Wireless Communication

University of Tehran, ECE

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HW3: Performance in fading, Diversity combining,
Coded systems, and Antenna arrays

Note : $\gamma = SNR = \frac{E_S}{N_0}$

Problem 1) Fading with diversity combining

Find the outage probability of QPSK modulation at $P_b = 10^{-3}$ for a Rayleigh fading channel with SC diversity for $M = 1$ (no diversity), $M = 2$, and $M = 3$. Assume branch SNRs of $\overline{\gamma}_1 = 10$ dB, $\overline{\gamma}_2 = 15$ dB, and $\overline{\gamma}_3 = 20$ dB.

Problem 2) MRC and SC

Consider a single branch Rayleigh faded signal has a 10% chance of being faded below SNR threshold (SNR_{thr}).

a) Determine the mean SNR in dB of the Rayleigh faded signal as reference to the SNR threshold.
b) Find the probability that a two branch selection diversity receiver will be 10 dB below the SNR threshold
c) Find the probability that a two branch MRC diversity receiver will be 10 dB below the SNR threshold.

Problem 3) Uniform Channel Gain

Consider the channel $h = \alpha e^{j\beta}$ between a transmitter and a receiver, where α is a uniformly distributed random variable in [0-A] interval, β takes the values $\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\}$ with equal probability, and we have $E[|h|^2] = 1$.

(a) If the receiver uses the selective combining method for combining the L independent branches, find the average SNR after combining and compare it with AWGN channel.
(b) Repeat part (a) for MRC and EGC methods.
(c) Using selective combining method, for $SNR_{thr} = 0$ dB and $P_{out} = 0.01$, find the minimum number of branches such that fading margin is less than 6 dB.

Problem 4) Array Gain in AWGN

This problem illustrates that, because of array gain, you can get performance gains from diversity combining even without fading.

Consider an AWGN channel with N branch antenna diversity at RX and γ_i =10 dB (fixed with no fading) per branch. Assume M-QAM modulation with M =4 and use the approximation $P_b = 0.2e^{-(1.5\,\gamma)/(M-1)}$ for bit error probability, where γ is the received SNR.

a) Find P_b for N =1.
b) Find N so that under MRC, P_b < 10^{-6}.

Problem 5: (Probability of Bit Error in Fading)

The probability of bit error for binary DPSK modulation over an AWGN channel is

$$P_b(\gamma) = \frac{1}{2}e^{-\gamma}$$

where $\gamma = \alpha^2 \frac{E_b}{N_0}$ is the received bit signal-to-noise ratio. Consider the performance of DPSK modulation over fading channels.

(a) If $E_b/N_0 = 12$ dB, sketch the distribution of γ and average probability of bit error of binary DPSK modulation over a fading channel with an amplitude gain α whose probability density function (PDF) is given by

$$P_\alpha(\alpha) = 0.3\delta\left(\alpha - \sqrt{0.3}\right) + 0.45\delta\left(\alpha - \sqrt{0.8}\right) + 0.25\delta\left(\alpha - \sqrt{1.5}\right)$$

(b) Determine the minimum required E_b/N_0 value for DPSK modulation in the fading channel given in part (a) such that for 80% of the time, $P_b < 10^{-4}$.
(c) Repeat part (b) if the channel is subject to Rayleigh fading.
(d) Find the average probability of bit error for binary DPSK modulation over a flat Rayleigh fading channel, i.e. assume $E[\alpha^2] = 1$.

Problem 6) Space-time codes

Find the coding rate, the diversity order, and the coding gain in a system with 2 transmitting antennas if the following space-time code is used. Assume that symbols are binary with values +A and –A. Also find an upper bound for the average BER of the code.

Rotation code: $X = \begin{bmatrix} s_0 & s_3 \\ s_1 & s_0 \\ s_2 & s_1 \\ s_3 & s_2 \end{bmatrix}$

Note: Normalize the average transmitted power first.

Problem 7) Simulation: Coded modulation system

In a coded modulation system, shown below, first a rate ½ block code encodes blocks of 4-bit input data bits into blocks of 8-bit coded bits. Then, the coded bits are separated to generate a block of coded symbols which are used for QPSK modulation. The symbol index n is mapped to QPSK symbol $e^{\frac{jn\pi}{2}}$.

a) Demonstrate the system performance in AWGN and fading channels. Assume that the coded symbols experience independent Raleigh fading with the same average SNR in fading channel. The channel response is perfectly known at the receiver and we use Maximum Likelihood (ML) decoding.

b) Determine the diversity order of this system by TWO different methods: using system analysis and using the performance curve.

Bits	→	coded bits	→	coded symbols
0000	→	00000000	→	0 0 0 0
0001	→	00010110	→	0 1 1 2
0010	→	00101101	→	0 2 3 1
0011	→	00111011	→	0 3 2 3
0100	→	01001001	→	1 0 2 1
0101	→	01011111	→	1 1 3 3
0110	→	01100100	→	1 2 1 0
0111	→	01110010	→	1 3 0 2
1000	→	10000111	→	2 0 1 3
1001	→	10010001	→	2 1 0 1
1010	→	10101010	→	2 2 2 2
1011	→	10111100	→	2 3 3 0
1100	→	11001110	→	3 0 3 2
1101	→	11011000	→	3 1 2 0
1110	→	11100011	→	3 2 0 3
1111	→	11110101	→	3 3 1 1

Problem 8) Simulation: Antenna array

We have a uniform linear array antenna (ULA) with 4 omni-directional antennas spacing $\lambda/2$ apart. The baseline of the array (the line connecting the antennas) is the reference for the angle of arrival of waves denoted by θ. The noise is AWGN with the same PSD for all antennas.

Plot the resulting antenna array pattern as a function of θ for each of the following cases, and determine the final SNR(SINR) of the received signal coming from the antenna array.

a) Three copies of the desired signal coming from -50 degrees, -10 degrees and 35 degrees with SNRs of 20 dB, 10 dB and 6 dB respectively. Assume that these three different rays are added constructively in the first antenna. The beamforming is done using MRC coefficients.

b) There are 4 independent complex Gaussian signals with relative SNRs of 20 dB(desired signal), 14 dB, 11 dB and 10 dB. The angles of arrival for the signals are 45, 90, 120 and -30 degrees respectively. The beamforming coefficients are chosen to provide perfect nulls in the direction of the interfering signals.

c) An incoming desired signal at an angle of 90-degrees with an SNR of 20 dB and an interfering signal (Gaussian) with equal power incident from -60 degrees. The beamforming coefficients are chosen to maximize the SINR.

Note: The given SNRs are measured on an antenna assuming that the waves are coming from only from one direction.