

- ② a) Find the transformation matrix of mirror reflection about the line $2x - 3y + 3 = 0$. you do not need multiply matrices. Use Homogenous.

Soln: $(1 \ 0 \ 0)^T$

Given,

$$2x - 3y + 3 = 0$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -2x-3 \\ 2x+3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -2x-3 \\ 2x+3 \end{bmatrix}$$

$$\therefore y = \frac{2}{3}x + 1$$

Here, $m = 2/3$, $c = 1$

Now, $\tan \theta = m$

$$\Rightarrow \tan \theta = 2/3$$

$$\Rightarrow \theta = \tan^{-1}(2/3)$$

$$\therefore \theta = 33.69^\circ$$

$$\therefore 90^\circ - \theta = (90^\circ - 33.69^\circ)$$

$$= 56.31^\circ$$

Now,

transformation matrix, $R(90^\circ - \theta)$ $T(0, 1)$

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos 56.31^\circ & -\sin 56.31^\circ & 0 \\ -\sin 56.31^\circ & \cos 56.31^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

 M_y

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

 $R(90^\circ - \theta)$

$$\begin{bmatrix} \cos 56.31^\circ & -\sin 56.31^\circ & 0 \\ \sin 56.31^\circ & \cos 56.31^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

 $T(0, -1)$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

A.

- b) Consider two straight line L_1 and L_2 which are represented by the equation $x + y - 6 = 0$ and $x - y + 4 = 0$ respectively. you want to align L_1 and L_2 by performing 2D translation and rotations. write down the corresponding composite transformation matrix for reflection.

1) (b)

Given,

$$A = (3, -9, 7, -1)$$

$$B = (0, -3, -7, 2)$$

$$C = (4, 4, 4, 4)$$

For Homogenous point A, $(1, 1, 1, 1) =$

$$= (3, -9, 7, -1) / (-1)$$

$$= (-3, 9, -7, 1)$$

For Homogenous point B, $(1, 1, 1, 1) =$

$$= (0, -3, -7, 2) / 2$$

$$= (0, -3/2, -7/2, 1)$$

For Homogenous point C, $(1, 1, 1, 1) =$

$$= (4, 4, 4, 4) / 4$$

$$= (1, 1, 1, 1)$$

Now, $\overline{AB} = B - A$

$$= (0, -3/2, -7/2, 1) - (-3, 9, -7, 1)$$

$$= (0+3, -3/2-9, -7/2+7, 1-1)$$

$$= (3, -2\frac{1}{2}, 7\frac{1}{2}, 0)$$

Again, $\vec{EB} = B - C$

$$= (0, -3\frac{1}{2}, -7\frac{1}{2}, 1) - (1, 1, 1, 1)$$

$$= (-1, -5\frac{1}{2}, -9\frac{1}{2}, 0)$$

And, $\vec{AC} = C - A$

$$= (1, 1, 1, 1) - (-3, 9, -7, 1)$$

$$= (4, -8, 8, 0)$$

Now, $3\vec{AB} - 2\vec{EB} + 5\vec{AC}$

$$= 3(3, -2\frac{1}{2}, 7\frac{1}{2}, 0) - 2(-1, -5\frac{1}{2}, -9\frac{1}{2}, 0)$$

$$+ 5(4, -8, 8, 0)$$

$$= (9, -\frac{63}{2}, 2\frac{1}{2}, 0) - (-2, -5, -9, 0)$$

$$+ (20, -40, 40, 0)$$

$$= (9+2+20, -\frac{63}{2}+5-40, \frac{21}{2}+9+40, 0-0+0)$$

$$= (31, -13\frac{1}{2}, 11\frac{1}{2}, 0)$$

2. $\overline{AB} \cdot \overline{CA}$

now, for previous solution, now work

$$\overline{CA} = A - C$$

$$= (-3, 9, -7, 1) - (1, 1, 1, 1)$$

$$= (-4, 8, -8, 0)$$

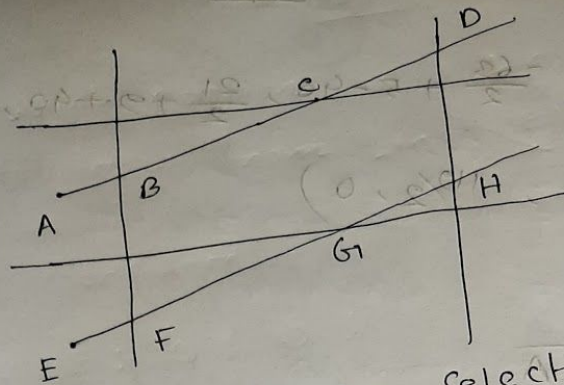
$$\overline{AB} = (3, -2\frac{1}{2}, 7\frac{1}{2}, 0)$$

now, $\overline{AB} \cdot \overline{CA}$

$$= (3, -2\frac{1}{2}, 7\frac{1}{2}, 0) \cdot (-4, 8, -8, 0)$$

$$= (-12 - 84 - 28 + 0)$$

$$= -124$$



→ which portion will be selected to draw from Line AD.

use
TBRL

Soln: Consider AD.

A: 0 0 0 1

D: 1 0 1 0

Both end point are not 0000.

So, it can not trivially accepted.

Again, ADD gives Zero with A and D.

$$\begin{array}{r}
 0001 \\
 1010 \\
 \hline
 0000
 \end{array}$$

So can not trivially rejected.

select the point A and set fourth bit
Zero. new point B is found.

B: 0 0 0 0

D: 1 0 1 0

Both end point are not 0000.
we can perform AND operation,

0 0 0 0

1 0 1 0

0 0 0 0

it can not trivially rejected.

Select the point D and set 1st bit

Zero. New point C is found.

B: 0 0 0 0

C: 0 0 0 0

so, it can accepted.

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* ii) which portion will be selected to draw from Line EH?

Soln: Consider EH,

E: 0 1 0 1

H: 0 0 1 0

Both end points are not accepted.
 it can not initially accepted.
 Now AND gives zero E with H.

0	1	0	1						
0	0	1	0	0	0	0	0	0	0
0	0	0	0						

So, it can not initially rejected.
 Select the point H and set third bit
 Zero. New point G is found.

E: 0 1 0 1

G: 0 0 0 0

Both end points are not accepted.
 So, it can not initially accepted.

now, ADD gives 10 with E & G.

0	1	0	1
0	0	0	0
<hr/>			
0	0	0	0

it can not trivially rejected. Select the End point

E: 0 0 0 1

G: 0 0 0 0

Both end point are not 0000 so it can not trivially accepted.

Again, AND gives Zero.

0	0	0	1
0	0	0	0
<hr/>			
0	0	0	0

So, it can not trivially rejected. now, select point E and set fourth bit is Zero.

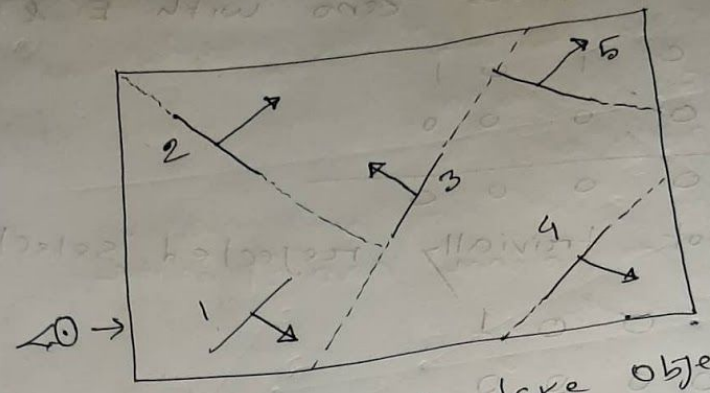
E: 0 0 0 0

G: 0 0 0 0

Both are Zero so it can trivially accepted.

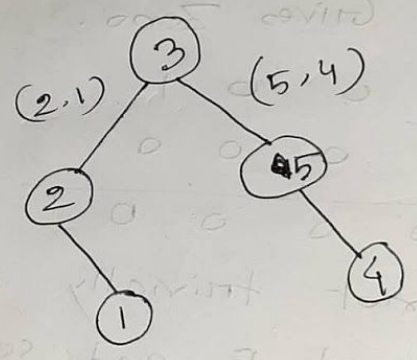
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④



1) Draw BSP tree and take object 3 as root. then display BSP tree.

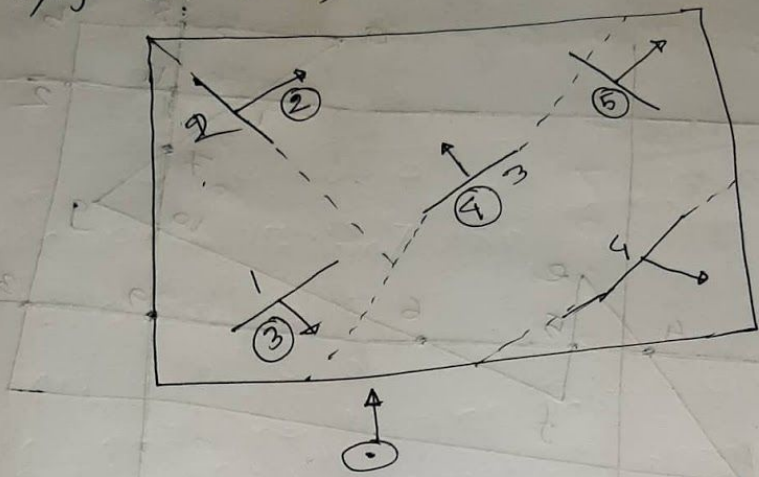
Soln:



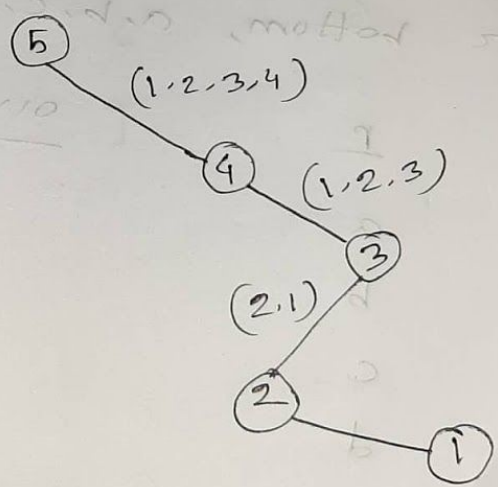
order: 5 → 4 → 3 → 2 → 1

(12)

Displaying BSP tree,



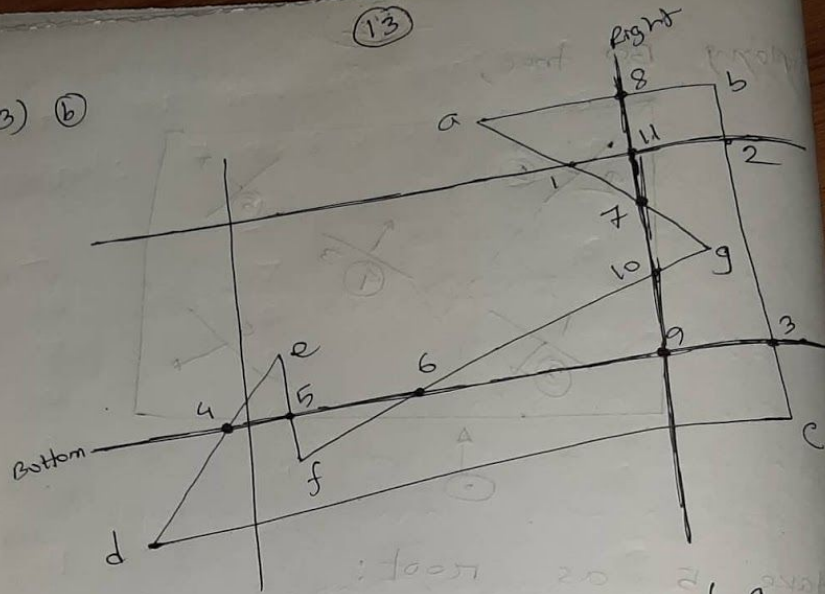
ii) take 5 as root:



order: $5 \rightarrow 4 \rightarrow 3 \rightarrow 2 \rightarrow 1$

(13)

3) (b)



soln: Forz bottom, a, b, c, d, e, f, g

s
g
a
b
c
d
e
f

r
a
b
c
d
e
f
g

output:

a
b
3
—
4, e
5
6, g

(14)

For right, not an iteration
output of previous iteration,
a, b, 3, 4, e, 5, 6, g

<u>S</u>	<u>P</u>	<u>output:</u>
g	a	7, a
a	b	8
b	3	-
3	4	9, 4
4	e	e
e	5	5
5	6	6
6	g	10

(15)

For previous iteration of
output: 7, a, 8, 9, 4, e, 5, 6, 10

<u>S</u>	<u>P</u>	<u>output</u>
10	7	7
7	a	a
a	8	-
8	9	11.9
9	4	4
4	e	e
e	5	5
5	6	6
6	10	10

(16)

(*)

Hence,

$$\vec{v} = \frac{1}{7}\hat{i} - 3\hat{j} + 6\hat{k}$$

$$\text{Now, } a = 6, b = -3, c = \frac{1}{7}$$

$$\begin{aligned}\text{Now, } \alpha &= \tan^{-1}\left(\frac{b}{c}\right) \\ &= \tan^{-1}\left(\frac{-3}{1/7}\right) \\ &= -87.28\end{aligned}$$

$$\begin{aligned}\text{Again, } \beta &= \tan^{-1}\left(\frac{-a}{\sqrt{b^2+c^2}}\right) \\ &= \tan^{-1}\left(\frac{-6}{\sqrt{(-3)^2+(\frac{1}{7})^2}}\right) \\ &= -63.41^\circ\end{aligned}$$