Detection and Estimation Theory

University of Tehran

Instructor: Dr. Ali Olfat Spring 2020

Homework 9

Due : 99/3/29

Problem 1

The discrete process $x(n)$ is a $MA(2)$ process defined as

$$x(n) = w(n) + 0.8w(n-1) + 1.5w(n-2)$$

where $w(n)$ is a zero mean stationary white noise with autocorrelation function $R_w(m) = \delta(m)$. we have access to noisy measurements of the process $y(n) = x(n) + v(n)$ where $v(n)$ is a zero mean stationary withe process, called measurement noise, with autocorrelation function $R_v(m) = 0.1\delta(m)$. $v(n)$ and $x(n)$ are uncorrelated, $R_{xv}(m) = 0$.

a. Derive an optimum filter for linear Minimum Mean Square Error (MMSE) estimation of $x(n)$ based on observations $y(k), -\infty < k < \infty$. Find the associated mean-square estimation error.

b. Derive an optimum filter for linear Minimum Mean Square Error (MMSE) estimation of $x(n)$ based on observations $y(n-k), k \geq 1$. Find the associated mean-square estimation error.

c. Derive an optimum filter for linear Minimum Mean Square Error (MMSE) estimation of $x(n)$ based on observations $y(n-k), k \geq -1$. Find the associated mean-square estimation error.

d. Determine the recursive Kalman filter equations for the linear MMSE estimate of $x(n)$ based on observations $y(n-k), 1 \leq k \leq n$. Find the associated mean-square estimation error as a function of n. Find the MSE as $n \to \infty$.

e. Determine the recursive Kalman filter equations for the linear MMSE estimate of $x(n)$ based on observations $y(n-k), -1 \leq k \leq n$. Find the associated mean-square estimation error as a function of n. Find the MSE as $n \to \infty$.

Problem 2

Suppose $x(n)$ as an $AR(2)$ Process

$$x(n) = 0.3x(n-1) + 0.18x(n-2) + w(n)$$

where $w(n)$ is a zero mean stationary white noise with autocorrelation function $R_w(m) = 1.44\delta(m)$. we have access to noisy measurements of the process $y(n) = x(n) + v(n)$ where $v(n)$ is a zero mean stationary withe process, called measurement noise, with autocorrelation function $R_v(m) = 0.1\delta(m)$. $v(n)$ and $x(n)$ are uncorrelated, $R_{xv}(m) = 0$.

a. Derive an optimum filter for linear Minimum Mean Square Error (MMSE) estimation of $x(n)$ based on observations $y(k), -\infty < k < \infty$. Find the associated mean-square estimation error.

b. Derive an optimum filter for linear Minimum Mean Square Error (MMSE) estimation of $x(n)$ based on observations $y(n-k), k \geq 0$. Find the associated mean-square estimation error.

c. Derive an optimum filter for linear Minimum Mean Square Error (MMSE) estimation of $x(n)$ based on observations $y(n-k), k \geq -2$. Find the associated mean-square estimation error.

d. Determine the recursive Kalman filter equations for the linear MMSE estimate of $x(n)$ based on observations $y(n-k), 0 \leq k \leq n$. Find the associated mean-square estimation error as a function of n. Find the MSE as $n \to \infty$.

e. Determine the recursive Kalman filter equations for the linear MMSE estimate of $x(n)$ based on observations $y(n-k), -2 \leq k \leq n$. Find the associated mean-square estimation error as a function of n. Find the MSE as $n \to \infty$.

Problem 3

Let $x(n)$ be a stationary process with

$$S_X(f) = \frac{1.01 + 0.2\cos(2\pi f)}{1.04 - 0.4\cos(2\pi f)}$$

a. Derive an optimum filter for linear Minimum Mean Square Error (MMSE) estimation of $x(n)$ based on observations $x(n-k), k \geq 3$. Find the associated mean-square estimation error.

b. Derive an optimum filter for linear Minimum Mean Square Error (MMSE) estimation of $x(n)$ based on observations $x(n-k), k \geq 1$. Find the associated mean-square estimation error.

c. Determine the recursive Kalman filter equations for the linear MMSE estimate of $x(n)$ based on observations $x(n-k), 3 \leq k \leq n$. Find the associated mean-square estimation error as a function of n. Find the MSE as $n \to \infty$.

d. Determine the recursive Kalman filter equations for the linear MMSE estimate of $x(n)$ based on observations $x(n-k), 1 \leq k \leq n$. Find the associated mean-square estimation error as a function of n. Find the MSE as $n \to \infty$.