1) $2\sigma^2 = -80\ \text{dBm} = 10^{-8}\ \text{mW}$, Rayleigh fading channel : $f(r) = \frac{1}{2\sigma^2}\exp\left(-\frac{r}{2\sigma^2}\right)$

$$\rightarrow f(r) = 10^{8}\exp(-10^{8}r) \rightarrow P\left\{\frac{P_R}{P_T} < P_o\right\} = \int_0^{P_o} 10^{8}\exp(-10^{8}r) = -e^{-10^{8}r}\Big]_0^{P_o}$$

$$= 1 - e^{-10^{8}P_o}$$

a. if $P_o = -100\ \text{dBm} = 10^{-10}\ \text{mW} \rightarrow P\left\{\frac{P_R}{P_T} < -100\ \text{dBm}\right\} = 1 - e^{-10^{8}(10^{-10})} = 1 - e^{-0.01}$

$$= 9.9502 \times 10^{-3}$$

b. if $P_o = -90\ \text{dBm} = 10^{-9}\ \text{mW} \rightarrow P\left\{\frac{P_R}{P_T} < -90\text{dBm}\right\} = 1 - e^{-10^{8}(10^{-9})} = 1 - e^{-0.1} = 0.09516$

2) $P_{out} = 0.01$ , $P_{thr} = -80\ \text{dBm}$ , Rayleigh fading channel : $f(r) = \frac{1}{2\sigma^2}\exp\left(-\frac{r}{2\sigma^2}\right)$

$$\rightarrow P\left\{\frac{P_R}{P_T} < P_o\right\} = \int_0^{P_o} \frac{1}{2\sigma^2}\exp\left(-\frac{r}{2\sigma^2}\right)dr = -e^{-\frac{r}{2\sigma^2}}\Big]_0^{P_o} = 1 - e^{-\frac{P_o}{2\sigma^2}}$$

$$\Rightarrow 1 - e^{-10^{-8}/2\sigma^2} = 0.01 \rightarrow e^{\frac{10^{-8}}{2\sigma^2}} = 0.99 \rightarrow -\frac{10^{-8}}{2\sigma^2} = -0.01 \rightarrow 2\sigma^2 = 10^{-6}\ W = -60\text{dBm}$$

The required average power for recieved signal is $-60$ dBm

3) a. $h_1(\tau;t) = \mathcal{F}^{-1}\left\{H_1(f,t) = \alpha_1(t)e^{j\theta_1(t)}\right\} = \alpha_1(t)e^{j\theta_1(t)}\,\delta(\tau)$

$$h_2(\tau;t) = \mathcal{F}^{-1}\left\{H_2(f;t) = \alpha_1(t)e^{j\theta_1(t)} + \alpha_2(t)\exp\left(-j2\pi f\tau_1 + j\theta_2(t)\right)\right\}$$

$$= \alpha_1(t)e^{j\theta_1(t)}\,\delta(\tau) + \alpha_2(t)\,e^{-j\theta_2(t)}\,\delta(\tau-\tau_1)$$

b. For the first fading channel : $r_1(t) = \alpha_1(t)e^{j\theta_1(t)}\,x(t)$

For the 2nd fading channel : $r_2(t) = \alpha_1(t)e^{j\theta_1(t)}\,x(t) + \alpha_2(t)\,e^{-j\theta_2(t)}\,x(t-\tau_1)$

c. For the first channel: $T_d = \tau_{max} - \tau_{min} = 0 - 0 = 0$ , $\bar{\tau} = 0$

For the 2nd channel: $T_d = \tau_{max} - \tau_{min} = \tau_1 - 0 = \tau_1$ , $\bar{\tau} = \frac{0+\tau_1}{2} = \frac{\tau_1}{2}$

d. For flat fading, signal's bandwidth must be less than coherence bandwidth.

Coherence bandwidth is related to $T_d$. Meaning $W_c \propto \frac{1}{T_d}$

So for the first channel $W_c$ is infinity, meaning no matter what the bandwidth of signal is, channel acts as a flat fading one.

For the second channel, $W_c \propto \frac{2}{\tau_1}$. So signal's bandwidth must be less than $1/\tau_1$ in order to face a flat fading channel.

4) a. $T_c \propto \left(\frac{1}{D_s}\right)$ , $D_s = f_{d_{max}} - f_{d_{min}}$ , $f_d = \frac{v}{\lambda}\cos\theta$ , $f = \frac{c}{\lambda}$ , $D_s \leqslant \frac{2v}{\lambda}$

Since we know driver's speed & sent signal's frequency, therefore $f_d$ can be found. This'll help us find $D_s$ & its upper bound.

In A. Goldsmith $T_c = \frac{1}{2D_s}$ so $T_c$ can be calculated
In Tse $T_c = \frac{1}{4D_s}$ so $T_c$'ll be found
} Either way calculating $T_c$ using the given information is possible.

$W_c \propto \frac{1}{T_d}$ , ① $T_d \simeq \tau_{max} - \tau_{min}$ , ② $\bar{\tau} = \frac{\int \tau A_c(\tau) d\tau}{\int A_c(\tau) d\tau}$ , $A_c(\tau) = R_h(\tau; \Delta t = 0)$

Finding $T_d$ using equation ① needs information about delays & using equation ② needs information about channel responce which again needs information about gains & delay. These 2 parameters are not given. As a result $W_c$ can't be found without knowing them.

Although if there'll be ~~nothing~~ no obstacles in rays' path & the only reason rays reach the reciever in different ways & angles is recievers movement, then $T_d$ is almost zero & $W_c$ can be found with the given information. (Here this info is not given.)

So to sum up, since given information about this reciever's environment is not enough or clear, we can't talk about what $W_c$ can be.

b. When in a environment range of angles is large, delay spread is bigger than environments with limited range of angles. Because the wider range of angles result in a bigger difference between $f_{d_{max}}$, $f_{d_{min}}$. So $D_s$ becomes larger. As part a. shows, with bigger value for $D_s$, smaller value for $T_c$ can be seen. As a result channel changes faster.

So to sum up in areas with reflectors & scatterers in all directions the channel changes faster than in areas with smaller angular range.

5) $A_c(\tau) = Ke^{-\tau/10} u(\tau)$

expected value of R.V with exp. dist.

a. $\bar{\tau} = \dfrac{\int_0^\infty \tau A_c(\tau) d\tau}{\int_0^\infty A_c(\tau) d\tau} = \dfrac{\int_0^\infty \tau K e^{-\frac{\tau}{10}} d\tau}{\int_0^\infty K e^{-\tau/10} d\tau} = \dfrac{\int_0^\infty \tau e^{-\tau/10} d\tau}{-10 e^{-\tau/10}\Big]_0^\infty = 10} = \int_0^\infty \frac{\tau}{10} e^{-\tau/10} d\tau = 10$ msec

$\tau_{rms}^2 = \dfrac{\int_0^\infty (\tau-\bar{\tau})^2 A_c(\tau) d\tau}{\int_0^\infty A_c(\tau) d\tau} = \dfrac{\int_0^\infty (\tau-10)^2 K e^{-\tau/10} d\tau}{10K} = \int_0^\infty \frac{(\tau-10)^2}{10} e^{-\tau/10} d\tau = 100$

variance of a R.V with exponential distribution

$\rightarrow \tau_{rms} = 10$ msec

b. Goldsmith: In time-varying channels the multipath delays vary with time, so the delay spread becomes random variable. $\rightarrow$ We use the average & rms delay spread for this question.

Goldsmith: The channel response is approximatly independent @ frequency separations $\Delta f$ where $A_c(\Delta f) \simeq 0$ $\rightarrow$ Coherent Bandwidth: $A_c(\Delta f) \simeq 0$ $\Delta f > W_c$

$\rightarrow$ Minimum value for $W_c \simeq \dfrac{1}{\tau_{rms}} = 0.1$ KHz $= 100$ Hz $\rightarrow \Delta f > 100$ : independent

In a more general way: channel correlation $> 0.9 \rightarrow B_c \simeq \dfrac{0.02}{\tau_{rms}} = 200$ Hz

channel correlation $> 0.5 \rightarrow B_c \simeq \dfrac{0.2}{\tau_{rms}} = 2$ KHz

c. 3 KHz $> B_c \rightarrow$ Frequency Selective

30 KHz $> B_c \rightarrow$