Fateme Noorzad 810198271 Wireless Communications HW# 4

3) a) Because neither transmitter nor the reciever know the state of jammer, they should concider the worst case which in this case means we always have interference:

$$C = W\log_2(1+SINR), \quad SINR = \frac{P_S}{N_0W+P_I} \rightarrow C = 10^6\log_2\left(1+\frac{10\times10^{-3}}{(10^{-7})(10^6)+8\times10^{-3}}\right) = 1.277\times10^5$$

b) In this part we need to solve an optimization problem: If we assume P_{S_1} is transmitted when jammer is off & P_{S_2} is transmitted when jammer is on then:

$$\begin{cases} C = W\max\left\{\log\left(1+\frac{P_{S_1}}{N_0W+0}\right)(0.6)+\log\left(1+\frac{P_{S_2}}{N_0W+P_I}\right)(0.4)\right\} \\ s.t.\ \ 0.6P_{S_1}+0.4P_{S_2} = P_T \end{cases}$$

$$\alpha \triangleq \frac{1}{N_0W}, \quad \beta \triangleq \frac{1}{N_0W+P_I} \rightarrow \mathcal{L} = \frac{3}{5}\log(1+\alpha P_{S_1})+\frac{2}{5}\log(1+\beta P_{S_2})+\lambda\left\{\frac{3}{5}P_{S_1}+\frac{2}{5}P_{S_2}-P_T\right\}$$

$$\rightarrow \frac{\partial\mathcal{L}}{\partial P_{S_1}} = \frac{\cancel{3}}{5\ln2}\cdot\frac{\alpha}{1+\alpha P_{S_1}}+\frac{\cancel{3}}{\cancel{5}}\lambda = 0 \rightarrow \frac{\alpha}{\ln2(1+\alpha P_{S_1})} = -\lambda \rightarrow 1+\alpha P_{S_1} = \frac{-\alpha}{\lambda\ln2}$$

$$\rightarrow P_{S_1} = -\frac{1}{\lambda\ln2}-\frac{1}{\alpha}$$

$$\rightarrow \frac{\partial\mathcal{L}}{\partial P_{S_2}} = \frac{\cancel{2}}{5\ln2}\cdot\frac{\beta}{1+\beta P_{S_2}}+\frac{\cancel{2}}{\cancel{5}}\lambda = 0 \rightarrow \frac{\beta}{\ln2(1+\beta P_{S_2})} = -\lambda \rightarrow 1+\beta P_{S_2} = -\frac{\beta}{\lambda\ln2}$$

$$\rightarrow P_{S_2} = -\frac{1}{\lambda\ln2}-\frac{1}{\beta}$$

$$\rightarrow -\frac{0.6}{\lambda\ln2}-\frac{0.6}{\alpha}-\frac{0.4}{\lambda\ln2}-\frac{0.4}{\beta} = P_T$$

$$\rightarrow -\frac{1}{\lambda\ln2} = \frac{0.6}{\alpha}+\frac{0.4}{\beta}+P_T \rightarrow \lambda = \frac{-1}{\left(P_T+\frac{0.6}{\alpha}+\frac{0.4}{\beta}\right)\ln2}$$

$\rightarrow P_{S_1} = P_T + \frac{0.6}{\alpha} + \frac{0.4}{\beta} - \frac{1}{\alpha} = P_T + 0.4\left(\frac{1}{\beta} - \frac{1}{\alpha}\right) = P_T + 0.4(N_oW + P_I - N_oW) = P_T + 0.4\,P_I$

$\rightarrow P_{S_2} = P_T + \frac{0.6}{\alpha} + \frac{0.4}{\beta} - \frac{1}{\beta} = P_T + 0.6\left(\frac{1}{\alpha} - \frac{1}{\beta}\right) = P_T + 0.6(N_oW - N_oW - P_I) = P_T - 0.6P_I$

$\Rightarrow P_{S_1} = 10 + 0.4(8) = 13.2 \rightarrow 0.6 \log_2\left(1 + \frac{13.2\times10^{-3}}{10^{-7}(10^6)}\right) = 0.1073$

$\Rightarrow P_{S_2} = 10 - 0.6(8) = 5.2 \rightarrow 0.4 \log_2\left(1 + \frac{5.2\times10^{-3}}{10^{-7}(10^6) + 8\times10^{-3}}\right) = 0.0271$

$\rightarrow C = 10^6(0.1073) = 1.073\times10^5$

C) first scenario: all power for sending signal: $C_1 = W\log_2\left(1 + \frac{P_T}{N_oW + P_I}\right)$

second scenario: interference power is cancelled: $C_2 = W\log_2\left(1 + \frac{P_T - P_I}{N_oW}\right)$

Using given values: $C_1 = 10^6\log_2\left(1 + \frac{10\times10^{-3}}{10^{-7}(10^6) + 8\times10^{-3}}\right) = 1.2776\times10^5$

$$C_2 = 10^6\log_2\left(1 + \frac{10\times10^{-3} - 8\times10^{-3}}{10^{-7}(10^6)}\right) = 2.8569\times10^{+4}$$

$\Rightarrow$ The first scenario results in bigger capacity.

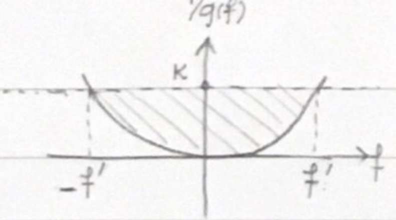
1) a) $g(f) = \frac{|H(f)|^2}{N_o} \rightarrow g(f) = \frac{A^2}{N_o(1 + (\frac{f}{W})^2)}$

$A = [-f', f']$

$$S_x^{best}(f) = \begin{cases} K - \frac{1}{g(f)} & \underset{A}{K > 1/g(f)} \\ 0 & \text{O.W} \end{cases} \rightarrow P = \int_A \left(K - \frac{1}{g(f)}\right)df = \int_{-f'}^{f'}\left(K - \frac{1}{g(f)}\right)df$$

$$= 2\int_0^{f'}\left(K - \frac{N_o}{A^2}\left(1 + \left(\frac{f}{W}\right)^2\right)\right)df$$

$$= 2Kf' - \frac{2N_o}{A^2}f' - \frac{2N_o}{3A^2W^2}f'^3$$



$$K = \frac{1}{g(f)}\Big|_{f=f'} \to K = \frac{N_0}{A^2}\left(1+\left(\frac{f'}{W}\right)^2\right)$$

∴

$$\to P = \frac{2N_0}{A^2}f' + \frac{2N_0}{A^2W^2}f'^3 - \frac{2N_0}{A^2}f' - \frac{2N_0}{3A^2W^2}f'^3 = \frac{4N_0}{3A^2W^2}f'^3 \to f'^3 = \frac{3A^2W^2}{4N_0}P \to f' = \left(\frac{3A^2W^2}{4N_0}P\right)^{1/3}$$

$$\to C = \int_{-f'}^{f'} \log_2\left(1+g(f)\left(K-\frac{1}{g(f)}\right)\right)df = \int_{-f'}^{f'}\log_2(Kg(f))\,df = \int_{-f'}^{f'}\log_2\left(\frac{KA^2}{N_0\left(1+\left(\frac{f}{W}\right)^2\right)}\right)df$$

$$= \int_{-f'}^{f'} \log_2(KA^2)\,df - \int_{-f'}^{f'}\log_2(N_0)\,df - \int_{-f'}^{f'} \log_2\left(1+\left(\frac{f}{W}\right)^2\right)df$$

$$= 2f'\log\left(\frac{KA^2}{N_0}\right) - \frac{1}{\ln 2}\underbrace{\int_{-f'}^{f'}\ln\left(1+\left(\frac{f}{W}\right)^2\right)df}_{\text{I}}$$

$$\text{I} \quad x \triangleq \frac{f}{W} \to dx = \frac{df}{W} \to 2\int_{x=0}^{\frac{f'}{W}} W\ln(1+x^2)\,dx = 2W\left\{x\ln(1+x^2)-2x+2\tan^{-1}(x)\right\}_{x=0}^{\frac{f'}{W}}$$

$$= W\left\{\frac{2f'}{W}\ln\left(1+\left(\frac{f'}{W}\right)^2\right) - 4\frac{f'}{W} + 4\tan^{-1}\left(\frac{f'}{W}\right)\right\}$$

$$\to C = \cancel{2f'\log_2\left\{1+\left(\frac{f'}{W}\right)^2\right\}} - \cancel{2f'\log_2\left\{1+\left(\frac{f'}{W}\right)^2\right\}} + \frac{4f'}{\ln 2} - 4W\frac{\tan^{-1}\left(\frac{f'}{W}\right)}{\ln 2} = \frac{4}{\ln 2}\left\{f' - W\tan^{-1}\left(\frac{f'}{W}\right)\right\}$$

$$f' = \left(\frac{3A^2W^2}{4N_0}P\right)^{1/3}$$

b) $f' = \left(\frac{3(10^{-3})^2(4\times10^3)^2}{4(10^{-12})}\,1\right)^{1/3} = (12\times10^{12})^{1/3} = 2.2894\times10^4$ Hz

$$C = \frac{4}{\ln 2}\left\{2.2894\times10^4 - 4\times10^3\underbrace{\tan^{-1}\left(\frac{2.2894\times10^4}{4\times10^3}\right)}_{1.3978}\right\} = \frac{4(17302.8)}{\ln 2} = 99850.65501$$

c) $10\times10^6 = \frac{4}{\ln 2}\left\{f' - 4\times10^3\tan^{-1}\left(\frac{1}{4}\times10^{-3}f'\right)\right\} \to 2.5\ln2\times10^6 - f' = 4\times10^3\tan^{-1}(25\times10^{-5}f')$

$\to f' = 1.79\times10^6$

$$\rightarrow 1.7\times10^{6} = \left(\frac{3(10^{-6})(4\times10^{3})^{2}}{4(10^{-12})}P\right)^{1/3} \rightarrow P = 440\ W$$

6) a) $P\{\log_2(1+|h|^2 SNR) < C_e\} = e \rightarrow P\{|h|^2 < \frac{2^{C_e}-1}{SNR}\} = e$

$$H \triangleq |h|^2 \rightarrow \mathcal{F}_H(h) = \begin{cases} 0 & h<0 \\ \frac{h}{2} & 0<h<2 \\ 1 & h>2 \end{cases} \rightarrow \frac{2^{C_e}-1}{SNR} = e \rightarrow C_e = \log_2(1+e\,SNR)$$

b) $$C_{ergodic} = E_h\{\log_2(1+|h|^2 SNR)\} = \int_0^2 \frac{1}{2}\log_2(1+x\,SNR)\,dx = \frac{1}{2\ln 2}\int_0^2 \ln(1+SNR\,x)\,dx$$

$$= \frac{1}{2\ln 2}\left\{ x\ln(1+x\,SNR)\Big]_0^2 - \int_0^2 \frac{x\,SNR}{1+x\,SNR}dx \right\}$$

$$= \frac{1}{2\ln 2}\left\{ 2\ln(1+2SNR) - \int_0^2 dx - \int_0^2 \frac{1}{1+x\,SNR}dx \right\}$$

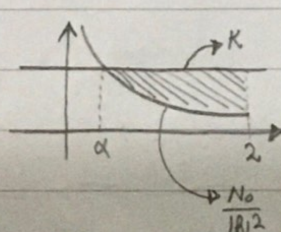
$$= \log_2(1+2SNR) - \frac{1}{\ln 2} - \frac{\ln(1+x\,SNR)}{(\ln 2)SNR}\Big]_0^2$$

$$= \log_2(1+2SNR) - \frac{1}{\ln 2} - \frac{1}{SNR}\log_2(1+2SNR) = \log_2(1+2SNR)\left(\frac{SNR-1}{SNR}\right) - \log_2^{e}$$

c) $$C_{ZO} = \log_2\left(1+\frac{SNR}{E\{1/|h|^2\}}\right)$$

$$E\left\{\frac{1}{|h|^2}\right\} = \int_0^2 \left(\frac{1}{2}\right)\left(\frac{1}{x}\right)dx = \frac{1}{2}\ln x\Big]_0^2 = \infty \rightarrow C_{ZO} = \log_2(1+0) = 0$$

d) $$g(h) = \frac{|h|^2}{N_0} \rightarrow P = E\left\{\left(K - \frac{1}{g(h)} = K - \frac{N_0}{|h|^2}\right)^{+}\right\}$$



$$= K - \int_\alpha^2 \frac{N_0}{2x}dx = K - \frac{N_0}{2}\ln x\Big]_\alpha^2 = K - \frac{N_0}{2}\ln\left(\frac{2}{\alpha}\right)$$

$$\frac{N_0}{|h|^2}\Bigg|_{|h|^2=\alpha} = K \rightarrow K = \frac{N_0}{\alpha} \rightarrow P = \frac{N_0}{\alpha} - \frac{N_0}{2}\ln\left(\frac{2}{\alpha}\right)$$

Based on the drawn plot : $P^{best}(h) = \begin{cases} \frac{N_0}{\alpha} - \frac{N_0}{|h|^2} & \alpha < |h|^2 < 2 \\ 0 & o.w \end{cases}$

$$\rightarrow C_{CSI} = E\{\log_2(1+P^{best}(h)g(h))\} = \int_\alpha^2 \log_2\left(K\frac{x}{N_0}\right)\frac{dx}{2} = \frac{1}{2\ln 2}\int_\alpha^2 \ln\left(\frac{K}{N_0}x\right)dx$$

$$= \frac{1}{2\ln 2}\left\{ x\ln\left(\frac{K}{N_0}x\right)\Big]_\alpha^2 - \int_\alpha^2 x\left(\frac{\frac{K}{N_0}}{\frac{K}{N_0}x}\right)dx = 2\ln\left(\frac{2K}{N_0}\right) - \alpha\ln\left(\frac{\alpha K}{N_0}\right) - (2-\alpha)\right\}$$

$$= \log_2\left(\frac{2K}{N_0}\right) - \frac{\alpha}{2}\log_2\left(\frac{\alpha K}{N_0}\right) + \frac{\alpha-2}{2\ln 2} \underset{K=\frac{N_0}{\alpha}}{\overset{\uparrow}{=}} \log_2\left(\frac{2}{\alpha}\right) - \overbrace{\frac{\alpha}{2}\log_2(1)}^{0} + \frac{\alpha-2}{2\ln 2}$$

$$, P = \frac{N_0}{\alpha} - \frac{N_0}{2}\ln\left(\frac{2}{\alpha}\right) \rightarrow \frac{N_0}{2}\ln\left(\frac{2}{\alpha}\right) = \frac{N_0}{\alpha} - P \rightarrow \ln\left(\frac{2}{\alpha}\right) = \frac{2}{\alpha} - \frac{2P}{N_0}$$

$$\rightarrow \log_2\left(\frac{2}{\alpha}\right) = \frac{2}{\alpha\ln 2} - \frac{2P}{N_0\ln 2}$$

$$\Rightarrow C_{CSI} = \frac{2}{\alpha\ln 2} - \frac{2P}{N_0\ln 2} + \frac{\alpha-2}{2\ln 2} = \frac{1}{\ln 2}\left\{\frac{2}{\alpha} + \frac{\alpha}{2} - \frac{2P}{N_0} - 1\right\}, \quad P = N_0\left(\frac{1}{\alpha} - \frac{\ln(2/\alpha)}{2}\right)$$

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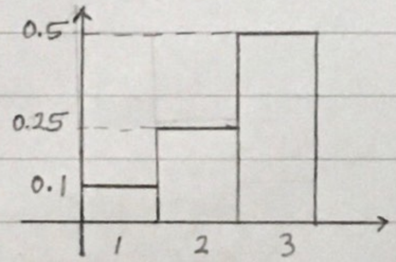
2) a) $\gamma_1 = 10dB = 10$, $\gamma_2 = 6dB = 3.98 \simeq 4$, $\gamma_3 = 3dB = 1.99 \simeq 2$

$SNR_i = \frac{|h_i|^2}{N_0}P$, $P = 0dBm = 10^{-3}W = 1mW$, $g_i = \frac{|h_i|^2}{N_0}$ $\Rightarrow$

$g_1 = 10\,\frac{1}{mW}$, $g_2 = 4\,\frac{1}{mW}$, $g_3 = 2\,\frac{1}{mW}$ $\longrightarrow$



if $K > 0.5 \rightarrow K = \frac{P + \sum_{i=1}^{3} 1/g_i}{N^* - 3} = \frac{1 + 0.1 + 0.25 + 0.5}{3} = 0.62 > 0.5 \checkmark$

$\rightarrow P_1^{best} = 0.62 - 0.1 = 0.52$, $P_2^{best} = 0.62 - 0.25 = 0.37$, $P_3^{best} = 0.62 - 0.5 = 0.12$

$$\rightarrow C = \sum_{i=1}^{3}\log_2(1 + g_i P_i^{best}) = \sum_{i=1}^{3}\log_2(Kg_i) = \log_2(0.62(10)) + \log(0.62(4)) + \log_2(0.62(2))$$

$= 4.252$

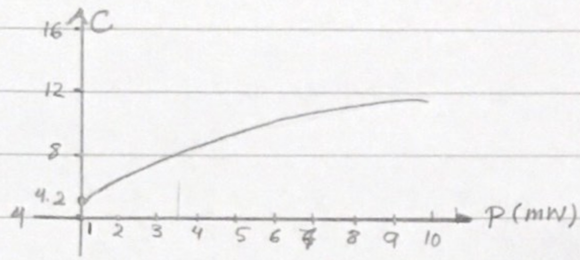
So this is the lower bound of capacity. For powers more than 0 dBm we have :

$$K = \frac{P + \sum_{i=1}^{3} 1/g_i}{N^* = 3} = \frac{1}{3}P + \frac{1}{3}(0.25+0.5+0.1) = \frac{1}{3}P + 0.28$$

Since for $P = 1$ mW, for finding K all channels participated, for $P \geq 1$ mW all channels should be in too.

$$\rightarrow C = \sum_{i=1}^{3} \log_2\left(\left(\tfrac{1}{3}P + 0.28\right)g_i\right) = \log_2\left(\tfrac{10}{3}P + 2.8\right) + \log_2\left(\tfrac{4}{3}P + 1.12\right) + \log_2\left(\tfrac{2}{3}P + 0.56\right)$$

Using matlab for $P = 1:0.1:10$ the plot look like:



b) $SNR = \frac{SNR_i}{|h_i|^2} \rightarrow |h_i|^2 = \frac{SNR_i}{SNR}$

$$SNR_i = \frac{|h_i|^2 P}{N_0} \xrightarrow[P = 1\,mW]{if} SNR_1 = 10 = \frac{|h_1|^2}{N_0} \xrightarrow{\text{for power} = P} SNR_1 = 10P \rightarrow SNR_2 = 4P,\ SNR_3 = 2P$$

$$|h|^2 = \begin{cases} \frac{10P}{SNR} & prob: 0.2 \\ \frac{4P}{SNR} & = 0.3 \\ \frac{2P}{SNR} & = 0.5 \end{cases} \rightarrow E\left\{\frac{1}{|h|^2}\right\} = 0.2\left(\frac{SNR}{10P}\right) + 0.3\left(\frac{SNR}{4P}\right) + 0.5\left(\frac{SNR}{2P}\right)$$

$$= 0.345\,\frac{SNR}{P} = \frac{1}{K}$$

$$\rightarrow C_{ZO} = \log_2\left(1 + \frac{SNR}{0.345\,\frac{SNR}{P}}\right) = \log(1 + 2.9P)$$

$$P_i = \frac{K}{|h_i|^2}P \rightarrow P_1 = \left(\frac{SNR}{10P}\right)\left(2.9\,\frac{P}{SNR}\right)P = 0.29P \text{ (mW)} \rightarrow P_2 = \frac{2.9}{4}P = 0.725P \text{ (mW)}$$

$$\rightarrow P_3 = \frac{2.9}{2}P = 1.45P \text{ (mW)}$$

c) $H \triangleq |h|^2 \rightarrow \mathcal{F}_H(h) = P\{H \leq h\}$, $P \geq 1$ mW :

if $h < \frac{2P}{SNR} \rightarrow \mathcal{F}_H(h) = 0$, if $\frac{2P}{SNR} < h < \frac{4P}{SNR} \rightarrow \mathcal{F}_H(h) = 0.5$

if $\frac{4P}{SNR} < h < \frac{10P}{SNR} \rightarrow \mathcal{F}_H(h) = 0.5 + 0.3 = 0.8$, if $h > \frac{10P}{SNR} \rightarrow \mathcal{F}_H(h) = 1$

$$\rightarrow \mathcal{F}_H(h) = \begin{cases} 0 & h < \frac{2P}{SNR} \\ 0.5 & \frac{2P}{SNR} < h < \frac{4P}{SNR} \\ 0.8 & \frac{4P}{SNR} < h < \frac{10P}{SNR} \\ 1 & h > \frac{10P}{SNR} \end{cases} \longrightarrow \begin{cases} \text{if } e \leq 0.5 \rightarrow C_e = \log_2(1+2P) \\ \text{if } 0.5 \leq e \leq 0.8 \rightarrow C_e = \log_2(1+4P) \\ \text{if } 0.8 \leq e \leq 1 \rightarrow C_e = \log_2(1+10P) \end{cases}$$

d) $C_{ergodic} = E\{\log_2(1+|h|^2 SNR)\} = 0.2\log(1+10P) + 0.3\log(1+4P) + 0.5\log(1+2P)$

e) $P_{out}(SNR) = P\{\frac{\|h_i\|^2}{L} SNR < SNR_e\} = e$, $C_e = \log_2(1+SNR_e)$

→ We assume here we have Tx diversity, CSI in unknown @ Tx & using the best space-time codes.

$\|h_i\|^2 \frac{SNR}{2} = (|h_{i1}|^2 + |h_{i2}|^2)\frac{SNR}{2}$ → we need to find CDF of $\|h\|^2$ →

$P\{|h|^2 = x\} = P\{|h_1|^2 = x_1\} P\{|h_2|^2 = x_2\} \Rightarrow$

$x_1 = h_1, x_2 = h_1 \rightarrow P_1^2 = (0.2)^2 = 0.04$: $\|h\|^2 = 2|h_1|^2 = 2\left(\frac{10 \times 10^3}{SNR}\right) = \frac{2 \times 10^4}{SNR}$

$x_1 = h_1, x_2 = h_2, x_1 = h_2, x_2 = h_1$: $2P_1P_2 = 2(0.2)(0.3) = 0.12$, $\|h\|^2 = \frac{10(10^3) + 4(10^3)}{SNR} = \frac{1.4 \times 10^4}{SNR}$

$x_1 = h_1, x_2 = h_3, x_1 = h_3, x_2 = h_1$: $2P_1P_3 = 2(0.2)(0.5) = 0.2$, $\|h\|^2 = \frac{10(10^3) + 2(10^3)}{SNR} = \frac{1.2 \times 10^4}{SNR}$

$x_1 = h_2,\ x_2 = h_3,\ x_1 = h_3,\ x_2 = h_2 :\ 2P_2P_3 = 2(0.3)(0.5) = 0.3,\ \|h\|^2 = \frac{4(10^3)+2(10^3)}{SNR} = \frac{6\times10^3}{SNR}$

$x_1 = h_2,\ x_2 = h_2 :\ P_2^2 = (0.3)^2 = 0.09,\ \|h\|^2 = 2\ \frac{4\times10^3}{SNR} = \frac{8\times10^3}{SNR}$

$x_1 = x_2 = h_3 :\ P_3^2 = (0.5)^2 = 0.25,\ \|h\|^2 = 2\ \frac{2\times10^3}{SNR} = \frac{4\times10^3}{SNR}$

$$\underline{H} \triangleq \|h\|^2,\ \alpha \triangleq \frac{10^3}{SNR} \rightarrow F_{\underline{H}}(h) = \begin{cases} 0 & h \leq 4\alpha \\ 0+0.25=0.25 & 4\alpha < h \leq 6\alpha \\ 0.25+0.3=0.55 & 6\alpha < h \leq 8\alpha \\ 0.55+0.09=0.69 & 8\alpha < h \leq 12\alpha \\ 0.69+0.2=0.84 & 12\alpha < h \leq 14\alpha \\ 0.84+0.12=0.96 & 14\alpha < h \leq 20\alpha \\ 0.96+0.04=1 & h > 20\alpha \end{cases}$$

$e < 0.25 \rightarrow C_e = \log_2(1+2\times10^3) = 10.9665$, $0.25 < e < 0.55 \rightarrow C_e = \log_2(1+3\times10^3) = 11.5512$

$0.55 < e < 0.64 \rightarrow C_e = \log_2(1+4\times10^3) = 11.9661$, $0.64 < e < 0.84 \rightarrow C_e = \log_2(1+6\times10^3) = 12.551$

$0.84 < e < 0.96 \rightarrow C_e = \log_2(1+7\times10^3) = 12.773$, $0.96 < e < 1 \rightarrow C_e = \log_2(1+1\times10^4) = 13.2879$

f) Part C : same as above : $e < 0.25 : C_e = \log_2(1+4\times10^3) = 11.9661$ / $0.25 < e < 0.55 : C_e = \log_2(1+6\times10^3) = 12.551$

$0.55 < e < 0.64 : C_e = \log(1+8\times10^3) = 12.966$, $0.64 < e < 0.84 : C_e = \log_2(1+12\times10^3) = 13.5509$

$0.84 < e < 0.96 : C_e = \log_2(1+14\times10^3) = 13.77$, $0.96 < e < 1 : C_e = \log_2(1+2\times10^4) = 14.2878$

Part d : above : $C_{ergodic} = E_h\{\log_2(1+\|h\|^2 SNR)\} = 0.04\log_2(1+2\times10^4) + 0.12\log_2(1+14\times10^3)$
$+ 0.2\log_2(1+12\times10^3) + 0.3\log_2(1+6\times10^3)$
$+ 0.09\log_2(1+8\times10^3) + 0.25\log_2(1+4\times10^3)$

$= 12.7386$

4) Since channel bandwidth is divided to subchannels of the B_c size, we can assume that each of the subchannels is independent, T.V. & flat fading.

If we assume Rayleigh fading channel:

$$\gamma_i = |h_i|^2 \, SNR \quad , \quad SNR = \frac{P}{N_0} = \frac{6}{0.2\times 10^{-6} \times 3\times 10^{6}} = 10 \rightarrow \gamma_i = 10\,|h_i|^2$$

$$\rightarrow P(\gamma_i) = E\{|h_i|^2\} \exp\{-E\{|h_i|^2\}\gamma_i\}$$

Using (4.36) of Goldsmith book: $\sum_{i=1}^{3} \int_{\gamma_0}^{\infty} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma_i}\right) P(\gamma_i) d\gamma_i = 1$

$$\rightarrow \int_{\gamma_0}^{\infty} \left(\frac{1}{\gamma_0} - \frac{1}{x}\right)\left(\frac{2}{3} e^{-\frac{2}{3}x}\right) dx + \int_{\gamma_0}^{\infty} \left(\frac{1}{\gamma_0} - \frac{1}{x}\right)\left(4/3\, e^{-4/3\,x}\right) dx + \int_{\gamma_0}^{\infty} \left(\frac{1}{\gamma_0} - \frac{1}{x}\right)\left(2e^{-2x}\right) dx$$

→ since there's no closed form for this integral, matlab is used. Its code can be found in the attached file.

s.a.m