University of Tehran, ECE
Wireless Communications, Spring 2020

Assignment Set #4

Wireless Channel Capacity

Problem 1

A telephone line channel has the following frequency response:

$$H(f) = \frac{A}{1 + j(\frac{f}{W})}$$

The PSD of the AWGN is $N_0/2$.

a) Find the Shannon capacity of this channel if the transmit power is P. **A simple closed-form** solution for **the bandwidth of the channel used** and **the capacity** should be derived using the parameters of the system.

b) Find the capacity of this channel for $A = 10^{-3}$, W = 4 KHz , P = 1W, and $N_0 = 10^{-12}$ mW/Hz. Also find the used bandwidth.

c) How much transmit power (P) do we need for a capacity of 10Mbps? How much more bandwidth we are using?

Note: $\int \boldsymbol{ln(1+x^2)dx = x\, ln(1+x^2) - 2x + 2\, tan^{-1}(x)}$

Problem 2

Consider a **flat-fading channel** where for a fixed transmit power of 0 dBm, the received SNR is one of three values: $SNR_1 = 10\ dB$, $SNR_2 = 6\ dB$, $SNR_3 = 3\ dB$. The probability associated with each state is $p_1 = 0.2$, $p_2 = 0.3$, $and\ p_3 = 0.5$.

a) If both transmitter and receiver have CSI, find the capacity of such channel as a function of transmit power (P) and draw its curve.

b) Find the channel inversion power control policy for this channel and associated zero-outage capacity per unit bandwidth as a function of transmit power (P).

c) Find the outage capacity of this system as a function of outage probability (e) when the transmit power is P.

d) Find the ergodic capacity of the system as function of transmit power (P).

e) If we use a coded time diversity by using **only 2 sub-blocks** spaced T_C seconds from each other, find the outage capacity when the transmit power P = 1W.

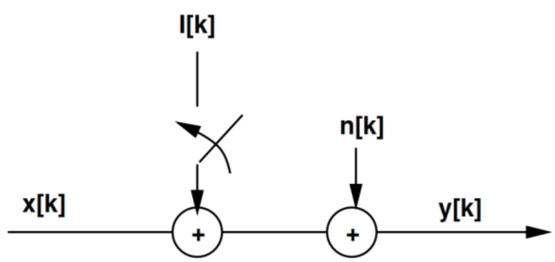
f) If there are 2 antennas at the RX with **2 independent fading channel** (with the specified fading statistics), repeat c) and d).

Note : In a) and b) both transmitter and receiver have CSI, but in c), d), e) and f) only receiver has CSI.

Problem 3 (problem 4.8 Goldsmith)

Time varying Interference: Consider following interference channel, which the jammer transmits interferer signal with probability of 0.4 and is off with probability of 0.6. Jammer's transmit power is 8mW. While the desired signal power at the receiver is 10mW. Noise also has a power spectral density of $0.1\frac{\mu W}{Hz}$. Bandwidth of the channel is 1MHz.

a) What is capacity of this channel if neither transmitter nor receiver know when interference is on (assume interference is like AWGN noise).
b) What is capacity of this channel if both transmitter and receiver know when the interference is on?
c) Suppose that the transmitter knows the interference signal perfectly. Consider two possible transmit strategies under this scenario: the transmitter can ignore the interference and use all its power for sending its signal, or it can use some of its power to cancel out the interferer (i.e. transmit the negative of the interference signal). In the first approach the interferer will degrade capacity by increasing the noise, and in the second strategy the interferer also degrades capacity since the transmitter sacrifices some power to cancel out the interference. Which strategy results in higher capacity?



Problem 4 (problem 4.13 Goldsmith)

Consider a frequency selective channel with coherence bandwidth of 3MHz, and total bandwidth of 9MHz. The channel's state at each of these sub-channels are as

$E\,|H_1|^2 = 1.5, E\,|H_2|^2 = 0.75,\ E\,|H_3|^2 = 0.5$. The total transmit power is 6W and noise spectral density is 0.2 $\mu W\ / Hz$.

a) What is optimal power allocated to each sub channel by assuming full CSI at both transmitter and receiver?
b) Compare the capacity of part a) with a scenario that allocates equal power to each of sub channels.

Problem 5

A 2x2 MIMO system uses Alamouti's scheme at the TX. Assume that the channels experience independent Rayleigh fading and only the receiver has CSI. Derive the best receiver for the system (to maximize the SNR of the symbols) and find the outage capacity and ergodic capacity of the system.

Problem 6 (Optional)

A SISO wireless system has a fading channel with a random power gain of $|h|^2$ that is distributed uniformly in [0-2] interval. Find the following capacities as a function of SNR = P/N_0, where P is the transmit power and $N_0/2$ is the PSD of the AWGN.

a) Outage capacity (for $P_{out} = e$)
b) Ergodic capacity
c) Zero-outage capacity
d) CSI capacity