Detection and Estimation Theory

University of Tehran

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Homework 1

Due : 98/11/28

Problem 1

Suppose Y is a random variable, that under hypothesis H_0, has pdf

$$P_0(y) = \begin{cases} \frac{2}{3}(y+1), & 0 \leq y \leq 1 \\ 0, & \text{Otherwise} \end{cases}$$

and under hypothesis H_1, has pdf

$$P_1(y) = \begin{cases} 1, & 0 \leq y \leq 1 \\ 0, & \text{Otherwise} \end{cases}$$

1. Find the Bayes rule and minimum Bayes risk for testing H_0 versus H_1 with uniform costs and equal priors.
2. Find the minimax rule and minimax risk for uniform costs.

Problem 2

For the following problem:

$$f_{Y|H_0}(y|H_0) = \frac{1}{2}e^{-|y|}$$
$$f_{Y|H_1}(y|H_1) = e^{-2|y|}$$

1. Find the Bayes rule when $\pi_0 = \frac{3}{4}$, $C_{00} = C_{11} = 0$, $C_{10} = 1$ and $C_{01} = 2$.
2. Evaluate and sketch the optimum Bayesian risk function $V(\pi_0)$.

Problem 3

Repeat Exercise 1 for the hypothesis pair

$$H_0 : Y = N - s \quad \text{vs.} \quad H_1 : Y = N + s$$

where $s > 0$ is a fixed real number and N is a continuous random variable with density

$$p_N(n) = \frac{1}{\pi(1+n^2)}, \quad n \in \mathcal{R}$$

Problem 4

Show that the minimum-Bayes-risk function V (defined in section II.C) is concave and continuous in $[0,1]$. [After showing that V is concave, you may use the fact that any concave function on $[0,1]$ is continuous on $(0,1)$.]

Problem 5

Suppose we have a real observation Y and binary hypotheses described by the following pair of pdfs

$$P_0(y) = \begin{cases} 1 - |y|, & |y| \le 1 \\ 0, & |y| \ge 1 \end{cases} \qquad \text{vs.} \qquad P_1(y) = \begin{cases} \frac{2-|y|}{4}, & |y| \le 2 \\ 0, & |y| \ge 2 \end{cases}$$

Assume that the costs are given by $C_{00} = C_{11} = 0$ and $C_{01} = 2C_{10} > 0$. Find the minimax test of H_0 versus H_1 and the corresponding minimax risk.

Problem 6

Consider the simple hypothesis testing problem for the real-valued observation Y

$$H_0 : p_0(y) = \frac{e^{-\frac{y^2}{2}}}{\sqrt{2\pi}}, y \in \mathcal{R} \qquad \text{vs.} \qquad H_1 : p_1(y) = \frac{e^{-\frac{(y-1)^2}{2}}}{\sqrt{2\pi}}, y \in \mathcal{R}$$

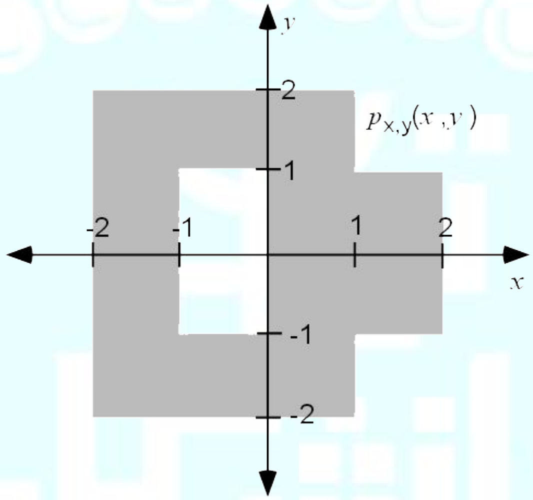
Suppose the cost assignment is given by $C_{00} = C_{11} = 0$, $C_{10} = 1$ and $C_{01} = N$. Investigate the behavior of the Bayes rule and risk for equally likely hypotheses and the minimax rule and risk when N is very large.

Problem 7

Suppose X and Y are random variables. Their joint density, depicted below, is costant in the shaded area and 0 elsewhere.

Let

$$H_0 : x \leq 0$$
$$H_1 : x > 0$$



1. Determine $P_0 = \Pr[H_0]$ and $P_1 = \Pr[H_1]$, and make fully labled sketches of $p_{Y|H_0}(y|H_0)$ and $p_{Y|H_1}(y|H_1)$
2. Find the rule $\delta(y)$ for deciding between H_0 and H_1 with minimum probability of error. Specify for which values of y your rule chooses H_1, and for which values it chooses H_0. What is the resulting probability of error?