# SQUARE ROOTS, NEWTON'S METHOD

## Review

Want millionth digit of $\sqrt{2}$

$\lfloor \sqrt{2} \cdot 10^{2d} \rfloor$

$d = 10^6$

↑ digits of precision

Compute $\lfloor \sqrt{a} \rfloor$ via Newton's method.

$x_0 = 1$ (initial guess)

$x_{i+1} = \frac{x_i + a/x_i}{2}$ ← division

## Error Analysis of Network's Method

$x_n = \sqrt{a}(1 + \varepsilon_n)$ $\varepsilon_n$ may +/−

$$x_{n+1} = \frac{\sqrt{a}(1+\varepsilon_n) + \frac{a}{\sqrt{a}(1+\varepsilon_n)}}{2} =$$

$$= \frac{\sqrt{a}\left[(1+\varepsilon_n) + \left(\frac{1}{1+\varepsilon_n}\right)\right]}{2} =$$

$$= \sqrt{a}\left(1 + \frac{\varepsilon_n^2}{2(1+\varepsilon_n)}\right)$$

Therefore

$$\varepsilon_{n+1} = \frac{\varepsilon_n^2}{2(1+\varepsilon_n)} \quad \text{with } 2(1+\varepsilon_n) \approx 1$$

$d$ digits of precision

$\Rightarrow \lg d$ iterations

**STRASSEN**

$\Theta(n \lg n \lg\lg n)$ time

uses FFT