University of Tehran, ECE
Wireless Communication, spring 2020

Due Date: ------------, 1398 Homework #2

Multi-path, Fading, Time-Varying Channel

1. Assume a Rayleigh fading channel with the average RX signal power $2\sigma^2 = -80$ dBm.
    a. What is the outage probability if the required power for a good reception is $P_0 = -100\ dBm$? How much we should increase the transmit power if we would like to decrease the outage probability by a factor of 10?
    b. Repeat a) for $P_0 = -90\ dBm$.

2. Assume an application that requires a power outage probability of 0.01 and the received power threshold is $P_{thr} = -80\ dBm$. For Rayleigh fading, what value of the average signal power is required at the receiver?

3. Consider the following frequency transfer functions of a fading channel:
$H_1(f,t) = \alpha_1(t)\mathrm{e}^{j\theta_1(t)}$,
$H_2(f,t) = \alpha_1(t)\mathrm{e}^{j\theta_1(t)} + \alpha_2(t)\mathrm{e}^{-j(2\pi f\tau_1+\theta_2(t))}$
    a. Derive the channels impulse response $h_1(\tau,t)$, $h_2(\tau,t)$.
    b. If $x(t)$ is the transmitted signal, express the received signal $r(t)$ in terms of x(t).
    c. Find the "mean delay" and "delay spread" for each case.
    d. Determine in which condition each of the above channels are flat fading? Explain.

4. A mobile receiver is moving at a speed of $\boldsymbol{v}$ and is receiving signals arriving along two reflected paths which make angles $\boldsymbol{\theta_1}$ and $\boldsymbol{\theta_2}$ with the direction of motion. The transmitted signal is a sinusoid at frequency f.
    a) Is the above information enough for estimating the coherence time (T_C) and the coherence bandwidth (W_C)? If so, express them in terms of the given parameters. If not, specify what additional information would be needed.
    b) Consider an environment in which there are reflectors and scatterers in all directions from the receiver and an environment in which they are clustered within a small angular range. Using part (a), explain how the channel would differ in these two environments.

5. In an environment with rich scattering the rays are arriving to the receiver antenna from all directions with positive delays. The average power gain of rays arriving with delay τ (in msec) is $Ke^{-\tau/10}$

a) Find the "mean delay" and "rms delay spread" of this channel.
b) If two sinusoids with frequencies f_c and $f_c + \Delta f$. What is the minimum value of Δf for which the channel response to the first sinusoid is approximately independent of the channel response to the second one?
c) Will this channel exhibit flat fading or frequency-selective fading for a typical voice channel with a 3 KHz bandwidth? How about for a cellular channel with a 30 KHz bandwidth?

6. Use MATLAB for the following questions:

In a multipath wireless channel, rays are arriving in clusters. Assume that there are 10 clusters that are coming from random angles with uniform distribution in [0-2π]. Each cluster can be modeled as Rayleigh fading channel with a random delay τ which is distributed uniformly in [1-10] μsec. The average power gain of each cluster ($2\sigma^2$) is determined by its delay τ as:
$2\sigma^2 = 10^{-3}\,\tau^{-4}$ (when τ is in μsec).

The mobile user is moving at a speed of 20 m/sec. The carrier frequency is f_C = 3GHz and we assume that we have a **narrowband** system.

a) Simulate 100,000 of such channels. For each realization of the channel find the overall channel gain, h, and compute the $|h|^2$ and find its average over all realizations to come up with an estimation of $E[|h|^2]$. Then, plot its CDF. What kind of distribution do you expect?
b) For one realization of the channel find the frequency response of the channel H(f) in frequency range [0-1 MHz]. What do you observe?
c) For one realization of the channel, compute the continuation of channel model for one second. Find the overall channel power gain, $|h|^2$, during this period for intervals of 100 μ seconds. Estimate the expected value of $|h|^2$ and plot its CDF. Explain the result.

   Note: Assume that during 1 second continuation of the channel, clusters are not changing much and only their phase change according to Doppler effect (gain amplitudes and delays are not changing.)