Change of variable, parametric estimation and hypothesis testing

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Exercise 1 :

Let V representing a directed celerity along an axis such that :

$$V \sim \mathcal{N}(0, \sigma^2),$$

and define the variable

$$E = \frac{1}{2} m V^2.$$

Given i.i.d. samples $E_1, \ldots, E_n$, we define an estimator of E as :

$$\hat{E}_n = \frac{1}{n} \sum_{i=1}^{n} E_i.$$

And an estimator :

$$\hat{\sigma}^2 = \frac{2}{m} \hat{E}_n.$$

1. Compute the probability density of E.
2. Show that $\hat{\sigma}^2$ is the maximum likelihood estimator for σ^2.
3. Find a confidence interval for σ^2 using $\hat{E}_n$.
4. Define a good statistics to perform hypothesis testing on σ^2.
5. What are associated rejection zones for unilateral ($H_0 : \sigma^2 > \sigma_0^2$) and bilateral ($H_0 : \sigma^2 = \sigma_0^2$) tests.