

HW#8
SIMULATING EQUALIZERS

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Zero Forcing Equalizer:

For simulating how this equalizer works, 2 different ways are used.

In the first way, bits are convolved with $h^*(-t)$, then AWGN goes through the same channel and is added to the resulting bits. (This signal is called y_i in code.) After finding channel response of equalizer, the resulting y_i signal goes through channel d(t).

In the second way, q(t) which is x(t)*d(t), and bits go through this channel. Then using the below equation, noise is calculated and added to the last signal, resulting in a new received signal, y_2 .

$$v = n * h^*(-t) . \eta = v * d(t)$$

$$S_{\eta}(z) = {}^{\Upsilon} N_o X(z) D(z) D^* \left(\frac{1}{z}\right) \to R_{\eta}(m) = {}^{\Upsilon} N_o x(m) * d(m) * d^*(-m)$$

Since in ZF we assume $q_i = 0$ for $i \in \{-k, -k + 1, \dots, -1, 1, 1, \dots, k\}$:

$$\sigma_n^{\dagger} = {}^{\dagger} N_o(q(m) * d(m)) = {}^{\dagger} N_o d_o$$

For calculating the probability of error based on the calculations in class, as was used above, $q_i = 0$ for $i \in \{-k, -k + 1, ..., -1, 1, 1, ..., k\}$, but as can be seen in simulations, some of them are not exactly zero. Their value is about 10⁻¹⁶ which is considered zero in matlab, but since assuming they are zero or not made difference in calculating pe, 2 different pe is calculated, the first one considering their exact value, and the other way is neglecting these values.

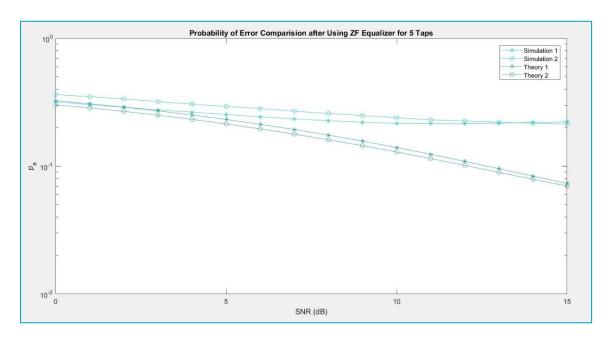
With considering their value, formula for finding pe is: (For 5 taps)

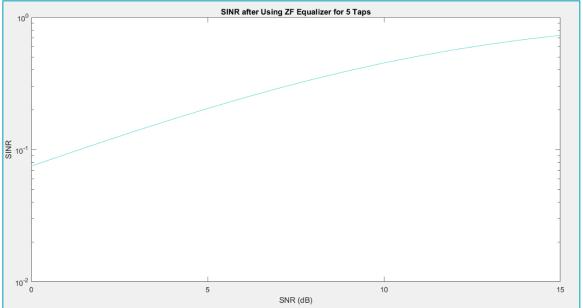
$$pe = \frac{1}{r^{4}} \sum_{\varepsilon: \varepsilon \{+1\}} Q(\frac{q_{o} + \varepsilon_{1}q_{1} + \varepsilon_{-1}q_{-1} + \varepsilon_{-r}q_{-r} + \varepsilon_{r}q_{r} + \varepsilon_{r}q_{r} + \varepsilon_{-r}q_{-r} + \varepsilon_{\ell}q_{\ell} + \varepsilon_{-\ell}q_{\ell}}{\cdot \cdot \circ \sigma_{\eta}^{r}}$$

But if we don't consider the almost zero ones we have: (For 5 taps)

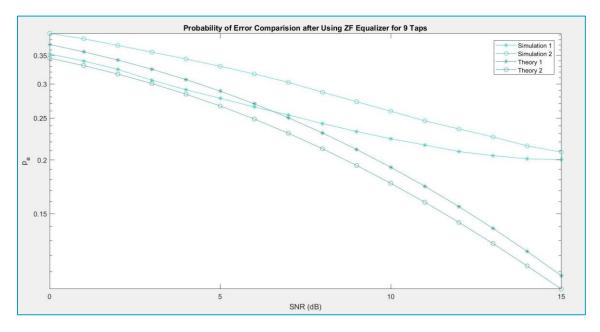
$$pe = \frac{1}{r'} \sum_{\varepsilon_i \in \{\pm^{1}\}} Q(\frac{q_o + \varepsilon_r q_r + \varepsilon_{-r} q_{-r} + \varepsilon_{\ell} q_{\ell} + \varepsilon_{-\ell} q_{\ell}}{\cdot \cdot \circ \sigma_{\eta}^{r}}$$

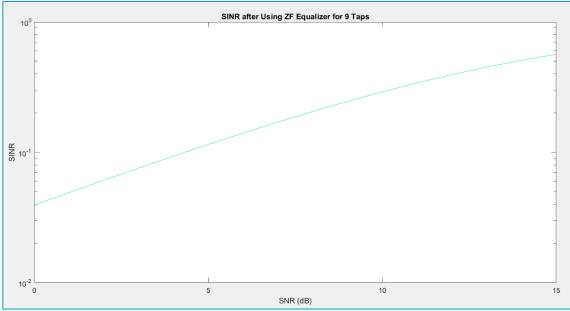
The first formula is labeled as Theory 1 and the second can be found under Theory 2 label:

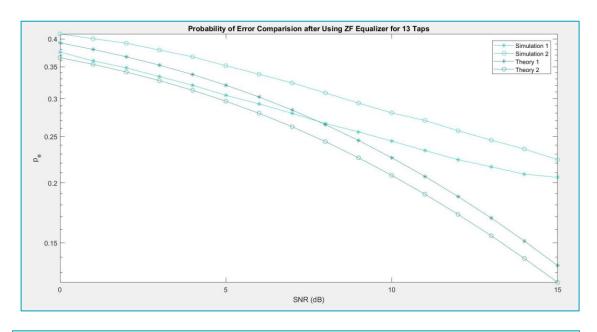


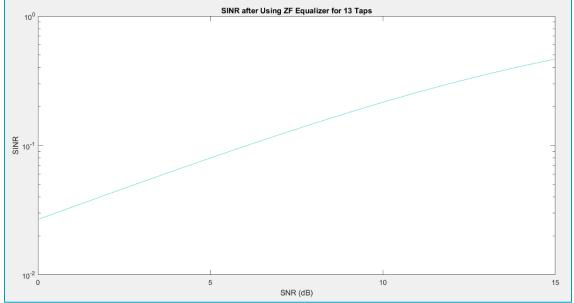


The same ways are done for the other taps too:

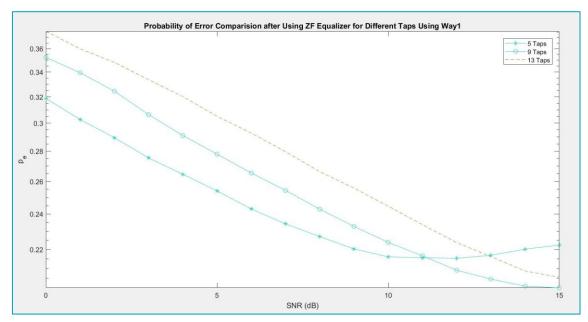


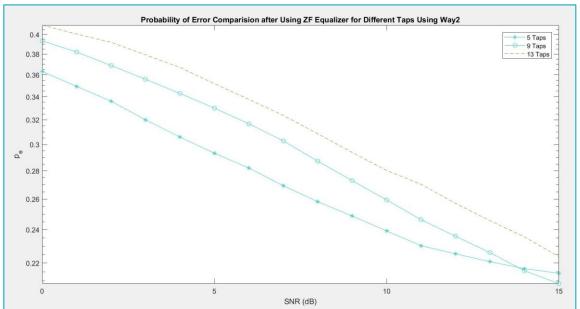


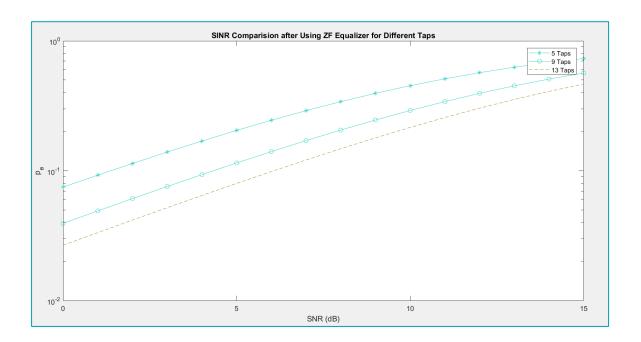




So each ones you want to compare, just commend the other way and check the plot.

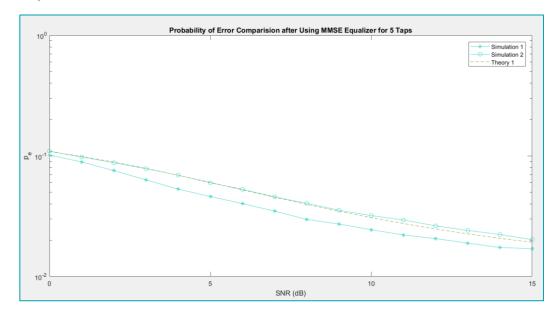


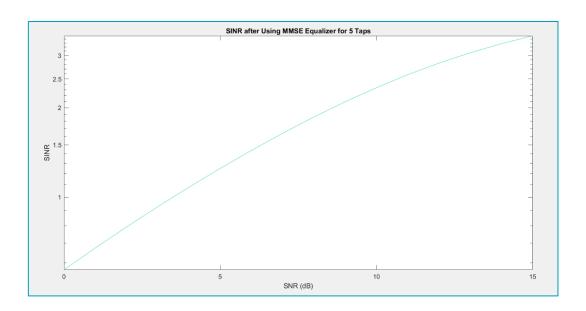


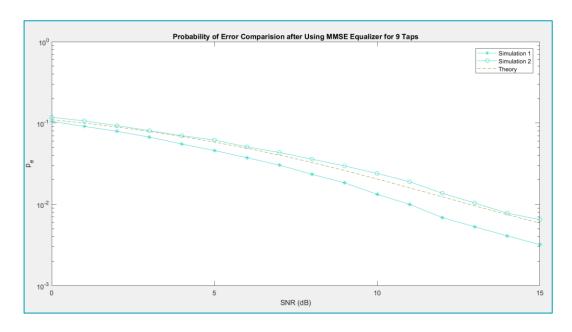


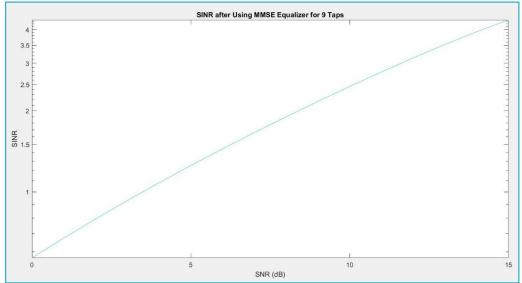
MMSE:

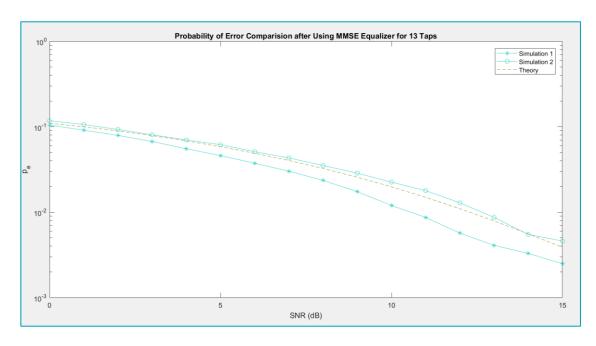
For this equalizer, same as the last one, 2 different methods of simulating noise is used. But since all q coefficients must be used, only one way for simulating pe using theory is used. The result is as below:

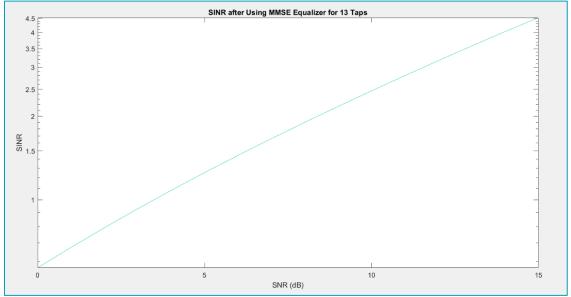




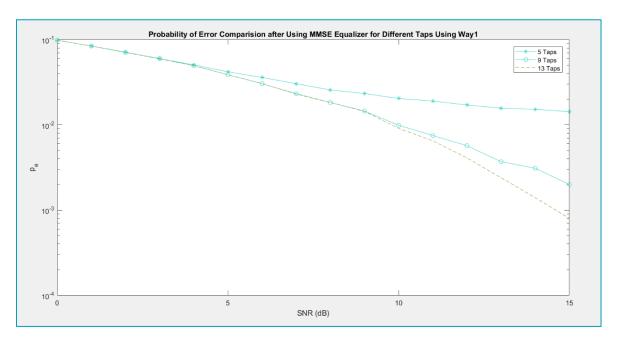


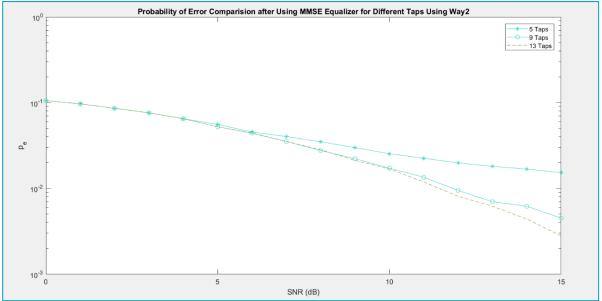


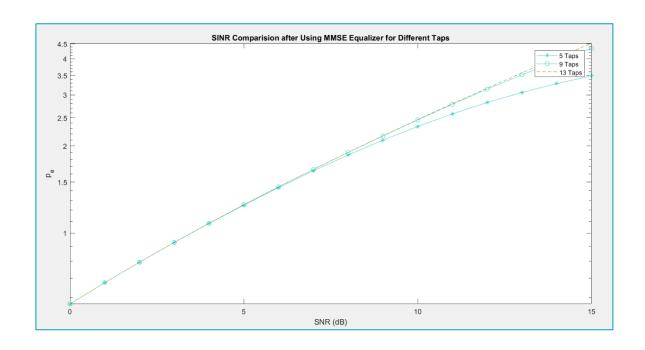




So each ones you want to compare, just commend the other way and check the plot.







DFE:

For this equalizer only the first way of noise is simulated and the results can be seen as below:

