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1) $f_c = 1\,GHz = 10^9\,Hz$, $P_T = 0\,dBm = 1\,mW = -30\,dB$, $P_L(dB) = 10 + 40\log_{10}^{d}$

Log-Normal Shadowing: $\log X \sim N(0, 36)$, outage $\triangleq P_r - P_{r_{min}}$, $P_{r_{min}} = -90\,dBm$

a. $P_{out} = P\{outage\} = P\{P_r < P_{r_{min}}\} = P\{\underbrace{P_r(dBm)}_{N(\bar{P}_R, 36)} < -90\,dBm\}$

$\bar{P}_R\,(dBm) = P_T(dBm) - P_L(dB) = 0 - \{10 + 40\log_{10}^{d}\} = -10 - 40\log_{10}^{d}$

$$P_{out} = \Phi\left(\frac{-90 + 10 + 40\log_{10}^{d}}{6}\right) = \Phi\left(\frac{-40}{3} + \frac{20}{3}\log_{10}^{d}\right) = 1 - Q\left(-\frac{40}{3} + \frac{20}{3}\log_{10}^{d}\right)$$

if $P_{out} = 1/2 \rightarrow Q\left(-\frac{40}{3} + \frac{20}{3}\log_{10}^{d}\right) = 1/2 \rightarrow -\frac{40}{3} + \frac{20}{3}\log_{10}^{d} = Q^{-1}(1/2) = 0$

$\rightarrow \log_{10}^{d} = \frac{40}{3}\left(\frac{3}{20}\right) = 2 \rightarrow d = 100\,m$

b. $C = E\left\{\frac{1}{\pi R^2}\int_{\text{Cell area}} \{P_r > P_{min} \text{ in } dA\}\,dA\right\} = \frac{1}{\pi R^2}\int_{\text{Cell area}} \underbrace{E\{P_r > P_{min} \text{ in } dA\}}_{P_A}\,dA$

$$= \frac{1}{\pi R^2}\int_{\theta=0}^{2\pi}\int_{r=0}^{R} P_A\, r\,dr\,d\theta \ , \quad P_A = P\{P_r > P_{min}\} = 1 - P_{out} = Q\left(\frac{P_{min} - P_T + P_L}{\sigma}\right)$$

$$= Q\left(\frac{P_{min} - P_T - 10\log_{10}^{K} + 10\gamma\log_{10}^{r/d_0}}{\sigma}\right)$$

$$\rightarrow C = \frac{1}{\pi R^2}\int_0^{2\pi} d\theta \int_0^R Q\left(\frac{P_{min} - P_T - 10\log_{10}^{K} + 10\gamma\log_{10}^{r/d_0}}{\sigma}\right) r\,dr$$

if $\bar{P}_R \triangleq P_T + 10\log_{10}^{K} - 10\gamma\log_{10}^{R/d_0} \rightarrow C = \frac{2}{R^2}\int_0^R Q\left(\frac{P_{min} - \bar{P}_R + 10\gamma\log_{10}^{r/R}}{\sigma}\right) r\,dr$

if $a \triangleq \frac{P_{min} - \bar{P}_R}{\sigma}$, $b \triangleq \frac{10\gamma}{\sigma}\log_{10}^{e} \rightarrow C = \frac{2}{R^2}\int_0^R r\,Q\left(a + b\ln\frac{r}{R}\right) dr$

$$= Q(a) + \exp\left(\frac{2 - 2ab}{b^2}\right) Q\left(\frac{2 - ab}{b}\right)$$

If we use last part's values: $P_L = -10\log_{10}^{K} + 10\gamma\log_{10}^{d/d_0} \equiv 10 + 40\log_{10}^{d} \rightarrow \begin{cases} K = -1 \\ d_0 = 1 \\ \gamma = 4 \end{cases}$

$\rightarrow \bar{P}_R = 0 + 10\log_{10}^{0.1} - 40\log_{10}^{\frac{100}{1}} = -10 - 80 = -90 \rightarrow \bar{P}_R = P_{min} \rightarrow a = 0$

$$b = \frac{10(4)}{6} \log_{10}^{e} = 2.8953 \rightarrow C = Q(0) + \exp\left(\frac{2}{(2.8953)^2}\right) Q\left(\frac{2}{2.8953}\right) = 0.8108$$

matlab

$$C = \frac{r^2}{R^2} \rightarrow r = R\sqrt{C} = 100\sqrt{0.8108} = 90.044 \rightarrow$$ If we suppose the total radius is 100 m then based on the fact that 81% is desirable, r is calculated

C. if $d = 100 \rightarrow P_L = 10 + 40 \log_{10}^{100} = 90 \rightarrow \bar{P}_R = P_T - 90$, $P_{min} = -90$ dBm $\rightarrow$

$$P_{out} = 1 - Q\left(\frac{-90 - P_T + 90}{6}\right) = 1 - Q\left(-\frac{P_T}{6}\right) = Q\left(\frac{P_T}{6}\right) \rightarrow P_T = 6Q^{-1}(P_{out})$$

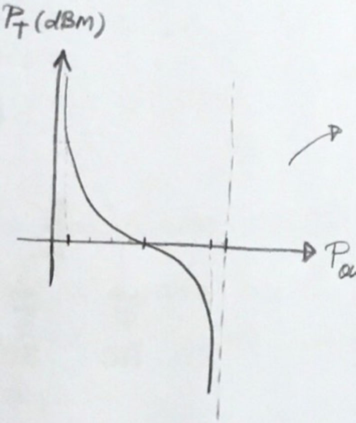
$P_{out} = 0 \rightarrow P_T = \infty$

$P_{out} = 0.1 \rightarrow P_T = 7.6893$

$P_{out} = 0.5 \rightarrow P_T = 0$

$P_{out} = 0.9 \rightarrow P_T = -7.6893$ $\rightarrow$

$P_{out} = 1 \rightarrow P_T = -\infty$



This plot is also created using matlab. It's codes are in codes file under the name "q1" & the result is in reports file

2) a. $P_{out} = P\{SINR < SINR_{min}\}$, $SINR_{min} = 10$ dB

$SINR = \{P_{T_1} - P_{L_1}\} - \{P_{T_2} - P_{L_2}\}$, if $P_{T_1} = P_{T_2} \rightarrow SINR = P_{L_2} - P_{L_1}$

$P_{L_1} = K + 10(4)\log_{10}^{d_1} + X_1$, $P_{L_2} = K + 10(4)\log_{10}^{d_2} + X_2$ $\rightarrow$

$$P_{out} = P\left\{X_2 - X_1 + 40\log_{10}^{d_2/d_1} < 10\right\} = P\left\{X_2 - X_1 < 10 - 40\log_{10}\frac{d_2}{d_1}\right\}$$

$$X_2, X_1 \sim N(0, 25) \rightarrow Y \triangleq X_2 - X_1 \rightarrow Var\{Y\} = E\{Y^2\} = E\{X_1^2\} + E\{X_2^2\} = 2(25) = 50$$

$$\rightarrow P_{out} = 1 - Q\left(\frac{10 - 40\log_{10}^{d_2/d_1} - 0}{\sqrt{50}}\right) < 0.05 \rightarrow Q\left(-\sqrt{2} + 4\sqrt{2}\log_{10}^{d_2/d_1}\right) < 0.05$$

$$\rightarrow 4\sqrt{2}\log_{10}^{d_2/d_1} - \sqrt{2} > 1.6449 \rightarrow \log_{10}^{d_2/d_1} > 0.54078 \rightarrow \frac{d_2}{d_1} > 3.4736$$

Worst case: $d_2 = d - d_1 = 1000 - d_1 \rightarrow \frac{1000}{d_1} - 1 > 3.4736 \rightarrow \frac{1000}{d_1} > 4.4736 \rightarrow d_1 < 223.53$

$\rightarrow$ max value for d_1 is 223.53 m.

b. (We suppose the same bound for outage probability is desired)

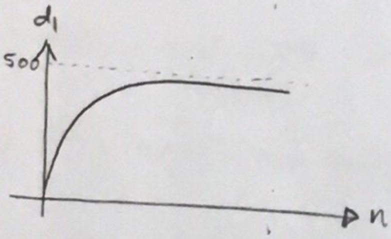
$P_{L_1} = K + 10n\log_{10}^{d_1} + X_1$, $P_{L_2} = K + 10n\log_{10}^{d_2} + X_2 \rightarrow SINR = X_2 - X_1 + 10n\log_{10}^{d_2/d_1}$

$\rightarrow P\{X_2 - X_1 < 10 - 10n\log_{10}^{d_2/d_1}\} = 1 - Q\left(\frac{10 - 10n\log_{10}^{d_2/d_1}}{\sqrt{50}}\right) < 0.05$

$\rightarrow Q\left(\sqrt{2}n\log_{10}^{d_2/d_1} - \sqrt{2}\right) < 0.05 \rightarrow n\sqrt{2}\log^{d_2/d_1} - \sqrt{2} > 1.6449$

$\rightarrow n\log^{d_2/d_1} > 2.163 \rightarrow n\log^{\frac{1000}{d_1}-1} = 2.163 \rightarrow \frac{1000}{d_1} - 1 = 10^{\frac{2.163}{n}}$

$\rightarrow d_1 = \frac{1000}{10^{\frac{2.163}{n}} + 1} \rightarrow \begin{cases} \text{if } n \rightarrow 0 : d_1 \rightarrow 0 \\ \text{if } n \rightarrow \infty : d_1 \rightarrow 500 \end{cases} \Rightarrow$



$\rightarrow$ This plot is also created using matlab. It's codes are in file under the name "q2" & the result is in reports file

C. $N_0 = 10^{-12} \rightarrow P_N = -120\ dB$

3) $f_c = 900$ MHz, Log-Normal Shadowing : $\sigma = 6$ dB , $SNR_{min} = 15$ dB , $P_T = 1W = 0$ dB

gain $= 3$ dB , $P_{N_R} = -10$ dBm $= 0.1$ mW $= 10^{-4}$ W $= -40$ dB

$$P_{out} = P\{SNR < SNR_{min}\} = P\{SNR < 15\} \geqslant 0.9$$

$$SNR = \frac{P_R}{P_{N_R}} \xrightarrow{dB} SNR(dB) = P_R(dB) - P_{N_R}(dB) = P_R + 40$$

$$P_R = \frac{G\lambda_c^2}{(4\pi d)^2} P_T \xrightarrow{dB} P_R = G(dB) + 20\log_{10}^{\lambda_c} - 20\log_{10}^{4\pi d} + P_T(dB)$$

$$= 3 + 20\log_{10}^{\frac{3\times10^8}{900\times10^6}} - 20\log_{10}^{4\pi} - 20\log_{10}^{d}$$

$$= 3 - 9.54 - 21.9842 - 20\log_{10}^{d} = -28.5242 - 20\log_{10}^{d}$$

without shadowing

$$\rightarrow SNR = -28.5242 - 20\log_{10}^{d} + 40 - X = 11.4758 - X - 20\log_{10}^{d}$$

$$\rightarrow P\{11.4758 - X - 20\log_{10}^{d} < 15\} = P\{X > -3.5242 - 20\log_{10}^{d}\}$$

$$= Q\left(-\frac{3.5242 + 20\log_{10}^{d}}{6}\right) \geqslant 0.9 \rightarrow -\frac{3.5242 + 20\log_{10}^{d}}{6} \geqslant -1.2816$$

$$\rightarrow \frac{3.5242 + 20\log^{d}}{6} \leqslant 1.2816 \rightarrow \log^{d} \leqslant 0.20827 \rightarrow d \leqslant \underline{\underline{1.6153}}$$

maximum value

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4) $P_0(dBm) = 0$, $d_0 = 1$, $\gamma = 3$, $P_X(x) = \begin{cases} 0.2 & x = 0, \pm1, \pm2 \text{ dB} \\ 0 & \text{O.W} \end{cases}$

$$P\{P_r > -90\} = 0.8 \rightarrow P\{-30\log_{10}^{d} + X > -90\} = 0.8 \rightarrow P\{X > -90 + 30\log^{d}\} = 0.8$$

$$0.8 = 4\times0.2 = P\{X=2\} + P\{X=1\} + P\{X=0\} + P\{X=-1\} \rightarrow$$

$$-90 + 30\log^{d} = -2 \rightarrow 30\log^{d} = 88 \rightarrow d = 857.695986$$