MIMO systems

Problem 1

Consider a MIMO system with 2 antennas at the transmitter and a 4-antenna ULA at the receiver. The spacing between adjacent antennas are $\lambda/2$ and AOA and AOD are arbitrary random angles. The receiver is very far from the transmitter and maintains line of sight (LOS) reception. Determine the capacity of the system (as a function of SNR) in the following scenarios:

a) Perfect CSI at both transmitter and receiver.
b) Perfect CSI at the receiver and no CSI at the transmitter. The system employs Alamouti's coding.

Note: Assume normalized channel gains, the total transmitted power = P, and SNR = P/N_0.

Problem 2 (problem 10.17 Goldsmith)

Consider a 2x2 MIMO system with channel gain matrix $\mathbf{H} = \begin{bmatrix} .7 & .5 \\ 0.3 & .9 \end{bmatrix}$. Assume **H** is known at both the transmitter and receiver, and that there is a total transmit power of P = 10 mW across the two transmit antennas, AWGN with power $N_0 = 10^{-9}$ W/Hz at each receive antenna, and bandwidth B = 100 KHz.

a) Find the SVD for **H**.
b) Find the capacity of this channel.
c) Assuming transmit precoding and receiver shaping is used to transform this channel into two parallel independent channels with a total power constraint P. Find the maximum data rate that can be transmitted over this parallel set assuming MQAM modulation on each channel with optimal power adaptation across the channels subject to power constraint P. Assume a target BER of 10^{-3} on each channel, the BER is bounded by $0.2\exp(-1.5\gamma/(M-1))$, and the constellation size of the MQAM is unrestricted.
d) Suppose now that the antennas at the transmitter and receiver are all used for diversity with optimal weighting at the transmitter and receiver to maximize the SNR of the combiner output. Find the SNR of the combiner output, and the BER of a BPSK modulated signal transmitted over this diversity system. Compare the data rate and BER of this BPSK signaling with diversity (assuming B = 1/Tb) to the rate and BER from part c).
e) Comment on the diversity/multiplexing tradeoffs between the systems in parts c) and d).

Problem 3

Consider a 2x4 MIMO system with channel gain matrix $\mathbf{H} = \begin{bmatrix} +1 & -1 \\ -1 & j \\ j & +1 \\ +1 & j \end{bmatrix}$. The total transmit power across the two transmit antennas is P, AWGN with PSD of $N_0/2$ at each receive antenna, and bandwidth of $B = 1/Ts = 1$ Hz.

If the transmitter knows H,

a) What is the capacity of this channel?
b) Suppose now that the antennas at the transmitter and receiver are all used for diversity with optimal weights at transmitter and receiver to maximize the SNR of the combiner output. Find the SNR of the combiner output, and the capacity of this system.
c) Comment on the diversity/multiplexing tradeoffs between the systems in parts a) and b).

If the transmitter does not know H,

d) What is the maximum data rate of any space-time code (without the knowledge of the channel) which is error-free for this channel?
e) What is the maximum data rate (error free for this channel) if Aamouti's STBC is used? Is Alamouti's optimum? Explain.
f) If we send two independent coded data streams directly from 2 TX antennas, with equal power, what is the capacity of this system when a decorrelator receiver is used?

Note : All answers should be in terms of $SNR = P/N_0$.

Problem 4

Determine the best diversity-multiplexing trade-off d(r) for the following channels:

a) A fading channel with diversity order (d*) of 4.
b) 2 parallel Rayleigh fading channels.

Hint: If X and Y are two independent exponential distributed random variables, we have

$\Pr\{ XY < a \} \propto a$ for small values of a

Problem 5 (Simulations)

Consider a 4×4 MIMO Rayleigh channel with uncorrelated distributions. We wish to simulate the performance of the system with the following configurations:

I. Coded modulation across antennas

In this scheme transmitter sends a block of coded 4-bit data onto 4 antennas. The coded symbols are $C_1 = (x_1 \quad x_2 \quad x_3 \quad x_4)^T$ and uses the following coding table.

Bits	→	coded bits	→	coded symbols
0000	→	00000000	→	0 0 0 0
0001	→	00010110	→	0 1 1 2
0010	→	00101101	→	0 2 3 1
0011	→	00111011	→	0 3 2 3
0100	→	01001001	→	1 0 2 1
0101	→	01011111	→	1 1 3 3
0110	→	01100100	→	1 2 1 0
0111	→	01110010	→	1 3 0 2
1000	→	10000111	→	2 0 1 3
1001	→	10010001	→	2 1 0 1
1010	→	10101010	→	2 2 2 2
1011	→	10111100	→	2 3 3 0
1100	→	11001110	→	3 0 3 2
1101	→	11011000	→	3 1 2 0
1110	→	11100011	→	3 2 0 3
1111	→	11110101	→	3 3 1 1

The modulation used is QPSK. So the overall spectral efficiency is 4bits/sec/Hz.

II. Alamouti-like STBC

The Alamouti-like STBC scheme encoding operation is given by

$$C_2 = \begin{pmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \\ x_3 & -x_4^* \\ x_4 & x_3^* \end{pmatrix}$$

In this STBC, the data are transmitted over two time slots. The modulation used is QPSK. So the overall spectral efficiency is again 4bits/sec/Hz.

III. Beamforming

In this system, we transmit only one data stream using the best possible beam-forming both at TX and RX. The channel gains (CSI) is known at TX and RX.

The modulation used here is 16-QAM. So the overall spectral efficiency is again 4bits/sec/Hz.

For all above cases, evaluate the performance of the system using Maximum-Likelihood (ML) decoder. Plot BER curve with respect to $SNR = \frac{E_x}{N_0}$. Compare different cases and comment on the diversity of the schemes.

Optional:

Problem 6 (problem 10.8 Goldsmith)

For the 4x4 MIMO channels given below, find the capacity per unit Hz, assuming both transmitter and receiver know the channel, for channel SNR = P/N_0 = 10 dB.

$$\mathbf{H}1 = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix} \qquad \mathbf{H}2 = \begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & -1 & -1 & -1 \end{bmatrix}$$