Dynamics of Neural Systems: Optional Problem Exercise 4

Martin Giese, Prerana Kumar, Margaux Asclipe

Email: martin.giese@uni-tuebingen.de

Note: *This problem requires some knowledge from the Advanced Theoretical Methods lectures. If you have problems, please contact me.*

1 Simple autoassociative memory

Assume the very simple autoassociative memory network that is given by the dynamics:

$$\tau \dot{\mathbf{u}}(t) = -\mathbf{u}(t) + [\mathbf{M}\mathbf{u}(t)]_+ \tag{1}$$

where the matrix $\mathbf{M}$ is given by:

$$\mathbf{M} = \begin{bmatrix} 1 & -0.1 & -0.1 \\ -0.1 & 1 & -0.1 \\ -0.1 & -0.1 & -0.1 \end{bmatrix}$$

1.1

Which equation determines the fixed points of the network? Find the patterns stored in this memory network. Remark that the fixed points are degenerate with respect to their scale. This means if $\mathbf{u}^*$ is a fixed point then $\alpha\mathbf{u}^*$ with $\alpha > 0$ is also a fixed point.

Hint: E.g., try to iterate the nonlinear fixed point equation with random initial conditions, or try informed self-chosen initial conditions for finding the fixed points.

1.2

Prove the stability of the stable states. Remark that the fixed point equation implies the inequality $u_i \geq 0$.

Hint: Prove inequalities for the components of the fixed points for three cases: $u_1 > 0$, $u_2 > 0$ *and* $u_3 > 0$.

1.3

Prove the stability of the fixed points by deriving a Lyapunov function using the Cohen-Grossberg theorem.

Hint: Transform the system using the new state variable $\mathbf{v} = \mathbf{M}\mathbf{u}$ *and remark that* $\mathbf{M}$ *is non-singular.*