_i)P_0(x_i)}{\sum_{i=0}^{4} P_0(x_i)^2} = \frac{12}{5}$

$a_1 = \frac{\langle f, P_1\rangle}{\langle P_1, P_1\rangle} = \frac{\sum_{i=0}^{4} f(x_i)P_1(x_i)}{\sum_{i=0}^{4} P_1(x_i)^2} = \frac{0}{26} = 0$

$a_2 = \frac{\langle f, P_2\rangle}{\langle P_2, P_2\rangle} = \frac{\sum_{i=0}^{4} f(x_i)\left(x_i^2 - \frac{26}{5}\right)}{\sum_{i=0}^{4} \left(x_i^2 - \frac{26}{5}\right)^2} = \frac{-\frac{2}{5}}{\frac{1470}{25}} = \frac{2}{5}\cdot\frac{25}{1470} = \frac{5}{735} = \frac{1}{147}$

Pamiętaj o zasadach nadsyłania rozwiązań!

$w_2^* = \frac{12}{5}\cdot 1 + 0\cdot x + \left(\frac{1}{147}\right)\cdot\left(x^2 - 5\frac{1}{5}\right) = \frac{12}{5} - \frac{1}{147}\left(x^2 - 5\frac{1}{5}\right)$