Detection and Estimation Theory

University of Tehran

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Homework 3

Due : 98/12/28

Problem 1

Consider the composite hypothesis testing problem:

$$\begin{aligned} H_0 &: \quad p_0(y) = \tfrac{1}{2}e^{-|y|} \quad , y \in \mathcal{R} \\ H_1 &: \quad p_1(y) = \tfrac{1}{2}e^{-|y-\theta|} \quad , y \in \mathcal{R}, \theta > 0 \end{aligned}$$

a) Describe the locally most powerful α-level test and derive its power function.

b) Does a uniformly most powerful test exist? If so, find it and derive its power function.

c) Design a GLRT test and derive its ROC.

Problem 2

Consider the following pair of hypotheses concerning a sequence $Y_1, Y_2, ..., Y_n$ of independent random variables

$$\begin{aligned} H_0 &: Y_k \sim \mathcal{N}(\mu_0, \sigma^2), k = 1, 2, ..., n \\ H_1 &: Y_k \sim \mathcal{N}(\mu_1, \sigma^2), k = 1, 2, ..., n \end{aligned}$$

where μ_0, μ_1 and σ^2 are known constants, and $\mu_1 > \mu_0 = 0$. Does there exist a uniformly most powerful test of these hypotheses under the assumption that μ_1 is known and σ^2 is not? If so, find it and show that it is UMP. If not, show why and find the generalized likelihood ratio test.

Problem 3

Let

$$Y = A + V$$

where the random variable V is uniformly distributed over $[-\frac{1}{2}, \frac{1}{2}]$ and A is an unknown amplitude. Given Y, we seek to test $H_1 : 0 < A \leq 1$ against $H_0 : A = 0$. The probability of false alarm must be less than or equal to α.

a) Does a UMP test exist? If so, determine it as a function of α.

b) If a UMP test exist, evaluate its power P_D.

Problem 4

Given an $\mathcal{N}(0, \nu)$ distributed observation Y, we wish to test $H_1 : \nu > \nu_0$ against $H_0 : \nu = \nu_0$ The probability of false alarm must be less than or equal to α.

a) Does a UMP test exist? If so, express the threshold in function of α, and evaluate the power of the test in function of the unknown variance ν.

b) Suppose that we wish to test H_1 against $H_0' : 0 < \nu \leq \nu_0$. It is required that the size of the test to be α, i.e. ,

$$\max_{0<\nu\leq\nu_0} P_F(\delta, \nu) \leq \alpha$$

Does a UMP test exist? If so, explain how its threshold can be selected.

c) Does the answer to last part change if the range of values for ν under H_0' is $\frac{\nu_0}{2} \leq \nu \leq \nu_0$ with $\nu_0 > 0$?

Problem 5

In the previous problem for testing H_1 against H_0, Design a GLRT test. Select its threshold so that the probability of false alarm is α.
